HW5

CPTS 453 | HW5

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1.

Give details – two paragraphs will suffice – about your choice for the end-of-semester project.

I am researching the application of graph theory in neural network models. I am planning to put together some kind of relationship between the following topics:

- Graph Theory
- Graph Neural Networks
- HyperNetworks
- Graph HyperNetworks for Neural Architecture Search

In recent years neural network modeling has become increasingly the go-to choice for solving high-dimensional problems which are highly computationally complex. Being able to relate graph theory and neural network is a very important to me in order to begin tackling these problems.

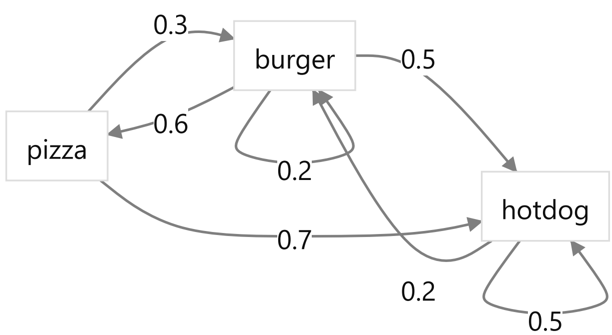
2.

In the current era, *media beyond written sources* have been effective in communicating complex mathematical ideas to a broad audience. Shining examples of this include YouTube videos such as the 3Blue1Brown videos produced by Grant Sanderson. **Find and provide a short synopsis of two YouTube videos each effectively explaining a problem that involves graph theory**. A “short synopsis” includes *the URL along with at least two paragraphs* of explanation and/or narrative.

[Markov Chain for Stochastic Decision Making](https://www.youtube.com/watch?v=i3AkTO9HLXo)

A Markov chain is a probability distribution wherein the next state is dependent only on the previous state, not the entire sequence of previous states.

A Markove chain is a very useful construct when we need to deal with non-determinism, i.e. stochastic decision making. For example, let's say we want to go on a pizza date, but our favorite restaurant only serve one type of food out of three everyday (pizza, hot dog, and burger). That means each day is different, with a caveat: the food served today is dependent on what is served on the previous day:



Using a Markov chain we can estimate the probability distribution for the three types of food possibly served every day.

To do that, we map these *states* and their probabilitistic relations to a directed graph, and thus have our Markov chain for this decision making problem. Given a Markov chain mapped to an adjacency matrix (also called a transition matrix), there exists a state vector π, called the *stationary distribution*, composed of the probability distributions of all nodes in the chain, and is the convergence of all the state vectors π_i where $i = 0, 1, \dots N$.

$$\pi \cdot A = \pi$$

Link: https://www.youtube.com/watch?v=i3AkTO9HLXo

Mapping of Manifolds to Decision Making

One important property associated with manifolds is *homeomorphism*, which is an isomorphism from some subdivision of a graph G to some subdivision of another graph G'. In other words, where the graphic structure is preserved in *isomorphism*, the topological structure is preserved in *homeomorphism*. For example, performing a subdivision (adding a new node) on some edge in graph G should mean that the same action happens on some edge in the graph G' and simultaneously preserve their isomorphic relation, and thus preserving their local topological mapping.

Another important thing to be noted about manifolds is that high-dimensional topologies are very often mapped to low-dimensional topologies. For example, the topological gradient of a 3D object is a simple point on a 2D plane in space (which can be obtained via a linear transformation). Similarly, some N-dimensional manifold mapped out from some N neurons can represent the overall neural activities (i.e. local states in space-time and various biological feedbacks such as temperature, humidity, and so on). This manifold in turn can be localized to a decision making process in Euclidian space.

Furthermore, the graph of N neurons are indeed composed of some M subgraphs of related neurons (neurons that are typically in close proximity and activate on the same input). Thus, it could be further be mapped to a sort of hypernetwork of connected components.

Link: https://www.youtube.com/watch?v=v6VJ2RO66Ag