

## TWO-STAGE INTEGRATED SQUID AMPLIFIER WITH SERIES ARRAY OUTPUT

Richard P. Welty

University of Colorado

Department of Electrical and Computer Engineering  
Boulder, CO 80309

John M. Martinis

National Institute of Standards and Technology

325 Broadway

Boulder, CO 80303

We have fabricated a 2-stage integrated dc SQUID amplifier which uses a compact series array of 100 dc SQUIDs as the readout device for a low-noise single SQUID. The output noise is dominated by the amplified noise of the input SQUID and substantially exceeds the input noise of a good room-temperature preamp. The input stage is a low-inductance double-loop SQUID with energy sensitivity of approximately 30-h (equivalent flux noise  $\phi_n \approx 0.3 \mu\phi_0/\sqrt{\text{Hz}}$ ), with an input transformer having input inductance  $L_{in} \approx 0.25 \mu\text{H}$  and net coupling to the SQUID of  $k^2 \approx 0.1$ . The bandwidth extends from dc to about 390 kHz. The series array has an output voltage swing of 3-4 mV, providing a dynamic range of over 50 dB at full bandwidth. The results suggest the general utility of series SQUID arrays as readout devices for SQUIDs.

### INTRODUCTION

Although dc SQUIDs are inherently capable of amplifying signals at frequencies from dc to a few GHz, the coupling techniques necessary to match the SQUID output signal to room temperature (RT) electronics severely limit the usable bandwidth. The voltage noise of a typical dc SQUID is about 2 orders of magnitude lower than the input noise of the best RT amplifiers. A transformer or resonant circuit, with ac flux modulation and lock-in detection, is usually used to step up the SQUID voltage to the required level.<sup>1</sup> This technique generally limits the bandwidth to tens of kHz, and requires complex RT electronics.

The voltage noise of a SQUID can be expressed as a flux noise multiplied by a transfer function:  $V_n = \phi_n dV/d\phi$ . Positive feedback has been used to make  $dV/d\phi$  large enough at certain flux bias points for  $V_n$  to exceed the input noise of an RT preamp when directly connected.<sup>2</sup> This technique suffers from the disadvantage that while  $V_n$  becomes larger, the total output voltage swing remains the same, so the dynamic range is reduced. Two-stage SQUID amplifiers in which a second SQUID is used to amplify the output of the first have been reported.<sup>3,4</sup> The gain of the second SQUID is sufficient to make the amplified noise of the first SQUID larger than the intrinsic noise of the second SQUID. A matching transformer and associated electronics are still required at the output of the second SQUID stage to achieve

the required total low-temperature gain. The step-up ratio required of the matching transformer is reduced, however, simplifying its design. This scheme suffers from the same loss of dynamic range as the high- $dV/d\phi$  SQUIDs, unless the output voltage swing of the second SQUID is substantially larger than that of the first SQUID. This latter approach is adopted in a 2-stage amplifier proposed by Foglietti,<sup>5</sup> and in the present work in which a series array of SQUIDs is used to provide a large output voltage swing, preserving the dynamic range of the input SQUID.

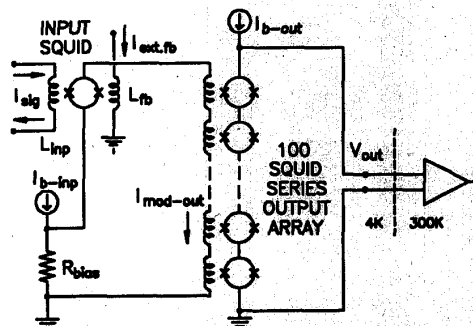


Fig. 1 - Schematic diagram of the 2-stage SQUID amplifier circuit.

In this paper we describe a 2-stage dc SQUID amplifier which consists of a single input SQUID modulating a 100-SQUID series output array through a 50-turn modulation coil. The output noise is dominated by the amplified noise of the input SQUID, and substantially exceeds the input noise of a good RT preamp. No additional RT electronics are required therefore, except a feedback circuit if flux-locked loop operation is desired. The amplifier has larger bandwidth than transformer-coupled SQUIDs, and larger dynamic range than high- $dV/d\phi$  SQUIDs. It is easy to use and requires only 2 more wires to RT than a single SQUID. Fabrication of the series array output stage was only slightly more demanding than fabrication of single SQUIDs. On-chip series arrays may be attractive as readout devices in a variety of SQUID applications.

