Lab #1: Practice in R

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The purpose of this lab is to get you comfortable working in R and trying a few things from the first lecture.

Simulating data:

We will simulate something like the weight vs. age data from a cubic function and add some noise and then apply a few methods.

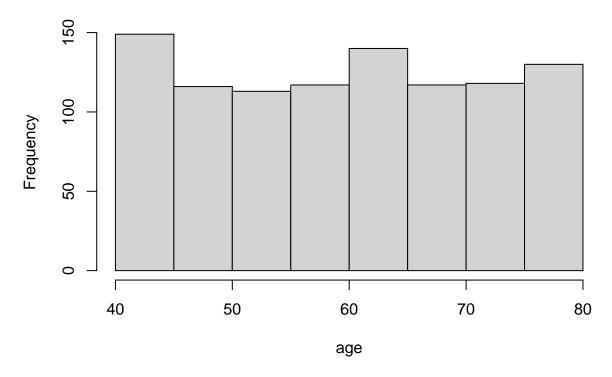
1. Write a function in R to input the value (age) and output, "noise-free" weight: $f(x) = -0.0004x^3 + 0.025x^2 + 2x + 50$

```
make_weight <- function(age) {
  -0.0004 * age^3 + 0.025 * age^2 + 2 * age + 50
}
```

2. Next we will simulate some values for x. Use the sample function to generate a random sample of integers between 40 and 80 of size 1000 (you will need the replace=TRUE option to make sure the numbers can be resampled otherwise you will run out of values quickly). Don't forget that you can type ?sample to get help as to how to use the function.

```
?sample
age <- sample(seq(40, 80), size=1000, replace=TRUE)
hist(age)</pre>
```

Histogram of age



3. Now generate a vector of the output of your function in part 1 to generate the noise-free weights for each of the 1000 individuals.

```
no_noise_weights <- make_weight(age)
no_noise_weights</pre>
```

```
##
      [1] 150.4222 168.2792 144.4000 173.6182 174.7688 166.6742 171.1478 174.3008
##
      [9] 173.6000 173.6000 160.9654 155.9656 159.3632 167.9144 173.6000 170.8096
     [17] 165.2000 175.5424 170.8096 170.8096 173.6182 175.5424 167.9144 174.3008
##
     [25] 172.8734 172.8104 154.1750 160.9654 169.6118 172.8734 175.9198 172.8734
##
##
     [33] 174.7688 154.1750 166.8094 175.5424 154.1750 168.2792 154.1750 175.2062
##
     [41] 174.8606 166.6742 146.4566 171.1478 174.2326 166.6742 169.6118 144.4000
##
     [49] 166.8094 154.1750 174.7688 154.1750 166.8094 168.2792 162.5000 170.1536
##
     [57] 171.8750 165.3568 166.8094 175.5424 175.5424 174.7688 172.8734 168.2792
##
     [65] 175.8272 165.3568 174.2326 175.8272 175.9198 174.3008 173.6000 175.3000
##
     [73] 169.6118 170.1536 159.3632 168.2792 174.7688 172.0552 175.7750 175.6214
##
     [81] 174.8606 168.2792 175.8272 173.6182 160.9654 144.4000 175.7750 167.9144
##
     [89] 175.8272 175.2062 163.9646 168.2792 175.2062 175.9198 174.2326 166.8094
##
     [97] 174.3008 172.0552 175.7750 172.8104 144.4000 175.8272 166.6742 174.2326
    [105] 154.1750 175.9198 165.2000 170.8096 148.4648 174.2326 172.8104 155.9656
##
    [113] 172.8104 165.2000 175.9016 155.9656 172.8734 175.2062 169.0750 172.0552
##
    [121] 171.8750 146.4566 144.4000 168.2792 165.2000 165.3568 175.7750 173.6182
##
##
    [129] 146.4566 172.0552 170.1536 173.6182 162.5000 175.3000 172.8104 172.8104
    [137] 172.0552 162.5000 169.0750 166.8094 159.3632 175.9198 175.2062 160.9654
    [145] 167.9144 170.8096 174.8606 170.8096 168.2792 175.3000 170.8096 175.8272
##
```

