DC SQUID FLUX FOCUSER

Mark B. Ketchen, W. J. Gallagher, A. W. Kleinsasser, S. Murphy*

IBM T. J. Watson Research Center
Yorktown Heights, NY 10598

John R. Clem

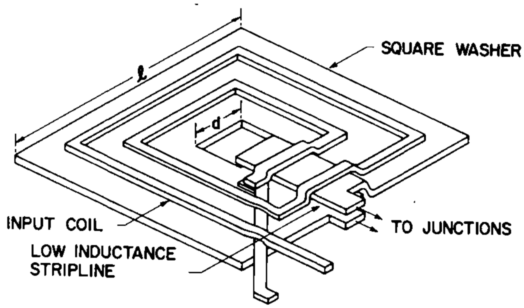
Ames Laboratory - U.S.D.O.E. and Dept. of Physics, Iowa State University,
Ames, Iowa 50011

Introduction

Over the past several years a number of workers have demonstrated low noise thin film dc SQUIDS utilizing planar coupling schemes (1-7). One of the most promising of these (1) makes use of a SQUID loop in the form of a superconducting square washer as shown in Fig. 1. The washer is slit in a low inductance fashion and connected to shunted tunnel junctions remotely located at the washer's outer edge. The input coil consists of an n-turn thin-film spiral insulated by a thin oxide layer from the underlying SQUID loop. From numerical calculations and experiments (1) we know that the SQUID inductance $L \approx 1.25\ \mu_0 d$, where d is the edge length of the square hole in the washer. The coupling between the washer and the spiral is extremely good, and the input coil inductance is $\approx n^2L$. The input coil inductance can thus be adjusted to match a particular load by varying the outer dimension of the washer to accommodate the required number of input coil turns. To use such a SQUID in applications where the device itself may experience changes in magnetic field (such as in an integrated thin-film gradiometer) it is important to understand the response of the washer geometry to an applied field. If a magnetic field is directed perpendicular to the plane of the washer, some flux will pass through the central hole, while most will be diverted around the outside. The flux coupled to the hole constitutes a direct input that will be sensed by the SQUID. The distortion resulting from the diversion of flux outside the washer contributes to a substantial dipole field that could, for example, differentially couple to otherwise balanced

* Present Address: Department of Physics, Cornell University, Ithaca, NY 14853

Fig. 1. Square washer SQUID design with spiral input coil.



SQUID '85 - Superconducting Quantum Interference Devices and their Applications

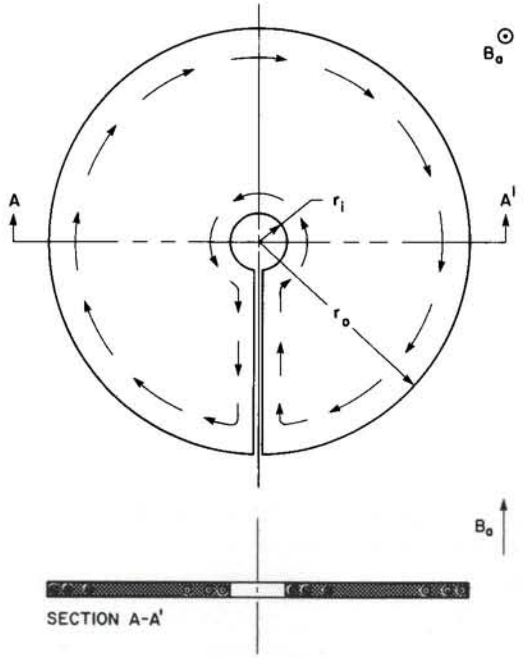


Fig. 2. Round superconducting washer with low inductance slit. Arrows indicate current flow pattern in response to the applied field B_a. The washer thickness (2000-4000Å for practical situations) has been drawn artificially large to permit indication of current flow.

pick-up loops. In this paper, we examine both theoretically and experimentally the response of the washer geometry to an applied field. We find that the structure does indeed tend to focus flux into the central hole by an amount proportional to the product of ℓ, the outside dimension of the washer, and d. We also find that about 40% of the flux displaced outside the washer can be found in the induced dipole field, while the rest is in higher moment fields of shorter range.

