CptS 415 Big Data

# Relational Algebra

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# Relational Query Languages

- Query Languages (QLs):
  - Allow manipulation and retrieval of data from a database
- Relational model supports simple, powerful QLs:
  - Strong formal foundation based on logic
  - Allows for much optimization
- Query Languages != Programming Languages!
  - QLs not expected to be "Turing complete"
  - QLs not intended to be used for complex calculations
  - QLs support easy, efficient access to large datasets

## Formal Relational Query Languages

- Two mathematical Query Languages form the basis for "real" languages (e.g. SQL), and for implementation
- Relational Algebra
  - More operational (procedural), very useful for representing execution plans
- Relational Calculus
  - Let users describe what they want, rather than how to compute it
  - Non-operational, Declarative

### Preliminary

- A query is applied to relation instances, and the result of a query is also a relation instance
- Schemas of input relations of a query are fixed
- The schema of the result of a given query is also fixed, determined by definition of query language constructs.

## Relational Algebra

- Basic Operations:
  - Selection  $\sigma$ : Selects a subset of rows from a relation
  - Projection  $\pi$ : Deletes unwanted columns from a relation
  - Cross-Product X: Combines two relations
  - Set-Difference −: Tuples in R₁ but not in R₂
  - Union U: Tuples in  $R_1$  and  $R_2$
- Additional operations
  - Intersection, Join, Division, Renaming: Optional, but very useful
- Since each operation returns a relation, operations can be composed
  - The algebra is "closed"

### **Example Instances**

#### Sailors and Reserves relations

#### Columns:

Sid: Sailor ID

Bid: Boat ID

 We will use positional or named field notations

 $R_1$ 

sid	bid	day
22	101	10/10/96
58	103	11/12/96

 $S_1$ 

sid	sname	rating	age
22	dustin	7	45
31	luber	8	55.5
58	rusty	10	35

 $S_2$ 

sid	sname	rating	age
28	yuppy	9	35
31	luber	8	55.5
44	guppy	5	35
58	rusty	10	35

### Selection $\sigma$

- Select rows that satisfy selection condition
- Schema of result is identical to schema of (only) input relation
- Result relation can be the input for another relational operation

$$\sigma_{rating>8}(S_2)$$

sid	sname	rating	age
28	yuppy	9	35
58	rusty	10	35

$$\pi_{sname,rating} \Big( \sigma_{rating>8} ig( S_2 ig) \Big)$$

sname	rating
yuppy	9
rusty	10

### Projection $\pi$

- Deletes attributes that are not in projection list
- Schema of result contains exactly the fields in the projection list, with the same names that they had in the (only) input relation
- Project operator has to eliminate duplicates!
  - Why?
  - What are the consequences?

 $S_2$ 

sid	sname	rating	age
28	yuppy	9	35
31	luber	8	55.5
44	guppy	5	35
58	rusty	10	35

 $\pi_{sname, \ rating}(S_2)$ 

sname	rating
yuppy	9
luber	8
guppy	5
rusty	10

 $\pi_{age}(S_2)$ 

age	
35	
55.5	

### Union, Intersection, Set-Difference

- All of these operations take two input relations, which must be union-compatible
  - Same number of fields
  - "Corresponding" fields have the same type
- What is the output schema?

 $S_1 - S_2$ 

sid	sname	rating	age
22	dustin	7	45

 $S_1 \cap S_2$ 

sid	sname	rating	age
31	luber	8	55.5
58	rusty	10	35

 $S_1$ 

sid	sname	rating	age
22	dustin	7	45
31	luber	8	55.5
58	rusty	10	35

 $S_2$ 

sid	sname	rating	age
28	yuppy	9	35
31	luber	8	55.5
44	guppy	5	35
58	rusty	10	35

### Cross-Product X

- lacksquare Each row of  $S_1$  is paired with each row of  $R_1$
- lacktriangle Result schema has one field per field of  $S_1$  and  $R_1$ , with field names "inherited" if possible.
  - Conflict: Both  $S_1$  and  $R_1$  have a field called sid  $S_1 \times R_1$

sid	sname	rating	age	sid	bid	day
22	dustin	7	45	22	101	10/10/96
22	dustin	7	45	58	103	11/12/96
31	luber	8	55.5	22	101	10/10/96
31	luber	8	55.5	58	103	11/12/96
58	rusty	10	35	22	101	10/10/96
58	rusty	10	35	58	103	11/12/96