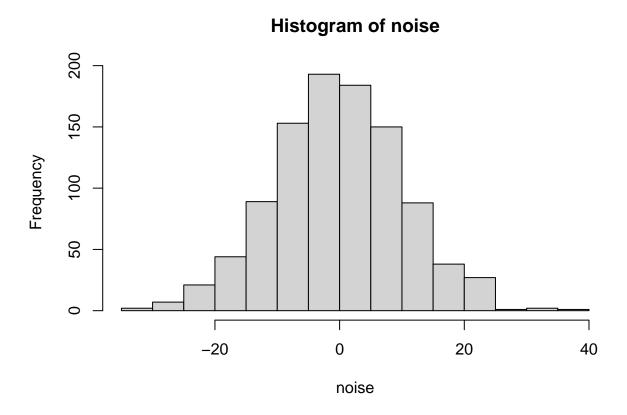
```
[153] 163.9646 166.6742 169.6118 169.0750 167.9144 146.4566 144.4000 148.4648
    [161] 159.3632 169.6118 170.1536 172.8734 152.3264 165.2000 175.9016 175.3000
##
    [169] 157.6958 171.8750 165.2000 175.8272 173.6000 144.4000 172.8734 175.2062
    [177] 174.2326 169.6118 169.0750 175.6214 175.8272 146.4566 157.6958 166.6742
##
    [185] 171.8750 175.7750 166.8094 170.8096 157.6958 155.9656 171.8750 171.8750
    [193] 159.3632 175.9016 165.2000 167.9144 169.6118 154.1750 150.4222 172.8104
##
    [201] 172.8104 165.3568 165.2000 155.9656 175.5424 175.3000 170.8096 175.9198
    [209] 168.2792 175.9198 172.8734 175.8272 166.8094 144.4000 175.9198 166.6742
##
##
    [217] 175.3000 163.9646 175.8272 169.0750 160.9654 173.6182 175.7750 174.3008
    [225] 150.4222 174.2326 175.5424 175.2062 170.8096 167.9144 166.6742 170.1536
##
    [233] 171.1478 174.3008 174.8606 157.6958 168.2792 155.9656 175.9016 159.3632
    [241] 172.8734 173.6182 170.8096 175.8272 162.5000 146.4566 148.4648 169.6118
##
    [249] 165.2000 167.9144 175.9198 152.3264 175.9198 166.6742 170.8096 170.1536
    [257] 144.4000 175.7750 174.3008 172.0552 175.7750 174.7688 150.4222 169.0750
##
    [265] 175.9016 168.2792 169.6118 169.0750 173.6182 175.8272 172.8104 160.9654
##
##
    [273] 154.1750 168.2792 174.3008 174.7688 174.3008 172.8104 162.5000 165.3568
    [281] 170.8096 175.7750 175.7750 166.8094 173.6182 150.4222 175.5424 169.6118
##
##
    [289] 170.1536 175.2062 159.3632 175.6214 159.3632 162.5000 174.2326 174.8606
    [297] 166.8094 148.4648 172.0552 174.3008 150.4222 165.3568 144.4000 166.8094
    [305] 152.3264 172.8104 174.7688 175.8272 172.8104 171.8750 159.3632 172.8104
    [313] 172.8104 172.8734 174.2326 165.2000 174.2326 172.0552 171.1478 168.2792
##
    [321] 175.8272 175.9016 171.8750 174.8606 175.3000 170.1536 170.8096 175.5424
    [329] 159.3632 173.6000 173.6000 167.9144 170.8096 175.2062 166.6742 155.9656
##
    [337] 175.9016 175.7750 175.6214 167.9144 154.1750 173.6182 173.6000 144.4000
##
    [345] 170.1536 165.3568 175.7750 166.8094 146.4566 165.2000 157.6958 175.5424
##
    [353] 175.3000 172.8734 172.8734 152.3264 160.9654 172.8734 167.9144 175.8272
    [361] 146.4566 144.4000 152.3264 175.5424 170.8096 174.8606 166.8094 165.3568
##
##
    [369] 175.5424 150.4222 169.6118 152.3264 144.4000 169.6118 165.3568 157.6958
    [377] 172.0552 175.9198 169.6118 162.5000 157.6958 174.7688 172.8734 152.3264
##
    [385] 146.4566 152.3264 163.9646 148.4648 175.7750 166.6742 165.3568 175.3000
##
    [393] 173.6182 175.6214 175.7750 155.9656 175.9016 159.3632 157.6958 174.2326
##
    [401] 154.1750 175.9016 175.6214 160.9654 175.6214 144.4000 154.1750 170.1536
##
    [409] 165.3568 175.7750 146.4566 174.8606 171.8750 152.3264 174.7688 169.6118
    [417] 172.8734 175.9198 175.9198 163.9646 172.8104 150.4222 175.7750 165.3568
##
    [425] 148.4648 167.9144 173.6182 169.6118 150.4222 175.7750 173.6000 173.6000
##
    [433] 169.0750 170.8096 175.2062 173.6000 148.4648 175.7750 174.7688 171.8750
##
    [441] 174.2326 148.4648 150.4222 171.8750 170.8096 144.4000 170.8096 174.2326
##
    [449] 173.6000 175.9016 162.5000 162.5000 154.1750 171.8750 154.1750 172.8734
    [457] 166.8094 174.3008 166.8094 162.5000 162.5000 173.6000 173.6000 170.1536
##
    [465] 159.3632 171.1478 175.3000 157.6958 174.7688 154.1750 162.5000 170.8096
##
    [473] 175.5424 146.4566 168.2792 174.8606 166.8094 174.2326 172.8104 162.5000
    [481] 165.3568 146.4566 166.8094 154.1750 167.9144 165.2000 175.9198 144.4000
##
##
    [489] 146.4566 159.3632 152.3264 175.9016 175.8272 146.4566 170.8096 175.3000
    [497] 166.6742 175.5424 173.6000 174.7688 154.1750 173.6000 152.3264 175.5424
##
    [505] 170.1536 174.3008 166.8094 172.0552 170.1536 172.8734 174.7688 172.8734
    [513] 175.3000 152.3264 175.9016 174.3008 144.4000 175.8272 173.6182 171.8750
##
##
    [521] 172.8734 173.6000 169.6118 175.2062 173.6182 172.8734 148.4648 172.8734
    [529] 173.6182 167.9144 175.6214 165.3568 169.0750 171.8750 175.6214 175.8272
##
    [537] 175.9198 165.3568 155.9656 171.1478 144.4000 150.4222 152.3264 171.8750
    [545] 173.6182 175.7750 174.3008 172.0552 170.1536 165.3568 168.2792 171.8750
##
    [553] 155.9656 174.2326 159.3632 174.8606 166.8094 174.3008 175.9198 163.9646
##
##
    [561] 175.6214 172.8104 175.7750 148.4648 167.9144 168.2792 174.7688 175.5424
##
    [569] 144.4000 175.9198 159.3632 166.6742 175.3000 169.0750 173.6000 163.9646
