Lab #1: Practice in R

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The purpose of this lab is to get you comfortable working in R and trying a few things from the first lecture.

Simulating data:

We will simulate something like the weight vs. age data from a cubic function and add some noise and then apply a few methods.

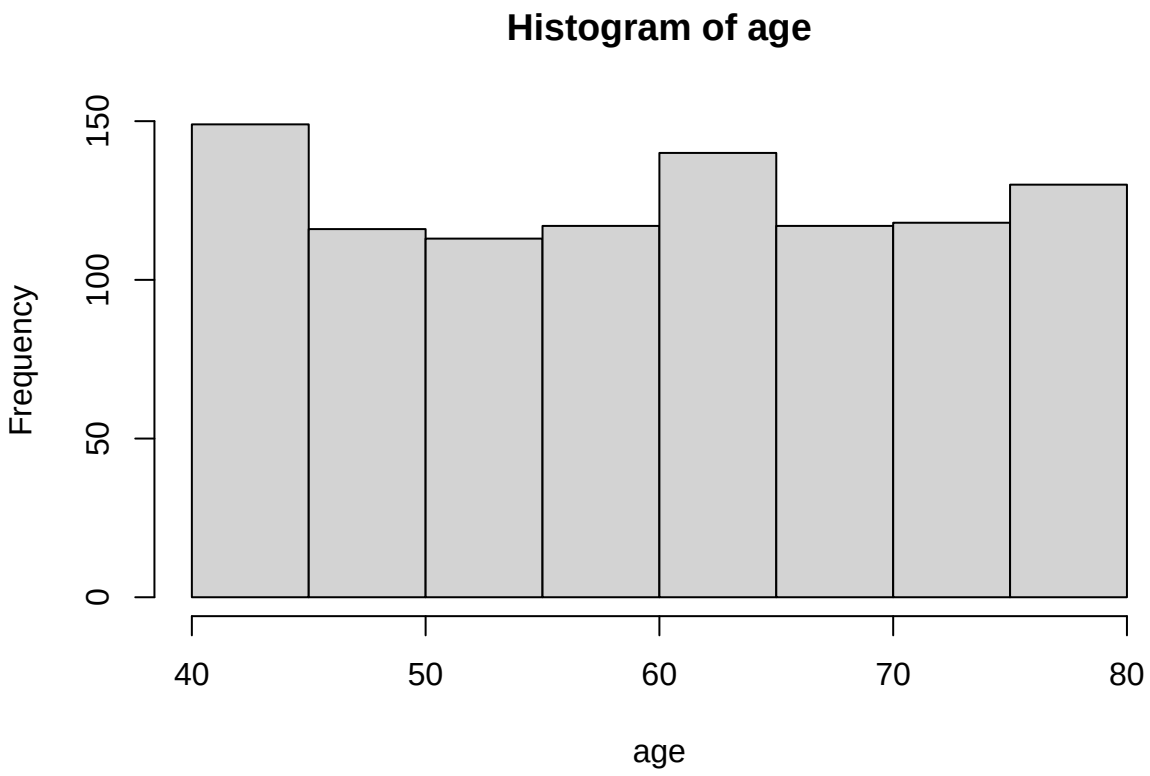
1. Write a function in R to input the value (age) and output, "noise-free" weight: $f(x) = -0.0004x^3 + 0.025x^2 + 2x + 50$

```
make_weight <- function(age) {
  -0.0004 * age^3 + 0.025 * age^2 + 2 * age + 50
}
```

2. Next we will simulate some values for x. Use the sample function to generate a random sample of integers between 40 and 80 of size 1000 (you will need the replace=TRUE option to make sure the numbers can be resampled otherwise you will run out of values quickly). Don't forget that you can type `?sample` to get help as to how to use the function.

```
?sample

age <- sample(seq(40, 80), size=1000, replace=TRUE)
hist(age)
```



3. Now generate a vector of the output of your function in part 1 to generate the noise-free weights for each of the 1000 individuals.

```
no_noise_weights <- make_weight(age)
no_noise_weights
```

```
##    [1] 150.4222 168.2792 144.4000 173.6182 174.7688 166.6742 171.1478 174.3008
##    [9] 173.6000 173.6000 160.9654 155.9656 159.3632 167.9144 173.6000 170.8096
##   [17] 165.2000 175.5424 170.8096 170.8096 173.6182 175.5424 167.9144 174.3008
##   [25] 172.8734 172.8104 154.1750 160.9654 169.6118 172.8734 175.9198 172.8734
##   [33] 174.7688 154.1750 166.8094 175.5424 154.1750 168.2792 154.1750 175.2062
##   [41] 174.8606 166.6742 146.4566 171.1478 174.2326 166.6742 169.6118 144.4000
##   [49] 166.8094 154.1750 174.7688 154.1750 166.8094 168.2792 162.5000 170.1536
##   [57] 171.8750 165.3568 166.8094 175.5424 175.5424 174.7688 172.8734 168.2792
##   [65] 175.8272 165.3568 174.2326 175.8272 175.9198 174.3008 173.6000 175.3000
##   [73] 169.6118 170.1536 159.3632 168.2792 174.7688 172.0552 175.7750 175.6214
##   [81] 174.8606 168.2792 175.8272 173.6182 160.9654 144.4000 175.7750 167.9144
##   [89] 175.8272 175.2062 163.9646 168.2792 175.2062 175.9198 174.2326 166.8094
##   [97] 174.3008 172.0552 175.7750 172.8104 144.4000 175.8272 166.6742 174.2326
##  [105] 154.1750 175.9198 165.2000 170.8096 148.4648 174.2326 172.8104 155.9656
##  [113] 172.8104 165.2000 175.9016 155.9656 172.8734 175.2062 169.0750 172.0552
##  [121] 171.8750 146.4566 144.4000 168.2792 165.2000 165.3568 175.7750 173.6182
##  [129] 146.4566 172.0552 170.1536 173.6182 162.5000 175.3000 172.8104 172.8104
##  [137] 172.0552 162.5000 169.0750 166.8094 159.3632 175.9198 175.2062 160.9654
##  [145] 167.9144 170.8096 174.8606 170.8096 168.2792 175.3000 170.8096 175.8272
```

