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# 1. Theoretical Background

This chapter provides a short introduction into Josephson junctions and their role in dc-SQUIDs (**d**irect **c**urrent **S**uperconducting **Q**Uantum **I**nterference **D**evice), which will be the main focus of this thesis. We start with a brief overview on macroscopic quantum phenomena such as the Josephson effect and explain the general working principle of superconductor-isolator-superconductor (SIS) tunnel contacts, followed by a summary of their basic properties. These tunnel contacts form the theoretical framework to describe the SQUIDs, which are developed in this group and optimized within the scope of this thesis. Lastly, we will take a closer look into their parasitic resonance behavior and investigate chapter different methods to reduce the quality factors. We will closely follow the derivations from the textbooks [Cla04] and [Gro16].

## 1.1 Josephson junctions

The *Josephson junction* named after Brian D. Josephson consists of two identical superconductors weakly coupled to each other. In the case of the junctions produced in this working group, such coupling is realized through a few nm thin insulating layer between the superconducting electrodes. Consequently, they are also referred to as SIS junctions. The resulting trilayer structure typically consists of Nb/Al-AlO<sub>x</sub>/Nb, with niobium being used for the superconductors and the insulating layer being provided by the aluminum oxide. Both superconductors are attributed with a macroscopic quantum mechanical phase  $\theta_1$  and  $\theta_2$ , respectively. The macroscopic nature of these phases will be explained in subsection 1.1.1. A schematic structure is shown in figure 1.1. By connecting the tunnel junction to a current source they exhibit a non-trivial current-voltage behavior, which will be covered in the following.

### 1.1.1 Josephson effect

According to the BCS theory developed by Bardeen, Cooper and Schrieffer in 1957 [Bar57], electrons in a superconductor form pairs below a material dependent critical temperature  $T_c$ . These composite particles are also referred to as *Cooper pairs* and they represent the superconducting charge carriers with twice the mass and charge of a single electron. Their dissipationless flow causes the current to have zero dc-resistance, which is alongside the Meissner-Ochsenfeld effect [Mei33] the most characteristic feature of a superconductor. The latter describes magnetic field expulsion at the temperature  $T_c$ , provided the external magnetic field is smaller than a

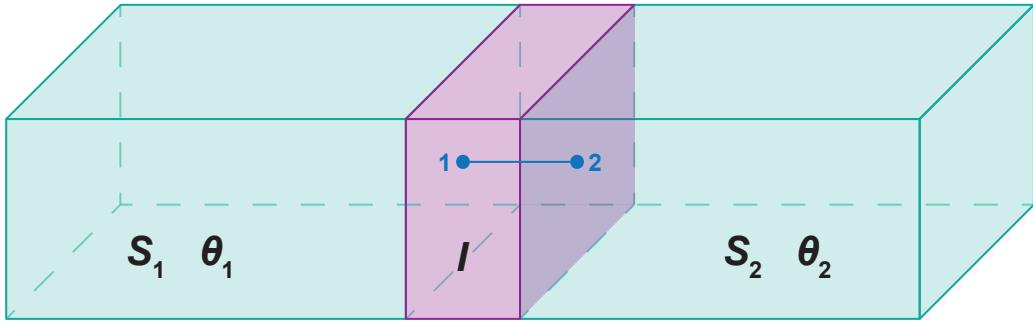


Figure 1.1: Schematic of a Josephson (SIS) junction. Both superconducting electrodes  $S_1$  and  $S_2$  are weakly coupled with each other through a thin tunnel barrier  $I$ .  $\theta_1$  and  $\theta_2$  represent the macroscopic phases of each superconductor.

critical field  $B_c$ . Further details on the microscopic theory of superconductivity can be found in [Bar57] and [Gin50].

For temperatures  $T < T_c$  ( $T_c = 9.3$  K [Ina80]) Cooper pairs are able to tunnel across a Nb/Al-AlO<sub>x</sub>/Nb Josephson junction despite the insulating barrier. In fact, their tunneling behavior resembles that of a single electron, which can be motivated by the *Macroscopic Quantum Model* that was formulated by Fritz London in 1953. This model heavily focuses on the quantum mechanical phase  $\theta$  of a Cooper pair, whose macroscopic nature can be understood by the following arguments. On one hand, the distance between both electrons in a Cooper pair is approximately 10 to 1000 nm which is significantly larger than the spacing between Cooper pairs, resulting in strongly overlapping wave functions. On the other hand, Cooper pairs have to obey Bose-Einstein statistics due to their total spin of 0. Thus, all Cooper pairs share the same ground state, and as a consequence, the energies and temporal evolutions of the phases are equal. These two effects lead to what is known as *phase-lock* [Gro16]. The phases of neighboring pairs synchronize such that this quantum mechanical property now holds on a macroscopic scale. This gives rise to the phase-dependent macroscopic wave function

$$\Psi(\mathbf{r}, t) = \Psi_0(\mathbf{r}, t)e^{i\theta(\mathbf{r}, t)} , \quad (1.1)$$

which describes all charge carriers of a bulk superconductor. Here, the charge carrier density is given by  $|\Psi_0(\mathbf{r}, t)|^2 = n_s$ . The phase of the Cooper pair ensemble depends on the time  $t$  and the position  $\mathbf{r}$ . As a result of sharing the same phase, both electrons of a Cooper pair consequently possess the same tunneling probability as an individual electron, enabling the supercurrent. This coherence phenomenon is referred to as the *Josephson effect* [Jos62]. Another significant consequence of the

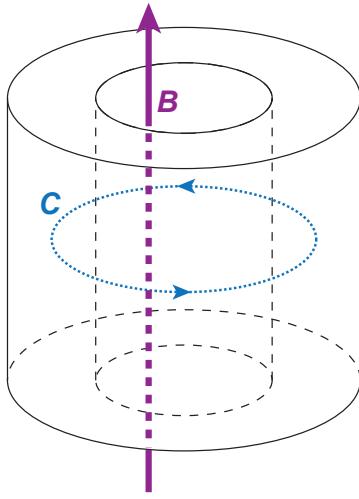


Figure 1.2: Superconducting ring-shaped cylinder threaded by an external magnetic field. By applying the field at low temperatures, shielding currents arise to expel the field from the superconductor. Upon turning off the external field the shielding currents will remain due to the lack of resistance, causing magnetic flux to be trapped. The dotted blue path  $C$  is situated at the center of the cylinder wall, which we assume to be current-free due to the London penetration depth  $\lambda_L$  being much smaller than the thickness of the cylinder wall.

macroscopic quantum model is flux quantization.

This phenomenon can be explained through the capture of an external magnetic flux within a superconducting cylinder (see figure 1.2). The wave function must remain unchanged after circumnavigating the cylinder due to  $e^{i\theta} = e^{i\theta+2\pi n}$ . As a result, upon integrating along the current-free center of the cylinder wall (path  $C$ ), the following equation holds for the captured flux [Dea61]

$$\Phi = \frac{h}{q_s} n = \frac{h}{2e} n \equiv \Phi_0 n . \quad (1.2)$$

Here,  $n \in \mathbb{Z}$  and  $\Phi_0 = 2.07 \times 10^{-15} \text{ T m}^2$  [Tie21] represents the so-called magnetic flux quantum. The captured flux is thus quantized, a consequence solely arising from the macroscopic nature of the phase. This quantity plays a crucial role in the theoretical description of Josephson junctions.

The current and voltage behavior in a SIS junction is described by the *Josephson equations*. Crucial to this description is a critical current  $I_c$  that is linearly proportional to the applied current  $I$ , which marks the boundary between two operational modes; the zero-voltage state and the voltage state. Additionally, due to the macroscopic nature of the phase,  $I$  oscillates with the gauge-invariant phase difference  $\varphi$ , leading to the **first Josephson equation** [Jos65]

$$I_s = I_c \sin(\varphi) . \quad (1.3)$$

The critical current  $I_c$  is proportional to the coupling strength  $\kappa$ , which describes the overlap of the wave functions  $\Psi_1$  and  $\Psi_2$  within the insulating layer. The relationship is given by

$$I_c = \frac{4e\kappa V n_s}{\hbar} , \quad (1.4)$$

where  $V$  represents the volume of the superconducting electrode and  $e$  denotes the elementary charge of an electron. We assume that the Cooper pair density  $n_s$  of the two superconductors  $S_1$  and  $S_2$  is identical, meaning  $n_{s1} = n_{s2} = n_s$ .

The gauge-invariant phase difference refers to the phases  $\theta_1$  and  $\theta_2$  of the respective electrodes at the boundary of the insulating layer (positions 1 and 2, see figure 1.1). Taking into account possible external electromagnetic fields within the barrier, the general form using the vector potential  $\mathbf{A}$  is given by [Gro16]

$$\varphi(\mathbf{r}, t) = \theta_2(\mathbf{r}, t) - \theta_1(\mathbf{r}, t) - \frac{2\pi}{\Phi_0} \int_1^2 \mathbf{A}(\mathbf{r}, t) \cdot d\mathbf{l} . \quad (1.5)$$

Assuming a constant supercurrent density  $J_s$  across the junction, taking the time derivative of equation (1.5) yields the **second Josephson equation** [Jos65]

$$\frac{\partial \varphi}{\partial t} = \frac{2\pi}{\Phi_0} V . \quad (1.6)$$

The first of the two above-mentioned operating modes describes the zero-voltage state, i.e.  $I < I_c$  (zero-voltage state). Here, the entire injected current is carried by Cooper pairs, so  $I = I_s = \text{const}$ . As a result,  $\varphi$  is constant over time, which, according to equation (1.6), leads to  $V = 0$ . This is known as the *dc Josephson effect*.

For  $I > I_c$  however, Cooper pairs begin to break up such that a portion of the current needs to be carried by quasiparticles, which will then lead to a voltage drop  $V$ . According to the second Josephson equation, the phase  $\varphi$  becomes time dependent, and after integration one obtains

$$\varphi = \frac{2\pi}{\Phi_0} V t + \varphi_0 = w_J t + \varphi_0 \quad \text{with} \quad w_J = \frac{2\pi}{\Phi_0} V . \quad (1.7)$$

Thus, if we insert equation (1.7) into equation (1.3), we observe that the current  $I_s$  oscillates with the *Josephson frequency*  $\frac{f_J}{V} = \frac{w_J}{2\pi V} = \frac{1}{\Phi_0} \approx 483.6 \frac{\text{MHz}}{\mu\text{V}}$ . Accordingly, this phenomenon is referred to as the *ac Josephson effect*.

### 1.1.2 Josephson Junctions in a Magnetic Field

To motivate the structure of a dc-SQUID, it is essential to first investigate the current behavior of an extended Josephson junction in the presence of an external magnetic field. So far, all previous formulae apply for point-like junctions, assuming a spatially constant phase difference  $\varphi$  and Josephson current density  $J_s$  across the junction area. This is not the case for three-dimensional (extended) junctions with a length  $L$  and width  $W$ . The *Josephson penetration depth*  $\lambda_J$  is a quantity used to classify an extended junction as short ( $W, L \leq \lambda_J$ ) or long ( $W, L \geq \lambda_J$ ) and is defined as

$$\lambda_J = \sqrt{\frac{\Phi_0}{2\pi\mu_0 J_{\text{c}} t_B}} . \quad (1.8)$$

Here, the magnetic thickness is defined as  $t_B = d + \lambda_{L,1} + \lambda_{L,2}$ , where  $d$  is the geometric thickness of the isolator. It describes how far an external magnetic field penetrates

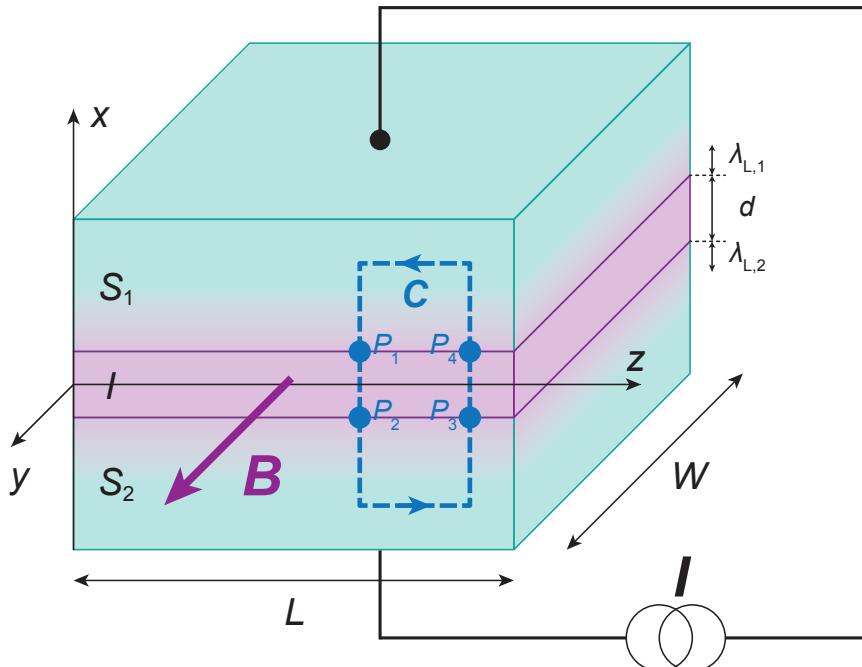


Figure 1.3: Short Josephson junction connected to a current source in the presence of an external  $B$ -field in  $y$ -direction, parallel to the junction area. Inside the electrodes the magnetic field decays exponentially according to the London penetration depths  $\lambda_{L,1}$  and  $\lambda_{L,2}$ , visually shown by the purple color gradient. The closed contour  $C$  is used to derive expressions for the spatially dependent phase difference  $\varphi$  and current density  $J_s$ .

both superconducting electrodes if applied parallel to the junction area, as depicted in figure 1.3. The respective London penetration depths are  $\lambda_{L,1}$  and  $\lambda_{L,2}$  and  $J_c = \frac{I_c}{WL}$  is the critical current density. This distinction is needed to determine whether the magnetic self-field generated by the supercurrent is negligible in comparison to the external field (short junctions) or not (long junctions). Within the scope of this thesis, we only use short junctions.

To analyze the current and phase distribution of such a junction we consider the setup shown in figure 1.3. A short junction is connected to a current source and is penetrated by an external B-field in y-direction, parallel to the junction area. Now, obtaining an expression for the phase requires a similar approach as the calculation for the quantized flux, where we assumed that the phase changes by  $2\pi n$  around a closed loop. Here, we again integrate over a closed contour  $C$ , with the points  $P_1 - P_4$  marking the transitions between superconductor and isolator. Using equation 1.5, we find

$$\frac{\partial \varphi}{\partial z} = \frac{2\pi}{\Phi_0} B_y t_B \quad \text{and} \quad \frac{\partial \varphi}{\partial y} = -\frac{2\pi}{\Phi_0} B_z t_B . \quad (1.9)$$

In this experiment, however, the magnetic field points in y-direction only, meaning  $\varphi$  will only vary along the z-axis. Integrating the first expression in equation 1.9 then leads to

$$\varphi(z) = \frac{2\pi}{\Phi_0} B_y t_B z + \varphi_0 . \quad (1.10)$$

Here, the integration constant  $\varphi_0$  represents the phase difference for the case  $z = 0$ . Inserting equation 1.10 into the first Josephson equation and using  $J_s = \frac{I_s}{WL}$  gives

$$J_s(y, z, t) = J_c(y, z) \sin(kz + \varphi_0) \quad \text{with} \quad k = \frac{2\pi}{\Phi_0} B_y t_B . \quad (1.11)$$

If we now assume the critical current density  $J_c$  to be constant across the junction area, we can integrate equation 1.11 to get a flux-dependent maximum Josephson current

$$I_s^m(\Phi) = I_c \left| \frac{\sin\left(\frac{kL}{2}\right)}{\frac{kL}{2}} \right| = I_c \left| \frac{\sin\left(\frac{\pi\Phi}{\Phi_0}\right)}{\frac{\pi\Phi}{\Phi_0}} \right| , \quad (1.12)$$

where  $\Phi = B_y t_B L$  is the total flux threading through the junction. This expression describes the so-called Fraunhofer diffraction pattern, shown in figure 1.4. The result resembles the single slit experiment, where the same pattern is found for the light

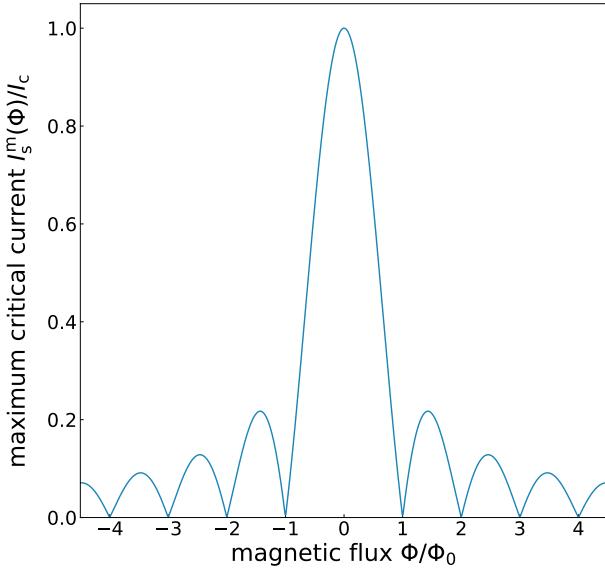


Figure 1.4: Normalized flux-dependent maximum Josephson current  $I_s^m(\Phi)$  showing a Fraunhofer pattern. It modulates with the flux quantum  $\Phi_0$ , peaking at  $\Phi = 0$  with subsequent maxima at  $\Phi = \pm(\frac{3}{2} + n)\Phi_0$  with  $n \in \mathbb{N}_0$ . For  $\Phi = \pm(n + 1)\Phi_0$  the total net current is zero.

intensity behind the slit. Here, the analogy works by considering the integral of the critical current density  $J_c$  as a transmission function which is constant inside the junction and zero outside.

### 1.1.3 RCSJ Model

The Fraunhofer pattern describes the flux-dependent current for the case of  $I < I_c$ , staying in the so-called zero-voltage state. In this regime, only the dc Josephson effect applies as discussed in subsection 1.1.1. Switching to the voltage stage, i.e.  $I > I_c$ , Cooper pairs start breaking up into quasiparticles if the electric energy  $eV$  exceeds the sum of both electrodes' gap energies  $\Delta_1(T) + \Delta_2(T)$  [Bar57]. Consequently, at the *gap-voltage*

$$V_g = \frac{\Delta_1(T) + \Delta_2(T)}{e} \quad (1.13)$$

quasiparticles start to cross the tunnel barrier resulting in a steep rise of a resistive normal current  $I_n$ . This process also occurs at finite temperatures for  $k_B T > \Delta_1(T) + \Delta_2(T)$ , leading to a reduction of  $I_c$  as well as  $V_g$ . Under a dc current source, the condition  $I = I_s + I_n$  must be constantly fulfilled. This results in an oscillating normal current and therefore voltage, since  $I_s$  oscillates with  $f_J$  according to the ac Josephson effect. According to the second Josephson equation (1.6) the oscillating voltage thus causes the term  $\frac{d\varphi}{dt}$  to vary sinusoidally, causing both  $I_s$  and  $I_n$  and

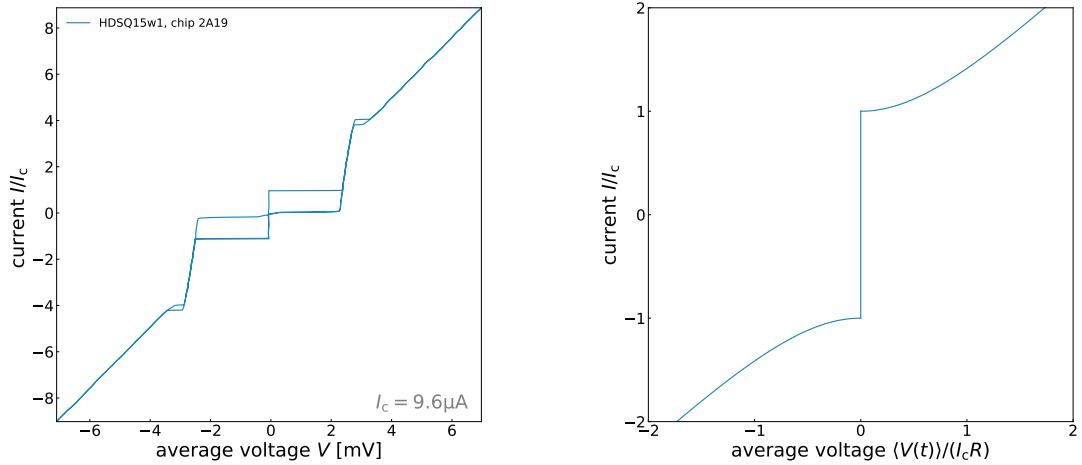


Figure 1.5: Left: Measured IVC of an underdamped junction manufactured in this working group, showing the typical hysteresis. Right: Theoretical IVC of an over-damped junction with a current-voltage shape that is independent of the current sweep direction.

in turn the resulting voltage to oscillate in a complex manner. As a voltage with such a high frequency cannot be measured, only the time-averaged voltage will be considered in the following discussion.