    [577] 175.6214 174.2326 144.4000 163.9646 175.9198 163.9646 175.5424 175.5424
```

```
[585] 157.6958 175.3000 152.3264 163.9646 175.6214 159.3632 146.4566 159.3632
    [593] 165.2000 165.2000 174.2326 175.9016 169.6118 174.7688 173.6182 148.4648
##
    [601] 144.4000 146.4566 172.0552 175.8272 152.3264 173.6182 165.2000 163.9646
##
    [609] 165.2000 171.1478 173.6182 165.3568 168.2792 150.4222 154.1750 174.7688
##
##
    [617] 175.5424 169.0750 174.2326 175.7750 166.8094 166.6742 154.1750 157.6958
    [625] 173.6000 175.2062 148.4648 166.8094 170.8096 166.6742 175.9016 165.3568
##
    [633] 172.0552 148.4648 175.8272 152.3264 175.5424 175.3000 166.6742 171.8750
    [641] 172.8734 175.3000 165.3568 148.4648 166.8094 175.2062 175.8272 160.9654
##
##
    [649] 165.3568 154.1750 160.9654 168.2792 172.0552 171.1478 173.6000 174.7688
    [657] 175.7750 167.9144 175.8272 175.8272 152.3264 174.2326 144.4000 174.2326
##
    [665] 165.3568 174.2326 154.1750 165.2000 174.3008 175.2062 174.2326 174.3008
    [673] 160.9654 172.0552 157.6958 163.9646 162.5000 169.0750 173.6000 154.1750
##
##
    [681] 175.2062 175.7750 157.6958 173.6000 155.9656 172.8104 174.3008 175.6214
    [689] 152.3264 152.3264 150.4222 162.5000 165.2000 146.4566 160.9654 173.6182
##
##
    [697] 146.4566 163.9646 154.1750 159.3632 168.2792 172.0552 152.3264 170.1536
##
    [705] 159.3632 175.5424 165.3568 163.9646 172.8104 157.6958 146.4566 159.3632
    [713] 175.3000 155.9656 152.3264 155.9656 168.2792 175.3000 146.4566 172.8734
##
##
    [721] 165.2000 157.6958 175.5424 172.8104 173.6000 174.8606 169.0750 155.9656
    [729] 175.5424 172.0552 172.8104 175.5424 165.3568 160.9654 173.6182 169.0750
##
##
    [737] 162.5000 175.2062 148.4648 155.9656 175.6214 174.2326 174.8606 174.7688
##
    [745] 175.2062 163.9646 175.9198 157.6958 174.7688 165.3568 171.1478 146.4566
    [753] 165.2000 163.9646 160.9654 174.7688 154.1750 157.6958 174.7688 165.3568
    [761] 175.6214 173.6182 157.6958 163.9646 174.2326 174.7688 148.4648 150.4222
##
    [769] 173.6000 163.9646 175.2062 165.3568 155.9656 175.9016 174.2326 172.8104
##
    [777] 174.8606 152.3264 169.6118 175.9016 160.9654 175.8272 165.3568 163.9646
##
    [785] 175.9016 168.2792 144.4000 159.3632 172.8104 162.5000 166.8094 171.8750
##
    [793] 170.8096 160.9654 173.6000 171.8750 174.7688 175.9198 174.8606 175.5424
    [801] 175.2062 163.9646 152.3264 175.3000 170.1536 154.1750 175.8272 172.0552
##
##
    [809] 175.7750 175.9016 144.4000 144.4000 159.3632 154.1750 171.8750 175.5424
    [817] 150.4222 159.3632 159.3632 168.2792 171.8750 172.0552 150.4222 172.8104
##
##
    [825] 175.8272 169.0750 148.4648 165.2000 171.8750 172.8734 166.6742 166.6742
##
    [833] 174.3008 144.4000 172.8104 172.0552 175.2062 162.5000 174.7688 146.4566
    [841] 168.2792 172.8734 166.6742 160.9654 173.6000 174.3008 174.8606 175.9198
##
    [849] 175.9198 155.9656 150.4222 175.5424 175.6214 175.6214 170.8096 174.8606
##
##
    [857] 166.8094 160.9654 174.7688 159.3632 174.3008 162.5000 172.8734 172.8734
    [865] 173.6182 159.3632 165.3568 152.3264 148.4648 175.7750 170.1536 144.4000
##
##
    [873] 157.6958 172.0552 171.1478 146.4566 174.2326 171.8750 150.4222 166.6742
##
    [881] 162.5000 169.0750 175.5424 171.8750 152.3264 175.9016 175.3000 175.9016
    [889] 146.4566 165.2000 174.8606 174.7688 172.8734 154.1750 152.3264 172.0552
##
##
    [897] 171.1478 173.6182 173.6000 168.2792 172.8734 175.7750 174.8606 168.2792
    [905] 166.8094 160.9654 162.5000 173.6000 165.3568 172.8734 169.6118 155.9656
##
    [913] 170.8096 175.8272 170.1536 157.6958 169.6118 174.8606 175.2062 154.1750
    [921] 174.7688 160.9654 162.5000 160.9654 166.6742 169.0750 175.9016 165.2000
##
    [929] 144.4000 150.4222 168.2792 174.7688 174.8606 169.0750 166.8094 172.8734
##
    [937] 150.4222 159.3632 169.0750 175.3000 160.9654 174.3008 166.8094 170.1536
##
    [945] 165.3568 175.5424 171.1478 175.5424 160.9654 175.8272 159.3632 150.4222
##
    [953] 175.7750 169.6118 171.1478 172.8104 144.4000 173.6000 166.6742 175.6214
##
    [961] 144.4000 174.7688 165.2000 172.0552 175.9016 155.9656 166.6742 174.8606
##
##
    [969] 166.6742 175.9198 172.8734 144.4000 155.9656 175.9198 175.2062 175.7750
##
    [977] 175.6214 170.8096 175.2062 175.9198 166.8094 169.6118 174.7688 175.8272
    [985] 166.8094 167.9144 160.9654 172.8104 175.5424 159.3632 166.6742 172.0552
##
    [993] 175.7750 166.8094 169.0750 175.3000 165.3568 165.2000 148.4648 162.5000
##
```

