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Formulas of dielectric and total attenuations of a microstrip line

Wenquan Che,[1] Yu Fang Tang,[1] Jie Zhang,[1] and Y. L. Chow[2]

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[1] This paper constructs a simple formula of distribute conductance of a microstrip line and then of the dielectric attenuation with only one arbitrary constant, p = 1.08. By adding the formulas of the dielectric and copper, a similarly simple formula of total attenuation is obtained. Both formulas of the dielectric, as well as the total losses, agree closely with the numerical results by HFSS software over wide ranges of frequency and microstrip parameters. The simplicity of the formulas gives an interesting insight; that is, a microstrip line is considered a high loss at high frequencies f because of the overtaking of the slower skin-effect loss of $\sqrt{f}$ dependence along the copper strip by the faster current-leakage loss of f dependence across the dielectric substrate from strip to ground. The formula is verified with simulation by field software but not by hardware measurements. Hardware experiments are actually unreliable in the case of microstrip line attenuation; the details, evidences, and quantification of the reliability are discussed in Appendix A.

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1. Introduction

[2] Loss is necessarily small, say 0.4 dB/λ, for a microstrip line [*Pucel et al.*, 1968; *Lee and Itoh*, 1989; *Waldow and Wolff*, 1985; *Cheng*, 1989; *Chow et al.*, 1999] of practical length. For a line of $\lambda/4$, this means an attenuation of say 0.1 dB: that is, a voltage drop of about 1% or a power drop of 2%. Such a small voltage drop is comparable to the error or tolerance, either in hardware measurements or in the computation with field software. Clearly, therefore, loss attenuation is difficult to determine accurately by these two popular approaches. By inspection, on the other hand, elementary textbooks have provided a simple and accurate analytical formula of the resistance of a conducting wire, in DC and in RF with skin depth; years ago, that stood the test of time. In fact, developed from this tradition of inspection, *Che et al.* [2008] constructed a simple formula of copper attenuation along a microstrip line. With the formula results that pass through the midpoints between the results of three popular software, HFSS, IE3D, and ADS, the copper formula was considered validated. The copper formula covers full ranges of the parameters concerned (e.g., from zero to infinity), but it needs only two arbitrary constants, q and κ; the formula is simple.

[3] Following the previous work, this paper continues with the by-inspection formula of dielectric attenuation for a microstrip line with substrate loss. The dielectric formula is checked with only HFSS software but, unlike the copper formula, the agreement is excellent. The dielectric formula also covers the full ranges of the parameters concerned but needs only one arbitrary constant p; the dielectric formula is simpler.

[4] The total attenuation of a microstrip line is the sum of the copper, the dielectric, and the radiation (including surface wave) leakage. It is clear, physically and from the formulas below, that the copper attenuation is proportional to the square root of the frequency, and the dielectric attenuation is proportional to the frequency itself. On the other hand, the radiation attenuation has been found to be highly nonlinear, at about the fourth power with frequency [*Chow and Tang*, 2001a]. Such nonlinearity effectively means that the radiation can be kept negligible until the frequency is impractically high, such that the next higher (nontransverse electromagnetic (non-TEM)) mode may be about to appear.

[1]Department of Electrical Engineering, Nanjing University of Science and Technology, Nanjing, China.
[2]Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, Ontario, Canada.

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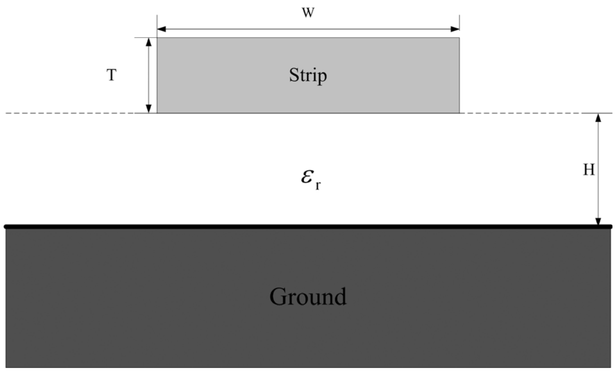

Figure 1. The geometry of a microstrip line.

[5] For this reason, we shall consider that the total attenuation is simply the sum of the attenuations of the copper and the dielectric. We shall neglect the radiation attenuation in this work.

[6] The formula is not verified by hardware measurements. In a practical microstrip line of copper, the copper surfaces cannot be perfectly smooth: that is, slightly rough. There is evidence that the roughness is of the order of 1 μm. At RF, say 10 GHz, the skin depth is about 0.6 μm, and this means that the surface current would actually flow around the bumps of the roughness and thus increase the effective surface resistance. Obviously, the surface roughness is essentially random, and the amplitudes of the randomness could be different, say from different manufactures of the copper-clad substrate. Because of this, microstrip line fabricated from the substrate, therefore, cannot be reliable enough to check the accuracy of the formula. The details, the evidences, and quantification of the reliability are added in the Appendix.

