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Physica C 351 (2001) 85–90

PHYSICA C

www.elsevier.nl/locate/physc

Resonance damping in tightly coupled d.c. SQUIDs via intra-coil resistors ☆

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Received 5 July 2000; accepted 14 September 2000

Abstract

Intra-coil damping is demonstrated to be an effective method for reducing resonance structure in well coupled d.c. SQUIDs with large input coils. A resistance of 1 Ω is placed across each turn of a 137 turn coil coupled to a planar washer d.c. SQUID. Non-ideal structure in the voltage–flux curve is greatly reduced, extending the range (in current bias) of satisfactory device operation. The resulting flux noise, while higher than the thermal noise limit, is less than that observed in undamped devices and can be predicted with a simple model. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: SQUID; Resonance; Damping; Planar coil; Distortion; Noise

We present a new technique for reducing resonance-induced structure in the output characteristics of a well-coupled d.c. SQUID without significantly degrading the noise performance relative to an equivalent undamped device. The d.c. SQUID with a tightly coupled input coil excels as a low-noise transducer for virtually any physical quantity that can be converted to a current. Modern well-coupled d.c. SQUIDs typically employ the design of Jaycox and Ketchen: a spiral input coil deposited over a washer-style SQUID [1]. One long recognized problem associated with this design is the presence of high frequency resonances resulting from the parasitic capacitance between the washer and the input coil. These resonances can induce nonideal structure on the current–voltage (I–V) and voltage–flux (V–Φ) output characteristics [2,3].

Our new circuit damps unwanted resonances using resistors connected in parallel with every turn of the superconducting input coil [4]. We numerically analyze circuit models to demonstrate that this damping technique is more effective than a single on-chip resistor placed across the entire input coil. Our experimental results demonstrate that the new damping scheme, which we call intra-coil damping, can produce dramatically improved output characteristics compared to nominally identical d.c. SQUIDs without intra-coil damping.