4. Use rnorm to generate some normally distributed noise with mean 0.0 and sd of 10.0 for each of the

values.

```
noise <- rnorm(1000, mean=0, sd=10)
hist(noise)</pre>
```



5. Add the noise-free weights to the noise to get your simulated outcome data.

```
weights <- no_noise_weights + noise</pre>
```

6. Put these together in a dataframe with columns for age, weight (the observed outcome with noise), trueFx (the noise-free weight value), and noise.

```
df <- data.frame(age=age, weight=weights, trueFx=no_noise_weights, noise=noise)
df</pre>
```

```
##
              weight
                        trueFx
                                      noise
## 1
         43 161.6581 150.4222
                                11.23585423
## 2
         78 160.6807 168.2792
                                -7.59845657
         40 155.8896 144.4000
                               11.48959103
## 4
         73 165.1934 173.6182
                               -8.42476259
## 5
         62 178.6829 174.7688
                                 3.91413340
         53 175.5880 166.6742
                                 8.91377242
## 6
## 7
         57 157.7952 171.1478 -13.35258713
         72 178.2820 174.3008
## 8
                                 3.98123478
```

```
## 9
         60 172.4841 173.6000 -1.11586803
## 10
         60 180.3574 173.6000
                                 6.75743917
## 11
         49 153.0794 160.9654
                               -7.88597935
## 12
         46 155.0957 155.9656
                               -0.86986334
## 13
         48 173.1860 159.3632
                                13.82284008
## 14
         54 169.5993 167.9144
                                 1.68490164
## 15
         60 181.8319 173.6000
                                 8.23190948
## 16
         76 168.6007 170.8096 -2.20894598
## 17
         80 154.9061 165.2000 -10.29391655
## 18
         64 175.4331 175.5424 -0.10925691
## 19
         76 158.5597 170.8096 -12.24991155
         76 144.8485 170.8096 -25.96111388
## 20
## 21
         73 185.3094 173.6182 11.69122592
## 22
         64 164.6733 175.5424 -10.86908817
## 23
         54 149.6536 167.9144 -18.26083013
## 24
         72 184.2536 174.3008
                                 9.95281807
                                -0.11861781
## 25
         59 172.7548 172.8734
## 26
         74 166.8141 172.8104
                                -5.99628395
## 27
         45 152.3955 154.1750
                                -1.77947987
## 28
         49 156.7056 160.9654
                                -4.25981342
## 29
         77 179.5784 169.6118
                                 9.96658776
## 30
         59 180.1500 172.8734
                                 7.27660709
## 31
         67 158.6535 175.9198 -17.26630596
## 32
         59 176.4074 172.8734
                                 3.53398496
## 33
         62 182.0369 174.7688
                                 7.26813666
## 34
         45 160.8576 154.1750
                                 6.68260976
## 35
         79 142.5662 166.8094 -24.24317309
## 36
         64 173.1888 175.5424
                               -2.35357425
## 37
         45 173.9713 154.1750
                               19.79633321
## 38
         78 176.2471 168.2792
                                 7.96794539
## 39
         45 137.0822 154.1750 -17.09276181
## 40
         63 158.5695 175.2062 -16.63668712
## 41
         71 179.7717 174.8606
                                 4.91109552
         53 164.9336 166.6742
## 42
                                -1.74055486
## 43
         41 156.0695 146.4566
                                 9.61290564
## 44
         57 174.0861 171.1478
                                 2.93826662
## 45
         61 175.0426 174.2326
                                 0.80999364
## 46
         53 168.5108 166.6742
                                 1.83661843
         77 171.2744 169.6118
## 47
                                 1.66255035
         40 131.7040 144.4000 -12.69599066
## 48
         79 190.3043 166.8094
## 49
                                23.49493321
## 50
         45 140.0549 154.1750 -14.12005407
## 51
         62 174.5992 174.7688
                                -0.16961493
## 52
         45 148.7318 154.1750
                               -5.44319353
## 53
         79 184.8105 166.8094
                                18.00112333
         78 178.3936 168.2792
## 54
                                10.11440176
## 55
         50 156.8628 162.5000
                                -5.63716556
## 56
         56 172.2078 170.1536
                                 2.05420795
## 57
         75 183.5296 171.8750
                                11.65461950
## 58
         52 187.7200 165.3568
                                22.36322840
## 59
         79 169.8321 166.8094
                                 3.02265076
## 60
         64 165.1173 175.5424 -10.42506602
## 61
         64 165.7070 175.5424
                               -9.83542313
## 62
         62 194.8260 174.7688 20.05718580
```

```
## 63
         59 152.1677 172.8734 -20.70571484
         78 198.8366 168.2792 30.55742369
## 64
## 65
         68 173.2137 175.8272
                                -2.61350594
## 66
         52 160.8129 165.3568
                               -4.54393259
## 67
         61 175.8082 174.2326
                                 1.57560555
## 68
         68 185.1611 175.8272
                                 9.33388728
## 69
         67 178.9481 175.9198
                                 3.02828276
## 70
         72 154.7393 174.3008 -19.56150222
## 71
         60 177.1354 173.6000
                                 3.53536709
## 72
         70 179.8042 175.3000
                                 4.50424514
## 73
         77 176.2073 169.6118
                                 6.59550871
## 74
         56 159.8394 170.1536 -10.31420728
## 75
         48 135.6530 159.3632 -23.71022881
## 76
         78 165.0334 168.2792
                               -3.24576308
## 77
         62 165.3258 174.7688
                                -9.44298751
## 78
         58 164.3963 172.0552
                                -7.65889998
## 79
         65 166.2372 175.7750
                                -9.53779267
## 80
         69 171.6414 175.6214
                                -3.98004450
## 81
         71 171.7484 174.8606
                                -3.11217063
## 82
         78 176.2401 168.2792
                                 7.96092713
## 83
         68 185.6915 175.8272
                                 9.86428344
## 84
         73 165.6729 173.6182
                               -7.94531662
         49 157.8772 160.9654
## 85
                                -3.08817972
         40 148.0144 144.4000
## 86
                                 3.61444766
## 87
         65 189.7629 175.7750
                               13.98791105
## 88
         54 167.3537 167.9144
                                -0.56070420
## 89
         68 158.8385 175.8272 -16.98873492
## 90
         63 177.5247 175.2062
                                 2.31852545
## 91
         51 162.7737 163.9646
                               -1.19090671
                               17.72492853
## 92
         78 186.0041 168.2792
## 93
         63 178.6404 175.2062
                                 3.43422165
## 94
         67 169.6893 175.9198
                                -6.23049782
## 95
         61 169.8374 174.2326
                                -4.39522294
## 96
         79 161.7564 166.8094
                                -5.05296790
## 97
         72 176.1612 174.3008
                                 1.86035137
## 98
         58 173.8194 172.0552
                                 1.76417798
## 99
         65 184.9335 175.7750
                                 9.15848207
## 100
         74 176.0122 172.8104
                                 3.20176726
## 101
         40 140.7331 144.4000
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         46 154.4255 155.9656
                                -1.54014786
## 741
         69 194.1638 175.6214
                                18.54244505
## 742
         61 155.1909 174.2326 -19.04167354
## 743
         71 178.4536 174.8606
                                 3.59299702
## 744
         62 195.9141 174.7688
                                21.14534946
## 745
         63 163.8487 175.2062 -11.35748987
## 746
                               -7.96255986
         51 156.0020 163.9646
## 747
         67 171.9441 175.9198
                                -3.97569450
## 748
         47 158.6130 157.6958
                                 0.91724042
## 749
         62 172.3819 174.7688
                                -2.38694074
## 750
         52 160.7372 165.3568
                                -4.61962364
## 751
         57 190.4112 171.1478
                                19.26340607
## 752
         41 147.6771 146.4566
                                 1.22054815
##
  753
         80 176.1205 165.2000
                                10.92050577