2. Formula of the Dielectric Attenuation

[7] The schematic geometry of a microstrip line is given in Figure 1, which is constructed on a substrate with dielectric constant ε_r and height h. The strip width is w, and its thickness t is assumed much thinner than the strip width w but still thicker than the skin depth δ_c in the usual RF range of interest. The ground plane and the substrate are assumed infinite in width.

[8] For the microstrip line of a substrate ε_r with loss tangent, through a C to G duality, the formula of distributed conductance G may be constructed from the formula of distributed capacitance C of the substrate. From the synthetic asymptote of fuzzy EM by *Chow and Tang* [2001b], the capacitance C per unit length of microstrip line is derived as

$$C = \varepsilon_0 \left\{ \left(\frac{\varepsilon_r w}{h}\right)^p + \left[2\pi \frac{(\varepsilon_r + 1)}{2} \left(\frac{1}{\ln(8h/w + 1)} - \frac{w}{8h}\right)\right]^p \right\}^{1/p}. \quad (1)$$

[9] To convert the C into G, it is necessary to understand the meaning of the terms in equation (1). The first term is the capacitance of the parallel plate field (i.e., the near asymptote term of $h \to 0$ of the synthetic asymptote); the second term is the capacitance of the fringe field (i.e., the far asymptote term of $h \to \infty$). At large h, as discussed by *Chow and Tang* [2001b], the effective dielectric of the fringe field becomes $\frac{(\varepsilon_r+1)}{2}$: that is, the average between the thick substrate ε_r and the air above. The resulted formula (1), in synthetic asymptote, is the pth power norm of the near and far asymptotes. The power p (equal to 1.08 away from unity) is a minor correction for h at the intermediate values.

[10] In a homogeneous infinite dielectric medium, the C to G duality means that the conversions of formula C to formula G is simply done by replacing the permittivity $\varepsilon_0\varepsilon_r$ with the loss conductivity σ_ε (equivalent to the loss tangent).

[11] In the media of substrate of finite thickness h, there is air above. As a result, in equation (1), the effective permittivity of the fringe field of the second term of C has already become $\frac{(\varepsilon_r+1)}{2}$: the average of substrate ε_r and air (of ε_r = 1). Air is a medium that has a conductivity of $\sigma_{\mathrm{air}} = 0$, while substrate has a conductivity of a nonzero σ_ε. The duality conversion of equation (1), therefore, is done by replacing $\varepsilon_0\varepsilon_r$ with σ_ε, as usual in the first term, but by replacing the effective $\varepsilon_0\frac{(\varepsilon_r+1)}{2}$ with $\frac{(\sigma_\varepsilon+0)}{2}$ in the second term. The result of such conversion is then

$$G = \sigma_\varepsilon \left\{ \left(\frac{w}{h}\right)^p + \left[\pi \left(\frac{1}{\ln(8h/w + 1)} - \frac{w}{8h}\right)\right]^p \right\}^{1/p}. \quad (2)$$

[12] From the definitions of $\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$, we can write

$$\sigma_\varepsilon = j\omega\varepsilon_r''. \quad (3)$$

[13] The commonly used term "loss tangent" is related to ε'' by

$$\tan\delta = \varepsilon_r''/\varepsilon_r'. \quad (4)$$

[14] From transmission line theory [*Pozar*, 1990], the conductance may be converted to dielectric attenuation as

$$\alpha_d = G/2Y_0 \tag{5}$$

[15] The characteristic admittance Y_0 is simply proportional to the ratio of C in equation (3), evaluated both in the dielectric ε_r and in air, of which $\varepsilon_r = 1$, and then taking the square root. That is,

$$Y_0 = \sqrt{\frac{\varepsilon_0}{\mu_0}}\sqrt{\frac{C|_{\varepsilon r}}{C|_{\varepsilon r=1}}}. \tag{6}$$

[16] The simple formula of equation (6) has been found [*Chow and Tang*, 2001b] to be just as accurate as the more common formula in text books (e.g., those by *Pozar* [1990]). That is, with an error of < 2% over the whole practical range (e.g., zero to infinity) of all parameters in the formula.

