6.825 Recitation Problems: Lec. 3-5

Exercises

October 6, 2001

1 Lecture 3: Logic

Which of these are legal sentences? Give fully parenthesized expressions.

- $(P \Rightarrow Q)$
- $P, R \Rightarrow Q$
- $(A \wedge (B \vee C \vee (\neg D)))$
- $P(Q)$

2 Lecture 4: Conjunctive Normal Form

2.1 Converting to CNF

1. $(A \rightarrow B) \rightarrow C$
2. $A \rightarrow (B \rightarrow C)$
3. $(A \rightarrow B) \vee (B \rightarrow A)$
4. $(\neg P \rightarrow (P \rightarrow Q))$
5. $(P \rightarrow (Q \rightarrow R)) \rightarrow (P \rightarrow (R \rightarrow Q))$
6. $(P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))$

2.2 DPLL

1. How would you modify DPLL so that it
   - returns a satisfying assignment if there is one, and false otherwise?
   - returns *all* satisfying assignments?
2. Would using DPLL to return all satisfying assignments be any more efficient that simply listing all the assignments and checking to see whether they're satisfying? Why or why not?

3 Lecture 5: FOL

3.1 Recitation Problems I

For each of the following sentences, determine whether it is true or false in the interpretation I we've been using:

1. $\forall x.above(x, Fred)$
2. $\forall x.above(x, Hat(x))$

3. $\forall x.oval(x) \rightarrow \exists y.above(y, x)$
4. $square(Hat(Hat(Fred)))$
5. $\forall x.above(x, Fred) \rightarrow square(x)$
6. $\exists x.\forall y.circle(y) \rightarrow above(y, x)$

3.2 Recitation Problems II

For each of the following English sentences, write a corresponding sentence in FOL.

1. Somebody loves Jane E x loves(x,Jane)
2. For every mountain in England, there is a higher mountain in Scotland v x (mountain(x) and InEngland(x)——> Ey InScotland(y) and higher(y,x)
3. There are at least two mountains in England Ex,y M(x) and M(y) and In(x) and In(y) and not x=y
4. You can fool all of the people some of the time
5. The Barber of Seville shaves all men who do not shave themselves 男理发师
6. The only good extraterrestrial is a dead extraterrestrial V x good(x) and dead(x) 文本
7. There is exactly one coin in the box not (Ex,y coin(x) and coin(y) and in(x) and in(y) and not x=y)
8. No mountain is higher than itself v x mountain(x) ——> not higher(x,x)
9. All students get good grades if they study vx study(x)——>good(x)
10. Some students get good grades if they study E x good(x) and study(x)

3.3 Recitation Problems III

For each group of sentences, write an interpretation under which the last sentence is false and all the rest are true.

1. $\forall x.h(x) \rightarrow g(x)$
   $\forall x.f(x) \rightarrow g(x)$
   $\exists x.f(x) \wedge h(x)$
2. $\forall x.\exists y.f(x, y)$
   $\exists y.\forall x.f(x, y)$
3. $\forall x.(f(x) \rightarrow g(A))$
   $(\forall x.f(x)) \rightarrow g(A)$

3.4 Recitation Problems IV

For each group of sentences, give an interpretation in which all sentences are true.

1. $(\forall x.p(x) \vee q(x)) \rightarrow \exists x.r(x)$
   $\forall x.r(x) \rightarrow q(x)$
   $\exists x.p(x) \wedge \neg q(x)$
2. $\forall x.\neg f(x, x)$
   $\forall x, y, z.f(x, y) \wedge f(y, z) \rightarrow f(x, z)$
   $\forall x.\exists y.f(x, y)$

3. $\forall x.\exists y.f(x,y)$

   $\forall x.(g(x) \rightarrow \exists y.f(y,x))$

   $\exists x.g(x)$

   $\forall x.\neg f(x,x)$