Now, further increasing the energy of the quasiparticles ( $T > T_c$  and/or  $V > V_g$ ) leads to a transition into normal-conducting electrons, which exhibit an ohmic dependence. This behavior can be seen in the typical current-voltage-characteristic (IVC) depicted in figure 1.5.

For real junctions, however, one needs to take into account that they are comprised of two electrodes separated by a thin insulating layer, which represent a parallel plate capacitor with the Al-AlO<sub>x</sub> layer being the dielectric material. Therefore, a junction capacitance  $C$  needs to be taken into account. A displacement current  $I_d$  will flow as a consequence, given we are in the voltage state. Lastly, thermal and 1/f noise cause a small fluctuating current  $I_f$ . All these current channels were defined in the so-called Resistively and Capacitively Shunted Junction (RCSJ) model [McC68], [Ste68], which models the total current of a lumped (0-dimensional) junction to a sufficiently high accuracy. A schematic of an effective circuit diagram is shown in figure 1.6 (left). Combining every current channel leads to the *Basic Junction Equation*, which is defined as [Gro16]

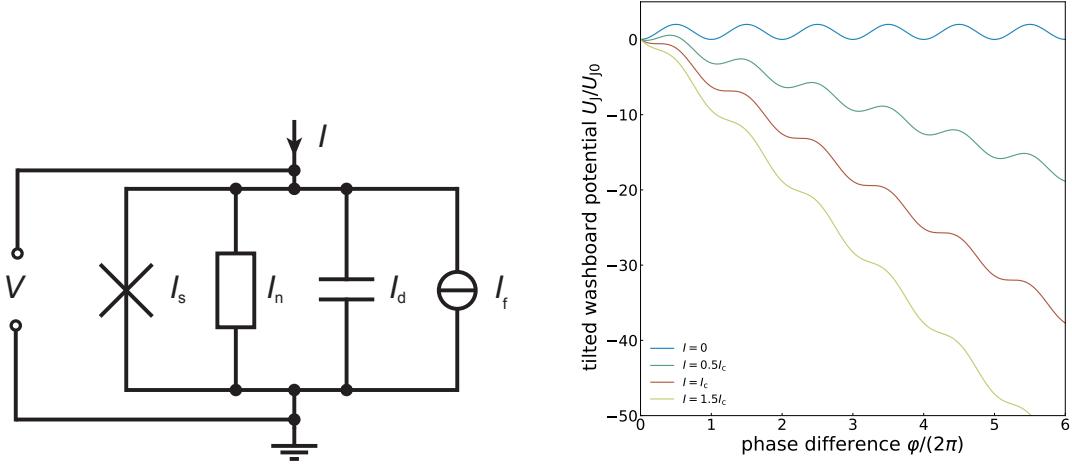


Figure 1.6: Left: Schematic circuit of a lumped Josephson junction with all four current channels connected in parallel, according to the RCSJ model. The junction is represented by the cross symbol on the left, marking the supercurrent  $I_s$ . The normal current  $I_n$  is realized with a resistance  $R$ , while the displacement current  $I_d$  and the noise  $I_f$  are attributed to a capacitor  $C$  and a current source, respectively. Right: Tilted washboard potential for different currents, ranging from 0 to  $1.5I_c$ . The tilt increases with the injected current  $I$ .

$$I = I_s + I_n + I_d + I_f = I_c \sin(\varphi) + \frac{1}{R(V)} \frac{\Phi_0}{2\pi} \frac{d\varphi}{dt} + C \frac{\Phi_0}{2\pi} \frac{d^2\varphi}{dt^2} + I_f . \quad (1.14)$$

By defining the Josephson coupling energy  $U_{j0} = \frac{\hbar I_c}{2e}$  and the normalized currents  $i = \frac{I}{I_c}$  and  $i_f(t) = \frac{I_f(t)}{I_c}$ , equation 1.14 can be rewritten to

$$\left( \frac{\hbar}{2e} \right)^2 C \frac{d^2\varphi}{dt^2} + \left( \frac{\hbar}{2e} \right)^2 \frac{1}{R(V)} \frac{d\varphi}{dt} + \frac{d}{d\varphi} \{ U_{j0} [1 - \cos \varphi - i\varphi + i_f(t)\varphi] \} = 0 . \quad (1.15)$$

The expression in the curly brackets represents the potential energy in the system  $U_J$ , allowing equation 1.14 to be compared to

$$M \frac{d^2x}{dt^2} + \eta \frac{dx}{dt} + \nabla U = 0 , \quad (1.16)$$

which describes a particle with mass  $M$  and damping  $\eta$  moving inside the potential  $U$ . This mechanical analogue therefore allows us to interpret a *phase particle*, where

it's motion corresponds to a change of the gauge-invariant phase difference  $\varphi$  within a potential  $U_J$  [Cla04]. Consequently, it is attributed with a mass  $M = \left(\frac{\hbar}{2e}\right)^2 C$  and damping  $\eta = \left(\frac{\hbar}{2e}\right)^2 \frac{1}{R(V)}$ . Figure 1.6 (right) visualizes how this phase particle behaves for different currents  $I$ . Given the shape of  $U_J(\varphi)$ , the potential is referred to as the *tilted washboard potential*.

For  $I = 0$ , the phase particle will remain within one of the potential minima. As the current increases, however, the potential starts to tilt such that the depth of the minima reduces until it vanishes for  $I = I_c$ , thus becoming a saddle point. Up until this point, the phase particle can't overcome the potential barrier to move downward, which agrees with the second Josephson equation as the phase difference  $\varphi$  should remain constant on average for  $I < I_c$ . Further increasing the current and therefore the tilt of the potential causes the phase particle to fall along the potential, resulting in a voltage drop across the junction ( $\frac{\partial\varphi}{\partial t} > 0$ ).

Reversing the current sweep showcases the importance of the particle's mass  $M$  and damping  $\eta$ , as they determine if the return path equals the current shape described above or not. For the case of a small mass (small  $C$ ) and large damping (small  $R$ ), the phase particle will, due to a lack of momentum, come to a halt as soon as minima reappear in the washboard potential by reducing the current below  $I_c$ . The current path will therefore remain unchanged as  $I$  is reduced back to 0, as shown in figure 1.5 (right). Such a junction is consequently called an *overdamped* junction.

The other case describes an *underdamped* junction (figure 1.5 (left)) and involves a large mass (large  $C$ ) and small damping (large  $R$ ). This allows the phase particle to continue to move downward as it now carries enough momentum to overcome the arising maxima and minima. The finite voltage drop despite the current being below  $I_c$  is displayed as the steep quasiparticle current curve, which ends with a return current  $I_R$  that arises with the recapture of the particle in a minimum. This leads to a hysteretic IVC, as depicted in figure 1.5 (left).  $I_R$  can be calculated via [Lik86]

$$I_R = \frac{4}{\pi\sqrt{\beta_C}} I_c , \quad (1.17)$$

with  $\beta_C$  being the dimensionless Stewart-McCumber parameter, that is used to quantitatively distinguish between both junction types. It is given by

$$\beta_C = \frac{2\pi}{\Phi_0} I_c R^2 C \quad (1.18)$$

with  $\beta_C \gg 1$  corresponding to a strongly underdamped junction, whereas  $\beta_C \ll 1$  represents a strongly overdamped junction. The junctions developed and produced within the scope of this thesis aim to be overdamped, which is why we take a closer

look on the time-averaged voltage for  $I > I_c$  in the case of  $\beta_C \ll 1$ . Neglecting the noise in equation 1.15, as well as assuming the resistance to be linear below and above the gap voltage  $V_g$ , i.e.  $R(V) = R$ , the time-averaged voltage can be derived to [Cla04]

$$\langle V(t) \rangle = I_c R \sqrt{\left(\frac{I}{I_c}\right)^2 - 1} \quad \text{for} \quad \frac{I}{I_c} > 1 . \quad (1.19)$$

This equation will be crucial to determine the voltage drop of a dc-SQUID, as its derivation is analogous to that of a single junction, which will be covered in the next section.

## 1.2 dc-SQUIDS

We have now covered the theoretical framework necessary to understand the working principle of a dc-SQUID, which consists of a superconducting ring intersected by two identical Josephson junctions with critical Josephson currents  $I_c$ , as depicted in figure 1.7. Both junctions are shunted with shunt resistors  $R_s$  to avoid hysteretic behavior in the respective IVCs. If the loop is then biased with a bias current  $I_b$  while being threaded by an external magnetic flux  $\Phi_e$ , it is possible to convert small flux variations into a measurable voltage change. dc-SQUIDs are therefore used as highly sensitive flux-to-voltage transducers.

### 1.2.1 Zero Voltage State

In order to fully understand the working principle of a dc-SQUID it is again necessary to first cover the zero voltage stage as we did for a single junction. The parallel connection of the two junctions allows the bias current to split into two supercurrents  $I_{s1}, I_{s2}$  with identical critical currents, i.e.  $I_{c,1} = I_{c,2} = I_c$ . Here we assume  $I_b < 2I_c$  to ensure that no voltage drop across both junctions occurs ( $V_s = 0$ ). Applying Kirchhoff's law we then obtain the following expression

$$I_b = I_s = I_c \sin \varphi_1 + I_c \sin \varphi_2 = 2I_c \cos\left(\frac{\varphi_1 - \varphi_2}{2}\right) \sin\left(\frac{\varphi_1 + \varphi_2}{2}\right) . \quad (1.20)$$

In chapter 1.1.2 we concluded that a magnetic flux  $\Phi$  causes the supercurrent to modulate with  $\Phi_0$ . A dc-SQUID can be considered as a single junction with a much larger effective area  $A_{\text{eff}}$  (loop area), that an external magnetic flux can penetrate. It is therefore reasonable to expect a similar behavior for a dc-SQUID. The same approach as with a single junction is used to determine the flux dependency of the

total supercurrent, where a closed loop integral is performed around the SQUID loop. The calculation leads to the relation [Gro16]

$$\varphi_2 - \varphi_1 = \frac{2\pi\Phi}{\Phi_0} , \quad (1.21)$$

which can be directly inserted into equation 1.20 to obtain

$$I_s = 2I_c \cos\left(\pi \frac{\Phi}{\Phi_0}\right) \sin\left(\varphi_1 + \pi \frac{\Phi}{\Phi_0}\right) . \quad (1.22)$$

In the most general case, however, one needs to take into account the inductance  $L_s$  of the SQUID loop and therefore a circulating current  $I_{\text{cir}} = \frac{I_{s1}-I_{s2}}{2}$  that induces the additional flux  $\Phi_{\text{cir}} = L_s I_{\text{cir}}$ . With the external flux  $\Phi_e$  we can thus write for the total flux

$$\Phi = \Phi_e + \Phi_{\text{cir}} \quad (1.23)$$

$$= \Phi_e - L_s I_c \sin\left(\pi \frac{\Phi}{\Phi_0}\right) \cos\left(\varphi_1 + \pi \frac{\Phi}{\Phi_0}\right) \quad (1.24)$$

$$= \Phi_e - \frac{1}{2} \beta_L \Phi_0 \sin\left(\pi \frac{\Phi}{\Phi_0}\right) \cos\left(\varphi_1 + \pi \frac{\Phi}{\Phi_0}\right) . \quad (1.25)$$

Here, we introduced the dimensionless screening parameter  $\beta_L = \frac{2L_s I_c}{\Phi_0}$ , which relates the maximum possible flux  $\Phi_{\text{cir}}^{\max} = L_s I_{\text{cir}}^{\max} = L_s I_c$  produced by screening currents to  $\frac{\Phi_0}{2}$ . This quantity describes the influence the screening currents have on the total flux  $\Phi$ , which in turn affects  $I_s$  in equation 1.22. We will now simplify the expression above by considering the limiting case for small currents, i.e.  $I_s \ll 2I_c$ .

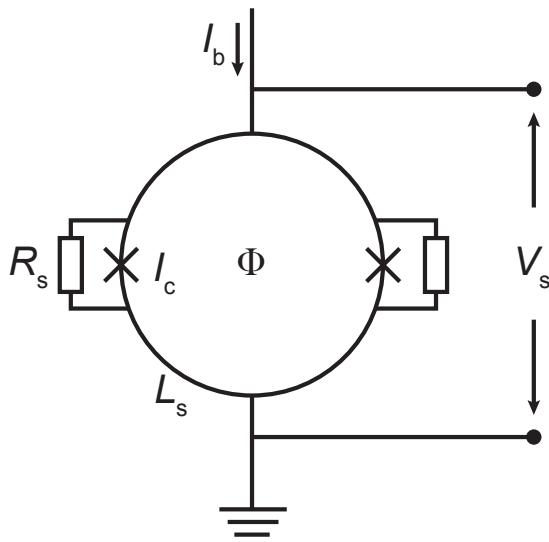


Figure 1.7: Schematic circuit diagram of a shunted dc-SQUID. A superconducting loop with inductance  $L_s$  is interrupted by two lumped Josephson junctions such that they form a parallel connection. Operation requires a bias current  $I_b$  and an external magnetic flux  $\Phi$ . To avoid hysteresis effects, a shunt resistance  $R_s$  is connected to each junction.

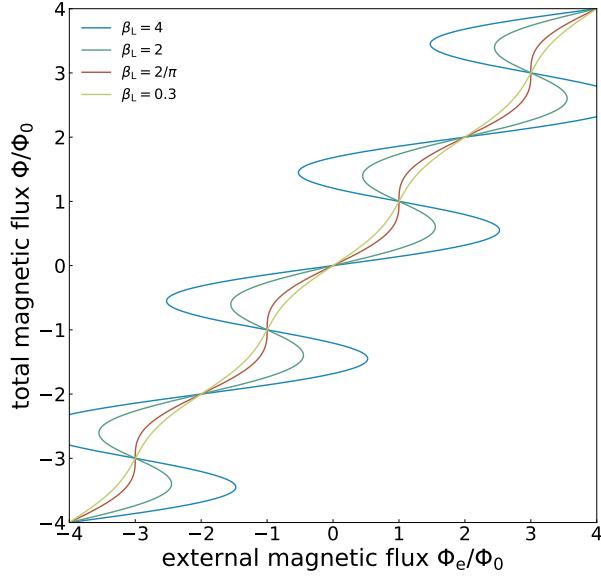


Figure 1.8: Normalized flux  $\Phi$  modulated by the external flux  $\Phi_e$ . The amplitude of the modulation depends on the screening parameter  $\beta_L$ , where  $\Phi(\Phi_e)$  remains a single-valued function for  $\beta_L \leq 2/\pi$ .

This condition implies that  $\sin \varphi_1 \approx -\sin \varphi_2$  and thus  $\varphi_1 \approx -\varphi_2$ , leading to a vanishing cosine argument  $\varphi_1 + \pi \frac{\Phi}{\Phi_0} \approx 0$ . This results in

$$\Phi = \Phi_e - \frac{1}{2}\beta_L \Phi_0 \sin\left(\pi \frac{\Phi}{\Phi_0}\right) . \quad (1.26)$$

Figure 1.8 showcases this relation for several values of  $\beta_L$ . High values ( $\beta_L > 2/\pi$ ) correspond to hysteretic characteristics, meaning there can be multiple values of total flux  $\Phi$  for the same applied flux  $\Phi_e$ . For practical dc-SQUIDs, it is therefore desirable to avoid this ambiguous behavior. The intersections of each curve represent the case for  $\Phi = n\Phi_0$ , such that the screening currents vanish and the total flux equals the external flux ( $\Phi = \Phi_e$ ). This is to be expected as the flux in a superconducting ring needs to be quantized (see equation 1.2). Consequently, the SQUID tries to maintain the total flux at integer values of  $\Phi_0$  for the limiting case of  $\beta_L \gg 1$ , where  $\Phi_{\text{cir}}$  dominates over any applied flux. This compensation is visualized by the strong modulation for high  $\beta_L$  in figure 1.8, where a wide range of  $\Phi_e$  values remain in the proximity of  $n\Phi$ . The other limiting case, i.e.  $\beta_L \ll 1$ , allows us to neglect the circulating currents such that we can write  $\Phi \approx \Phi_e$ . From equation 1.22 we then obtain the maximum possible supercurrent

$$I_s^m(\Phi_e) = 2I_c \left| \cos\left(\pi \frac{\Phi_e}{\Phi_0}\right) \right| . \quad (1.27)$$

The modulation of this current quickly diminishes for increasing  $\beta_L$ , as was derived

in [Cla04] to

$$\frac{\Delta I_s^m(\Phi_e)}{2I_c} \approx 1 - \frac{2\Phi_e}{\Phi_0\beta_L} . \quad (1.28)$$

For the SQUIDs produced within the scope of this thesis, values of  $\beta_L \approx 1$  were considered optimal to minimize resonant behavior without reducing the SQUID inductance  $L_s$  too much. In subsection 1.2.3 we will discuss how various parameters are chosen to ensure an optimal SQUID performance.