3. Formulas of the Copper Attenuation and the Total Attenuation: Copper Plus Dielectric

[17] The formula of copper attenuation of a microstrip has been derived by inspection before [*Che et al.*, 2008]. The formula is

$$\alpha_c = R/2Z_0, \tag{7}$$

where

$$R = \frac{1}{\sigma_c}\left[\left(\frac{1}{tw}\right)^q + \left[\frac{1}{\delta}\left(\frac{1}{w} + \frac{1}{w + 2\pi h e^{-\alpha_t \kappa 2\pi_t h}}\right)\right]^q\right]^{1/p}. \tag{8}$$

[18] with the constants $q = 5$ and $\kappa = 30$; also Z_0 is the characteristic impedance, the reciprocal Y_0 of equation (6); σ_c is the conductivity of copper, and δ_c is its skin depth of copper; that is,

$$\delta_c = \sqrt{\frac{2}{\omega\sigma_c\mu_0}}. \tag{9}$$

[19] Also, α_t is the transverse attenuation resulting from the slow wave propagation β_g along the microstrip line in respect to the surrounding air; that is,

$$\alpha_t = \sqrt{\beta_g^2 - \omega^2\mu_0\varepsilon_0}. \tag{10}$$

[20] Assuming that the radiation attenuation is negligible, the total attenuation α in the microstrip is then simply the sum of the dielectric attenuation of equation (5) above and the copper attenuation in equation (7), which is

$$\alpha = \alpha_d + \alpha_c = G/2Y_0 + R/2Z_0. \tag{11}$$

4. Verification of the Formula of Dielectric Attenuation

[21] Figure 2 checks the dielectric attenuation formula in equations (5) and (2) with the numerical solution from the HFSS software. There should be five parameters of importance affecting the dielectric loss of the microstrip: namely, the (substrate) dielectric loss tangent tan δ, the dielectric constant ε_r, the substrate height h, the ratio w/h (of the strip width to substrate height), and the operating frequency f. In turn, through the transmission line equations, the width-to-height ratio w/h is related to other parameters of importance, such as the impedance Z_0 and the propagation velocity v_g of the microstrip line.

[22] Figure 2 keeps the impedance Z_0 at 50 Ω while changing the dielectric ε_r; this requires the ratio w/h to change. This means that the formula checking with the HFSS software in Figure 2 varied four out of the five important parameters: namely, the dielectric constant ε_r, the substrate height h, the ratio w/h, and the frequency f. The only parameter that is not varied is the loss tangent tan δ proportional to $1/\sigma_\varepsilon$, and σ_ε is only a simple multiplication factor to the line conductance G in equation (2) and the dielectric attenuation α_d in equation (5); therefore, σ_ε needs to not be varied for the checking.

[23] Figure 2 checks the dielectric attenuation with the HFSS software. More checks are available when the attenuations of the dielectric and copper are added together to the total attenuation. These are given later in Figures 4 to 7.

[24] G of equation (2) is a TEM approximation and, therefore, is dependent on only the ratio w/h and not on h itself. The TEM assumption may be inaccurate, since the microstrip line may only be quasi-TEM under the usual operating frequency range. Figure 2 therefore checks α_d of equation (5) and G of equation (2) with the HFSS software results by using two common values of h at 0.5 and 1 mm. The little change in the two values of h in Figure 2 indicates that G of equation (2), with its TEM assumption, is indeed accurate in the usual range of operating frequencies.

[25] Figure 2 of the dielectric attenuation keeps Z_0 at 50 Ω, but the dielectric attenuation α_d of equation (5) is a function of the strip width to substrate height ratio of w/h.

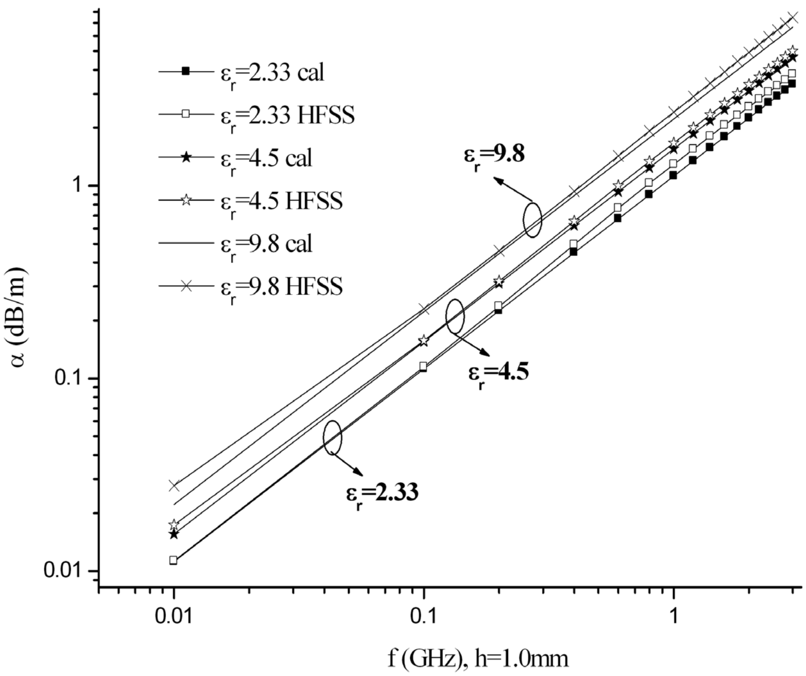

Figure 2. Comparison of dielectric losses from formula calculation (solid markers) and HFSS software simulations (hollow markers). $Z0$ = 50 Ω; ε_r = 2.33, 4.5, and 9.8; tan δ = 0.01; substrate h = 1 mm; metallization t = 0.018 mm, and line length l = 100 mm for HFSS software computation.