## 754
         51 134.2420 163.9646 -29.72257475
## 755
         49 172.2503 160.9654
                                11.28489051
## 756
         62 187.8888 174.7688
                                13.12000129
## 757
         45 149.3489 154.1750
                                -4.82607973
## 758
         47 155.4472 157.6958
                                -2.24858997
## 759
         62 191.4465 174.7688
                                16.67765311
## 760
         52 165.7690 165.3568
                                 0.41218532
## 761
         69 186.0031 175.6214
                                10.38174846
## 762
         73 179.4946 173.6182
                                 5.87642933
## 763
         47 156.1128 157.6958
                               -1.58299577
## 764
         51 161.0437 163.9646 -2.92085415
```

```
## 765
         61 160.0336 174.2326 -14.19898963
## 766
         62 153.3856 174.7688 -21.38318025
                               -0.22500105
## 767
         42 148.2398 148.4648
## 768
         43 153.1199 150.4222
                                 2.69770954
##
  769
         60 168.5703 173.6000
                                -5.02969399
## 770
         51 170.2727 163.9646
                                 6.30807390
## 771
         63 179.9519 175.2062
                                 4.74565208
## 772
         52 161.9502 165.3568
                                -3.40659062
## 773
         46 156.4616 155.9656
                                 0.49601560
## 774
         66 188.8542 175.9016
                                12.95263191
## 775
         61 177.3788 174.2326
                                 3.14624205
## 776
         74 180.8261 172.8104
                                 8.01572504
##
  777
         71 179.7212 174.8606
                                 4.86058896
## 778
                                17.25561057
         44 169.5820 152.3264
## 779
         77 170.6361 169.6118
                                 1.02432861
## 780
         66 166.0032 175.9016
                                -9.89837075
## 781
         49 156.3610 160.9654
                                -4.60441209
## 782
         68 178.2038 175.8272
                                 2.37661492
## 783
         52 162.6628 165.3568
                                -2.69397264
## 784
         51 162.2691 163.9646
                                -1.69554070
## 785
         66 182.5503 175.9016
                                 6.64868973
## 786
         78 160.5237 168.2792
                                -7.75550915
## 787
         40 125.2074 144.4000 -19.19261131
## 788
         48 179.5565 159.3632
                                20.19332540
## 789
         74 177.1625 172.8104
                                 4.35210515
## 790
         50 173.0017 162.5000
                                10.50168832
## 791
         79 163.6883 166.8094
                                -3.12114841
##
  792
         75 172.8112 171.8750
                                 0.93617617
## 793
         76 149.2777 170.8096 -21.53187751
## 794
         49 164.5453 160.9654
                                 3.57993071
## 795
         60 178.2786 173.6000
                                 4.67864811
## 796
         75 162.8081 171.8750
                                -9.06688835
## 797
         62 169.4753 174.7688
                                -5.29349916
## 798
         67 189.3556 175.9198
                                13.43576717
##
  799
         71 179.5549 174.8606
                                 4.69434057
## 800
         64 173.0113 175.5424
                               -2.53105088
## 801
         63 178.8972 175.2062
                                 3.69103715
## 802
         51 150.4975 163.9646 -13.46707190
## 803
         44 164.0458 152.3264
                               11.71935426
## 804
         70 156.2126 175.3000 -19.08736345
## 805
         56 160.1097 170.1536 -10.04390811
## 806
         45 160.9419 154.1750
                                 6.76687514
##
  807
         68 187.3504 175.8272
                               11.52321744
##
  808
         58 154.7276 172.0552 -17.32759918
## 809
         65 181.7881 175.7750
                                 6.01308857
## 810
         66 179.3181 175.9016
                                 3.41650314
## 811
         40 144.0472 144.4000
                                -0.35281009
## 812
         40 138.1383 144.4000
                                -6.26165020
## 813
         48 150.8035 159.3632
                                -8.55971210
## 814
         45 155.6928 154.1750
                                 1.51777842
## 815
                               12.06372694
         75 183.9387 171.8750
## 816
         64 160.6566 175.5424 -14.88577883
## 817
         43 139.9185 150.4222 -10.50374123
## 818
         48 175.7419 159.3632 16.37874117
```

```
## 819
         48 147.2899 159.3632 -12.07334181
         78 142.0176 168.2792 -26.26155085
## 820
## 821
         75 172.0496 171.8750
                                 0.17463266
## 822
         58 175.4971 172.0552
                                 3.44194027
## 823
         43 150.5494 150.4222
                                 0.12719837
## 824
         74 164.0759 172.8104
                               -8.73450133
## 825
         68 179.2552 175.8272
                                 3.42800284
## 826
         55 167.3011 169.0750
                               -1.77387754
## 827
         42 157.6791 148.4648
                                 9.21433255
## 828
         80 168.2094 165.2000
                                 3.00944402
## 829
         75 178.8089 171.8750
                                 6.93386753
## 830
         59 176.1281 172.8734
                                 3.25471906
## 831
         53 170.7547 166.6742
                                 4.08051023
## 832
         53 174.2726 166.6742
                                 7.59842256
## 833
         72 151.4293 174.3008 -22.87153520
## 834
         40 149.5784 144.4000
                                 5.17838262
## 835
         74 159.3380 172.8104 -13.47236258
## 836
         58 175.6868 172.0552
                                 3.63159859
## 837
         63 188.3537 175.2062
                                13.14752565
## 838
         50 158.0169 162.5000
                                -4.48305911
## 839
         62 166.6872 174.7688
                                -8.08164468
## 840
         41 145.5939 146.4566
                                -0.86268499
## 841
         78 184.1334 168.2792
                               15.85424273
## 842
         59 173.8840 172.8734
                                 1.01057448
## 843
         53 168.1618 166.6742
                                 1.48762212
## 844
         49 155.5011 160.9654
                               -5.46434668
## 845
         60 168.3232 173.6000
                               -5.27675587
## 846
         72 172.9021 174.3008
                               -1.39871559
## 847
         71 158.9665 174.8606 -15.89411414
## 848
         67 188.2608 175.9198
                               12.34104166
## 849
         67 172.5653 175.9198
                               -3.35454848
## 850
         46 135.9556 155.9656 -20.01002507
## 851
         43 150.2476 150.4222
                               -0.17458403
## 852
         64 173.1327 175.5424 -2.40966896
## 853
         69 157.7015 175.6214 -17.91989399
## 854
         69 196.1183 175.6214 20.49691511
## 855
         76 159.6374 170.8096 -11.17217612
## 856
         71 161.3251 174.8606 -13.53549545
## 857
         79 171.3224 166.8094
                                 4.51302311
## 858
         49 171.2977 160.9654
                               10.33227425
## 859
         62 172.8099 174.7688
                                -1.95885695
## 860
         48 161.6363 159.3632
                                 2.27310387
##
  861
         72 163.2788 174.3008 -11.02199750
##
  862
         50 172.0528 162.5000
                                 9.55277369
## 863
         59 176.0017 172.8734
                                 3.12830446
## 864
         59 173.8835 172.8734
                                 1.01005713
## 865
         73 181.0994 173.6182
                                 7.48115414
## 866
         48 169.3528 159.3632
                                 9.98960895
## 867
         52 158.0316 165.3568
                                -7.32515309
## 868
         44 157.1354 152.3264
                                 4.80895240
         42 138.5703 148.4648
## 869
                               -9.89450308
## 870
         65 198.0706 175.7750
                               22.29559048
## 871
         56 174.7210 170.1536
                                 4.56736600
## 872
         40 144.0418 144.4000 -0.35815444
```

```
## 873
         47 152.7914 157.6958 -4.90442706
## 874
         58 169.8495 172.0552 -2.20566611
## 875
         57 160.1357 171.1478 -11.01209361
## 876
         41 147.0913 146.4566
