On the Design of Multi-Dimensional Compactly Supported Parseval Framelets with Directional Characteristics Supplementary File

N. Atreas, N. Karantzas, M. Papadakis, T. Stavropoulos

June 22, 2018

In this file, we provide the filter matrices for all high-pass filter sets created in Section 4 of the manuscript.
Example 4.3: The following high-pass filter matrices pre-define a high-pass filter subset H we want to extend to a set of high-pass filters defining a Parseval framelet. These pre-defined filters will be incorporated in the high-pass filter set up to scalar multiplications with the components of the vector λ^* derived by the optimization problem set in Theorem 3.2(a).

$$u_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, u_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, u_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, u_4 = \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$
$$u_5 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 0 \end{pmatrix}, u_6 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{pmatrix}, u_7 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, u_8 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

The optimal vector λ^* obtained is

$$\lambda^* = (0.0442, 0.0884, 0.0442, 0.0884, 0.0234, 0.0293, 0.0088, 0.0316)\,.$$

Our algorithm produces the following matrix of high-pass filter coefficients

$$B = 10^{-2}\begin{pmatrix} -17.7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17.7 \\ 0 & -25 & 0 & 0 & 0 & 0 & 0 & 25 & 0 \\ 0 & 0 & -17.7 & 0 & 0 & 0 & 17.7 & 0 & 0 \\ 0 & 0 & 0 & -25 & 0 & 25 & 0 & 0 & 0 \\ 0 & 0 & 0 & -6.63 & 13.3 & -6.63 & 0 & 0 & 0 \\ 0 & 0 & -11.7 & 0 & 23.5 & 0 & -11.7 & 0 & 0 \\ 0 & -2.5 & 0 & 0 & 4.99 & 0 & 0 & -2.5 & 0 \\ -12.6 & 0 & 0 & 0 & 25.3 & 0 & 0 & 0 & -12.6 \\ 0.002 & 0 & 0.001 & 0.0003 & -0.008 & 0.0003 & 0.001 & 0 & 0.002 \\ -8.52 & 0.0288 & 9.59 & 0.233 & -2.66 & 0.233 & 9.59 & 0.0288 & -8.52 \\ 5.46 & -0.939 & 5.69 & -19 & 17.5 & -19 & 5.69 & -0.939 & 5.46 \\ 3.39 & -21.5 & 3.4 & 8.1 & 13.2 & 8.1 & 3.4 & -21.5 & 3.39 \end{pmatrix}$$

The corresponding high-pass filter matrices ($\times 100$) are given by

$$h_1 = \begin{pmatrix} 0 & 0 & 18 \\ 0 & 0 & 0 \\ -18 & 0 & 0 \end{pmatrix} h_2 = \begin{pmatrix} 0 & 25 & 0 \\ 0 & 0 & 0 \\ 0 & -25 & 0 \end{pmatrix}$$

$$h_3 = \begin{pmatrix} 18 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -18 \end{pmatrix} h_4 = \begin{pmatrix} 0 & 0 & 0 \\ -25 & 0 & 25 \\ 0 & 0 & 0 \end{pmatrix}$$

$$h_5 = \begin{pmatrix} 0 & 0 & 0 \\ -6.6 & 13 & -6.6 \\ 0 & 0 & 0 \end{pmatrix} h_6 = \begin{pmatrix} -12 & 0 & 0 \\ 0 & 23 & 0 \\ 0 & 0 & -12 \end{pmatrix}$$

$$h_7 = \begin{pmatrix} 0 & -2.5 & 0 \\ 0 & 5 & 0 \\ 0 & -2.5 & 0 \end{pmatrix} h_8 = \begin{pmatrix} 0 & 0 & -13 \\ 0 & 25 & 0 \\ -13 & 0 & 0 \end{pmatrix}$$

$$h_9 = \begin{pmatrix} 0.0017 & 0 & 0.0023 \\ 0.00033 & -0.0087 & 0.00033 \\ 0.0023 & 0 & 0.0017 \end{pmatrix} h_{10} = \begin{pmatrix} 9.6 & 0.029 & -8.5 \\ 0.23 & -2.7 & 0.23 \\ -8.5 & 0.029 & 9.6 \end{pmatrix}$$

$$h_{11} = \begin{pmatrix} 5.7 & -0.94 & 5.5 \\ -19 & 17 & -19 \\ 5.5 & -0.94 & 5.7 \end{pmatrix} h_{12} = \begin{pmatrix} 3.4 & -21 & 3.4 \\ 8.1 & 13 & 8.1 \\ 3.4 & -21 & 3.4 \end{pmatrix}$$

