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Towards noise engineering: Recent insights in low-frequency excess flux noise of superconducting quantum devices

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The comprehensive analysis of low-frequency excess flux noise both in terms of magnetic flux noise $S_{\Phi,1/f}$ and energy sensitivity $\epsilon_{1/f}$ of 84 superconducting quantum devices studied at temperatures below 1 K reveals a universal behavior. When analyzing data in terms of $\epsilon_{1/f}$, we find that noise spectra of independent devices cross each other all at certain crossing frequencies f_c. Besides this main result of our paper, we further show that superconducting quantum interference device (SQUID) arrays systematically feature higher noise exponents than single SQUIDs and give evidence for a material and device type dependence of low-frequency excess flux noise. The latter results facilitate to engineer the shape of magnetic flux noise spectra and thus to experimentally modify key properties such as coherence or measurement times of superconducting quantum devices. *Published by AIP Publishing.* [http://dx.doi.org/10.1063/1.4965293]

Low-frequency 1/*f* noise is a common phenomenon in solid state devices. Nevertheless, its underlying physics is often not well understood. This applies in particular, to superconducting quantum devices for which the origin of low-frequency noise is known for only a very few specific cases: In charge-sensitive devices such as charge qubits,[1] it arises from the stochastic hopping of electrons between neighboring trapping sites.[2,3] In Josephson junction based devices such as flux[4,5] or phase qubits[6] as well as superconducting quantum interference devices (SQUIDs),[7] low-frequency noise is caused by critical current fluctuations resulting from the trapping and subsequent release of electrons in the tunnel barrier.[8–11] Both, charge noise and critical current fluctuations, can be kept small by an optimization of the fabrication processes. And since their amplitude additionally decreases with temperature, both contributions hardly matter at very low temperatures in practice.[12,13] In contrast, low-frequency excess flux noise is the dominant low-frequency noise contribution for magnetic flux sensitive superconducting quantum devices at temperatures below 1 K. Its origin has been an unresolved puzzle since its first experimental observation about 30 years ago.[14,15] Its existence limits, for example, the coherence time of superconducting qubits[16–18] or makes high-precision measurements of low-frequency signals using SQUIDs challenging.

Recent experiments indicate that low-frequency excess flux noise[19] with power spectrum $S_{\Phi,1/f}(f) = S_{\Phi,1/f}(1\,\mathrm{Hz})/f^{\alpha}$ originates from the random reversal of interacting surface spins with an areal density of $\sigma \sim 5 \times 10^{17}\,\mathrm{m}^{-2}$.[20] The spins might result from metal-induced gap states,[21] dangling bonds,[22] adsorbed oxygen molecules,[23] or nuclear spins[24] and are most likely located at the metal-substrate or metal-surface oxide interface.[25] They might interact via the Ruderman-Kittel-Kasuya-Yoshida (RKKY)-interaction,[25] the dipole-dipole-interaction,[26] or hyperfine interactions.[27]

Anton *et al.* showed that the noise spectra of a device that were taken at different temperatures T cross each other all at a certain crossing frequency f_c.[28] This indicates a correlation between the noise amplitude $S_{\Phi,1/f}(f)$ and the noise exponent α that can be expressed by

$$S_{\Phi,1/f}(f) = S_{\Phi,1/f}(f_c) \times (f_c/f)^{\alpha}. \quad (1)$$

This correlation also applies when comparing the low-temperature values of the noise amplitude $S_{\Phi,1/f}(f)$ and the noise exponent α of different SQUIDs.[29] Anton *et al.* also pointed out that the dephasing times of flux qubits are very sensitive to the noise exponent α and that the coherence times may be increased substantially by raising α.[30] Similarly, the measurement time for SQUIDs might be significantly reduced by having the possibility to engineer α. For this, a detailed understanding of what sets α experimentally is required. However, neither a clear systematic material nor a geometry or device type dependence of the noise amplitude and noise exponent has been found so far.