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PII: S0921-4534(00)01627-0

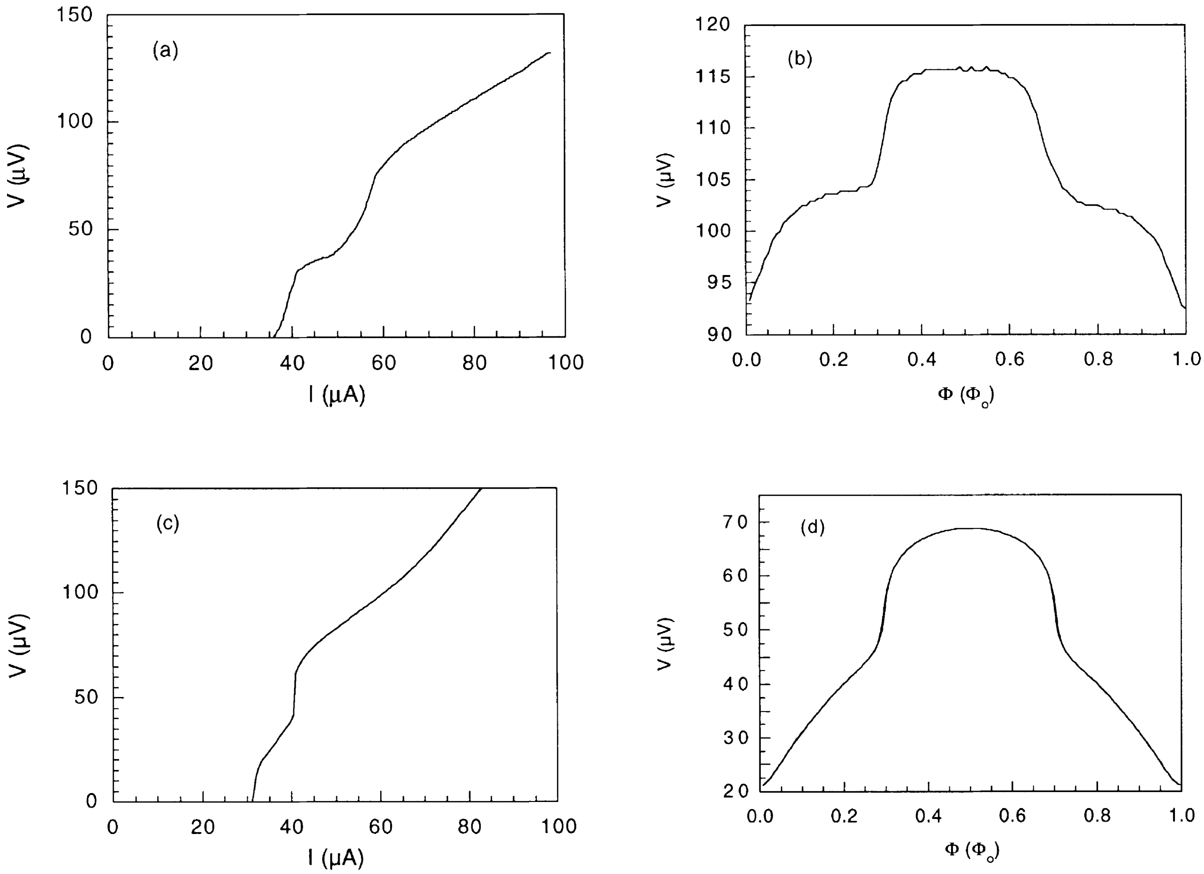

Fig. 1. Experimental and modeled operating characteristics for a typical undamped, tightly coupled d.c. SQUID. (a) Distortions are clearly present in the experimental current–voltage characteristic of a 137 turn input-coil device with no added damping. (b) The voltage–flux characteristics of the same experimental device are also distorted in the operating region of interest. (c) Qualitatively similar features are observed in the current–voltage characteristics of a modeled device in which the input coil is treated as a single *L–C–R* circuit inductively coupled to the SQUID inductance. (d) Qualitatively similar features are also observed in the voltage–flux characteristics of the modeled device.

Not only is the output characteristic improved, but also less noise is injected into the SQUID compared to some other damping techniques.

In Fig. 1a and b, we show the experimental $I–V$ and $V–\Phi$ characteristics of a d.c. SQUID with a tightly coupled 137 turn input coil. The damping in this device consists of only the junction damping and a resistive shunt across the washer inductance. The evident distortions in the output characteristics are typical of those seen in a d.c. SQUID with a tightly coupled, many-turn input coil. Without an input coil, a washer-style SQUID has resonances due to the slot and washer inductances shunted by stray capacitance; in principle, these washer resonances can also modify the output characteristics. However, it is possible to design the device so that the washer resonances are above the operating frequency and therefore do not significantly degrade the output characteristics. In our devices, the distorted output characteristics seen in Fig. 1a and b are not a result of washer resonances. This is confirmed by the observation that otherwise identical SQUIDs fabricated without an input coil *do not* exhibit such distortions. Moreover, a model of the SQUID coupled to a simple resonant circuit *does* exhibit similar features, as shown in Fig. 1c and d. This model will be described subsequently.

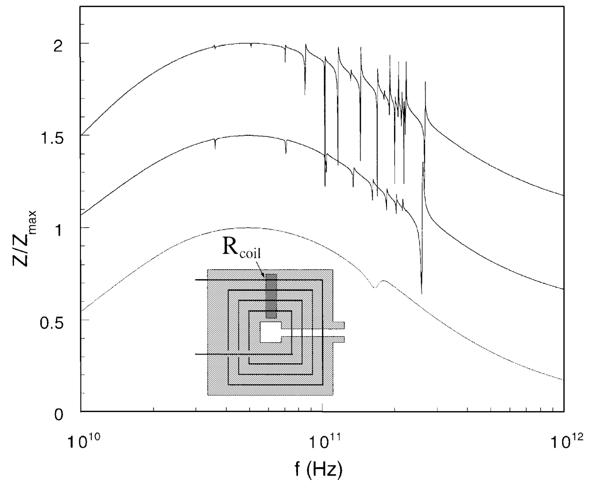

Fig. 2. Modeled frequency-dependent impedance $Z_C(f)$ of the input coil seen by the junctions for various damping techniques. In the top curve, with no coil damping and only a single resistor shunting the washer inductance, strong resonances are clearly apparent in $Z_C(f)$. In the middle curve, adding a single resistor to the model across the input coil reduces the Q of the resonances, but peaks are still apparent with resistor values typically used in practice. If, instead, a resistor of approximately 4 Ω is placed across each turn of the input coil (inset), resonances disappear, as in the bottom curve. This value of damping resistance does not significantly affect the device noise. The impedance is normalized to its maximum value Z_{max} and the curves are offset vertically by 0.5 for clarity.

