Data Storage

- What is stored: Schemas, Relation metadata (e.g. indexes, statistical info), Log files

Secondary Storage

- DBMS storage includes:
  - Data: Stored in disk blocks/pages
  - File layer: Organisation and data retrieval
  - Buffer manager: Reading/writing of disk pages
  - Disk space manager: Keeps track of pages used by file layer

Magnetic Hard-Disk Drive (HDD)

- Disk Access time:
  1. Command Processing time (negligible)
  2. Seek time: Move disk head on track
  3. Rotational delay: Rotate to put head on start of correct sector
     a. Avg rotational delay = time for ½ revolution = $\frac{1}{2} \times \frac{60}{\text{RPM}}$ s
  4. Transfer time: Rotate along the correct sector(s) to move data to/from disk = $\frac{\text{\# sectors read}}{\text{Total \# sectors on track}} \times \frac{60}{\text{RPM}}$ s
- Sequential I/O:
  - Pages stored contiguously on one track, then move on to next surface of the cylinder (i.e. same track across different surfaces), then move on to next cylinder
- **Solid-State Drive (SDD)**
  - Per block: Avg seek time + Avg rotational delay + Transfer time

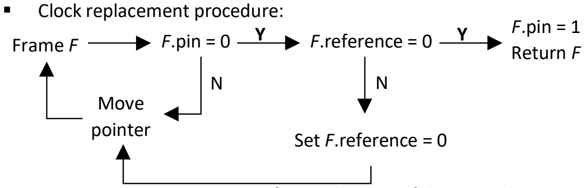
Buffer Manager

- Maintains a buffer pool in RAM (memory allocated for DBMS) for caching. Each unit of memory is called a **frame**
  - Each frame has:
  1. Pin count: # clients using the page
  2. Dirty flag: Whether page has been modified but not updated on disk
- Disk pages are fetched into/release from the buffer pool
- Page request procedure:
  1. Client requests page *P*
  2. Is page *P* already in memory?
     a. Yes: Pin frame *F*. Return address of *F*. **End**
     b. No: continue to 3
  3. Find free frame or evict a page if buffer pool is full (i.e. Find some frame *F*.**pin = 0**)
  4. Pin frame *F*
  5. Write frame *F* into disk if *F* is dirty
  6. Return address of frame *F*

Page Replacement Policies

Clock Replacement Policy

- Each frame has an additional reference bit
- Pointer moves in FIFO manner
- Only frame with reference bit = 0 AND pin = 0 will be replaced
- Reference bit set to 0 when pin drops to 0
- Clock replacement procedure:

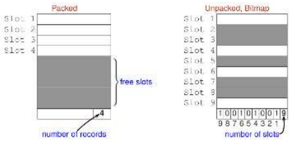


- **DO NOT** increment pointer after each successful page replacement

Page/Record Format

Fixed-length Records

- Each record is of fixed length.
- Each record is idenfitied by a RID = (page id, slot #)
- Packed: Contiguous; Unpacked: Not Contiguous
- Record Format: Fields in a recor



Variable-length Records (Slotted Page Organisation)

- Slot directory:
  - Pointer to start of free space: Memory location for next record to be inserted
  - Number of slots: Increments whenever a new record is inserted
  - Pointers to each record: Address to the start of a record in the page + size of the record (byte offset)
- Record Format:
  - Delimit fields with special symbols (e.g. F1 $ F2 $ .... $ Fn)
  - Use an array of field offsets

Tree-based Indexing

- **Index**: Data structure to speed up data retrieval; stored as a file.)
- **Search key:** Sequence of *k* data attributes e.g. (name, age)
- **Unique index**: Search key is a candidate key
- **Data entry**: (Key Value, RID), stored in index

B+ Tree Index

- **Leaf nodes** store data entries. Leaf nodes are doubly-linked and all of them are on the same level
- **Height** h: # levels of internal nodes (root is level 0). Leaf nodes are at level h
- **Internal nodes** store index entries in the form (p_0, k_1, p_1, k_2, p_2, …p_n)
- **Order** of a B+ Tree, *d*: Root node must contain [1, 2d] entries and non-root nodes must contain [d, 2d] entries
- A B+ Tree with n level of internal nodes has with order d has $2(d+1)^{n-1} \leq$ # leafs $\leq (2d+1)^n$

Search

1. At each internal node, find largest k_i s.t. target $k_i \leq k$.
   a. Search subtree at p_i . if k_i exists
   b. Otherwise, search subtree at p_0
2. Continue until leaf node and return all entries with search key = *k*.
   a. If range search, traverse along leaf nodes and return all entries within the bound

Insert (Handling overflows)

- **Node splitting**
  - **Overflowing leaf node**
    1. Distribute the $d+1$ largest entries into new leaf node
    2. Create and insert new index entry using smallest key in new leaf node into parent node
    3. If parent node overflows, split parent node
  - **Overflowing internal node**
    1. Split at the middle key, and push it up to the parent node
    2. Propagate node splitting until no overflows/reached root
- **Redistribution of data entries**
  1. Redistribute entries in overflowed leaf node *N* by putting the largest /smallest entry (among the $2d+1$ entries) into adjacent right/left sibling N'
  2. Then, update the separating key in parent

Delete (Handling underflows)

- **Node Merging**
  - **Underflowing leaf node**
  1. Merge underflowed leaf node N with adjacent sibling N' by moving all entries from N' to N.
  2. Then, delete N' and the separating key in parent
  3. Update parent index if needed
- **Underflowing internal node** (Pre-condition: N' must have d entries)
  1. Merge underflowed internal node N with adjacent sibling N' by pulling down separating key in parent, combining N and N'
  2. Propagate node merging until no underflows/reached root
- **Redistribution of data entries** (Pre-condition: N' must have > d entries)
  1. Redistribute entries by moving the data entry with the smallest/largest key from right/left sibling N' to underflowing leaf node N
  2. Update separating key in parent with the smallest key in N'
- **Redistribution of internal entries**
  1. Merge leaf nodes, causing internal node N to be underflowed
  2. "Pull down" the separating parent key K between N and sibling N' and join with N
  3. Then, replace the K in the affected index entry in parent node with $N'.k_i$ (i.e. left/right-most key in right/left sibiling)
  4. Remove k_i from sibling

Data Formats

- **Format-1**: Leaves store data records
- **Format-2**: Leaves store (k, rid)
- **Format-3**: Leaves store (k, rid-list)

Bulk Loading

1. Sort data entries to be inserted by search key
2. Load the leaf pages with those sorted entries
3. Initialize the B+ tree with an empty root page
4. For each leaf page (in sequential order), insert its index entry into the rightmost parent-of-leaf level page of the B+ tree

- Advantages:
  - Efficient construction
  - Leaf pages are allocated sequentially

