Boolean Algebra

Boolean Algebra Identities

1. $A + B = B + A \qquad A * B = B * A$ (Communicative Property)
2. $A + (B + C) = (A + B) + C \quad A * (B * C) = (A * B) * C$ (Associative Property)
3. $A * (B + C) = A * B + A * C$ (Distributive Property)
4. $\overline{A + B} = \overline{A} * \overline{B}$ (DeMorgan's Law)
5. $\overline{A * B} = \overline{A} + \overline{B}$ (DeMorgan's Law)
6. $A + 0 = A \qquad A * 0 = 0$
7. $A + 1 = 1 \qquad A * 1 = A$
8. $A + \overline{A} = 1 \qquad A * \overline{A} = 0$
9. $A + A = A \qquad A * A = A$
10. $\overline{\overline{A}} = A$

11. $A + \overline{A} * B = A + B$
12. $(A + B) * (A + C) = A + B * C$
13. $(A + B) * (C + D) = A * C + A * D + B * C + B * D$
14. $A * (A + B) = A$
15. $A \oplus B = A * \overline{B} + \overline{A} * B$
16. $\overline{A \oplus B} = \overline{A} \oplus B = A \oplus \overline{B}$

Sample Programs

Simplify the following expression as much as possible:	Using various elementary identities, the expression simplifies as follows:
$\overline{A(A + B)} + B\overline{A}$	$\overline{A(A + B)} + B\overline{A} = \overline{\overline{A(A + B)}} * \overline{B} * \overline{\overline{A}}$ $= A(A + B) * (\overline{B} + A)$ $= (A + AB)(\overline{B} + A)$ $= A(1 + B)(\overline{B} + A)$ $= A(1)(\overline{B} + A)$ $= A(\overline{B} + A)$ $= A\overline{B} + AA$ $= A\overline{B} + A$ $= A(\overline{B} + 1)$ $= A(1) = A$

Find all ordered pairs (A, B) that make the following expression TRUE. $\overline{\overline{A+B}+\overline{A}*B}$	$\overline{\overline{A+B}+\overline{A}*B}=(\overline{\overline{A+B}})(\overline{\overline{A}B})$ $=(A+B)(A+\overline{B})$ $=AA+A\overline{B}+BA+B\overline{B}$ $=A+A(B+\overline{B})+0$ $=A+A(1)=A+A=A$ This yields the solutions (1, 0) and (1, 1). This problem, like most Boolean Algebra problems, could also be solved by drawing a truth table with the following seven column headings: A, B, A+B, $\overline{A+B}$, $\overline{A}B$, $\overline{A+B}+\overline{A}B$, $\overline{\overline{A+B}+\overline{A}B}$.
Simplify the following expression to one that uses only two operators. $(\overline{\overline{A}+\overline{B}}*\overline{C})+(A*\overline{(B+\overline{C})})$	The evaluation is as follows: $(\overline{\overline{A}+\overline{B}}*\overline{C})+(A*\overline{(B+\overline{C})})$ $=(A*B*\overline{C})+(A*\overline{B}*C)$ $=A*(B*\overline{C}+\overline{B}*C)$ $=A*(B\oplus C)$ To realize this equation as a circuit, two gates are used: an XOR (input is B and C) and an AND (inputs are A and the output of the XOR gate). The output of the AND is the output of the circuit.