Theoretical Considerations

For simplicity we will concentrate our theoretical attention on the circular washer structure shown in Fig. 2. Results obtained should be applicable to square washers apart from numerical factors of order unity. We will not include the interaction of the washer with an input coil, which can be substantial, particularly if the input coil is connected to a superconducting pick-up loop structure. Such situations can be handled as a straight forward extension of the present work together with the coupling considerations discussed in Ref. 1. If a magnetic field B_a is applied perpendicular to the circular washer as shown in Fig. 2, supercurrents will flow as indicated by the arrows to screen the magnetic field from the superconducting film. These currents will at the same time support the magnetic flux passing through the hole of radius $r = r_i$ and around the outside edge of radius $r = r_0$. We will assume that the slit is of negligible inductance. (This can be realized by using an overlapping stripline configuration as in Fig. 1 or by using a floating groundplane.) Intuitively we expect the current flow to be concentrated near the hole and around the outside edge as indicated in Fig. 2.

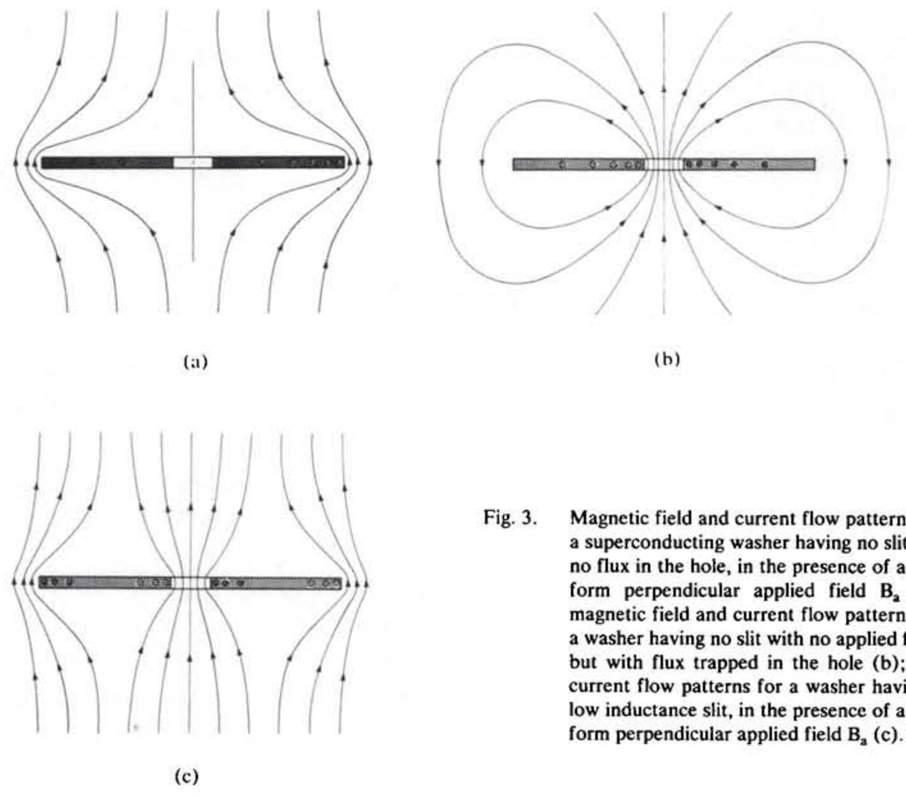
To analyze this situation, we will first consider two special cases and then make use of superposition. In the first special case shown in Fig. 3(a), the washer is assumed to have no slit so that when the magnetic field is applied, no flux passes through the hole. We will assume that the film is very thin compared to the size of the washer and that the superconducting penetration depth is somewhat less than half the film thickness. For $r_i = 0$ this is a classic mixed boundary condition problem closely related to the charged conducting disk discussed by Jackson (8) and directly analogous to a hydrodynamics problem discussed by Lamb (9). Solving for $K_{\theta 1}$, the circumferential current per radial unit length, we find for $r \lesssim r_0$:

$$K_{\theta 1} = \frac{-4B_a}{\pi\mu_0}\left(\frac{r}{\sqrt{r_0^2 - r^2}}\right), \tag{1}$$

which exhibits the expected square root divergence at the outside edge. Integrating Eq. 1 from $r = 0$ to $r = r_0$ we obtain the total circulating current,

$$I_{\theta 1} = \frac{-4B_a\, r_0}{\pi\, \mu_0}. \tag{2}$$

If $0 < r_i << r_0$, the boundary conditions are only minimally perturbed, and since for $r_i = 0$ almost no current flows over the center of the disk, Eq. 2 remains a good approximation.