### DESIGN AND FABRICATION

Figure 1 shows a schematic of the amplifier circuit. An input signal  $I_{sig}$  couples flux into the input SQUID, which is voltage-biased<sup>5</sup> with a 25 mΩ resistor so that the SQUID

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current is modulated by variations in the applied flux. The flux modulation coil of the output array is connected in series with the input SQUID, so that variations in the SQUID current change the flux applied to the output array. The series array is biased at constant current, so the output voltage  $V_{out}$  is modulated by this applied flux.

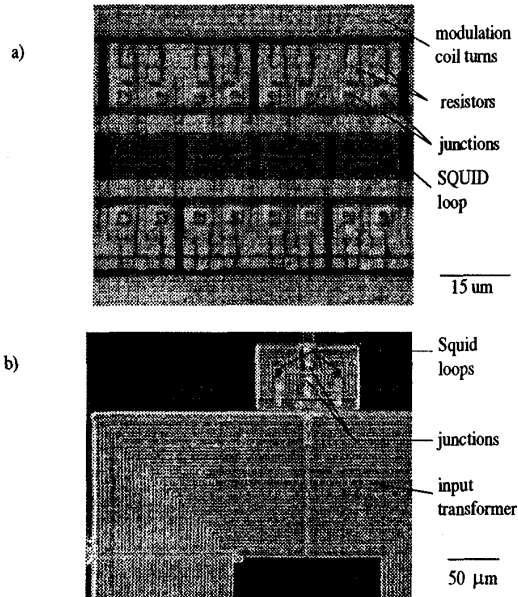


Figure 2 - (a) An 8-SQUID section of the 100-SQUID series output array. (b) The input SQUID and part of its input transformer.

#### A. Output stage

The gain  $G_{out} = dV_{out}/dI_{mod-out}$  required of the output stage is determined by the amplitude of the input SQUID current noise and the desired amplitude of the output voltage noise. The current noise  $I_n$  of a voltage-biased SQUID having flux noise  $\phi_n \approx 0.2 \mu\phi_0/\sqrt{\text{Hz}}$  and current modulation  $\Delta I \approx 100 \mu\text{A}$  is about  $40 \text{ pA}/\sqrt{\text{Hz}}$ . The output voltage  $V_{n-out}$  due to current  $I_{mod-out} = I_n$  in the output stage modulation coil should exceed the input voltage noise of a RT preamp. If we want, for example, a RT preamp with input noise of  $2 \text{ nV}/\sqrt{\text{Hz}}$  to contribute less than 5% to the total noise power,  $V_{n-out}$  should be at least  $9 \text{ nV}/\sqrt{\text{Hz}}$ , and  $G_{out}$  should therefore be at least  $225 \text{ V/A}$ .

The gain is proportional to  $N_{sq}N_t$ , where  $N_{sq}$  is the number of SQUIDs in the series array and  $N_t$  is the number of turns in its modulation coil. Increasing  $N_{sq}$  linearly increases the maximum output voltage swing  $\Delta V_{out}$ , and hence the gain, since the SQUIDs in the array are modulated coherently.<sup>6</sup> The noise of the array increases only as  $N_{sq}^{1/2}$  however, since the noise voltages are expected to add incoherently, so the total output noise of the amplifier is dominated by the amplified noise of the input stage. Within the constraint of the required gain,  $N_{sq}$  and  $N_t$  may be chosen to maximize the bandwidth BW and the dynamic range. A 100-SQUID series array similar to the present design was shown previously<sup>6</sup> to have bandwidth of at least 175 MHz when operated alone.

The bandwidth of the 2-stage amplifier is determined by the lowest cutoff frequency in the system,  $f_c = R_{dyn}/2\pi L_{in}$ , where  $R_{dyn}$  is the dynamic resistance of the input SQUID (around  $0.5 \Omega$ ) and  $L_{in}$  is the input inductance of the output array modulation coil. This input inductance is proportional to  $N_t^2 N_{sq} L_{sq}$ , where  $L_{sq}$  is the inductance of a single SQUID in the output array. Since  $L_{in}$  increases more rapidly with  $N_t$  than with  $N_{sq}$ , it is best to achieve the required gain by increasing  $N_{sq}$  rather than  $N_t$ . Increasing  $N_{sq}$  likewise maximizes the dynamic range  $\Delta V_{out}/(V_{n-out} BW^{1/2})$  since  $\Delta V_{out}$  increases, while  $V_{n-out}$  remains the same for a given gain. We chose 100 SQUIDs as a practical large  $N_{sq}$ , although larger arrays may well be feasible. Based on the mutual inductance per turn of the modulation coil to the SQUID array ( $\approx 3.2 \text{ pH}$ ) and the total voltage swing ( $\approx 3 \text{ mV}$ ), about 25 turns are required to achieve the required gain. We used more than the calculated required number of modulation coil turns to ensure adequate gain.

