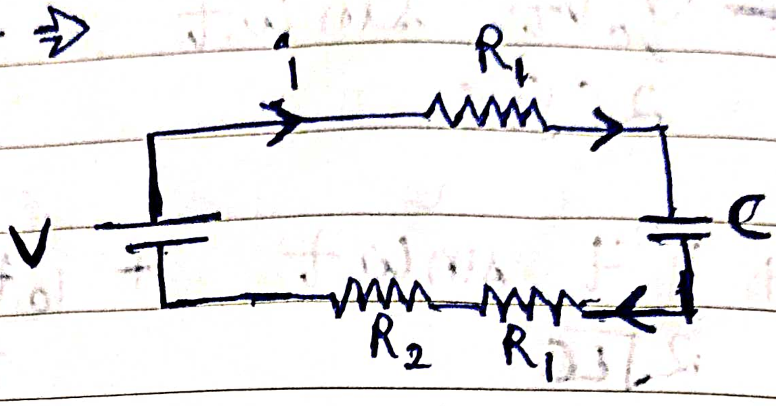
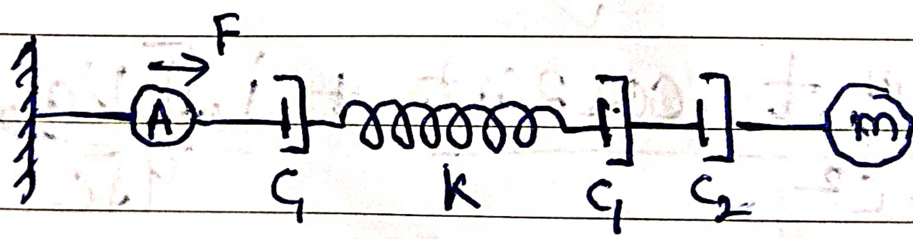
Modelling of HASEL actuator + Leg system as relation b/w mechanical and electrical parameters

- Electrical circuit ⇒



- Here C is the capacitance of ~~an ac~~ the HASEL actuator
- R_1 is the resistance of carbon electrodes
- R_2 is the ~~an~~ external resistor.

- Mechanical circuit



Here Ⓐ is the actuator exerting force, the rest of the circuit is the leg system.

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- For equivalent circuit

Lets have some analogy between electrical and mechanical circuit.

Force	x = displacement	v = velocity	c = damping constant
Voltage	Q = charge	i = current	R = resistance

c = capacitance
k = spring constant

- Approach :- Taking reference from extreme mechanics letters (EML) paper, I have added external resistor to take into account of dissipative effects. I have tried to use quasistatic equation approach by writing total potential energy and differentiating it, equating the differential equation to 0 for a static equilibrium state. The analogy between mechanical and electrical parameters has been inspired by coupling equation of piezoelectric.

- Objective :- To make some connection between parameters of mechanical circuit and electrical circuit. So that we can control mechanical characteristics of HASELs by changing electrical parameters which are easier to control and experiment.