The analytical link between the two is given by Y_0 of equation (6) and C of equation (1). Evaluated from these two equations, Figure 3 is plotted to provide the graphical link for easy inspection, with the impedance Z_0 and the velocity of propagation v_g as a function of the ratio w/h. It is easy to realize as well, as observed in Figure 2, that the dielectric attenuation is a linear function with frequency.

5. Verification of the Total Attenuation: Dielectric Plus Copper

[26] The total attenuation is what one observes in practice and not the individual attenuations of dielectric and copper. This verification is now done. Two dielectrics are chosen: namely, ε_r = 2.33 and 9.8 for the substrate. The substrate height h is kept at 1 mm. To each dielectric, two characteristic impedances Z_0 are chosen: namely, 50 Ω and 25 Ω. The results of these choices are plotted in Figures 4 to 7 as examples.

[27] It is to be noted in Figures 4 to 7 that the added dash-dot-dot line indicates the dielectric loss at tan δ = 0.001. Also, the dielectric loss, conductor loss and total loss of the microstrip line from calculation and HFSS simulation for case of tan δ = 0.001, are all illustrated and compared.

[28] The frequency range of Figures 4–7 is from 10 MHz to 10 GHz. Ten MHz is the frequency at which the copper resistance of the strip is essentially DC: that is, nearly independent of frequency; 10 GHz is the frequency above which the microstrip line is not much used because of high RF losses.

[29] More importantly, beyond 10 GHz, radiation loss should become significant; that is, with the substrate height h being at 1 mm and the dielectric constant up to 9.8, the substrate h could be as large as $\lambda/10$ in the dielectric. However, a simple formula of radiation loss of a finite-length microstrip is beyond the scope of this paper. Therefore, 10 GHz is the upper limit of the examples here.

[30] For ease of discussion, we have chosen a nominal loss tangent of the dielectric substrate at tan δ = 0.01. This is the loss tangent of an inexpensive substrate, such as FR-4 (used widely for low-cost RF devices). This choice makes the dielectric attenuation large enough to become comparable and therefore interact with the copper attenuation within the usual frequency range of up to 10 GHz.

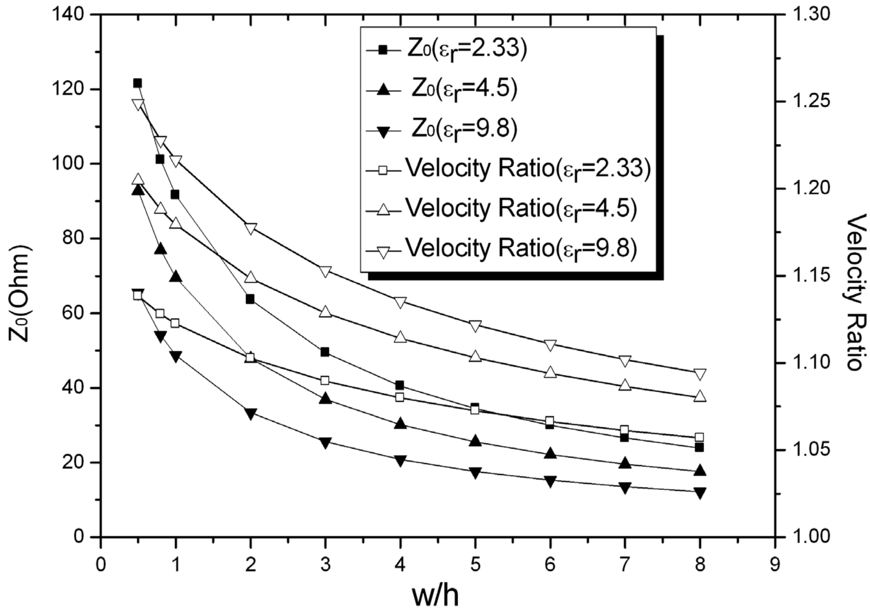

Figure 3. Strip ratio w/h versus velocity ratio v_g / $\left(c/\sqrt{\varepsilon_r}\right)$: the ratio between that of the microstrip and that in a homogeneous dielectric, and ratio w/h versus the impedance Z_0. Other parameters are the same as those in Figure 2.