Hash-based Indexing

- Numbers in the buckets are the **HASH VALUES**, NOT THE KEY VALUES

Linear Hashing

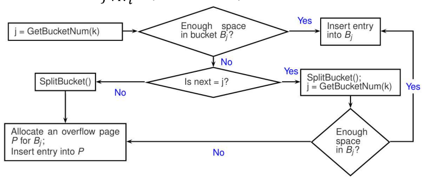
- N_0 = initial # of buckets = 2^m
- N_i = # buckets in at start of level $i = 2^i N_0 = \mathbf{2^{m+i}}$
- **Split image** of B_j = B_{j+N_i}

Insert

- Hash function: $h_i(k) = h(k)\ mod\ \boldsymbol{N_i}$ (look at the **last $\boldsymbol{m+i}$ bits**)

$$\text{Bucket \#} = \begin{cases} h_i(k), & h_i(k) \geq next \\ h_{i+1}(k), & otherwise \end{cases}$$

- **Splitting**: occurs when any bucket overflows:
  - Split the bucket B_j pointed to by *'next'*
  - Redistribute entries:
    - Entries in B_j : $(m+i+1)^{\text{th}}$ bit is **0**
    - Entries in B_{j+N_i} : $(m+i+1)^{\text{th}}$ bit is **1**



Delete

- If last bucket $B_{N_i+next-1}$ is empty:
  - If next > 0
    1. Decrement next
    2. Delete last bucket
  - If next = 0 and level > 0
    1. Decrement level
    2. Update next to point to last bucket in previous level ($B_{N_{i-1}-1}$)
- Delete overflow pages that become empty after redistribution

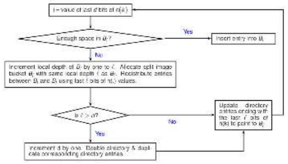
Performance

- Best case: No overflow pages – 1 disk I/O per insertion
- Worst case: All hashed to the same bucket – linear I/O cost
- Average: 1.2 disk I/O per insertion

Extendible Hashing

- Global depth (directory) = d; # directory entries = 2^d
- Local depth (bucket) = $l \leq d$
- Directory entry # = **last $\boldsymbol{d}$ bits** of $h(k)$; points to bucket
- All entries in a bucket have same l bits in their $h(k)$
- Corresponding entries: differ only by the d^{th} bit (indexed 1)
- # directory entries pointing to a bucket = 2^{d-l}
- **Splitting**: occurs when target bucket B_i overflows
  1. Increment l of B_i
  2. Allocate new bucket B_j (split image) with same l
  3. Redistribute:
     - Entries in B_i : l^{th} bit is 0
     - Entries in B_j : l^{th} bit is 1
  4. Using the last l bits, redistribute pointer(s) between B_i and B_j

- Bucket B_i overfows:
  - If $l = d$
    1. Increment d and l of B_i
    2. Double the number of directory entries
    3. Split B_i; redistribute
    4. Redistribute pointer(s)
  - If $l \leq d$
    1. Increment l of B_i
    2. Split B_i; redistribute
    3. Redistribute pointer(s)



Delete

- If entries in B_i and B_j (corresponding entry) can fit into 1 bucket:
  1. Merge B_i and B_j into one bucket e.g. B_i
  2. Delete B_j
  3. Decrement l of B_i
  4. Move directory entries pointing to B_j to point to B_i
  5. If each pair of corresponding entries point to the same bucket, decrement d and halve the directory

Performance

- At most 2 disk I/O per insertion

Index Formats

Clustered vs Unclustered

- Clustered: Order of data records is close to order of the data entries
  - Format-1 index is a clustered index
  - Cost of RID lookups become 0 (format-1) or $\|\sigma(R)\|$ / b_d
- Each relation can have at most one clustered index

Dense vs Sparse

- Dense: There is an index record for every search key value; otherwise, it is sparse
- Unclustered index must be dense
- Format-1 B+ Tree index is sparse

Query Evaluation: Sort

External Mergesort

- Assume N data records need to be sorted
- Assume that data records are on the disk
- Only B memory pages are allocated for sorting
  - 1 page is allocated for **output**
  - $B-1$ pages are allocated for **input**

1. Read the N data records into $\lceil N/B \rceil$ initial sorted runs
2. Recursively merge the sorted runs in a $B-1$ way merge
   a. With $B-1$ input pages, we can sort $B-1$ sorted runs by allocating one page for each of the $B-1$ sorted runs
   b. Each input pages has a pointer to the current smallest value
   c. When output page is full, write the page back to disk

Analysis

- N_0 = # sorted runs in initial pass (pass 0) = $\lceil N/B \rceil$
- Total # passes = $\lceil log_{B-1}(N_0) \rceil + 1$
- Total # I/O = $2N(\lceil log_{B-1}(N_0) \rceil + 1)$
  - Each pass has N reads and writes

External Mergesort with Blocked I/O

- Instead or reading/writing one page at a time, read/write in units of **buffer blocks** of b_i and b_o pages respectively
  - This allows for sequential I/O over random I/O
- Assume B memory pages are allocated for sorting
  - **Output** block size = b_o pages; **Input** block size = b_i pages
  - 1 block is allocated for **output**; $\left\lfloor \frac{B-b_o}{b_i} \right\rfloor$ blocks are allocated for **input**

1. Read the N data records into $\lceil N/B \rceil$ sorted runs
2. Recursively merge the sorted runs in a $\left\lfloor \frac{B-b_o}{b_i} \right\rfloor$ way merge
   a. With $\left\lfloor \frac{B-b_o}{b_i} \right\rfloor$ input blocks, we can sort $\left\lfloor \frac{B-b_o}{b_i} \right\rfloor$ sorted runs by allocating one block for each of the sorted runs
   b. When output block is full, write the block back to disk

Analysis

- N_0 = # sorted runs in initial pass (pass 0) = $\lceil N/B \rceil$
- F = # sorted runs that can be merged at each merge pass = $\left\lfloor \frac{B-b_o}{b_i} \right\rfloor$
- Total # passes = $\lceil log_F(N_0) \rceil + 1$
- Reduced merge factor, but more sequential I/O

Query Evaluation: Select σ

- Access paths:
  - Table scan: Scan all data pages
  - Index scan: Scan index pages + RID lookup (if needed)
  - Index intersection: Combine results from multiple index scans + RID lookup (if needed)
- More selective access path → fewer pages need to be accessed
- **Include Columns**: Specify the attributes whose values are also stored in the data entries of the index, on top of the index key. Can be used to avoid RID lookups
- **Covering Index**: Index I is a covering index for query Q if all attributes rein Q are part of the key or include columns(s) of I
  - Then, Q is evaluated with index-only plan

Matching Predicates

- **B+ Tree**: $I = (K_1, K_2, \dots, K_n)$. I matches predicate p if p is in the form:

$$(K_1 = c_1) \wedge \ \dots \ \wedge (K_{i-1} = c_{i-1}) \wedge (K_i\ op_i\ c_i), \quad i \in [1, n]$$

