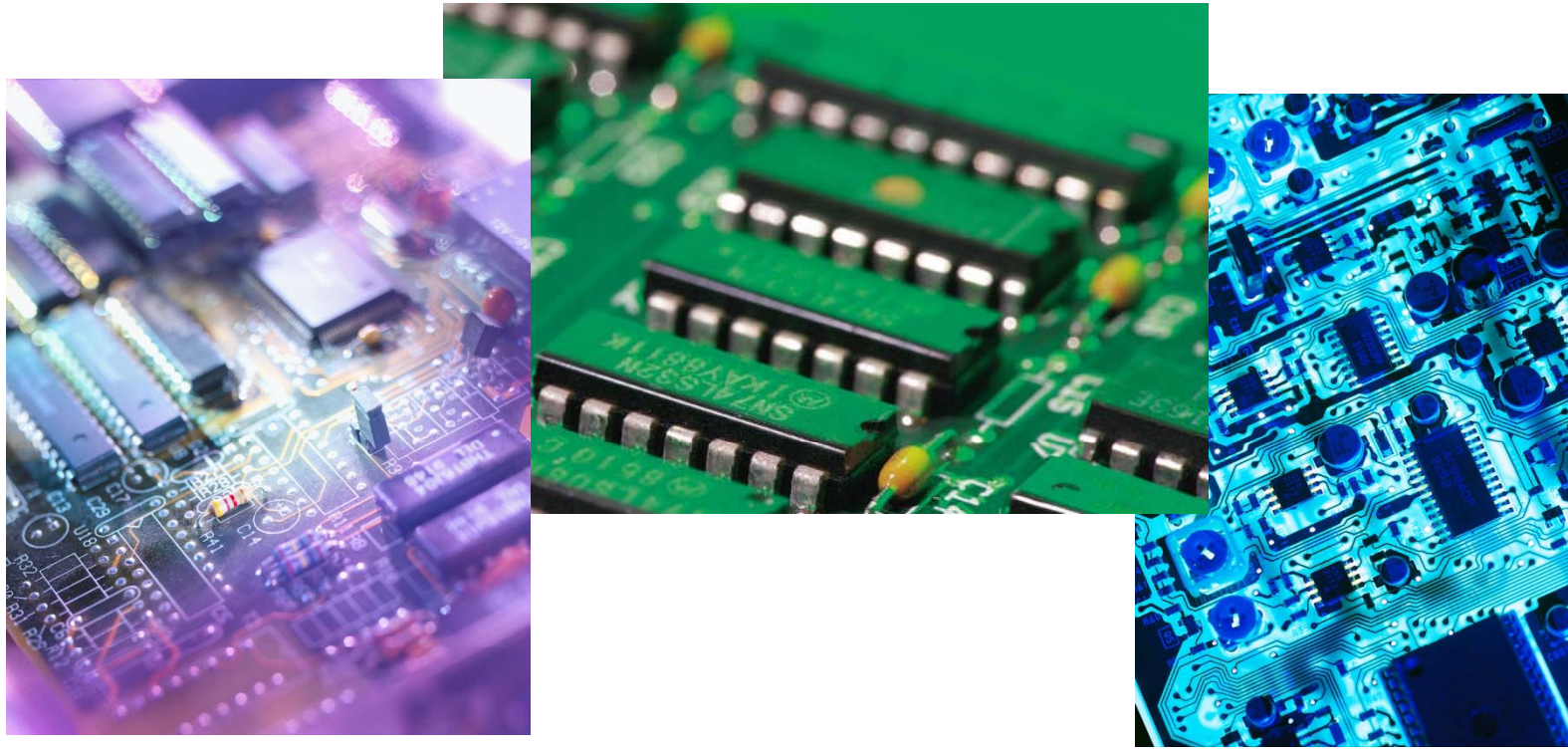


## Chapter 2

### Parallel Hardware and Parallel Software

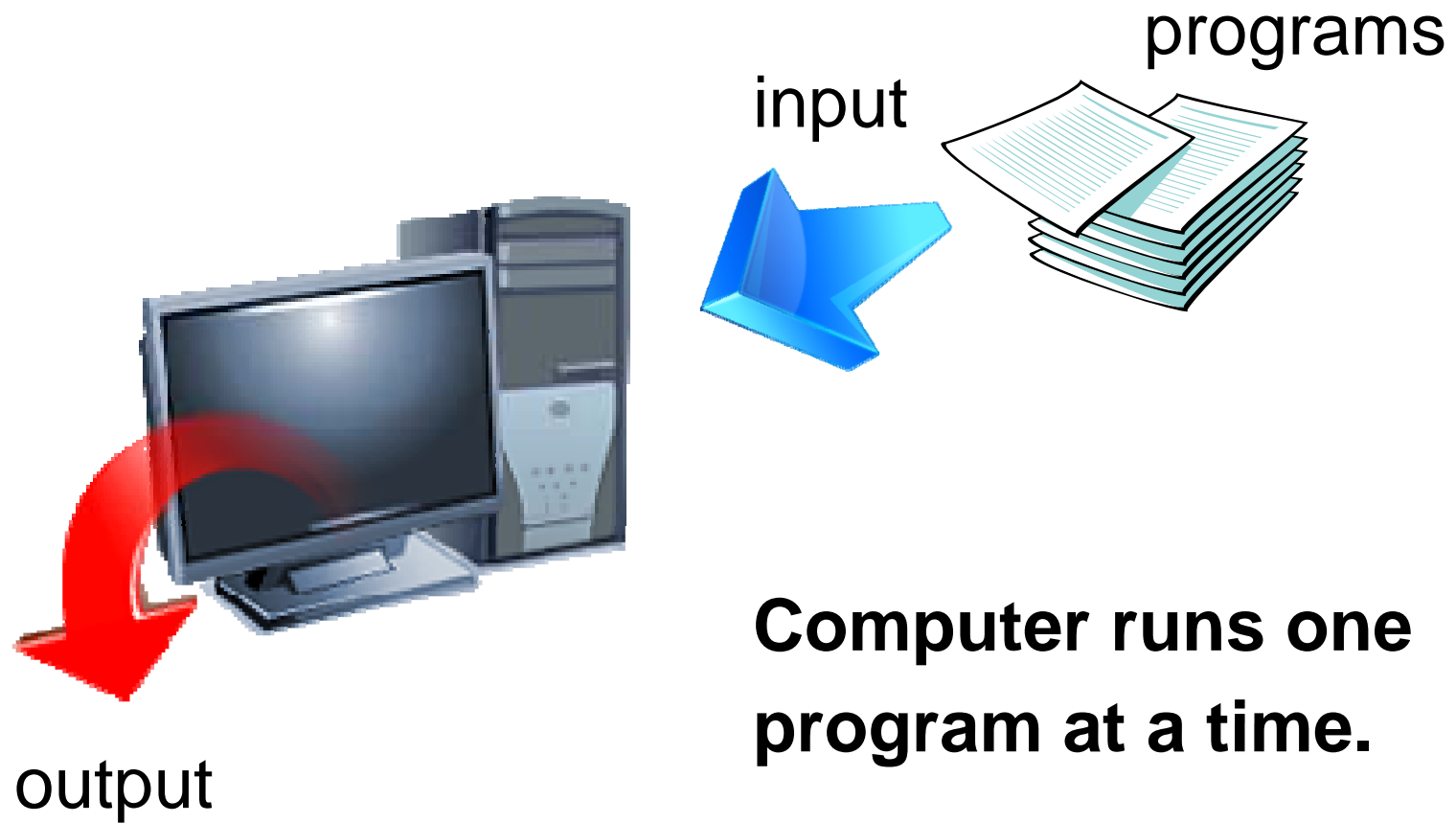
# Roadmap

- Some background
- Modifications to the von Neumann model
- Parallel hardware
- Parallel software
- Input and output
- Performance
- Parallel program design
- Writing and running parallel programs
- Assumptions



# SOME BACKGROUND

# Serial hardware and software



# The von Neumann Architecture

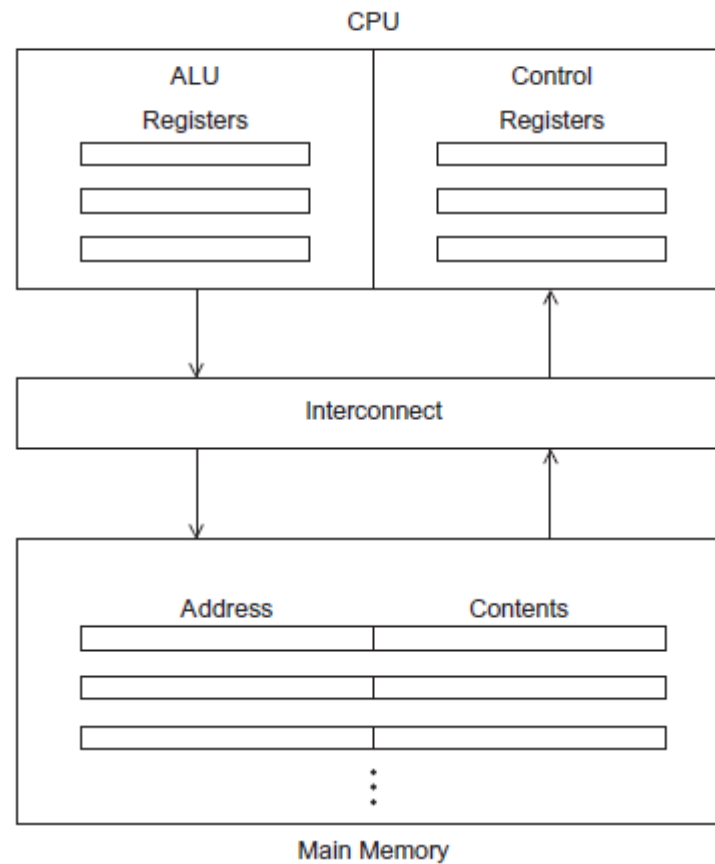
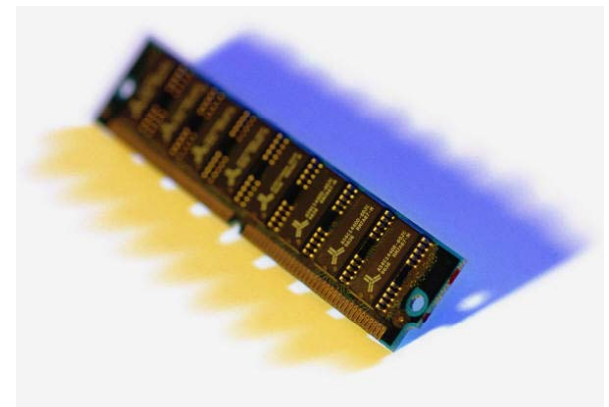


Figure 2.1

# Main memory

- This is a collection of locations, each of which is capable of storing both instructions and data.
- Every location consists of an address, which is used to access the location, and the contents of the location.



# Central processing unit (CPU)

- Divided into two parts.
- **Control unit** - responsible for deciding which instruction in a program should be executed. (*the boss*)
- **Arithmetic and logic unit (ALU)** - responsible for executing the actual instructions. (*the worker*)



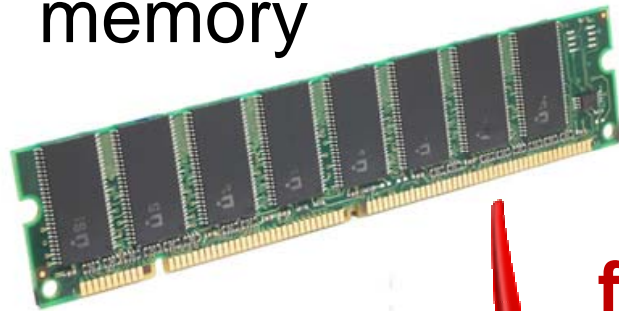
# Key terms

- **Register** – very fast storage, part of the CPU.
- **Program counter** – stores address of the next instruction to be executed.
- **Bus** – wires and hardware that connects the CPU and memory.