### 1.2.2 Voltage State

To utilize dc-SQUIDs as sensitive magnetometers, it is necessary to operate them in the voltage state by applying a large enough current bias  $I_b$ , such that  $I_b > 2I_c$ . In the case of negligible screening ( $\beta_L \ll 1, \Phi \approx \Phi_e$ ) and strong damping ( $\beta_C \ll 1$ ), i.e. by choosing a small junction capacitance  $C$  and SQUID inductance  $L_s$ , it is possible to derive the flux dependency of the resulting voltage drop across the SQUID. Following the RCSJ model, we are only left with the supercurrent  $I_s$  and the resistive current  $I_n$ , such that by using equation 1.22 we can write for the bias current

$$I_b = 2I_c \cos\left(\pi \frac{\Phi_e}{\Phi_0}\right) \sin\left(\varphi_1 + \pi \frac{\Phi_e}{\Phi_0}\right) + 2 \frac{V_s}{R} , \quad (1.29)$$

where we again assumed identical junctions, each shunted by a small shunt resistor  $R_s \ll R_n$ . Here  $R_n$  denotes the normal resistance of a single, unshunted junction. Therefore the total normal resistance  $R$  for each parallel connection is approximately  $R \approx R_s$ . Additionally, we can define a new phase  $\varphi = \varphi_1 + \pi \frac{\Phi}{\Phi_0}$  and with the maximum supercurrent from equation 1.27 we obtain a current relation that resembles that of a single junction:

$$I_b = I_s^m(\Phi_e) \sin(\varphi) + \frac{2}{R_s} \frac{\Phi_0}{2\pi} \frac{\partial \varphi}{\partial t} . \quad (1.30)$$

This equivalence of a dc-SQUID and a single junction stems from the above-mentioned fact that the SQUID loop represents a single Josephson contact that provides a larger effective area external fields can penetrate. It is therefore possible to derive the voltage drop across the SQUID in the same manner as in subsection 1.1.3. With the critical current  $I_s^m(\Phi_e)$  now being flux-dependent with a modulation of  $\Phi_0$ , we can

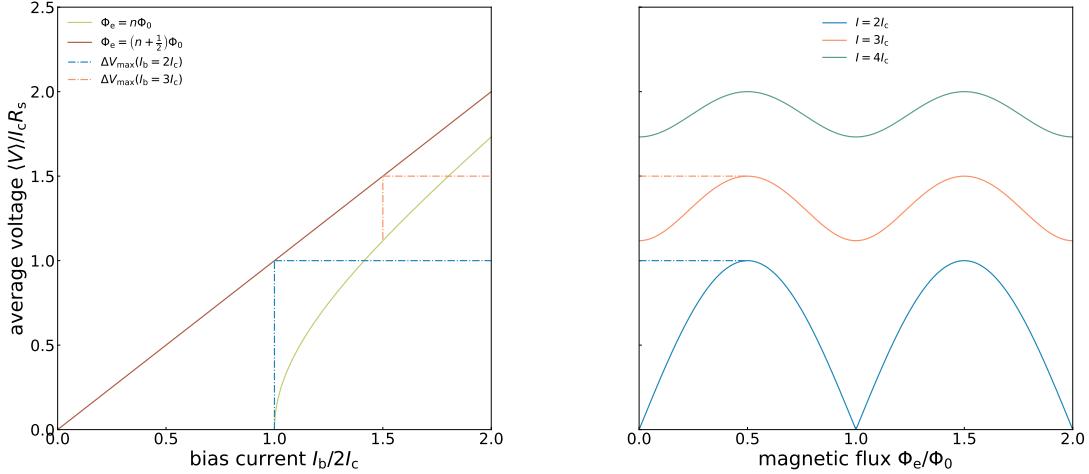


Figure 1.9: Left: IV-characteristics for the total flux  $\Phi \approx \Phi_e$  being an integer and half integer number of flux quanta, given that  $\beta_C \ll 1$  and  $\beta_L \ll 1$ . The maximum voltage swing  $\Delta V_{\max}$  is approximately at  $I_b \approx 2I_c$  and corresponds to  $I_c R_s$  for a resistively shunted dc-SQUID. Right: The projection of equation 1.31 onto the  $V\Phi$ -plane shows the flux dependency of the voltage at the bias current values  $I_b = 2I_c$ ,  $I_b = 3I_c$  and  $I_b = 4I_c$ . The amplitude of the modulation decreases for increasing  $I_b$ .

compare to equation 1.19 and obtain for the time averaged voltage [Cla04]

$$\langle V(t) \rangle = \frac{R_s}{2} \sqrt{I_b^2 - I_s^m(\Phi_e)^2} \quad (1.31)$$

$$= I_c R_s \sqrt{\left(\frac{I_b}{2I_c}\right)^2 - \left[\cos\left(\pi \frac{\Phi_e}{\Phi_0}\right)\right]^2}. \quad (1.32)$$

Evidently, both the current and the voltage are flux dependent and are modulated by  $\Phi_0$ . Figure 1.9 (left) showcases this behavior by considering the case for the minimum and maximum critical current, i.e. for  $\Phi_e = n\Phi_0$  and  $\Phi_e = (n + \frac{1}{2})\Phi_0$ , with  $n \in \mathbb{Z}$ . The current-voltage-characteristics at these flux values are particularly interesting, as they can be used to extract crucial SQUID parameters like the voltage swing  $\Delta V_{\max}$ . This property describes how the voltage varies with the applied flux  $\Phi_e$ , at a given current  $I_b$ . It is maximal at  $I_b \approx 2I_c$ , as depicted in figure 1.9 (right).

It is, however, important to note that equation 1.31 doesn't hold for practical SQUIDS, as they are typically not fabricated to fulfill the limiting case of  $\beta_C \ll 1$  and  $\beta_L \ll 1$ . The conclusions reached here will nevertheless be applicable to practical SQUIDS, only needing a few adjustments.

### 1.2.3 Optimal Parameters

Negligible screening is not reasonable, as it would require to choose an extremely small SQUID inductance  $L_s$ , which in turn deteriorates the sensitivity for magnetic fields. The main reason to construct a dc-SQUID was to obtain a highly sensitive magnetometer by creating a large area for magnetic fields to thread through. Also, the fabrication process doesn't allow to produce an arbitrarily small junction capacitance  $C$ . The parameter  $\beta_C$  will therefore reach a lower limit as well, since also decreasing  $R_s$  too much reduces the voltage swing  $\Delta V_{\max}$  and increases the energy sensitivity  $\epsilon(f)$ , as we will see in subsection 1.2.4. Now by allowing displacement and fluctuation currents, the current and voltage expressions become analytically unsolvable and therefore have to be solved numerically. In [Tes77] such numerical simulations lead to optimal values of  $\beta_C \approx 1$  and  $\beta_L \approx 1$  to minimize the energy sensitivity.

To further fine-tune various parameters it is essential to look at how dc-SQUIDs are typically operated to achieve the highest possible flux sensitivity. Here, we distinguish between a current and a voltage bias, where the former was assumed in figure 1.9. Maximizing sensitivity in this mode is done by maintaining the flux thorough a constant offset at the steepest point in the  $V\Phi$ -curve, which is referred to as the working point and corresponds to  $\Phi_e = (2n + 1)\frac{\Phi_0}{4}$ . This allows for the largest possible voltage change  $\Delta V$  at a given flux change  $\Delta\Phi$ . Similarly, at a voltage bias the working point will mark the steepest point in the  $I\Phi$ -curve. To quantify this, we introduce the transfer parameters

$$V_\Phi \equiv \left| \left( \frac{\partial V}{\partial \Phi_e} \right) \right| \quad (1.33)$$

$$I_\Phi \equiv \left| \left( \frac{\partial I}{\partial \Phi_e} \right) \right| . \quad (1.34)$$

As mentioned above, at  $I_b \approx 2I_c$  (current bias) the amplitude of the voltage modulation is maximal. This needs to be modified for practical SQUIDs, where thermal fluctuations can't be neglected. The resulting thermal current  $I_{\text{th}}$  causes a rounding of the edge at  $I_b = 2I_c$  (figure 1.9 (left)), thereby reducing  $\Delta V_{\max}$  and  $V_\Phi$  [Iva68]. To minimize this effect, numerical simulations were made that lead to the condition [Cla88]

$$\frac{I_c}{5} \geq I_{\text{th}} \equiv \frac{2\pi k_B T}{\Phi_0} . \quad (1.35)$$

A lower bound for  $I_c$  at  $T = 4.2 \text{ K}$  will therefore be approximately  $1 \mu\text{A}$ . This effect

shifts the current  $I_{b,\max}$ , at which the voltage swing is maximal, according to [Dru96b] by a temperature correction factor leading to

$$I_{b,\max} \approx 2I_c(1 - \sqrt{\Gamma/\pi}) \quad , \quad (1.36)$$

where  $\Gamma$  is the noise parameter defined as  $\Gamma = I_{\text{th}}/I_c$ . Lastly, the thermal current can also be used to set an upper limit to the SQUID inductance. We can define a thermal inductance  $L_{\text{th}} = \frac{\Phi_0}{2I_{\text{th}}}$  for the thermal current inducing half a flux quantum. This should be significantly larger than the SQUID inductance  $L_s$  to minimize the impact of these thermal fluctuations. Again, simulations provide a constraint for optimization, giving the relation [Cla88]

$$5L_s \leq L_{\text{th}} \equiv \frac{\Phi_0}{2I_{\text{th}}} = \frac{\Phi_0^2}{4\pi k_B T} \quad . \quad (1.37)$$

For  $T = 4.2\text{ K}$  we would obtain  $L_s \leq 1\text{ nH}$ , which is typically fulfilled for practical dc-SQUIDs.

#### 1.2.4 Noise

The above-mentioned energy sensitivity, also called spectral noise energy density or energy resolution, is defined as the flux noise per SQUID inductance  $L_s$  and is typically expressed through a power spectral density as

$$\epsilon(f) = \frac{S_\Phi(f)}{2L_s} \quad . \quad (1.38)$$

This conveniently allows to compare noise properties from SQUIDs with different loop inductances. The flux noise power spectral density  $S_\Phi(f)$  cannot be measured directly and is therefore calculated with the equivalent voltage noise using the transfer coefficient introduced above:

$$S_\Phi(f) = \frac{S_V(f)}{V_\Phi^2} \quad . \quad (1.39)$$

The flux noise in SQUIDs is typically separated into a high frequency constant white noise and a low frequency  $1/f$ -noise component [Koc07]. To derive an expression for  $S_V(f)$  we consider the white noise regime only, limiting ourselves to higher frequencies to avoid any significant influence of  $1/f$ -noise. For this we need to distinguish between two noise currents, one flowing across the SQUID and the other circulating within the SQUID loop. The former sees the total resistance of the SQUID which corresponds

to  $\frac{R_s}{2}$  for  $I \gg I_{b,\max}$  and to the dynamic resistance  $R_{\text{dyn}} = \frac{\partial V}{\partial I}$  for operation at the working point. The circulating current, however, sees two resistances in series, i.e.  $2R_s$ . By taking into account noise inducing resonances due to the simulated optimal values of  $\beta_C \approx 1$  and  $\beta_L \approx 1$  that exhibit hysteretic behavior, both noise currents can be used to obtain the voltage noise given by [Tes77], [Bru82]

$$S_V(f) = \frac{4k_B T}{R_s} \left[ 2R_{\text{dyn}}^2 + \frac{L_s^2 V_\Phi^2}{2} \right] . \quad (1.40)$$

Here, we used the approximation  $V_\Phi \approx \frac{I_c R_s}{\Phi_0/2} \approx \frac{R_s}{L_s}$  for  $\beta_L \approx 1$ . These calculations also derived the relation  $R_{\text{dyn}} \approx \sqrt{2}R_s$ , which can be used to rewrite equation 1.40 to

$$S_V(f) = 18k_B T R_s . \quad (1.41)$$

Inserting this into equations 1.39 and 1.38 leads to the expression

$$\epsilon(f) \approx 16k_B T \sqrt{\frac{L_s C}{\beta_C}} \approx 16k_B T \sqrt{L_s C} \quad \text{for } \beta_C \approx 1 . \quad (1.42)$$

For a moderately damped dc-SQUID ( $\beta_C \approx 1$ ) at a temperature  $T = 100 \text{ mK}$  and with a suitably small inductance  $L_s = 100 \text{ pH}$  we would therefore obtain a theoretical value of  $\epsilon \approx 0.32 \text{ h}$ .

Lastly, to get the full picture we will look at the total flux noise spectrum by considering the  $1/f$  component. This not entirely understood noise behavior is added to the above derived white noise and dominates the low frequency regime. It is parameterized by a power density at 1 Hz, giving a total noise power density of

$$S_\Phi(f) = S_{\Phi,w} + \frac{S_{\Phi,1/\text{f}}(1 \text{ Hz})}{f^\alpha} , \quad (1.43)$$

with  $S_{\Phi,w}$  being the white noise component and  $\alpha$  ranging between 0.5 and 1 [Dru11], which can be determined through experiment.

### 1.2.5 Parasitic Resonances

We have seen that optimizing dc-SQUID properties requires careful fine-tuning of various parameters to ensure high sensitivity and low noise at the same time. Another constraint in this regard involves the presence of resonances in the system that we need to take into account. As discussed in subsection 1.1.3, a Josephson contact

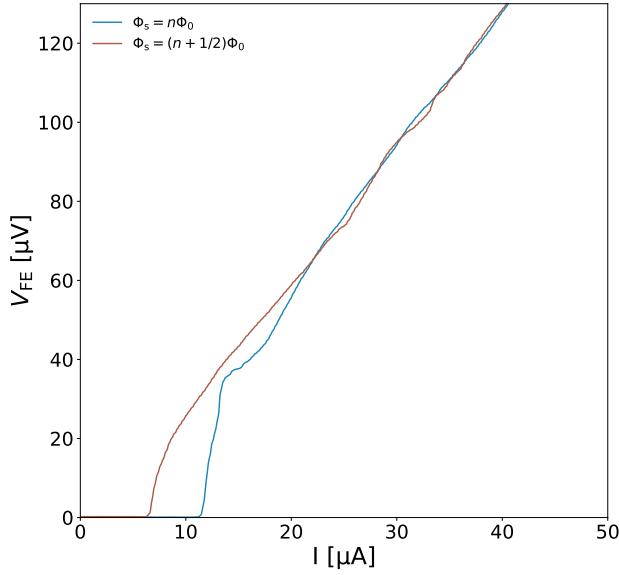


Figure 1.10: Measured IV-characteristic of a dc-SQUID developed within the scope of this thesis. The first current step at  $V_s \approx 40 \mu\text{V}$  corresponds to the fundamental SQUID resonance for  $L_s = 119 \text{ pH}$  and  $C = 0.95 \text{ pF}$ .

represents a parallel plate capacitor with capacitance  $C$ . Due to the series connection of the two junctions within the SQUID loop, we obtain a total capacitance of  $C/2$ . The SQUID with its loop inductance  $L_s$  will therefore form an  $LC$  resonator that can be excited if the Josephson currents oscillate with the resonance frequency  $f_{L_sC} = \frac{1}{2\pi\sqrt{L_sC/2}}$ . This is fulfilled for the condition

$$\frac{V_s}{\Phi_0} = \frac{1}{2\pi\sqrt{L_sC/2}} , \quad (1.44)$$

where  $V_s$  denotes the voltage drop across the SQUID that is associated with the Josephson frequency  $f_J = \frac{V_s}{\Phi_0}$  resulting from the ac Josephson effect. With optimal  $\beta_C$  and  $\beta_L$  values the voltage corresponding to this resonance frequency will move towards the vicinity of the working point, thus becoming relevant for the SQUID's performance. This so-called *fundamental SQUID resonance* negatively affects the operation range by manifesting itself through a current step in the IVC, as shown in figure 1.10. The IV curves for integer and half integer flux quanta intersect as a result multiple times, due to higher harmonics of the resonance causing additional current steps [Cla04]. The voltage swing  $\Delta V$  and consequently  $V_\Phi$  are therefore limited by this resonance, which favors the choice of small values for  $L_s$  and  $C$ .

It is important to note, that the fundamental resonance does not depend on any geometrical structures of the circuit, which typically play a significant role in practical SQUIDS. Resonance inducing structures are however necessary for practical reasons.

Those include the need for effectively coupling external flux changes into the SQUID in order to take advantage of its high sensitivity. For this an external input coil with inductance  $L_i$  is usually fabricated on top of the SQUID loop, separated by an insulating layer to ensure the coupling to be solely inductive (see section 2.1). The SQUID can therefore be used as a current sensor by converting small current signals in the input coil into small magnetic flux changes in the SQUID. This added coil, however, provides another parasitic capacitance  $C_p$  resulting from its fabrication on top of the SQUID loop. Consequently, rf currents can couple inductively from one system into the other, causing further parasitic effects. Particularly, problematic  $LC$  resonances arise from the added inductance  $L_i$  and capacitance  $C_p$ , namely the  $f_{L_s C_p}$  and  $f_{L_i C_p}$  resonance. The latter can be excited for  $f_J = f_{L_i C_p}$ , since the Josephson currents are able to couple into the input coil. The parasitic capacitance  $C_p$  and consequently both resonance frequencies depend on geometrical factors such as the length of the input coil, as well as the widths and heights of the individual components that form the resonator. It has been shown, that these resonances lead to the energy sensitivity increasing proportional to  $\sqrt{1 + 2C_p/C}$ , as long as  $C_p/C \leq 2$ . For  $C_p/C \geq 2$  the energy sensitivity saturates due to the resonance frequencies falling below the working point [Ryh92]. It is therefore desirable to minimize  $C_p$  as much as possible in order to reduce the resulting voltage noise.

The fabrication of the input coil on top of the SQUID loop also represents a microstrip transmission line, consisting of a conductor carrying the signal (input coil) and a ground plane (SQUID loop), separated by a dielectric layer. This forms a waveguide allowing electromagnetic waves to propagate alongside it, which undergo reflections where impedance mismatches occur. These would arise whenever the input coil exits the SQUID loop, thereby leaving the underlying ground plane. Standing waves occur if the length  $l$  of the signal carrying line above the ground plane corresponds to an integer number of half the wavelength of the Josephson frequency. This is consequently also called the  $\lambda/2$  resonance. The corresponding resonance frequency

$$f_l = \frac{mc_{\text{str}}}{2l} , \quad (1.45)$$

where  $m \in \mathbb{Z}$ , depends on  $l$  and the material-dependent wave propagation velocity  $c_{\text{str}}$ . This resonance behavior also emerges if we consider the SQUID loop to be the signal carrying line with the input coil acting as the corresponding ground plane. Here it is again possible to move the resonance further away from the operation frequency at the working point by choosing adequate geometric proportions, e.g. by varying the length of the input coil or the SQUID loop. These methods to mitigate the influence of resonances can be complemented by direct damping through attenuators, which will be discussed in section 3.3. In chapter 3 we will cover how

various parameters are chosen in the SQUID design to suppress and avoid possible resonances, given the constraints we derived in subsection 1.2.3 and 1.2.4.



## 2. Experimental Setup

So far we discussed general aspects of dc-SQUIDs and how their working principle allows for highly sensitive magnetic flux measurements. As already briefly seen in subsection 1.2.5, when it comes to practical SQUIDs many theoretical considerations regarding parameter optimization need to be reevaluated to account for usability in practical experiments. We begin this chapter with general concepts of a practical SQUID design and introduce a typical low-noise setup with a room temperature readout electronic. In this working group, SQUIDs are mainly developed for the readout of *Metallic Magnetic Calorimeters (MMCs)* (see section 2.3). We will see in the following how those SQUIDs need to be designed to optimize their coupling to these detectors. Furthermore, this chapter will cover various methods to reduce quality factors of parasitic resonances, such as adding shunt resistors or coupling to normal conducting gold layers.