                                 0.63470471
## 877
         61 180.0679 174.2326
                                 5.83525933
## 878
         75 166.7414 171.8750
                               -5.13358846
## 879
         43 142.2967 150.4222 -8.12552122
## 880
         53 150.3810 166.6742 -16.29319906
## 881
         50 168.1891 162.5000
                                 5.68905707
## 882
         55 172.4128 169.0750
                                 3.33783307
## 883
         64 173.4292 175.5424
                               -2.11322559
## 884
         75 166.3640 171.8750
                               -5.51097930
##
  885
         44 154.9100 152.3264
                                 2.58361129
         66 162.1495 175.9016 -13.75210446
## 886
  887
         70 160.5725 175.3000 -14.72754149
##
##
  888
         66 161.4495 175.9016 -14.45209673
  889
##
         41 142.8437 146.4566
                               -3.61288425
## 890
         80 173.1071 165.2000
                                 7.90709965
         71 182.0072 174.8606
## 891
                                 7.14657846
## 892
         62 169.1173 174.7688
                               -5.65153517
## 893
         59 187.0535 172.8734
                               14.18014303
## 894
         45 142.7174 154.1750 -11.45756781
## 895
         44 147.3982 152.3264
                               -4.92817823
## 896
         58 164.7247 172.0552
                                -7.33052274
## 897
         57 172.7401 171.1478
                                 1.59232080
## 898
         73 156.1063 173.6182 -17.51192959
## 899
         60 185.4483 173.6000
                               11.84830867
## 900
         78 180.3495 168.2792
                               12.07031601
## 901
         59 182.6499 172.8734
                                 9.77649327
                               -0.81991744
## 902
         65 174.9551 175.7750
## 903
         71 186.4232 174.8606
                                11.56263423
## 904
         78 173.1472 168.2792
                                 4.86796337
## 905
         79 176.3979 166.8094
                                 9.58848663
## 906
         49 162.7433 160.9654
                                 1.77792371
## 907
         50 165.4225 162.5000
                                 2.92250881
## 908
         60 152.4666 173.6000 -21.13340685
## 909
         52 159.9763 165.3568
                               -5.38047911
## 910
         59 183.0137 172.8734
                               10.14025585
## 911
         77 170.6245 169.6118
                                 1.01269641
## 912
         46 143.7785 155.9656 -12.18707372
## 913
         76 161.2258 170.8096
                               -9.58376793
## 914
         68 170.2692 175.8272
                               -5.55797485
## 915
         56 191.3724 170.1536
                               21.21875912
## 916
         47 155.8999 157.6958
                               -1.79589988
## 917
         77 163.1644 169.6118 -6.44735408
## 918
         71 153.9348 174.8606 -20.92583772
## 919
         63 156.1875 175.2062 -19.01870838
## 920
         45 166.3975 154.1750 12.22250276
## 921
         62 166.2566 174.7688
                               -8.51220441
## 922
         49 177.1774 160.9654
                               16.21202883
## 923
         50 159.2090 162.5000
                               -3.29102802
## 924
         49 137.7013 160.9654 -23.26409527
         53 188.6042 166.6742 21.92998000
## 925
## 926
         55 158.2502 169.0750 -10.82479976
```

```
## 927
         66 170.8380 175.9016 -5.06360967
## 928
         80 166.1592 165.2000
                                 0.95918957
         40 143.1983 144.4000 -1.20165258
## 929
## 930
         43 135.2355 150.4222 -15.18667630
## 931
         78 153.1733 168.2792 -15.10585063
## 932
         62 188.7248 174.7688
                               13.95604928
## 933
         71 167.7764 174.8606
                               -7.08421944
## 934
         55 177.3544 169.0750
                                 8.27938353
## 935
         79 183.8870 166.8094
                                17.07755837
## 936
         59 165.0817 172.8734
                               -7.79166557
## 937
         43 143.3532 150.4222
                               -7.06899069
## 938
         48 139.4342 159.3632 -19.92902511
## 939
         55 188.0738 169.0750
                               18.99876533
                               -7.56206936
## 940
         70 167.7379 175.3000
         49 157.1858 160.9654
## 941
                               -3.77955780
## 942
         72 160.6613 174.3008 -13.63954801
## 943
         79 174.2290 166.8094
                                 7.41958159
## 944
         56 186.2634 170.1536
                                16.10975996
         52 167.6307 165.3568
## 945
                                 2.27387447
## 946
         64 171.5372 175.5424
                               -4.00515674
## 947
         57 154.4520 171.1478 -16.69580400
## 948
         64 190.4494 175.5424
                                14.90701097
## 949
         49 167.8803 160.9654
                                 6.91487433
         68 175.4174 175.8272
## 950
                                -0.40977477
## 951
         48 171.4066 159.3632
                               12.04344853
## 952
         43 163.4073 150.4222
                               12.98514964
## 953
         65 171.1528 175.7750
                               -4.62217021
## 954
         77 157.3541 169.6118 -12.25773748
## 955
         57 180.7805 171.1478
                                 9.63266754
## 956
         74 179.7425 172.8104
                                 6.93211228
## 957
         40 139.3861 144.4000
                                -5.01393166
## 958
         60 205.1397 173.6000
                                31.53971417
## 959
         53 141.1831 166.6742 -25.49106895
## 960
         69 175.5400 175.6214
                                -0.08138085
## 961
         40 155.7878 144.4000
                                11.38780641
## 962
         62 189.5923 174.7688
                               14.82346468
## 963
         80 174.9585 165.2000
                                 9.75845009
## 964
         58 173.5387 172.0552
                                 1.48350293
## 965
         66 183.7907 175.9016
                                 7.88908757
## 966
         46 161.5414 155.9656
                                 5.57581220
## 967
         53 166.4003 166.6742
                               -0.27394393
## 968
         71 166.0899 174.8606
                               -8.77074959
## 969
         53 178.8366 166.6742
                               12.16243471
## 970
         67 172.4073 175.9198
                               -3.51249812
## 971
         59 169.9295 172.8734
                               -2.94393117
## 972
         40 146.9314 144.4000
                                 2.53137809
## 973
         46 156.0806 155.9656
                                 0.11497986
## 974
         67 156.1620 175.9198 -19.75783962
## 975
         63 170.9429 175.2062
                               -4.26326724
## 976
         65 181.8980 175.7750
                                 6.12302560
## 977
         69 187.9295 175.6214 12.30814109
## 978
         76 158.1015 170.8096 -12.70807808
## 979
         63 168.7006 175.2062 -6.50555664
## 980
         67 151.4881 175.9198 -24.43166237
```

```
## 981
         79 176.7749 166.8094
                                 9.96548595
## 982
         77 169.6629 169.6118
                                 0.05113528
## 983
         62 182.6790 174.7688
                                 7.91024764
## 984
         68 170.1953 175.8272
                                -5.63191537
## 985
         79 168.9477 166.8094
                                 2.13826203
## 986
         54 177.6852 167.9144
                                 9.77082664
         49 158.6238 160.9654
## 987
                                -2.34161103
## 988
         74 181.2970 172.8104
                                 8.48660554
## 989
         64 182.0821 175.5424
                                 6.53970891
## 990
         48 143.4575 159.3632 -15.90570026
## 991
         53 169.7684 166.6742
                                 3.09423133
## 992
         58 147.1954 172.0552 -24.85982190
## 993
         65 172.9906 175.7750
                                -2.78439107
## 994
         79 161.2851 166.8094
                               -5.52433759
## 995
         55 177.8354 169.0750
                                 8.76042073
## 996
         70 178.2348 175.3000
                                 2.93483694
## 997
         52 159.3846 165.3568
                                -5.97222047
## 998
         80 163.6280 165.2000
                                -1.57203603
         42 150.3067 148.4648
## 999
                                 1.84185060
## 1000
        50 152.3697 162.5000 -10.13032014
```