Example 4.4: We want to include in the construction the following high-pass filter matrices (modulo scalar multiplications as in the Example 4.3).

$$u_1 = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} u_2 = \begin{pmatrix} 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$u_3 = \begin{pmatrix} 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 \end{pmatrix} u_4 = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$

$$u_5 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 \end{pmatrix} u_6 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$u_7 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} u_8 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$u_9 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} u_{10} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$u_{11} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} u_{12} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$u_{13} = \begin{pmatrix} 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \end{pmatrix} u_{14} = \begin{pmatrix} 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \end{pmatrix}$$

$$u_{15} = \begin{pmatrix} 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 \end{pmatrix} u_{16} = \begin{pmatrix} 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 \end{pmatrix}$$

$$u_{17} = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} u_{18} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -2 & 0 & 1 & 0 & 0 \\ 1 & 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$u_{19} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -2 & -2 & -2 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} u_{20} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -2 \\ 1 & 0 & -2 & 0 & 1 \\ -2 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$u_{21} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} u_{22} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$u_{23} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} u_{24} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & -2 & -2 & -2 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The corresponding high-pass filter matrices ($\times 100$) are given by

$$h_1 = \begin{pmatrix} 0 & 0 & 0 & -4.6 & 0 \\ 0 & 0 & -4.6 & 0 & 4.6 \\ 0 & -4.6 & 0 & 4.6 & 0 \\ -4.6 & 0 & 4.6 & 0 & 0 \\ 0 & 4.6 & 0 & 0 & 0 \end{pmatrix} h_2 = \begin{pmatrix} 0 & 0 & -2.1 & 0 & 2.1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -2.1 & 0 & 2.1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -2.1 & 0 & 2.1 & 0 & 0 \end{pmatrix}$$

$$h_3 = \begin{pmatrix} 0 & -4.4 & 0 & 4.4 & 0 \\ 0 & -4.4 & 0 & 4.4 & 0 \\ 0 & -4.4 & 0 & 4.4 & 0 \\ 0 & -4.4 & 0 & 4.4 & 0 \\ 0 & -4.4 & 0 & 4.4 & 0 \end{pmatrix} h_4 = \begin{pmatrix} -2.2 & 0 & 2.2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -2.2 & 0 & 2.2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2.2 & 0 & 2.2 \end{pmatrix}$$

$$h_5 = \begin{pmatrix} 0 & 4.7 & 0 & 0 & 0 \\ -4.7 & 0 & 4.7 & 0 & 0 \\ 0 & -4.7 & 0 & 4.7 & 0 \\ 0 & 0 & -4.7 & 0 & 4.7 \\ 0 & 0 & 0 & -4.7 & 0 \end{pmatrix} h_6 = \begin{pmatrix} 2.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.8 & 0 & 0 \\ -2.8 & 0 & 0 & 0 & 2.8 \\ 0 & 0 & -2.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2.8 \end{pmatrix}$$

$$h_7 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 4.1 & 4.1 & 4.1 & 4.1 & 4.1 \\ 0 & 0 & 0 & 0 & 0 \\ -4.1 & -4.1 & -4.1 & -4.1 & -4.1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} h_8 = \begin{pmatrix} 0 & 0 & 0 & 0 & 3.4 \\ 0 & 0 & 3.4 & 0 & 0 \\ 3.4 & 0 & 0 & 0 & -3.4 \\ 0 & 0 & -3.4 & 0 & 0 \\ -3.4 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$h_9 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -11 & 0 & 0 \\ 0 & -11 & 0 & 11 & 0 \\ 0 & 0 & 11 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} h_{10} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -7.2 & 0 & 7.2 & 0 \\ 0 & -7.2 & 0 & 7.2 & 0 \\ 0 & -7.2 & 0 & 7.2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$h_{11} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9.7 & 0 & 0 \\ 0 & -9.7 & 0 & 9.7 & 0 \\ 0 & 0 & -9.7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} h_{12} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 5.8 & 5.8 & 5.8 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -5.8 & -5.8 & -5.8 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$h_{13} = \begin{pmatrix} 0 & 0 & 0 & 2.3 & -2.3 \\ 0 & 0 & 2.3 & -4.6 & 2.3 \\ 0 & 2.3 & -4.6 & 2.3 & 0 \\ 2.3 & -4.6 & 2.3 & 0 & 0 \\ -2.3 & 2.3 & 0 & 0 & 0 \end{pmatrix} h_{14} = \begin{pmatrix} 0 & 0 & 1.7 & -3.3 & 1.7 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1.7 & -3.3 & 1.7 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1.7 & -3.3 & 1.7 & 0 & 0 \end{pmatrix}$$