Linear-circuit simulations can provide a qualitative understanding of the resonances caused by the presence of a tightly coupled, many-turn input coil. We use a lumped-element model to approximate the washer-to-input coil structure. In this model, each turn of the input coil is represented by an inductor with appropriate mutual inductances coupling it to all other input coil turns and to the washer inductance, which itself is modeled by a single inductor. Individual capacitors representing the distributed capacitance of the washer-insulator layer-input coil structure connect every coil inductor to the washer inductor. We ran simulation program with integrated circuit emphasis (SPICE) [5] simulations on this model to determine the impedance $Z_C(f)$ presented to the Josephson junctions as a function of frequency.

In Fig. 2, the top curve shows $Z_C(f)$ for a 30 turn input coil with no damping resistors anywhere in the input coil, but with one resistor in parallel with the washer inductor to account for the presence of the junction damping and washer shunt. The initial rise in impedance represents the large inductance of the input coil, and the decrease in impedance at higher frequencies is due to the capacitive shorting of the coil inductance. In between, there are a large number of abrupt changes in impedance – these are resonances due to the complicated interaction of inductance and capacitance in the model circuit. With only junction damping in the model, these coil resonances clearly have a very high quality factor Q. If realistic values of inductance and capacitance are used in the SPICE model, the lowest frequency resonances in Fig. 2 appear at frequencies roughly equal to Josephson frequencies in our d.c. SQUIDs (typically between 20 and 100 GHz).

The coil resonances modify the I–V and V–Φ characteristics by causing high frequency currents to flow in the coil when the frequency of the Josephson oscillations is close to one of the resonant frequencies. These high frequency currents in turn perturb the Josephson oscillations and shift the frequency of the Josephson oscillations (and hence the d.c. voltage) toward the resonance frequency, resulting in a modification of the d.c. output characteristic. We carried out resistively shunted junction (RSJ) simulations using personal superconductor circuit analyzer (PSCAN) [6] of a d.c. SQUID connected through a transformer to a load circuit with a single resonance near typical Josephson frequencies; the circuit consisted of a parallel inductance L_L, capacitance C_L, and resistor R_L. Using this simplest possible resonant circuit we have generated output characteristics (Fig. 1c and d) with qualitatively similar features to experimental characteristics (Fig. 1a and b). Although the actual circuit is obviously much more complex, the qualitative agreement with such a simple model indicates that the basic properties of the observed resonances are present in the model.

Significant previous work has been conducted with the goal of reducing the effect of these input coil resonances. Efforts at damping have included placing a resistor across the washer slot [7] and increasing the junction damping by decreasing the value of β_c $(=2\pi I_c R_S^2 C/\Phi_0$, where I_c is the junction critical current, R_S is the junction shunt resistance, C is the junction capacitance, and Φ_0 is the flux quantum). However, neither of these changes directly reduces the Q of the resonances due to the input coil. Drastically increasing junction damping can help by making the junctions less sensitive to resonances, but at the cost of lower responsivity and greatly increased noise. A number of authors [3,8–10] have obtained improved voltage–flux characteristics by placing a resistor, or a resistor/capacitor network, either across the entire input coil or between the input coil and the washer. If only a resistor is used, the noise performance of the d.c. SQUID is typically degraded, and, if a resistor/capacitor network is used, the complexity and size of the device increase. An alternative approach has been to push the resonances above the range of Josephson frequencies by reducing the parasitic capacitance. This has been done, for example, by coupling the SQUID to the signal via an intermediate superconducting transformer [8,11].

SPICE simulations demonstrate that intra-coil damping can be more effective than a single resistor of the same value shunting the input coil as a whole. The middle curve of Fig 2 shows $Z_C(f)$ for a 30 turn input coil with a single 4 Ω damping resistor across the entire input coil (we call this single-resistor damping). Clearly, the Q of the resonances has been greatly reduced, but their individual Q's are still greater than 1. Our RSJ simulations show that the damping provided by this single 4 Ω resistor is insufficient to completely eliminate significant perturbations from the SQUID's d.c. output characteristics. The bottom curve in Fig. 2 shows $Z_C(f)$ for a 30 turn input coil with a 4 Ω resistor in parallel with *each turn* of the superconducting coil. The Q's are now well below one, and the resonances are no longer visibly present in the RSJ simulations. We emphasize that this comparison is not intended to demonstrate that single-resistor damping is necessarily ineffective; a *lower* value of the resistance *can* provide sufficient damping. Rather, our simulations demonstrate that for *corresponding* values of damping resistance, intra-coil damping is more effective than simple single-resistor damping.