Here, we present a comprehensive analysis of the low-frequency excess flux noise both in terms of magnetic flux noise $S_{\Phi,1/f}$ and energy sensitivity $\epsilon_{1/f}$ of superconducting quantum devices at temperatures T below 1 K. We include not only our own measurements but also a rather large data set published by several other groups. In contrast to most previous studies, we investigated more sophisticated devices such as *N*-SQUID series arrays having a more complex device geometry compared to the commonly studied devices with simple washer geometry. Our analysis includes 373 individual noise spectra taken at $T < 1$ K. We find that noise spectra of independent devices cross each other all at certain crossing frequencies f_c. Our analysis further reveals evidence for a material and device type dependence of low-frequency excess flux noise. In addition, we show that the noise exponent α of dc-SQUIDs covers different value ranges depending on the device material or type. Flux and phase qubits always feature noise spectra with $\alpha \sim 1$ and have significantly smaller noise amplitudes than SQUIDs with the same noise exponent.

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We measured the temperature dependence of magnetic flux noise of 21 dc-SQUIDs in a temperature range between 20 mK and 800 mK. The SQUIDs that we normally use to read out metallic magnetic calorimeters[31] include single second-order parallel gradiometers as well as N-SQUID series arrays consisting of N first-order series gradiometers. Mostly, SQUID arrays with 14 or 16 SQUID cells were used but to check a scaling with N we also measured a tiny number of smaller arrays with 2, 4, or 8 cells. Fifteen devices were fabricated in our cleanroom facilities. Design and performance of these SQUIDs are described in detail in Ref. 32. The other six devices were fabricated and provided by Physikalisch-Technische Bundesanstalt (PTB) Berlin.[33] Measurements were performed in a dilution refrigerator with a base temperature of 20 mK using an FFT spectrum analyzer to acquire noise spectra in the frequency range between 100 mHz and 100 kHz. Single-SQUIDs were operated with a voltage-bias in a two-stage SQUID setup. SQUID arrays were current-biased in a single-stage setup. All SQUIDs were shielded by soft-magnetic and superconducting material. For determining the intrinsic noise spectra of the sample SQUID for a given temperature, the noise contributions from the amplifier SQUID and the readout electronics were subtracted. The amplitude $S_{\Phi,1/f}(1\,\mathrm{Hz})$ and exponent α of the low-frequency excess flux noise were then determined by fitting the intrinsic noise spectrum with the equation

$$S_\Phi(f) = S_{\Phi,\mathrm{w}} + S_{\Phi,1/f}(1\,\mathrm{Hz}) \times (f/1\,\mathrm{Hz})^{-\alpha}. \quad (2)$$

Here, $S_{\Phi,\mathrm{w}}$ denotes the SQUID white noise contribution. It is worth mentioning that the intrinsic noise spectra nicely follow Eq. (2) in the frequency range between 100 mHz and 100 kHz, i.e., there is no change of noise exponent in the acquired frequency range (supplementary material) After fitting, flux noise was converted to energy sensitivity $\epsilon_{1/f}(f) = S_{\Phi,1/f}/2L$ with L being the device inductance. Besides our own measurements, we include data from 63 other devices that were studied by Drung *et al.*,[29] Anton *et al.*,[28,30] Wellstood *et al.*,[15,34] Lanting *et al.*,[35] Bialczak *et al.*,[18] Sank *et al.*,[36] and Harris *et al.*[37] using different experimental setups and techniques as described in detail the given references.

We analyzed low-frequency excess flux noise both in terms of magnetic flux noise $S_{\Phi,1/f}$ and energy sensitivity $\epsilon_{1/f}$. While the first approach is currently state-of-the-art, i.e., all present studies concerning low-frequency excess flux noise are using flux noise data, we surprisingly found some interesting correlations when analyzing our data in terms of energy sensitivity as we will show below. In addition, using energy sensitivity directly allows to compare single SQUIDs with SQUID arrays. This is very helpful when it comes to certain applications for which low-frequency noise always has to be compared to actual magnetic field sensitivity of the SQUID.