[31] Better RF devices use substrates with a low loss tangent, such as tan δ = 0.001. The dielectric attenuation of this low loss substrate is added as a dash-dot line to complete the discussion leading to the reason why a microstrip line becomes more lossy at high radio frequencies. In the log-log plot, in the dielectric attenuation of Figure 4, the dash-dot line of 45^o for tan δ = 0.001 is parallel to the solid line of 45^o for tan δ = 0.01. The

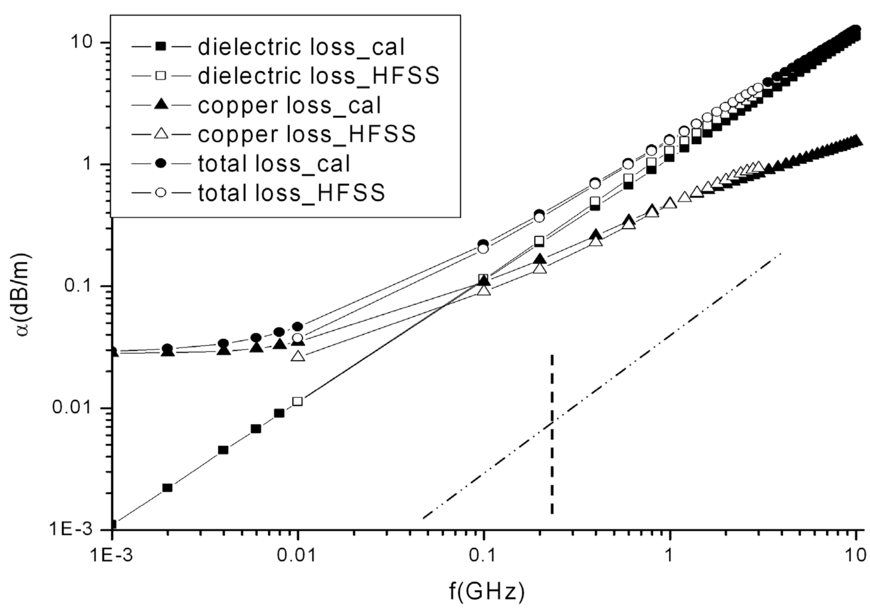

Figure 4. Calculated and simulated attenuations α versus f at Z_0 = 50 Ω and ε_r = 2.33, with h = 1 mm, tan δ = 0.01, t = 0.018 mm, and l =100 mm for HFSS software.

reason is that tan δ is proportional to $1/\sigma_\varepsilon$ in equations (3) and (4); σ_ε is a simple multiplier to give the conductance G in equation (2), therefore, to give the dielectric attenuation α_d in equation (5). A simple multiplier gives a 45° line in the log-log plot.

[32] In Figures 4 to 7, solid markers indicate formula-calculated results, and hollow markers indicate HFSS software-simulated results. Between Figures 4–7, together with formulas α of equation (11), α_d of equation (7), and α_c of equation (5), one can observe the following properties: namely, 1 to 9 in the following sections.

5.1. Errors of Formulas, as Compared to HFSS Software

[33] 1. On the dielectric attenuation α_ε from the formula and the HFSS software, the agreement is excellent; that is, the error is always < 3%. The formula error is small as the current field of G colocates in space with the electric field of C; therefore, as C is accurate, so are G and α_ε.

[34] 2. On the copper attenuation α_c, the agreement is not as good, especially at higher frequencies of 3 GHz, at up to 20% error. The formula error is generally larger as the fields are not colocated, as the current field of R is on the skin of the strip and the magnetic field of L is in the space surrounding the strip. Then, at higher frequencies, leaking radiation starts in HFSS software simulations but is neglected in the formula.

[35] 3. The formula error of the total attenuation therefore comes only from the error in the copper attenuation.

5.2. Attenuations as Functions of Frequency and Loss Tangent of Substrate

[36] 4. At low frequency f, the copper loss dominates, meaning that the total attenuation stays constant at DC then increases as $\sqrt{f}$ due to skin effect. After a crossover point (indicated in Figures 4–7 by a vertical dotted line) to high frequency, the dielectric attenuation dominates, meaning that the total attenuation α increases as f: that is, a faster rise in loss.

[37] 5. With a higher loss tangent (e.g., tan $\delta = 0.01$) in Figures 4–7, the crossover point between the copper and dielectric attenuations is at about 0.1 GHz. With a lower loss tangent (e.g., tan $\delta = 0.001$), the added dash-dot line shows that the dielectric loss is 10 times lower, meaning that the crossover point is much higher, at about 4 to 12 GHz, depending on the substrate ε_r and the impedance Z_0.

[38] 6. An attenuation rise of $\sqrt{f}$ of copper may be considered lower loss, an attenuation rise of f, of dielectric, may be considered higher loss. It is possible that the microstrip line is considered high loss and unsuitable for use beyond 5 GHz because the higher attenuation of the dielectric sets in, even with expensive substrates of tan $\delta = 0.001$.