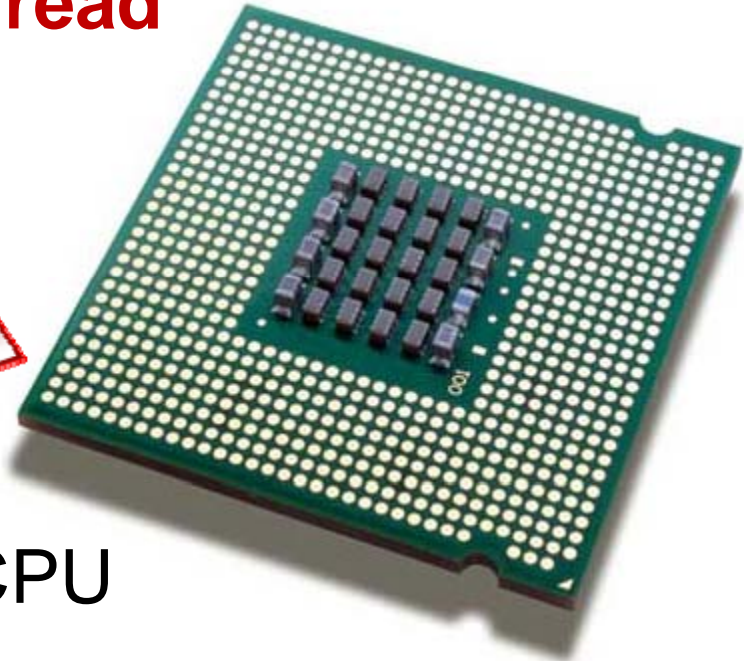
memory



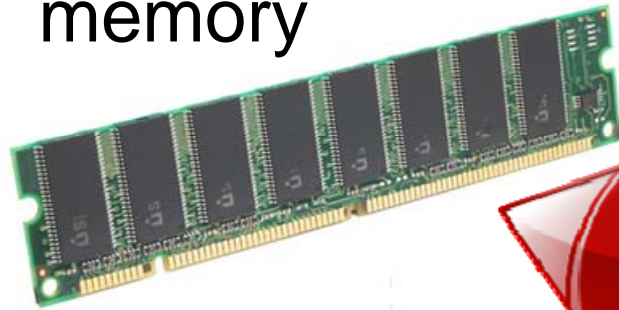
**fetch/read**



CPU



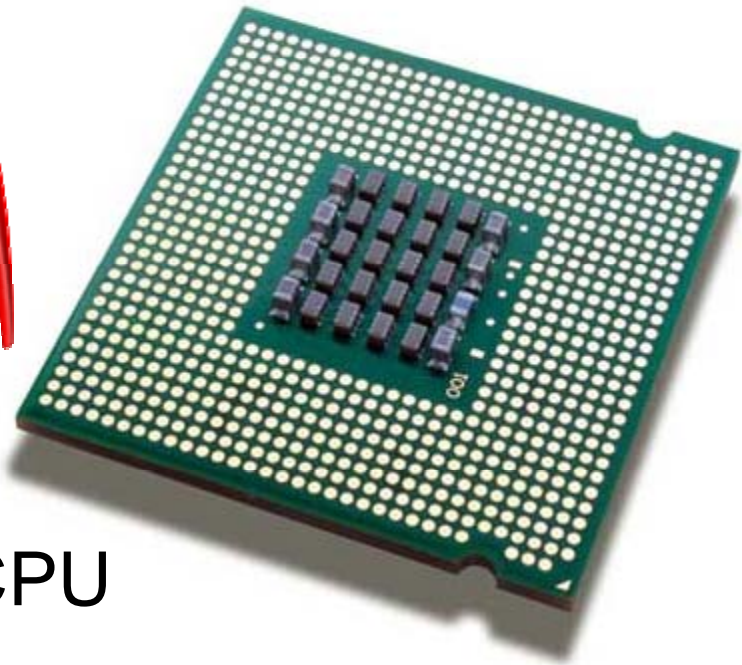
memory



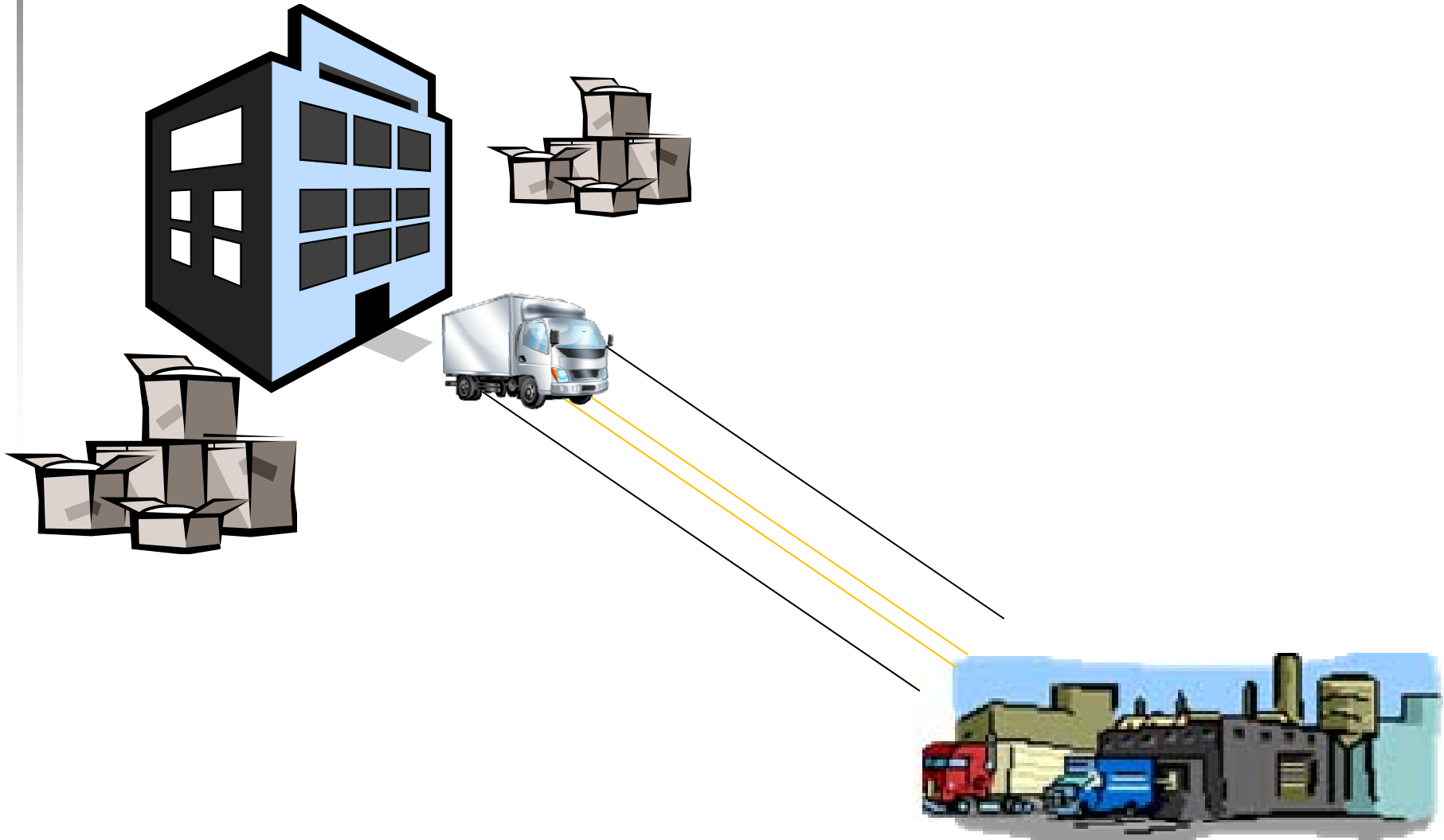
**write/store**



CPU



# von Neumann bottleneck



# An operating system “process”

- An instance of a computer program that is being executed.
- Components of a process:
  - The executable machine language program.
  - A block of memory.
  - Descriptors of resources the OS has allocated to the process.
  - Security information.
  - Information about the state of the process.

# Multitasking

- Gives the illusion that a single processor system is running multiple programs simultaneously.
- Each process takes turns running. (**time slice**)
- After its time is up, it waits until it has a turn again. (**blocks**)

# Threading

- Threads are contained within processes.
- They allow programmers to divide their programs into (more or less) independent tasks.
- The hope is that when one thread blocks because it is waiting on a resource, another will have work to do and can run.

# A process and two threads

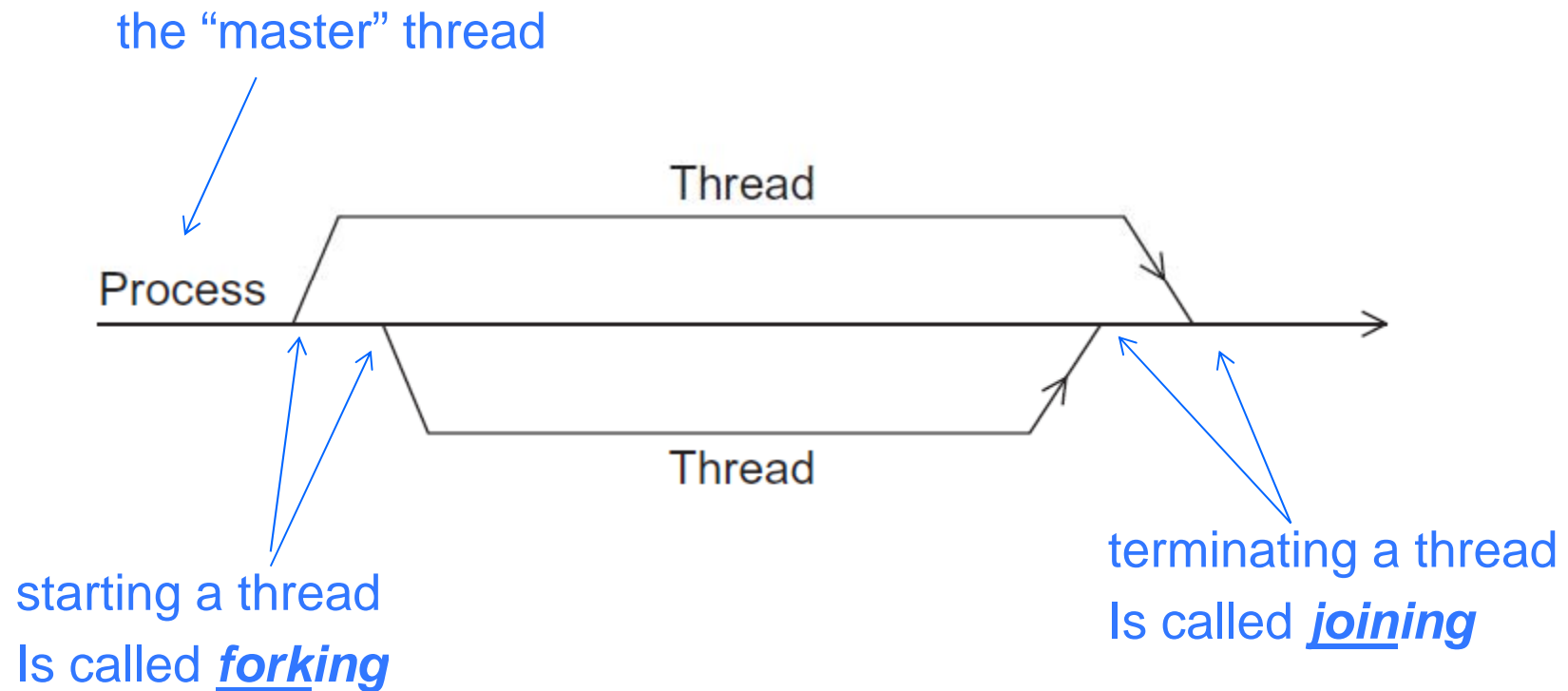
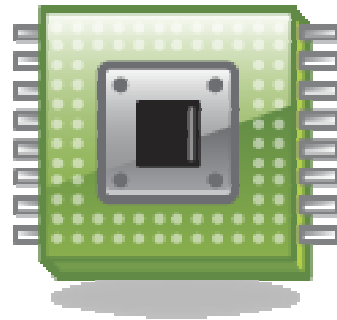


Figure 2.2

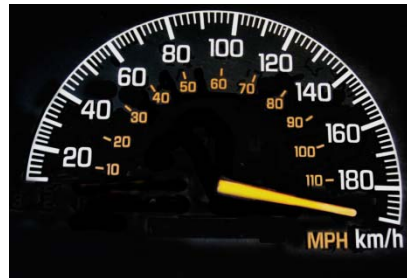


# MODIFICATIONS TO THE VON NEUMANN MODEL



# Basics of caching

- A collection of memory locations that can be accessed in less time than some other memory locations.
- A CPU cache is typically located on the same chip, or one that can be accessed much faster than ordinary memory.



# Principle of locality

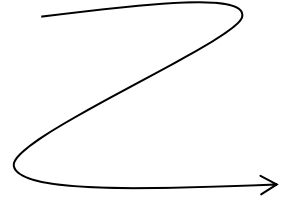
- Accessing one location is followed by an access of a nearby location.
- **Spatial locality** – accessing a nearby location.
- **Temporal locality** – accessing in the near future.

# Principle of locality

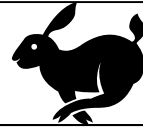
```
float z[1000];  
...  
sum = 0.0;  
for (i = 0; i < 1000; i++)  
    sum += z[i];
```

# Levels of Cache

smallest & fastest

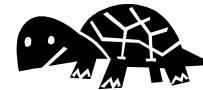


L1



L2

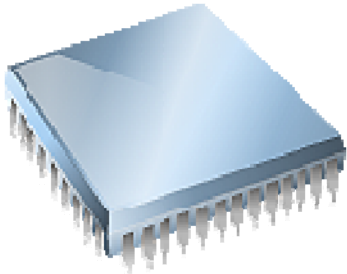
L3



largest & slowest



# Cache hit



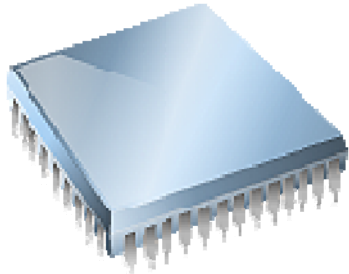
fetch x

L1    x    sum

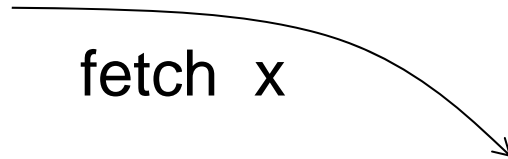
L2            y    z    total

L3            A[ ]    radius    r1    center

# Cache miss



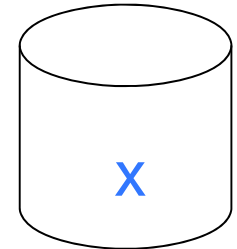
fetch x



L1    y    sum

L2            r1    z    total

L3            A[ ]    radius    center



main  
memory

# Issues with cache

- When a CPU writes data to cache, the value in cache may be inconsistent with the value in main memory.
- **Write-through** caches handle this by updating the data in main memory at the time it is written to cache.
- **Write-back** caches mark data in the cache as **dirty**. When the cache line is replaced by a new cache line from memory, the **dirty** line is written to memory.

# Cache mappings

- **Full associative** – a new line can be placed at any location in the cache.
- **Direct mapped** – each cache line has a unique location in the cache to which it will be assigned.
- **$n$ -way set associative** – each cache line can be place in one of  $n$  different locations in the cache.



# *n*-way set associative

- When more than one line in memory can be mapped to several different locations in cache we also need to be able to decide which line should be replaced or **evicted**.



# Example

Memory Index	Cache Location		
	Fully Assoc	Direct Mapped	2-way
0	0, 1, 2, or 3	0	0 or 1
1	0, 1, 2, or 3	1	2 or 3
2	0, 1, 2, or 3	2	0 or 1
3	0, 1, 2, or 3	3	2 or 3
4	0, 1, 2, or 3	0	0 or 1
5	0, 1, 2, or 3	1	2 or 3
6	0, 1, 2, or 3	2	0 or 1
7	0, 1, 2, or 3	3	2 or 3
8	0, 1, 2, or 3	0	0 or 1
9	0, 1, 2, or 3	1	2 or 3
10	0, 1, 2, or 3	2	0 or 1
11	0, 1, 2, or 3	3	2 or 3
12	0, 1, 2, or 3	0	0 or 1
13	0, 1, 2, or 3	1	2 or 3
14	0, 1, 2, or 3	2	0 or 1
15	0, 1, 2, or 3	3	2 or 3

Table 2.1: Assignments of a 16-line main memory to a 4-line cache

# Caches and programs

```
double A[MAX][MAX], x[MAX], y[MAX];  
.  
.  
.  
/* Initialize A and x, assign y = 0 */  
.  
.  
.  
/* First pair of loops */  
for (i = 0; i < MAX; i++)  
    for (j = 0; j < MAX; j++)  
        y[i] += A[i][j]*x[j];  
.  
.  
.  
/* Assign y = 0 */  
.  
.  
.  
/* Second pair of loops */  
for (j = 0; j < MAX; j++)  
    for (i = 0; i < MAX; i++)  
        y[i] += A[i][j]*x[j];
```

Cache Line	Elements of A			
0	A[0][0]	A[0][1]	A[0][2]	A[0][3]
1	A[1][0]	A[1][1]	A[1][2]	A[1][3]
2	A[2][0]	A[2][1]	A[2][2]	A[2][3]
3	A[3][0]	A[3][1]	A[3][2]	A[3][3]

# Virtual memory (1)

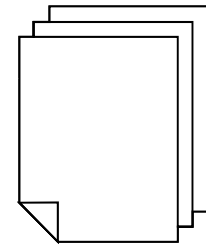
- If we run a very large program or a program that accesses very large data sets, all of the instructions and data may not fit into main memory.
- Virtual memory functions as a cache for secondary storage.

# Virtual memory (2)

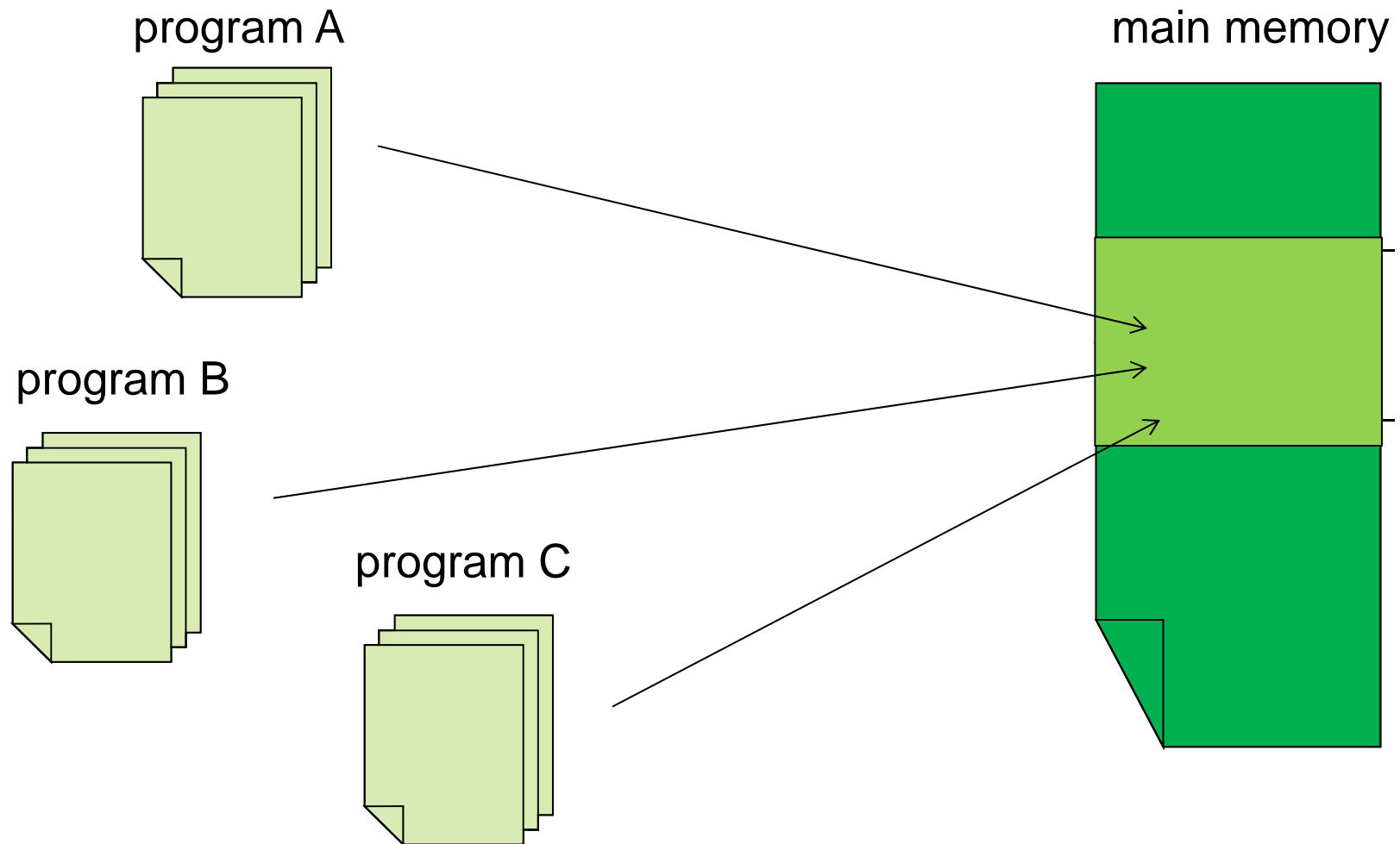
- It exploits the principle of spatial and temporal locality.
- It only keeps the active parts of running programs in main memory.