### 2.1 Practical dc-SQUIDs

The SQUIDs developed in this working group are used as current sensors for the MMC readout by sending the detected signals from the *pickup coil* of the MMC to the input coil of the SQUID. The requirement of the latter entails a parasitic capacitance resulting in numerous resonances, as discussed in subsection 1.2.5. To achieve high inductive coupling between input coil and SQUID loop, it is necessary to fabricate them closely on top of each other, only separated by a thin insulating layer. The coupling strength is given by the dimensionless parameter

$$k_{\text{is}} = \frac{M_{\text{is}}}{\sqrt{L_{\text{i}} L_{\text{s}}}} , \quad (2.1)$$

where  $M_{\text{is}} = \Delta\Phi_{\text{s}}/\Delta I_{\text{i}}$  is the mutual inductance, describing how much flux  $\Delta\Phi_{\text{s}}$  is generated in the SQUID loop for a current change  $\Delta I_{\text{i}}$  in the input coil. This allows us to define the so-called coupled energy sensitivity  $\epsilon_{\text{c}}(f)$  with respect to the input coil, which by using equation 1.38 is given as

$$\epsilon_{\text{c}}(f) = \frac{\epsilon(f)}{k_{\text{is}}^2} = \frac{L_{\text{i}} S_{I,\text{i}}}{2} . \quad (2.2)$$

This expression refers to the apparent current noise  $S_{I,\text{i}} = S_{\Phi_{\text{s}}} / M_{\text{is}}^2$ , which is generated by the flux noise from the SQUID loop through the coupling  $M_{\text{is}}$ . A strong

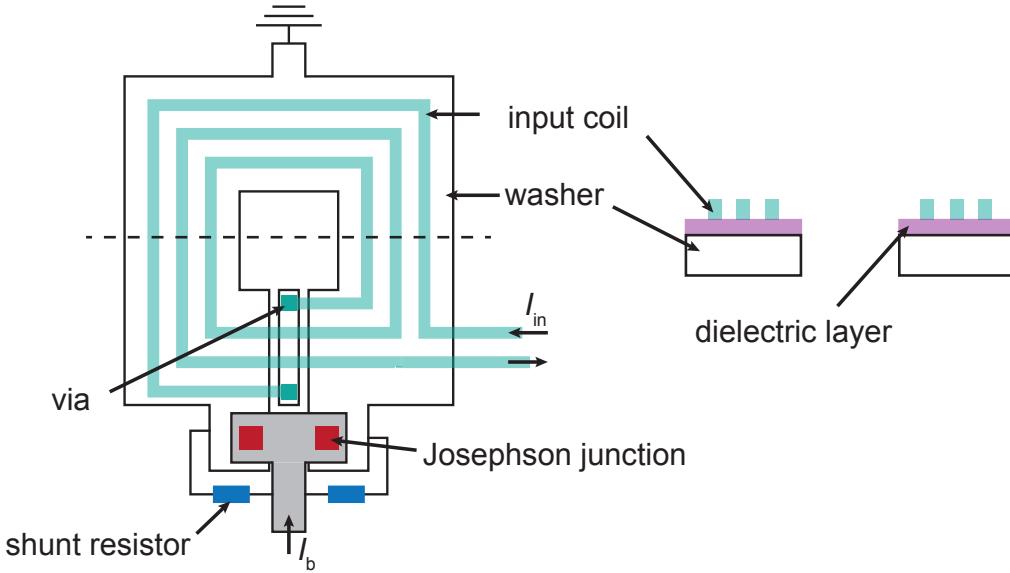


Figure 2.1: Schematic drawing of a typical planar thin-film dc-SQUID. The SQUID loop is realized as a square *washer*-geometry interrupted by a narrow slit, only connected at the junction area. A thin insulating layer separates the washer from the planar multi-turn input coil above. Left: View from the top. Right: Cross section marked by the dashed line.

coupling can be achieved by the commonly used square *washer*-geometry with a planar input coil [Jay81], as shown in figure 2.1.

Here, the SQUID loop is represented by the washer, whereas each turn of the input coil is symmetrically located on top of it to maximize the coupling between each system. A cross section of this setup is depicted in figure 2.1 (right), showing the insulating dielectric layer separating each coil. The washer is intersected by a slit, which starts at the square hole in the middle and ends at the remotely situated junction area that connects each side of the loop. The total inductance of the SQUID loop can be calculated by adding the dominating washer hole inductance  $L_h$ , the slit inductance  $L_t \approx 0.3 \frac{\text{pH}}{\mu\text{m}}$  and the much smaller parasitic inductance  $L_j$  associated with the junction area, giving [Ket91]

$$L_s = L_h + L_t + L_j . \quad (2.3)$$

The latter is referred to as parasitic due to its position outside of the input coil, thus not contributing to the coupling. By neglecting  $L_t$  and  $L_j$ , we can approximate the washer inductance in the limit of  $d \ll w$  to  $L_s \approx L_h \approx 1.25\mu_0 d$ , where  $d$  and  $w$  are the inner and outer side lengths, respectively [Jay81]. This is a reasonable result considering that the supercurrent will only flow along the inner edge of the washer

[Ket82], thereby being independent of the outer side length  $w$ . The effective area  $A_{\text{eff}}$  of the SQUID loop has been calculated to  $A_{\text{eff}} \approx dw$  [Ket85], showing that this geometry allows for high sensitivity while keeping the SQUID inductance small. The input coil inductance on the other hand can be approximated by  $L_i = L_{\text{str}} + n^2 L_s$ , where  $L_{\text{str}}$  is the stripline inductance (see section 3.3) and  $n$  is the number of input coil turns [Jay81]. The dc-SQUID designs used in this working group, however, are too complex to provide such analytical expressions and therefore need to be calculated numerically using simulation softwares such as *InductEX*.

### 2.1.1 Gradiometer

The high flux sensitivity of a SQUID makes it prone to detect unwanted magnetic bias fields and/or gradients that may be present during its operation. Typical SQUIDs are therefore built in a gradiometric design to counteract this effect [Ket78]. A first order gradiometer consists of two identical conducting loops connected in series or parallel, with opposing orientation as shown in figure 2.2 (left, middle). Under the presence of a homogeneous bias field  $\mathbf{B}$  in x-direction (perpendicular to the gradiometer plane), this configuration produces a zero net current after a field change  $\Delta B_x$ , due to the opposing currents induced in each turn. To also achieve the same effect for a field gradient  $\frac{\partial \mathbf{B}}{\partial z}$  or  $\frac{\partial \mathbf{B}}{\partial y}$ , a second order gradiometer composed of four loops in series or parallel is required, see figure 2.2 (right), where only the currents induced in the upper loops are drawn for the sake of overview. In order to incorporate this into a practical SQUID, the input coil and the SQUID loop will consist of four serial and parallel turns, respectively. This configuration enables to combine a small SQUID inductance with a large input coil inductance while maintaining a strong coupling between the two, as each turn of both coils can be produced with similar dimensions. The low SQUID inductance results from the reciprocal summation over each loop inductance  $L_l$  due to the parallel connection, giving

$$L_s = \frac{L_l}{4} . \quad (2.4)$$

Whereas a serial gradiometer gives

$$L_i = 4L_l \quad (2.5)$$

for the input coil. This gradiometric setup allows for adapting the input coil to the pickup coil of an MMC by choosing a large enough inductance  $L_i$ , which will be discussed in chapter 3.

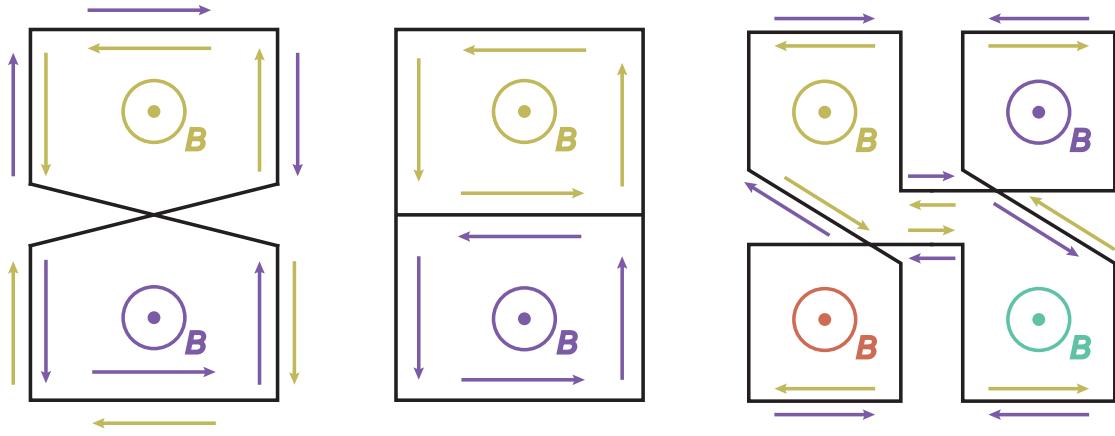


Figure 2.2: Schematic examples of a gradiometric dc-SQUID configuration threaded by a homogeneous magnetic field  $\mathbf{B}$ . A first order gradiometer can be realized by either connecting two loops in series (left) or in parallel (middle). A magnetic field change  $\Delta B_x$  induces two opposing currents that cancel each other out. Right: Second order gradiometer consisting of four loops connected in series. This geometry results in a net zero current also for an applied field gradient  $\frac{\partial \mathbf{B}}{\partial z}$ .

## 2.2 Operation of a dc-SQUID

We have seen in section 1.2.2 that the periodic  $V\Phi$ -characteristic provides an approximately linear dependency at  $\Phi = (2n + 1)\frac{\Phi_0}{4}$ , which only holds for  $\Delta\Phi \approx \Phi_0/4$ . This restricts the dynamic range greatly, as the linearity vanishes for larger flux changes and for  $\Delta\Phi > \Phi_0/2$  the voltage even becomes ambiguous. Such behavior is unsuitable for MMC readout, as they require the highest possible signal to noise ratio and therefore a linearized output voltage.

### 2.2.1 Flux-Locked Loop

The standard readout method involves a flux feedback circuit to maintain the operation at the working point independently of the flux change amplitude [Dru02]. This so-called flux-locked loop (FLL) readout technique first amplifies the output signal of the SQUID  $V_s$  with a differential amplifier operated at room temperature, where the voltage  $V_b$  corresponding to the working point is provided by a voltage source on the second amplifier input. This voltage compensation at the working point ensures that only variations  $\Delta V = V_s - V_b$  that correspond to the flux change  $\Delta\Phi$  are amplified. The signal is then fed into an integrator, which integrates it over time and thus creates a rising output voltage  $V_{\text{out}}$ . By now connecting a feedback resistance  $R_{\text{fb}}$

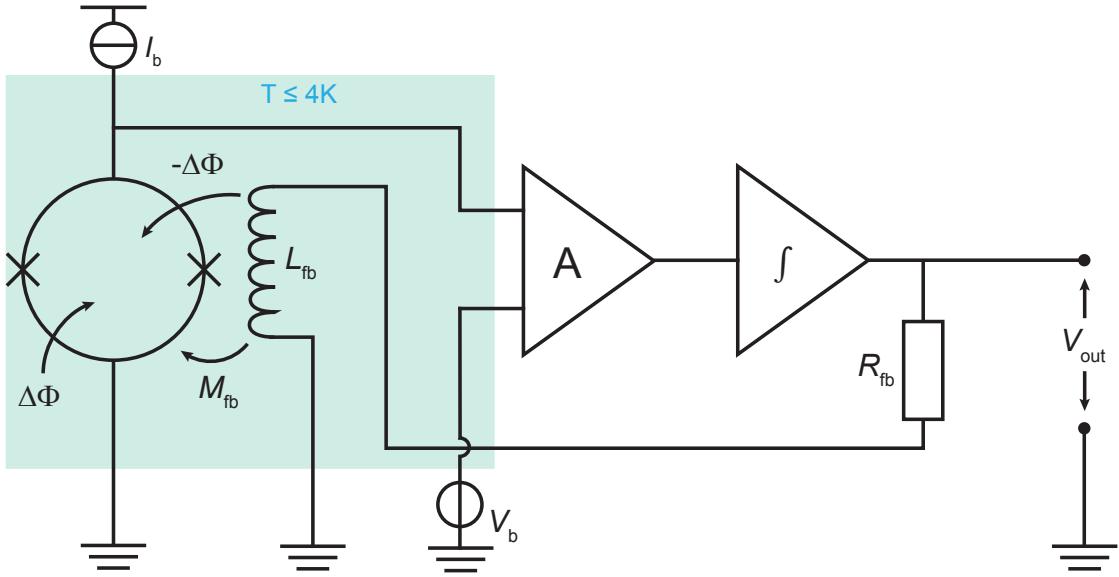


Figure 2.3: Schematic circuit diagram of a flux-locked loop dc-SQUID readout. The amplified and integrated SQUID voltage signal  $V_s$ , which is caused by a detected flux change  $\Delta\Phi$ , is fed back to a feedback coil with inductance  $L_{fb}$ , creating a compensating flux  $-\Delta\Phi$ . This enables the operation at the working point, while flux changes far greater than  $\Phi_0/4$  can be linearized.

to the output circuit, a rising feedback current  $I_{fb}$  emerges that flows to a feedback coil with inductance  $L_{fb}$ . This coil is coupled to the SQUID analogous to the input coil (see chapter 3), but with the opposite orientation. A compensation flux  $-\Delta\Phi$  is generated up until the initial flux change is fully canceled out, i.e.  $V_s \rightarrow 0$ . The integrator will therefore approach a constant value due to the vanishing voltage at the input circuit. This voltage signal is proportional to the current that was needed to completely compensate for the input signal that induced  $\Delta\Phi$ , leading to the relation

$$V_{out} = \frac{R_{fb}}{M_{fb}} \Delta\Phi . \quad (2.6)$$

A schematic for this readout process is shown in figure 2.3. With this setup the SQUID is used as a null-detector that allows for the linearization of the quantity of interest, while also providing a large dynamic range. A state-of-the-art, low-noise SQUID readout electronic by the company Magnicon<sup>1</sup> of the type XXF-1, which is used in this working group, provides the necessary current and voltage sources, as well as the room temperature amplifiers within the FLL circuit described above.

<sup>1</sup>Magnicon GmbH, Barkhausenweg 11, 22339 Hamburg

This SQUID electronic exhibits an intrinsic voltage noise of  $\sqrt{S_{V,\text{el}}} \approx 0.33 \frac{\mu\text{V}}{\sqrt{\text{Hz}}}$  and intrinsic current noise of  $\sqrt{S_{I,\text{el}}} \approx 2.6 \frac{\mu\text{A}}{\sqrt{\text{Hz}}}$  [Dru06]. Furthermore, a large amplifier bandwidth of 6 MHz is provided to ensure high sensitivity for short signal rise times. The intrinsic noise of the SQUIDs produced in this working group, however, typically reaches values of  $\sqrt{S_{\Phi_s}} \leq 1 \frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$ . Adding these terms together leads to the total apparent flux noise in the SQUID, which is expressed as the spectral power density

$$S_{\Phi_s,\text{SQ}} = S_{\Phi_s} + \frac{S_{V,\text{el}}}{V_{\Phi_s}^2} + \frac{S_{I,\text{el}}}{I_{\Phi_s}^2} . \quad (2.7)$$

Typical values for the transfer coefficients of SQUIDs produced within the scope of this thesis, are  $V_{\Phi_s} = 80 \frac{\mu\text{V}}{\Phi_0}$  and  $I_{\Phi_s} = 20 \frac{\mu\text{A}}{\Phi_0}$ , leading to the SQUID electronic having a total noise contribution of  $4.13 \frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$ . The amplifier noise therefore dominates the noise level, thereby deteriorating the signal to noise ratio. To avoid this effect, a second SQUID is typically added to act as a low temperature amplifier [Wel93]. This method significantly reduces the apparent flux noise in the detector SQUID, which is crucial for MMC readout, as the intrinsic noise of a MMC detector should not be lower than that of the readout electronics.

### 2.2.2 Two-Stage Configuration

Implementing a low temperature amplifier is usually realized through a second stage SQUID, situated between the first stage (detector) SQUID and the room temperature amplifier, as depicted in figure 2.4. Here, the subscripts 's' and 'x' refer to the first and second stage, respectively. The first stage SQUID, also referred to as a Front-End SQUID, needs to be operated in a voltage bias for this two-stage setup. This can be achieved by connecting a gain resistor  $R_g$  in parallel with both the Front-End and the input coil of the amplifier SQUID. If a bias current  $I_{b,s}$  is injected into the circuit, all the current will flow through the Front-End, as long as it stays superconducting. Once it becomes normal conducting by further increasing  $I_{b,s}$ , the current will start shifting to  $R_g$ , whose resistance is chosen to be much smaller than the dynamic resistance  $R_{\text{dyn}}$ , until most of the current flows through  $R_g$ . At this point, the resulting voltage across both components becomes approximately constant. This behavior can be visualized through a loadline created by the parallel resistances, which intersects the IVC of the Front-End. The loadline voltage  $V_s$  between both extremal IV curves will then remain nearly constant, as the slope is given by the small gain resistance  $R_g$ . If a detector signal is now coupled into the Front-End through the input coil with  $M_{is}$ , the current in the SQUID will move along the loadline in the  $I\Phi$ -plane, corresponding to the externally induced flux  $\Phi_s$ . The attached input coil of the second stage SQUID would experience these current changes, hence creating a

flux change  $\Delta\Phi_x$  in the amplifier SQUID, which is being operated in a current bias. To maximize the amplification, the second stage SQUID is typically realized as a  $N$ -SQUID series array consisting of  $N$  serially connected identical SQUID cells. This results in a large voltage drop across the array, given by  $V_{\text{array}} = NV_{\text{cell}}$ . Analogous to the single stage readout, the signal will then be amplified at room temperature and fed back to a feedback coil with mutual inductance  $M_{fs}$  to compensate for the initial flux change  $\Delta\Phi_s$ . An additional feedback coil with mutual inductance  $M_{fx}$ , spanning symmetrically across every SQUID cell couples a constant flux offset through a bias current  $I_{\Phi_x}$  into the array in order to maintain it at its working point. The resulting two-stage  $V_x\Phi_s$ -characteristic will strongly depend on the flux gain defined as

$$G_\Phi = \frac{\partial\Phi_x}{\partial\Phi_s} = \frac{M_{ix}}{R_g + R_{\text{dyn}}} V_{\Phi_s} \approx \frac{M_{ix}}{R_{\text{dyn}}} V_{\Phi_s} , \quad (2.8)$$

which relates the flux change induced in the second stage SQUID for a given flux change in the detector SQUID. For  $\Delta\Phi_x = \Delta I_s M_{ix} > \Phi_0/2$ , additional minima and maxima emerge. These start to overlap for  $\Delta\Phi_x > \Phi_0$ , thereby creating multiple working points that prevent a practical FLL operation. This sets an upper limit for the flux gain, however, it should be chosen as large as possible to reduce the apparent flux noise of the Front-End SQUID. An optimal flux gain has been calculated to  $G_\Phi \approx \pi$ , corresponding to  $\Delta\Phi_x \approx \Phi_0/2$  [Dru96a].

The two-stage setup contributes additional noise sources to equation 2.7, namely the gain resistor and the amplifier SQUID. However, the resulting conversion to the flux  $\Phi_s$  of the detector SQUID significantly reduces the influence of the room temperature amplifier, which in turn strongly improves the overall signal to noise ratio. The total apparent flux noise of the Front-End then reads [Dru96a]<sup>2</sup>

$$S_{\Phi_s,\text{SQ}} = S_{\Phi_s} + \frac{4k_B T R_g}{G_\Phi^2 (R_g + R_{\text{dyn}})^2} M_{ix}^2 + \frac{S_{\Phi_x}}{G_\Phi^2} + \frac{S_{V,\text{el}}}{G_\Phi^2 V_{\Phi_x}^2} + \frac{S_{I,\text{el}}}{G_\Phi^2 I_{\Phi_x}^2} . \quad (2.9)$$

The second term describes the Nyquist current noise caused by the gain resistor, which becomes negligible with a voltage biased detector SQUID where  $R_g \ll R_{\text{dyn}}$ . The low and room temperature amplifier terms are reduced by the flux gain parameter, which can't be chosen arbitrarily large as mentioned above. However, using a SQUID array for the second stage increases the voltage swing and thus the transfer coefficient by an  $N$ -fold, i.e.  $V_{\Phi_x} = NV_{\text{cell}}$ , where the subscript 'cell' refers to a single array SQUID cell. Consequently, the total noise level can be further reduced by choosing a high number  $N$  of SQUID cells. Here it is noteworthy, however, that

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<sup>2</sup>Auch Stromquellen, siehe F. Kaap?