  - Prefix key with all equality operator, and at most one non-equality operator on last attribute of prefix
- **Hash Index**: $I = (K_1, K_2, \dots, K_n)$. I matches predicate p if p is in the form:

$$(K_1 = c_1) \wedge (K_2 = c_2) \wedge \ \dots \ \wedge (K_n = c_n)$$

  - All attributes must appear in p and only equality operator

Primary Conjuncts p'

- The subset of conjuncts in p that I matches

Covered Conjuncts p_c

- The subset of conjuncts p_c in p such that all attributes in p_c appear in the key or include column(s) of I

Cost Evaluation

B+ Tree Index

$$Cost_{internal} = \begin{cases} \left\lceil log_F \left\lceil \frac{\|R\|}{b_d} \right\rceil \right\rceil, & \text{format} - 1 \\ \left\lceil log_F \left\lceil \frac{\|R\|}{b_i} \right\rceil \right\rceil, & \text{otherwise} \end{cases}$$

$$Cost_{leaf} \quad = \begin{cases} \left\lceil \frac{\|\sigma_{p'}(R)\|}{b_d} \right\rceil, & \text{format} - 1 \\ \left\lceil \frac{\|\sigma_{p'}(R)\|}{b_i} \right\rceil, & \text{otherwise} \end{cases}$$

$$Cost_{RID} \quad = \begin{cases} 0, & \text{format} - 1 \text{ or covering} \\ \|\sigma_{p_c}(R)\|, & \text{otherwise (worst case)} \\ \frac{\|\sigma_{p_c}(R)\|}{b_d}, & \text{otherwise (clustered)} \end{cases}$$

$$Cost_{total} \quad = Cost_{internal} + Cost_{leaf} + Cost_{RID}$$

Hash Index

- Ranged query: Table scan = $|R|$
- Format 1
  - $Cost_{total} = Cost_{records} \geq \left\lceil \frac{\|\sigma_{p'}(R)\|}{b_d} \right\rceil$
- Format 2
  - $Cost_{entries} \geq \left\lceil \frac{\|\sigma_{p'}(R)\|}{b_i} \right\rceil$, due to possible long overflow chain
  - $Cost_{records} = \begin{cases} 0, & I \text{ is covering index} \\ \|\sigma_{p_c}(R)\|, & \text{otherwise} \end{cases}$
  - $Cost_{total} = Cost_{records} + Cost_{entries}$

Query Evaluation: Project π

- $\pi_L(R)$ – No duplicates (select DISTINCT) ; $\pi_L^*(R)$ – Keep duplicates

Sort-Based Projection

Unoptimized Approach

1. Extract attributes L from records $R \rightarrow \pi_L^*(R)$
2. Sort $\pi_L^*(R)$ using L as sort key → sorted $\pi_L^*(R)$ [External Mergesort]
3. Scan $\pi_L^*(R)$ to remove duplicates → $\pi_L(R)$

- **Analysis**

1. Read I/O (table scan) = $|R|$; Write I/O = $|\pi_L^*(R)|$ → $|R| + |\pi_L^*(R)|$
2. Given B buffer pages for sorting:
   - Merge factor = $B - 1$; $N_0 = \lceil \pi_L^*(R)/B \rceil$
   - Step 2 total = $2|\pi_L^*(R)|(\log_{B-1} N_0 + 1)$
3. Step 3 total = $|\pi_L^*(R)|$ (ignore write I/O as cost)
4. Total = $|R| + 2|\pi_L^*(R)|(log_{B-1}N_0 + 2)$

Optimized Approach

1. Create sorted runs with attributes L
   - Write only attributes L to output page
2. Merge sorted runs and remove duplicates simultaneously

- **Analysis**

1. Read I/O = $|R|$; Write I/O = $|\pi_L^*(R)|$ → Step 1 total = $|R| + |\pi_L^*(R)|$
2. Given B buffer pages for sorting:
   - Merge factor = $B - 1$; $N_0 = \lceil \pi_L^*(R)/B \rceil$
   - Step 2 total = $2|\pi_L^*(R)|(log_{B-1}N_0) - |\pi_L^*(R)|$
3. Total = $|R| + 2|\pi_L^*(R)|(log_{B-1}N_0)$

- If $B > \sqrt{|\pi_L^*(R)|}$,
  - # initial sorted runs = $\left\lceil \frac{|R|}{B} \right\rceil \approx \sqrt{|\pi_L^*(R)|}$
  - # merging passes = $log_{B-1}\left(\sqrt{|\pi_L^*(R)|}\right) \approx 1$
  - Total = $|R| + 2|\pi_L^*(t)|$

Hash-Based Projection

1. **Partition** all the tuples in R into $R_1, R_2, \ldots, R_{B-1}$
   - Only B memory pages
     - 1 page is allocated for **input** buffer
     - $B - 1$ pages for **output** buffers/partitions $R_1, R_2, \ldots, R_{B-1}$
   - For each t in R,
     1.1 $h(\pi_L(t))$, then output $\pi_L(t)$ into output buffer $R_{h(\pi_L(t))}$
     1.2 If $R_{h(\pi_L(t))}$ is full, flush to disk.
   - Output of partition phase is $\pi_L^*(R_1), \pi_L^*(R_2), \ldots, \pi_L^*(R_{B-1})$
2. **Duplicate Elimination**: For each $\pi_L^*(R_i)$ (possibly done in parallel),
   2.1 Initialize hash table T of size $B - 1$
   2.2 For each tuple t in $\pi_L^*(R_i)$:
      2.2.1 Perform $h'(t) = j$ (NOTE: $h' \neq h$)
      2.2.2 If t not in B_j, then insert t into B_j
   2.3 Output all the tuples in T as$\pi_L(R_i)$
3. Combine all $\pi_L(R_i)$

- Entire T must fit in main memory. If not, recursively partition $\pi_L^*(R_i)$ into $\pi_L^*(R_{i_1}), \ldots, \pi_L^*(R_{i_{B-1}})$

Analysis

- Assuming no partition overflow:
  1. Partitioning = $|R| + |\pi_L^*(R)|$
     - Read I/O (table scan) = $|R|$.
     - Each t is projected before writing. So, write I/O = $|\pi_L^*(R)|$.
  2. Duplicate elimination = $|\pi_L^*(R)|$
     - Each of the projected tuples is read once. Ignore write I/O
  3. Total = $|R| + 2|\pi_L^*(t)|$
- If $B > \sqrt{f|\pi_L^*(R)|}$, then there will not be partition overflow
  1. Assume h hashes every t in R uniformly
  2. Each R_i will have $\approx \frac{|\pi_L^*(R)|}{B-1}$ pages → $B > f\,\frac{|\pi_L^*(R)|}{B-1} \approx \sqrt{f|\pi_L^*(R)|}$