# Virtual memory (3)

- **Swap space** - those parts that are idle are kept in a block of secondary storage.
- **Pages** – blocks of data and instructions.
  - Usually these are relatively large.
  - Most systems have a fixed page size that currently ranges from 4 to 16 kilobytes.



# Virtual memory (4)



# Virtual page numbers

- When a program is compiled its pages are assigned *virtual* page numbers.
- When the program is run, a table is created that maps the virtual page numbers to physical addresses.
- A **page table** is used to translate the virtual address into a physical address.



# Page table

Virtual Address									
Virtual Page Number					Byte Offset				
31	30	...	13	12	11	10	...	1	0
1	0	...	1	1	0	0	...	1	1

Table 2.2: Virtual Address Divided into Virtual Page Number and Byte Offset

# Translation-lookaside buffer (TLB)

- Using a page table has the potential to significantly increase each program's overall run-time.
- A special address translation cache in the processor.

# Translation-lookaside buffer (2)

- It caches a small number of entries (typically 16–512) from the page table in very fast memory.
- **Page fault** – attempting to access a valid physical address for a page in the page table but the page is only stored on disk.

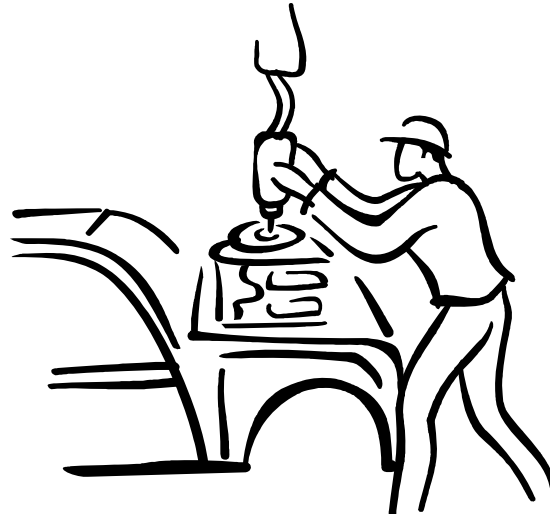
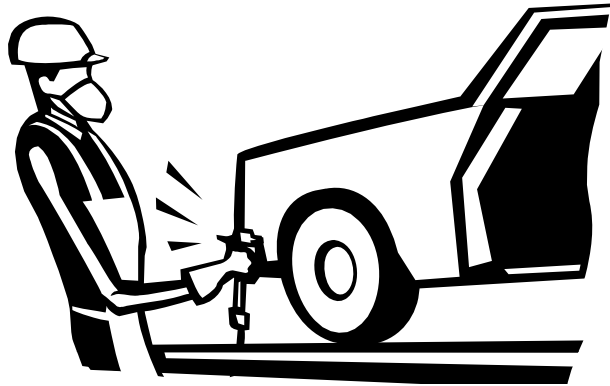
# Instruction Level Parallelism (ILP)

- Attempts to improve processor performance by having multiple processor components or **functional units** simultaneously executing instructions.

# Instruction Level Parallelism (2)

- **Pipelining** - functional units are arranged in stages.
- **Multiple issue** - multiple instructions can be simultaneously initiated.

# Pipelining



# Pipelining example (1)

Time	Operation	Operand 1	Operand 2	Result
1	Fetch operands	$9.87 \times 10^4$	$6.54 \times 10^3$	
2	Compare exponents	$9.87 \times 10^4$	$6.54 \times 10^3$	
3	Shift one operand	$9.87 \times 10^4$	$0.654 \times 10^4$	
4	Add	$9.87 \times 10^4$	$0.654 \times 10^4$	$10.524 \times 10^4$
5	Normalize result	$9.87 \times 10^4$	$0.654 \times 10^4$	$1.0524 \times 10^5$
6	Round result	$9.87 \times 10^4$	$0.654 \times 10^4$	$1.05 \times 10^5$
7	Store result	$9.87 \times 10^4$	$0.654 \times 10^4$	$1.05 \times 10^5$

Add the floating point numbers  
 $9.87 \times 10^4$  and  $6.54 \times 10^3$

# Pipelining example (2)

```
float x[1000], y[1000], z[1000];
```

```
for (i = 0; i < 1000; i++)  
    z[i] = x[i] + y[i];
```

- Assume each operation takes one nanosecond ( $10^{-9}$  seconds).
- This for loop takes about 7000 nanoseconds.



# Pipelining (3)

- Divide the floating point adder into 7 separate pieces of hardware or functional units.
- First unit fetches two operands, second unit compares exponents, etc.
- Output of one functional unit is input to the next.

# Pipelining (4)

Time	Fetch	Compare	Shift	Add	Normalize	Round	Store
0	0						
1	1	0					
2	2	1	0				
3	3	2	1	0			
4	4	3	2	1	0		
5	5	4	3	2	1	0	
6	6	5	4	3	2	1	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
999	999	998	997	996	995	994	993
1000		999	998	997	996	995	994
1001			999	998	997	996	995
1002				999	998	997	996
1003					999	998	997
1004						999	998
1005							999

Table 2.3: Pipelined Addition.

Numbers in the table are subscripts of operands/results.

# Pipelining (5)

- One floating point addition still takes 7 nanoseconds.
- But 1000 floating point additions now takes 1006 nanoseconds!

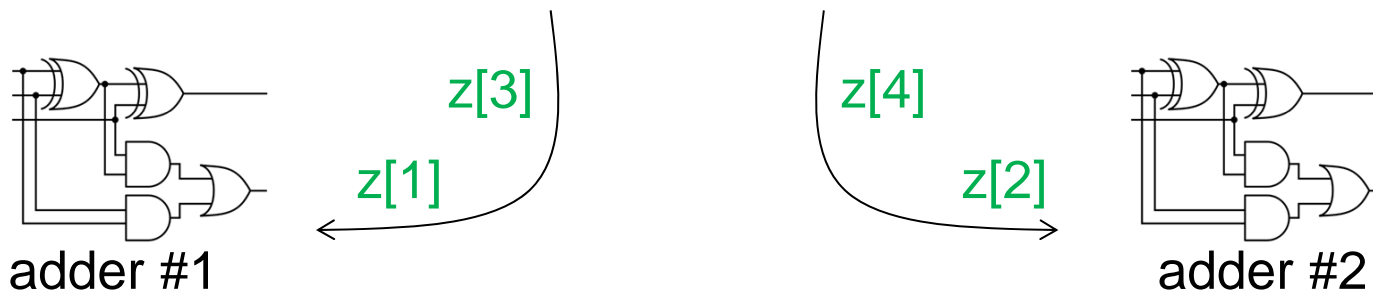


# Multiple Issue (1)

- Multiple issue processors replicate functional units and try to simultaneously execute different instructions in a program.

for (i = 0; i < 1000; i++)

$z[i] = x[i] + y[i];$



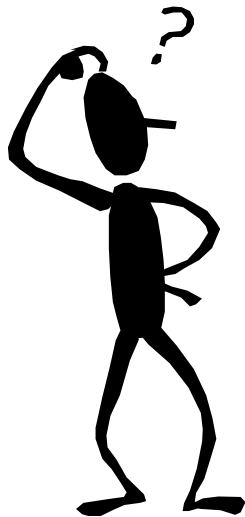
# Multiple Issue (2)

- **static** multiple issue - functional units are scheduled at compile time.
- **dynamic** multiple issue – functional units are scheduled at run-time.

↑  
superscalar

# Speculation (1)

- In order to make use of multiple issue, the system must find instructions that can be executed simultaneously.



- In speculation, the compiler or the processor makes a guess about an instruction, and then executes the instruction on the basis of the guess.

# Speculation (2)

```
z = x + y ;  
if ( z > 0 )  
    w = x ;  
else  
    w = y ;
```



If the system speculates incorrectly,  
it must go back and recalculate  $w = y$ .

# Hardware multithreading (1)

- There aren't always good opportunities for simultaneous execution of different threads.
- Hardware multithreading provides a means for systems to continue doing useful work when the task being currently executed has stalled.
  - Ex., the current task has to wait for data to be loaded from memory.



# Hardware multithreading (2)

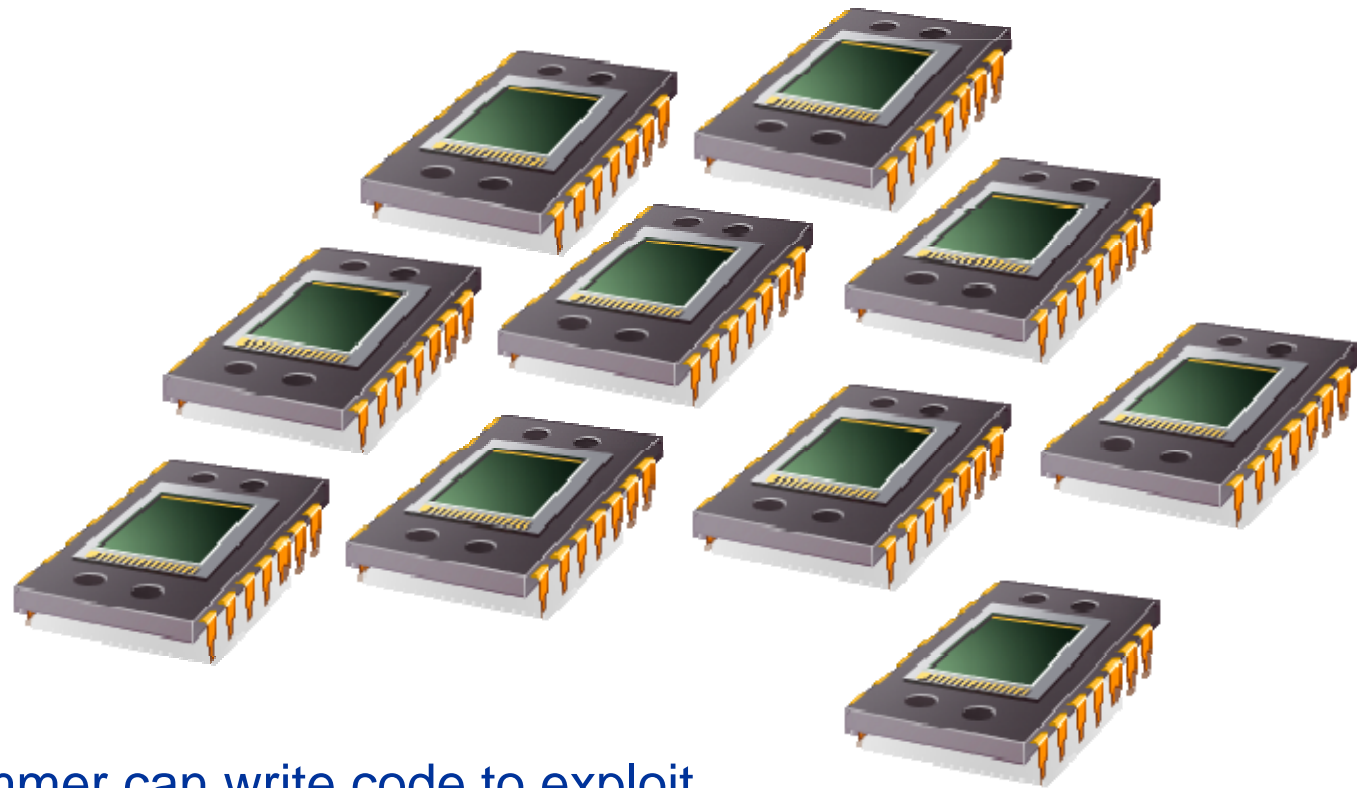
- **Fine-grained** - the processor switches between threads after each instruction, skipping threads that are stalled.
  - Pros: potential to avoid wasted machine time due to stalls.
  - Cons: a thread that's ready to execute a long sequence of instructions may have to wait to execute every instruction.

# Hardware multithreading (3)

- **Coarse-grained** - only switches threads that are stalled waiting for a time-consuming operation to complete.
  - Pros: switching threads doesn't need to be nearly instantaneous.
  - Cons: the processor can be idled on shorter stalls, and thread switching will also cause delays.

# Hardware multithreading (3)

- **Simultaneous multithreading (SMT)** - a variation on fine-grained multithreading.
- Allows multiple threads to make use of the multiple functional units.



A programmer can write code to exploit.

# PARALLEL HARDWARE

# Flynn's Taxonomy

*classic von Neumann*

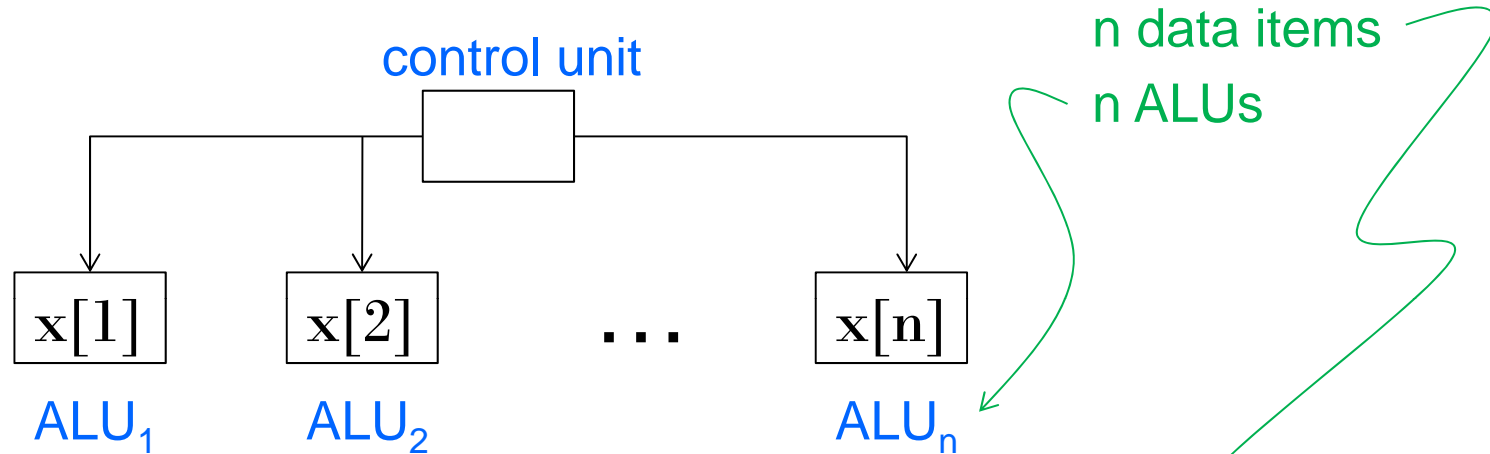
<p><b>SISD</b></p> <p>Single instruction stream Single data stream</p>	<p><b>(SIMD)</b></p> <p>Single instruction stream Multiple data stream</p>
<p><b>MISD</b></p> <p>Multiple instruction stream Single data stream</p>	<p><b>(MIMD)</b></p> <p>Multiple instruction stream Multiple data stream</p>

*not covered*

# SIMD

- Parallelism achieved by dividing data among the processors.
- Applies the same instruction to multiple data items.
- Called data parallelism.

# SIMD example



```
for (i = 0; i < n; i++)  
    x[i] += y[i];
```

# SIMD

- What if we don't have as many ALUs as data items?
- Divide the work and process iteratively.
- Ex.  $m = 4$  ALUs and  $n = 15$  data items.

Round	ALU <sub>1</sub>	ALU <sub>2</sub>	ALU <sub>3</sub>	ALU <sub>4</sub>
1	X[0]	X[1]	X[2]	X[3]
2	X[4]	X[5]	X[6]	X[7]
3	X[8]	X[9]	X[10]	X[11]
4	X[12]	X[13]	X[14]	



# SIMD drawbacks

- All ALUs are required to execute the same instruction, or remain idle.
- In classic design, they must also operate synchronously.
- The ALUs have no instruction storage.
- Efficient for large data parallel problems, but not other types of more complex parallel problems.

# Vector processors (1)

- Operate on arrays or vectors of data while conventional CPU's operate on individual data elements or scalars.
- Vector registers.
  - Capable of storing a vector of operands and operating simultaneously on their contents.

# Vector processors (2)

- Vectorized and pipelined functional units.
  - The same operation is applied to each element in the vector (or pairs of elements).
- Vector instructions.
  - Operate on vectors rather than scalars.