(a) (b) (c)

Fig. 3. Magnetic field and current flow patterns for a superconducting washer having no slit and no flux in the hole, in the presence of a uniform perpendicular applied field B_a (a); magnetic field and current flow patterns for a washer having no slit with no applied field, but with flux trapped in the hole (b); and current flow patterns for a washer having a low inductance slit, in the presence of a uniform perpendicular applied field B_a (c).

The second special case is shown in Fig. 3(b). Here we again have the same circular washer with no slit. This time $B_a = 0$, but there is some magnetic flux ϕ trapped in the hole that is supported by a circulating supercurrent $I_{\theta 2}$. In the limit that $r_0 = \infty$, we have solved for $K_{\theta 2}$, the circumferential current per radial unit length, and find, for $r > r_i$:

$$K_{\theta 2} = \frac{\phi}{\mu_0 \pi r \sqrt{r^2 - r_i^2}}, \tag{3}$$

again exhibiting a square root singularity at the edge. Integrating Eq. 3 from $r = r_i$ to ∞ we obtain:

$$I_{\theta 2} = \frac{\phi}{2 \mu_0 r_i}. \tag{4}$$

Since $K_{\theta 2}$ falls off like r^{-2} for large r, there is very little current flowing out beyond several r_i, and Eq. 4 should remain valid for any $r_0 >> r_i$. From Eq. 4 it is evident that the inductance L of the washer is $2 \mu_0 r_i$. Expressing this in terms of the circumference of the hole, $C = 2 \pi r_i$, we find $L = \mu_0 C/\pi$. It is interesting to compare this with the numerical result for a square washer (1), $L = 5 \mu_0 C/16$, where C is now equal to 4ℓ, the perimeter of the hole. These expressions for L are within 2% of each other. Furthermore, numerical results for the square washer suggest that Eq. 5 will likely be very accurate for $r_0 \geq 2 r_i$.

To solve the original problem posed in Fig. 2, we superpose the solutions to the two special cases analyzed. The resulting magnetic field pattern will be as shown in Fig. 3(c). Because of the presence of the slit in the washer, the net circulating current must vanish. Since the slit is assumed to be of negligible inductance, its primary influence is to allow flux to enter the hole as B_a is applied. The condition of zero net circulating current implies $I_{\theta 1} + I_{\theta 2} = 0$. Solving for ϕ we find:

$$\phi = \frac{8}{\pi} B_a r_0 r_i = \frac{8}{\pi^2} B_a A_h \left(\frac{A_w}{A_h} \right)^{1/2}, \tag{5}$$

where $A_h = \pi r_i^2$ and $A_w = \pi r_0^2$. This indicates an effective pickup area of

$$A_{eff} = \frac{8}{\pi^2} A_h \left(\frac{A_w}{A_h} \right)^{1/2}. \tag{6}$$

Thus, while the area occupied by the washer A_w increases like r_0^2, the effective pickup area increases only as $A_w^{1/2} \propto r_0$.

Since we know the current flow patterns in the washer, we are in a position to calculate the induced dipole moment m. In the plane of the washer the induced dipole field in the direction of B_a can be written as $- mr^{-3}$ where

$$m = m_1 + m_2 = \frac{\mu_0}{4\pi} \int_{r_i}^{r_0} \pi r^2 (K_{\theta 1} + K_{\theta 2})\, dr. \tag{7}$$

Integrating we find

$$m_1 = \frac{-2 B_a r_0^3}{3 \pi} = \left(\frac{-4}{3 \pi} \right) \left(\frac{B_a r_0^3}{2} \right) \tag{8}$$

$$m_2 = \frac{\phi\, r_0}{4\,\pi} = \frac{2\, B_a\, r_0^2\, r_i}{\pi^2}. \tag{9}$$