5.3. Attenuation Dependence on Impedance Z_0, Dielectric ε_r, and Substrate Height h

[39] 7. Between Figures 4 and 6 of $\varepsilon_r = 2.33$ and Figures 5 and 7 of $\varepsilon_r = 9.8$, there is a reduction of impedance Z_0 from the nominal 50 Ω to 25 Ω. It appears that a reduction of characteristic impedance Z_0 would cause a reduction of copper attenuation but little change in the dielectric attenuation. This means there is little change in the total attenuation of the microstrip line at the higher radio frequencies with dielectric attenuation being dominant.

[40] 8. Between Figures 4 and 5 of $Z_0 = 50\ \Omega$ and Figures 6 and 7 of $Z_0 = 25\ \Omega$, there is an increase of the dielectric constant ε_r from 2.33 to 9.8. It appears that an increase of dielectric constant ε_r does increase the attenuations of both copper and dielectric. Together they cause a general increase of the total attenuation over the whole frequency range.

[41] 9. Finally, if the substrate height h is increased, for the same impedance Z_0, the strip width w is proportionally increased. Equations (7) and (8) indicate that there is a decrease of the copper attenuation. On the other hand, equations (5) and (2) indicate that there is no change in the dielectric attenuation as the ratio w/h remains unchanged. Similar to property 7, therefore, there is little change in the total attenuation of the microstrip line at the higher radio frequencies. Numerically, the HFSS software in Figure 2 also shows that the dielectric attenuation has little dependence on the substrate height h.

5.4. Summary of Properties

[42] The microstrip line remains at high loss at high radio frequencies because of the dominance of the dielectric attenuation. The way of reducing the loss is to reduce the loss tangent or the dielectric constant or, preferably, both. The change of the characteristic impedance does not help.

6. Conclusions

[43] By inspection, the formulas of the conductance and dielectric attenuation in a microstrip line are derived. Adding to the similarly derived formulas of resistance and copper attenuation, the formula of the total attenuation is found. Very good agreement with the numerical HFSS software is observed for the formulas of both dielectric attenuation and the total attenuation. The formulas are simple and cover the complete ranges of parameter values

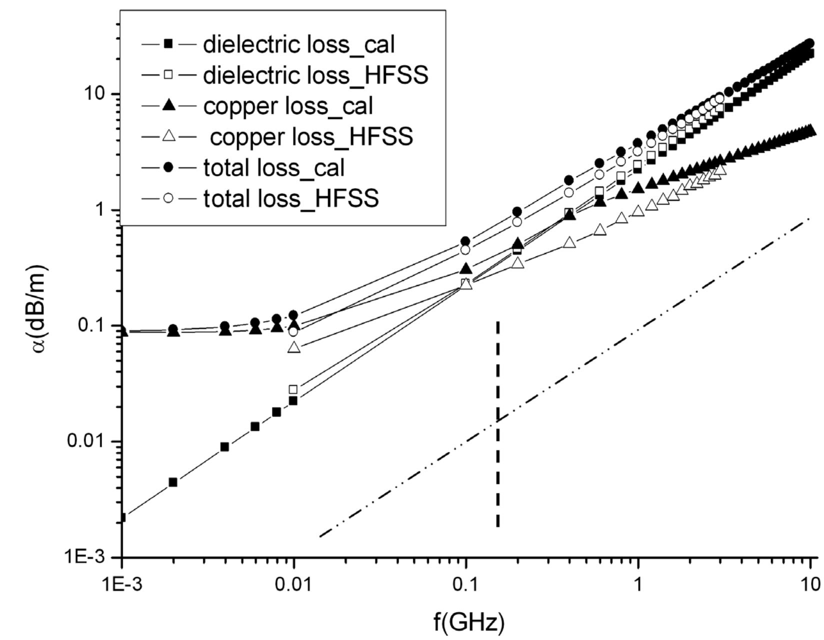

Figure 5. Calculated and simulated attenuations α versus f at Z_0 = 50 Ω and ε_r = 9.8, with h =1 mm, tan δ = 0.01, t = 0.018 mm, and l =100 mm for HFSS software.