Figs. 1 and 2 show the low-frequency excess flux noise amplitudes in terms of magnetic flux noise $S_{\Phi,1/f}(1\,\mathrm{Hz})$ and energy sensitivity $\epsilon_{1/f}(1\,\mathrm{Hz})$ versus the noise exponents α of all single devices as well as of all SQUID arrays that we include in our analysis. The corresponding values were either determined by fitting our measured noise spectra according

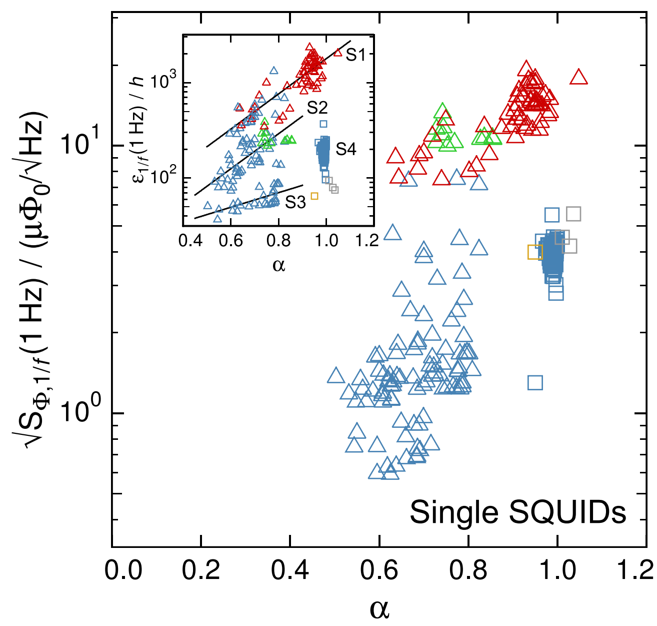

FIG. 1. Amplitudes of low-frequency excess flux noise expressed in terms of magnetic flux noise $S_{\Phi,1/f}(1\,\mathrm{Hz})$ or low-frequency energy sensitivity $\epsilon_{1/f}(1\,\mathrm{Hz})$ (inset) versus noise exponents α of single dc-SQUIDs (△) and flux or phase qubits (□). The devices were made of Nb (blue), PbIn (green), Al (yellow), 50% Nb, and 50% PbIn (red) or of unknown material (grey). Each point in this plot represents a single noise measurement of a sample device taken at a fixed temperature below 1 K. Most of the devices were measured at different temperatures. These measurements show up as individual data points without temperature or sample device encoding. Solid lines in the inset are fits to SQUID data according to Eq. (1). Data are either taken from own measurements or from literature.[15,18,28–30,34–37]

to Eq. (2) or were taken from the literature. Each point in these figures represents a single noise measurement of a sample device at given temperature T. Since we took noise measurements at different temperatures for each device, Figs. 1 and 2 include data for different devices and temperatures. However, we do not want to discuss the temperature dependence of the low-frequency noise contribution. Therefore, we summarize all data points in a single plot.

Though the devices were fabricated in various places using independent source materials and fabrication equipment and have quite different designs, they show a universal behavior and agree qualitatively and quantitatively with each other. Using energy sensitivity to describe the noise amplitude, all data points can be sorted into several groups each showing a correlation between the noise amplitude $\epsilon_{1/f}(1\,\mathrm{Hz})$ and the noise exponent α. Although this arrangement in groups might look arbitrary, it is triggered by the temperature dependence of the corresponding parameters for a certain device. More precisely, when we plot temperature dependent values of $\epsilon_{1/f}(1\,\mathrm{Hz})$ versus the corresponding α values for a certain device, the points are aligned along the corresponding solid lines in Fig. 1 and 2. It is interesting to note that these correlations do not show up when analyzing the data in terms of magnetic flux noise. Though we cannot explain at the moment why this is the case, we think that these observations point towards an universal behavior of low-frequency excess flux noise and are therefore worth mentioning to probably give some new input for future theoretical descriptions. It is further worth mentioning that the existence of a certain number of groups does not necessarily