In the case of a superconducting sphere of radius r_0, all of the displaced flux, $\pi\, r_0^2\, B_a$, can be found in the dipole field (9), and the dipole moment is $-\, B_a\, r_0^3\,/2$. Thus for a washer with no slit, about 40% of the displaced flux goes into dipole field while the rest must be in shorter range fields. For the washer with the low inductance slit, the counter-circulating current reduces the magnitude of the dipole moment somewhat to give

$$m \approx -\,0.2\, B_a\, r_0^3 \left(1 - \frac{r_i}{r_0}\right). \tag{10}$$

For the square washer geometry, we expect the results to be very similar to those calculated for the circular washer. In particular, we would predict

$$A_{eff} = a\, A_h \left(\frac{A_w}{A_h}\right)^{1/2} = a\, d^2 \left(\frac{\ell^2}{d^2}\right)^{1/2} = a\, d\, \ell \tag{11}$$

and

$$m = -\, b\, B_a\, \ell^3 \left[1 - c\left(\frac{d}{\ell}\right)\right] \approx -\, b\, B_a\, \ell^3, \tag{12}$$

where a and c are numerical constants of order unity and $b \approx 0.04$.

Experiments and Results

To verify our understanding of the response of a superconducting washer to an applied magnetic field, we fabricated and tested a number of different square washer SQUIDS. All had a hole size d of 24 μm corresponding to an inductance $L \approx 40$ pH. We made washers having an outer dimension of 40 μm, 74 μm, 150 μm, 250 μm, 500 μm, 1000 μm and 2000 μm. A photograph of the $\ell = 500$ μm design is shown in Fig. 4. The washer

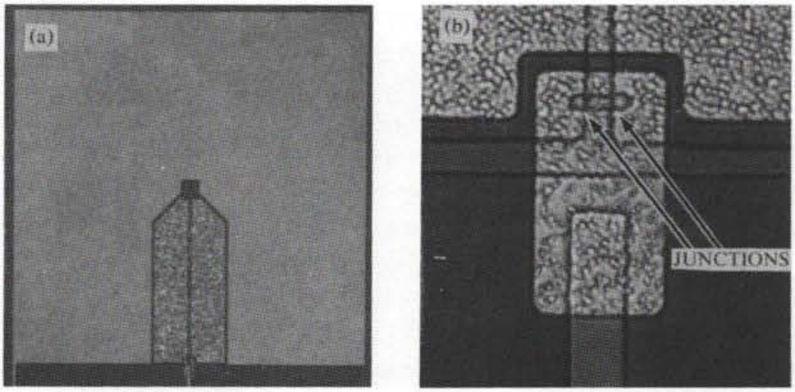


Fig. 4. Photograph of experimental square washer SQUID having $\ell = 500$ μm and $d = 24$ μm (a); blowup of junction area showing two 1.8 μm long unshunted edge junctions (b). Note in (a) the floating groundplane structure over the slit used to reduce the stray inductance.

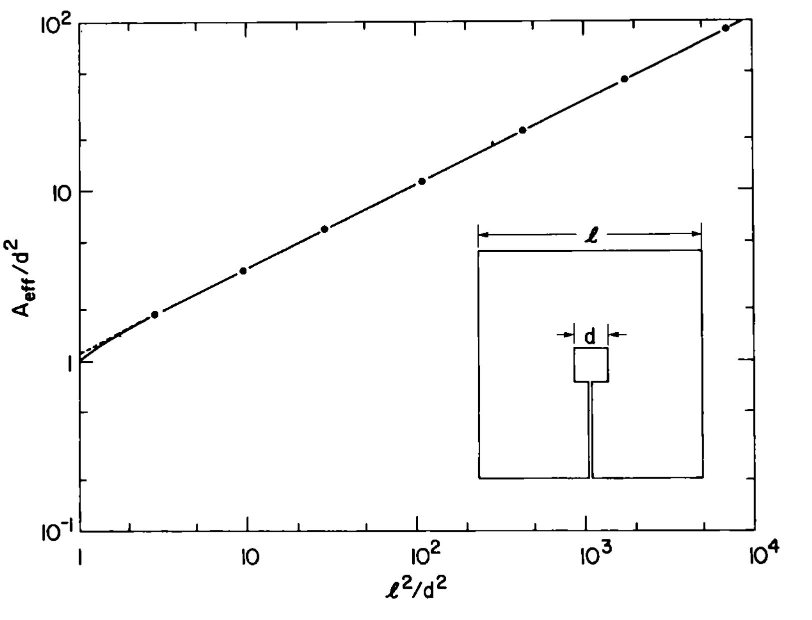


Fig. 5. A_{eff}/d^2 versus ℓ^2/d^2. The data points represent the seven different SQUIDs evaluated.