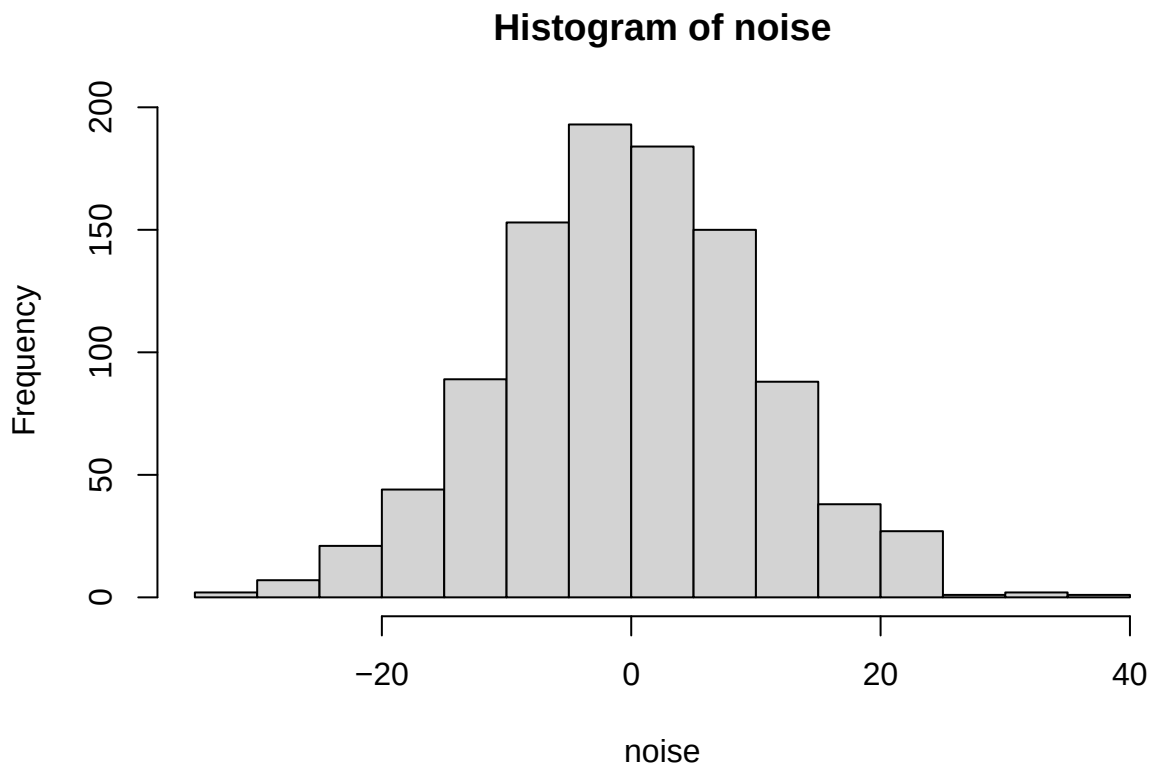
```
##  [153] 163.9646 166.6742 169.6118 169.0750 167.9144 146.4566 144.4000 148.4648
##  [161] 159.3632 169.6118 170.1536 172.8734 152.3264 165.2000 175.9016 175.3000
##  [169] 157.6958 171.8750 165.2000 175.8272 173.6000 144.4000 172.8734 175.2062
##  [177] 174.2326 169.6118 169.0750 175.6214 175.8272 146.4566 157.6958 166.6742
##  [185] 171.8750 175.7750 166.8094 170.8096 157.6958 155.9656 171.8750 171.8750
##  [193] 159.3632 175.9016 165.2000 167.9144 169.6118 154.1750 150.4222 172.8104
##  [201] 172.8104 165.3568 165.2000 155.9656 175.5424 175.3000 170.8096 175.9198
##  [209] 168.2792 175.9198 172.8734 175.8272 166.8094 144.4000 175.9198 166.6742
##  [217] 175.3000 163.9646 175.8272 169.0750 160.9654 173.6182 175.7750 174.3008
##  [225] 150.4222 174.2326 175.5424 175.2062 170.8096 167.9144 166.6742 170.1536
##  [233] 171.1478 174.3008 174.8606 157.6958 168.2792 155.9656 175.9016 159.3632
##  [241] 172.8734 173.6182 170.8096 175.8272 162.5000 146.4566 148.4648 169.6118
##  [249] 165.2000 167.9144 175.9198 152.3264 175.9198 166.6742 170.8096 170.1536
##  [257] 144.4000 175.7750 174.3008 172.0552 175.7750 174.7688 150.4222 169.0750
##  [265] 175.9016 168.2792 169.6118 169.0750 173.6182 175.8272 172.8104 160.9654
##  [273] 154.1750 168.2792 174.3008 174.7688 174.3008 172.8104 162.5000 165.3568
##  [281] 170.8096 175.7750 175.7750 166.8094 173.6182 150.4222 175.5424 169.6118
##  [289] 170.1536 175.2062 159.3632 175.6214 159.3632 162.5000 174.2326 174.8606
##  [297] 166.8094 148.4648 172.0552 174.3008 150.4222 165.3568 144.4000 166.8094
##  [305] 152.3264 172.8104 174.7688 175.8272 172.8104 171.8750 159.3632 172.8104
##  [313] 172.8104 172.8734 174.2326 165.2000 174.2326 172.0552 171.1478 168.2792
##  [321] 175.8272 175.9016 171.8750 174.8606 175.3000 170.1536 170.8096 175.5424
##  [329] 159.3632 173.6000 173.6000 167.9144 170.8096 175.2062 166.6742 155.9656
##  [337] 175.9016 175.7750 175.6214 167.9144 154.1750 173.6182 173.6000 144.4000
##  [345] 170.1536 165.3568 175.7750 166.8094 146.4566 165.2000 157.6958 175.5424
##  [353] 175.3000 172.8734 172.8734 152.3264 160.9654 172.8734 167.9144 175.8272
##  [361] 146.4566 144.4000 152.3264 175.5424 170.8096 174.8606 166.8094 165.3568
##  [369] 175.5424 150.4222 169.6118 152.3264 144.4000 169.6118 165.3568 157.6958
##  [377] 172.0552 175.9198 169.6118 162.5000 157.6958 174.7688 172.8734 152.3264
##  [385] 146.4566 152.3264 163.9646 148.4648 175.7750 166.6742 165.3568 175.3000
##  [393] 173.6182 175.6214 175.7750 155.9656 175.9016 159.3632 157.6958 174.2326
##  [401] 154.1750 175.9016 175.6214 160.9654 175.6214 144.4000 154.1750 170.1536
##  [409] 165.3568 175.7750 146.4566 174.8606 171.8750 152.3264 174.7688 169.6118
##  [417] 172.8734 175.9198 175.9198 163.9646 172.8104 150.4222 175.7750 165.3568
##  [425] 148.4648 167.9144 173.6182 169.6118 150.4222 175.7750 173.6000 173.6000
##  [433] 169.0750 170.8096 175.2062 173.6000 148.4648 175.7750 174.7688 171.8750
##  [441] 174.2326 148.4648 150.4222 171.8750 170.8096 144.4000 170.8096 174.2326
##  [449] 173.6000 175.9016 162.5000 162.5000 154.1750 171.8750 154.1750 172.8734
##  [457] 166.8094 174.3008 166.8094 162.5000 162.5000 173.6000 173.6000 170.1536
##  [465] 159.3632 171.1478 175.3000 157.6958 174.7688 154.1750 162.5000 170.8096
##  [473] 175.5424 146.4566 168.2792 174.8606 166.8094 174.2326 172.8104 162.5000
##  [481] 165.3568 146.4566 166.8094 154.1750 167.9144 165.2000 175.9198 144.4000
##  [489] 146.4566 159.3632 152.3264 175.9016 175.8272 146.4566 170.8096 175.3000
##  [497] 166.6742 175.5424 173.6000 174.7688 154.1750 173.6000 152.3264 175.5424
##  [505] 170.1536 174.3008 166.8094 172.0552 170.1536 172.8734 174.7688 172.8734
##  [513] 175.3000 152.3264 175.9016 174.3008 144.4000 175.8272 173.6182 171.8750
##  [521] 172.8734 173.6000 169.6118 175.2062 173.6182 172.8734 148.4648 172.8734
##  [529] 173.6182 167.9144 175.6214 165.3568 169.0750 171.8750 175.6214 175.8272
##  [537] 175.9198 165.3568 155.9656 171.1478 144.4000 150.4222 152.3264 171.8750
##  [545] 173.6182 175.7750 174.3008 172.0552 170.1536 165.3568 168.2792 171.8750
##  [553] 155.9656 174.2326 159.3632 174.8606 166.8094 174.3008 175.9198 163.9646
##  [561] 175.6214 172.8104 175.7750 148.4648 167.9144 168.2792 174.7688 175.5424
##  [569] 144.4000 175.9198 159.3632 166.6742 175.3000 169.0750 173.6000 163.9646
##  [577] 175.6214 174.2326 144.4000 163.9646 175.9198 163.9646 175.5424 175.5424
```