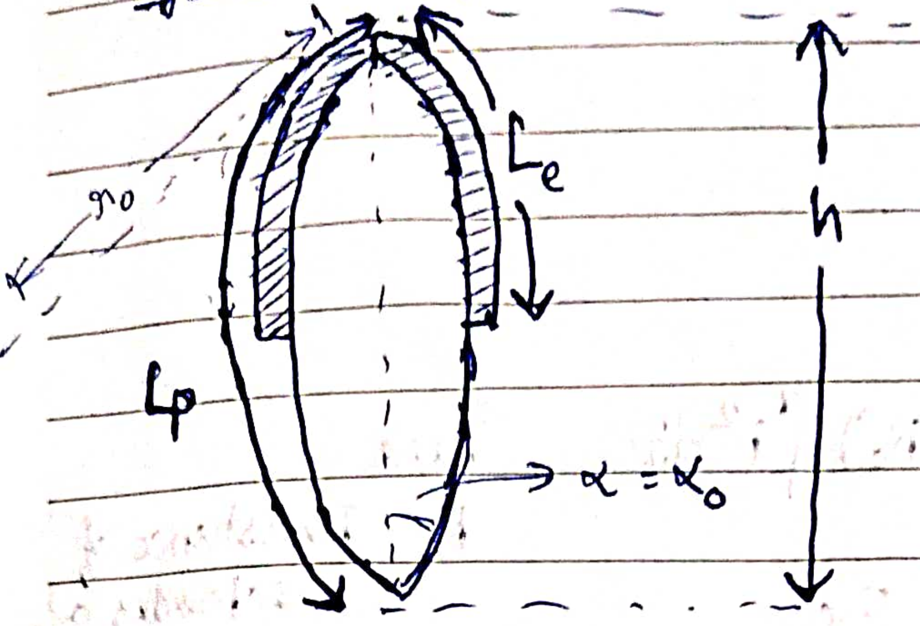
- Experimentally, we get relations like
  (i) $k \propto V^2$
  (ii) $c \propto R^2$
  (iii) $m \propto L^2$

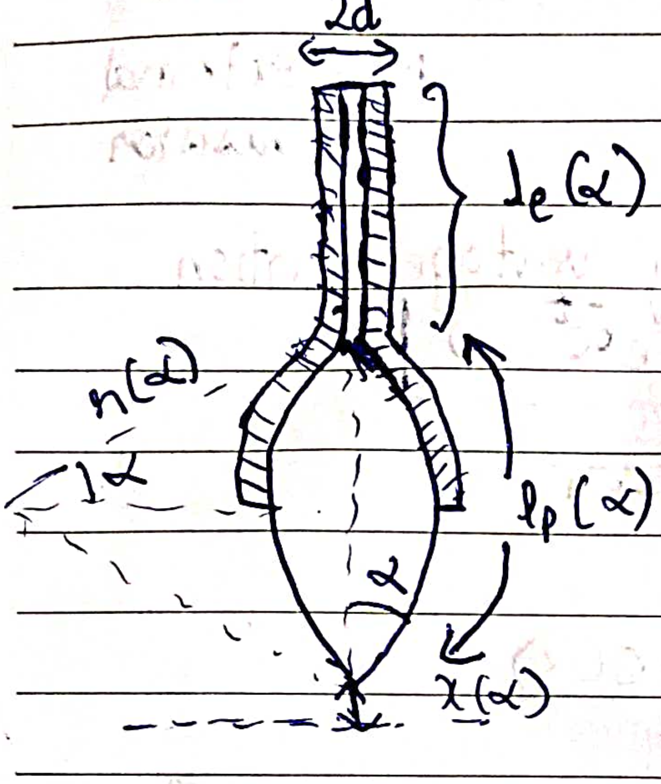
  I have tried to verify these results analytically.

- Assumptions for (i) result $k \propto V^2$

  : I have neglected energy contributed by mass, damper and inductor. I am writing the potential energy equation for a RC circuit having account of an equivalent spring constant k of HASEL actuator and leg system
  ↳ stiffness of whole leg

- **Geometric parameters**





Here 2d is the distance between carbon electrodes

- **Calculations for $k \propto V^2$**

Total potential energy = $E_{capacitor} - E_{battery} - E_{resistor} + E_{displacement} + E_{spring}$

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U = total potential energy

$$E_{capacitor} = \frac{1}{2}\frac{Q^2}{C}$$

$$E_{battery} = QV$$

$$E_{resistor} = (2R_1 + R_2)\int i^2 dt$$

here

R_1 = Resistance of electrodes of HASEL

R_2 = External resistor

$$E_{displacement} = Fx$$

$$E_{spring} = \frac{1}{2}kx^2$$

From electrical circuit, using voltage equation, we get ⇒ $\boxed{Q = CV - C(2R_1+R_2)\, e^{\frac{-t}{C(2R_1+R_2)}}}$