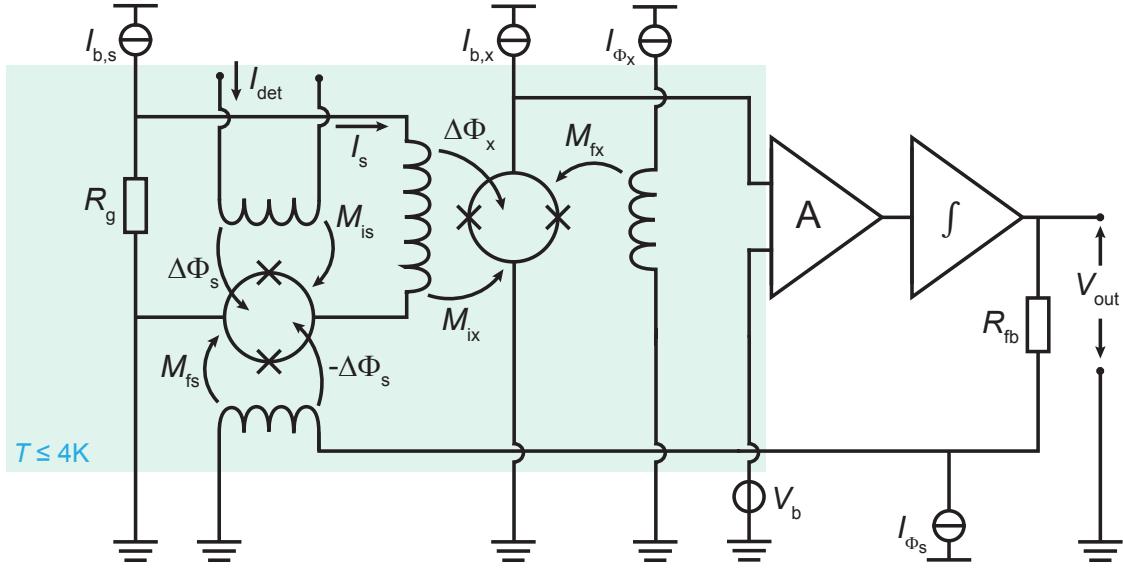


Figure 2.4: Schematic circuit diagram of a two-stage dc-SQUID based readout. The first stage consisting of the detector SQUID is voltage biased for  $R_g \ll R_{dyn}$ . The detector signal  $I_{det}$  induces a current change  $\Delta I_s$  and therefore a flux change  $\Delta\Phi_x$  in the second stage SQUID, which acts as a low temperature preamplifier. In FLL mode, the signal will be fed back to compensate for the initial flux change  $\Delta\Phi_s$ , thereby linearizing the detector signal. Choosing an N-SQUID series array amplifies the voltage swing and transfer coefficient, thus significantly reducing the overall apparent flux noise in the detector SQUID.

these considerations only account for the magnetic flux in a single cell, as otherwise the transfer coefficient would remain constant [Sta93],[Fog93]. Now, using equation 1.39 we obtain for the SQUID array flux noise

$$\sqrt{S_{\Phi_x}} = \frac{\sqrt{S_{V_x}}}{V_{\Phi_x}} = \frac{\sqrt{N S_{V_{cell}}}}{N V_{\Phi_{cell}}} = \frac{1}{\sqrt{N}} \sqrt{S_{\Phi_{cell}}} , \quad (2.10)$$

hence the intrinsic noise of the second stage gets reduced by a factor of  $\frac{1}{\sqrt{N}}$  [Sta93]. This also has a consequence for the coupled energy sensitivity of the SQUID array, which is calculated by summing the array flux noise over all  $N$  cells, giving

$$\epsilon_{c,x} = N \frac{S_{\Phi_x}}{2L_{cell}k_{i,cell}} = \frac{S_{\Phi_{cell}}}{2L_{cell}k_{i,cell}} . \quad (2.11)$$

with the parameter  $k_{i,cell}$  denoting the coupling of a cell with inductance  $L_{cell}$  to its respective input coil segment. Connecting  $N$  SQUIDs in series did therefore not affect the energy sensitivity, provided that  $k_{i,cell}$  remains constant across the array. The

arrays produced in this working group either contain 16 [Kem15] or 18 [Kra23] cells. Applying this to equation 2.9, we would reduce the above-mentioned contribution of the room temperature amplifier flux noise of  $4.13 \frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$  to  $0.29 \frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$  and  $0.26 \frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$  for  $N = 16$  and  $N = 18$ , respectively. Provided an optimal flux gain of  $G \approx \pi$ , the noise contribution would further decrease to  $0.09 \frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$  or  $0.08 \frac{\mu\Phi_0}{\sqrt{\text{Hz}}}$ , well below the typical noise level of a detector SQUID.

Besides the improvement of the noise behavior, a major advantage of a two-stage readout setup consists of the low power dissipation produced by the detector SQUID. As opposed to the current bias operation in a single-stage setup, the Front-End is held at a constant, low voltage, such that the dissipated power given by  $P = UI$  only consists of approximately 1 nW [Dru07]. However, this amount poses a problematic heat source for low temperature detectors consisting of up to millions of temperature-sensitive calorimeter pixels, such that using a two-stage setup for each pixel is unfavorable. For these, a different approach using time- or frequency-division dc-SQUID multiplexing is followed. These readout techniques constitute a research focus in this working group in order to optimize the readout of several experiments such as the ECHo experiment [Gas17].

## 2.3 Metallic Magnetic Calorimeters

Low temperature detectors used in this working group exhibit detector arrays consisting of MMC pixels, which are able to absorb and detect heavy particles or photons such as X-rays. The focus of this thesis was to improve the dc-SQUID based MMC readout, which is why we first briefly cover the working principle of such calorimeters.

A general overview of the detector setup can be seen in figure 2.5. The upper part consists of a particle absorber, which, following the law of energy conservation, converts the energy  $E$  of an incoming particle into a change in temperature  $\Delta T$  in the absorber. A paramagnetic sensor is directly attached below, submerged in a weak magnetic field. Strong thermal coupling allows for an effective temperature increase in the sensor, which is given by the total heat capacity  $C_{\text{tot}}$  of both the sensor and the absorber, resulting in

$$\Delta T \simeq \frac{E}{C_{\text{tot}}} . \quad (2.12)$$

Not shown in figure 2.5 is an underlying thermal bath weakly linked to the sensor, which reverts the temperature change by an exponential decay ensures a thermal equilibrium. The paramagnetic nature of the sensor changes its magnetization  $M(T)$

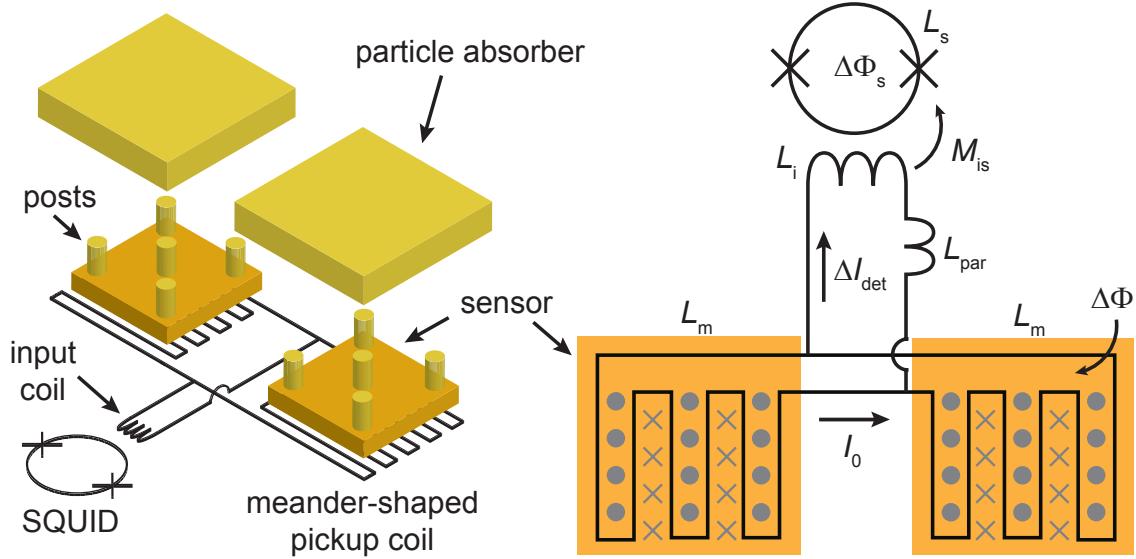


Figure 2.5: Schematic depiction (left) and circuit diagram (right) of a common MMC channel consisting of two pixels. The planar particle absorber is fabricated on top of the paramagnetic sensor, separated by cylinder-shaped posts to prevent energy losses through athermal phonons, while still providing good thermal coupling. Below the sensors two meander-shaped pickup coils connected in parallel detect the flux change  $\Delta\Phi$ , which can be precisely measured with a SQUID as a flux transformer. The grey dots and crosses within the meander mark a magnetic field induced by the bias current  $I_0$ .

upon a temperature rise, according to the Curie law  $M \propto \frac{1}{T}$  for low temperatures. This induces a magnetic flux change  $\Delta\Phi$  linearly proportional to  $\Delta M$ , which is detected by a typically meander-shaped pickup coil that is strongly coupled to the sensor. The input coil of the SQUID is directly connected to the pickup coil forming a flux transformer, thereby enabling the readout of the created current signal  $\Delta I_{\text{det}}$ . With equation 2.12 we can write for the induced flux in the SQUID

$$\Delta\Phi_s \propto \Delta\Phi \propto \Delta M \simeq \frac{\partial M}{\partial T} \frac{E}{C_{\text{tot}}} , \quad (2.13)$$

provided the deposited energy  $E$  is small. Evidently, it is desirable to both reduce the heat capacity and increase the magnetization gradient in order to maximize the flux change  $\Delta\Phi$ . Additionally, short signal rise times can be realized by a high thermal conductivity of the sensor and the absorber, which is particularly crucial for coincidence measurements. All these material-dependent requirements are fulfilled for the normal conducting metal gold as an absorber material and gold or silver for the sensor. The latter needs to be doped with a few hundred ppm of the rare-earth

metal erbium in order to obtain paramagnetic properties. State-of-the-art MMCs of this kind were successfully developed in this working group, providing an exceptional energy resolution, a large energy bandwidth and fast signal rise times well below 100 ns [Kem18].

The detector geometry shown in figure 2.5 is widely used in this working group. Two meander-shaped pickup coils, each with an inductance  $L_m$ , are connected as a parallel gradiometer and are located beneath their respective sensor pixel, with a connection to the input coil of the flux transformer. As the SQUID is typically placed far away to mitigate the influence of its heat dissipation, a parasitic inductance  $L_{\text{par}}$  needs to be taken into account due to the wiring between the SQUID and the detector (figure 2.5, right). A constant small bias current  $I_0$  is injected into the pickup coils to provide the weak bias field needed for the paramagnet. If now a particle gets detected by one of the absorbers, a flux change  $\Delta\Phi$  and thus a current is induced in the meander. By applying Kirchhoff's first law and knowing that magnetic flux in a closed superconducting loop is conserved [Lan60], we can calculate the resulting current signal in the input coil to [Bur04]

$$\Delta I_{\text{det}} = \frac{\Delta\Phi}{L_m + 2(L_i + L_{\text{par}})} . \quad (2.14)$$

A significant advantage of this setup lies in the fact, that only one SQUID is needed to readout two pixels. The gradiometer geometry ensures, that depending on which absorber gets hit, the current  $\Delta I_{\text{det}}$  will flow in one direction or the other. Thus, every pixel can be assigned a polarity of the resulting flux in the SQUID loop, making them distinguishable. Together with the mutual inductance  $M_{\text{is}} = \frac{\Delta\Phi}{\Delta I_{\text{det}}}$  between the input coil and the SQUID loop, we can now introduce the flux-to-flux coupling

$$\frac{\Delta\Phi_s}{\Delta\Phi} = \frac{M_{\text{is}}}{L_m + 2(L_i + L_{\text{par}})} , \quad (2.15)$$

which is maximal for  $L_i = \frac{L_m}{2} + L_{\text{par}}$ . This crucial parameter is a measure for the flux gain and gives therefore information about the quality of the readout setup. Consequently, it is desirable to choose the optimal input coil inductance  $L_i$  for a given pickup coil inductance  $L_p = \frac{L_m}{2}$ .

### 2.3.1 Extrinsic Energy Sensitivity

In this regard it is reasonable to discuss one aspect of the noise behavior for this readout setup. Just like the flux change originating in the detector induces a flux change in the SQUID, it is also possible to happen the other way around. The

intrinsic flux noise of the SQUID can therefore also couple into the pickup coil, creating the apparent flux noise  $\sqrt{S_{\Phi_{s,p}}}$  in the detector. To quantify this noise influence, we define analogous to equation 1.38 the extrinsic energy sensitivity with respect to a given pickup coil inductance  $L_p$  as [Knu88]

$$\epsilon_p = \frac{S_{\Phi_{s,p}}}{2L_p} \quad (2.16)$$

Here, it is necessary to point out that the choice of a single coil with inductance  $L_p$  adds a factor of 2 to equations 2.14 and 2.15, such that the flux-to-flux coupling reads

$$\frac{\Delta\Phi_s}{\Delta\Phi_p} = \frac{M_{is}}{L_p + L_i + L_{par}} \quad . \quad (2.17)$$

This advantage, however, does not outweigh the above-mentioned benefit to readout two pixels with one read-out electronic, as the gradiometric design also cancels out unwanted signals through changes in external magnetic field or substrate temperature [Man21]. In the following we consider the case for a single pickup coil with inductance  $L_p = \frac{L_m}{2}$ . The presence of a flux transformer is accompanied with a shielding effect with respect to the SQUID loop, such that the geometric inductance is reduced to an effective inductance given by [Gro16]

$$L'_s = L_s(1 - k_{is}^2 s_{in}) \quad , \quad (2.18)$$

where  $s_{in} = \frac{L_i}{L_i + L_p + L_{par}}$  denotes the screening factor [Cla04], which reaches the value of 0.5 for the condition  $L_i = L_p + L_{par}$  that maximizes the flux-to-flux coupling. By now using the relation  $\frac{\Delta\Phi_s}{\Delta\Phi_p} = \frac{\sqrt{S_{\Phi_s}}}{\sqrt{S_{\Phi_{s,p}}}}$ , we finally obtain with equation 2.19 to 2.18 the extrinsic energy sensitivity [Knu88]

$$\epsilon_p = \frac{S_{\Phi_s}(L_i + L_{par} + L_p)^2}{2L_p M_{is}^2} = \frac{S_{\Phi_s}(L_i + L_{par} + L_m/2)}{L_s k_{is}^2 s_{in} L_m/2} \quad . \quad (2.19)$$

This parameter is minimized by maximizing the flux-to-flux coupling  $\frac{\Delta\Phi_s}{\Delta\Phi_p}$  as well as minimizing the intrinsic flux noise of the SQUID  $S_{\Phi_s}$  given by the expressions derived in subsection 1.2.4. Finding the minimum for the latter provides optimal values for the shunt resistors  $R_s$  and the SQUID loop inductance  $L_s$ , as we will see in the following section.

### 3. dc-SQUID Design

The main objective for this thesis was the optimization of an existing Front-End SQUID design for an improved coupling to one of the detectors being developed in this working group. As we have seen in the previous section 2.3, adjusting  $L_i$  to the detector coil ensures the maximization of the flux-to-flux coupling and therefore minimizes the extrinsic energy sensitivity. The previous SQUID design exhibits a design value of  $L_i = 1.64$  for the input coil inductance [Bau22], which is well suited for the pickup coil inductance of the ECHo-100k detector of  $L_p = 1.14\text{ nH}$  [Man21]. The parasitic inductance  $L_{\text{par}}$ , that arises from the aluminum bonds between the SQUID and the detector substrate to form the flux transformer, has been estimated to  $0.5\text{ nH}$  [Hen17]. Other MMC detectors from this working group such as the 4k-pixel molecule camera MOCCA and the X-ray detector maXs100 require higher input inductances, as their pickup coil inductances are  $L_p = 8.8\text{ nH}$  and  $L_p = 6.65\text{ nH}$ , respectively. In [Bau22] SQUIDs with matching input inductances for the MOCCA and maXs100 detector were developed for the first time using an intermediary coupling transformer. These improved the energy resolution  $\Delta E_{\text{FWHM}}$  of the detectors, although the effect was minimal for the latter due to a reduction of the effective coupling constant  $k'_{\text{is}}$ , compensating the gain through an increased input inductance of  $L'_i = 5.47\text{ nH}$ . Specifically for the maXs100 detector, a different approach was followed in the framework of this thesis to achieve a better coupling while avoiding a significant increase of the detector noise.

#### 3.1 dc-SQUID with a Two-Turn Input Coil

Increasing the input coil inductance can be realized either by changing the geometry of the coil itself, or by implementing a double flux transformer structure, with the benefit of easily adapting the inductance independently of the SQUID design. The latter, however, was accompanied with a reduction of the effective coupling constant  $k'_{\text{is}}$  in the work of [Bau22] regarding the maXs100 detector readout, which led to a lower flux-to-flux coupling despite the higher inductance of  $L'_i = 5.47\text{ nH}$ . Only the white noise reduction of the SQUID, which was caused by the shielding effects of the added flux transformer, resulted in a small overall improvement of  $\epsilon_p$  and  $\Delta E_{\text{FWHM}}$ . In this work we designed a new detector SQUID with *window-type*<sup>1</sup> Josephson junctions, which is based on the design developed in [Bau22]. A schematic of this SQUID is shown in figure 3.1. Four large oval loops form the second order gradiometer de-

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<sup>1</sup>A well-established microfabrication technique to produce Josephson junctions for SQUIDs

scribed in subsection 2.1.1, where the lower fabricated niobium layer (Nb1) contains the SQUID loop as a parallel gradiometer. The feed line coming from the top supplies the serially connected input coil, which was fabricated as a second niobium layer (Nb2) on top of the SQUID loop, only separated by an insulating  $\text{SiO}_2$  layer. This geometry allows to combine a small SQUID loop inductance  $L_s = \frac{L_1}{4}$  with a large input inductance  $L_i = 4L_1$ . Also in the Nb2 layer, another serial gradiometer of second order representing the feedback coil is located below the input coil, whose feed lines are shown on the bottom left. This coil has a design inductance of  $L_f = 336 \text{ pH}$  with a line width of  $3 \mu\text{m}$ . Both coils exhibit the same geometry as the corresponding underlying washer strip to maximize the overlap and therefore the coupling. At the same time, the proximity to the input coil was kept at a minimum to minimize cross talk between the two coils. The feed lines for the SQUID loop at the bottom center lead to the junction area, which is shown in the zoomed in section. Both square-shaped Nb/Al-AlO<sub>x</sub>/Nb junctions are realized with the dimensions  $4.5 \mu\text{m} \times 4.5 \mu\text{m}$  and a targeted critical current of  $I_c = 6 \mu\text{A}$ , leading to a critical current density of  $j_c = 30 \frac{\text{A}}{\text{cm}^2}$ . Two AuPd junction shunt resistors  $R_s$  are located on the left and right side of the junction area, respectively. Both are attached to a large heat sink made of two gold layers, a thick galvanized layer on top of a sputtered, thin one. These so-called *cooling fins* provide thanks to their large volume a better electron-phonon coupling, which reduces the electron temperature of the normal-conducting shunts and thus mitigates the corresponding thermal noise [Maz24]. A third AuPd resistor  $R_d$  with the same dimensions is placed above the junction area and connected in parallel with the washer loop. This so-called *washer shunt* provides damping properties to reduce quality factors of parasitic resonances, as will be discussed in section 3.3.