```
##  [585] 157.6958 175.3000 152.3264 163.9646 175.6214 159.3632 146.4566 159.3632
##  [593] 165.2000 165.2000 174.2326 175.9016 169.6118 174.7688 173.6182 148.4648
##  [601] 144.4000 146.4566 172.0552 175.8272 152.3264 173.6182 165.2000 163.9646
##  [609] 165.2000 171.1478 173.6182 165.3568 168.2792 150.4222 154.1750 174.7688
##  [617] 175.5424 169.0750 174.2326 175.7750 166.8094 166.6742 154.1750 157.6958
##  [625] 173.6000 175.2062 148.4648 166.8094 170.8096 166.6742 175.9016 165.3568
##  [633] 172.0552 148.4648 175.8272 152.3264 175.5424 175.3000 166.6742 171.8750
##  [641] 172.8734 175.3000 165.3568 148.4648 166.8094 175.2062 175.8272 160.9654
##  [649] 165.3568 154.1750 160.9654 168.2792 172.0552 171.1478 173.6000 174.7688
##  [657] 175.7750 167.9144 175.8272 175.8272 152.3264 174.2326 144.4000 174.2326
##  [665] 165.3568 174.2326 154.1750 165.2000 174.3008 175.2062 174.2326 174.3008
##  [673] 160.9654 172.0552 157.6958 163.9646 162.5000 169.0750 173.6000 154.1750
##  [681] 175.2062 175.7750 157.6958 173.6000 155.9656 172.8104 174.3008 175.6214
##  [689] 152.3264 152.3264 150.4222 162.5000 165.2000 146.4566 160.9654 173.6182
##  [697] 146.4566 163.9646 154.1750 159.3632 168.2792 172.0552 152.3264 170.1536
##  [705] 159.3632 175.5424 165.3568 163.9646 172.8104 157.6958 146.4566 159.3632
##  [713] 175.3000 155.9656 152.3264 155.9656 168.2792 175.3000 146.4566 172.8734
##  [721] 165.2000 157.6958 175.5424 172.8104 173.6000 174.8606 169.0750 155.9656
##  [729] 175.5424 172.0552 172.8104 175.5424 165.3568 160.9654 173.6182 169.0750
##  [737] 162.5000 175.2062 148.4648 155.9656 175.6214 174.2326 174.8606 174.7688
##  [745] 175.2062 163.9646 175.9198 157.6958 174.7688 165.3568 171.1478 146.4566
##  [753] 165.2000 163.9646 160.9654 174.7688 154.1750 157.6958 174.7688 165.3568
##  [761] 175.6214 173.6182 157.6958 163.9646 174.2326 174.7688 148.4648 150.4222
##  [769] 173.6000 163.9646 175.2062 165.3568 155.9656 175.9016 174.2326 172.8104
##  [777] 174.8606 152.3264 169.6118 175.9016 160.9654 175.8272 165.3568 163.9646
##  [785] 175.9016 168.2792 144.4000 159.3632 172.8104 162.5000 166.8094 171.8750
##  [793] 170.8096 160.9654 173.6000 171.8750 174.7688 175.9198 174.8606 175.5424
##  [801] 175.2062 163.9646 152.3264 175.3000 170.1536 154.1750 175.8272 172.0552
##  [809] 175.7750 175.9016 144.4000 144.4000 159.3632 154.1750 171.8750 175.5424
##  [817] 150.4222 159.3632 159.3632 168.2792 171.8750 172.0552 150.4222 172.8104
##  [825] 175.8272 169.0750 148.4648 165.2000 171.8750 172.8734 166.6742 166.6742
##  [833] 174.3008 144.4000 172.8104 172.0552 175.2062 162.5000 174.7688 146.4566
##  [841] 168.2792 172.8734 166.6742 160.9654 173.6000 174.3008 174.8606 175.9198
##  [849] 175.9198 155.9656 150.4222 175.5424 175.6214 175.6214 170.8096 174.8606
##  [857] 166.8094 160.9654 174.7688 159.3632 174.3008 162.5000 172.8734 172.8734
##  [865] 173.6182 159.3632 165.3568 152.3264 148.4648 175.7750 170.1536 144.4000
##  [873] 157.6958 172.0552 171.1478 146.4566 174.2326 171.8750 150.4222 166.6742
##  [881] 162.5000 169.0750 175.5424 171.8750 152.3264 175.9016 175.3000 175.9016
##  [889] 146.4566 165.2000 174.8606 174.7688 172.8734 154.1750 152.3264 172.0552
##  [897] 171.1478 173.6182 173.6000 168.2792 172.8734 175.7750 174.8606 168.2792
##  [905] 166.8094 160.9654 162.5000 173.6000 165.3568 172.8734 169.6118 155.9656
##  [913] 170.8096 175.8272 170.1536 157.6958 169.6118 174.8606 175.2062 154.1750
##  [921] 174.7688 160.9654 162.5000 160.9654 166.6742 169.0750 175.9016 165.2000
##  [929] 144.4000 150.4222 168.2792 174.7688 174.8606 169.0750 166.8094 172.8734
##  [937] 150.4222 159.3632 169.0750 175.3000 160.9654 174.3008 166.8094 170.1536
##  [945] 165.3568 175.5424 171.1478 175.5424 160.9654 175.8272 159.3632 150.4222
##  [953] 175.7750 169.6118 171.1478 172.8104 144.4000 173.6000 166.6742 175.6214
##  [961] 144.4000 174.7688 165.2000 172.0552 175.9016 155.9656 166.6742 174.8606
##  [969] 166.6742 175.9198 172.8734 144.4000 155.9656 175.9198 175.2062 175.7750
##  [977] 175.6214 170.8096 175.2062 175.9198 166.8094 169.6118 174.7688 175.8272
##  [985] 166.8094 167.9144 160.9654 172.8104 175.5424 159.3632 166.6742 172.0552
##  [993] 175.7750 166.8094 169.0750 175.3000 165.3568 165.2000 148.4648 162.5000
```