# Vector processors (3)

- Interleaved memory.
  - Multiple “banks” of memory, which can be accessed more or less independently.
  - Distribute elements of a vector across multiple banks, so reduce or eliminate delay in loading/storing successive elements.
- Strided memory access and hardware scatter/gather.
  - The program accesses elements of a vector located at fixed intervals.

# Vector processors - Pros

- Fast.
- Easy to use.
- Vectorizing compilers are good at identifying code to exploit.
- Compilers also can provide information about code that cannot be vectorized.
  - Helps the programmer re-evaluate code.
- High memory bandwidth.
- Uses every item in a cache line.



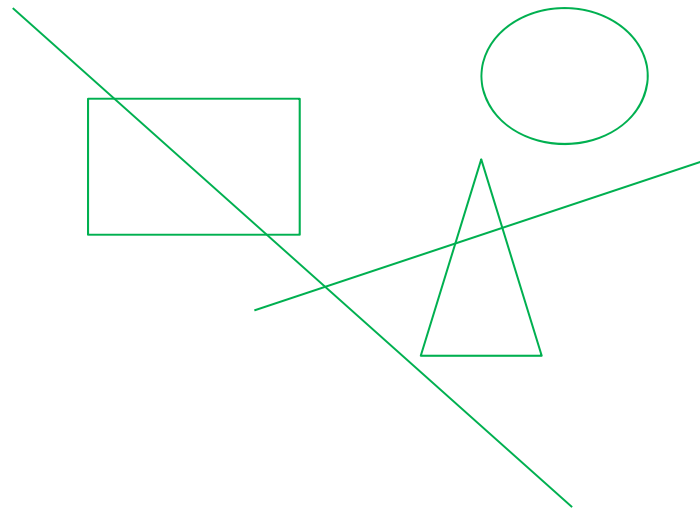
# Vector processors - Cons

- They don't handle irregular data structures as well as other parallel architectures.
- A very finite limit to their ability to handle ever larger problems. (scalability)



# Graphics Processing Units (GPU)

- Real time graphics application programming interfaces or API's use points, lines, and triangles to internally represent the surface of an object.



# GPUs

- A graphics processing pipeline converts the internal representation into an array of pixels that can be sent to a computer screen.
- Several stages of this pipeline (called **shader functions**) are programmable.
  - Typically just a few lines of C code.





# GPUs

- Shader functions are also implicitly parallel, since they can be applied to multiple elements in the graphics stream.
- GPU's can often optimize performance by using SIMD parallelism.
- The current generation of GPU's use SIMD parallelism.
  - Although they are not pure SIMD systems.

# MIMD

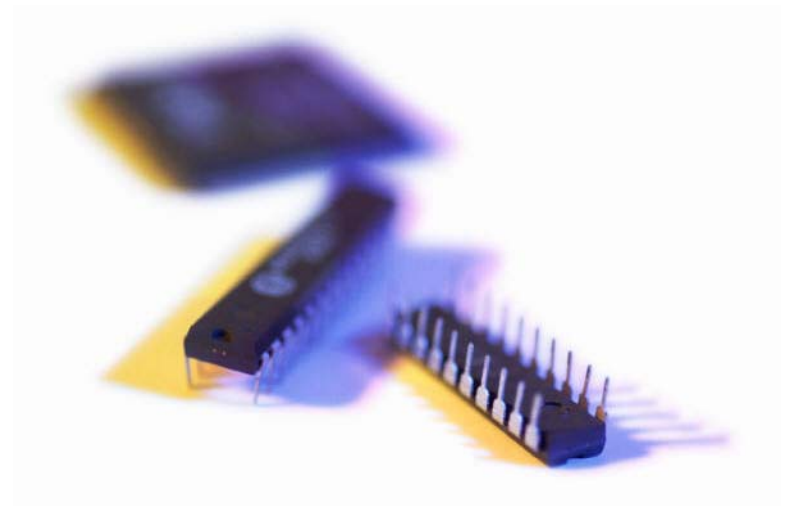
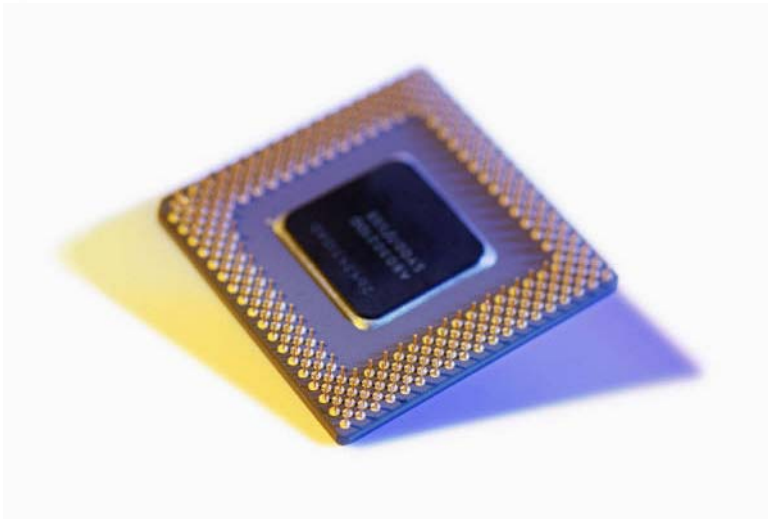
- Supports multiple simultaneous instruction streams operating on multiple data streams.
- Typically consist of a collection of fully independent processing units or cores, each of which has its own control unit and its own ALU.

# Shared Memory System (1)

- A collection of autonomous processors is connected to a memory system via an interconnection network.
- Each processor can access each memory location.
- The processors usually communicate implicitly by accessing shared data structures.

# Shared Memory System (2)

- Most widely available shared memory systems use one or more multicore processors.
  - (multiple CPU's or cores on a single chip)



# Shared Memory System

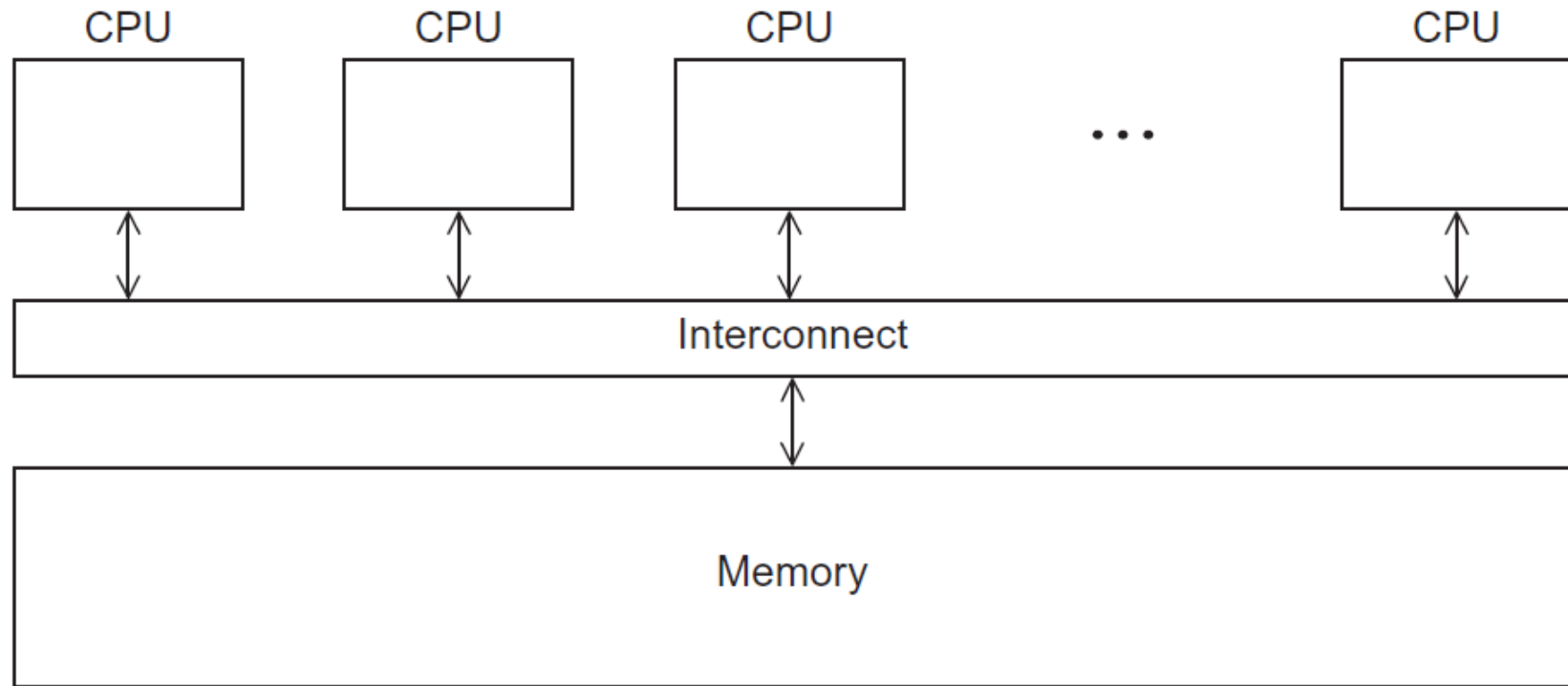
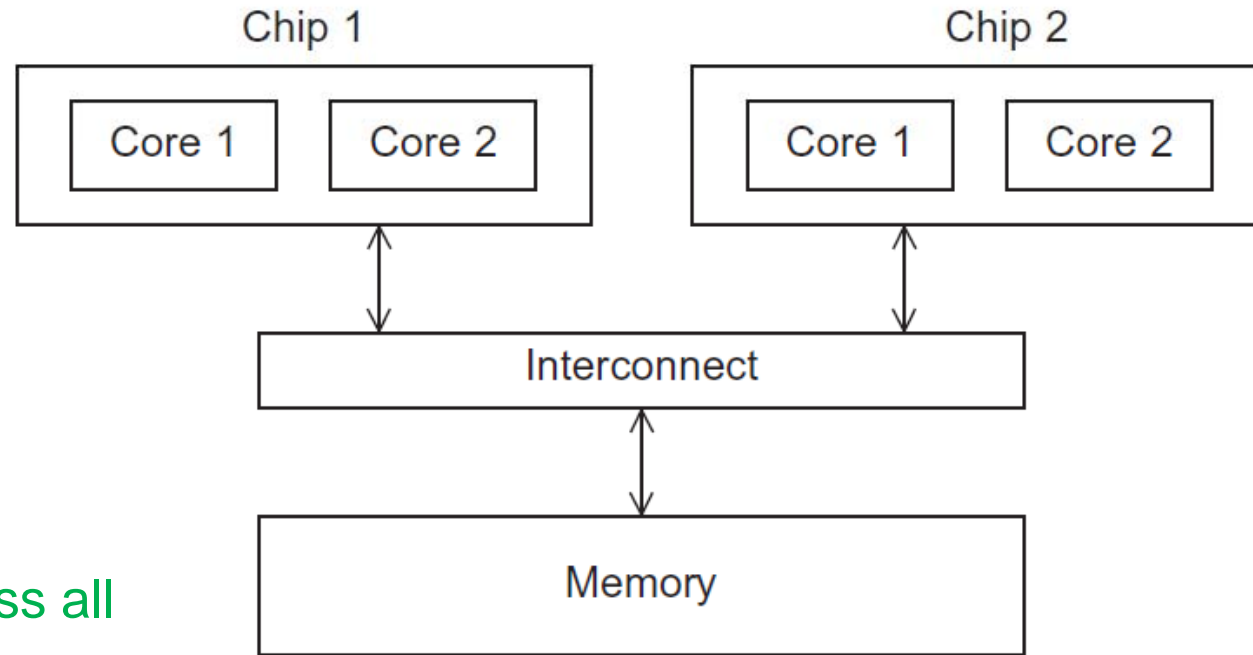


Figure 2.3

# UMA multicore system



Time to access all the memory locations will be the same for all the cores.

Figure 2.5

# NUMA multicore system

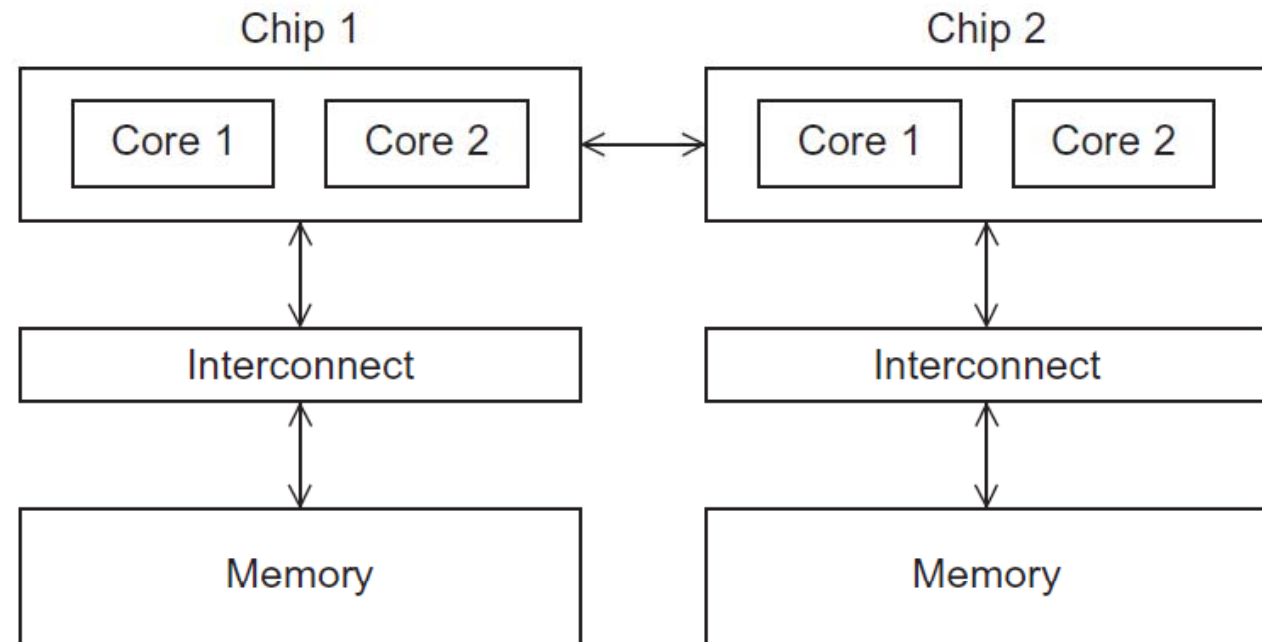


Figure 2.6

A memory location a core is directly connected to can be accessed faster than a memory location that must be accessed through another chip.

# Distributed Memory System

- Clusters (most popular)
  - A collection of commodity systems.
  - Connected by a commodity interconnection network.
- **Nodes** of a cluster are individual computations units joined by a communication network.