 $S_1$ 

sid	sname	rating	age
22	dustin	7	45
31	luber	8	55.5
58	rusty	10	35

 $R_1$ 

sid	bid	day
22	101	10/10/96
58	103	11/12/96

# Renaming $\rho$

Renaming the attributes in the schema

$$\rho(C(1 \rightarrow sid1, 5 \rightarrow sid2), S_1 \times R_1)$$

sid1	sname	rating	age	sid2	bid	day
22	dustin	7	45	22	101	10/10/96
22	dustin	7	45	58	103	11/12/96
31	luber	8	55.5	22	101	10/10/96
31	luber	8	55.5	58	103	11/12/96
58	rusty	10	35	22	101	10/10/96
58	rusty	10	35	58	103	11/12/96

### Join ⋈

- Conditional Join:  $R_1 \bowtie_c S_1 = \sigma_c(R_1 \times S_1)$
- Result schema same as that of cross-product
- Fewer tuples than cross-product.
  - Filters tuples not satisfying the join condition
- Sometimes called a theta-join

$$R_1 \bowtie_{S_1.sid < R_1.sid} S_1$$

(sid)	sname	rating	age	sid	bid	day
22	dustin	7	45	58	103	11/12/96
31	luber	8	55.5	58	103	11/12/96

 $S_1$ 

sid	sname	rating	age
22	dustin	7	45
31	luber	8	55.5
58	rusty	10	35

 $R_1$ 

sid	bid	day
22	101	10/10/96
58	103	11/12/96

# Join (Cont'd)

#### Inner Join:

Specifies a join between two relations with an explicit join clause

#### Left Outer Join

- Specifies a join between two relations with an explicit join clause
- preserving unmatched rows from the first table

### Right Outer Join

- Specifies a join between two tables with an explicit join clause
- preserving unmatched rows from the second table

## Equi-Join $\bowtie_c$

- A special case of condition join where the condition only contains only equalities
- Result schema similar to cross-product, but only one copy of fields for which equality is specified
- Natual Join: Equi-Join on all common fields

$$\pi_{sid, \ldots, age, bid, \ldots}(R_1 \bowtie_{sid} S_1)$$

sid	sname	rating	age	bid	day
22	dustin	7	45	101	10/10/96
58	rusty	10	35	103	11/12/96

 $S_1$ 

sid	sname	rating	age
22	dustin	7	45
31	luber	8	55.5
58	rusty	10	35

 $R_1$ 

sid	bid	day
22	101	10/10/96
58	103	11/12/96

### Division /

- Not supported as a primitive operator, but useful for expressing queries like
  - Find sailors who have reserved all boats
- Precondition: in A/B the attributes in B must be included in the schema for A, also, the results has attributes A-B
  - SALES(supId, prodId)
  - PRODUCTS(prodId)
  - Relations SALES and PRODUCTS must be built using projections
  - SALES/PRODUCTS: the ids of the suppliers supplying all products

# Example of Division

 $\boldsymbol{A}$ 

sid	pid
s1	p1
s1	p2
s1	р3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

 $\boldsymbol{B}_1$ 

pid	
p2	

 $A/B_1$ 

sid	
s1	
s2	
s3	
s4	

 $\boldsymbol{B}_1$ 

pid
p2
p4

 $A/B_2$ 

sid
s1
s4

 $B_1$ 

pid	
p1	
p2	
p4	

 $A/B_3$ 

sid	
s1	

# Expressing A/B Using Basic Operators

- Devision is not an essential operator, i.e. composite
  - Also true of joins, but joins are so common that almost all systems implement joins.
  - Division is NOT implemented in SQL
- Idea: For SALES/PRODUCTS, compute all products such that there exists at least one supplier not supplying it
  - X value is disqualified if by attaching y value from B, we obtain an xy tuple that is not in C

$$C = \pi_{sid} \Big( \big( \pi_{sid}(Sales) \times Products \big) - Sales \Big)$$

$$Sales/Products = \pi_{sid}(Sales) - C$$

### Find names of sailors who have reserved boat #103

#### Solution 1:

$$\pi_{sname}igg(igg(\sigma_{bid=103}ig(R_1ig)igg)oxtimes Sailorsigg)$$

Solution 2:

