CPTS 453 Graph Theory -- Assessment

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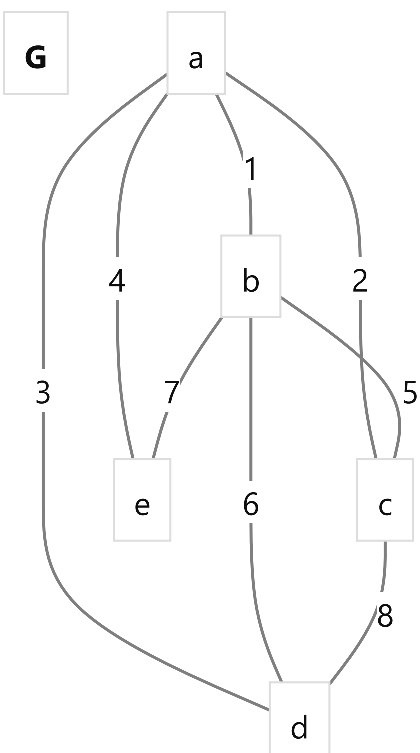
1. Incidence Matrices

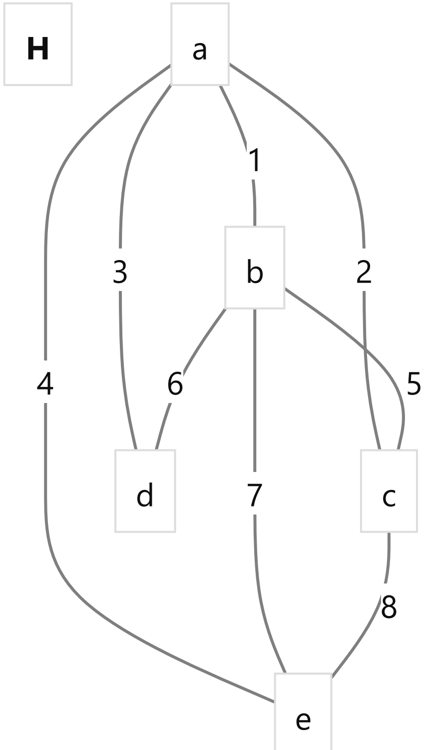
A. Draw graphs from incidence matrix G and H.

Given the matrices:

$$M_G = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}; \quad M_H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Their graphs:

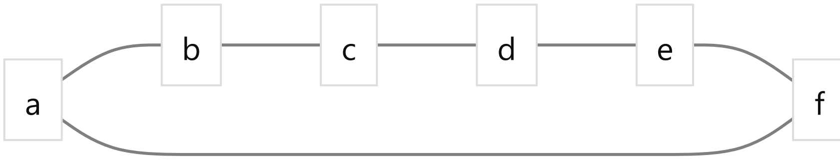


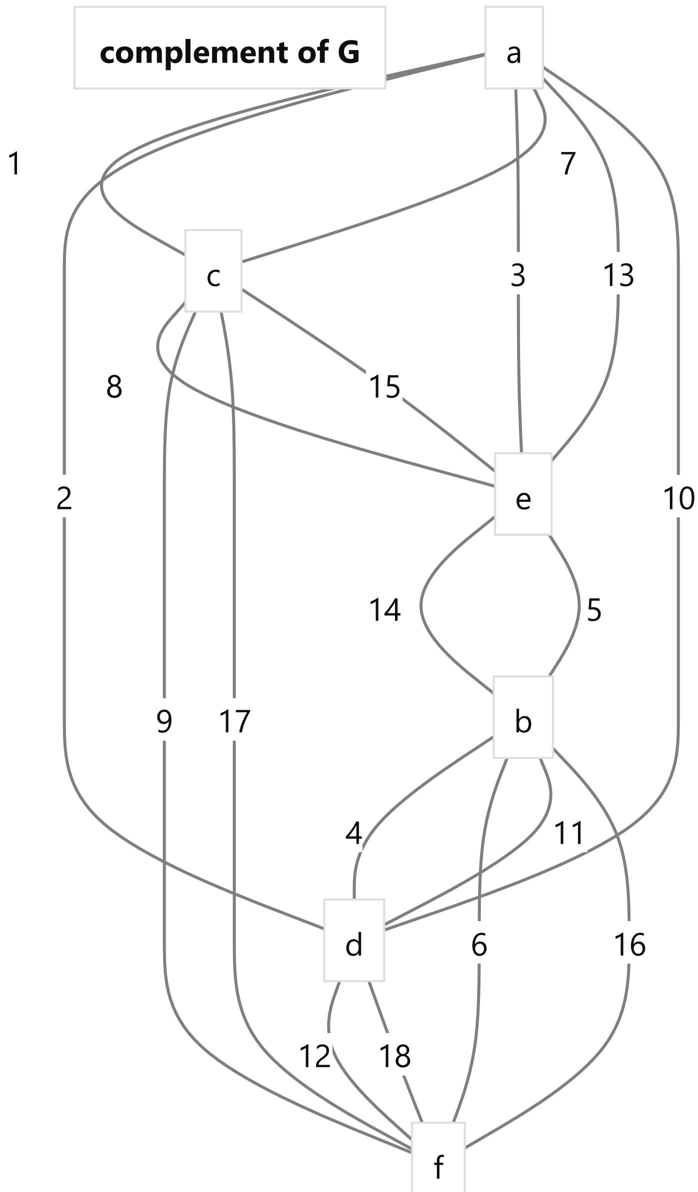


B. The two graph G and H are not isomorphic because there is no bijection between the vertex set of G and vertex set of H. Counter evident is the number of edges incident on $G.d = 3$ while $H.d = 2$.

2. Complement Graph

A. Suppose G is the six-cycle drawn below. Draw its complement $\overline{G}$.





B. Suppose G is an r-regular simple graph of order n. Explain why $\overline{G}$ is an s-regular simple graph of order n and determine the value of s in terms of r.

C. If $G = K_{p,q}$ where p and q are positive integers, describe $\overline{G}$.

3.

1. Write incidence matrix and adjacency matrix for G.
2. Find the radius and diameter of G.
3. This is the graph from question 3 in homework 6. Explain why $(x-k)$ is a factor of its chromatic polynomial for every $k \in \{0, 1, 2, 3\}$. You are not required to find the chromatic polynomial of G.

4. Circuitry

Recall that a **circuit** is a closed walk (one in which the starting and ending vertices are the same) that does not repeat an edge. Explain why if G has a nontrivial circuit, then it must have a nontrivial cycle.

5. Boundaries of n-ary tree

Let T be a full ternary (3-ary) tree of height 7.

A. Determine, with justification, a tight upper bound on the number of vertices T can have.
B. Determine, with justification, a tight upper bound on the number of edges T can have.
C. Determine, with justification, a tight upper bound on the number of leaves T can have.
D. Determine, with justification, whether it is possible for T to have exactly 100 leaves.

6.

A. Determine, in terms of q and r, the number of vertices in $S_{q,r}$
B. Determine, in terms of q and r, the number of edges in $S_{q,r}$
C. Show that r is even if and only if $S_{q,r}$ is bipartite.