HW6

CPTS 453 | HW6

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1. Tree Tight Bounds

Non-parent vertices are leaf nodes. Thus the tree has $N = 10^{12}$ leaves. This tree has height H.

Given some m-ary tree, where $m \in \mathbb{N}^+$.

A. Tight lowerbound $\Omega(H) = 0$ for any 0-ary trees. This is an ensemble of totally disconnected nodes.

B. H is maximal for 1-ary tree, however the tight upperbound must also obey the condition that the trees has $N = 10^{12}$ leaves. Since any 1-ary trees have only a single leaf node, we choose $m = 2$ for binary trees. Thus, the upperbound for H is the height of complete binary trees of N leaves:

$$\theta(H) = \lceil log_2(N + V) \rceil, \text{where V are missing leaves of the full tree.}$$

Then, the tight upperbound of H is:

$$\Theta(H) = \lceil log_2(N) \rceil$$

C. Tight lowerbound for H for rooted binary trees with N leaves is also:

$$\Omega(H) = \lceil log_2(N) \rceil$$

D. Tight upperbound for H for n-ary rooted binary trees with N leaves. Assuming that the tree is nontrivial like in (A), for $n = N$, all N leaves are connected to a single root node. Thus, the tight lowerbound is the height of n-ary tree is:

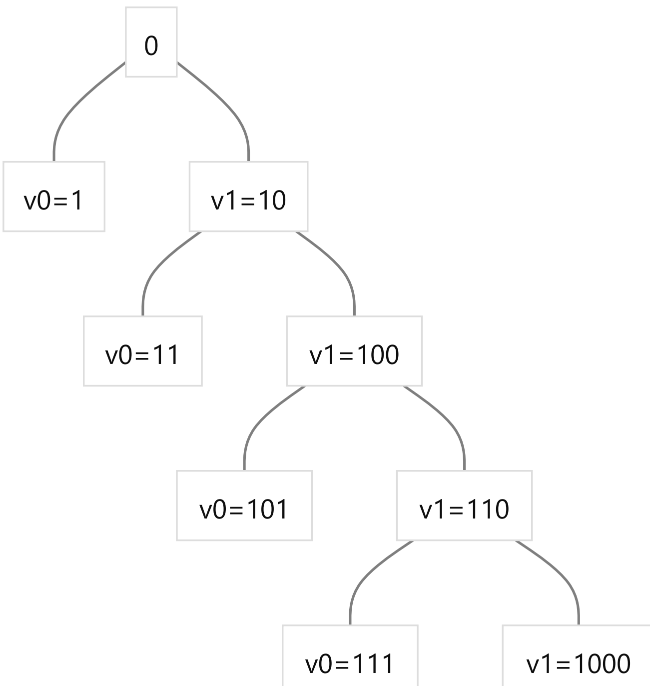
$$\Omega(H) = 1.$$

2. Tree Shape

⚠ **Rules**

- Binary root tree (T, r)
- each tree's vertex is a finite string over $\{0, 1\}$, i.e. some binary number
- root is v_0
- if v_i ends at 0, then left-child is v_0 and right-child is v_1.
- if v_i ends at 1, then the only child is v_0.

A. Draw levels 0 through 4of this tree, with the vertices labeled properly.



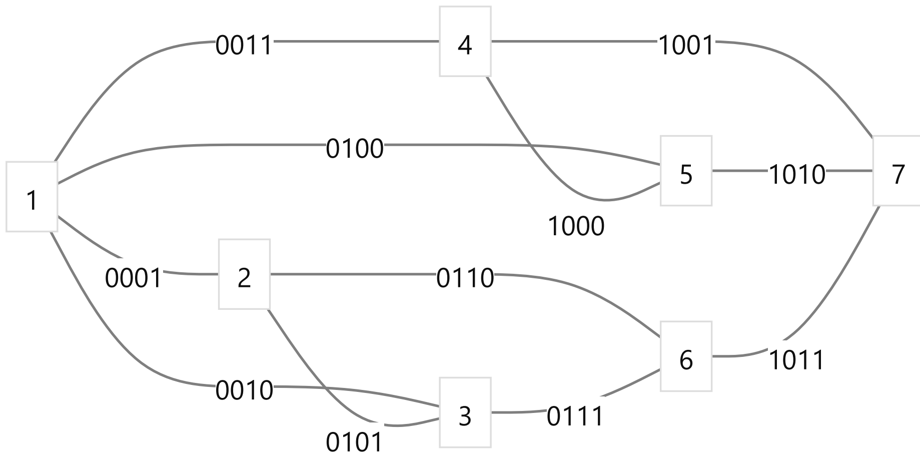
B. Make a conjecture of how to compute the number of vertices at a given level.

Because this tree is amputated on exactly one branch for every level, the number of vertices in the tree can be calculated as:

$$V = 2l + 1, \quad \text{where } l = \text{levels}$$

3. k-Colorable

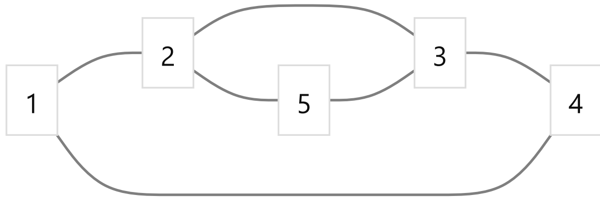
Where k=3 for the given graph,



The graph contains cycles so no k-coloring can exists for k. Thus, it's not 3-colorable.

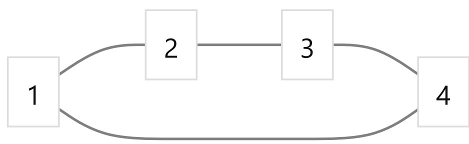
4. Chromatic Polynomial

Given the graph, we call it X:

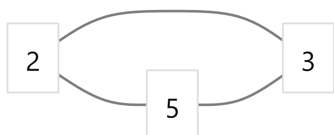


Solution

This graph is the disjoint union $X = G \sqcup H$ of the following two graphs:



G



H

where G is a 2-regular graph of order 4 (K_4), and H is a 2-regular graph of order 3 (K_3).

We then have,

$$p_{G \sqcup H}(k) = p_G(k) \cdot p_H(k)$$

where,

$$p(k) = \begin{cases} \frac{k!}{(k-n)!} & k \geq n \ (1) \\ 0 & k < n \ (2) \end{cases}$$

Beacuse $(k = 3) < (n_G = 4)$ and $(k = 3) \geq (n_H = 3)$, we have,

$$p_{G \sqcup H}(k) = 0 \cdot p_H(k) = 0$$