Graph Theory Fall 2022

Assignment 2

1. Consider simple bipartite graphs (i.e., no loops or parallel edges) with $n = 200$. What are bounds on the possible values of m for such a graph? It's enough to provide a tight lower bound and a tight upper bound. Here, the word "tight" means that an example of a graph with that number of edges exists.

2. Let p and q be positive integers with $p < q$. Find tight lower and upper bounds on the length of a path in $K_{p,q}$, the complete bipartite graph on p and q vertices. Recall that the length of a path is the number of edges in the path.

3. Let $\mathbb{Z}_{10} = \{0,1,2,3,4,5,6,7,8,9\}$ with arithmetic modulo 10. For $k \in \mathbb{Z}_{10}$, let G_k be the graph where $V = \mathbb{Z}_{10}$ and vertices i and j are joined by an edge if and only if i and j differ by k mod 10. As examples, G_1 is the cycle graph C_{10} and G_3 is sketched in figure 1 below. For which values of k is G_k connected?

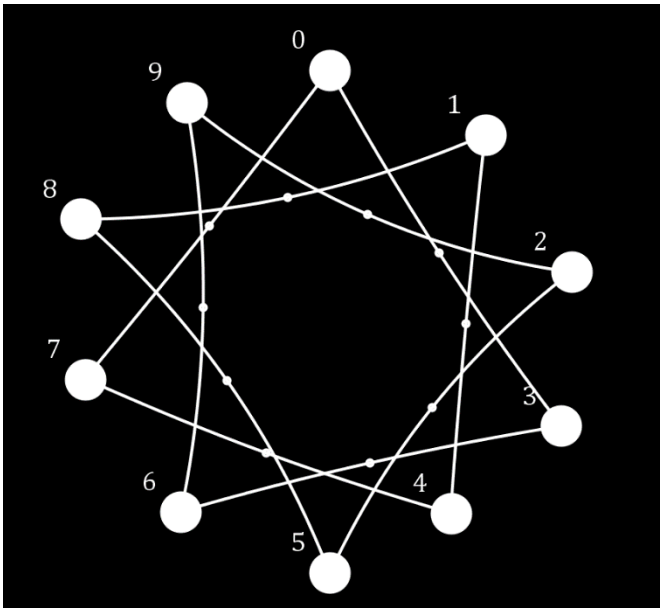

Figure 1. A sketch of G_3

4. Recall that a graph G is "cubic" if and only if it is 3-regular (i.e., every vertex has degree 3.)
   A. Show that there exists no cubic graph with an odd number of vertices.
   B. Show that there exists a simple cubic graph with 4 vertices.
   C. For every integer $n \geq 3$, show how to construct a simple cubic graph with $2n$ vertices. You can do this using a sketch of small examples with an obvious generalization, or by specifying a set V with $2n$ elements and a recipe for joining them that produces the desired graph.