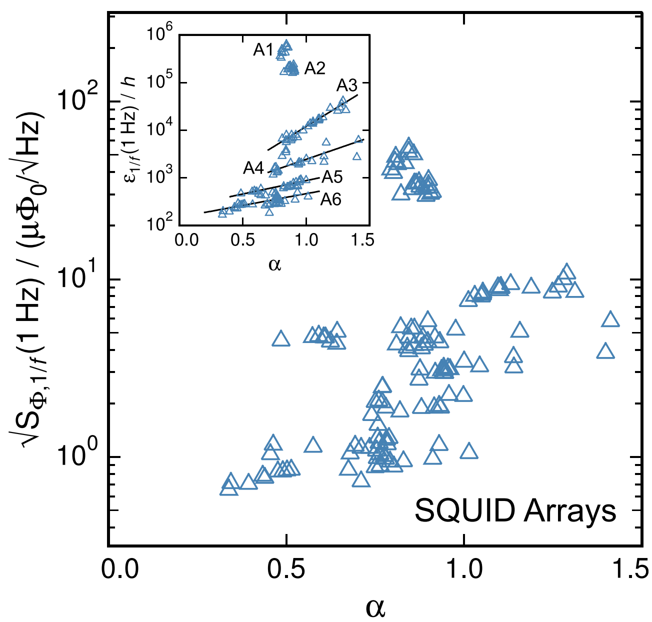

FIG. 2. Amplitudes of low-frequency excess flux noise expressed as magnetic flux noise $S_{\Phi,1/f}(1\,\mathrm{Hz})$ or low-frequency energy sensitivity $\epsilon_{1/f}(1\,\mathrm{Hz})$ (inset) versus noise exponents α of N-SQUID series arrays made of Nb. Each point in this plot represents a single noise measurement of a sample device taken at a fixed temperature below 1 K. Most of the devices were measured at different temperatures. These measurements show up as individual data points without temperature or sample device encoding. Solid lines in the inset are fits to data according to Eq. (1). Due to the limited number of data points in a quite small α range, the fits for groups A1 and A2 are not well-defined and have large error bars. They are therefore omitted in the further analysis. Data are taken either from own measurements or from literature.[29]

have to have a physical meaning. When looking at an even larger data set, it might turn out that either more groups exist or that the arrangement of the data is completely washed out. The correlation between $\epsilon_{1/f}(1\,\mathrm{Hz})$ and α can be described by Eq. (1). This indicates that not only the noise spectra of a certain device but also the noise spectra of completely different devices intersect each other at a crossing frequency f_c. When taking into account that the data points within a group are not perfectly following Eq. (1), which is indicated as a straight line in the plots, this correlation can also be interpreted as the appearance of a crossing band as predicted by Lanting *et al.*[35]

For the further discussion, we first want to concentrate on the difference between the noise properties of Nb-based single SQUIDs and Nb-based N-SQUID series arrays and its implications for actual measurements. It is obvious from Table I that single SQUIDs behave systematically different than N-SQUID series arrays. Single SQUIDs have noise exponents $0.50 \leq \alpha \leq 0.82$ with a median of 0.74 while SQUID arrays show $0.34 \leq \alpha \leq 1.41$ with a median of 0.84 (remember that these ranges include data in a temperature range from 20 mK to 800 mK). In addition, when looking at energy sensitivity, the noise amplitudes $\epsilon_{1/f}(f_c)$ of SQUID arrays are about a factor of ten higher as compared to single SQUIDs. Due to the different noise exponent, the corner frequency of SQUID arrays at which the low-frequency excess noise intersects the white noise contribution is much lower. This directly implies that SQUID arrays are more suited for measurements in the frequency range of some tens of hertz

TABLE I. Summary of the number of SQUIDs N_{SQ}, the number of spectra N_{spec}, crossing frequency f_c as well as the noise amplitude $\epsilon_{1/f}(f_c)$ for groups S1–S3 and A3–A7 in Figs. 1 and 2. For determining f_c and $\epsilon_{1/f}(f_c)$, the N_{spec} spectra acquired from N_{SQ} different SQUIDs at various temperatures were fitted according to Eq. (2).

Group	N_{SQ}	N_{spec}	f_c/Hz	$\epsilon_{1/f}(f_c)/h$
S1	14	70	68.8 ± 18.4	25.5 ± 5.7
S2	15	58	67.9 ± 36.7	10.0 ± 3.8
S3	7	28	5.8 ± 2.1	17.2 ± 4.5
A3	6	30	44.2 ± 7.1	269.0 ± 45.2
A4	6	17	8.3 ± 2.7	294.8 ± 89.5
A5	3	26	3.7 ± 0.5	237.8 ± 26.4
A6	5	37	3.1 ± 0.6	148.1 ± 19.4

or above, while the use of single SQUIDs is advantageous for measurements in the range of some hertz or below. This argument holds at least down to a frequency of 100 mHz, which was the lowest frequency of our noise spectra. However, this frequency range can likely be extended by future measurements.