Figure 2a shows an 8-SQUID segment of the output array. The individual SQUID inductances are defined by the T-shaped slots in the base electrode. A 50-turn modulation coil is wound around the outside of the array. Coupling between the input coil and SQUID array is improved by use of a flux-focusing washer, similar geometrically to those used for washer-type single SQUIDs,<sup>7</sup> but electrically isolated from the SQUIDs in the array. A  $2 \Omega$  resistor is connected in parallel with the washer to damp high frequency resonances.

#### B. Input SQUID

The input stage consists of a low-inductance double-loop SQUID with a matched input transformer. Double-loop SQUIDs<sup>8</sup> and low-inductance SQUIDs with input transformers<sup>9</sup> have been investigated previously. Figure 2b shows the double-loop SQUID and part of the input transformer. The two SQUID loops are connected in parallel, with the junctions located between them. The SQUID inductance is therefore half of the individual loop inductance. Two 4-turn modulation coils are wound (in opposite directions) on the SQUID loops, one on each side, and connected in series. This double coil is connected to the washer (primary) of an input transformer having 36 input turns. The washer inductance is approximately equal to the input inductance of the modulation coil. A  $2 \Omega$  damping resistor is connected in parallel with the washer and the input coil. The input transformer has an additional 2-turn coil for external feedback, to permit operation in a flux-locked loop. While the use of a separate input transformer results in significant coupling losses<sup>10</sup> compared to a coil wound directly on the SQUID washer, it allows the SQUID inductance to be made smaller and helps isolate the SQUID oscillations from input coil resonances.

#### E. Fabrication

The circuit uses  $3.5 \times 3.5 \mu\text{m}^2$  trilayer Josephson junctions with critical current density  $J_c \approx 1000 \text{ A/cm}^2$ . The circuit was fabricated by a 6-layer process with Nb base and counter electrodes,  $\text{AlO}_x$  tunneling barrier, Pd/Au resistors, an  $\text{SiO}_2$  insulating layer, and a Pb/Au/In wiring layer. The input coils were made with  $2 \mu\text{m}$  lines and spaces. The overall dimen-

sions of the 100-SQUID array with modulation coil are about  $500 \times 1500 \mu\text{m}^2$ , and of the input SQUID with transformer about  $500 \times 500 \mu\text{m}^2$ . Since the junction size, minimum linewidth, and number of layers are the same for the series array as for single SQUIDs, the array is no more difficult to fabricate, except for increased exposure to contamination due to its larger area.

## RESULTS AND DISCUSSION

### A. Biasing and operation

The amplifier is biased beginning with the output stage. A triangle-wave modulation signal is applied through the bias line of the input SQUID with amplitude less than its maximum critical current  $I_{c\text{-inp}}$ . The output stage bias current is then adjusted to maximize the output voltage swing, which generally occurs just above  $I_{b\text{-out}} = I_{c\text{-out}}$ , the maximum critical current of the array. Figure 3a shows the output voltage vs. modulation current for an output stage alone, with maximum output voltage swing of about 4 mV. The shape and modulation depth of the  $V$ - $\phi$  curve are sensitive to trapped flux. In our moderately shielded test probe this often required the array to be heated momentarily above its  $T_c$  during initial biasing to expel or at least redistribute the trapped flux. We used a resistor embedded in the chip holder for this purpose.

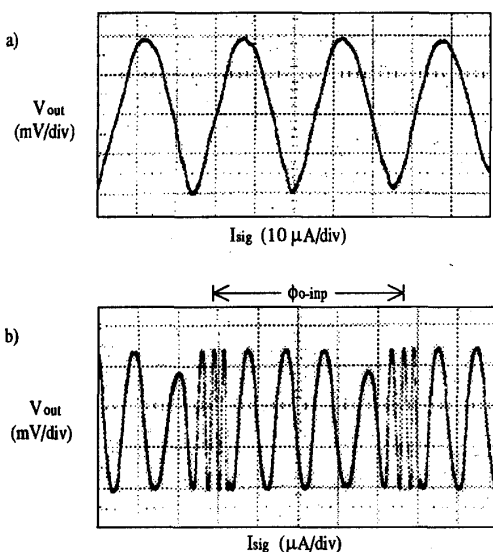


Figure 3 - (a) Output voltage vs. output stage modulation current. (b) Output voltage vs. input stage signal current. The interval labeled  $\phi_{0\text{-inp}}$  indicates 1 period of input SQUID modulation.