Indexing

- If there is a **covering index** I for the projection, then replace table scan (i.e. read I/O = $|R|$) in both projection schemes with index scan.
  - Since the index is likely smaller than the relation, this will incur lower I/O costs
- If index is ordered (e.g. B+-tree) whose search key (e.g. (A, B, C) includes all wanted attributes (e.g. $\pi_{A,B}(R)$) as a **prefix**
  1. Scan all the data entries in order
  2. Compare adjacent data entries for duplicates

Query Evaluation: Join $\bowtie$

- In general, the smaller relation should be the outer relation

Tuple-based Nested Loop Join

- For each tuple r in R, for each tuple s in S, check if r matches s

Analysis

- Scan R = |R|; Scan S = $\|R\| \times |S|$
- Total = $|R| + \|R\| \times |S|$

Page-based Nested Loop Join

- For each page P_r in R, for each page P_s in S, for each tuple r in P_r, for each tuple s in P_s, check if r matches s

Analysis

- Scan $R = |R|$; Scan S = $|R| \times |S|$
- Total = $|R| + |R| \times |S|$

Block Nested Loop Join

- Allocate 1 buffer page for output, 1 buffer page for S and $B - 2$ buffer pages for R. Read pages from R in blocks of $B - 2$
- For each block B_i from R, for each page P_S in S, for each tuple r in B_i, for each tuple s in P_s, check if r matches s
- In general, if $|R| \leq |S|$, use R as outer relation

Analysis

- Scan $R = |R|$; Scan $S = \left\lceil \frac{|R|}{B-2} \right\rceil \times |S|$
- Total = $|R| + \left\lceil \frac{|R|}{B-2} \right\rceil \times |S|$

Index Nested Loop Join

- Precondition: There is an index on the join attribute(s) of **inner** relation
- For each tuple r in R, use r to probe S's index to find matching tuples

Analysis

- Assuming uniform distribution, one tuple in R matches with $\left\lceil \frac{\|S\|}{\|\pi_{B(S)}\|} \right\rceil$ tuples
- Scan R = |R|; Scan S = $\|R\| \times \left(log_F \left\lceil \frac{\|S\|}{b_i} \right\rceil + \left\lceil \frac{\|S\|}{b_i \|\pi_{B(S)}\|} \right\rceil \right)$
- Total = $|R| + \|R\| \times \left(log_F \left\lceil \frac{\|S\|}{b_i} \right\rceil + \left\lceil \frac{\|S\|}{b_i \|\pi_{B(S)}\|} \right\rceil \right)$ + RID Lookup (if needed)

Sort-Merge Join

- Assume $|R| > |S|$; R is outer relation
- If join key is a primary key of S, use S as inner relation

Unoptimized Approach

1. **Sort**: Sort both R and S based on $\bowtie$ attributes, if not already sorted
   - This partitions R into $R_1, \ldots R_k$ and S into $S_1, \ldots S_l$ each containing tuples with the same join attribute value(s)
2. **Merge**:
   2.1. Initialize pointers p_r and p_s for R and S, each pointing to the first tuple. Let r and s be the tuples pointed to by p_r and p_s
   2.2. r and s do not match: Advance smaller pointer
      2.2.1 If p_r is advanced and r matches s at p_s', **rewind**: $p_s \leftarrow p_s'$
   2.3. r and s match:
      2.3.1 Remember position p_s : $p_s' \leftarrow p_s$
      2.3.2 Output $r \bowtie s$ and advance p_s until r and s do not match

- **Analysis**
  - Sort R and S:
    - External Mergesort = $2|R|(log_m(N_R) + 1) + 2|S|(log_m(N_S) + 1)$
    - Internal Mergesort = $|R| + |S|$
  - Merge:
    - Best case: No rewinds = $|R| + |S|$
    - Worst case: All tuples in R and S match = $|R| + \|R\| \times |S|$

Optimized Approach

- Start merging as soon as sorted runs from R and S can fit into memory
  - $B > N(R, i) + N(S, j)$, i, j = # passes from sorting R, S

1. Create sorted runs of R and merge partially to get (R, i)
2. Create sorted runs of S and merge partially to get (S, j)
3. Merge (R, i) and (S, j)

- **Analysis:** If $B > \sqrt{2|S|}$
  - # initial sorted runs of $S < \sqrt{\frac{|S|}{2}}$
  - Total # initial sorted runs of R and $S < \sqrt{2|S|}$
  - 1 pass is sufficient to merge and join the initial sorted runs
  - Total cost = $2(|R| + |S|) + (|R| + |S|) = 3(|R| + |S|)$

Grace Hash Join

- Assume $|R| < |S|$; S is probe relation, R is build relation

1. Partition R into $R_1, \dots, R_{B-1}$ by h
2. Partition S using same h into $S_1, \dots, S_{B-1}$
3. For each R_i, **build hashtable**
   - 3.1. Allocate 1 buffer page for input, 1 page for output and $B - 2$ pages for hashtable T
   - 3.2. For each tuple r in R_i, read it into input buffer and hash it into T using h', $h \neq h'$
   - 3.3. For each tuple s in S_i, read it into input buffer and **probe** T: If s matches with any r in bucket $h'(s)$, write $r \bowtie s$ into output buffer

Analysis

- By UHA: $|R_i| = \frac{|R|}{B-1}$. Let size of T be $f \times \frac{|R|}{B-1}$, f = fudge factor
  - Hence, $B > \frac{f|R|}{B-1} + 2 \approx \sqrt{f|R|}$ (1 input buffer, 1 output buffer) to prevent partition overflow
- Assuming no partition overflow:
  1. Partitioning = $2(|R| + |S|)$
  2. Probing = $|R| + |S|$
     - Read each page of R_i to build T
     - Read each page of S_i to probe
  3. Total = $3(|R| + |S|)$

Query Evaluation: Misc Operations

Set Operations

- **Intersection** $R \cap S$
  - Join with join predicate involving all columns of R and S
- **Cross Product** $R \times S$
  - Join with join predicate = true (trivial)
- **Union** $R \cup S$
  - Sorting approach:
    1. Sort R, sort S (on all attrs)
    2. Combine R and S and removing duplicates
  - Hashing approach: ≈ Grace Hash Join
    1. Partition (on all attrs) R into $R_1, \dots, R_{B-1}$,
    2. Partition S into $S_1, \dots, S_{B-1}$
    3. Build hash table T_i for each R_i (suppose R is build relation)
    4. For each $t \in S_i$, probe T_i and discard t if t in T_i, otherwise insert t into T_i
- **Difference** $R - S$
  - Sorting approach: ≈ Sort-Merge Join using R as outer relation
    1. Sort R, sort S (on all attrs)
    2. Remove $t \in R$ if $t \in S$
  - Hashing approach: ≈ Grace Hash, using R as build relation
    1. Partition (on all attrs) R into $R_1, \dots, R_{B-1}$,
    2. Partition S into $S_1, \dots, S_{B-1}$
    3. Build hash table T_i for each R_i
    4. For each $t \in S_i$, probe T_i and discard t from T_i if t in T_i