The reason for the different behavior of single SQUIDs and SQUID arrays is not yet clear. One might argue that the noise spectrum of SQUID arrays is the incoherent sum of N noise spectra of single SQUID cells that add up to a spectrum with a higher noise exponent. However, since we observe similar noise exponents for short arrays with 2 or 4 SQUID cells as compared to long arrays with 14 or 16 cells, we have not yet been able to find a distribution of noise amplitudes and noise exponents that are consistent with single SQUID data but that allows at the same time adding up noise spectra to a smooth noise spectrum consistent with our SQUID array data. Alternatively, there might be some kind of interaction between different SQUID cells, e.g., mediated by high-frequency Josephson currents, which increases the noise exponent. Although the SQUID arrays are magnetically well-screened during the measurements, they might be disturbed by external background fields. SQUID arrays are in general more sensitive to spatial inhomogeneities of residual fields due to their rather long extension on the chip.

Besides the difference between single SQUIDs and SQUID arrays, Fig. 1 gives evidence for a material and device type dependence of low-frequency excess flux noise. It is obvious that flux or phase qubits (group S4) show a different behavior than dc-SQUIDs (groups S1–S3 and A1–A6). In particular, flux or phase qubits always show noise exponents $\alpha \simeq 1$ and have systematically lower noise amplitudes than dc-SQUIDs with the same noise exponent. This device type dependence not only implies that studies on low-frequency excess flux noise have to be carried out on that kind of device that one is interested in and that experimental findings cannot be transferred between different kinds of devices. It also gives another possibility to engineer the shape of the flux noise spectra.

Finally, our analysis reveals a weak material dependence of low-frequency excess flux noise for SQUIDs. Fig. 1 shows that Nb-based SQUIDs have noise exponents $0.50 \leq \alpha \leq 0.82$ while PbIn-based SQUIDs show $0.73 \leq \alpha \leq 0.86$. In contrast, "mixed" SQUIDs having SQUID loops consisting half of Nb and half of PbIn show $0.64 \leq \alpha \leq 1.05$ and tend to show the highest noise amplitudes compared to the other materials. While a material dependence of low-frequency noise has already been observed for other noise sources, e.g., frequency noise in superconducting microwave resonators (see, for example,[38]), such a material dependence has not been reported for SQUIDs so far. Therefore, this might be a starting point for a more detailed understanding of the origin of surface spins or their mutual interaction. However, taking into account that Nb-based qubits show completely different noise amplitudes and exponents than Nb-based SQUIDs, it is obvious that also the device type or the kind of noise measurement has a great influence on the noise properties. Thus, they allow for engineering the low-frequency part of the flux noise spectra of superconducting quantum devices by a proper choice of the device material and device type.

The reason for the observed dependencies has neither yet been understood nor is predicted by any available theory. One might argue, for example, that the material dependence arises from the difference of adsorption potentials of oxygen molecules or other adsorbates on the SQUID surface[23] or that the coupling between spins due to hyperfine interactions changes with the SQUID material.[27] However, both theories do not predict a material dependence of the noise exponent α. Therefore, further studies of the noise properties of superconducting quantum devices made of different materials and varying device geometries are required. When performing these measurements, one should take into account not only the most simple devices such as simple washer SQUIDs but should also look at more sophisticated devices that might reveal up to now unknown features of low-frequency excess flux noise.

In conclusion, we performed a comprehensive analysis of low-frequency excess flux noise both in terms of magnetic flux noise $S_{\Phi,1/f}$ and energy sensitivity $\epsilon_{1/f}$ of superconducting quantum devices at temperatures below 1 K. Our analysis revealed a universal behavior of low-frequency excess flux noise, i.e., devices fabricated in various places using different source materials and fabrication equipment show qualitatively and quantitatively the same results. When analyzing data in terms of $\epsilon_{1/f}$, we found that noise spectra of independent devices cross each other all at certain crossing frequencies f_c. Besides this main result, we showed that SQUID arrays systematically feature higher noise exponents compared to single SQUIDs and give evidence for a material and device type dependence of low-frequency excess flux noise. We particularly showed that flux or phase qubits exhibit always noise exponents $\alpha \simeq 1$ independent of the device material and feature lower noise amplitudes compared to SQUIDs with the same noise exponent. Our observations somehow allow to engineer the low-frequency flux noise of superconducting quantum devices with respect to a certain application.

See supplementary material for several example noise spectra.

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