The input SQUID is biased by passing current  $I_{b\text{-inp}} > I_{c\text{-inp}}$  through the bias resistor  $R_{\text{bias}}$ . For  $I_{b\text{-inp}} \gg I_c$ , the bias voltage  $V_{b\text{-inp}} \approx I_{b\text{-inp}} R_{\text{bias}}$  is nearly constant despite variations in the SQUID current, since the SQUID resistance  $R_{\text{sq}} \gg R_{\text{bias}}$ . Figure 3b shows the output stage voltage vs. input signal current for a 2-stage amplifier. The multiple output voltage oscillations within one input SQUID period occur because the modulation depth  $\Delta I \approx 100 \mu\text{A}$  of the input SQUID

current is larger than the input period ( $\approx 15 \mu\text{A}/\phi_0$ ) of the output stage. The output voltage therefore undergoes several modulation periods for each period of input SQUID modulation. Since  $\Delta I$  is a function of  $V_{b\text{-inp}}$ , the number of oscillations per input stage period varies with  $V_{b\text{-inp}}$ .

### B. Noise

Noise measurements were made by connecting the output voltage of the series array directly to the input of a low noise preamp with 1000x gain, the output of which was connected to the spectrum analyzer. The input SQUID was flux-biased at a point of high responsivity, and the slope  $dV_{\text{out}}/dI_{\text{sig}}$  of the curve was measured at that point. No feedback was used. The noise varied with input SQUID bias voltage, tending to be lowest at higher bias voltages. Figure 4 shows a voltage noise spectrum at  $V_{\text{bias}} \approx 120 \mu\text{V}$  and  $dV_{\text{out}}/dI_{\text{sig}} \approx 4300 \text{ V/A}$ . The value in the white noise region is of  $V_{n\text{-out}} \approx 7 \text{ nV}/\sqrt{\text{Hz}}$ , corresponding to flux noise of about  $\phi_n \approx 0.3 \mu\phi_0/\sqrt{\text{Hz}}$  and intrinsic energy sensitivity  $\epsilon = \phi_n^2/2L_{\text{sq}} \approx 30\text{-h}$  (after subtracting the RT preamp noise). The flux noise at 1 Hz is about  $2.5 \mu\phi_0/\sqrt{\text{Hz}}$ . The spikes in the spectrum appear to be due to power frequency harmonics and microphonic resonances in the wiring, which we think can be greatly reduced by improved shielding and wiring.

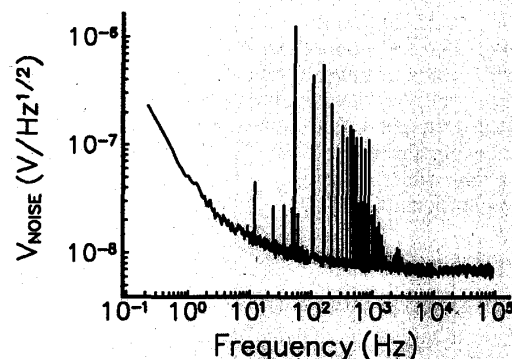


Figure 4 - Voltage noise spectrum of the 2-stage SQUID amplifier and RT preamp, referred to the RT preamp input. The voltage noise corresponds to an input SQUID flux noise of about  $0.3 \mu\phi_0/\sqrt{\text{Hz}}$  at 10 kHz, and about  $2.5 \mu\phi_0/\sqrt{\text{Hz}}$  at 1 Hz.

The RT preamp input noise was  $4 \text{ nV}/\sqrt{\text{Hz}}$  in the white noise region. This contribution to the total noise could easily be reduced by use of a quieter RT preamp. The amplified value of the SQUID noise was lower than expected because the output stage gain was reduced due to a fabrication problem. The mutual inductance of the 50-turn coil on the output array, and therefore the gain, varied from chip to chip by as much as a factor of 4, and was at best around 25% lower than expected based on the mutual inductance per turn measured for an identical array with a 3-turn coil. We suspect that pinholes in the insulator layer may have short-circuited some of the turns in the rather large modulation coil.

### C. Bandwidth and dynamic range

The amplifier bandwidth BW was determined by measuring the rise time  $t_r$  of the output voltage in response to a

square wave input signal, and calculating  $BW \approx 0.35/\tau$ . The output voltage was connected through wires on the chip and chip-holder to a 50  $\Omega$  coaxial cable, which was terminated with 50  $\Omega$  at the oscilloscope. The dynamic resistance of the output stage was approximately 70  $\Omega$ , and so was reasonably well matched to the transmission line. The input square wave signal was also coupled to the chip holder through 50  $\Omega$  coax. A 50  $\Omega$  resistor was fabricated on-chip in series with the input coil to improve matching.