7. Generate a plot of weight against age.

```
plot(weights, age, pch=19, cex=0.4)
```

Applying smoothing kernels:

1. Fit a nearest neighbors curve with ksmooth using a bandwidth of 10 and the box kernel.

```
box_curve <- ksmooth(df$age, df$weight, kernel = "box", bandwidth = 10)
plot(box_curve)</pre>
```

2. Fit another curve this time using the normal kernel.

```
normal_curve <- ksmooth(df$age, df$weight, kernel = "normal", bandwidth = 10)
```

3. Plot the data with the two fitted curves and compare them. Hint: look at the object you have generated with ksmooth (i.e. type name_of_the_object or print(name_of_the_object). Also, try names(name_of_the_object). Use the command lines to plot the curves.

```
plot(age, weights, pch=19, cex=0.4)
lines(box_curve$x, box_curve$y, lwd=3, col="red")
lines(normal_curve$x, normal_curve$y, lwd=3, col="blue")
legend("bottomright", cex=0.8, lwd=3, lty=c(1,1,2,3),c("box bw=10", "normal bw=10"), col=c("red", "blue")
```

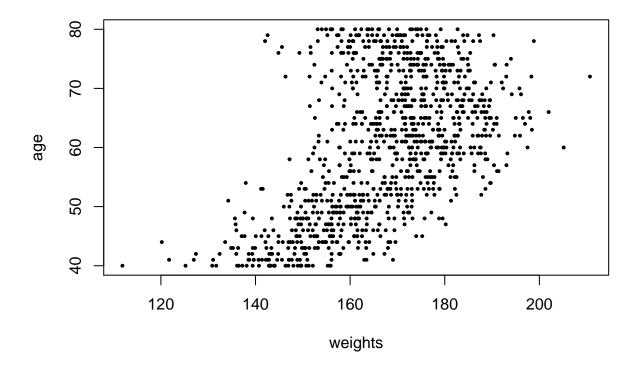


Figure 1: Figure: Age vs Weights

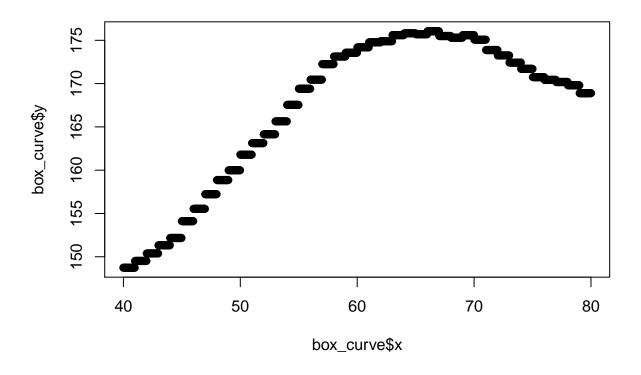
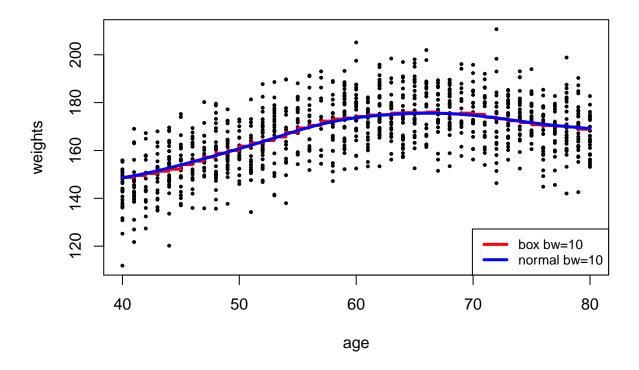


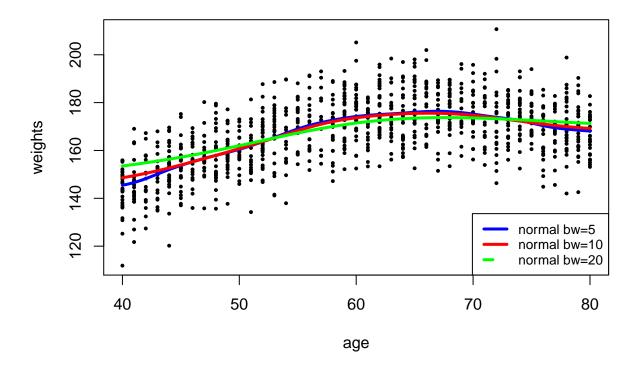
Figure 2: ksmooth curve of age against weight



