CptS 453 — Homework-04

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Problem 1:

A.

Because M is a simple graph, by definition the Cartesian product of M with its own *transpose* produces a square matrix of m ($m \times m$) where m is the number of vertices. Each diagonal entry i of this matrix denote a *relation* of a vertex on itself. The *relation* here is the number of edges incident on each vertex.

B.

For every entry (i, i) where i is a unique vertex, there is a relation as shown in part A. Thus, for every entry (i, j) where $i \neq j$, there is a relation between two unique vertices. In part A, we showed that each entry (i, i) denotes the number of edges incident on some vertex i. Thus, here we see that (i, j) shows the number of edges incident on the pair of vertices (i, j).

C.

Matrix M:

	14	15	24	25	34	35
1	1	1	0	0	0	0
2	0	0	1	1	0	0
3	0	0	0	0	1	1
4	1	0	1	0	1	0
5	0	1	0	1	0	1

Matrix *transposed* M^T:

	1	2	3	4	5
14	1	0	0	1	0
15	1	0	0	0	1
24	0	1	0	1	0
25	0	1	0	0	1
35	0	0	1	1	0
35	0	0	1	0	1

Matrix $M \cdot M^T$:

	2	0	0	1	1
	0	2	0	1	1
	0	0	2	1	1
	1	1	1	3	0
	1	1	1	0	3

Matrix D:

	2	0	0	0	0
	0	2	0	0	0
	0	0	2	0	0
	0	0	0	3	0
	0	0	0	0	3

Matrix A:

	0	0	0	1	1
	0	0	0	1	1
	0	0	0	1	1
	1	1	1	0	0
	1	1	1	0	0

Problem 2:

A.

Given that the diameters have been found for v_{10}, similarly we have:

$v_5 : d(v_{13}, v_5) = 6$
$v_6 : d(v_{13}, v_6) = 6$
$v_7 : d(v_{13}, v_7) = 5$
$v_8 : d(v_{13}, v_8) = 4$
$v_9 : d(v_5, v_9) = d(v_6, v_9) = d(v_(13), v_9) = 3$
$v_{14} : d(v_5, v_{14}) = d(v_6, v_{14}) = d(v_{13}, v_{14}) = 4$
$v_{11} : d(v_5, v_{11}) = d(v_6, v_{11}) = 4$
$v_{12} : d(v_5, v_{12}) = d(v_6, v_{12}) = 5$
$v_{13} : d(v_5, v_{13}) = d(v_6, v_{13}) = 6$

B.

The **diameter** of G_A is 3 and radius 3 for the same set of vertices.

Because all eccentricities for any given vertex have the same value, the central vertices are: v_9 and v_{10}.

C.

Similarly, the peripheral vertex is thus v_9 and v_{10}.

I have no idea how to prove u and v are peripheral given $d(u, v) = diameter(G)$.

D.

Proof by induction:

For a graph G with $V = \sum V_i$. A path from $i = 1$ to $i = 2$ has length 1. Thus, a path from $i = 1$ to $i = k$ has length $k - 1$. Thus, a path from $i = 1$ to $i = k + 1$ has length k.

Thus, by induction adjacent vertex of an eccentricity has length differs at most by 1.

E.

For any connected graph, radius and diameter are pulled from the same set of eccentricities as defined by the problem statement, where diamiater is the upperbound of distance from two vertex i and j, and radius is the lowerbound.

F.

Proof by induction. Because diameter and radius are drawn from the same set of equidistant eccentricities, lowerbound and upperbound overlap. Thus, diameter of H must also be less than twice the radius of H.

Problem 3:

A.

Given that G is simple, there is a path from r to itself, thus this relation is *reflexive*.

B.

Because G is simple and connected, for any adjacent $u, w \in V_G$, $e_{u,w}, e_{w,u} \in G$. Thus this relation is symmetric.

C.

Because of (A) and (B), this relation is also transitive given that the equimagnitude holds for $d(r, u) = d(u, w)$.

D.

The equivelence class of r, denoted $[r]$ is the set of all vertices on which the relation $\approx$ can be applied.

E.

$[u]$ is the class of all vertices connected to r and are equdistant to one another.