As opposed to the previous design, this input coil is realized with two turns instead of one. Neglecting the stripline inductance  $L_{\text{str}}$  (see section 4.2), the input inductance becomes approximately proportional to the number of turns squared  $n^2$  [Ket81, Jay81], giving a theoretical value of  $L_i^{\text{theo}} \approx 2^2 \cdot 1.64 \text{ nH} \approx 6.56 \text{ nH}$ . To implement the second turn a wider washer loop line width of  $w_s = 10 \mu\text{m}$  was needed. The line width of the input coil  $w_i$  with two turns remained at  $3 \mu\text{m}$ , which would have not been possible to fabricate on top of the previous washer width of  $w_s = 5 \mu\text{m}$  without sacrificing coupling strength. A general increase in  $w_i$ , however, entails the risk of capturing noise inducing flux vortices. These form if the B-field pointing perpendicular to the SQUID plane exceeds a critical field given by [Kui08]

$$B_{v,\text{crit}} = 1.65 \frac{\Phi_0}{w_s^2} . \quad (3.1)$$

At the earths surface, its magnetic field reaches a maximum amplitude of  $65 \mu\text{T}$ ,

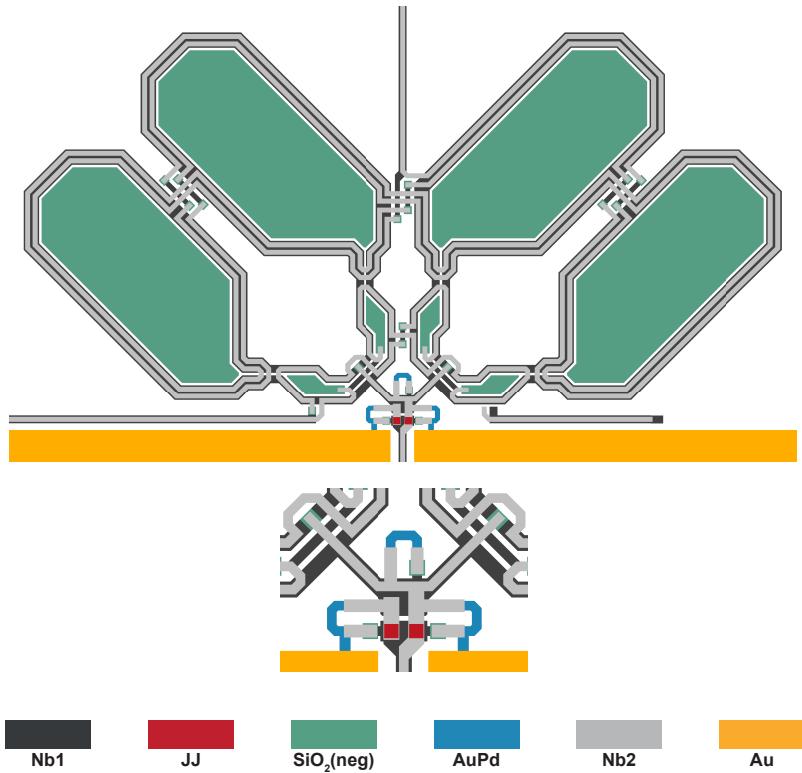


Figure 3.1: Halb klayout-halb foto wie bei fabian?.. Schematic of the new planar dc-SQUID design with a two-turn input coil, including a zoom into the junction area (bottom). Four large oval-shaped loops form the microstrip transmission line structure given by the washer SQUID loop in the lower niobium layer Nb1 and the input coil in the upper niobium layer Nb2. Fabricated in the same manner is a small feedback coil between the junction area and the input coil. Both coils are free of  $\text{SiO}_2$ , visualized through the inverse drawing of the  $\text{SiO}_2$  layer.

which would give a threshold width of  $w_s = 7.2 \mu\text{m}$ . The dilution refrigerators used to cool down both the SQUIDs and the detectors typically provide magnetic shielding, such that we consider a width of  $w_s = 10 \mu\text{m}$  to have a negligible impact on the flux noise attributed to flux vortices. As the input inductance was the only parameter necessary to adjust, we attempted to keep the other design parameters unaltered. The widened washer width would therefore need to be compensated with an approximately 15 % larger washer hole circumference in order to maintain the same SQUID loop inductance. For the sake of safety, the circumference was increased only by 10 % to prevent possible hysteretic behavior through a too large SQUID loop inductance and therefore screening parameter  $\beta_L$ . The increase was estimated by modeling the rather complicated oval washer loop geometry as a ring-shaped structure whose in-

ductance can be calculated with the relation  $L = \mu_0 R \left( \ln \left( \frac{8R}{a} \right) - 2 \right)$ , where  $R$  denotes the loop radius and  $a$  the radius of the wire [Den16]. In addition to these geometric adjustments, a comprehensive structuring of the SiO<sub>2</sub> layer in the area of the washer loops was also omitted, allowing the empty interior to remain free from the insulator layer. This is intended to prevent potential flux noise induced by unavoidable magnetic impurities in the SiO<sub>2</sub>. The same method was applied to the smaller loops formed by the feedback coil. The absence of insulation within the loops is visualized by the green-colored areas in figure 3.1.

The design values for  $R_s$  and  $L_s$  can be obtained by minimizing the extrinsic energy sensitivity given in equation 2.19, which requires the maximization of the flux-to-flux coupling ( $L_i = L_p + L_{\text{par}}$ ) and the minimization of the intrinsic white noise of the SQUID. In [Bau22] this numerical calculation led to the optimal parameters  $\beta_C = 0.7$  and  $\beta_L = 0.86$ , which were given the restraints  $\beta_C \leq 0.7$  and  $\beta_L \leq 1$  to avoid hysteretic behavior. Consequently, additional noise through voltage jumps caused by hysteretic IVCs as well as Nyquist noise from higher harmonics of the Josephson frequencies [Cla96] have been neglected, which was not the case for the derivation of equation 1.40. The intrinsic white noise of the SQUID used for the numerical calculation is therefore given by [Knu88]

$$S_{\Phi_s} = 2k_B T \frac{L_s^2}{R_s} \left[ (1 - k_{is}^2 s_{\text{in}})^2 + \frac{\sqrt{2}(1 + \beta_L)^2}{\beta_L} \right] . \quad (3.2)$$

The targeted critical current of  $I_c = 6 \mu\text{A}$  provides with the given junction dimensions a critical current density of  $j_c = 29.63 \frac{\text{A}}{\text{cm}^2}$ , which can be used to calculate the junction capacitance  $C$  by using the empirical relation  $\frac{1}{C'} = p_1 + p_2 \log_{10} j_c$  [Mae95]. Here, the intrinsic capacitance  $C'$  excludes any parasitic capacitances arising from the window-type fabrication technique. For simplicity reasons, we assume  $C \approx C'$  and thus obtain  $C = 0.95 \text{ pF}$ . The optimal Stewart McCumber and screening parameter then provide the values  $R_s = 6.3 \Omega$  and  $L_s = 147 \text{ pH}$ , respectively. The designed shunt resistor in both the previous and the new design was rounded to  $6 \Omega$ , which results with Ohm's circuit law in a normal resistance of  $R_n = 3 \Omega$  for the whole SQUID. This consequently corresponds to a slightly lower damping parameter of  $\beta_L = 0.62$ . The coupling constant was set to an upper limit of  $k_{is} = 0.75$ , which is typically the highest achievable value for the SQUIDs produced in this working group. Under the assumption of  $k_{is}$  being maximal and  $L_i = L_p + L_{\text{par}} = 7.15 \text{ nH}$  for the maXs100 detector read-out, the theoretically obtainable flux-to-flux coupling regarding a single pickup coil is  $\frac{\Delta\Phi_s}{\Delta\Phi_p} = 5.38\%$ . Lastly, for the extrinsic energy sensitivity we would obtain with equation 3.2  $\epsilon_p = 0.53 \text{ h}$ , given the typical detector operation temperature of  $T = 20 \text{ mK}$ .

## 3.2 Integrated Two-Stage Chip

2

*Figure:* Int. 2stage chip (klayout and microscope?)

Briefly cover its features

## 3.3 Damping Methods

As discussed in subsection 1.2.5, several SQUID parameters can be optimized to mitigate the influence of various resonances in the circuit. However, most of these were determined by the minimization of the extrinsic energy sensitivity (see section 3.1), which imposes substantial limitations on the extent to which additional parameter modifications could be implemented. For instance, increasing the length of the input coil  $l_i$  would shift the corresponding strip line resonance given by equation 1.45 away from the operation frequency, which in turn would lead to a larger input inductance, thus impeding the maximization of the flux-to-flux coupling. Furthermore, even resonances far away from the operation frequency can result problematic as thermally activated transitions between different states increase the noise level [Sep87]. This motivates to follow a more practical approach to suppress  $LC$  resonances, which can be realized through damping with attenuators, such as the damping resistor  $R_d$  shown in the top of junction area in figure 3.1. These are typically connected in parallel to the resonant circuit, as this reduces the quality factor  $Q$  of the corresponding  $LC$  resonance given an appropriate dimensioning of the resistor. Consequently, the  $L_sC_p$  and  $L_sC$  resonances can be damped by implementing a *washer shunt*  $R_d$  [Ono97, Ryh92]. Although the IVC intersection caused by the latter cannot be eliminated, previous works in this group showed a significant smoothing of the curves as the accompanying step structures decreased [Bau18]. The current noise introduced through this resistor, on the other hand, deteriorates the energy sensitivity and thus limits the damping benefit. However, this effect is minimal for the condition  $R_d \approx R_s$  with  $\beta_L = 1$  [Enp86, Ryh92], which is why we choose  $R_d = 6\Omega$ . The input circuit is shunted with an  $R_xC_x$  attenuator to damp the  $L_iC_p$  resonance, where the added capacitance  $C_x$  blocks low frequency current noise [Sep87]. Both damping techniques to suppress  $C_p$ -related resonances were proven to be effective both in literature and our working group [Sep87, Knu87, Enp86, Can91, Bau18, Bau22]<sup>3</sup>. As for the  $\lambda/2$  resonances, both the  $R_d$  and the  $R_xC_x$  attenuator provide good damping as well [Can91], since they terminate the microstrip lines and thus avoid impedance mis-

<sup>2</sup>wird evtl weg gelassen und dann nur in ein, zwei sätzen erwähnt

<sup>3</sup>overkill? besser weniger/keine quellen oder satz ganz streichen?

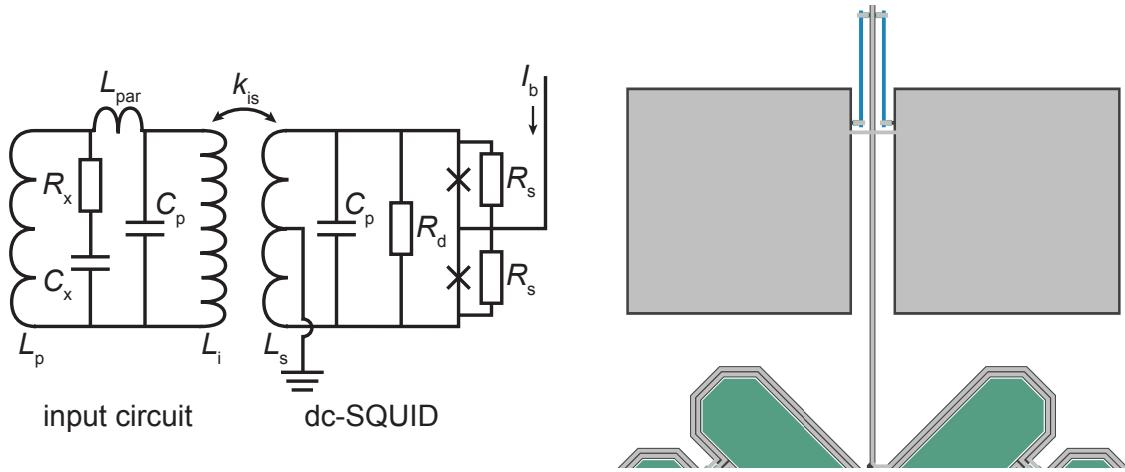


Figure 3.2: Halb layout-halb foto wie bei fabian? ... Left: Schematic circuit of a coupled dc-SQUID with damping components  $R_d$  and  $R_xC_x$ . The input coil forms a flux transformer with a pickup coil of inductance  $L_p$ . The parasitic capacitance between the input circuit and the SQUID loop is connected in parallel to both  $L_i$  and  $L_s$ . Right: Structure design of the  $R_xC_x$  attenuator located above the dc-SQUID. Both devices  $R_x$  and  $C_x$  are each realized as two parallel components.

matches given a suitable dimensioning of the resistors (see section 4.2). A schematic of the resulting circuit diagram of the coupled dc-SQUID with all damping components is depicted in figure 3.2 (left). Shown on the right is the design of the  $R_xC_x$  shunt, which has been adapted from [Bau22]. As the needed capacitance  $C_x = 10 \text{ pF}$  is rather large, two parallelly connected square-shaped capacitors with  $\frac{C_x}{2}$  are used to reduce the needed space. The top and bottom plate of each capacitor are fabricated in the Nb2 and Nb1 layer, respectively. The shunt  $R_x$  is split as well into two parallel AuPd resistors with  $2R_x$  each. Optimal values for these components will be discussed in chapter 4.

### 3.3.1 Lossy Input Coil

Several approaches to reduce  $Q$  values of  $\lambda/2$  resonances associated with microstrip lines have been investigated in [Boy22]. Two parallel meanders fabricated on top of each other in a direct-coupled MMC setup<sup>4</sup> produced high  $Q$  resonances at integer and half-integer wavelengths. Whereas placing an individual resistor in parallel to one meander only damped the half-integer modes, a more distributed damping scheme in the form of an insulated gold layer between both meanders provided strong

<sup>4</sup>As opposed to the flux transformer setup in section 2.3, the SQUID is directly coupled to the sensor.

damping of all microstrip resonances while maintaining a low detector noise level. The noise and the  $Q$  values have been further reduced by structuring the Au layer with the same geometry as the microstrip lines and electrically connecting it to one of the meanders, thereby preventing large noise inducing, normal-conducting loops. The attenuation constant  $\alpha$  of this lossy microstrip line increased with frequency and thickness of the Au layer, while the noise stayed insensitive upon thickness changes. Within the scope of this thesis, such damping techniques were implemented and tested on the dc-SQUID described in section 3.1. For this, we sputtered a gold layer between  $\text{SiO}_2$  and Nb2 with the same geometry as the input coil running above the SQUID loop. This gold layer was omitted around the vias between the washer loops to avoid additional normal resistances. For fabrication reasons, gold and niobium were structured together as a bilayer in the same microfabrication step by sputtering the upper niobium layer directly after the gold. To connect the four input coil segments over vias, the original second niobium layer Nb2 was used. A schematic of our Front-End design with this lossy input coil, including an image obtained with an optical microscope, is shown in figure 3.3. Our Front-End SQUIDs with this gold layer adjacent to the input coil will be referred to as 'lossy' for the upcoming discussions.

### 3.3.2 Inductive Damping

Together with the attempt of suppressing resonances through a lossy microstrip line, we introduce a second new damping technique denoted as inductive damping. This method is based on the principle of magnetic damping, where a change in magnetic flux creates eddy currents in a nearby conductor, which following Lenz's law induces a flux trying to compensate the initial one. The flux change is therefore damped by effectively transferring part of the magnetic energy to the kinetic energy of the induced current, which in the case of a normal conductor emits heat. This phenomenon also causes the reduction of the geometric inductance of the SQUID loop through the shielding effect of the flux transformer (see subsection 2.3.1). A strong indicator that this mechanism could be applied to SQUIDs showed the first experiment in [Boy22], where a square gold layer representing the MMC sensor was placed at a height of 300 nm above an isolated meander. This lead to a significant reduction of the high  $Q$  values of the meander modes. The concept is now applied to the feed lines on our SQUID chip, where large gold pads have been placed across the SQUID washer and feedback coil feed lines, as shown in figure 3.4. The sharp voltage spikes associated with high  $Q$  resonances, that might be present within those lines would therefore be damped by partly converting their energy into heat in the normal conducting gold layer. The gold pads are fabricated in the same layers as the heat sinks for the shunt resistors and are consequently sputtered first before being

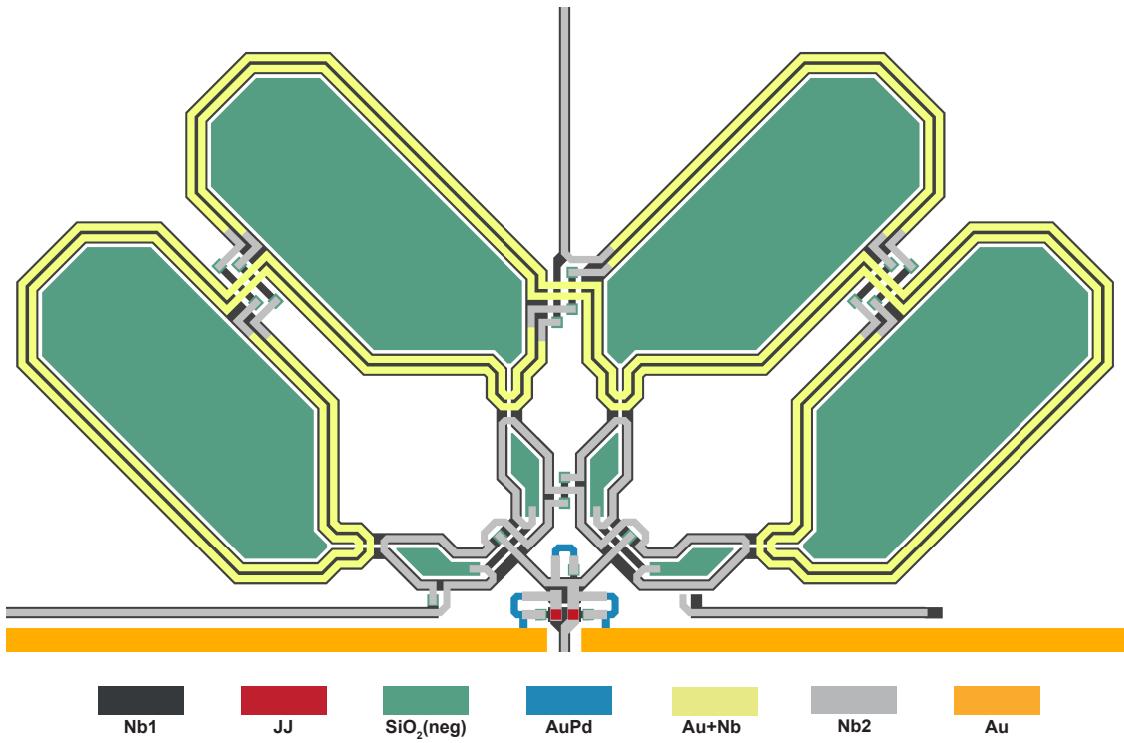


Figure 3.3: Halb klayout-halb foto wie bei fabian? ... dc-SQUID design with a two-turn input coil realized as a lossy microstrip line. A gold layer is structured between the insulating SiO<sub>2</sub> and an upper niobium layer and has been fabricated in a single step as a bilayer containing both gold and niobium. The overlying and gold-free niobium in the Nb2 layer connects all four input coil segments over their respective washer loop to prevent the vias to acquire a normal resistance.

electroplated. The latter step significantly increases their height and therefore volume, which allows for larger and more effective eddy currents. Additionally, the generated heat is expected to better dissipate into the chip substrate as the electron-phonon coupling increases with volume. All feed lines are typically realized as microstrip lines, which for the gold pads needed to be adjusted into a coplanar structure as there is no insulation layer after Nb2. Figure 3.4 shows the chip design consisting of four distinct Front-End variants, each provided with the inductive damping scheme. The first channel at the top is realized without an input coil in order to better allocate possible resonance structures visible in the SQUID's IVCs. The design introduced in 3.1, also referred to as 'non-lossy', is represented in channel 2, followed by the lossy variant in channel 3. The last channel contains a lossy Front-End as well, however, the washer loop interiors were not kept SiO<sub>2</sub>-free. This allows to investigate the influence of possible magnetic impurities in the insulation material and assess whether it can be regarded as negligible or not. The same chip design

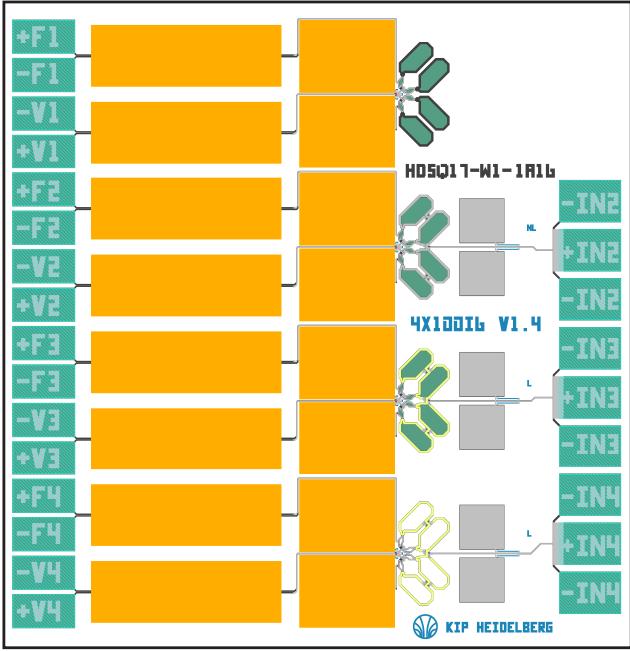


Figure 3.4: Inductive damping scheme on the Front-End SQUID chip of the type 4x100i6 v1.4. All feedback coil and SQUID loop feed lines are covered with large rectangular, insulated gold pads. Each SQUID channel is occupied with a different SQUID variant. Channel 1: Front-End without an input coil. Channel 2: Non-lossy SQUID design presented in section 3.1. Channel 3: Lossy SQUID design from section 3.3.1. Channel 4: Lossy design with SiO<sub>2</sub> inside the washer loop interiors.

has been used without gold pads on the feed lines, resulting in 8 different Front-End SQUIDs that were developed and tested within the scope of this thesis. Some first results are presented in the following chapter.



