V. Conclusion

The first theoretical and experimental results concerning analog avalanche diode frequency dividers have been presented. The fundamental principle of operation relies on the use of an idler signal. The potential of these dividers has been demonstrated. Indeed, experimental results on frequency dividers with division ratios of 2, 3, and 4 in the centimeter-wave range have been presented with reproducible operation, wide bandwidth, and high conversion efficiencies. Both theoretical and experimental work will be extended to determine the possibilities of direct high rank frequency division of such avalanche diode frequency dividers. Moreover, such a division principle is expected to be valid up to the millimeter-wave range.

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A Parametric Study of the Attenuation Constant of Lossy Microstrip Lines

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***Abstract*—In high-density, high-speed electronic circuits, the conductor and dielectric losses in microstrip-type interconnections are of particular concern. The attenuation constant of a microstrip line with finite strip conductivity and strip thickness comparable to the skin depth is investigated at a frequency of 1 GHz, and its dependence on the width of the strip and the thickness of the dielectric substrate ($\epsilon_r = 11$) is examined. It is found that the minimum in the attenuation constant predicted by earlier studies, when the conductor thickness is about two skin depths, occurs only for microstrips with impractical strip width to substrate thickness ratios.**

I. Introduction

The microstrip transmission line has become very popular in modern microwave integrated circuit technology not only because of its simple construction, but also because it offers the wide range of characteristic impedances demanded by integrated circuits. Furthermore, the propagation characteristics of such microstrip structures are of particular concern in today's chip-to-chip interconnects for high-speed electronic systems. As we are moving toward higher device densities and smaller conductor dimensions, the attenuation associated with the ohmic losses in the strip conductors becomes a crucial parameter. Recent advances in wafer scale packaging technologies are expected to lead to dense interconnects with conductor widths and substrate thicknesses below 10 μm, and to conductor thicknesses below 5 μm. With conductor thicknesses of the order of a few microns, the skin depth becomes comparable to the thickness of the conductor, and Wheeler's incremental rule [1] and the perturbation techniques [2], [3] used for the evaluation of conductor losses in the strip become inapplicable.

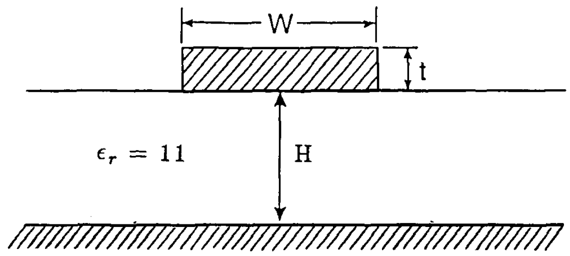

Fig. 1. Cross-sectional view of the microstrip line.

The attenuation constant for wide strips with thicknesses comparable to the skin depth was first studied by Welch and Pratt [4]. They modified the expressions derived by Assadourian and Rimai [5], for conductors with thicknesses comparable to the skin depth, using a skin effect resistance modification factor that predicts a minimum in the skin effect resistance when the thickness to skin depth ratio (t/δ) is equal to $\pi/2$. However, for the case they studied, this minimum in the attenuation did not appear. Horton *et al.* [6] studied the attenuation in thin microstrip lines by relating the longitudinal components of the current to the charge profiles of a static solution. Their results showed that, for very wide strips, a minimum attenuation occurs when t/δ is in the range from 2 to 3.

In this paper, we investigate in more detail the effects of the thickness to skin depth ratio (t/δ) on the attenuation constant α for a single aluminum microstrip line for various widths (W) and distances (H) above ground as shown in Fig. 1. It is assumed that all cross-sectional dimensions of the structure are sufficiently small compared to the minimum wavelength of interest, so that a quasi-TEM analysis is justified. Our analysis is based on a magneto-quasi-static approximation to calculate the frequency-dependent per-unit-length (p.u.l.) inductance and resistance matrices for a system consisting of N parallel, uniform rectangular strip conductors above either a finite or an infinite ground [7], [8]. The capacitance and conductance matrices are computed by solving an electrostatic problem as discussed in [9]. For the case of the microstrip transmission line, the propagation constant γ can be computed using the well-known expression

$$\gamma = \alpha + j\beta = \left[(R + j\omega L)(G + j\omega C)\right]^{1/2} \tag{1}$$

where R, L, C, and G are the p.u.l. resistance, inductance, capacitance, and conductance of the microstrip line, respectively.