$$h_{15} = \begin{pmatrix} 0 & 2.3 & -4.6 & 2.3 & 0 \\ 0 & 2.3 & -4.6 & 2.3 & 0 \\ 0 & 2.3 & -4.6 & 2.3 & 0 \\ 0 & 2.3 & -4.6 & 2.3 & 0 \\ 0 & 2.3 & -4.6 & 2.3 & 0 \end{pmatrix} h_{16} = \begin{pmatrix} 1.9 & -3.8 & 1.9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1.9 & -3.8 & 1.9 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.9 & -3.8 & 1.9 \end{pmatrix}$$

$$h_{17} = \begin{pmatrix} -2.1 & 2.1 & 0 & 0 & 0 \\ 2.1 & -4.2 & 2.1 & 0 & 0 \\ 0 & 2.1 & -4.2 & 2.1 & 0 \\ 0 & 0 & 2.1 & -4.2 & 2.1 \\ 0 & 0 & 0 & 2.1 & -2.1 \end{pmatrix} h_{18} = \begin{pmatrix} 1.8 & 0 & 0 & 0 & 0 \\ -3.7 & 0 & 1.8 & 0 & 0 \\ 1.8 & 0 & -3.7 & 0 & 1.8 \\ 0 & 0 & 1.8 & 0 & -3.7 \\ 0 & 0 & 0 & 0 & 1.8 \end{pmatrix}$$

$$h_{19} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 2.1 & 2.1 & 2.1 & 2.1 & 2.1 \\ -4.2 & -4.2 & -4.2 & -4.2 & -4.2 \\ 2.1 & 2.1 & 2.1 & 2.1 & 2.1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} h_{20} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1.5 \\ 0 & 0 & 1.5 & 0 & -3.1 \\ 1.5 & 0 & -3.1 & 0 & 1.5 \\ -3.1 & 0 & 1.5 & 0 & 0 \\ 1.5 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$h_{21} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6.8 & -6.8 & 0 \\ 0 & 6.8 & -14 & 6.8 & 0 \\ 0 & -6.8 & 6.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} h_{22} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 5.3 & -11 & 5.3 & 0 \\ 0 & 5.3 & -11 & 5.3 & 0 \\ 0 & 5.3 & -11 & 5.3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$h_{23} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -4 & 4 & 0 & 0 \\ 0 & 4 & -7.9 & 4 & 0 \\ 0 & 0 & 4 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} h_{24} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 5.3 & 5.3 & 5.3 & 0 \\ 0 & -11 & -11 & -11 & 0 \\ 0 & 5.3 & 5.3 & 5.3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$h_{25} = \begin{pmatrix} 0 & 0.0001 & 0 & 0 & 0 \\ 0 & 0.00012 & 0.00026 & 0 & 0 \\ 0 & 0.0001 & 0 & -0.0001 & 0 \\ 0 & 0 & -0.00026 & -0.00012 & 0 \\ 0 & 0 & 0 & -0.0001 & 0 \end{pmatrix} h_{26} = \begin{pmatrix} -0.0029 & 0.0034 & -0.0021 & 0.0027 & -0.0022 \\ 0.0031 & -0.0053 & 0.0047 & -0.0067 & 0.0023 \\ -0.0016 & 0.0054 & -0.0013 & 0.0054 & -0.0016 \\ 0.0023 & -0.0067 & 0.0047 & -0.0053 & 0.0031 \\ -0.0022 & 0.0027 & -0.0021 & 0.0034 & -0.0029 \end{pmatrix}$$

$$h_{27} = \begin{pmatrix} 0.02 & 0.21 & 0.036 & -0.25 & -0.16 \\ -0.048 & 0.31 & -0.3 & -0.39 & 0.062 \\ -0.13 & 0.51 & 0 & -0.51 & 0.13 \\ -0.062 & 0.39 & 0.3 & -0.31 & 0.048 \\ 0.16 & 0.25 & -0.036 & -0.21 & -0.02 \end{pmatrix} h_{28} = \begin{pmatrix} -0.44 & 0.62 & -0.48 & -0.069 & 0.23 \\ 0.52 & 0.92 & -1.8 & 2 & -0.14 \\ -0.33 & -1.4 & 0.51 & -1.4 & -0.33 \\ -0.14 & 2 & -1.8 & 0.92 & 0.52 \\ 0.23 & -0.069 & -0.48 & 0.62 & -0.44 \end{pmatrix}$$