*a.k.a. hybrid systems*



# Distributed Memory System

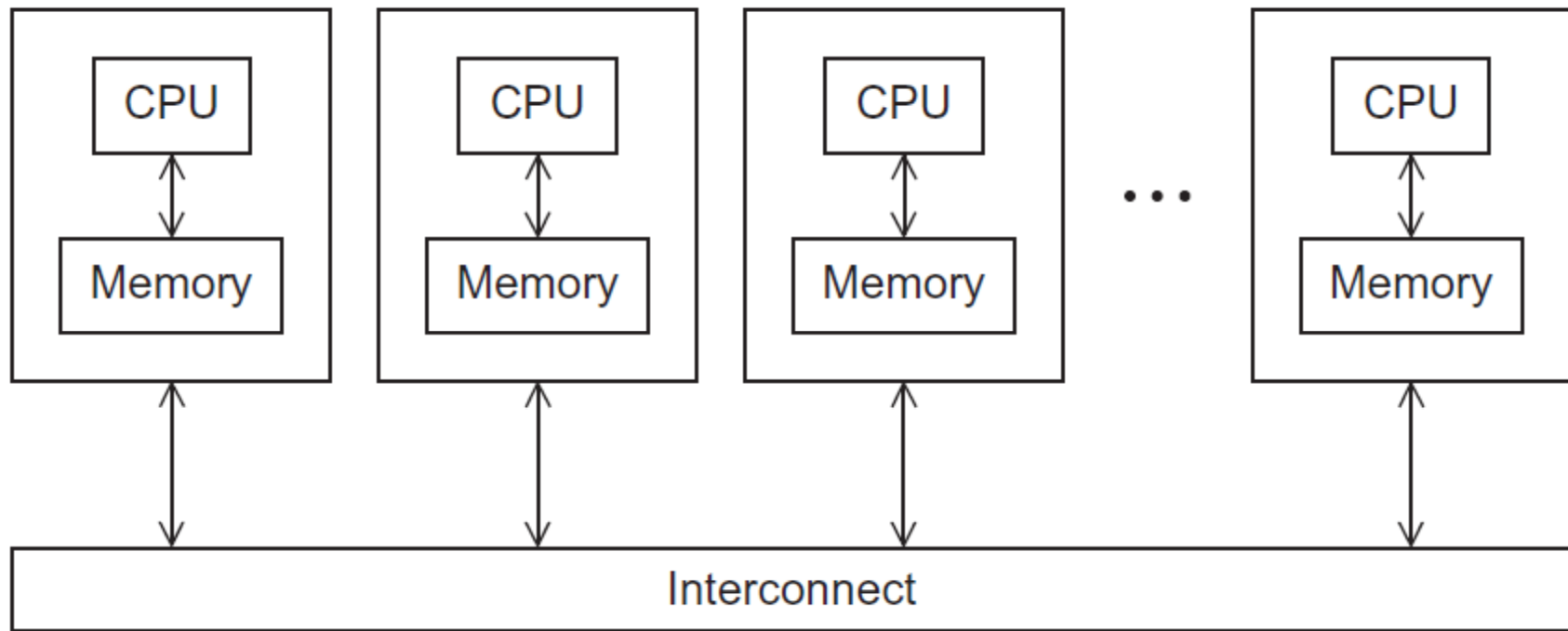


Figure 2.4

# Interconnection networks

- Affects performance of both distributed and shared memory systems.
- Two categories:
  - Shared memory interconnects
  - Distributed memory interconnects

# Shared memory interconnects

- Bus interconnect
  - A collection of parallel communication wires together with some hardware that controls access to the bus.
  - Communication wires are shared by the devices that are connected to it.
  - As the number of devices connected to the bus increases, contention for use of the bus increases, and performance decreases.

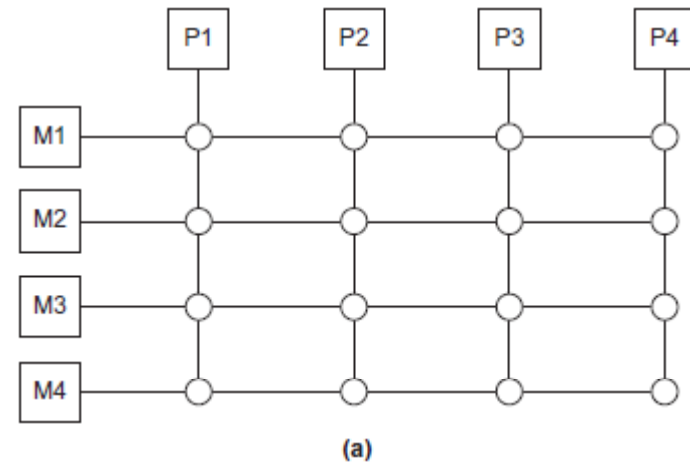
# Shared memory interconnects

- Switched interconnect
  - Uses switches to control the routing of data among the connected devices.
- Crossbar –
  - Allows simultaneous communication among different devices.
  - Faster than buses.
  - But the cost of the switches and links is relatively high.

Figure 2.7

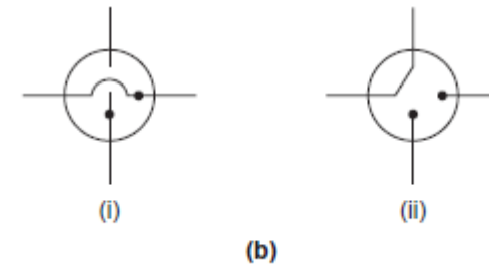
(a)

A crossbar switch connecting 4 processors ( $P_i$ ) and 4 memory modules ( $M_j$ )

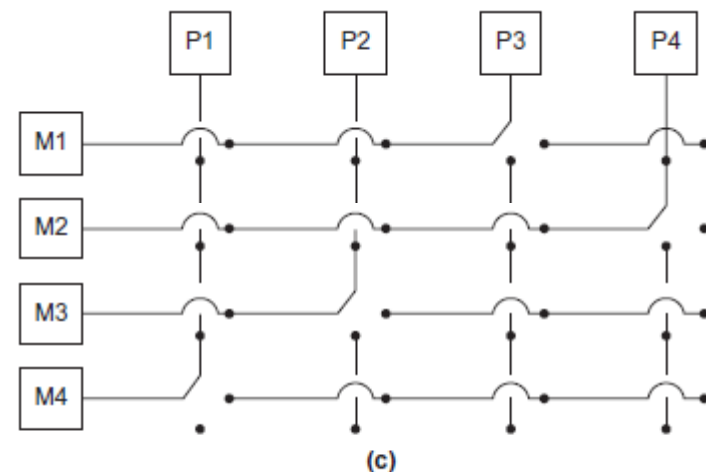


(b)

Configuration of internal switches in a crossbar



(c) Simultaneous memory accesses by the processors

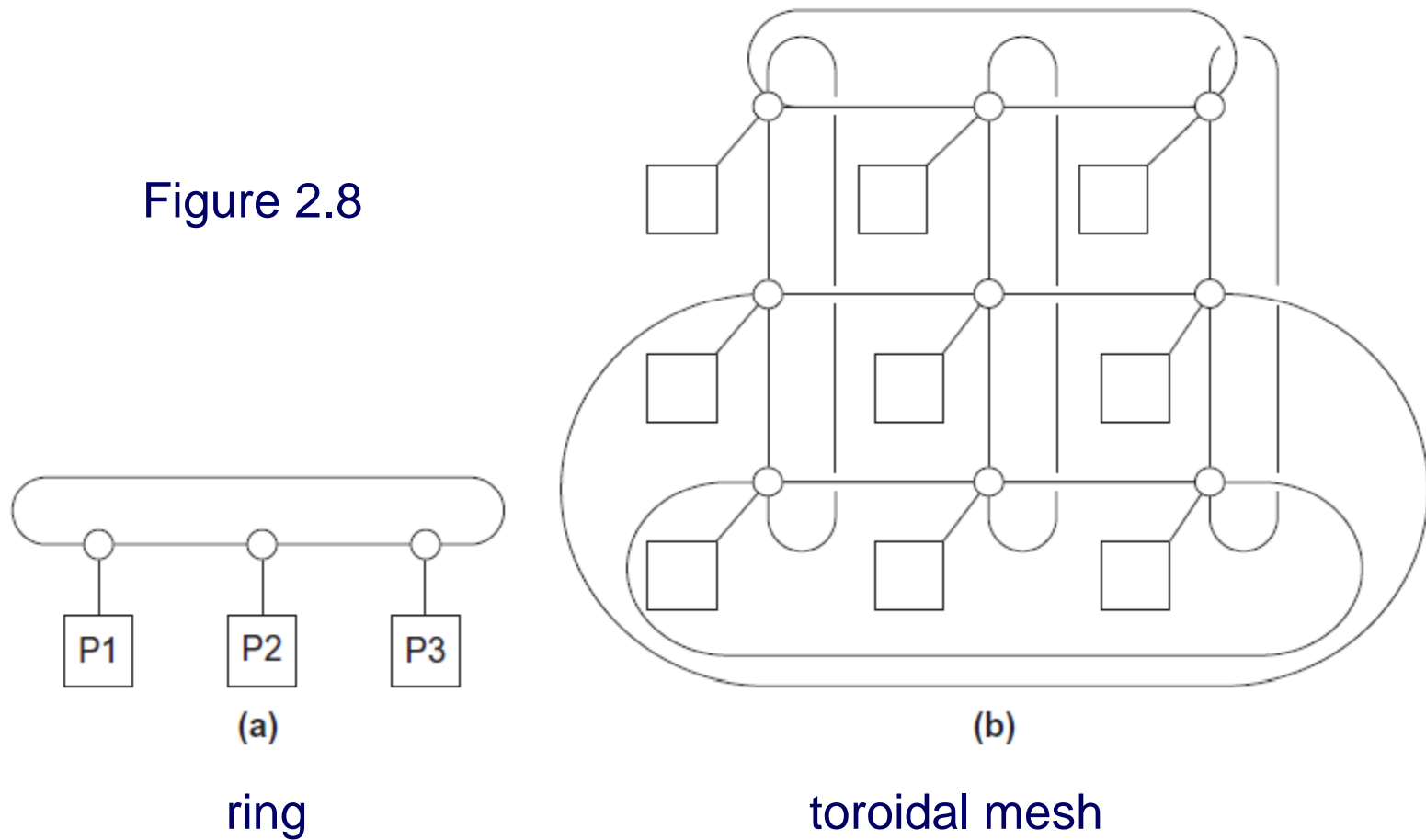


# Distributed memory interconnects

- Two groups
  - Direct interconnect
    - Each switch is directly connected to a processor memory pair, and the switches are connected to each other.
  - Indirect interconnect
    - Switches may not be directly connected to a processor.

# Direct interconnect

Figure 2.8



# Bisection width

- A measure of “number of simultaneous communications” or “connectivity”.
- How many simultaneous communications can take place “across the divide” between the halves?





# Two bisections of a ring

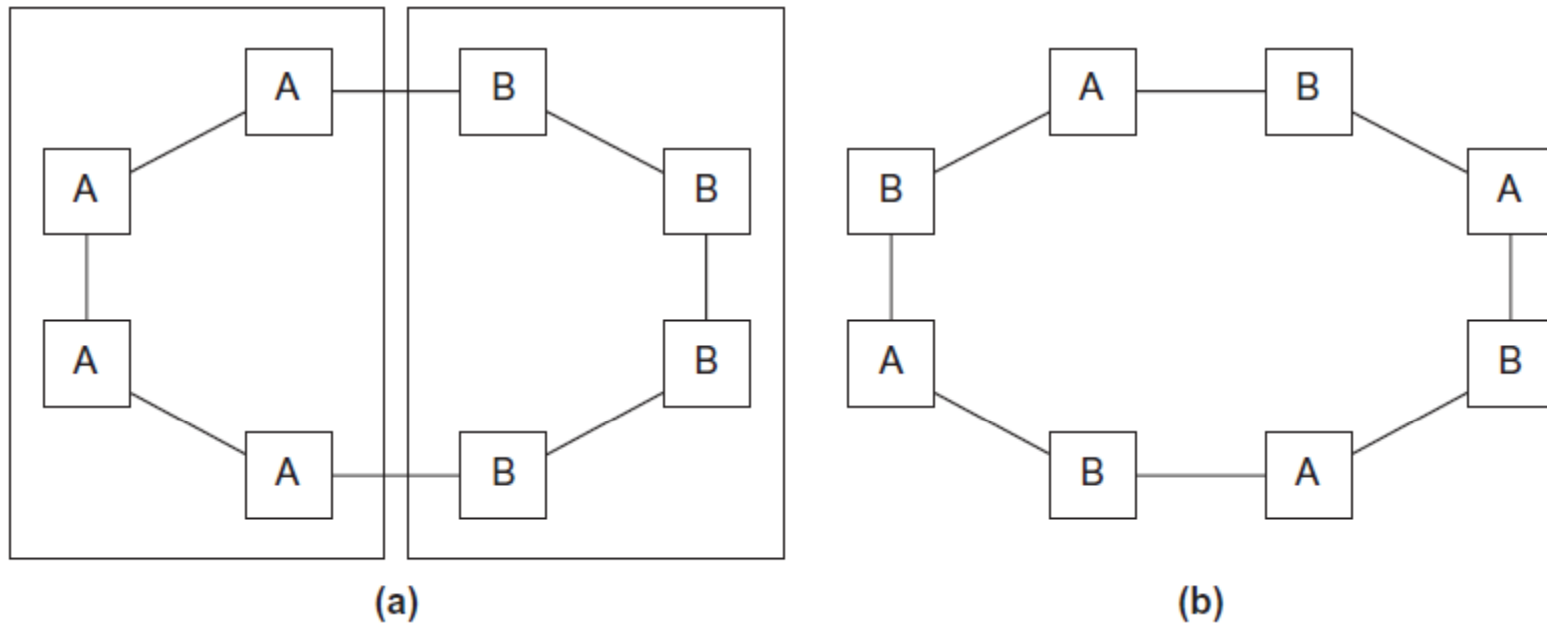


Figure 2.9

# A bisection of a toroidal mesh

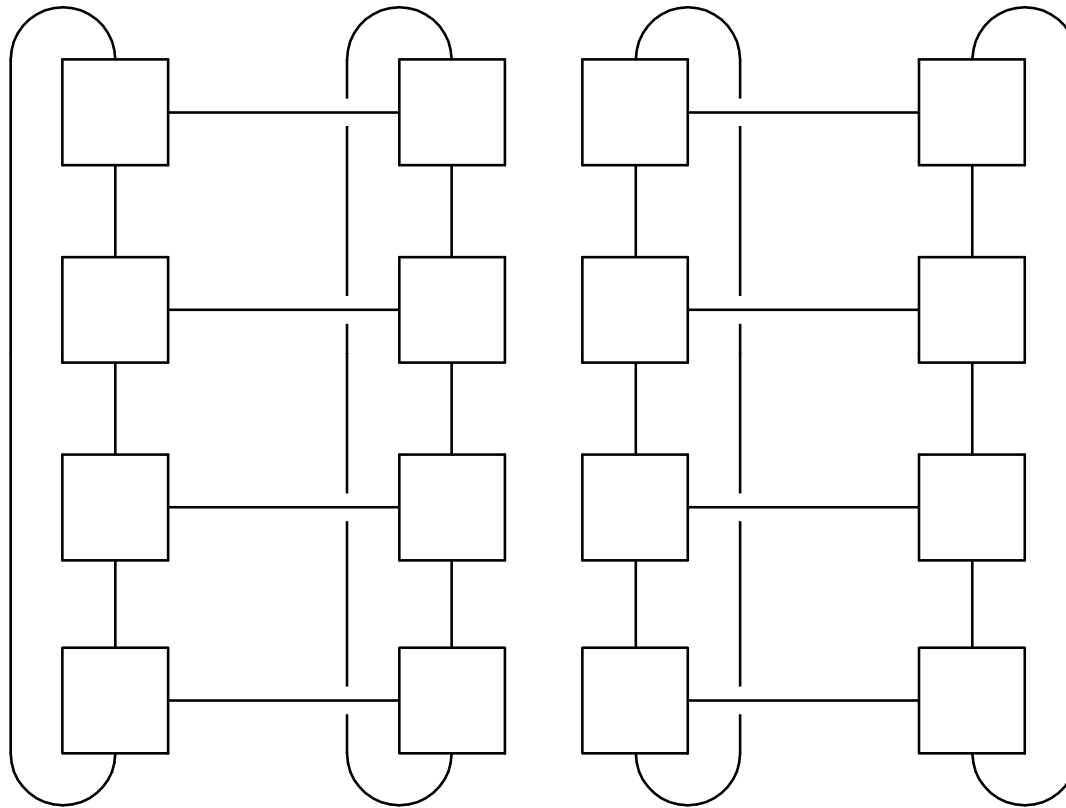


Figure 2.10

# Definitions

- Bandwidth
  - The rate at which a link can transmit data.
  - Usually given in megabits or megabytes per second.
- Bisection bandwidth
  - A measure of network quality.
  - Instead of counting the number of links joining the halves, it sums the bandwidth of the links.

# Fully connected network

- Each switch is directly connected to every other switch.