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II. Theory

In the following, the procedure for determining the p.u.l. inductance and resistance matrices for a system of parallel, uniform microstrip lines is outlined. A detailed discussion of the theory can be found in [7] and [8]. It is shown that, under a quasi-TEM approximation with the transverse component of the current neglected, the z component of the electric field inside the conductors is given by

$$E = -j\omega A - \frac{d\Phi}{dz} \tag{2}$$

where A is the z component of the vector magnetic potential and Φ is the scalar electric potential. Following Costache [10], using Ohm's law, and defining a new quantity $\tilde{A}_m$ as the average magnetic potential over the cross-sectional area of the mth conductor, we arrive at the following expression for the current density:

$$J = -j\omega\sigma A + j\omega\sigma\tilde{A}_m + \frac{I_{0m}}{S_m} \tag{3}$$

where I_{0m} is the net forcing current inside the mth conductor, S_m is its cross-sectional area, and σ is its conductivity. By expressing A and $\tilde{A}_m$ in terms of the current density J using the magneto-quasi-static Green's function for the magnetic vector potential, an integral equation is obtained for J, which is then solved using the method of moments. Once the current distribution inside each conductor is known, the p.u.l. inductance and resistance matrices are calculated from the time-averaged magnetic energy stored in the system and the ohmic losses in the conductor, respectively. The capacitance and conductance matrices are calculated using a moment-method parameter calculator program developed at the University of Arizona [11].

To verify the results obtained for the attenuation and phase constant in the case of very wide strips, the microstrip geometry was approximated by the infinitely wide layered structure formed by a dielectric layer of thickness H above a perfect ground, followed by a conductive layer of finite conductivity and thickness t and then air. The eigenvalue problem for the above layered structure was formulated and solved for the propagation constant of the dominant TM mode [12].

III. Results

The structure under consideration is that of Fig. 1, where W denotes the strip width, t its thickness, and H the thickness of the dielectric substrate. The conductor material is aluminum with a conductivity $\sigma = 4\times10^7$ S/m, and the dielectric is silicon with a relative dielectric constant $\epsilon_r = 11$ and a loss tangent equal to 2.5×10^{-4}. The ground is assumed to be perfect and infinite. However, losses due to a ground of finite conductivity can be accounted for in an approximate fashion for those cases where the thickness of the ground conductor is greater than three skin depths at the frequency of interest. For such cases, it is assumed that the current distribution in the ground plane can be represented equivalently by a uniform current distribution that extends some distance Q on either side of the strip's projection on the ground. Thus, the p.u.l. resistance associated with losses in the ground is given by $R_g = R_s/(2Q+W)$, where $R_s = 1/(\sigma\delta)$ is the surface resistance and δ is the skin depth. The overall p.u.l. resistance of the microstrip is found as the sum of R_g and the p.u.l. strip resistance. While the substrate thickness H has been suggested as an appropriate value for the distance Q [13], our calculations show that such a choice overestimates R_g, and the value $Q = 1.25\,(W+H)$ seems to be a more realistic choice. To be more specific, the magneto-quasi-static integral equation formulation, described in the first paragraph of Section II, was used for predicting the current distribution on the conductors of a two-conductor line formed by a conducting strip of finite width W parallel to a second strip of variable width and at a distance H from the first strip. It was found that the current distribution in the variable strip, as well as its p.u.l. resistance, showed negligible variations for values of the variable strip width greater than $8W$. Furthermore, it was found that by choosing $Q = 1.25\,(W+H)$ the p.u.l resistance R_g obtained from the above approximate formula was in very good agreement with the result from the integral equation solution.