4. Use `rnorm` to generate some normally distributed noise with mean 0.0 and sd of 10.0 for each of the

values.

```
noise <- rnorm(1000, mean=0, sd=10)
hist(noise)
```



5. Add the noise-free weights to the noise to get your simulated outcome data.

```
weights <- no_noise_weights + noise
```

6. Put these together in a dataframe with columns for age, weight (the observed outcome with noise), trueFx (the noise-free weight value), and noise.

```
df <- data.frame(age=age, weight=weights, trueFx=no_noise_weights, noise=noise)
df
```

```
##      age   weight   trueFx        noise
## 1     43 161.6581 150.4222  11.23585423
## 2     78 160.6807 168.2792  -7.59845657
## 3     40 155.8896 144.4000  11.48959103
## 4     73 165.1934 173.6182  -8.42476259
## 5     62 178.6829 174.7688   3.91413340
## 6     53 175.5880 166.6742   8.91377242
## 7     57 157.7952 171.1478 -13.35258713
## 8     72 178.2820 174.3008   3.98123478
```

```
## 9     60 172.4841 173.6000  -1.11586803
## 10    60 180.3574 173.6000   6.75743917
## 11    49 153.0794 160.9654  -7.88597935
## 12    46 155.0957 155.9656  -0.86986334
## 13    48 173.1860 159.3632  13.82284008
## 14    54 169.5993 167.9144   1.68490164
## 15    60 181.8319 173.6000   8.23190948
## 16    76 168.6007 170.8096  -2.20894598
## 17    80 154.9061 165.2000 -10.29391655
## 18    64 175.4331 175.5424  -0.10925691
## 19    76 158.5597 170.8096 -12.24991155
## 20    76 144.8485 170.8096 -25.96111388
## 21    73 185.3094 173.6182  11.69122592
## 22    64 164.6733 175.5424 -10.86908817
## 23    54 149.6536 167.9144 -18.26083013
## 24    72 184.2536 174.3008   9.95281807
## 25    59 172.7548 172.8734  -0.11861781
## 26    74 166.8141 172.8104  -5.99628395
## 27    45 152.3955 154.1750  -1.77947987
## 28    49 156.7056 160.9654  -4.25981342
## 29    77 179.5784 169.6118   9.96658776
## 30    59 180.1500 172.8734   7.27660709
## 31    67 158.6535 175.9198 -17.26630596
## 32    59 176.4074 172.8734   3.53398496
## 33    62 182.0369 174.7688   7.26813666
## 34    45 160.8576 154.1750   6.68260976
## 35    79 142.5662 166.8094 -24.24317309
## 36    64 173.1888 175.5424  -2.35357425
## 37    45 173.9713 154.1750  19.79633321
## 38    78 176.2471 168.2792   7.96794539
## 39    45 137.0822 154.1750 -17.09276181
## 40    63 158.5695 175.2062 -16.63668712
## 41    71 179.7717 174.8606   4.91109552
## 42    53 164.9336 166.6742  -1.74055486
## 43    41 156.0695 146.4566   9.61290564
## 44    57 174.0861 171.1478   2.93826662
## 45    61 175.0426 174.2326   0.80999364
## 46    53 168.5108 166.6742   1.83661843
## 47    77 171.2744 169.6118   1.66255035
## 48    40 131.7040 144.4000 -12.69599066
## 49    79 190.3043 166.8094  23.49493321
## 50    45 140.0549 154.1750 -14.12005407
## 51    62 174.5992 174.7688  -0.16961493
## 52    45 148.7318 154.1750  -5.44319353
## 53    79 184.8105 166.8094  18.00112333
## 54    78 178.3936 168.2792  10.11440176
## 55    50 156.8628 162.5000  -5.63716556
## 56    56 172.2078 170.1536   2.05420795
## 57    75 183.5296 171.8750  11.65461950
## 58    52 187.7200 165.3568  22.36322840
## 59    79 169.8321 166.8094   3.02265076
## 60    64 165.1173 175.5424 -10.42506602
## 61    64 165.7070 175.5424  -9.83542313
## 62    62 194.8260 174.7688  20.05718580
```

```
## 63    59 152.1677 172.8734 -20.70571484
## 64    78 198.8366 168.2792  30.55742369
## 65    68 173.2137 175.8272  -2.61350594
## 66    52 160.8129 165.3568  -4.54393259
## 67    61 175.8082 174.2326   1.57560555
## 68    68 185.1611 175.8272   9.33388728
## 69    67 178.9481 175.9198   3.02828276
## 70    72 154.7393 174.3008 -19.56150222
## 71    60 177.1354 173.6000   3.53536709
## 72    70 179.8042 175.3000   4.50424514
## 73    77 176.2073 169.6118   6.59550871
## 74    56 159.8394 170.1536 -10.31420728
## 75    48 135.6530 159.3632 -23.71022881
## 76    78 165.0334 168.2792  -3.24576308
## 77    62 165.3258 174.7688  -9.44298751
## 78    58 164.3963 172.0552  -7.65889998
## 79    65 166.2372 175.7750  -9.53779267
## 80    69 171.6414 175.6214  -3.98004450
## 81    71 171.7484 174.8606  -3.11217063
## 82    78 176.2401 168.2792   7.96092713
## 83    68 185.6915 175.8272   9.86428344
## 84    73 165.6729 173.6182  -7.94531662
## 85    49 157.8772 160.9654  -3.08817972
## 86    40 148.0144 144.4000   3.61444766
## 87    65 189.7629 175.7750  13.98791105
## 88    54 167.3537 167.9144  -0.56070420
## 89    68 158.8385 175.8272 -16.98873492
## 90    63 177.5247 175.2062   2.31852545
## 91    51 162.7737 163.9646  -1.19090671
## 92    78 186.0041 168.2792  17.72492853
## 93    63 178.6404 175.2062   3.43422165
## 94    67 169.6893 175.9198  -6.23049782
## 95    61 169.8374 174.2326  -4.39522294
## 96    79 161.7564 166.8094  -5.05296790
## 97    72 176.1612 174.3008   1.86035137
## 98    58 173.8194 172.0552   1.76417798
## 99    65 184.9335 175.7750   9.15848207
## 100   74 176.0122 172.8104   3.20176726
## 101   40 140.7331 144.4000  -3.66687297
## 102   68 166.4211 175.8272  -9.40611295
## 103   53 173.0212 166.6742   6.34702931
## 104   61 173.6077 174.2326  -0.62488482
## 105   45 156.0034 154.1750   1.82837868
## 106   67 186.9562 175.9198  11.03641018
## 107   80 182.7204 165.2000  17.52035619
## 108   76 161.2714 170.8096  -9.53816460
## 109   42 164.9056 148.4648  16.44080457
## 110   61 165.5653 174.2326  -8.66733529
## 111   74 175.4739 172.8104   2.66352192
## 112   46 158.1893 155.9656   2.22370495
## 113   74 170.0413 172.8104  -2.76908510
## 114   80 179.1425 165.2000  13.94253041
## 115   66 169.3125 175.9016  -6.58912000
## 116   46 162.5709 155.9656   6.60531281
```