*impractical*

bisection width =  $p^2/4$

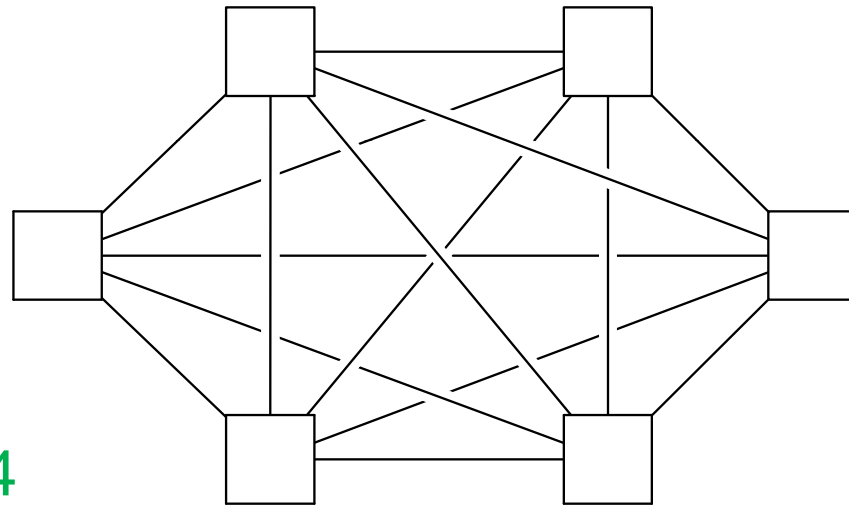


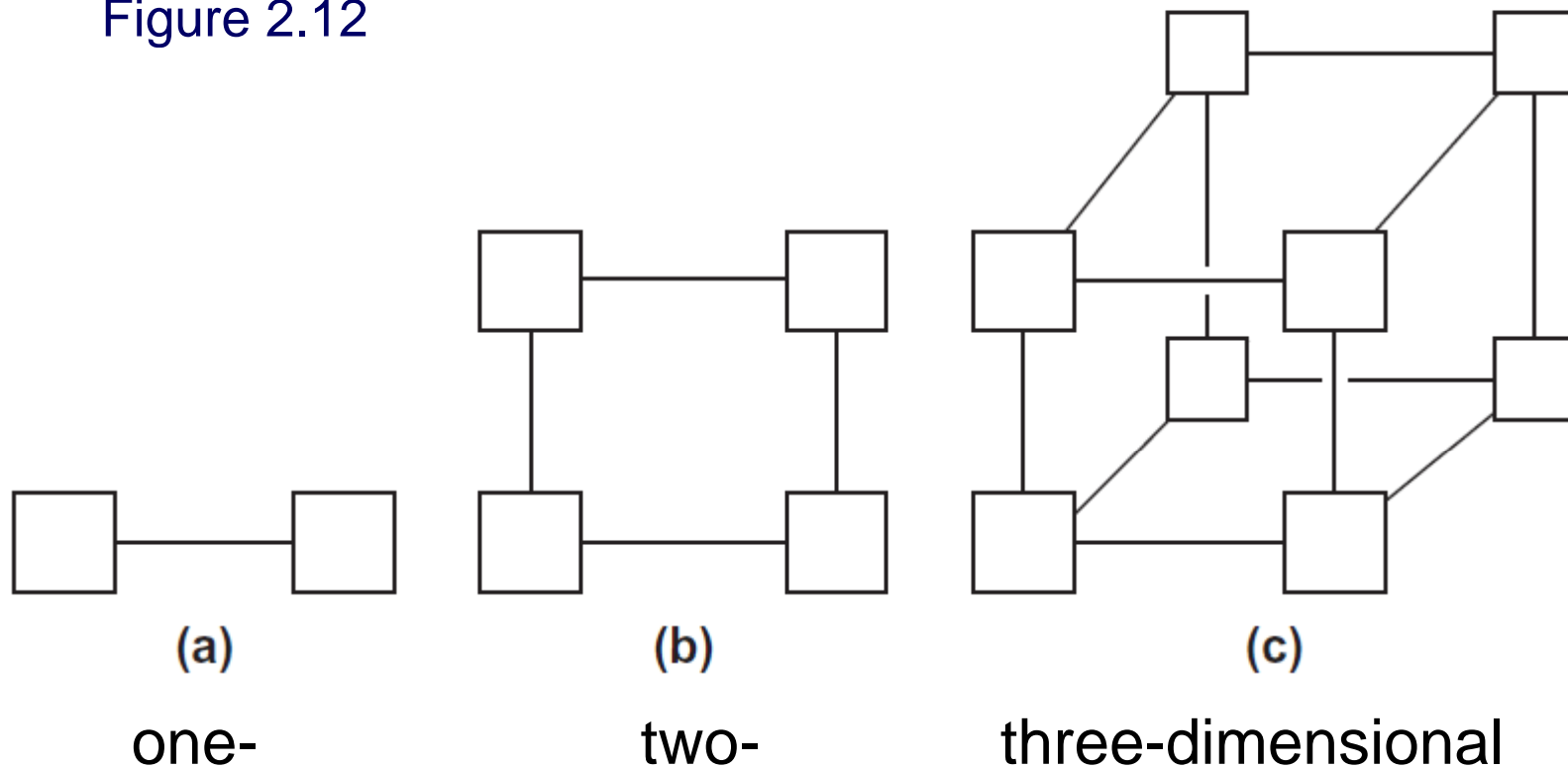
Figure 2.11

# Hypercube

- Highly connected direct interconnect.
- Built inductively:
  - A one-dimensional hypercube is a fully-connected system with two processors.
  - A two-dimensional hypercube is built from two one-dimensional hypercubes by joining “corresponding” switches.
  - Similarly a three-dimensional hypercube is built from two two-dimensional hypercubes.

# Hypercubes

Figure 2.12



# Indirect interconnects

- Simple examples of indirect networks:
  - Crossbar
  - Omega network
- Often shown with unidirectional links and a collection of processors, each of which has an outgoing and an incoming link, and a switching network.

# A generic indirect network

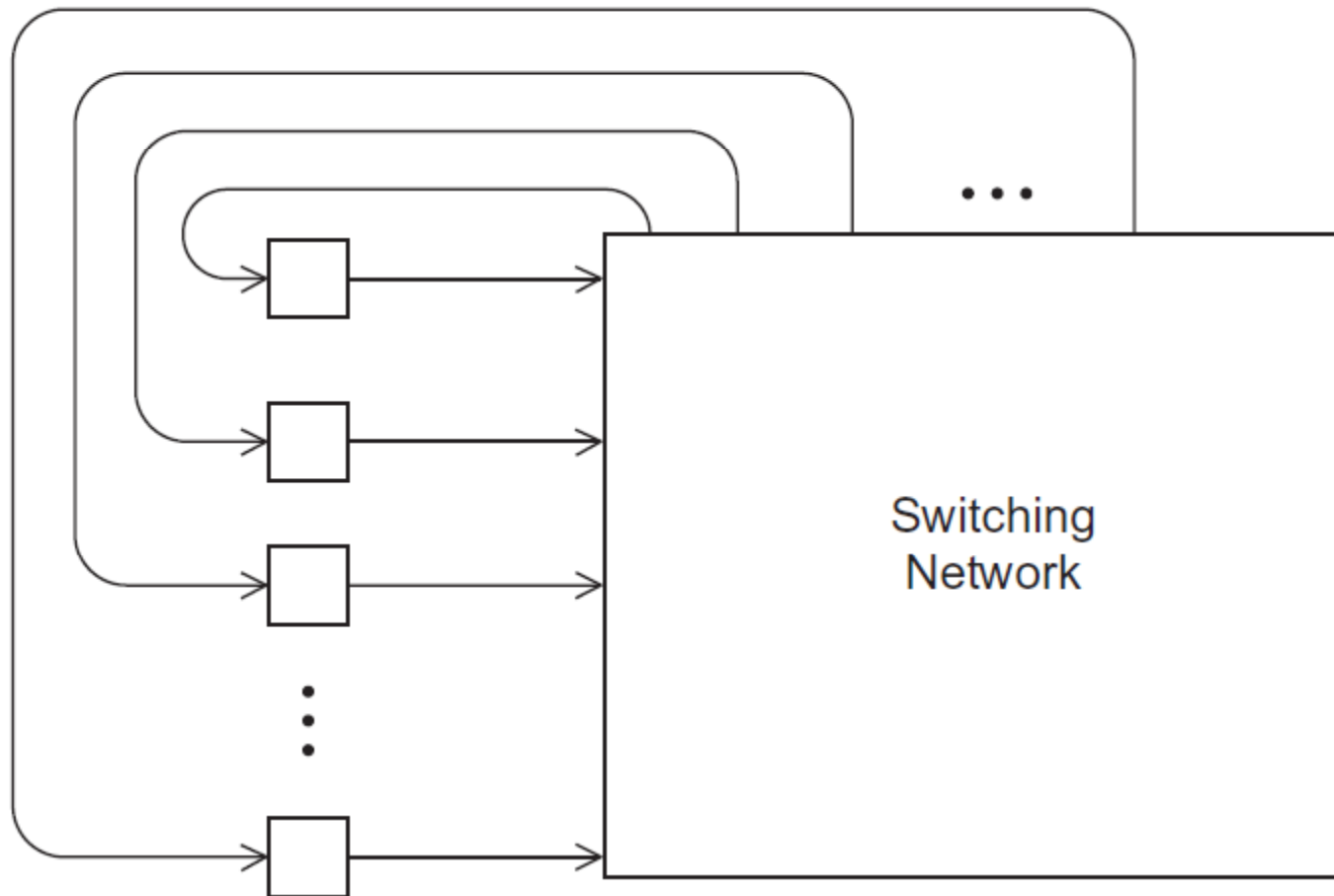


Figure 2.13



# Crossbar interconnect for distributed memory

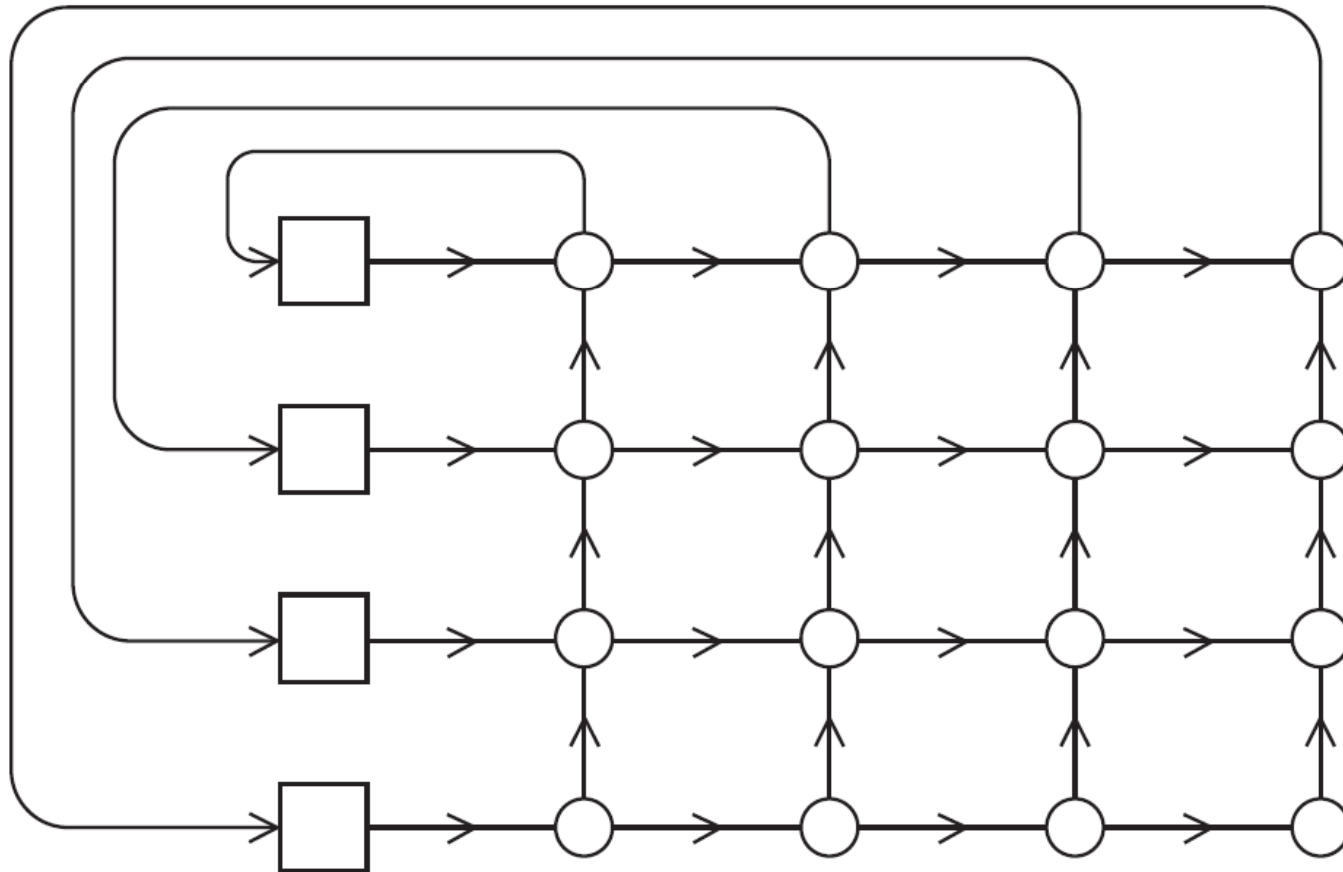


Figure 2.14

# An omega network

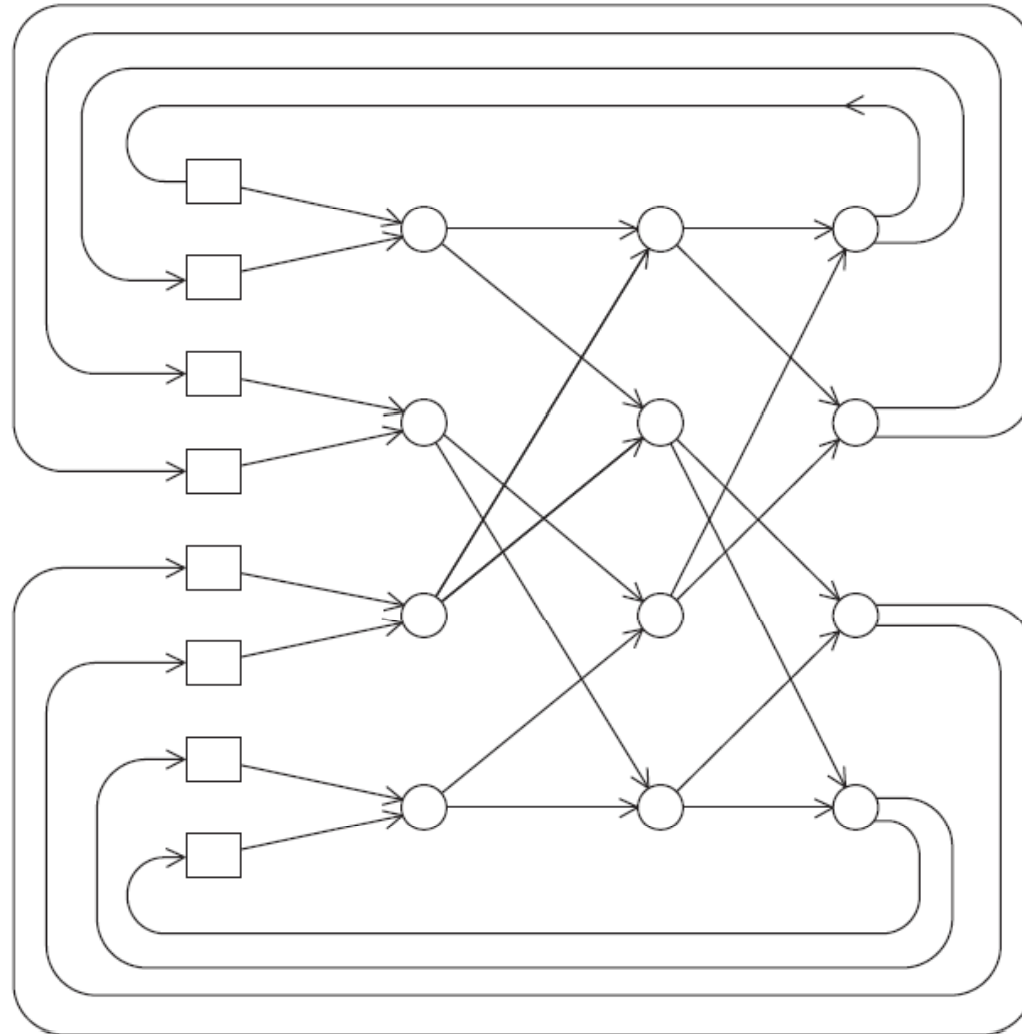


Figure 2.15

# A switch in an omega network

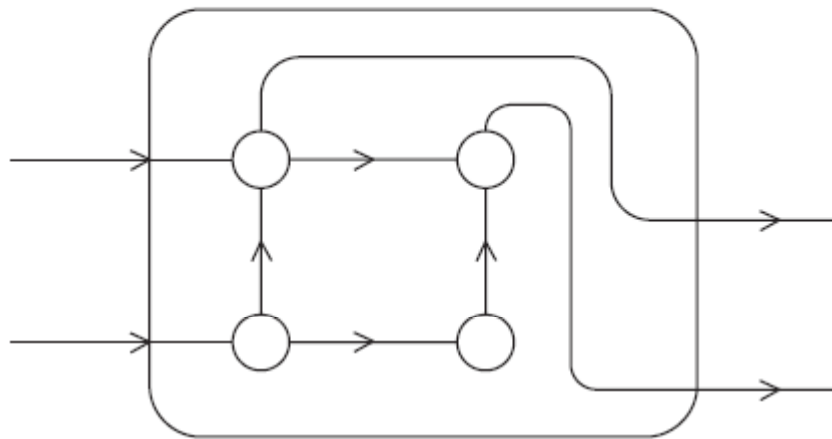


Figure 2.16

# More definitions

- Any time data is transmitted, we're interested in how long it will take for the data to reach its destination.
- **Latency**
  - The time that elapses between the source's beginning to transmit the data and the destination's starting to receive the first byte.
- **Bandwidth**
  - The rate at which the destination receives data after it has started to receive the first byte.