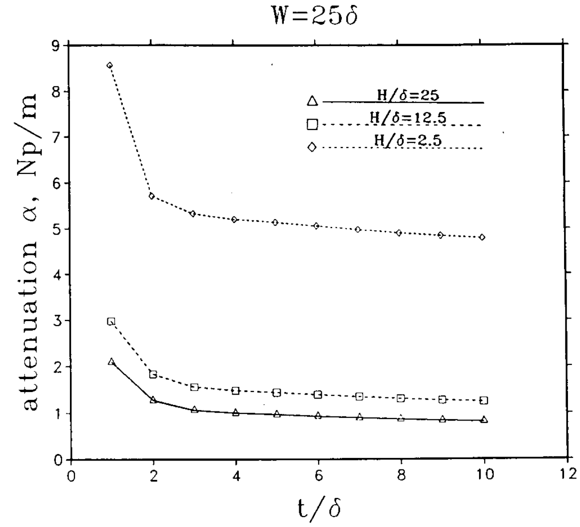

Fig. 2. Attenuation constant for a microstrip of width 25δ plotted versus the thickness to skin depth ratio, t/δ, with H/δ as a parameter.

For the results presented in Figs. 2–6, a fixed frequency of 1 GHz was assumed. The skin depth δ for aluminum at this frequency is approximately 2.5 μm. Conductors of three different widths were studied. The first had a width $W = 62.5\ \mu\text{m} = 25\delta$, the second had a rather large width $W = 250\ \mu\text{m} = 100\delta$, while the third had an even larger width $W = 400\ \mu\text{m} = 160\delta$. Figs. 2 and 3 show the attenuation constant α due to strip conductor losses and substrate dielectric losses as a function of t/δ, with H/δ as a parameter, for the two conductor widths $W = 25\delta$ and $W = 100\delta$, respectively. In all cases the attenuation decreases as the thickness of the strip is increased, and it is larger for the conductor with the smaller width ($W = 25\delta$). For the conductor with the larger width ($W = 100\delta$), the attenuation appears to decrease only slightly beyond $t/\delta = 4$. Thus far, ground losses have not been included in the calculations. In Fig. 4, the attenuation constant due to conductor losses, predicted by our formulation accounting for losses in the ground, is compared with the attenuation constant predicted by a closed-form expression [14], [15] for the case $W = H = 100\delta$. Good agreement is observed.

The above results do not indicate the existence of a minimum in the attenuation. However, if the ratio W/H is increased further, the minimum appears as predicted by the η'/η modification factor discussed in [4]. The results are shown in Figs. 5 and 6. Once again, the ground losses have not been included in the results shown in these two figures. In the first example, the

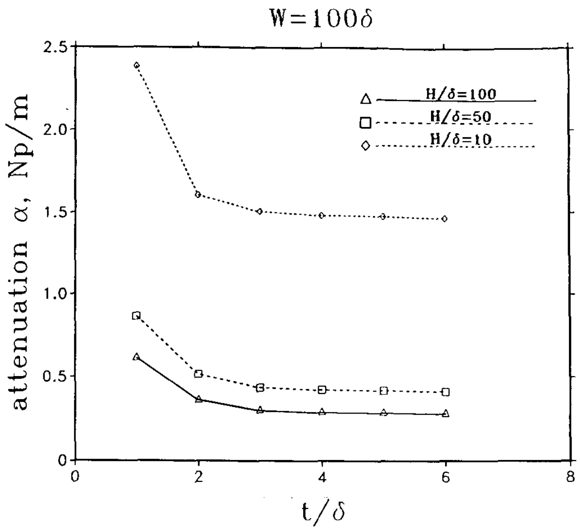

Fig. 3. Attenuation constant for a microstrip of width 100δ plotted versus the thickness to skin depth ratio, t/δ, with H/δ as a parameter.