```
## 117   59 172.7408 172.8734  -0.13255700
## 118   63 165.8914 175.2062  -9.31480299
## 119   55 181.2219 169.0750  12.14689143
## 120   58 151.1718 172.0552 -20.88340366
## 121   75 166.6135 171.8750  -5.26152469
## 122   41 131.0426 146.4566 -15.41402567
## 123   40 146.3432 144.4000   1.94321070
## 124   78 170.9234 168.2792   2.64422549
## 125   80 154.0126 165.2000 -11.18735169
## 126   52 171.8663 165.3568   6.50952956
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## 141   48 178.7986 159.3632  19.43536312
## 142   67 183.2919 175.9198   7.37213734
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## 896   58 164.7247 172.0552  -7.33052274
## 897   57 172.7401 171.1478   1.59232080
## 898   73 156.1063 173.6182 -17.51192959
## 899   60 185.4483 173.6000  11.84830867
## 900   78 180.3495 168.2792  12.07031601
## 901   59 182.6499 172.8734   9.77649327
## 902   65 174.9551 175.7750  -0.81991744
## 903   71 186.4232 174.8606  11.56263423
## 904   78 173.1472 168.2792   4.86796337
## 905   79 176.3979 166.8094   9.58848663
## 906   49 162.7433 160.9654   1.77792371
## 907   50 165.4225 162.5000   2.92250881
## 908   60 152.4666 173.6000 -21.13340685
## 909   52 159.9763 165.3568  -5.38047911
## 910   59 183.0137 172.8734  10.14025585
## 911   77 170.6245 169.6118   1.01269641
## 912   46 143.7785 155.9656 -12.18707372
## 913   76 161.2258 170.8096  -9.58376793
## 914   68 170.2692 175.8272  -5.55797485
## 915   56 191.3724 170.1536  21.21875912
## 916   47 155.8999 157.6958  -1.79589988
## 917   77 163.1644 169.6118  -6.44735408
## 918   71 153.9348 174.8606 -20.92583772
## 919   63 156.1875 175.2062 -19.01870838
## 920   45 166.3975 154.1750  12.22250276
## 921   62 166.2566 174.7688  -8.51220441
## 922   49 177.1774 160.9654  16.21202883
## 923   50 159.2090 162.5000  -3.29102802
## 924   49 137.7013 160.9654 -23.26409527
## 925   53 188.6042 166.6742  21.92998000
## 926   55 158.2502 169.0750 -10.82479976
```
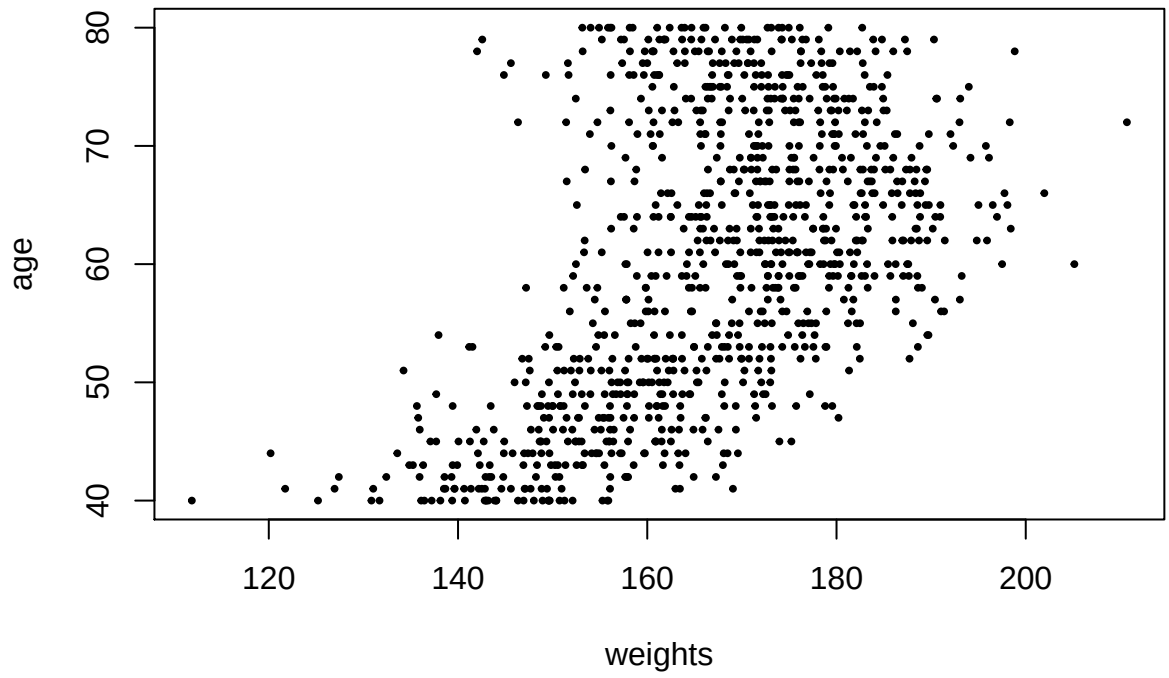

```
## 927   66 170.8380 175.9016  -5.06360967
## 928   80 166.1592 165.2000   0.95918957
## 929   40 143.1983 144.4000  -1.20165258
## 930   43 135.2355 150.4222 -15.18667630
## 931   78 153.1733 168.2792 -15.10585063
## 932   62 188.7248 174.7688  13.95604928
## 933   71 167.7764 174.8606  -7.08421944
## 934   55 177.3544 169.0750   8.27938353
## 935   79 183.8870 166.8094  17.07755837
## 936   59 165.0817 172.8734  -7.79166557
## 937   43 143.3532 150.4222  -7.06899069
## 938   48 139.4342 159.3632 -19.92902511
## 939   55 188.0738 169.0750  18.99876533
## 940   70 167.7379 175.3000  -7.56206936
## 941   49 157.1858 160.9654  -3.77955780
## 942   72 160.6613 174.3008 -13.63954801
## 943   79 174.2290 166.8094   7.41958159
## 944   56 186.2634 170.1536  16.10975996
## 945   52 167.6307 165.3568   2.27387447
## 946   64 171.5372 175.5424  -4.00515674
## 947   57 154.4520 171.1478 -16.69580400
## 948   64 190.4494 175.5424  14.90701097
## 949   49 167.8803 160.9654   6.91487433
## 950   68 175.4174 175.8272  -0.40977477
## 951   48 171.4066 159.3632  12.04344853
## 952   43 163.4073 150.4222  12.98514964
## 953   65 171.1528 175.7750  -4.62217021
## 954   77 157.3541 169.6118 -12.25773748
## 955   57 180.7805 171.1478   9.63266754
## 956   74 179.7425 172.8104   6.93211228
## 957   40 139.3861 144.4000  -5.01393166
## 958   60 205.1397 173.6000  31.53971417
## 959   53 141.1831 166.6742 -25.49106895
## 960   69 175.5400 175.6214  -0.08138085
## 961   40 155.7878 144.4000  11.38780641
## 962   62 189.5923 174.7688  14.82346468
## 963   80 174.9585 165.2000   9.75845009
## 964   58 173.5387 172.0552   1.48350293
## 965   66 183.7907 175.9016   7.88908757
## 966   46 161.5414 155.9656   5.57581220
## 967   53 166.4003 166.6742  -0.27394393
## 968   71 166.0899 174.8606  -8.77074959
## 969   53 178.8366 166.6742  12.16243471
## 970   67 172.4073 175.9198  -3.51249812
## 971   59 169.9295 172.8734  -2.94393117
## 972   40 146.9314 144.4000   2.53137809
## 973   46 156.0806 155.9656   0.11497986
## 974   67 156.1620 175.9198 -19.75783962
## 975   63 170.9429 175.2062  -4.26326724
## 976   65 181.8980 175.7750   6.12302560
## 977   69 187.9295 175.6214  12.30814109
## 978   76 158.1015 170.8096 -12.70807808
## 979   63 168.7006 175.2062  -6.50555664
## 980   67 151.4881 175.9198 -24.43166237
```

```
## 981   79 176.7749 166.8094   9.96548595
## 982   77 169.6629 169.6118   0.05113528
## 983   62 182.6790 174.7688   7.91024764
## 984   68 170.1953 175.8272  -5.63191537
## 985   79 168.9477 166.8094   2.13826203
## 986   54 177.6852 167.9144   9.77082664
## 987   49 158.6238 160.9654  -2.34161103
## 988   74 181.2970 172.8104   8.48660554
## 989   64 182.0821 175.5424   6.53970891
## 990   48 143.4575 159.3632 -15.90570026
## 991   53 169.7684 166.6742   3.09423133
## 992   58 147.1954 172.0552 -24.85982190
## 993   65 172.9906 175.7750  -2.78439107
## 994   79 161.2851 166.8094  -5.52433759
## 995   55 177.8354 169.0750   8.76042073
## 996   70 178.2348 175.3000   2.93483694
## 997   52 159.3846 165.3568  -5.97222047
## 998   80 163.6280 165.2000  -1.57203603
## 999   42 150.3067 148.4648   1.84185060
## 1000  50 152.3697 162.5000 -10.13032014
```