Message transmission time =  $l + n / b$

latency (seconds)

length of message (bytes)

bandwidth (bytes per second)

# Cache coherence

- Programmers have no control over caches and when they get updated.

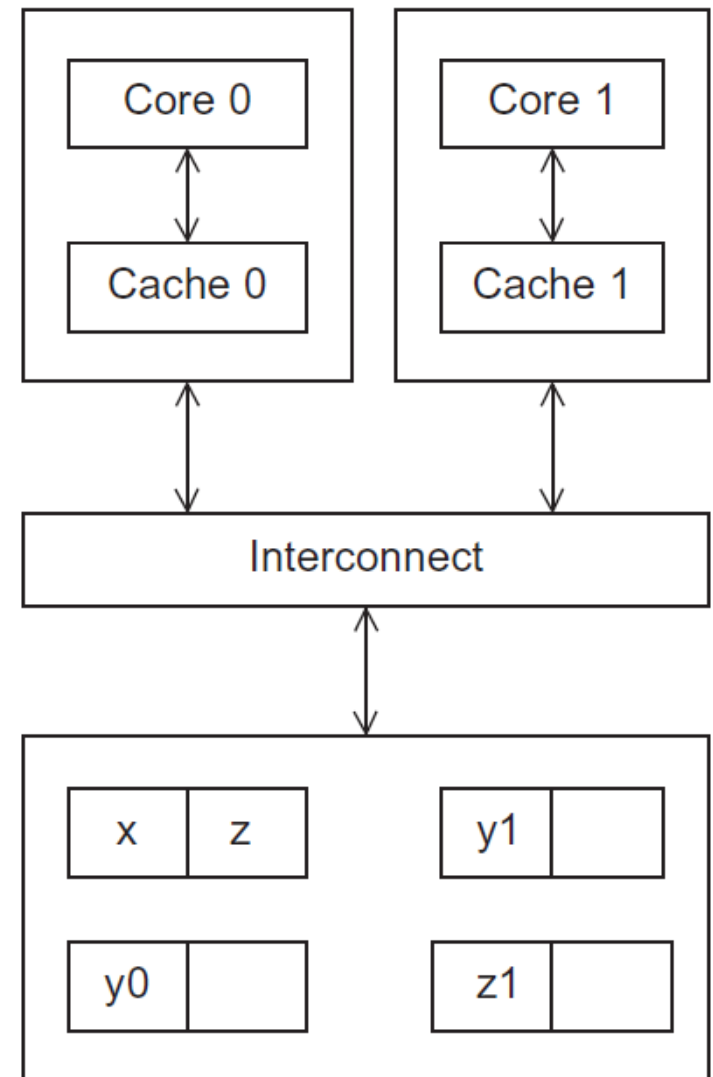


Figure 2.17

A shared memory system with two cores and two caches

# Cache coherence

y0 privately owned by Core 0

y1 and z1 privately owned by Core 1

x = 2; /\* shared variable \*/

Time	Core 0	Core 1
0	y0 = x;	y1 = 3*x;
1	x = 7;	Statement(s) not involving x
2	Statement(s) not involving x	z1 = 4*x;

y0 eventually ends up = 2

y1 eventually ends up = 6

z1 = ???

# Snooping Cache Coherence

- The cores share a bus .
- Any signal transmitted on the bus can be “seen” by all cores connected to the bus.
- When core 0 updates the copy of x stored in its cache it also broadcasts this information across the bus.
- If core 1 is “snooping” the bus, it will see that x has been updated and it can mark its copy of x as invalid.



# Directory Based Cache Coherence

- Uses a data structure called a **directory** that stores the status of each cache line.
- When a variable is updated, the directory is consulted, and the cache controllers of the cores that have that variable's cache line in their caches are invalidated.



# PARALLEL SOFTWARE

# The burden is on software

- Hardware and compilers can keep up the pace needed.
- From now on...
  - In shared memory programs:
    - Start a single process and fork threads.
    - Threads carry out tasks.
  - In distributed memory programs:
    - Start multiple processes.
    - Processes carry out tasks.

# SPMD – single program multiple data

- A SPMD programs consists of a single executable that can behave as if it were multiple different programs through the use of conditional branches.

```
if (I'm thread process i)
    do this;
else
    do that;
```



# Writing Parallel Programs

1. Divide the work among the processes/threads
  - (a) so each process/thread gets roughly the same amount of work
  - (b) and communication is minimized.

```
double x[n], y[n];  
...  
for (i = 0; i < n; i++)  
    x[i] += y[i];
```

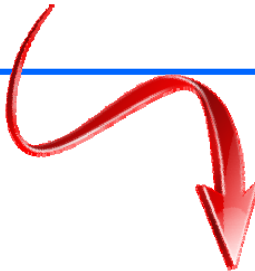
2. Arrange for the processes/threads to synchronize.
3. Arrange for communication among processes/threads.

# Shared Memory

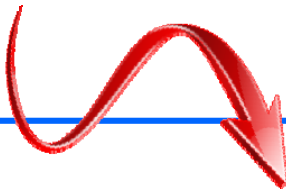
- Dynamic threads
  - Master thread waits for work, forks new threads, and when threads are done, they terminate
  - Efficient use of resources, but thread creation and termination is time consuming.
- Static threads
  - Pool of threads created and are allocated work, but do not terminate until cleanup.
  - Better performance, but potential waste of system resources.

# Nondeterminism

```
...  
printf ( "Thread %d > my_val = %d\n" ,  
        my_rank , my_x ) ;  
...
```



Thread 1 > my\_val = 19  
Thread 0 > my\_val = 7



Thread 0 > my\_val = 7  
Thread 1 > my\_val = 19

# Nondeterminism

```
my_val = Compute_val ( my_rank ) ;  
x += my_val ;
```

Time	Core 0	Core 1
0	Finish assignment to my_val	In call to Compute_val
1	Load x = 0 into register	Finish assignment to my_val
2	Load my_val = 7 into register	Load x = 0 into register
3	Add my_val = 7 to x	Load my_val = 19 into register
4	Store x = 7	Add my_val to x
5	Start other work	Store x = 19



# Nondeterminism

- Race condition
- Critical section
- Mutually exclusive
- Mutual exclusion lock (mutex, or simply lock)

```
my_val = Compute_val ( my_rank ) ;  
Lock(&add_my_val_lock ) ;  
x += my_val ;  
Unlock(&add_my_val_lock ) ;
```

# busy-waiting

```
my_val = Compute_val ( my_rank ) ;  
i f ( my_rank == 1 )  
    w h i l e ( ! ok_for_1 ) ; /* Busy-wait loop */  
x += my_val ; /* Critical section */  
i f ( my_rank == 0 )  
    ok_for_1 = true ; /* Let thread 1 update x */
```

# message-passing

```
char message [ 1 0 0 ] ;
```

```
...
```

```
my_rank = Get_rank ( ) ;
```

```
i f ( my_rank == 1) {
```

```
    sprintf ( message , "Greetings from process 1" ) ;
```

```
    Send ( message , MSG_CHAR , 100 , 0 ) ;
```

```
} e l s e i f ( my_rank == 0) {
```

```
    Receive ( message , MSG_CHAR , 100 , 1 ) ;
```

```
    printf ( "Process 0 > Received: %s\n" , message ) ;
```

```
}
```

# Partitioned Global Address Space Languages

```
shared i n t n = . . . ;
shared double x [ n ] , y [ n ] ;
private i n t i , my_first_element , my_last_element ;
my_first_element = . . . ;
my_last_element = . . . ;
/* Initialize x and y */
. . .
f o r ( i = my_first_element ; i <= my_last_element ; i++)
    x [ i ] += y [ i ] ;
```

# Input and Output

- In distributed memory programs, only process 0 will access *stdin*. In shared memory programs, only the master thread or thread 0 will access *stdin*.
- In both distributed memory and shared memory programs all the processes/threads can access *stdout* and *stderr*.

# Input and Output

- However, because of the indeterminacy of the order of output to *stdout*, in most cases only a single process/thread will be used for all output to *stdout* other than debugging output.
- Debug output should always include the rank or id of the process/thread that's generating the output.

# Input and Output

- Only a single process/thread will attempt to access any single file other than *stdin*, *stdout*, or *stderr*. So, for example, each process/thread can open its own, private file for reading or writing, but no two processes/threads will open the same file.



# PERFORMANCE



# Speedup

- Number of cores =  $p$
- Serial run-time =  $T_{\text{serial}}$
- Parallel run-time =  $T_{\text{parallel}}$



linear speedup

$$T_{\text{parallel}} = T_{\text{serial}} / p$$

# Speedup of a parallel program

$$S = \frac{T_{\text{serial}}}{T_{\text{parallel}}}$$

# Efficiency of a parallel program

$$E = \frac{S}{p} = \frac{\left( \frac{T_{\text{serial}}}{T_{\text{parallel}}} \right)}{p} = \frac{T_{\text{serial}}}{p \cdot T_{\text{parallel}}}$$

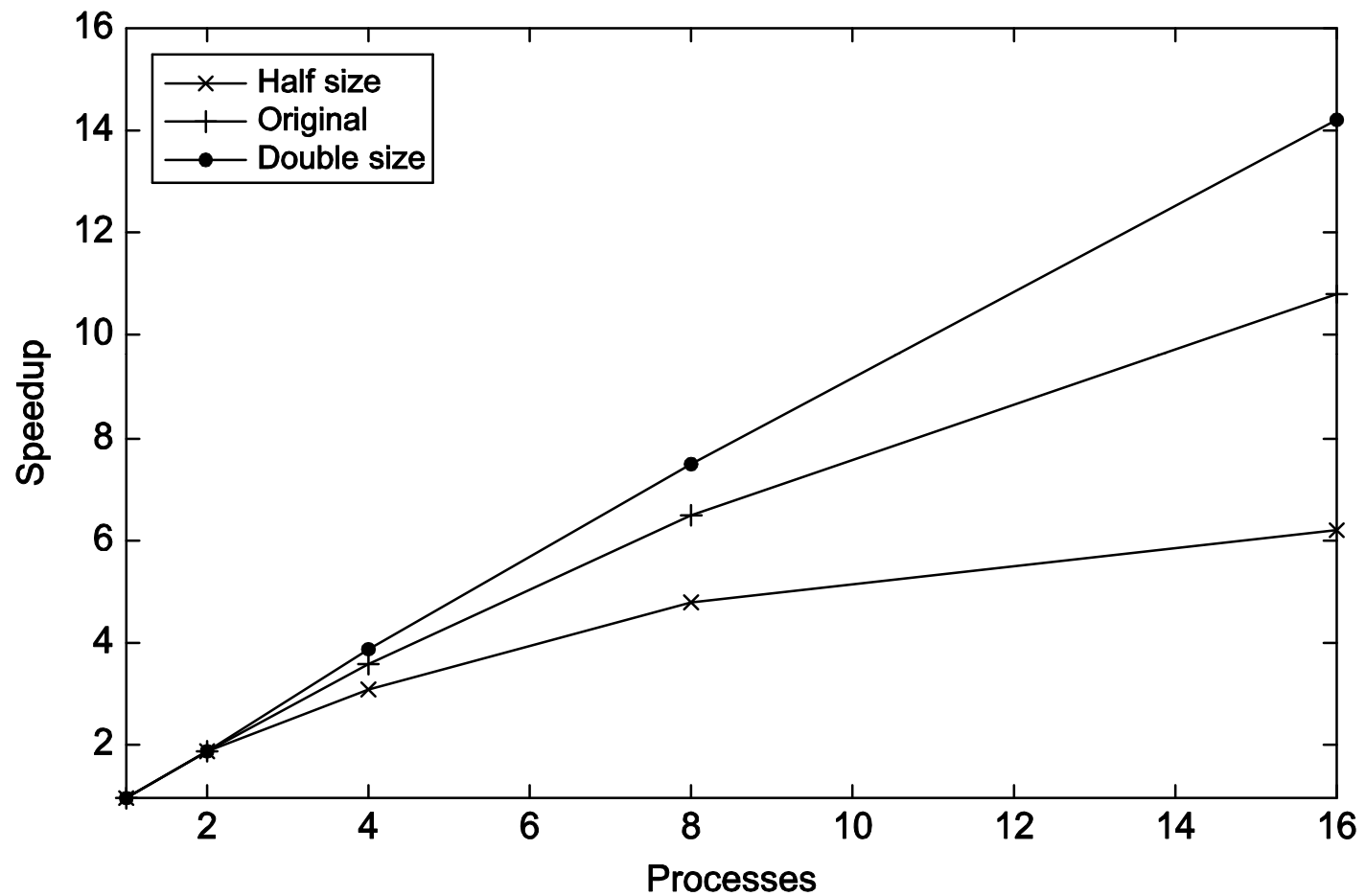
# Speedups and efficiencies of a parallel program

$p$	1	2	4	8	16
$S$	1.0	1.9	3.6	6.5	10.8
$E = S/p$	1.0	0.95	0.90	0.81	0.68

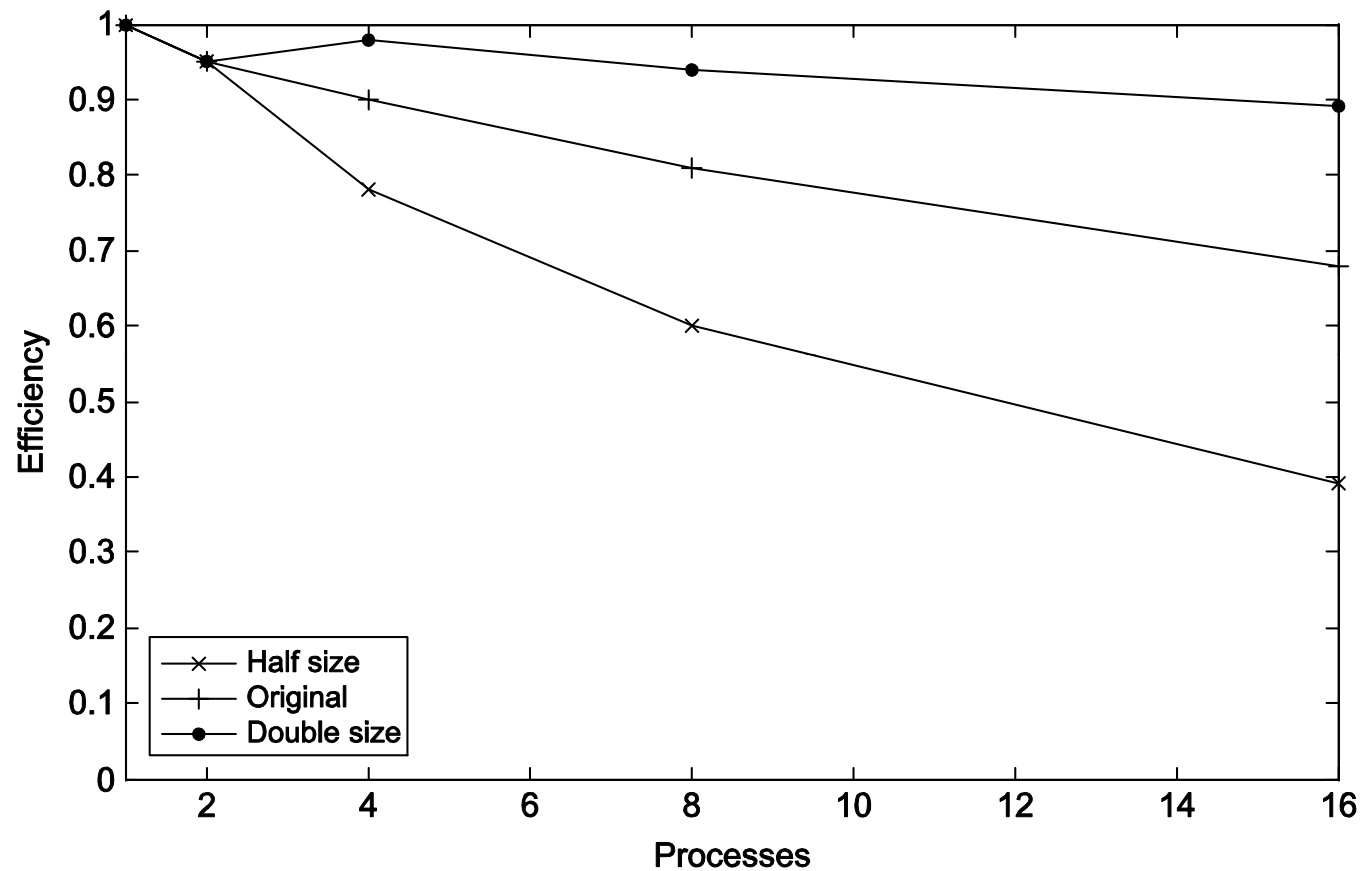
# Speedups and efficiencies of parallel program on different problem sizes

	$p$	1	2	4	8	16
Half	$S$	1.0	1.9	3.1	4.8	6.2
	$E$	1.0	0.95	0.78	0.60	0.39
Original	$S$	1.0	1.9	3.6	6.5	10.8
	$E$	1.0	0.95	0.90	0.81	0.68
Double	$S$	1.0	1.9	3.9	7.5	14.2
	$E$	1.0	0.95	0.98	0.94	0.89

# Speedup



# Efficiency



# Effect of overhead

$$T_{\text{parallel}} = T_{\text{serial}} / p + T_{\text{overhead}}$$



# Amdahl's Law

- Unless virtually all of a serial program is parallelized, the possible speedup is going to be very limited — regardless of the number of cores available.