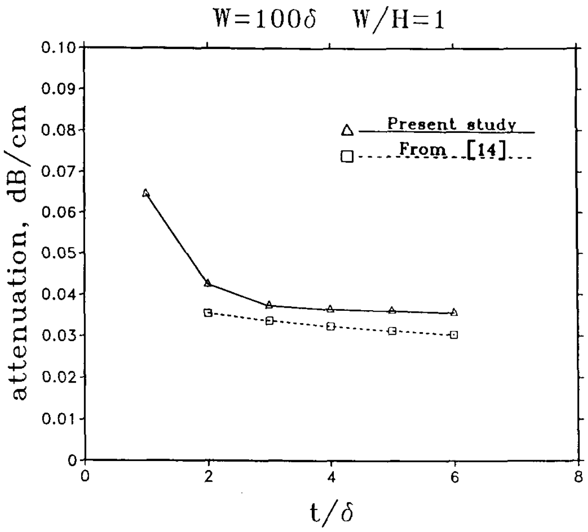

Fig. 4. Comparison of attenuation constant predicted by this method with the one obtained from the closed-form expressions in [14] ($W = 100\delta$).

substrate thickness was set to 10 μm and the width of the strip was increased until a minimum in the attenuation constant was observed. This occurred for $W/H = 40$, and the plot of the attenuation constant versus t/δ is shown in Fig. 5. A minimum in the attenuation is observed at $t/\delta = 2$. In the same figure, the dashed line, labeled $W =$ infinite, shows the results from the eigenvalue problem for the attenuation constant of the dominant TM mode for the infinitely wide layered structure described in the last paragraph of Section II. Observe that for the infinitely wide strip the attenuation minimum occurs at $t/\delta = 1.5$. Fig. 6 provides a comparison of the attenuation constant for a microstrip of $W/H = 100$ and $H = \delta = 2.5$ μm with the results obtained for the attenuation constant for the dominant TM mode of the corresponding infinitely wide layered structure. Once again, the curve labeled $W =$ infinite is for the infinitely

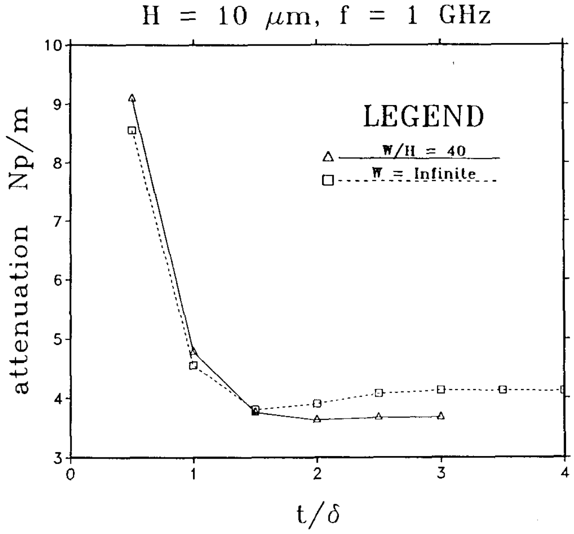

Fig. 5. Comparison of the attenuation predicted by this method for $W/H = 40$ and $H = 4\delta$ with that obtained for the corresponding layered structure of infinite width.

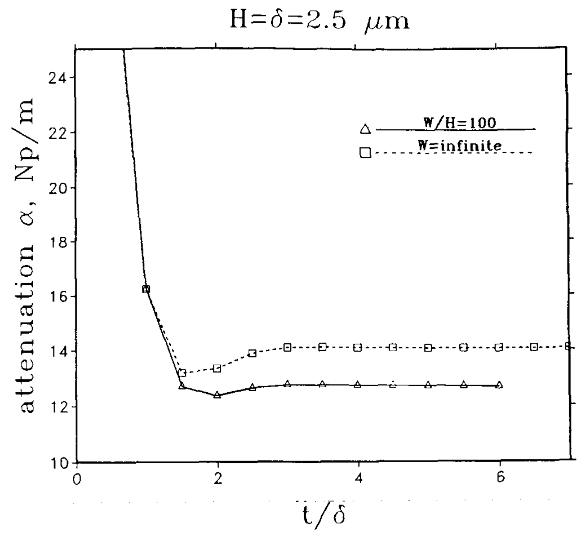

Fig. 6. Comparison of the attenuation predicted by this method for $W/H = 100$ and $H = \delta$ with that obtained for corresponding layered structure of infinite width.

wide layered structure, while the solid line is for the microstrip line. These results seem to support the thesis that the minimum of the attenuation constant can be attributed to a sort of resonance in the thickness of the conductor and the substrate.