## 4. Experimental Results

This chapter will provide an overview over the general performance of the first stage dc-SQUIDs developed within the framework of this thesis. Particularly, the overall resonance behavior and the noise spectra were investigated. We will begin with a summary of characteristic parameters obtained by our measurements and compare them with the target values. The SQUIDs were produced in the institute's cleanroom and then tested both in a single- and a two-stage setup as described in section 2.2. The single-stage measurement was carried out at  $T = 4.2\text{ K}$  in a liquid helium transport vessel as well as in a dilution cryostat with  $T = 10\text{ mK}$ . The former submerges the SQUIDs in liquid helium via a dipstick, which provides a sample holder for PCBs. The SQUIDs are glued onto those PCBs and can be electrically connected to them through aluminum bond wires, by utilizing the bond pads shown around the border of the chip in figure 3.4. The sample holder is equipped with both a superconducting (niobium) and a soft-magnetic shield to suppress external magnetic fields. The read-out is enabled by connecting the broadband SQUID electronic of the type XXF-1 (see subsection 2.2.1) to the dipstick and using it to both supply the necessary bias and ramp current signals to the SQUID as well as provide the FLL feedback circuit. The SQUID electronic is then controlled via software and the voltage output observed on a keysight InfiniiVision<sup>1</sup> oscilloscope. The noise measurement has been conducted with the two-stage setup at  $T = 10\text{ mK}$  in the cryostat, whereas the single-stage measurements to obtain characteristic Front-End properties have been done both in the helium vessel and the cryostat.

### 4.1 Characteristic dc-SQUID Parameters

Despite all above-mentioned distinctions between the Fron-End variants, most characteristic parameters such as the SQUID loop inductance  $L_s$  or shunt resistance  $R_s$  are unaffected by these variations. We therefore consider the following measurements to be representative for all variants and assume the errors to stem from fabrication-related deviations across the wafer.

To qualitatively explain the following considerations, we present the current-voltage as well as the voltage-flux characteristics of one of the measured SQUIDs, shown in figure 4.1. This SQUID with the label HDSQ17-W1-3C16 is of the type 'non-lossy' with inductive damping and has been measured at  $T = 10\text{ mK}$ . We start the characterization by measuring the slope of the IV curves across the ohmic regime, which

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<sup>1</sup>Keysight Technologies Deutschland GmbH, Herrenberger Straße 130, 71034 Böblingen

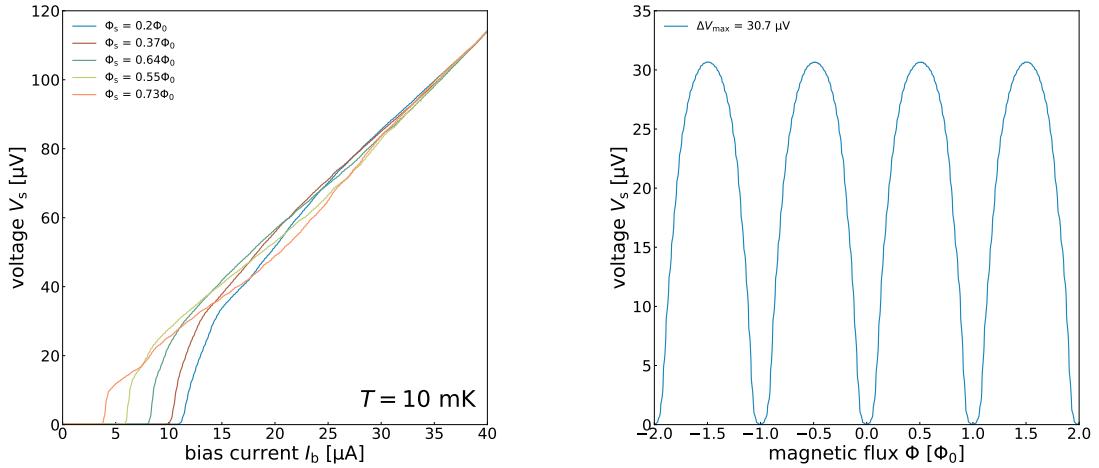


Figure 4.1: Current-voltage (left) and voltage-flux characteristics (right) of a non-lossy SQUID with inductive damping, labelled HDSQ17-W1-3C16. The family of IV curves are measured by varying the bias current for either the input or the feedback coil, while a current ramp signal is driven through the SQUID. To obtain the corresponding  $V\Phi$  characteristic, the roles for the bias and ramp signal are switched. The curve was measured at  $I_b^{\max} = 11.33 \mu\text{A}$ , resulting in a maximal voltage swing  $\Delta V_{\max} = 30.7 \mu\text{V}$ . Note that both plots have different scaling on the y-axis.

corresponds to the normal resistance  $R_n$  of the SQUID. Including the SQUID chosen for figure 3.4, we measured 10 SQUIDs from the wafer HDSQ17-W1 covering all above-mentioned variants. The median normal resistance taken from these SQUIDs yielded the value  $R_n = 3 \Omega$ . This corresponds to a shunt resistance  $R_s = 2R_n$  of  $R_s = 6 \Omega$ , which exactly corresponds to the targeted value of  $R_s = 6 \Omega$ . The maximum voltage swing as seen in figure 4.1 (right) is obtained by applying a bias current  $I_b^{\max}$  supplied by the SQUID electronic, while driving a ramp signal through one of the coupled coils to provide a varying external flux. This yielded median values of  $\Delta V_{\max} = 29.95 \mu\text{V}$  and  $I_b^{\max} = 11.62 \mu\text{A}$ . For  $T = 0 \text{ K}$ ,  $I_b^{\max}$  should correspond to twice the critical current  $I_c$  (compare figure 1.9). Due to the finite temperature we obtain noticeable thermal smoothening visible in the IVC of figure 4.1 (left), such that the critical current is better approximated by [Dru96b]

$$I_c \approx \frac{I_b^{\max}}{2} + \frac{k_B T}{\Phi_0} \left( 1 + \sqrt{1 + \frac{I_b^{\max} \Phi_0}{k_B T}} \right) . \quad (4.1)$$

We therefore calculate the critical current to<sup>2</sup>  $I_c = 5.84 \mu\text{A}$ . This corresponds to a critical current density of  $j_c = 28.8 \frac{\text{A}}{\text{cm}^2}$ , such that both values only deviate 2.7% from the design values. The junction capacitance thus becomes slightly smaller than the design value according to the empirical relation introduced in section 3.1. Using the fitted function  $\frac{1}{C'} = 0.132 - 0.053 \log_{10} j_c$  obtained in [Bau22] and by again assuming  $C' = C$ , the capacitance yields  $C = 0.948 \text{ pF}$ , which is in excellent agreement with the target value of 0.95 pF. The McCumber parameter can now be calculated to  $\beta_C = 0.61$ , also fitting well with the design value of 0.62. The screening parameter is typically derived by using the  $\beta_L$ -dependent normalized current swing  $\frac{\Delta I_{\max}}{2I_c}$  at  $V = 0$ , which has been numerically simulated in [Tes77]. We estimated the current swing by extrapolating both extremal IV curves for the theoretical case of  $T = 0$ , thus neglecting the thermal rounding and obtaining a minimal and maximal critical current  $I_{c,1}$  and  $I_{c,2}$ . For our SQUIDs, the extremal IV curve with the lower critical current  $I_{c,1}$  does not correspond exactly to  $\Phi = (n + \frac{1}{2}\Phi_0)$  (compare figure 4.1), which is a consequence of an asymmetric current injection (see below). The median of the maximal current swing  $\Delta I_{\max}$  therefore yields  $7.18 \mu\text{A}$ , ranging from  $6.09 \mu\text{A}$  to  $7.73 \mu\text{A}$  for the lowest and highest measured value. The value for  $\Delta I_{\max}$ , which approximately corresponds to the current swing of the voltage-biased SQUID in a two-stage configuration, indicates that the SQUID is well adapted to the arrays produced in this working group [Kem15]<sup>3</sup>. These arrays have a typical reciprocal mutual inductance of  $\frac{1}{M_{ix}} = 11.7 \frac{\mu\text{A}}{\Phi_0}$ , such that the flux  $\Delta\Phi_x$  coupled to the array is  $\Delta\Phi_x = M_{ix}\Delta I_{\max} = 0.61\Phi_0$ , which is close to the optimal value of  $\frac{\Phi_0}{2}$  to achieve a flux gain of  $G_\Phi \approx 3$  (see section 2.2.2). The current swing allows us to calculate the screening parameter to  $\beta_L = 0.60$ , which is rather low compared to the design value of  $\beta_L = 0.86$ . Hysteretic behavior related to the obtained  $\beta_L$  and  $\beta_C$  should nevertheless not occur and thus be absent in the IVCs. The intersections and current steps seen in figure 4.1 likely stem from resonances, which will be discussed in section 4.2.

With  $I_c = 5.84$  and  $\beta_L = 0.60$  we obtain a SQUID loop inductance of  $L_s = 103 \text{ pH}$ , which fits very well with a simulated value of  $L_s = 106 \text{ pH}$ . This significantly deviates from the design value of  $L_s = 147 \text{ pH}$ , which can be partly explained by the conservative estimation for the geometric adjustment of the washer loop size, tendentially resulting in a smaller inductance. Another reason is that the SQUID loop inductance of the previous design resulted in a lower value than intended in [Bau22]. On the wafer HDSQ17-W1 we included for comparison the original design with a

<sup>2</sup>Calculated with the median of all 10 Ic's. If I calculate Ic once with the median of Ib,max, then the result is Ib,max/2 ≈ 5.81. Which is better? For the above argument, 5.84 fits better.

<sup>3</sup>Die arrays in diesem paper sind andere als die die wir nutzen und haben  $M^{-1} = 12.9 \mu\text{A}/\phi_0$ . Wo finde ich veröffentlichtungen zu HDSQ15w3 arrays?

single turn input coil, where we measured two channels of the chip HDSQ17-W1-3A09 alongside the other 10 new SQUIDs discussed in this section. These provided the respective values  $L_s = 127 \text{ pH}$  ( $\beta_L = 0.80$ ) and  $L_s = 138 \text{ pH}$  ( $\beta_L = 0.87$ ), which further supports the assumption of the original SQUID loop design being too small. A better adjustment of  $L_s$  can be realized in future works.

The mutual inductances of the input and feedback coil were obtained by both sending a bias current to the SQUID loop and a current ramp signal to the coils. The input coil was only connected to the SQUID electronic during the measurements in the helium vessel at 4.2 K, where we calculated  $M_{is}$  of 22 Front-End channels from 9 chips, covering all non-lossy and lossy variants. Counting the obtained voltage oscillations per given current ramp amplitude leads to the measured median values  $M_{is} = 611 \text{ pF}$  and  $M_{fs} = 51 \text{ pF}$ . The latter coincides well with the measured value of 50 pF from [Bau22]. The input coil current sensitivity is almost twice as large as the value determined with the previous design with a single-turn input coil, which yielded  $M_{is} = 328 \text{ pF}$  [Bau22]. This result can be understood by the general linear dependence of the mutual inductance from the number of turns  $n$  of the input coil and the SQUID loop inductance, i.e.  $M_{is} \approx nL_s$  [Ket81]. This expression would need to be multiplied by a factor of 4 due to the parallel connection of the four washer loops. The result doesn't match the lower measured value of the previous design for  $n = 1$ , however, one needs to account for the parasitic inductance at the junction area that doesn't contribute to  $M$ , which in turn reduces the mutual inductance. The fact that  $M_{is} \approx 2 \cdot 328 \text{ pF} > 611 \text{ pF}$  can be explained by two reasons. On one hand, the inductance  $L_s$  has been reduced by approximately 20 % as compared to the previous design, which according to equation 2.1 leads to a decreased current sensitivity  $M_{is}$ . On the other hand, the geometric two-turn input coil structure might deteriorate the coupling constant  $k_{is}$ , as compared to a single turn design. This parameter will be determined together with the input coil inductance in the following subsection 4.1.1.

As for the transfer coefficients, we obtained different values for the positive and negative slopes of the  $I\Phi$  curve. This is attributed to the intended asymmetric current injection, which is realized by differing SQUID loop inductances between each arm of the loop, ultimately leading to a unilaterally larger transfer coefficient  $I_\Phi$  [Fer15]. The values are  $I_{\Phi,+} = 10.2 \frac{\mu\text{A}}{\Phi_0}$  and  $I_{\Phi,-} = 23.1 \frac{\mu\text{A}}{\Phi_0}$  for the positive and negative slope, respectively. The voltage transfer coefficient is approximately symmetric and yielded  $V_{\Phi,+} = 94.0 \frac{\mu\text{V}}{\Phi_0}$  and  $V_{\Phi,-} = 93.0 \frac{\mu\text{V}}{\Phi_0}$ . Except for  $I_{\Phi,+}$ , the variance of these transfer parameters was rather large, with deviations from the median of up to 39 %. These results are nevertheless comparable to previous SQUIDs produced in this working group.

### 4.1.1 Input Coil Inductance

To determine whether the new SQUID design with increased input inductance exhibits better coupling to the maXs100 detector, it is essential to measure  $L_i$ . For this measurement, the input coil is electrically shorted via its bond pads with aluminum bond wires while the Front-End is in a two-stage setup at 4.2 K. In this case, the detector SQUID is in an integrated chip that contains both the new non-lossy Front-End design as well as an array for the second stage, which has been developed and tested by [Kra23]. These integrated chips were produced on the wafer labeled as HDSQ16-W1 and were designed with the same material thicknesses, such that we expect the following considerations to be representative for the non-lossy SQUIDs from HDSQ17-W1. The aluminum bond wires are normal conducting at these temperatures and thus provide a resistance  $R_{\text{bond}}$ . This closed loop couples via  $M_{is}$  thermal noise from the resistive wires into the SQUID loop, which is added to the apparent noise of the first stage SQUID. The resistance  $R_{\text{bond}}$  forms together with the total inductance  $L_{\text{tot}}$  consisting of the input coil, its feed line and the wire inductance an  $RL$  lowpass filter. The attributed cut-off frequency damps higher noise frequencies up to the point where this noise contribution becomes negligible, as can be seen in the measured noise spectrum of figure 4.2. The second drop at even higher frequencies represents the lowpass characteristic of the SQUID electronic, as it provides a limited bandwidth of up to 7 MHz<sup>4</sup>. The total apparent noise of the Front-End SQUID is then given by

$$S_{\Phi_s, \text{SQ}} = M_{is}^2 \frac{4k_B T}{R_{\text{bond}}} \left[ \frac{1}{1 + (\frac{2\pi f L_{\text{tot}}}{R_{\text{bond}}})^2} \right] + S_{\Phi_s, \text{w}} , \quad (4.2)$$

where the first term describes the added current noise of the shorted circuit, which is schematically shown in the inset of figure 4.2. The second term represents the apparent white noise of the SQUID. This expression is numerically fitted to the measured data to obtain both the white noise component  $M_{is}^2 \frac{4k_B T}{R_{\text{bond}}}$  and the cut-off frequency  $f_c = \frac{R_{\text{bond}}}{2\pi L_{\text{tot}}}$ . Two measurements from separate chips (2C14 and 2C23) were carried out, where the one from 2C14 is depicted in figure 4.2. The fits resulted in  $R_{\text{bond}}^{2C14} = 2.07 \text{ m}\Omega$  and  $L_{\text{tot}}^{2C14} = 6.86 \text{ nH}$  for the chip 2C14, whereas  $R_{\text{bond}}^{2C23} = 1.86 \text{ m}\Omega$  and  $L_{\text{tot}}^{2C23} = 6.74 \text{ nH}$  for the chip 2C23. It has been shown in [Hen17], that the aluminum wires exhibit an inductance of  $L_{\text{bond}} = 0.14 \frac{\text{nH}}{\text{m}\Omega}$ , which consequently results in  $L_{\text{bond}}^{2C14} = 0.29 \text{ nH}$  and  $L_{\text{bond}}^{2C23} = 0.26 \text{ nH}$ . The other parasitic inductance that stems from the input coil feed lines can be estimated with the microstrip inductance per unit length [Cha79]

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<sup>4</sup>6 oder 7?