Aggregate Operations

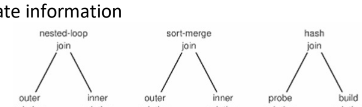
- **Simple Aggregation**: Maintain running info while scanning table
  - Valid for SUM, COUNT, AVG, MIN, MAX
- **Group-by Aggregation**
  - Sorting approach
    1. Sort relation by 'GROUP BY' attributes
    2. Scan relation and compute aggregate for each group
  - Hashing approach
    1. Scan relation to build hash table on 'GROUP BY' attributes
    2. Maintain running info for each group
- **Using Index**
  - Use index I over table scan if I is a covering index for aggregation operation (index is likely smaller than table)
  - Scan index leaves sequentially if 'GROUP BY' attributes is prefix of I's search key

Query Evaluation Approaches

- **Materialized evaluation**
  - An operator is evaluated only when all of its operands has been completely evaluated/materialized
  - Materialize intermediate results to disk
- **Pipelined evaluation**
  - Pass the output directly to its parent operator (no materialize)
  - Execution of operators is interleaved
  - Blocking operator: Operator that is unable to produce any output until it has received all the tuples from its child operators
- **Iterator Interface of Pipelined evaluation**
  1. open – initialization; allocates resources and operators' args
  2. getNext – generates next output tuple/null if no more output
  3. close: deallocate state information

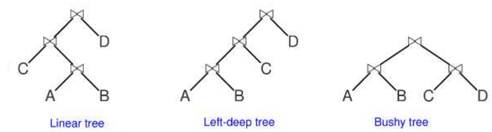
Query Optimization

- Join Plan Notation:



Commutative 1.1. $R \times S \equiv S \times R$ 1.2. $R \bowtie S \equiv S \bowtie R$	**Commutating σ with π** 4.1. $\pi_L(\sigma_p(R)) \equiv \pi_L(\sigma_p(\pi_{L \cup \text{attr}(p)}(R)))$
Associative 2.1. $(R \times S) \times T \equiv R \times (S \times T)$ 2.2. $(R \bowtie S) \bowtie T \equiv R \bowtie (S \bowtie T)$	**Commutating σ with $\times/\bowtie$** 5.1. $\sigma_p(R \times S) \equiv \sigma_p(R) \times S$, $attr(p) \subseteq \text{attr}(R)$ 5.2. $\sigma_p(R \bowtie_q S) \equiv \sigma_p(R) \bowtie_q S$, $attr(p) \subseteq \text{attr}(R)$ 5.3. $\sigma_p(R \cup S) \equiv \sigma_p(R) \cup \sigma_p(S)$
Idempotence 3.1. $\pi_A(\pi_B(R)) \equiv \pi_A(R)$, $A \subseteq B \subseteq \text{attr}(R)$ 3.2. $\sigma_{p_1}\left(\sigma_{p_2}(R)\right) \equiv \sigma_{p_1 \wedge p_2}(R)$	**Commutating π with $\times/\bowtie$** 6.1. $\pi_L(R \times S) \equiv \pi_{L_R}(R) \times \pi_{L_S}(S)$ 6.2. $\pi_L(R \bowtie_p S) \equiv \pi_{L_R}(R) \bowtie_p \pi_{L_S}(S)$, $\text{attr}(p) \cap \text{attr}(R) \subseteq L_R$ & $\text{attr}(p) \cap \text{attr}(S) \subseteq L_S$ 6.3. $\pi_L(R \cup S) \equiv \pi_L(R) \cup \pi_L(S)$

- Query Plan Trees:
  - Linear: At least 1 operand for each join operation is a base relation
  - Bushy: There is a join operation where no operand is a base relation
  - Left-deep: Every right join operand is a base relation
  - Right-deep: Every left join operand is a base relation



Query Plan Enumeration

- Uses a bottom-up Dynamic Programming approach starting with size-1 relations and memoizing the best plan
- For every possible combination S of R_i, for every possible pair of paritions S_i and S_j in S, find the optPlan(S) of joining S_i and S_j using the memoized optPlan(S_i) and optPlan(S_j)

```
Input: A SPJ query q on relations R1, R2, ..., Rn
Output: An optimal query plan for q
for i = 1 to n do
    optPlan({Ri}) = best access plan for Ri
for i = 2 to n do
    for each S ⊆ {R1, ..., Rn}, |S| = i do
        bestPlan = dummy plan with cost(bestPlan) = ∞
        for each Sj, Sk, |Sj| ∈ [1, i), S = Sj ∪ Sk do
            p = best way to join optPlan(Sj) and optPlan(Sk)
            if (cost(p) ≤ cost(bestplan)) then
                bestPlan = p
        optPlan(S) = bestPlan
return optPlan({R1, ..., Rn})
```

System R Optimizer

- Uses an enhanced Dynamic Programming approach that also considers sort order of query plan's output
  - Maintains optPlan(S_i, o_i), o_i = sort order of output by query plan wrt S_i
  - $o_i = \text{null}$ if output is unordered or a sequence of attributes
- Prunes search space:
  - Considers only left-deep query plans – query plans become fully pipelined; no materialization need
  - Avoids cross-product query plans – avoids high I/O cost
  - Considers early selections and projections

Query Plan Cost Estimation

- Reduction factor $rf(t_i)$: fraction of tuples in e that satisfy t_i i.e. $rf(t_i) = \frac{\|\sigma_{t_i}(e)\|}{\|e\|}$

Assumptions		
Uniformity	Independence	Inclusion
Uniform distribution of attribute values	Independent distribution of values in diff attributes	For $R \bowtie_{R.A=S.B} S$, if $\|\pi_A(R)\| \leq \|\pi_B(S)\|$, then $\pi_A(R) \subseteq \pi_B(S)$
$rf(A = c) \approx \frac{1}{\|\pi_A(R)\|}$	$rf(t_i \wedge t_j) \approx rf(t_i) \times rf(t_j)$	$rf(R.A = S.B) \approx \frac{1}{max\{\|\pi_A(R)\|, \|\pi_B(S)\|\}}$

Equiwidth Histogram
- Each bucket has almost equal number of values
- All buckets have the same width/range size of B
- $\|\sigma_{A=c}(R)\| = \frac{\|b_i\|}{B}$
- $\|\sigma_{A\in[x,y]}(R)\| = \frac{f_1\|b_i\|}{B} + \|b_{i+1}\| + \cdots + \frac{f_2\|b_{i+k}\|}{B}$