Our d.c. SQUID fabrication process (detailed elsewhere [12]) uses Nb/AlO_x/Nb trilayer Josephson junctions, PdAu resistors, SiO_2 interlayer, and Nb wiring. Typical parameters of SQUIDs used in this study are: 20 μA junction critical currents, 80 pH SQUID washer inductance, 137 turn input coil with an estimated coupling of 0.85 to the washer, 4 Ω junction shunt resistance, and 0.15 Ω washer shunt resistance. This gives, nominally, $\beta_L (= 2I_cL/\Phi_0) = 1.5$ and $\beta_C = 0.5$. We implemented the new damping scheme, intra-coil damping, by placing a 1 Ω resistor across *each turn* of the superconducting input coil. These resistors were fabricated using a PdAu planar wire extending radially outward and in contact with all turns of the Nb input coil (see inset of Fig. 2). Since this can be done in the same lithographic layer used to fabricate the PdAu junction shunt resistors, it does not add any steps to the fabrication process.

To test the effectiveness of the intra-coil damping technique experimentally, we have fabricated (on the same trilayer wafer) d.c. SQUIDs both with no damping in the input coil and with intra-coil damping. The V–Φ characteristics of the SQUID with the undamped coil (dashed lines in Fig. 3) are distorted over a range of d.c. bias currents, limiting useful operation to a small range at relatively high d.c. bias currents and low sensitivity. In contrast, the characteristics of an otherwise identical SQUID with intra-coil damping (solid lines in Fig. 3) are exceptionally smooth at high d.c. bias currents (approximately twice the maximum critical current, Fig. 3a). Even at low d.c. bias currents (only slightly greater than the maximum critical current, Fig. 3b), useful low-noise operation can be achieved on the smooth side of the curve. This performance has been repeatably achieved on several independent fabrication runs and on various device designs.

We now discuss an important aspect of resonance damping: the resulting noise levels seen by the SQUID. There is a trade-off between damping and noise; typically, greater damping also leads to greater noise. The advantage of our technique is that, for a given allowable noise injected by the damping resistors (set by the equivalent input noise of the SQUID), the d.c. characteristics can be smoothed more effectively by intra-coil damping than by single-resistor damping.

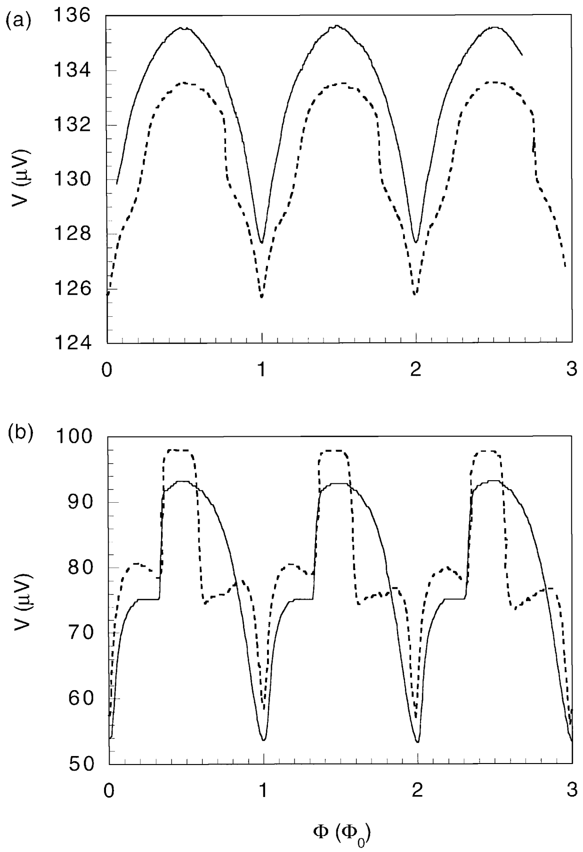

Fig. 3. Experimental voltage–flux characteristics for two SQUIDs with and without intra-coil damping. (a) At high d.c. current bias points, there are weak distortions present in the V–Φ characteristics of the SQUID with no coil damping (- - -); however, the operation in a flux-lock loop is still degraded. At the same bias points, the V–Φ characteristics of the SQUID with intra-coil damping (—) are nearly ideal. (b) At low current bias, the characteristics of SQUIDs with no coil damping are too irregular to allow standard flux-locked operation. The characteristics for the SQUID with intra-coil damping are remarkably smooth in comparison.