$$\rho\bigg(Temp1, \Big(\sigma_{bid=103}\big(R_1\big)\Big)\bigg)$$

$$\rho(Temp2, Temp1 \bowtie S_1)$$

$$\pi_{sname}(Temp2)$$

Solution 3:

$$\pi_{sname} \Big( \sigma_{bid=103} \big( R_1 \bowtie S_1 \big) \Big)$$

- Which one is fastest?
- Optimization: push-down selection

### Find names of sailors who've reserved a red boat

Information about boat color only available in Boats; so need an extra join:

$$\pi_{sname}\Big(\Big(\sigma_{color='red'}(B)\Big)\bowtie R_1\bowtie S_1\Big)$$

A more efficient solution

$$\pi_{sname} \bigg( \pi_{sid} \Big( \big( \pi_{bid} \sigma_{color='red'}(B) \big) \bowtie R_1 \bigg) \bowtie S_1 \bigg)$$

A query optimizer can find this, given the first solution

## Find sailors who've reserved a red or a green boat

Can identify all red or green boats, then find sailors who've reserved one of these boats:

$$\pi_{\mathit{sname}}\bigg(\pi_{\mathit{bid}}\bigg(\sigma_{\mathit{color}='\mathit{red'}\,\vee\,\mathit{color}='\mathit{green'}}(B)\bigg)\bowtie R\bowtie S\bigg)$$

Or, use union operator

$$\pi_{\mathit{sname}}\bigg(\pi_{\mathit{bid}}\Big(\sigma_{\mathit{color}='\mathit{red}'}(B)\cup\sigma_{\mathit{color}='\mathit{green}'}(B)\bigg)\bowtie R\bowtie S\bigg)$$

• What if  $\vee$  is replaced by  $\wedge$ ?

# Find sailors who've reserved a red and a green boat

- Previous approach won't work!
- Must identify
  - sailors who have reserved red boats
  - sailors who have reserved green boats
- Then Find intersection

$$\rho \Big( S_{red}, \ \pi_{sid} \Big( \sigma_{color='red'}(B) \bowtie R \Big) \Big)$$

$$\rho \bigg( S_{green}, \ \pi_{sid} \bigg( \sigma_{color='green'}(B) \bowtie R \bigg) \bigg)$$

$$\pi_{sname}igg(ig(S_{red}\cap S_{green}ig)owtiendsig)$$

### Find the names of sailors who've reserved all boats

- Uses division
  - Schemas of the input relations to division must be carefully chosen!

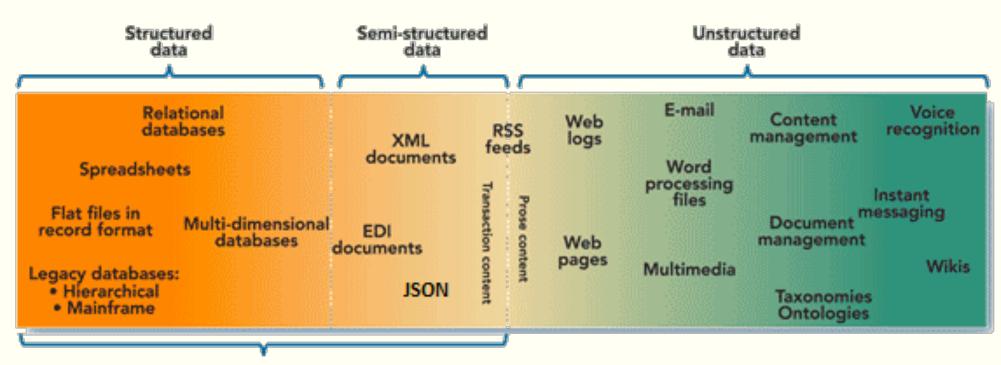
$$\rho \Big( S_{sids}, \ \Big( \pi_{sid, \ bid}(R) / \pi_{bid}(B) \Big) \Big)$$

$$\pi_{sname}(S_{sids} \bowtie S)$$

■ To find sailors who've reserved all 'interlake' boats:

$$.../\pi_{bid}(\sigma_{bname='interlake'}(B))$$

## Summary



Relational Data Lake