7. Generate a plot of weight against age.

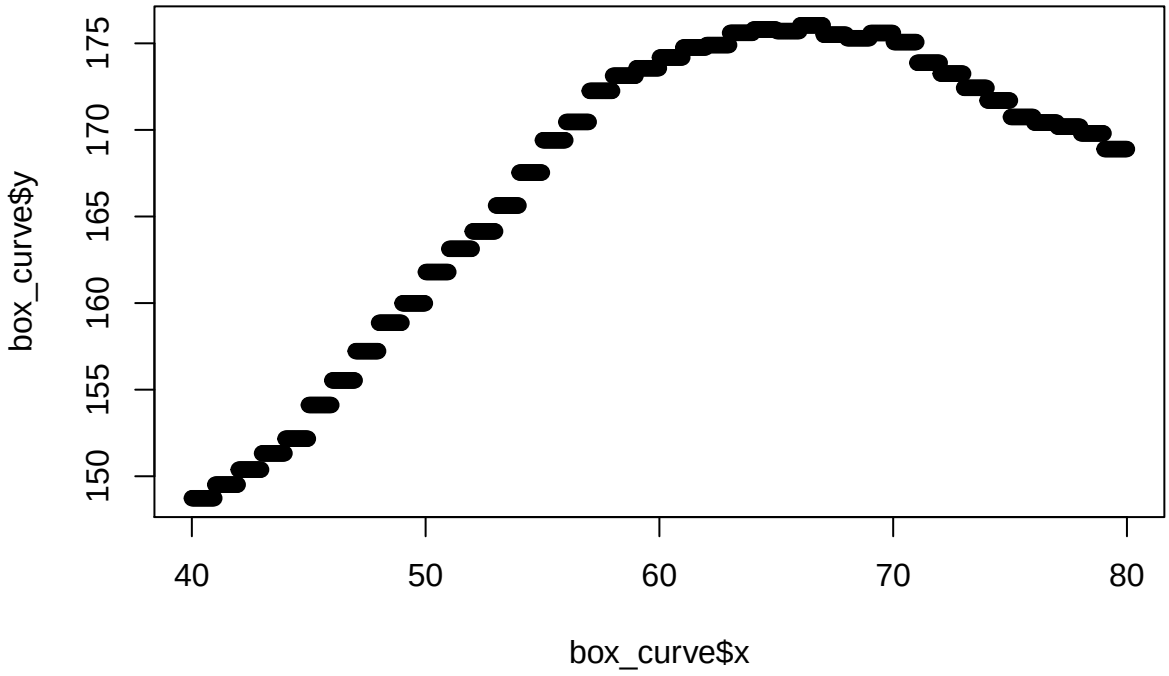
```
plot(weights, age, pch=19, cex=0.4)
```



Applying smoothing kernels:

1. Fit a nearest neighbors curve with ksmooth using a bandwidth of 10 and the `box` kernel.

```
box_curve <- ksmooth(df$age, df$weight, kernel = "box", bandwidth = 10)
plot(box_curve)
```