As a last test, our results were compared with some experimental data reported by Hyltin [16]. Hyltin reported a number of measurements on the attenuation of aluminum microstrips of various thicknesses, evaporated over silicon with resistivity $\rho = 16$ $\Omega\cdot$m, at 9 GHz. The width of the strip was 152.4 μm with $W/H = 0.6$. Tables I and II show the experimental results as well as the results obtained in this study for two cases. In the first case, Table I, the attenuation constant due to losses in the strip and the dielectric is shown with the ground losses ne-

TABLE I
COMPARISON OF THEORETICAL RESULTS FOR THE MICROSTRIP ATTENUATION CONSTANT, EXCLUDING GROUND LOSSES, WITH EXPERIMENTAL RESULTS REPORTED BY HYLTIN [16]

Thickness,t (μm)	Atten., α(dB/cm) (Experiment [16])	Atten., α(dB/cm) (Present Method) (No ground losses)
0.254	1.00	0.881
0.508	0.58	0.566
1.524	0.39	0.354
1.778	0.35	0.340
2.286	0.34	0.324
10.160	0.27	0.283

TABLE II
COMPARISON OF THEORETICAL RESULTS FOR THE MICROSTRIP ATTENUATION CONSTANT, INCLUDING GROUND LOSSES, WITH EXPERIMENTAL RESULTS REPORTED BY HYLTIN [16]

Thickness,t (μm)	Atten., α(dB/cm) (Experiment [16])	Atten., α(dB/cm) (Present Method) (With ground losses)
0.254	1.00	0.893
0.508	0.58	0.577
1.524	0.39	0.364
1.778	0.35	0.350
2.286	0.34	0.335
10.160	0.27	0.293

glected. In the second case, Table II the ground losses were included. Our results in these tables are in good agreement with Hyltin's measurements.

IV. CONCLUSION

The effect of the strip conductor thickness on the attenuation constant of microstrip transmission lines has been investigated. Our numerical results are in good agreement with experimental ones obtained from the literature. In agreement with previous studies, it is found that the attenuation is indeed minimized when the thickness to skin depth ratio is about 2, but this is true only for microstrip lines with very large W/H ratios, which are very rarely of any practical interest. We conclude that, for most practical microstrip structures, one cannot rely on a thickness of two skin depths to minimize the attenuation due to conductor losses.

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Frequency-Domain Nonlinear Microwave Circuit Simulation Using the Arithmetic Operator Method

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***Abstract*—A frequency-domain spectral balance technique for the analysis of microwave circuits with analytically modeled nonlinear devices is developed. The technique uses linear matrix transformation of spectra to perform basic arithmetic operations—multiplication and division—in the frequency domain, and is termed the arithmetic operator method. A single MESFET amplifier described by the Curtice model is simulated with one- and two-tone excitations using this novel technique. Excellent agreement is obtained when compared to the results simulated using the conventional harmonic balance method.**

I. INTRODUCTION

The analysis of nonlinear microwave circuits using frequency-domain spectral balance (FDSB) has been investigated in several different ways, which have included Volterra series expansions [1], [2], algebraic functional expansions [3], [4], and power-series expansions [5], [6]. In general, compared with the conventional harmonic balance (HB) hybrid methods, the FDSB techniques have a larger dynamic range and can be practically used with multitone excitations. However, most FDSB methods are restricted to series representations of nonlinear elements. This has been the major restriction to the widespread use of FDSB techniques. In this paper, we demonstrate a newly developed frequency-domain nonlinear circuit analysis technique, the arith-

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