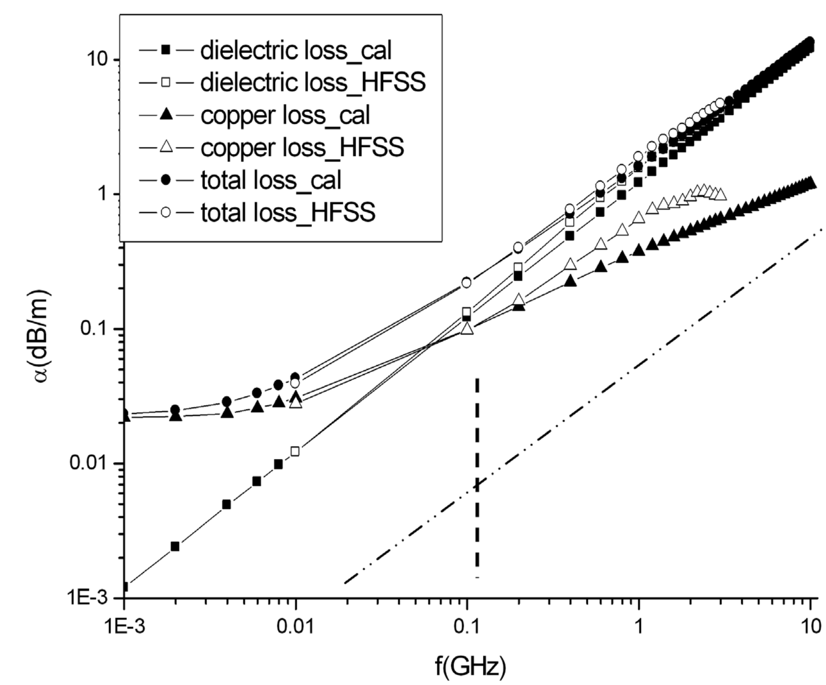

Figure 6. Calculated and simulated loss α versus f at Z_0 = 25 Ω and ε_r = 2.33, with h =1 mm, tan δ = 0.01, t = 0.018 mm, and l =100 mm for HFSS software.

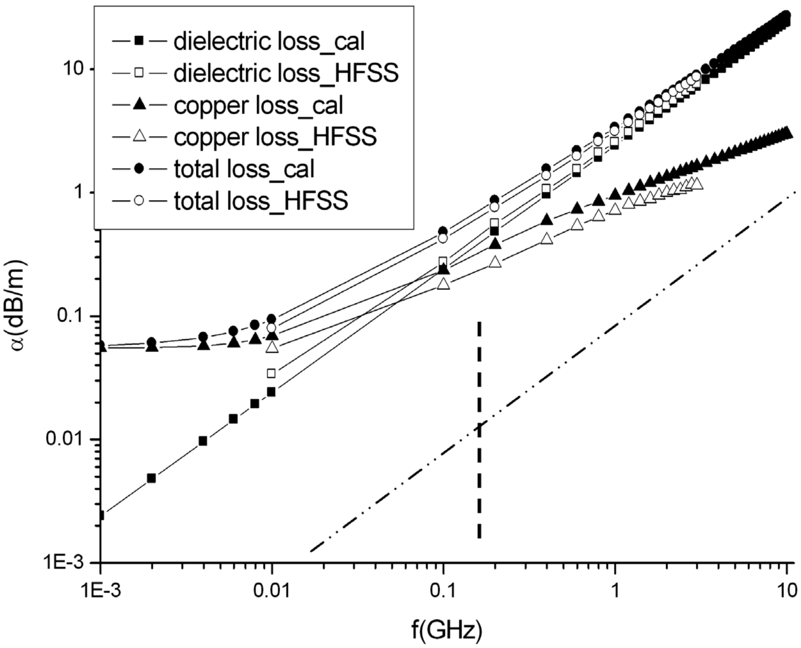

Figure 7. Calculated and simulated loss α versus f, at Z_0 = 25 Ω, and ε_r = 9.8, with h = 1 mm, tan δ = 0.01, h = 1, t = 0.018 mm, and l =100 mm for HFSS software.

(e.g., from zero to infinity) but need only three arbitrary constants: that is, p for the dielectric attenuation of equation (2), and q and κ for the copper attenuation of equation (8). By inspection from the first principle, the resulting formula therefore has no truncation errors commonly found in numerical field computations.

[44] The simplicity of the formulas and their applicability over the complete parameter ranges mean much insight can be gained. An interesting insight in the last section is that the copper attenuation from the skin effect is proportional to the square root of frequency, i.e., $\sqrt{f}$, and may be considered a slower rise in attenuation; the dielectric attenuation from the leakage current through the substrate to ground is proportional to the frequency f itself and may be considered a faster rise in attenuation. At low frequency, the slower rise of copper attenuation dominates in the total attenuation, after a crossover point to higher frequency, the faster rise of the dielectric attenuation dominates. This may be the reason that the microstrip line is widely used in the lower radio frequencies, but at higher radio frequencies, say beyond 5 GHz, it is considered high loss and therefore much less used.

[45] Section 5 and the formulas above show that the dominance of the dielectric attenuation would always cause a high attenuation in a microstrip line at high radio frequencies. The only ways of loss reduction seem to be to reduce the loss tangent, to lower the dielectric constant, or to have a larger portion of the electric field outside the substrate dielectric, i.e., in air.