$$h_{29} = \begin{pmatrix} 0.29 & -0.74 & -0.054 & 0.82 & -0.11 \\ 0.77 & 0.9 & 1.9 & 0.0047 & -0.43 \\ -0.21 & 3.9 & 0 & -3.9 & 0.21 \\ 0.43 & -0.0047 & -1.9 & -0.9 & -0.77 \\ 0.11 & -0.82 & 0.054 & 0.74 & -0.29 \end{pmatrix} h_{30} = \begin{pmatrix} 0.062 & 0.45 & 0.052 & 0.93 & -0.25 \\ -1.5 & -1.4 & 0.24 & -0.89 & -1.7 \\ -0.23 & 0.044 & 0 & -0.044 & 0.23 \\ 1.7 & 0.89 & -0.24 & 1.4 & 1.5 \\ 0.25 & -0.93 & -0.052 & -0.45 & -0.062 \end{pmatrix}$$

$$h_{31} = \begin{pmatrix} 0.58 & -0.2 & -0.91 & 1.4 & -1 \\ -0.44 & 2.7 & -2.4 & 1.4 & 1.1 \\ -0.42 & -0.98 & -1.7 & -0.98 & -0.42 \\ 1.1 & 1.4 & -2.4 & 2.7 & -0.44 \\ -1 & 1.4 & -0.91 & -0.2 & 0.58 \end{pmatrix} h_{32} = \begin{pmatrix} 0.99 & 0.64 & -0.096 & 0.08 & -0.74 \\ -0.22 & 0.25 & 5.5 & 3 & 0.81 \\ -1.3 & -3.8 & 0 & 3.8 & 1.3 \\ -0.81 & -3 & -5.5 & -0.25 & 0.22 \\ 0.74 & -0.08 & 0.096 & -0.64 & -0.99 \end{pmatrix}$$

$$h_{33} = \begin{pmatrix} -0.045 & -0.71 & 1.2 & -0.44 & -0.25 \\ 0.81 & 0.52 & 6.6 & 0.56 & 0.96 \\ -1.5 & -8.2 & 0.79 & -8.2 & -1.5 \\ 0.96 & 0.56 & 6.6 & 0.52 & 0.81 \\ -0.25 & -0.44 & 1.2 & -0.71 & -0.045 \end{pmatrix} h_{34} = \begin{pmatrix} -2.1 & 0.56 & 1.2 & 0.63 & -0.82 \\ 0.079 & 1.9 & 5.2 & 1.9 & 0.062 \\ 0.78 & 0.079 & 0 & -0.079 & -0.78 \\ -0.062 & -1.9 & -5.2 & -1.9 & -0.079 \\ 0.82 & -0.63 & -1.2 & -0.56 & 2.1 \end{pmatrix}$$

$$h_{35} = \begin{pmatrix} -0.0034 & 0.49 & -0.42 & -0.14 & 0.57 \\ 0.42 & 3.2 & 4.2 & 0.85 & -0.084 \\ -0.41 & 2.7 & -23 & 2.7 & -0.41 \\ -0.084 & 0.85 & 4.2 & 3.2 & 0.42 \\ 0.57 & -0.14 & -0.42 & 0.49 & -0.0034 \end{pmatrix} h_{36} = \begin{pmatrix} 0.09 & 2.6 & -3.2 & 1.9 & 0.13 \\ -2.7 & -0.43 & 5.4 & -0.3 & -2.1 \\ 3.2 & -5.1 & 1.1 & -5.1 & 3.2 \\ -2.1 & -0.3 & 5.4 & -0.43 & -2.7 \\ 0.13 & 1.9 & -3.2 & 2.6 & 0.09 \end{pmatrix}$$

$$h_{37} = \begin{pmatrix} 0.15 & -0.19 & -0.32 & 2.2 & 0.06 \\ 1.3 & 11 & -2.2 & -5 & -1.7 \\ -0.76 & -8.7 & 0 & 8.7 & 0.76 \\ 1.7 & 5 & 2.2 & -11 & -1.3 \\ -0.06 & -2.2 & 0.32 & 0.19 & -0.15 \end{pmatrix} h_{38} = \begin{pmatrix} -0.18 & -2.8 & 1 & -1.8 & -0.25 \\ 0.9 & -4.8 & 8.4 & -11 & 0.25 \\ -0.44 & -4 & 0 & 4 & 0.44 \\ -0.25 & 11 & -8.4 & 4.8 & -0.9 \\ 0.25 & 1.8 & -1 & 2.8 & 0.18 \end{pmatrix}$$