Equidepth Histogram
- Each bucket has almost equal number of tuples. Let B_i denote the width of bucket b_i
- A value can be contained in multiple buckets
- $\|\sigma_{A=c}(R)\| = \frac{f_1\|b_i\|}{B_i} + \frac{f_2\|b_{i+1}\|}{B_{i+1}} + \cdots + \frac{f_k\|b_{i+k}\|}{B_{i+k}}$, for all b_j containing c
- $\|\sigma_{A\in[x,y]}(R)\| = \frac{f_1\|b_i\|}{B_i} + \frac{f_2\|b_{i+1}\|}{B_{i+1}} + \cdots + \frac{f_k\|b_{i+k}\|}{B_{i+k}}$

Histogram with MCV
- Separately keep track of the frequencies of the top-k most common values and exclude them from the histogram

Transactions – Serializability & Recoverability

Atomicity – Either all or none of the actions in Xact happen
Consistency – If DB starts consistent, it ends up consistent
Isolation – Execution of one Xact is isolated from other Xacts
Durability – If a Xact commits, its effects persist

View Serializable Schedules
- Schedules S and S' are view equivalent ($S \equiv_v S'$) if:
  1. If T_j reads A from T_i/T_0 in S, then T_j must also read A from T_i/T_0 in S'
  2. For each object A, if T_i performs final write in S, then T_i must also perform final write in S'
- S is a VSS if $S \equiv_v$ some serial schedule over the set of Xacts

VSS Test – VSG(S)
- In VSG(S), an edge between two Xacts exists if it satisfies any of the following conditions:
  1. If T_j reads from T_i, then $(T_i \to T_j) \in VSG(S)$
  2. If $W_i(O), W_j(O) \in S$ and T_i performs the final write on O, then $(T_j \to T_i) \in VSG(S)$
  3. If T_j reads O from T_i and there is some T_k that $W_k(O)$, then either $(T_k \to T_i) \in VSG(S)$ or $(T_j \to T_k) \in VSG(S)$
     a. This implies that if T_i reads O from initial DB and then later T_j performs $W_j(O)$, $(T_i \to T_j) \in VSG(S)$
- S is VSS iff VSG(S) is acyclic

Conflict Serializable Schedules
- Two actions on the same object conflict if any of the conditions hold:
  1. At least one of the actions is a write
  2. The actions are from different Xacts

Conflicting Actions
- WR conflicts – Dirty Read
  - $W_1(O), R_2(O), Abort_1$
  - If T_1 aborts, then the O's value read by T_2 will be incorrect
- RW conflicts – Unrepeatable Read
  - $R_1(O), W_2(O), Commit_2, \ldots, R_1(O)$
- WW conflicts – Lost Update
  - $W_1(O), W_2(O)$
- Schedules S and S' are view equivalent ($S \equiv_c S'$) if every pair of conflicting actions are ordered the same way in both schedules

CSS Test – CSG(S)
- In CSG(S), each node is a committed Xact in S and there is an edge $T_i \to T_j$ if there is an action in T_i that precedes and conflicts with some action in T_j
- S is CSS iff CSG(S) is acyclic; otherwise, it is not CSS
- If S is CSS, then S is also VSS
- **Blind Writes**: $W_i(O)$ is a blind write if T_i did not read O prior to the write
  - If S is VSS and S has no blind writes, then S is also CSS
  - If S is VSS and S has blind writes, then S may or may not be CSS
  - Contrapositive: If S is not CSS but VSS , then there must be blind write(s)

Recoverable/Cascadeless/Strict Schedules
- Cascading Aborts: If T_j reads from T_i and T_i aborts, then T_j must also abort
  - Undesirable – high cost of bookkeeping
- **Recoverable Schedule**: S is recoverable if for every T that commits in S, T must commit after T' if T reads from T'
- **Cascadeless Schedule**: S is cascadeless whenever T_j reads from T_i, $Commit_i$ precedes this read action (i.e. $W_i(O) \ldots Commit_i \ldots R_j(O)$)
  - If S is cascadeless, then it is also recoverable
- Before-Images: Before T_i performs $W_i(O)$, log O's previous value as the before-image. If T_i aborted, restore O back to its before-image
  - Before-images are specific to a Xact. If a Xact aborts, restore the before-image(s) of that specific Xact
  - Does not always work, e.g. $W_1(O), W_2(O), Abort_1$
- **Strict Schedule**: S is strict if for every $W_i(O)$, O is not read or written by another Xact until T_i commits/aborts
  - Recovery becomes more efficient but concurrent executions become more restrictive
  - If S is strict, then S is also cascadeless

Locked-Based Concurrency Control
- Read O: Acquire Shared Lock S or Exclusive Lock X for O
- Write O: Acquire Exclusive Lock X for O
- If T's lock request for O is rejected, T will be placed in O's request queue
- If lock for O is released, lock manager grants the lock to first Xact in O's request queue, if possible
- When T commits/aborts, it releases all held locks and is removed from any request queue

2PL Protocol
1. To read O, T must hold S or X for O
2. To write O, T must X for O
3. Once T releases any lock (U), T cannot request any more locks

- Growing/Shrinking phrase: Before/After releasing 1st lock
- 2PL schedules are conflict serializable

Strict 2PL Protocol
1. To read O, T must hold S or X for O
2. To write O, T must X for O
3. T can only release all its locks once it has committed/aborted

- Strict 2PL schedules are strict and conflict serializable

Deadlocks
Deadlock Detection
- Waits-For Graph (WFG)
  - Nodes: Active Xacts
  - Edges: $T_i \to T_j$ if T_i must wait for T_j to release a lock
- Deadlock is present iff WFG has a cycle
- To handle a deadlock: Breaks deadlock by aborting a Xact in the cycle
  - When a Xact aborts/commits, release all its held locks and all its adjacent edges are removed from WFG

Deadlock Prevention
- Older Xacts T_i have higher priority than younger Xacts T_j ($T_i > T_j$)
- **Wait-die Policy**: Lower priority Xact never waits for higher priority Xact
  - If T_i requests for a lock that conflicts with a lock held by T_j
    - $T_i > T_j$: T_i waits for T_j
    - $T_i < T_j$: T_i aborts itself
  - Non-preemptive: Only requesting Xact can abort
- **Wound-wait Policy**: Higher priority Xact never waits for lower priority Xact
  - If T_i requests for a lock that conflicts with a lock held by T_j
    - $T_i > T_j$: T_i forces T_j to abort
    - $T_i < T_j$: T_i waits for T_j
  - Preemptive
- Aborted Xact must restart with its original timestamp to prevent starvation

Lock Conversion
- Changes lock mode, allowing interleaved execution, increasing concurrency
- **Lock Upgrade** ($UG_i(O)$)
  - Conditions:
    1. No other Xact is holding S on O (to maintain lock compability)
    2. T_i has not release any lock (to satisfy 2PL and thus serializability)
- **Lock Downgrade** ($DG_i(O)$)
  - Conditions:
    1. T_i has not modified O
    2. T_i has not release any lock (to satisfy 2PL and thus serializability)