$$i = V\frac{dC}{dt} + e^{\frac{-t}{C(2R_1+R_2)}}$$

here $$\frac{dc}{dt} \approx \frac{C(\frac{\pi}{2}) - C(\alpha)}{t}$$

$$\frac{dc}{dt} \Rightarrow \frac{\varepsilon_r \varepsilon_0 w}{2dt}\left[\sqrt{\frac{2A\alpha^2}{\alpha - \sin\alpha\cos\alpha}} - \sqrt{A\pi}\right]$$

$$\boxed{i = V\left[\frac{\varepsilon_r\varepsilon_0 \omega}{2dt}\left(\sqrt{\frac{2A\alpha^2}{\alpha - \sin\alpha\cos\alpha}} - \sqrt{A\pi}\right) + e^{\frac{-t}{C(2R_1+R_2)}}\right]}$$

$$\int i^2 dt \approx V^2 Z^2 \ln t - \frac{Y e^{-\frac{2t}{Y}}}{2} + \underbrace{2VZ \int \frac{e^{-\frac{t}{Y}}}{t} dt}_{*\text{neglected}} + X$$

here X = constant for indefinite integration

$Y = C(2R_1 + R_2)$

$$Z = \frac{\varepsilon_r \varepsilon_0 w}{2d}\left(\sqrt{\frac{2A\alpha^2}{\alpha - \sin\alpha\cos\alpha}} - \sqrt{A\pi}\right)$$

* I neglected $2VZ \int \frac{e^{-\frac{t}{Y}}}{t} dt$ as at $t = 0$, this term tends to ∞ and just at $t > 0$, this term tends to 0.

Next I performed minimization of potential energy U with respect to $\alpha \Rightarrow \frac{dU}{d\alpha} = 0$

$$\Rightarrow \boxed{(2R_1+R_2)^2 e^{\frac{-2t}{C(2R_1+R_2)}} + \frac{2(2R_1+R_2)\, e^{\frac{-2t}{C(2R_1+R_2)}}\, t}{C} - \frac{(2R_1+R_2) V^2 \ln t\, \varepsilon_r \varepsilon_0 w}{d}\left(\sqrt{\frac{2A\alpha^2}{\alpha - \sin\alpha\cos\alpha}} - \sqrt{A\pi}\right) + (F+kx)\left(\frac{2d}{\varepsilon_r\varepsilon_0 w}\left(\frac{\sin\alpha - 1}{\sin\alpha}\right)\right) - \frac{V^2}{2} + f(\alpha) = 0} \quad (i)$$

here $f(\alpha) = \frac{(\sin\alpha - \alpha\cos\alpha)}{\alpha^2} \frac{lp(\alpha)}{\frac{dc}{d\alpha}}$ which will remain constant at specific α.

- Conclusion from eq (i) is at a particular instant i.e. at a specific time t and at a specific angle α, we will get $\boxed{k \propto V^2}$.

- <u>Assumptions for (ii) result $c \propto R^2$</u>

- I have neglected energy contributed by mass and inductor.
- I have added energy dissipated by a mechanical damper. All the terms used in eq (i) are used here also.

- <u>Calculations for $c \propto R^2$</u>

Energy dissipated by mechanical damper ⇓

$$-c\int v^2 dt$$

where c = damping constant
v = velocity.

- $v(\alpha) = \frac{dx(\alpha)}{dt} \approx \frac{x(\frac{\pi}{2}) - x(\alpha)}{t}$

$$v(\alpha) = \frac{\sqrt{\frac{2A\alpha^2}{\alpha - \sin\alpha\cos\alpha}}\left(\frac{\sin\alpha}{\alpha} - 1\right) - \sqrt{A\pi}\left(\frac{2-\pi}{\pi}\right)}{t}$$