$$h_{39} = \begin{pmatrix} -1.6 & -0.46 & 5 & -1.3 & -1.6 \\ -1.7 & 0.69 & -1.9 & 3.2 & -0.68 \\ 5.1 & -1.5 & -6.3 & -1.5 & 5.1 \\ -0.68 & 3.2 & -1.9 & 0.69 & -1.7 \\ -1.6 & -1.3 & 5 & -0.46 & -1.6 \end{pmatrix} h_{40} = \begin{pmatrix} 0.17 & 3.9 & 0.035 & -4.1 & 0.14 \\ -3.7 & 0.47 & -0.46 & -0.81 & 4.4 \\ -1.2 & 0.55 & 1.1 & 0.55 & -1.2 \\ 4.4 & -0.81 & -0.46 & 0.47 & -3.7 \\ 0.14 & -4.1 & 0.035 & 3.9 & 0.17 \end{pmatrix}$$

$$h_{41} = \begin{pmatrix} -0.023 & 2.5 & 0.074 & -2.7 & 0.017 \\ 5 & -2.9 & 0.52 & 2.6 & -5.1 \\ -0.44 & -2.2 & 0 & 2.2 & 0.44 \\ 5.1 & -2.6 & -0.52 & 2.9 & -5 \\ -0.017 & 2.7 & -0.074 & -2.5 & 0.023 \end{pmatrix} h_{42} = \begin{pmatrix} -0.71 & -0.067 & 0.34 & -0.63 & 0.21 \\ 0.21 & 12 & -1.6 & -10 & -1.1 \\ 0.43 & -1.6 & 5 & -1.6 & 0.43 \\ -1.1 & -10 & -1.6 & 12 & 0.21 \\ 0.21 & -0.63 & 0.34 & -0.067 & -0.71 \end{pmatrix}$$

$$h_{43} = \begin{pmatrix} -0.0046 & -4.6 & 0.33 & -4.5 & -0.014 \\ -3 & 5.1 & 0.96 & 4.9 & -2.8 \\ -0.026 & -0.15 & 0 & 0.15 & 0.026 \\ 2.8 & -4.9 & -0.96 & -5.1 & 3 \\ 0.014 & 4.5 & -0.33 & 4.6 & 0.0046 \end{pmatrix} h_{44} = \begin{pmatrix} 1.1 & 0.32 & 9.5 & 0.3 & 0.033 \\ 0.12 & 0.61 & -1.3 & 0.34 & -0.063 \\ 3.6 & -0.4 & 0 & 0.4 & -3.6 \\ 0.063 & -0.34 & 1.3 & -0.61 & -0.12 \\ -0.033 & -0.3 & -9.5 & -0.32 & -1.1 \end{pmatrix}$$

$$h_{45} = \begin{pmatrix} 0.73 & -0.13 & -3.9 & -0.18 & -1.4 \\ 0.28 & 0.21 & 0.72 & -0.72 & -0.3 \\ 9 & -1.3 & 0 & 1.3 & -9 \\ 0.3 & 0.72 & -0.72 & -0.21 & -0.28 \\ 1.4 & 0.18 & 3.9 & 0.13 & -0.73 \end{pmatrix} h_{46} = \begin{pmatrix} 0.072 & -2.5 & -5.3 & -2.8 & 0.075 \\ 2.9 & -0.22 & 0.27 & -0.36 & 3.2 \\ 6.2 & -1.3 & -0.41 & -1.3 & 6.2 \\ 3.2 & -0.36 & 0.27 & -0.22 & 2.9 \\ 0.075 & -2.8 & -5.3 & -2.5 & 0.072 \end{pmatrix}$$

$$h_{47} = \begin{pmatrix} 1.1 & 3 & 4.2 & 3 & 1.2 \\ 2.6 & -2.5 & -5.7 & -4.1 & 2.4 \\ 3.2 & -5.6 & -5.4 & -5.6 & 3.2 \\ 2.4 & -4.1 & -5.7 & -2.5 & 2.6 \\ 1.2 & 3 & 4.2 & 3 & 1.1 \end{pmatrix} h_{48} = \begin{pmatrix} 1.8 & 3.6 & 0 & -3.6 & -1.8 \\ 3.6 & 0 & 0 & 0 & -3.6 \\ 0 & 0 & 0 & 0 & 0 \\ -3.6 & 0 & 0 & 0 & 3.6 \\ -1.8 & -3.6 & 0 & 3.6 & 1.8 \end{pmatrix}$$