ANSI SQL Isolation Levels
Phantom Read Problem
- Defn: 2 identical queries return different collection of rows
  - Unrepeatable read results in different row values. Phantom read results in different set of rows, but the values in those rows remain the same
- $R_i(p), W_j(O)$ is conflicting if O satisfies selection predicate p
- **Predicate Locking**: T_i locks p and T_j's request for $X_j(O)$ is blocked

Isolation Levels

Isolation Level	Dirty Read	Unrepeatable Read	Phantom Read
READ UNCOMMITTED	✓	✓	✓
READ COMMITTED	✗	✓	✓
REPEATABLE READ	✗	✗	✓
SERIALIZABLE	✗	✗	✗

- Short duration lock: Lock can be released after end of operation before Xact commits/aborts
- Long duration lock: Lock released only after Xact commits/aborts

Isolation Level	Write Locks	Read Locks	Predicate Locking
READ UNCOMMITTED	Long	-	✗
READ COMMITTED	Long	Short	✗
REPEATABLE READ	Long	Long	✗
SERIALIZABLE	Long	Long	✓

Multigranularity Locking

- Highest (coarsest) = database; Lowest (finest) = tuple
- Intention Shared (**IS**-lock): Intent to set S-locks at finer granularity
- Intention Exclusive (**IX**-lock): Intent to set X-locks at finer granularity
- Protocol:
  - Locks acquired top-down, released bottom-up
  - To hold **S**-lock or **IS**-lock on a node, need to hold **IS**-locks or **IX**-lock on all of its ancestors
  - To hold **X**-lock or **IX**-lock on a node, need to hold **IX**-lock on all of its ancestors

Lock Requested	Lock Held				
	-	**IS**	**IX**	**S**	**X**
IS	✓	✓	✓	✓	✗
IX	✓	✓	✓	✗	✗
S	✓	✓	✗	✓	✗
X	✓	✗	✗	✗	✗

Multiversion Concurrency Control

- No locks; instead, maintain multiple versions of each object: Write creates a new version of the object and Read an appropriate version
- Advantages:
  - Read-only Xacts and update Xacts do not block each other
  - Read-only Xacts are never aborted

Multiversion View Serializable Schedules (MVSS)

- Schedules S and S' are multiversion view equivalent ($S \equiv_{mv} S'$) if they have the same set of read-from relationships
  - i.e. $R_i(x_j) \in S$ iff $R_i(x_j) \in S'$
- **Monoversion** schedules: Each read action reads the most recently created version of that object
- S is a MVSS if $S \equiv_{mv}$ some serial monoversion schedule over the set of Xacts
- MVSS test: Apply VSS test, but exclude the Final Write condition

Snapshot Isolation (SI)

- Instead of having every Xact see the same snapshot of the DB, SI ensures that every Xact T sees a snapshot that consists of updates by Xacts COMMITTED BEFORE T starts
- Xacts T and T' are concurrent if they overlap

Concurrent Update Property

- Under SI, if multiple concurrent Xacts update the same object, **only one of the Xacts is allowed to commit**
  - Otherwise, the schedule may not be MVSS
- **First Committer Wins (FCW)**
  - Before T_i commits, check if $\exists\, T_j$ that is
    1. concurrent and committed, and
    2. updated some object that T_i also updated
  - T_j exists: Abort T_i. Otherwise, commit T_i
- **First Updater Wins (FUW)**
  - Before every $W_i(O)$, T_i requests for X-lock on O
  - X-locks are released when Xact commits/aborts
  - If X-lock is not held by any concurrent Xact, it is granted to T_i:
    - If O has been updated by any committed concurrent Xact: Abort T_i
  - If X-lock is held by some concurrent Xact T_j:
    - If T_j aborts, then grant X-lock to T_i
      - If O has been updated by any concurrent Xact: Abort T_i
    - If T_j commits: Abort T_i

Garbage Collection

- A version O_i of object O can be garbage collected if it will not be in any current **active**/**future** Xact's snapshot
- More formally, O_i may be deleted if $\exists$ newer version O_j ($Commit_i < Commit_j$) s.t. for every active T_k that started after T_i, $Commit_j < Start_k$

Serializable Snapshot Isolation Protocol (SSI)

- S is a SSI schedule if it is produced by SI and it is MVSS
- **SSI Test – DSG(S)**:
  - There is an edge from T_1 to T_2 if either dependencies exist:
    1. **ww**: T_1 writes X and T_2 later writes the immediate successor version of X
    2. **wr**: T_1 writes X and T_2 later reads the same version of X
    3. **rw**: T_1 reads X and T_2 later creates the immediate successor version of X
    - → if the Xact pair is non-concurrent
    - ⇢ if the Xact pair is concurrent
  - If S is a SSI schedule that is not MVSS, then
    1. DSG(S) contains some cycle, and
    2. Every cycle in DSG(S) contains Xacts T_i, T_j, T_k such that
       - $T_i \dashrightarrow_{rw} T_j \dashrightarrow_{rw} T_k$
       - T_i and T_k are possibly the same Xact
  - Can result in false positives (incorrectly classify as not serialisable when it actually is)

Crash Recovery

- **Steal/No-steal policy**
  - **Steal**: Allow dirty pages from T to be written to disk before T commits → UNDO + Pre-image required
  - **No steal**: Only write dirty pages back to disk after T commits → UNDO is not required
- **Force/No-force policy**
  - **Force**: All dirty pages from T must be written to disk before T commits → REDO is not required
  - **No force**: Some dirty pages can remain in buffer pool before T commits → REDO is required; During a system crash, the dirty pages in the buffer pool of a committed Xact are erased

	Force	**No-force**
Steal	UNDO & No REDO	UNDO & REDO
No-steal	No UNDO & No REDO	No UNDO & REDO

- **Log records**
  - Log records are appended to tail of **log file** in stable storage → allows fast sequential access
- **Write-ahead Logging (WAL) Protocol**
  - Log record containing its before-image must be flushed to the log before the update can be flushed to the DB
  - **pageLSN**: Most recent update log record for that page
  - HOW: Before page P can be flushed to disk, all log records with LSN ≤ P's pageLSN must be flushed to the log
  - WHY: Ensures that all before images can be retrieved to perform an abort/UNDO after a system crash. This allows steal
- **Force-at-commit Protocol**
  - T can only commit after the after-images of all its updated records are in stable storage (DB or log)
  - HOW: Write **commit log record** only after all log records by T have been flushed to the disk. Then, T is considered to be committed only if commit log record has been flushed to the log
  - WHY: Ensures that if T has committed (but not all dirty pages have been flushed), if the system crashes after the commit, the after images can be retrieved for REDO
- **Transaction Table (TT)**
  - In-memory, one entry for each active Xact
  - This table will be useful for UNDO

