# Contents

# 1. Theoretical Background

## 1.1 Josephson junctions

The *Josephson junctions* named after Brian D. Josephson consist of two identical superconductors weakly coupled to each other. In the case of the junctions produced in this working group, such coupling is realized through a few Ångström thin insulating layer between the superconducting electrodes. Consequently, they are referred to as SIS (Superconductor-Insulator-Superconductor) junctions. The resulting trilayer structure typically consists of $Nb/Al$-$AlO_x/Nb$, with niobium being used for the superconductors and the insulating layer being provided by the aluminum oxide. A schematic structure is shown in Fig. ?. When the junction is maintained at very cold temperatures ($\leq 4\,\mathrm{K}$) and connected to a current source a supercurrent is measurable.

### 1.1.1 Josephson effect

The flow of current implies the tunneling of Cooper pairs, as at these temperatures, niobium is predominantly superconducting ($T_c = 9.3\,\mathrm{K}$). Since the tunneling probability of an individual electron is approximately $p = 10^{-4}$, for a Cooper pair consisting of two electrons, a much lower probability is to be expected. However, Josephson predicted that the tunneling behavior of Cooper pairs and individual conduction electrons must be the same. This is justified by the so-called *Macroscopic Quantum Model*, formulated in 1953 by Fritz London.

The main focus here lies on the quantum mechanical phase $\theta$. On one hand, the distance between both electrons in a Cooper pair is approximately 10 to $1000\,\mathrm{nm}$ which is significantly larger than the spacing between Cooper pairs, resulting in strongly overlapping wave functions. On the other hand, Cooper pairs have to obey Bose-Einstein statistics due to their total spin of 0. Thus, all Cooper pairs share the same ground state, and as a consequence, the energies and temporal evolutions of the phases are equal. These two effects lead to what is known as *phase-lock*. The phases of neighboring pairs synchronize such that this quantum mechanical property now holds on a macroscopic scale. This gives rise to a macroscopic wave function

$$\Psi(\mathbf{r}, t) = \Psi_0(\mathbf{r}, t) e^{i\theta(\mathbf{r}, t)} \quad , \tag{1.1}$$

which describes all charge carriers of the superconductor. As a result of sharing the

same phase, both electrons of a Cooper pair consequently possess the same tunneling probability as an individual electron, enabling the supercurrent. This coherence phenomenon is referred to as the *Josephson effect*. Another significant consequence of the macroscopic quantum model is flux quantization. Together with the Josephson effect, this forms the basis for Josephson junctions and their applications.

Flux quantization is derived through the capture of an external magnetic flux within a superconducting cylinder. The wave function must remain unchanged after circumnavigating the cylinder due to $e^{i\theta} = e^{i\theta + 2\pi n}$. As a result, upon integrating along the current-free center of the cylinder wall, the following equation holds for the captured flux

$$\Phi = \frac{h}{q_{\mathrm{s}}}n = \frac{h}{2e}n \equiv \Phi_0 n \quad .\tag{1.2}$$

Here, $n \in \mathbb{Z}$ and $\Phi_0 = 2.07 \times 10^{-15}\,\mathrm{T\,m^2}$ represents the so-called magnetic flux quantum. The captured flux is thus quantized, a consequence solely arising from the macroscopic nature of the phase. This quantity plays a crucial role in the theoretical description of Josephson junctions.

The current and voltage behavior in a SIS junction is described by the *Josephson equations*. Crucial to this description is a critical current $I_{\mathrm{c}}$ that is linearly proportional to the applied current $I$, which marks the boundary between two operational modes. Additionally, due to the macroscopic nature of the phase, $I$ oscillates with the gauge-invariant phase difference $\varphi$, leading to the **first Josephson equation**

$$I_{\mathrm{s}} = I_{\mathrm{c}}\sin(\varphi) \quad .\tag{1.3}$$

$I_{\mathrm{c}}$ is proportional to the coupling strength $\kappa$, which describes the overlap of the wave functions $\Psi_1$ and $\Psi_2$ within the insulating layer. The relationship is given by

$$I_{\mathrm{c}} = \frac{4e\kappa V n_{\mathrm{s}}}{\hbar} \quad ,\tag{1.4}$$

where $V$ represents the volume of the superconducting electrode and $e$ denotes the elementary charge of an electron. It was assumed that the Cooper pair density $n_{\mathrm{s}}$ of the two superconductors $S_1$ and $S_2$ is identical, meaning $n_{\mathrm{s}1} = n_{\mathrm{s}2} = n_{\mathrm{s}}$.

The gauge-invariant phase difference refers to the phases $\theta_1$ and $\theta_2$ of the respective electrodes at the boundary of the insulating layer (positions 1 and 2, see Fig. ?). Taking into account potential external electromagnetic fields within the barrier, the general form using the vector potential $\mathbf{A}$ is given by

$$\varphi = \theta_2(\mathbf{r}, t) - \theta_1(\mathbf{r}, t) - \frac{2\pi}{\Phi_0} \int\limits_1^2 \mathbf{A}(\mathbf{r}, t) \cdot \mathrm{dl} \quad . \tag{1.5}$$

Assuming a constant supercurrent density $J_\mathrm{s}$ across the junction, taking the time derivative of equation (1.5) yields the **second Josephson equation**

$$\frac{\partial \varphi}{\partial t} = \frac{2\pi}{\Phi_0} U \quad . \tag{1.6}$$

The first operating mode describes the case for $I < I_\mathrm{c}$. Here, the entire injected current is carried by Cooper pairs, so $I = I_\mathrm{s} = \mathrm{const}$. As a result, $\varphi$ is temporally constant, which, according to equation (1.6), leads to $U = 0$. This voltage-free state is known as the *dc Josephson effect*.

For $I > I_\mathrm{c}$ however, Cooper pairs begin to break up such that a portion of the current needs to be carried by quasiparticles, which will then lead to a voltage drop $V$. According to the second Josephson equation, the phase $\varphi$ becomes time dependent, and after integration one obtains

$$\varphi = \frac{2\pi}{\Phi_0} V t + \varphi_0 = w_\mathrm{J} t + \varphi_0 \quad \text{with} \quad w_\mathrm{J} = \frac{2\pi}{\Phi_0} V \quad . \tag{1.7}$$

Thus, the current $I_\mathrm{s}$ oscillates after inserting equation (1.7) into equation (1.3) with the *Josephson frequency* $\frac{f_\mathrm{J}}{V} = \frac{w_\mathrm{J}}{2\pi V} = \frac{1}{\Phi_0} \approx 483.6 \, \frac{\mathrm{MHz}}{\mathrm{\mu V}}$. Accordingly, this phenomenon is referred to as the *ac Josephson effect*.

### 1.1.2 Josephson Kontakte im Magnetfeld

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

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After this fourth paragraph, we start a new paragraph sequence. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

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### 1.1.3   RCSJ Modell

## 1.2   dc-SQUIDs

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### 1.2.1 Spannungszustand

### 1.2.2 Rauschen

### 1.2.3 Inbetriebnahme eines dc-SQUIDs

## 1.3 Resonanzen eines dc-SQUIDs

### 1.3.1 Parasitäre Resonanzen

### 1.3.2 Dämpfungsmethoden

# 2. Metallisch Magnetische Kalorimeter