The rise time was approximately 0.9  $\mu$ s, corresponding to  $BW \approx 390$  kHz. The rise time was independent of signal amplitude up to the maximum output voltage. The measured BW is consistent with the expected  $f_c = R_{dyn}/2\pi L_{in}$ , where  $L_{in}$  of the output stage modulation coil was measured as discussed below. The dynamic range  $\Delta V_{out}/(V_{n-out} BW^{1/2})$  was determined to be about 57 dB over the full bandwidth, for a typical output voltage swing of 3 mV.

The bandwidth was larger than expected for a 50-turn output stage modulation coil since  $L_{in}$  was lower than expected, as was the mutual inductance, due to the fabrication problem mentioned previously. A further factor contributing to larger BW was a larger junction shunt resistance than intended in the input SQUID, resulting in higher  $R_{dyn}$ . The bandwidth for a 2-stage amplifier using the same output array with a 25-turn modulation coil is expected to be about 350 kHz. Further increases in BW are possible by reducing the number of output stage modulation coil turns, at the price of reduced gain. Larger bandwidth could also be obtained, at no cost in gain and with improvement in the dynamic range, by increasing the number of SQUIDs in the output stage series array, while reducing the number of modulation coil turns proportionally.

#### D. Input inductances and coupling efficiency

The coupled energy sensitivity  $\epsilon_c = L_{in} I_{n-eq}^2/2$  describes the usable sensitivity of the SQUID, where  $L_{in}$  is the input inductance of the input SQUID modulation coil and  $I_{n-eq} = \Phi_n/M_{sig} \approx 1.3$  pA/ $\sqrt{\text{Hz}}$  is the current noise equivalent of the SQUID flux noise, referred to the signal input. Here  $M_{sig}$  is the net mutual inductance coupling the signal to the input SQUID. We measured the input inductance by a simple technique, and could thus determine  $\epsilon_c$  directly. A resistor  $R_n \approx 25$  m $\Omega$  was fabricated in parallel with the modulation coil. This resistor induces current noise  $(4k_B T/R_n)^{1/2}$  in the coil. Since  $R_n$  is rather small, the current noise is large enough to produce an output voltage substantially larger than that due to the intrinsic noise of the SQUID. Measuring the magnitude  $V_n$  and cutoff frequency  $f_c$  of this noise (and  $dV/dI_{mod}$  at the chosen flux bias point) allows us to calculate  $R_n$  and the input inductance  $L_{in} = R_n/2\pi f_c$ . The resistor  $R_n$  was connected using a scratchable link, which was broken before subsequent measurements.

The input inductance of the input stage modulation coil was determined to be about 245 nH, implying that  $\epsilon_c \approx 310$ -h. The net coupling constant  $k^2 = \epsilon/\epsilon_c$  is therefore about 0.1. The coupling is worse than we expected, we think due at least in part to a design error in inductance matching between the

input stage transformer and the SQUID input coil. We are currently improving the design to provide tighter coupling. The input inductance of the output stage was measured similarly, using the current noise induced in its modulation coil by the input stage bias resistor (at zero bias current). Table 1 shows this result and other measurements of SQUID parameters.

Table 1. SQUID parameters

	INPUT	OUTPUT
$L_{squad}$	10 pH	15 pH
$I_{lc}$	250 $\mu$ A	250 $\mu$ A
$R_{shunt}$ (junction)	1.2 $\Omega$	0.8 $\Omega$
$\beta_c = 2\pi I_{lc} C R^2/\Phi_0$	0.5	0.25
$\beta_L = I_{lc} L_{eq}/\Phi_0$	1.4	1.9
$1/M_{mod}$	4.5 $\mu$ A/ $\Phi_0$	18 $\mu$ A/ $\Phi_0$ *
$N_t$	36	50*
$L_{in}$	250 nH	850 nH*

\* varied from chip to chip due to a fabrication problem

#### CONCLUSION

In summary, we have shown that it is possible to make an integrated dc SQUID amplifier using a series SQUID array to provide a large output voltage swing. The amplifier offers large dynamic range and bandwidth, and can be directly connected to RT electronics. Fabrication and operation are only slightly more difficult than for single SQUIDs. While results were reported for a specific input SQUID, we think that this readout technique may be useful generally for other SQUIDs. In continuing work we are extending the bandwidth by introducing 1 or 2 additional single SQUID amplification stages, connected in cascade with the input stage, to reduce the gain required of the output stage. The number of output stage modulation coil turns may then be reduced, reducing the input inductance and increasing the bandwidth.

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