6.825 Week 4 Exercises

October 3, 2004

1 Quantifiers

1.1 Part 1

- Prove that the following two statements are equivalent (this is the infant-fare ticket example from class in a more general form):
  - $\forall x, y, z \; P(x,z) \wedge Q(z,y) \wedge R(y) \rightarrow S(x)$
  - $\forall x \; (\exists y, z \; P(x,z) \wedge Q(z,y) \wedge R(y)) \rightarrow S(x)$

1.2 Part 2

- Would the two statements still be equivalent if the $\rightarrow$ were replaced by $\leftrightarrow$ in both cases?

2 Resolution-refutation

1. Using resolution refutation, prove the last sentence in each group from the rest of the sentences in the group.

   (a) $P \rightarrow Q$
   $\neg P \rightarrow R$
   $\neg Q \rightarrow R$

   (b) $(P \rightarrow Q) \vee (R \rightarrow S)$
   $(P \rightarrow S) \vee (R \rightarrow Q)$

   (c) $\neg(P \wedge \neg Q) \vee \neg(\neg S \wedge \neg T)$
   $\neg(T \vee Q)$
   $U \rightarrow (\neg T \rightarrow (\neg S \wedge P))$
   $\neg U$

2. Use resolution refutation to do problem 7.9 from R&N.

3 Unification

For each pair of sentences, give an MGU.

• Color(Tweety, Yellow)	Color(x, y)
• Color(Tweety, Yellow)	Color(x, x)
• Color(Hat(John), Blue)	Color(Hat(y), y)
• R(F(x), B)	R(y, z)
• R(F(y), x)	R(x, F(B))
• R(F(y), y, x)	R(x, F(A), F(v))

•	Loves(x, y)	Loves(y, x)
•	F(G(w), H(w, J(x, y)))	F(G(v), H(u, v))
•	F(G(w), H(w, J(x, u)))	F(G(v), H(u, v))
•	F(x, F(u, x))	F(F(y, A), F(z, F(B, z)))

4 Formalization and Resolution-refutation

4.1 A silly recitation problem

Symbolize the following argument, and then derive the conclusion from the premises using resolution-refutation.

- Nobody who really appreciates Beethoven fails to keep silence while the Moonlight sonata is being played.
- Guinea pigs are hopelessly ignorant of music.
- No one who is hopelessly ignorant of music ever keeps silence while the moonlight sonata is being played.
- Therefore, guinea pigs never really appreciate Beethoven.

(Taken from a book by Lewis Carroll, logician and author of *Alice in Wonderland*.)

4.2 Another, sillier problem

You don't have to do this one. It's just for fun. Same type as the previous one. Also from Lewis Carroll.

- The only animals in this house are cats
- Every animal that loves to gaze at the moon is suitable for a pet
- When I detest an animal, I avoid it
- No animals are carnivorous unless they prowl at night
- No cat fails to kill mice
- No animals ever like me, except those that are in this house
- Kangaroos are not suitable for pets
- None but carnivorous animals kill mice
- I detest animals that do not like me
- Animals that prowl at night always love to gaze at the moon
- Therefore, I always avoid a kangaroo

5 Add in some paramodulation...

Formalize each group of sentences (using the given function and predicate symbols), then prove the last from the others using resolution and paramodulation.

1. ($L(x)$ = the lover of x; $D(x)$ = x drives a red car)
   - Jane's lover drives a red car
   - Fred is the only person who drives a red car
   - Therefore, Fred is Jane's lover

2. ($T(x)$ = the teacher of x; $G(x) = x$ is a good student)

   - Mrs. Abbot only teaches good students
   - John and Mary have the same teacher
   - Mrs. Abbot is Mary's teacher
   - Therefore, John is a good student

3. ($M(x)$ = the manufacturer of part x; $W(x, y)$ = part x is stored in the warehouse of company y; $T(x)$ = part x is made of titanium; $F(x)$ = part x is fragile; use a constant for "the part I need")

   - Every part is either made by FooCorp or BarCorp
   - All fragile parts are stored in the warehouse of their manufacturer
   - BarCorp can't manufacture titanium parts
   - The part I need is fragile and made of titanium
   - Therefore, the part I need is the FooCorp's warehouse

6 Entailment

- If $\mathtt{KB} \not\models S$, does this mean $\mathtt{KB} \models \neg S$? If so then prove it, otherwise give a counterexample.