[46] The coplanar waveguide (CPW) is one example in this direction which has half of the field in air and therefore lower dielectric loss in higher radio frequencies. One may still come to other ways to move more electric field into the surrounding air and therefore reduce the loss of the lines, the CPW, the microstrip, or otherwise.

[47] Whatever is the shape of the transmission line, the skin effect requires that the copper loss is always proportional to $\sqrt{f}$ and the dominant loss at lower frequencies f; the loss tangent effect requires that the dielectric loss is always proportional to f and the dominant loss at a higher frequencies. This is the limit of the practical range of frequencies of the microstrip before the loss becomes too high. For the examples of the microstrip line in this paper, Figures 4 to 7, the limit is of the order of 10 GHz.

[48] Beyond this frequency limit, from antenna theory, the radiation loss should be proportional to f^2 and become the dominant loss. Because of the dependence of f^2, the radiation loss rises up rapidly with frequency f and becomes impractical.

[49] The formula is verified with simulation by field software but not by hardware measurements. The reason is that hardware experiments are actually unreliable in the case of microstrip line attenuation; because of the uncer-

tainty of the surface roughness of the copper conductor, the details, evidences, and quantification of the reliability are given in the Appendix.

Appendix A: Unreliability of Attenuation Measurements and the Conductor Roughness

[50] Normally, the ultimate verification of an EM property, from computation or from formula, is a carefully done experimental measurement. Exception does occur to attenuation measurement of a practical microstrip line. The reason is below.

[51] Practically, the attenuation measured on a microstrip line is always the total, as the attenuations of dielectric and copper (conductor) always coexist and cannot be separated while, theoretically, the attenuations are completely separable in derivations of their formulas. This ought not to be a problem, as two attenuation formulas are added together to form the total and then verified by the experimental attenuation measurement. This is given in detail in this paper.

[52] Still, for a long time [e.g., by *Chow et al.*, 1999] it has been known that the RF resistance calculated from the known copper resistivity (by field software or by simple formulas) is always lower than that from the experimental measurements by, say, 20%. This discrepancy has been attributed to the roughness of the conductor surface that has been idealized to be smooth in both the software and the formula. The roughness being random has not been quantified. In fact, it can be quite difficult to quantify the roughness at RF.

[53] For example, let the operating frequency be at x band, say 10 GHz; the skin depth of copper is about 0.6 μm. Of a commercial dielectric substrate, the copper cladding on both sides usually has a matte look instead of a mirror look. With the optical wavelength averaging 0.5 μm, the matte look means that there is, on average, 1 μm on the surface undulation amplitude and its period. This is in contrast with the mirror look which requires that the undulation be 1/20th of a wavelength: i.e., 0.025 μm or 25 nm.

[54] With the matte look, the surface undulations must be of the order of 1 μm or more; this means that at a skin depth of 0.6 μm, the surface current on the copper should follow the surface undulation (roughness) of the copper, taking a longer path and therefore resulting in a larger effective surface resistance, say, by the 20%, as observed by *Chow et al.* [1999].

[55] Unfortunately, an undulation of 1 μm is unobservable by a microscope with an optical wavelength of about 0.5 μm. Of course, an electron microscope would observe the undulation, but it would be too expensive and impractical for daily measurements in industry. With the surface undulation (roughness) unobserved, from sample to sample, the copper resistance and attenuation of the microstrip line are unreliable, and this extends to the total attenuation which includes the dielectric attenuation.

[56] In contrast, the physical insight as provided by the derivation of the dielectric and copper attenuation in this paper and the last [*Che et al.*, 2008], and their agreement with the computed results of the field software, make formulas in this paper quite reliable provided, of course, that the copper surface is smooth.

[57] In practice, the copper surface is not smooth (i.e., rough) at 1 μm or more. With the formulas as the basis, in comparing with the measurement result, a quantitative study of the property of the roughness effects of the copper surface may be performed in the future paper.

[58] The table now appears to be turned for the microstrip line case; that is, instead of using the experimental measurement to verify the validity of the formulas and computation, the formulas and computation may now be used to estimate the surface roughness of the microstrip line from different manufacturers, on the different production runs from the same manufacturer, and especially on the dependence on frequency of each.

[59] **Acknowledgments.** The authors express their sincere thanks to the financial support of the National Natural Science Foundation of China, under grant 60971013, and the Natural Science Foundation of Jiangsu Province, under grant BK2008402.

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W. Che, Y. F. Tang, and J. Zhang, Department of Electrical Engineering, Nanjing University of Science and Technology, 210094 Nanjing, China. (yeeren_che@yahoo.com.cn)

Y. L. Chow, Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON N2L 3G1, Canada. (ylchow@maxwell.uwaterloo.ca)