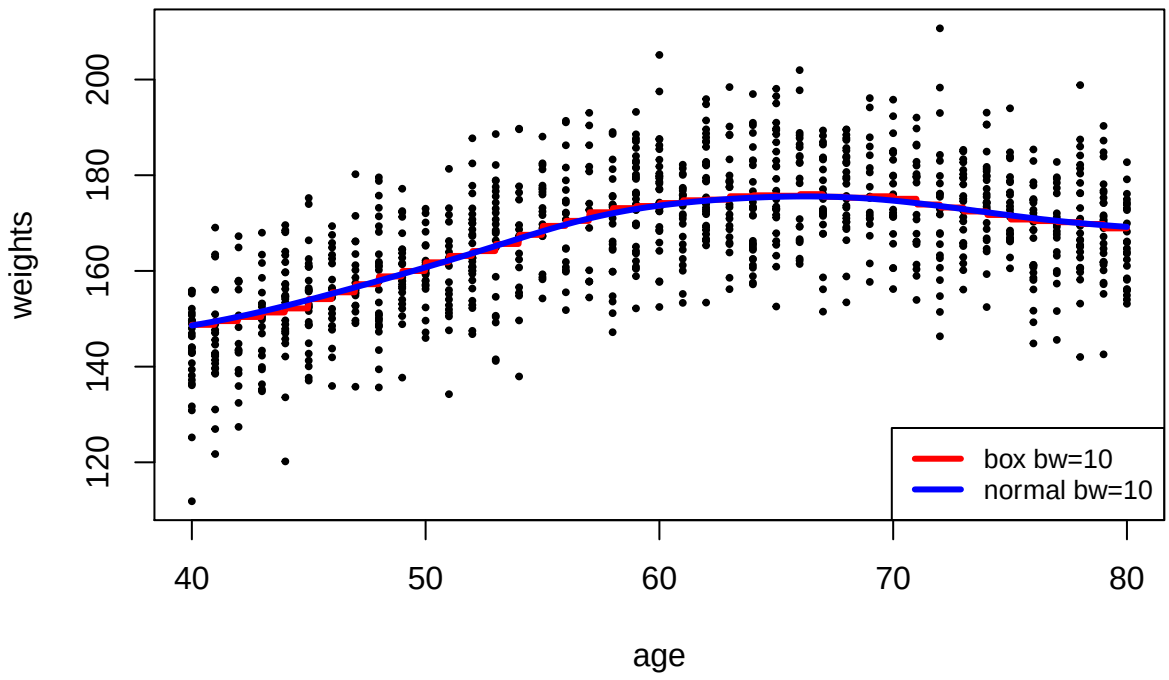


2. Fit another curve this time using the `normal` kernel.

```
normal_curve <- ksmooth(df$age, df$weight, kernel = "normal", bandwidth = 10)
```

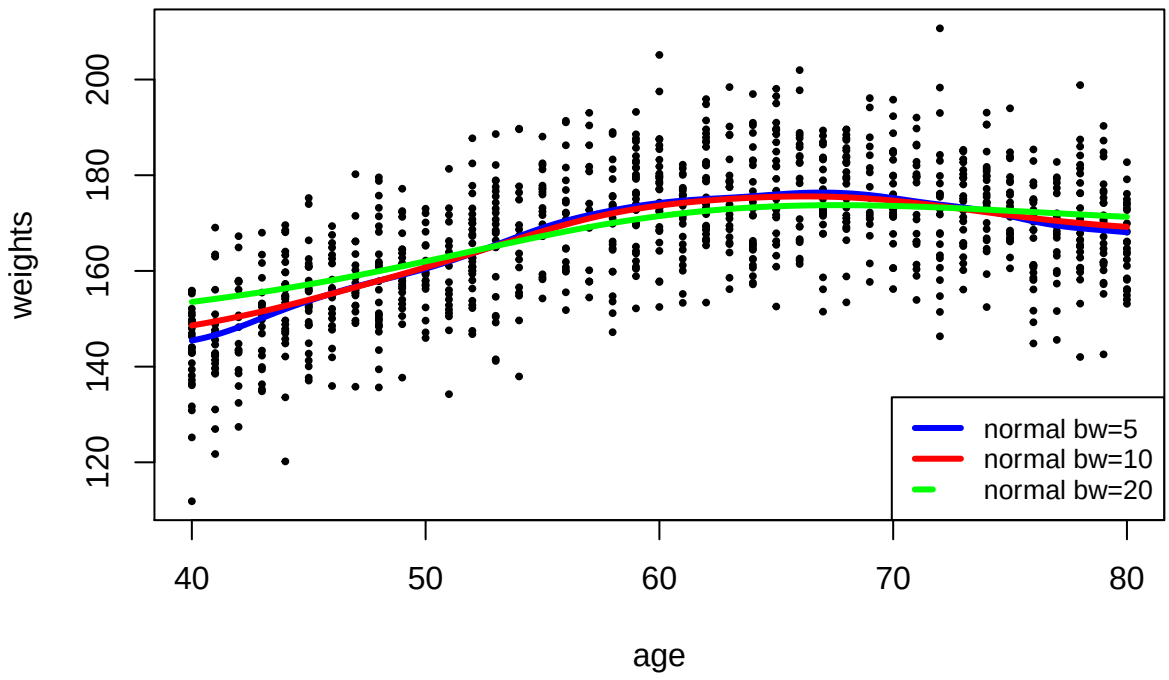
3. Plot the data with the two fitted curves and compare them. Hint: look at the object you have generated with ksmooth (i.e. type `name_of_the_object` or `print(name_of_the_object)`. Also, try `names(name_of_the_object)`. Use the command `lines` to plot the curves.

```
plot(age, weights, pch=19, cex=0.4)
lines(box_curve$x, box_curve$y, lwd=3, col="red")
lines(normal_curve$x, normal_curve$y, lwd=3, col="blue")
legend("bottomright", cex=0.8, lwd=3, lty=c(1,1,2,3),c("box bw=10", "normal bw=10"), col=c("red", "blue
```



4. Fit another 2 curves with the "normal" kernel using bandwidths of size 5, 10, and 20. How do they compare?

```
normal_curve5 <- ksmooth(df$age, df$weight, kernel = "normal", bandwidth = 5)
normal_curve10 <- ksmooth(df$age, df$weight, kernel = "normal", bandwidth = 10)
normal_curve20 <- ksmooth(df$age, df$weight, kernel = "normal", bandwidth = 20)

plot(age, weights, pch=19, cex=0.4)
lines(normal_curve5$x, normal_curve5$y, lwd=3, col="blue")
lines(normal_curve10$x, normal_curve10$y, lwd=3, col="red")
lines(normal_curve20$x, normal_curve20$y, lwd=3, col="green")
legend("bottomright", cex=0.8, lwd=3, lty=c(1,1,2,3), c("normal bw=5","normal bw=10","normal bw=20"), c
```



Fitting linear models:

1. Fit a linear regression to the data using the `lm` command.

```
head(df)
```

```
##   age   weight   trueFx     noise
## 1  43 161.6581 150.4222 11.235854
## 2  78 160.6807 168.2792 -7.598457
## 3  40 155.8896 144.4000 11.489591
## 4  73 165.1934 173.6182 -8.424763
## 5  62 178.6829 174.7688  3.914133
## 6  53 175.5880 166.6742  8.913772
```

```
linear_model <- lm(weight ~ age, data=df)
```

2. Run `summary(your_linear_model_name)` to get an idea of the fit

```
summary(linear_model)
```

```
##
## Call:
## lm(formula = weight ~ age, data = df)
```

```
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -42.605  -8.218  -0.198   8.245  38.326
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 129.79005    1.96776   65.96   <2e-16 ***
## age           0.61706    0.03204   19.26   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12.06 on 998 degrees of freedom
## Multiple R-squared:  0.271,  Adjusted R-squared:  0.2702
## F-statistic: 370.9 on 1 and 998 DF,  p-value: < 2.2e-16
```

3. Now fit quadratic and cubic models: you will need the `I` function to set quadratic and cubic terms in the regression, e.g. `I(x^2)`

```
#quadratic model
quatratic_model <- lm(weight ~ age + I(age^2), data=df)
summary(quatratic_model)
```

```
##
## Call:
## lm(formula = weight ~ age + I(age^2), data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -31.209  -6.762   0.176   7.299  36.560
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -36.543988   9.044877   -4.04 5.75e-05 ***
## age           6.385785   0.309374   20.64  < 2e-16 ***
## I(age^2)     -0.048051   0.002567  -18.72  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.38 on 997 degrees of freedom
## Multiple R-squared:  0.4606, Adjusted R-squared:  0.4595
## F-statistic: 425.7 on 2 and 997 DF,  p-value: < 2.2e-16
```

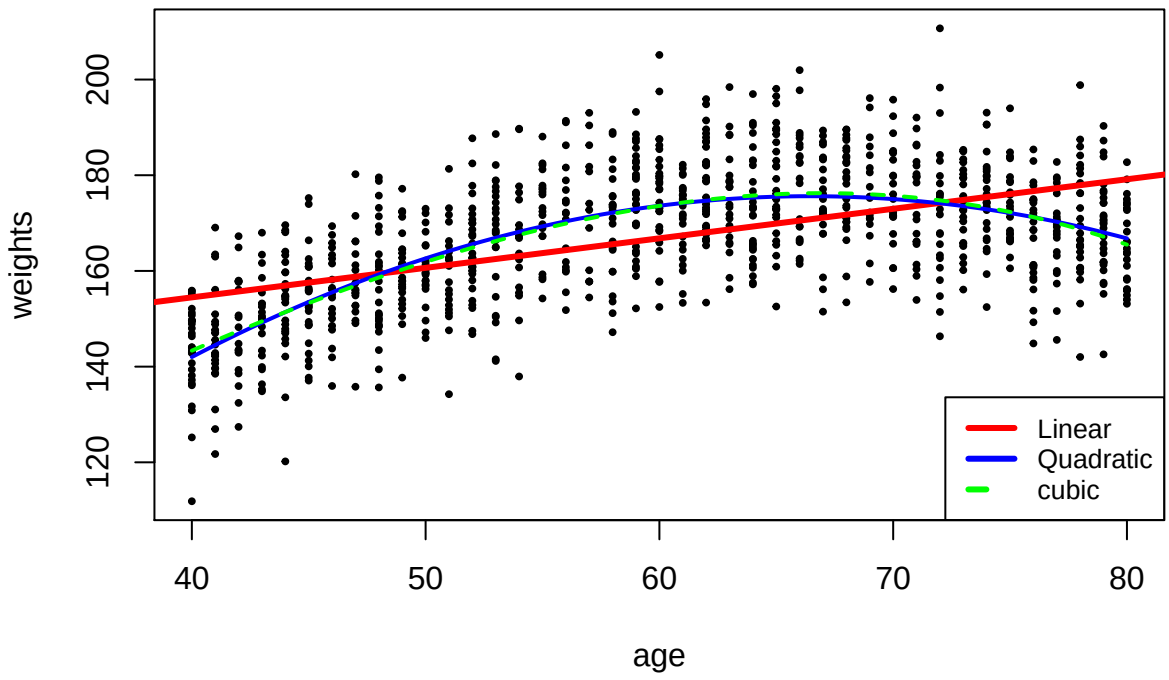
```
#cubic model
cubic_model <- lm(weight ~ age + I(age^2) + I(age^3), data=df)
summary(cubic_model)
```

```
##
## Call:
## lm(formula = weight ~ age + I(age^2) + I(age^3), data = df)
##
## Residuals:
```

```
##     Min      1Q  Median      3Q     Max
## -31.472  -6.905  -0.044   7.288  35.946
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 56.1267168 51.0925279   1.099   0.2722
## age          1.5209614  2.6579063   0.572   0.5673
## I(age^2)     0.0348917  0.0450815   0.774   0.4391
## I(age^3)    -0.0004602  0.0002497  -1.843   0.0657 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.36 on 996 degrees of freedom
## Multiple R-squared:  0.4624, Adjusted R-squared:  0.4608
## F-statistic: 285.6 on 3 and 996 DF,  p-value: < 2.2e-16
```

4. Add the fitted models to the plots. For the linear model, use the `abline` command. For the polynomial fits, you can use the `curve` command (you will need the option `add = TRUE`).

```
?predict
?predict.lm
?curve

quad_predict <- function(age) {predict(quatratic_model, newdata = data.frame(age=age))}
cubic_predict <- function(age) {predict(cubic_model, newdata = data.frame(age=age))}

plot(age, weights, pch=19, cex=0.4)
abline(linear_model, col="red", lwd=3)
curve(quad_predict, col="blue", lwd=2, add=T)
curve(cubic_predict, col="green", lwd=2, lty=2, add=T)
legend("bottomright", cex=0.8, lwd=3, lty=c(1,1,2), c("Linear", "Quadratic", "cubic"), col = c("red", "
```



5. Which curve do you prefer? How would you perform a hypothesis test for this?

```
anova(linear_model, quatratic_model, cubic_model)
```

```
## Analysis of Variance Table
##
## Model 1: weight ~ age
## Model 2: weight ~ age + I(age^2)
## Model 3: weight ~ age + I(age^2) + I(age^3)
##   Res.Df    RSS Df Sum of Sq       F  Pr(>F)
## 1    998 145050
## 2    997 107323  1     37727 351.315 < 2e-16 ***
## 3    996 106958  1       365   3.396 0.06565 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Practice installing an R package

6. R packages are easy to install using the `install.packages` command. Try installing the `e1071` package. We'll use this package later in the course for fitting support vector machines.

```
# install.packages("e1071")
```