# Example

- We can parallelize 90% of a serial program.
- Parallelization is “perfect” regardless of the number of cores  $p$  we use.
- $T_{\text{serial}} = 20$  seconds
- Runtime of parallelizable part is

$$0.9 \times T_{\text{serial}} / p = 18 / p$$

## Example (cont.)

- Runtime of “unparallelizable” part is

$$0.1 \times T_{\text{serial}} = 2$$

- Overall parallel run-time is

$$T_{\text{parallel}} = 0.9 \times T_{\text{serial}} / p + 0.1 \times T_{\text{serial}} = 18 / p + 2$$

## Example (cont.)

- Speed up

$$S = \frac{T_{\text{serial}}}{0.9 \times T_{\text{serial}} / p + 0.1 \times T_{\text{serial}}} = \frac{20}{18 / p + 2}$$

# Scalability

- In general, a problem is *scalable* if it can handle ever increasing problem sizes.
- If we increase the number of processes/threads and keep the efficiency fixed without increasing problem size, the problem is *strongly scalable*.
- If we keep the efficiency fixed by increasing the problem size at the same rate as we increase the number of processes/threads, the problem is *weakly scalable*.

# Taking Timings

- What is time?
- Start to finish?
- A program segment of interest?
- CPU time?
- Wall clock time?



# Taking Timings

```
double start, finish;  
....  
start = Get_current_time();  
/* Code that we want to time */  
....  
finish = Get_current_time();  
printf("The elapsed time = %e seconds\n", finish-start);
```

theoretical  
function



MPI\_Wtime



omp\_get\_wtime



# Taking Timings

```
private double start, finish;  
...  
start = Get_current_time();  
/* Code that we want to time */  
...  
finish = Get_current_time();  
printf("The elapsed time = %e seconds\n", finish-start);
```



# Taking Timings

```
shared double global_elapsed;  
private double my_start, my_finish, my_elapsed;  
.  
.  
.  
/* Synchronize all processes/threads */  
Barrier();  
my_start = Get_current_time();  
  
/* Code that we want to time */  
.  
.  
.  
  
my_finish = Get_current_time();  
my_elapsed = my_finish - my_start;  
  
/* Find the max across all processes/threads */  
global_elapsed = Global_max(my_elapsed);  
if (my_rank == 0)  
    printf("The elapsed time = %e seconds\n", global_elapsed);
```



# PARALLEL PROGRAM DESIGN

# Foster's methodology

1. **Partitioning**: divide the computation to be performed and the data operated on by the computation into small tasks.

The focus here should be on identifying tasks that can be executed in parallel.

# Foster's methodology

2. **Communication**: determine what communication needs to be carried out among the tasks identified in the previous step.



# Foster's methodology

3. **Agglomeration or aggregation:** combine tasks and communications identified in the first step into larger tasks.

For example, if task A must be executed before task B can be executed, it may make sense to aggregate them into a single composite task.

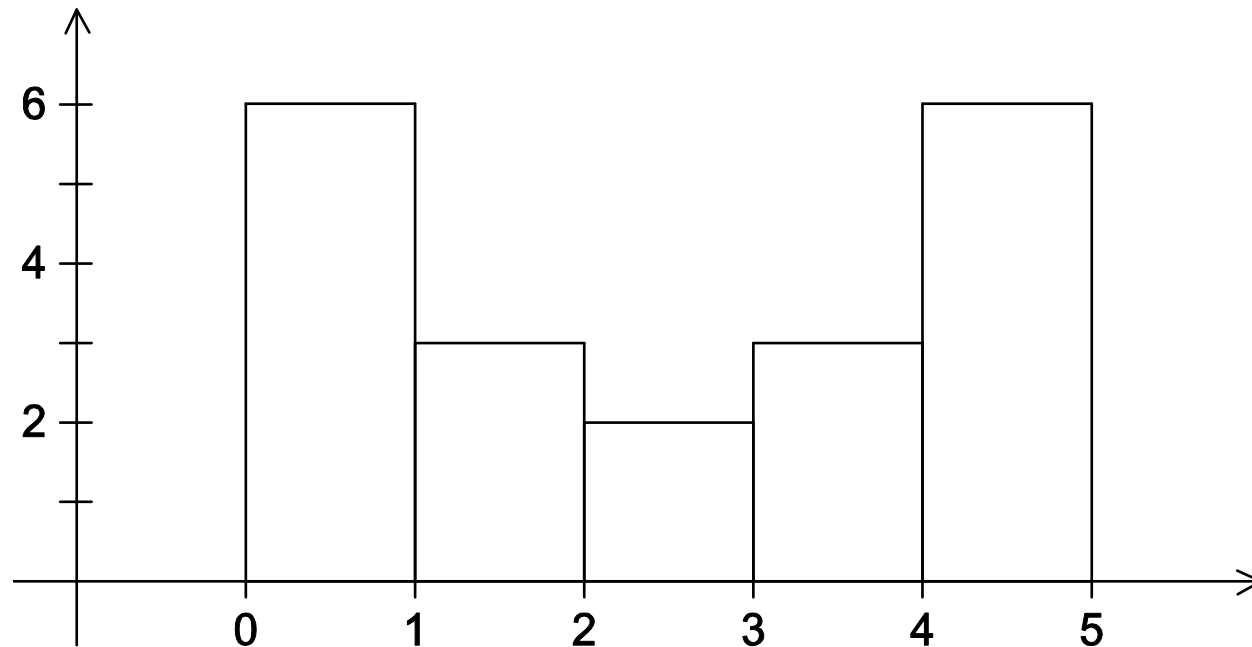
# Foster's methodology

4. **Mapping**: assign the composite tasks identified in the previous step to processes/threads.

This should be done so that communication is minimized, and each process/thread gets roughly the same amount of work.

# Example - histogram

- 1.3, 2.9, 0.4, 0.3, 1.3, 4.4, 1.7, 0.4, 3.2, 0.3, 4.9, 2.4, 3.1, 4.4, 3.9, 0.4, 4.2, 4.5, 4.9, 0.9



# Serial program - input

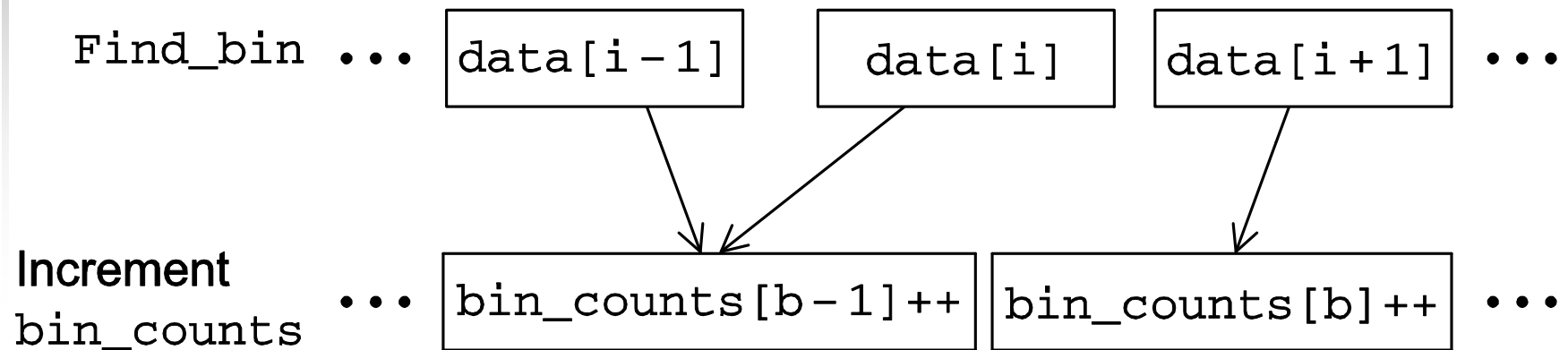
1. The number of measurements: `data_count`
2. An array of `data_count` floats: `data`
3. The minimum value for the bin containing the smallest values: `min_meas`
4. The maximum value for the bin containing the largest values: `max_meas`
5. The number of bins: `bin_count`



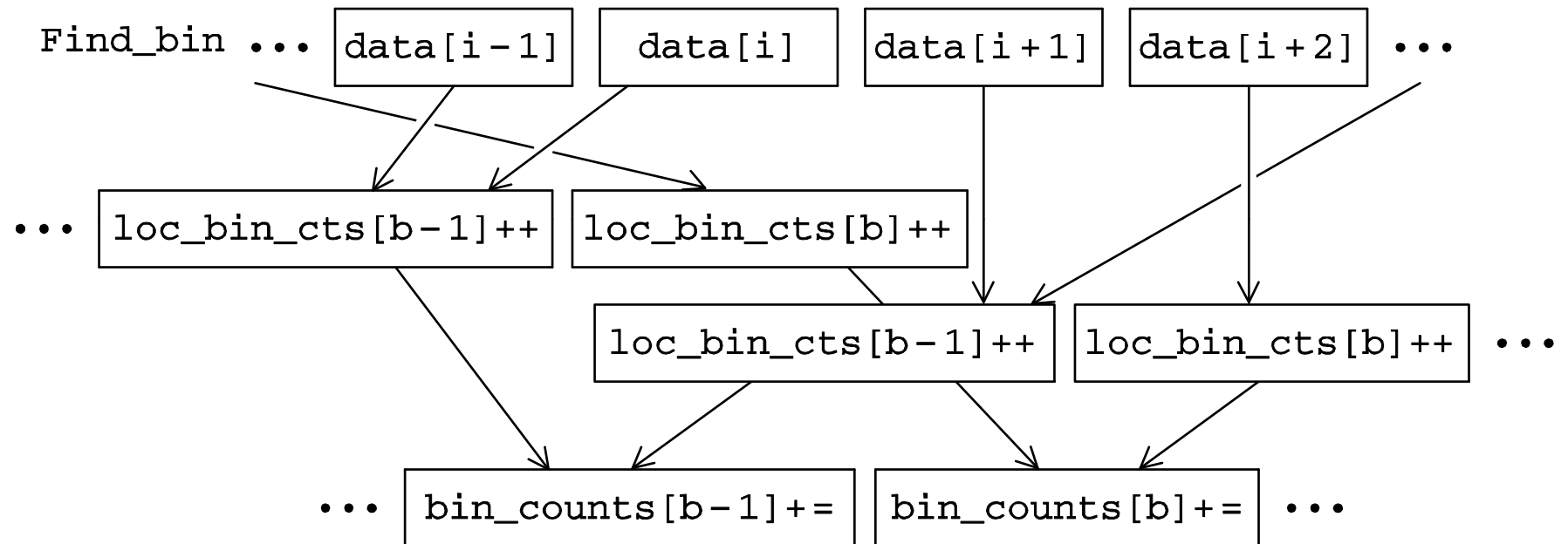
# Serial program - output

1. `bin_maxes` : an array of `bin_count` floats
2. `bin_counts` : an array of `bin_count` ints

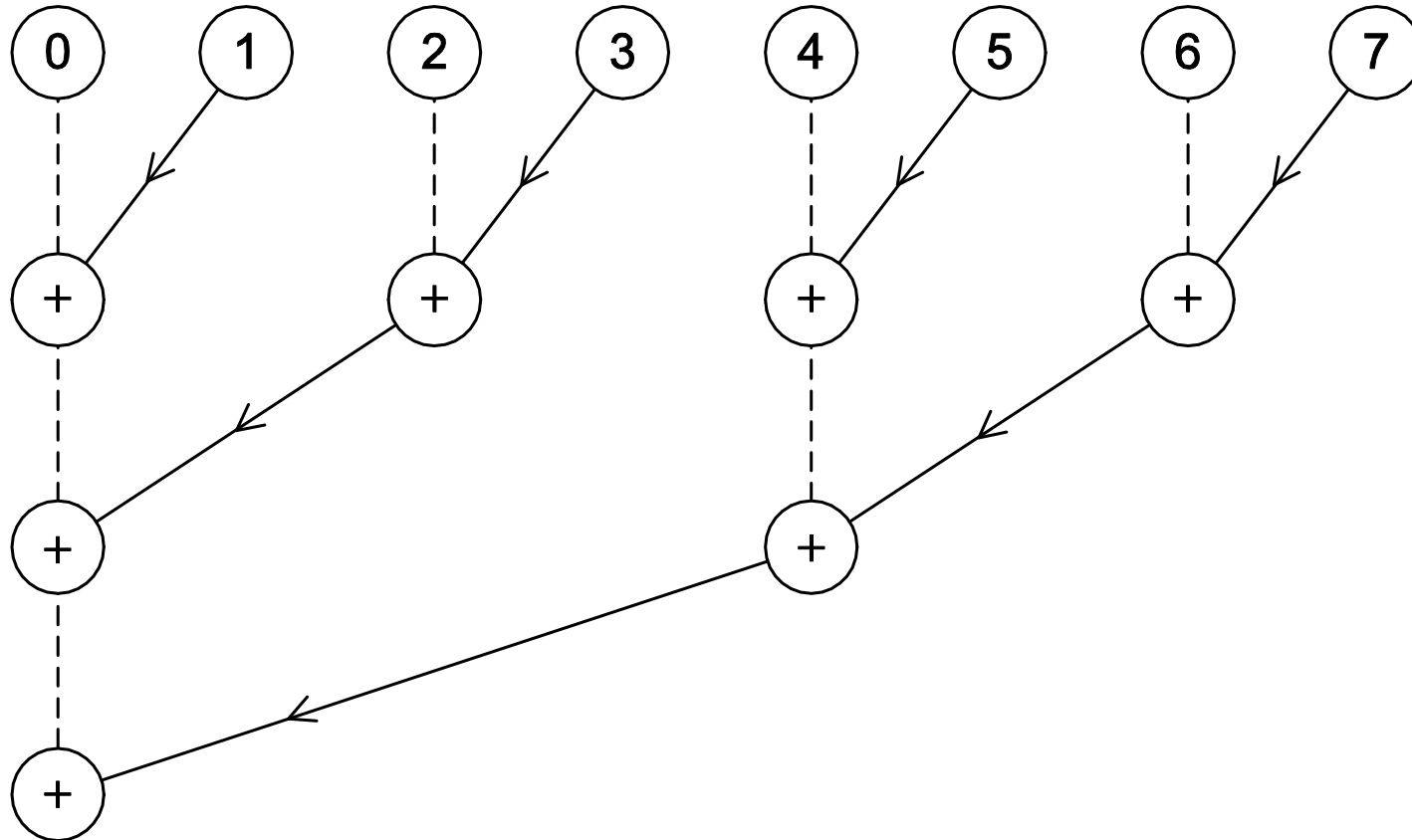
# First two stages of Foster's Methodology



# Alternative definition of tasks and communication



# Adding the local arrays



# Concluding Remarks (1)

- Serial systems
  - The standard model of computer hardware has been the von Neumann architecture.
- Parallel hardware
  - Flynn's taxonomy.
- Parallel software
  - We focus on software for homogeneous MIMD systems, consisting of a single program that obtains parallelism by branching.
  - SPMD programs.

# Concluding Remarks (2)

- Input and Output
  - We'll write programs in which one process or thread can access stdin, and all processes can access stdout and stderr.
  - However, because of nondeterminism, except for debug output we'll usually have a single process or thread accessing stdout.

# Concluding Remarks (3)

- Performance
  - Speedup
  - Efficiency
  - Amdahl's law
  - Scalability
- Parallel Program Design
  - Foster's methodology