was made of 2000Å thick Nb. The junctions were 1.8 μm long Nb - NbO_x - Pb alloy edge junctions with no resistive shunts. A wide floating groundplane (Pb alloy) was situated over the slit area to minimize the slit inductance. This was insulated from the washer by 700Å of Nb_2O_5 and a 2750Å thick layer of SiO (11). The SQUIDs were individually loaded into a low-noise rf SQUID readout sample holder (12) and cooled to 4.2 K in a low field ($\lesssim$ 10 μG) dewar. A calibrated magnetic field B_a was applied perpendicular to the plane of the washer by means of saddle coils built into the dewar. The magnitude of B_a required to couple one flux quantum through the hole of the washer was determined from the periodicity of the SQUID's threshold curve. The effective pickup area A_{eff} could then be determined. Fig. 5 is a plot of A_{eff}/d^2 versus ℓ^2/d^2 for the seven different SQUIDs. The straight line through the data points has a slope of 1/2 in perfect agreement with Eq. 11. From the straight line extrapolation back through $\ell^2/d^2 = 1$ (dotted line), we find a $\approx$ 1.1. From geometrical considerations the actual curve must of course pass through the point (1, 1) as shown by the solid line. This constraint is probably "responsible" for the good fit at lower values of ℓ^2/d^2 than would be expected from the original derivation. It is noteworthy that at the largest value of ℓ, 2000 μm, A_{eff} is over 90 times the hole area, but ℓ^2 is 7000 times the hole area. Thus, flux is focused into the hole, but the efficiency for this is rather low. Finally, while this experiment is a direct verification of Eq. 11 for A_{eff}, it is also a strong indirect verification for Eq 12. The current flowing around the hole which we are effectively measuring must return around the outside edge of the washer, and the sum of these currents inescapably leads to a dipole moment on the order of that predicted in Eq. 12.

Conclusions

In summary, we have studied both theoretically and experimentally the response of the superconducting washer to an applied field. We have found that the washer SQUID has an effective pickup area equal to ~ $A_h (A_w/A_h)^{1\ 2}$. We also deduce that about 40% of the flux displaced outside the washer is reshaped into a dipole field. Both of these results are important considerations in the design of an integrated SQUID system that may experience changes in applied magnetic field.

Acknowledgements

We wish to acknowledge the following individuals for contributions to the fabrication of the SQUID chips used in this work: B. Ek, C.D. Jessen, S.I. Raider, R.L. Sandstrom, J.L. Speidell, and J.W. Stasiak.

References

1. Jaycox, J.M. and M.B. Ketchen. 1981. IEEE Trans. Magn. *MAG-17*, 400; Ketchen, M. B. and J.M. Jaycox. 1982. Appl. Phys. Lett. *40*, 736.
2. Cromar, M.W. and P.Carelli. 1981. Appl. Phys. Lett. *38*, 723.
3. de Waal, V.J., P. van den Hamer, and T.M. Klapwijk. 1983. Appl. Phys. Lett. *42*, 389.
4. Muhlfelder, B., W.W. Johnson, and M.W. Cromar. 1983. IEEE Trans. Magn. *MAG-19*, 303.
5. Wellstood, F., C. Heiden, and J. Clarke. 1984. Rev. Sci. Instrum. *55*, 952.
6. Tesche, C.D., K.H. Brown, A.C. Callegari, M.M. Chen, J.H. Greiner, H.C. Jones, M.B. Ketchen, K.K. Kim, A.W. Kleinassaser, H.A. Notarys, G. Proto, R.H. Wang, and T. Yogi. 1985. IEEE Trans. Magn. *MAG-21* (2), 1032.
7. Pegrum, C.M., D. Hutson, G.B. Donaldson, and T. Tugwell. 1985. IEEE Trans. Magn. *MAG-21*, 1036.
8. Jackson, J.D. 1962. *Classical Electrodynamics*, Wiley, New York, p. 89.
9. Lamb, A. 1945. *Hydrodynamics*, Dover, New York, p. 138.
10. Jackson, J.D. ibid., p.156.
11. Details of the process will be published in a future paper by Gallagher, W.J., A.W. Kleinsasser, R.L. Standstrom, S.I. Raider, and M.B. Ketchen.
12. Ketchen, M.B., and C.C. Tsuei. 1980. *SQUID'80*, (H. D. Hahlbohm and H. Lubbig, eds.) Walter de Gruyter, Berlin, p. 227.