Transaction Table		
XID	**lastLSN**	**status**
Xact ID	Most recent log record for this Xact	C = committed; U = uncommitted

- **Dirty Page Table (DPT)**
  - In-memory, one entry for each dirty page in buffer pool
  - This table will be useful for REDO

Dirty Page Table	
PID	**recLSN**
Page ID	LSN of first log record that caused it to be dirty

ARIES Recovery Algorithm

- Uses **steal**, **no-force** policies
- Uses strict 2PL concurrency control
- Recovery phases: Analysis → Redo (Repeat History) → Undo

Normal Execution

- Updates to Transaction Table
  - If T_i not in **TT**, insert <i, LSN ,U> into TT
  - If T_i in **TT**, update the **lastLSN** with the new log record's LSN
  - If T_i commits, update **status** in TT to be C
  - If end log record is creates for T_i, remove T_i's entry from TT
- Updates to Dirty Page Table
  - If page P_i is updated and an entry for P is not in DPT, insert <i, LSN> into DPT
  - When P_i is flushed to disk, remove P_i's entry from DPT
- Update pageLSN after an update log recorded is created

Log Record Types

- All log records have **LSN**, **prevLSN, type, XID** fields

Update log record (additional fields)				
pageID	**offset**	**length**	**before-image**	**after-image**
Page updated	Byte offset in page to indicate start of updated portion	Number of bytes for updated portion	Value before	Value after

- **Compensation** log record (CLR): Created from an UNDO action

Compensation log record (additional fields)		
pageID	**undoNextLSN**	**action**
Page updated	LSN of next log record to be undone = prevLSN in the ULG being undone	Action taken to undo update

- **Commit** log record: Created when Xact commits
  - Following Force-at-Commit, flush all log records for Xact (including this commit log record) to the log
  - Xact is committed once this commit log has been written to log
- **Abort** log record: Created when Xact aborts during normal execution
  - Initiate undo for the aborted Xact
- **End** log record: Created after the follow-up processing from commit/abort has completed
- ULRs and CLRs are **redoable** log records

Abort

- Retrieve the log records of aborted Xact in reverse order using prevLSN, starting from lastLSN for the Xact in the TT and undo each action in the corresponding log record
- Create CLR for each undo action
- Create end log record after abort completes

Checkpointing

- **Simple Checkpointing**
  1. Stop accepting new actions and wait for all active actions to finish
  2. Flush all dirty pages in buffer pool to the disk
  3. Write **checkpoint log record (CPLR)** containing the TT
  4. Resume operations
  - During analysis phase, start from CPLR and initialize TT to be the CPLR's TT (DPT initialized to be empty)
  - Problems:
    1. Reduced concurrency since Xacts are frozen
    2. Flushing dirty pages incurs I/O – high overhead, slow
- **Fuzzy Checkpointing**
  1. Write **begin_checkpoint log record (BCPLR)**
  2. Write **end_checkpoint log record (ECPLR)** containing the DPT and TT at BCPLR (NOT ECPLR)
  3. Write **master record** containing LSN of BCPLR to known place on stable storage
  - In this course, assume that there are no other log records between BCPLR & ECPLR

Analysis Phase

- PURPOSE: Restore the DPT and TT as they were at time of crash
- Initialize TT and DPT:
  - Retrieve **BCPLR** from master record
  - Retrieve corresponding **ECPLR**
  - Initialize TT and DPT = ECPLR's TT and DPT
  - If no checkpoint records, initialize TT and DPT to be empty
- Scan the log in forward direction starting from record after **ECPLR** (or start, if no checkpoints), processing each log record r for T_i
  - If r is **end** log record:
    - Remove T_i from TT
  - If r is **not** an **end** log record:
    - If T_i not in TT: Insert <T_i, r's LSN ,U> in TT
    - If T_i in TT: Update lastLSN to be r's LSN
    - If r is a **commit** log record, update status to be C
  - If r is a **redoable** log record for P_j and P_j **not in DPT**:
    - Insert <P_j, r's LSN> into DPT
- At the end, TT lists all active Xacts at time of crash and DPT contains superset of dirty pages at time of crash
  - Superset because some of the dirty pages have actually already been flushed to disk before crash

Redo Phase

- PURPOSE: Repeat history; Redo uncommitted actions and CLRs
- Initialize r to be log record with LSN = **smallest recLSN** in DPT = **RedoLSN**
- Scan the log in forward direction, starting from r
  - If (r is **redoable**) AND (P **in DPT**) AND (P's **recLSN ≤ r's LSN**)*
    - Fetch page P from disk
    - If P's pageLSN (in **disk**) < r's LSN:
      - Redo action in r to P
      - Update P's pageLSN = r's LSN (so that redo isn't applied twice if it crashes again in redo phase)
    - Else:
      - Update P's recLSN = P's pageLSN + 1
- At end of Redo Phase:
  - Create **end** log records for Xacts with status = C in TT and remove them from TT
- *If P is not in DPT, then P's pageLSN ≥ r's LSN:
  - P is not in DPT → r must be before BCPLR, otherwise Analysis Phase would've saw r and added P into DPT → But DPT saved in ECPLR did not contain P → P must be flushed before BCPLR
- *If P's recLSN > r's LSN, then then P's pageLSN ≥ r's LSN:
  - Let r_2 be record with LSN = P's recLSN → r_2 must occur after r since r_2's LSN > r's LSN → r must be also be before BCPLR, if not, Analysis Phase will see r first and set P's recLSN = r's LSN instead of r_2's LSN → The fact that DPT contains r_2's LSN and not r's LSN means that there was no record for P in DPT during checkpointing → P was flushed after r's update and before r_2's update

Undo Phase

- PURPOSE: Abort active Xacts at time of crash ("loser" Xacts)
- Initialize L = lastLSNs of all Xacts in TT with status = U
- Repeat until L is empty:
  - Find and remove largest lastLSN in L
  - Let r be log record with LSN = this lastLSN
  - If r is **ULG**:
    - Create CLR r_2 for T, setting its **undoNextLSN** = r's prevLSN
    - Update lastLSN = r_2's LSN in **TT** for T
    - Create **DPT** entry for P (with recLSN = r_2's LSN) if P not in DPT
    - Undo the logged action on page P
    - Update P's **pageLSN** = r_2's LSN
    - If r's prevLSN is NULL: Entire T has been undone
      - Create **end** record for T
      - Remove T from TT
    - If r's prevLSN is not NULL: add it to L
  - If r is **CLR**:
    - If r's undoNextLSN is NULL: Entire T has been undone
      - Create **end** record for T
      - Remove T from TT
    - If r's undoNextLSN is not NULL: add it to L
  - If r is **abort** log record:
    - If r's prevLSN is NULL: No actions to undo
      - Create **end** record for T
      - Remove T from TT
    - If r's prevLSN is not NULL: add it to L