In a simple system in which a single damping resistor is placed across the input coil, the resistor will inject its Johnson noise into the SQUID loop. For a d.c. SQUID with an n-turn input coil with coupling α to the SQUID loop of inductance L_{SQ}, the equivalent flux noise due to a damping resistor R_D across the input coil is $\varphi_n = \alpha n L_{SQ}(4k_BT/R_D)^{1/2}$ at temperature T, where this noise is referred to the input of the bare SQUID loop. In contrast, the equivalent flux noise (referred to the input) due to intra-coil damping by a resistor $R_{i\text{-}c}(\neq R_D)$ across every turn of the input coil results in $\varphi_n = \alpha\sqrt{n}L_{SQ}\sqrt{4k_BT/R_{i\text{-}c}}$. Here, we have assumed that the impedance of the signal source across the input coil is high enough to approximate as an open circuit at signal frequencies (if not, the additional noise is even less than given above). The square root dependence of n comes about because the noise from each resistor adds incoherently.

The two equations above cannot be compared directly, because the choice of values for R_D and $R_{i\text{-}c}$ must be made based on the effectiveness of each approach to resonance damping. For example, with our device, a realistic external damping resistor value is $R_D = 4\ \Omega$; this should be considered a maximum value for even marginally effective damping as determined by SPICE simulation and experimental measurements. At 4 K, with a coupling near unity, and $L_{SQ} = 80$ pH, the flux noise due to this damping is approximately 0.3 $\mu\Phi_0/\sqrt{\mathrm{Hz}}$ *per turn* of the input coil. For a 100 turn input coil, this damping scheme would thus increase the noise floor of the SQUID to roughly 30 $\mu\Phi_0/\sqrt{\mathrm{Hz}}$, well above the noise floor of a good low-noise d.c. SQUID. In contrast, a d.c. SQUID with 4 Ω intra-coil damping (resulting in even more effective damping than in the external case) and a 100 turn coil would have a damping noise contribution of only 3 $\mu\Phi_0/\sqrt{\mathrm{Hz}}$.

The ideal thermal noise limit for the flux noise of a given SQUID is determined primarily by the junction shunt resistances [13,14] (for our SQUIDs, the limit is approximately 0.5 $\mu\Phi_0/\sqrt{\mathrm{Hz}}$). In practice, however, the effects of resonances in multi-turn, well-coupled SQUIDs prevent this limit from being reached. The actual noise in our undamped devices was high enough to prevent our secondary SQUID preamplifier from operating in flux-locked-loop mode. With intra-coil damping of 1 Ω/turn on our 137 turn coils, our simple model above predicts a flux noise of 7 $\mu\Phi_0/\sqrt{\mathrm{Hz}}$, compared to the 5 $\mu\Phi_0/\sqrt{\mathrm{Hz}}$ observed [15]. We have also measured the flux noise in a related system, d.c. SQUID series array amplifiers (SSAAs). In SSAAs with 100 SQUIDs, $L_{SQ} = 20$ pH, $R_{i\text{-}c} = 0.8$ Ω/turn, and $n = 10$ turns on the input coil, we have measured an equivalent flux noise of 1.0 $\mu\Phi_0/\sqrt{\mathrm{Hz}}$

for the individual SQUIDs [16], again close to the value expected from our simple model (1.4 $\mu\Phi_0/\sqrt{\mathrm{Hz}}$). We have not yet studied the noise in undamped SSAAs.

In conclusion, intra-coil damping is an effective technique for reducing SQUID resonances. It is fundamentally different than existing techniques in that the damping elements are distributed within the coil, and results in large improvements in V–Φ characteristics with acceptable noise levels. With this technique, the device may be operated with low noise over a larger range of bias currents, including low-current bias conditions with large sensitivity. Intra-coil damping also has the advantage (particularly with respect to resistor–capacitor networks) that it can be easily implemented in the fabrication process and does not require external circuit elements.

This work was supported in part by the Stanford University Relativity Gyroscope project.

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