Let numerator in above term as $g(\alpha)$

i.e. $$\boxed{g(\alpha) = \sqrt{\frac{2A\alpha^2}{\alpha - \sin\alpha\cos\alpha}}\left(\frac{\sin\alpha}{\alpha} - 1\right) - \sqrt{A\pi}\left(\frac{2-\pi}{\pi}\right)}$$

so $$v(\alpha) = \frac{g(\alpha)}{t}$$

$$v^2(\alpha) = \frac{g^2(\alpha)}{t^2}$$

$$\int v^2(\alpha)\,dt = \int \frac{g^2(\alpha)}{t^2}\,dt = -\frac{g^2(\alpha)}{t} + X$$

here X = constant for indefinite integration

$$c\int v^2(\alpha)\,dt = -\frac{c\,g^2(\alpha)}{t} + cX$$

$$\boxed{\frac{d\left(c\int v^2(\alpha)\,dt\right)}{d\alpha} = -\frac{c\,2g(\alpha)g'(\alpha)}{t}}$$

By adding the above term to eq (i), we will get

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minimization of potential energy U with respect to α for this case (ii),

$$\Rightarrow (2R_1+R_2)^2 e^{\frac{-2t}{C(2R_1+R_2)}} + \frac{2(2R_1+R_2)e^{\frac{-2t}{C(2R_1+R_2)}}t}{C}$$

$$- \frac{(2R_1+R_2)V^2 \ln t\, \varepsilon_r \varepsilon_0 \omega}{d}\left(\sqrt{\frac{2A\alpha^2}{\alpha - \sin\alpha\cos\alpha}} - \sqrt{A\pi}\right)$$

$$+ (F+kx)\left(\frac{2d}{\varepsilon_r\varepsilon_0 \omega}\left(\frac{\sin\alpha - 1}{\sin\alpha}\right)\right) - \frac{V^2}{2} + f(\alpha)$$

$$+ \frac{c}{t} j(\alpha) = 0$$

here $j(\alpha) = \dfrac{2g(\alpha)\,g'(\alpha)}{\frac{dc}{d\alpha}}$

$f(\alpha) = \dfrac{(\sin\alpha - \alpha\cos\alpha)}{\alpha^2}\dfrac{l_p(\alpha)}{\frac{dC}{d\alpha}}$

which will remain constant at specific α

- for simplicity, let $R = 2R_1 + R_2$,

$\cancel{R(\alpha) = \sqrt{\frac{2A\alpha^2}{\alpha - \sin\alpha\cos\alpha}} - \sqrt{A\pi}}$

$$z(\alpha) = \frac{\varepsilon_r \varepsilon_0 \omega}{2d}\left(\sqrt{\frac{2A\alpha^2}{\alpha - \sin\alpha\cos\alpha}} - \sqrt{A\pi}\right)$$

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$$\cancel{R^2 e^{-\frac{2t}{RC}} + \frac{2Re^{-\frac{2t}{RC}}t}{C} - \frac{RV^2 \ln t\, \varepsilon_r \varepsilon_o \omega\, k(\alpha)}{d}}$$

$$\Rightarrow \boxed{R^2 e^{-\frac{2t}{RC}} + \frac{2Re^{-\frac{2t}{RC}}}{C}t - 2RV^2 \ln t\, z(\alpha) + (F+kx)\left(\frac{2d}{\varepsilon_r \varepsilon_o \omega}\left(\frac{\sin\alpha - 1}{\sin\alpha}\right)\right) - \frac{V^2}{2} + f(\alpha) + \frac{C}{t} j(\alpha) = 0} \quad \text{(ii)}$$

- Conclusion from eq (ii) is at a particular instant i.e. at a specific time t and at a specific angle α, we will get

$$\cancel{C \propto R^2}$$

$$\boxed{C \propto 2R\ln t - R^2 e^{-\frac{2t}{RC}} - 2Re^{-\frac{2t}{RC}} t}$$

As $e^{-\frac{2t}{RC}}$ is a diminishing function with respect to increase in time t, after some initial time $2R\ln t$ will be dominant over other two terms on right side. so after some initial time $\boxed{C \propto R}$