4. Fit another 2 curves with the "normal" kernel using bandwidths of size 5, 10, and 20. How do they compare?

```
normal_curve5 <- ksmooth(df$age, df$weight, kernel = "normal", bandwidth = 5)
normal_curve10 <- ksmooth(df$age, df$weight, kernel = "normal", bandwidth = 10)
normal_curve20 <- ksmooth(df$age, df$weight, kernel = "normal", bandwidth = 20)

plot(age, weights, pch=19, cex=0.4)
lines(normal_curve5$x, normal_curve5$y, lwd=3, col="blue")
lines(normal_curve10$x, normal_curve10$y, lwd=3, col="red")
lines(normal_curve20$x, normal_curve20$y, lwd=3, col="green")
legend("bottomright", cex=0.8, lwd=3, lty=c(1,1,2,3), c("normal_bw=5", "normal_bw=10", "normal_bw=20"), c</pre>
```



Fitting linear models:

1. Fit a linear regression to the data using the lm command.

head(df)

```
## age weight trueFx noise
## 1 43 161.6581 150.4222 11.235854
## 2 78 160.6807 168.2792 -7.598457
## 3 40 155.8896 144.4000 11.489591
## 4 73 165.1934 173.6182 -8.424763
## 5 62 178.6829 174.7688 3.914133
## 6 53 175.5880 166.6742 8.913772
linear_model <- lm(weight ~ age, data=df)
```

2. Run summary(your_linear_model_name) to get an idea of the fit

summary(linear_model)

```
##
## Call:
## lm(formula = weight ~ age, data = df)
```

```
##
## Residuals:
##
       Min
                1Q Median
                                3Q
                                       Max
## -42.605 -8.218 -0.198
                             8.245 38.326
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 129.79005
                            1.96776
                                      65.96
                                              <2e-16 ***
## age
                 0.61706
                            0.03204
                                      19.26
                                              <2e-16 ***
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
##
## Residual standard error: 12.06 on 998 degrees of freedom
## Multiple R-squared: 0.271, Adjusted R-squared: 0.2702
## F-statistic: 370.9 on 1 and 998 DF, p-value: < 2.2e-16
  3. Now fit quadratic and cubic models: you will need the I function to set quadratic and cubic terms in
    the regression, e.g. I(x^2)
#quadratic model
quatratic_model <- lm(weight ~ age + I(age^2), data=df)</pre>
summary(quatratic_model)
##
## lm(formula = weight ~ age + I(age^2), data = df)
##
## Residuals:
##
       Min
                10 Median
                                3Q
                                       Max
## -31.209 -6.762
                    0.176
                             7.299
                                    36.560
## Coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept) -36.543988
                            9.044877
                                       -4.04 5.75e-05 ***
## age
                 6.385785
                            0.309374
                                       20.64 < 2e-16 ***
                            0.002567 -18.72 < 2e-16 ***
## I(age^2)
                -0.048051
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 10.38 on 997 degrees of freedom
## Multiple R-squared: 0.4606, Adjusted R-squared: 0.4595
## F-statistic: 425.7 on 2 and 997 DF, p-value: < 2.2e-16
#cubic model
cubic_model <- lm(weight ~ age + I(age^2) + I(age^3), data=df)</pre>
summary(cubic_model)
##
## Call:
## lm(formula = weight ~ age + I(age^2) + I(age^3), data = df)
##
```

Residuals:

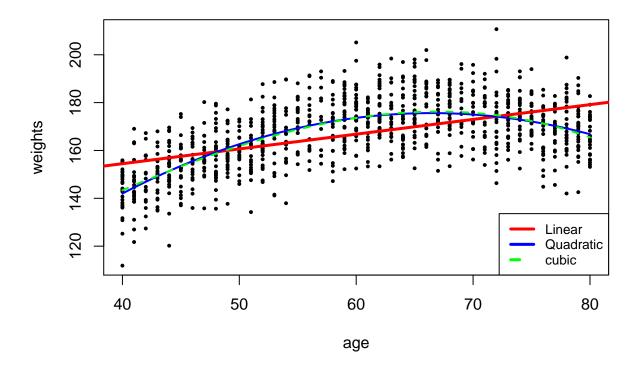
```
##
      Min
               1Q Median
                               3Q
                                      Max
## -31.472 -6.905 -0.044
                            7.288 35.946
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
                                      1.099
                                              0.2722
## (Intercept) 56.1267168 51.0925279
                                              0.5673
## age
               1.5209614 2.6579063
                                      0.572
## I(age^2)
               0.0348917 0.0450815
                                      0.774
                                              0.4391
## I(age^3)
              -0.0004602 0.0002497 -1.843
                                              0.0657 .
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
## Residual standard error: 10.36 on 996 degrees of freedom
## Multiple R-squared: 0.4624, Adjusted R-squared: 0.4608
## F-statistic: 285.6 on 3 and 996 DF, p-value: < 2.2e-16
```

4. Add the fitted models to the plots. For the linear model, use the abline command. For the polynomial fits, you can use the curve command (you will need the option add = TRUE).

```
?predict
?predict.lm
?curve

quad_predict <- function(age) {predict(quatratic_model, newdata = data.frame(age=age))}
cubic_predict <- function(age) {predict(cubic_model, newdata = data.frame(age=age))}

plot(age, weights, pch=19, cex=0.4)
abline(linear_model, col="red", lwd=3)
curve(quad_predict, col="blue", lwd=2, add=T)
curve(cubic_predict, col="green", lwd=2, lty=2, add=T)
legend("bottomright", cex=0.8, lwd=3, lty=c(1,1,2), c("Linear", "Quadratic", "cubic"), col = c("red", "</pre>
```



5. Which curve do you prefer? How would you perform a hypothesis test for this?

```
anova(linear_model, quatratic_model, cubic_model)
```

```
## Analysis of Variance Table
##
## Model 1: weight ~ age
## Model 2: weight ~ age + I(age^2)
  Model 3: weight ~ age + I(age^2) + I(age^3)
##
     Res.Df
               RSS Df Sum of Sq
                                       F Pr(>F)
## 1
        998 145050
## 2
        997 107323
                           37727 351.315 < 2e-16 ***
        996 106958
## 3
                             365
                                   3.396 0.06565 .
                     '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
```

Practice installing an R package

6. R packages are easy to install using the install.packages command. Try installing the e1071 package. We'll use this package later in the course for fitting support vector machines.

```
# install.packages("e1071")
```