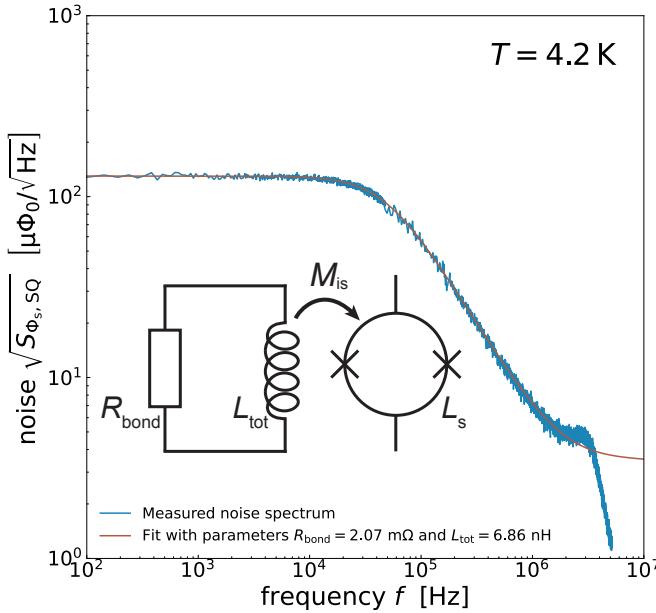


Figure 4.2: Noise spectrum of the input coil inductance  $L_i$  measurement. The input coil is resistively shorted through aluminum bond wires with resistance  $R_{\text{bond}}$ , which results in thermal noise that couples into the SQUID. The added noise has a cut-off frequency  $f_c = \frac{R_{\text{bond}}}{2\pi L_{\text{tot}}}$  that allows to extract the added white noise component by a numerical fit. The input inductance can be derived by subtracting the parasitic inductances from  $L_{\text{tot}}$ , which is a fit parameter alongside  $R_{\text{bond}}$ .

$$L_{\text{str}} = \mu_0 \frac{h + 2\lambda}{w_i + 2h + 4\lambda} , \quad (4.3)$$

with  $\lambda$  being the London penetration depth of the superconductor,  $w_i$  the width of the upper line and  $h$  the thickness of the insulating layer. The length of the feed lines is approximately  $724 \mu\text{m}$ , the width  $3 \mu\text{m}$  and  $\text{SiO}_2$  has been fabricated as a  $375 \text{ nm}$  thick layer. We also assume  $\lambda = 90 \text{ nm}$ , which is a typical value for the fabricated niobium in this working group. This yields a smaller contribution of  $0.12 \text{ nH}$  for the feed line inductance. Thus, we finally obtain the input inductances  $L_i^{2C14} = 6.45 \text{ nH}$  and  $L_i^{2C23} = 6.36 \text{ nH}$ . Even though these are close to the in the beginning of section 3.1 approximated  $L_i^{\text{theo}} \approx n^2 1.64 \text{ nH} \approx 6.56 \text{ nH}$ , we should take into account the smaller measured inductance  $L_i = 1.27 \text{ nH}$  for the single turn input coil [Bau22]. Furthermore, the microstrip inductance should not be neglected anymore as the length  $l_i$  of the input coil with two turns is now approximately twice as large.  $L_i$  is only linear proportional to the total microstrip inductance  $l_i L_{\text{str}}$  [Ket91], which would further reduce the increase of  $L_i$  upon adding a second turn. However, the enlargement of the washer loop has a stronger influence on  $L_i$  than on  $L_s$  due to the series connection of the input coil, such that the inductance increase per washer loop is multiplied by 4. This effect seems to narrowly compensate the two reductions, such that the measured values are only slightly below  $L_i^{\text{theo}}$ . With InductEx we obtained a simulated value of  $L_i = 5.7 \text{ nH}$ ?, which deviates 20 %? and 20 %? from  $L_i^{2C14}$  and  $L_i^{2C23}$ , respectively. These large deviations are in accordance with previous inductance simulations in several works of this working group [Fer15, Bau22]. For

the following discussions, we will use the average  $L_i = 6.40 \text{ nH}$  of the two measurements.

We have now all parameters to determine the coupling constant  $k_{is}$ . Using equation 2.1, we obtain  $k_{is} = 0.75$ , which corresponds to the target value that has been set as a realistic upper limit for the minimization of the extrinsic energy sensitivity  $\epsilon_p$ . This suggests that the reduction of the input coil current sensitivity  $M_{is}$  solely stems from the small measured SQUID loop inductance  $L_s$ . It is noteworthy, however, that the variance of both the measured critical currents and the current swings was rather large with deviating up to 11 % and 15 %, respectively. This of course provides an uncertainty for  $\beta_L$  and  $L_s$  that needs to be taken into account. Such variances have also been observed in [Bau22], where it was shown that SQUIDs with *cross-type* junctions exhibit a smaller variance across the wafer, making them more reliable than window-type junctions regarding  $I_c$ .

An overview of the most relevant measured parameters compared to their respective design values is shown in table 4.1.

| Parameter | $R_s$<br>[ $\Omega$ ] | $I_c$<br>[ $\mu\text{A}$ ] | $M_{is}$<br>[pH] | $M_{fs}$<br>[pH] | $L_s$<br>[pH] | $L_i$<br>[nH] | $\beta_L$ | $\beta_C$ | $k_{is}$ |
|-----------|-----------------------|----------------------------|------------------|------------------|---------------|---------------|-----------|-----------|----------|
| Measured  | 6                     | 5.84                       | 611              | 51               | 103           | 6.40          | 0.60      | 0.61      | 0.75     |
| Design    | 6                     | 6                          | -                | 50               | 147           | 6.56          | 0.86      | 0.62      | 0.75     |

Table 4.1: Summary of measured characteristic parameters of the new dc-SQUID design with a two-turn input coil, which are compared with the corresponding target values.

## 4.2 Resonance Behavior

Discuss in more detail all types of resonances with formulae and calculate the frequencies (there are still two of them that are unclear/need to be discussed in private or in a meeting)

*Figures:* All measured FE IVCs, like non-lossy, lossy, damping variants, iso, no-iso, no input coil, Fabiennes FE... (should I also show the corresponding VPhi curves?)

Try to identify the resonances in the plots. Discuss what actually helped damping and what not.

### 4.3 Noise Performance

#### 4.3.1 Lumped Element Two-Stage Setup

*Figures:* Show noise measurements at mK (3 setups from SQUID Cryo, 1 setup from Mocca Cryo)

Discuss all contributions, especially from the FE -> Difference if input coil is shorted or not?

Mention how it would change with a cross JJ SQUID (lower white noise) -> here or in the summary?

Are 2stage VPhi curves interesting or do we restrict ourselves to noise plots?

#### 4.3.2 Integrated Two-Stage Setup

*Figure:* Int. 2stage noise measurement

Summarize Fabians measurements and compare with ours.

## **5. Summary**



## Bibliography

- [Bar57] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Theory of Superconductivity, *Phys. Rev.*, **108**(5), 1175–1204, 1957.
- [Bau18] F. Bauer, *Optimierung von gekoppelten dc-SQUIDs durch die Unterdrückung parasitärer Resonanzen und die Etablierung eines neuen Herstellungsprozesses für Nb/Al-AlO<sub>x</sub>/Nb-Josephson-Kontakte*, Masterarbeit, Kirchhoff-Institut für Physik, Universität Heidelberg, 2018.
- [Bau22] Fabienne Bauer, *Rauscharme Stromsensor-dc-SQUIDs mit Impedanzanpassung für metallische magnetische Kalorimeter*, Dissertation, Kirchhoff-Institut für Physik, Universität Heidelberg, 2022.
- [Boy22] STP Boyd, G-B Kim, and S Friedrich, Radio-frequency resonances and damping in metallic magnetic calorimeter sensors, *Journal of Low Temperature Physics*, **209**(3-4), 534–539, 2022.
- [Bru82] J. J. P. Bruines, V. J. de Waal, and J. E. Mooij, Comment on: “DC SQUID: Noise and optimization” by Tesche and Clarke, *J. Low Temp. Phys.*, **46**, 383–386, 1982.
- [Bur04] Andreas Burck, *Entwicklung großflächiger magnetischer Kalorimeter zur energieaufgelösten Detektion von Röntgenquanten und hochenergetischen Teilchen*, Diplomarbeit, Kirchhoff-Institut für Physik, Universität Heidelberg, 2004.
- [Can91] R. Cantor, T. Ryhänen, D. Drung, H. Koch, and H. Seppä, Design and optimization of DC SQUIDs fabricated using a simplified four-level process, *IEEE Transactions on Magnetics*, **2**(2), 2927–2931, 1991.
- [Cha79] W. H. Chang, The inductance of a superconducting strip transmission line, *Journal of Applied Physics*, **50**(12), 8129–8134, 1979.
- [Cla88] J. Clarke and R. Koch, The Impact of High Temperature Superconductivity on SQUIDS, *Science*, **242**, 217–223, 1988.
- [Cla96] John Clarke, Squid fundamentals, in H. Weinstock (Ed.), *SQUID Sensors: Fundamentals, Fabrication and Applications*, 4–5, Kluwer Academic, Dordrecht, 1996.

- [Cla04] J. Clarke and A. I. Braginski (Ed.), *The SQUID Handbook: Vol. 1, Fundamentals and Technology of SQUIDs and SQUID Systems*, Wiley-VCH, 2004.
- [Dea61] B. S. Deaver and W. M. Fairbank, Experimental Evidence for Quantized Flux in Superconducting Cylinders, *Phys. Rev. Lett.*, **7**, 43–46, 1961.
- [Den16] R. Dengler, Self inductance of a wire loop as a curve integral, 2016.
- [Dru96a] D. Drung, Advanced SQUID read-out electronics, in H. Weinstock (Ed.), *SQUID sensors: Fundamentals, Fabrication and Application*, NATO ASI Series E329. Dordrecht: Kluwer, 1996.
- [Dru96b] D. Drung, F. Ludwig, W. Müller, U. Steinhoff, L. Trahms, H. Koch, Y. Q. Shen, M. B. Jensen, P. Vase, T. Holst, T. Freloft, and G. Curio, Integrated  $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$  magnetometer for biomagnetic measurements, *Appl. Phys. Lett.*, **68**, 1421–1423, 1996.
- [Dru02] D. Drung, High-performance DC SQUID read-out electronics, *Physica C: Superconductivity*, **368**(1-4), 134–140, 2002.
- [Dru06] D. Drung, C. Hinnrichs, and H.-J. Barthelmess, Low-noise ultra-high-speed dc SQUID readout electronics, *Supercond. Sci. Tech.*, **19**(5), S235, 2006.
- [Dru07] D. Drung, C. Abmann, J. Beyer, A. Kirste, M. Peters, F. Ruedel, and Th. Schurig, Highly sensitive and easy-to-use squid sensors, *IEEE Transactions on Applied Superconductivity*, **17**(2), 699–704, 2007.
- [Dru11] D. Drung, J. Beyer, J.-H. Storm, M. Peters, and T. Schurig, Investigation of Low-Frequency Excess Flux Noise in DC SQUIDS at mK Temperatures, *IEEE T. Appl. Supercon.*, **21**, 340–344, 2011.
- [Enp86] K. Enpuku, K. Yoshida, and S. Kohjiro, Noise characteristics of a dc SQUID with a resistively shunted inductance. II. Optimum damping, *Journal of Applied Physics*, **60**(12), 4218–4223, 1986.
- [Fer15] A. Ferring, *Entwicklung von dc-SQUIDs zur Auslesung von metallischen magnetischen Kalorimetern*, Masterarbeit, Kirchhoff-Institut für Physik, Universität Heidelberg, 2015.
- [Fog93] V. Foglietti, K.G. Stawiasz, M.B. Ketchen, and R.H. Koch, Performance of a flux locked series SQUID array, *IEEE T. Appl. Supercon.*, **3**(4), 3061–3065, 1993.

- [Gas17] L. Gastaldo, K. Blaum, K. Chrysalidis, T. Day Goodacre, A. Domula, M. Door, H. Dorrer, Ch. E. Düllmann, K. Eberhardt, S. Eliseev, C. Enss, A. Faessler, P. Filianin, A. Fleischmann, D. Fonnesu, L. Gamer, R. Haas, C. Hassel, D. Hengstler, J. Jochum, K. Johnston, U. Kebschull, S. Kempf, T. Kieck, U. Köster, S. Lahiri, M. Maiti, F. Mantegazzini, B. Marsh, P. Neroutsos, Yu. N. Novikov, P. C. O. Ranitzsch, S. Rothe, A. Rischka, A. Saenz, O. Sander, F. Schneider, S. Scholl, R. X. Schüssler, Ch. Schweiger, F. Simkovic, T. Stora, Z. Szücs, A. Türler, M. Veinhard, M. Weber, M. Wegner, K. Wendt, and K. Zuber, The electron capture in  $^{163}\text{Ho}$  experiment – ECHo, *The European Physical Journal Special Topics*, **226**(8), 1623–1694, 2017.
- [Gin50] V. L. Ginzburg and L. Landau, On the theory of superconductivity, *Zh. Èksper. Teoret. Fiz.*, **20**, 1064–1082, 1950.
- [Gro16] R. Gross, A. Marx, and F. Deppe, *Applied Superconductivity: Josephson Effect and Superconducting Electronics*, De Gruyter Textbook Series, Walter De Gruyter Incorporated, 2016.
- [Hen17] Daniel Hengstler, *Development and characterization of two-dimensional metallic magnetic calorimeter arrays for the high-resolution X-ray spectroscopy*, Dissertation, Kirchhoff-Institut für Physik, Universität Heidelberg, 2017.
- [Ina80] Akira Inaba, Superconductive transition point of tantalum and niobium as a reference temperature, *Japanese Journal of Applied Physics*, **19**(8), 1553, 1980.
- [Iva68] Yu. M. Ivanchenko and L. A. Zil’berman, Destruction of Josephson Current by Fluctuations, *ZhETF*, **8**, 113–115, 1968.
- [Jay81] J. M. Jaycox and M. B. Ketchen, Planar coupling scheme for ultra low noise dc SQUIDs, *IEEE Transactions on Magnetics*, **17**(1), 400–403, 1981.
- [Jos62] B. D. Josephson, Possible new effects in superconductive tunnelling, *Physics Letters*, **1**(7), 251–253, 1962.
- [Jos65] B. D. Josephson, Supercurrents through barriers, *Advances in Physics*, **14**(56), 419–451, 1965.
- [Kem15] S. Kempf, A. Ferring, A. Fleischmann, and C. Enss, Direct-current superconducting quantum interference devices for the readout of metallic magnetic calorimeters, *Superconductor Science and Technology*, **28**(4), 45008, 2015.

- [Kem18] S. Kempf, A. Fleischmann, L. Gastaldo, and C. Enss, Physics and Applications of Metallic Magnetic Calorimeters, *Journal of Low Temperature Physics*, 1–15, 2018.
- [Ket78] M. B. Ketchen, W. M. Goubau, J. Clarke, and G. B. Donaldson, Superconducting thinfilm gradiometer, *J. Appl. Phys.*, **49**, 4111–4116, 1978.
- [Ket81] M. Ketchen, Dc squids 1980: The state of the art, *IEEE Transactions on Magnetics*, **17**(1), 387–394, 1981.
- [Ket82] M. B. Ketchen and J. M. Jaycox, Ultra-low-noise tunnel junction dc SQUID with a tightly coupled planar input coil, *Applied Physics Letters*, **40**(8), 736–738, 1982.
- [Ket85] A. W. Kleinsasser et al. Ketchen, W. J. Gallagher, Dc squid flux focuser, in H. D. Hahlbohm und H. Lübbig (Ed.) (Ed.), *SQUID'85 Superconducting Quantum Interference Devices and their Applications: Proceedings of the Third International Conference on Superconducting Quantum Devices, Berlin (West), June 25-28, 1985*, 865–871, Walter de Gruyter, 1985.
- [Ket91] M.B. Ketchen, Design considerations for DC SQUIDs fabricated in deep sub-micron technology, *IEEE T. Magn.*, **27**, 2916–2919, 1991.
- [Knu87] J. Knuutila, A. Ahonen, and C. D. Tesche, Effects on DC SQUID characteristics of damping of input coil resonances, *Journal of Low Temperature Physics*, **68**(3-4), 269–284, 1987.
- [Knu88] J. Knuutila, M. Kajola, H. Seppä, R. Mutikainen, and J. Salmi, Design, Optimization, and Construction of a DC SQUID with Complete Flux Transformer Circuits, *Journal of Low Temperature Physics*, **71**(5-6), 369–392, 1988.
- [Koc07] Roger H. Koch, David P. DiVincenzo, and John Clarke, Model for  $1/f$  flux noise in squids and qubits, *Phys. Rev. Lett.*, **98**, 267003, 2007.
- [Kra23] Fabian Kraemer, *Development and characterization of an integrated two-stage dc-SQUID for the read-out of metallic magnetic calorimeters*, Bachelor thesis, Kirchhoff-Institut für Physik, Universität Heidelberg, 2023.
- [Kui08] K. H. Kuit, J. R. Kirtley, W. van der Veur, C. G. Molenaar, F. J. G. Roesthuis, A. G. P. Troeman, J. R. Clem, H. Hilgenkamp, H. Rogalla, and J. Flokstra, Vortex trapping and expulsion in thin-film  $\text{yba}_2\text{cu}_3\text{o}_{7-\delta}$  strips, *Phys. Rev. B*, **77**, 134504, 2008.

- [Lan60] L.D. Landau, L.P. Bell, E.M. Lifshitz, and J.B. Sykes, *Electrodynamics of Continuous Media*, Volumen 8 in *Course of Theoretical Physics*, Ney York: Pergamon Press, 1960.
- [Lik86] K. K. Likharev, *Dynamics of Josephson Junctions and Circuits*, Gordon and Breach, New York, 1986.
- [Mae95] M. Maezawa, M. Aoyagi, H. Nakagawa, I. Kurosawa, and S. Takada, Specific capacitance of Nb/AlO<sub>x</sub>/Nb Josephson junctions with critical current densities in the range of 0.1 – 18 kA/cm<sup>2</sup>, *Applied Physics Letters*, **66**(16), 2134–2136, 1995.
- [Man21] Federica Mantegazzini, *Development and characterisation of high-resolution metallic magnetic calorimeter arrays for the ECHo neutrino mass experiment*, Dissertation, Kirchhoff-Institut für Physik, Universität Heidelberg, 2021.
- [Maz24] David Mazibrada, *Title*, Master thesis, Kirchhoff-Institut für Physik, Universität Heidelberg, 2024.
- [McC68] D. E. McCumber, Effect of ac Impedance on dc Voltage-Current Characteristics of Superconductor Weak-Link Junctions, *Journal of Applied Physics*, **39**(7), 3113–3118, 1968.
- [Mei33] W. Meissner and R. Ochsenfeld, Ein neuer Effekt bei Eintritt der Supraleitfähigkeit, *Naturwissenschaften*, **21**(44), 787–788, 1933.
- [Ono97] R. H. Ono, J. A. Koch, A. Steinbach, M. E. Huber, and M. W. Cromar, Tightly coupled dc SQUIDs with resonance damping, *IEEE Transactions on Applied Superconductivity*, **7**(2), 2538–2541, 1997.
- [Ryh92] Tapani Ryhänen, Heikki Seppä, and Robin Cantor, Effect of parasitic capacitance and inductance on the dynamics and noise of dc superconducting quantum interference devices, *Journal of Applied Physics*, **71**(12), 6150–6166, 1992.
- [Sep87] H Seppä and T Ryhänen, Influence of the signal coil on DC-squid dynamics, *IEEE Transactions on Magnetics*, **23**(2), 1083–1086, 1987.
- [Sta93] K. G. Stawiasz and M. B. Ketchen, Noise Measurements of Series SQUID Arrays, *IEEE T. Appl. Supercon.*, **3**, 1808–1811, 1993.
- [Ste68] W. C. Stewart, Current-Voltage Characteristics of Josephson Junctions, *Applied Physics Letters*, **12**(8), 277–280, 1968.

- [Tes77] C. D. Tesche and J. Clarke, dc SQUID: Noise and Optimization, *Journal of Low Temperature Physics*, **294**(3), 1–9, 1977.
- [Tie21] Eite Tiesinga, Peter J. Mohr, David B. Newell, and Barry N. Taylor, Co-data recommended values of the fundamental physical constants: 2018, *Rev. Mod. Phys.*, **93**, 025010, 2021.
- [Wel93] R.P. Welty and J.M. Martinis, Two-stage integrated squid amplifier with series array output, *IEEE Transactions on Applied Superconductivity*, **3**(1), 2605–2608, 1993.