CptS 453 — Homework-02

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Problem 1:

The bounds for a *biparte* graph are as follows:

- The lower bound is when the graph is extremely skewed to one side, where $n_i == 1$ for either side of the biparte graph and $n_j = 200 - n_i$ is the remaining side. Thus, the lower bound is $m = 200$.

- The upper bound is when the graph is perfectly symmetrically, where $n_i == n_j$. In this case,

$$n_i = n_j = 100$$

Thus,

$$m = n_i \cdot n_j, \text{ where} n_i = n_j$$

Problem 2:

Similarly, for p and q as integers, where $p < q$. The lower bound of $K_{p,q}$ is the product $p \cdot q$, where $p == 1$. The upper bound is $((p + q)/2)^2$.

Problem 3:

The set of value k for which G_k is connected is controlled by the following conditions:

- k where k is prime

- k where $\|j - i\| == k\%10$

- k where $10\%k \neq 0$

Since I'm not used to set theory, I am just listing the related subsets. There should be a relation among these subsets. I am suspecting the following:

$$\{k \text{ is prime}\} \cup (\{\|j - i\| == k\%10\} \cap \{10\%k \neq 0\})$$

The set of k is $k = \{1, 3, 7\}$.

Problem 4:

A cubic graph is a *3-regular* graph, i.e. where all vertices have *degree* $k = 3$.

a.

Proof:

A complete graph has every *pair* of distinct vertices connected by a unique edge. A complete graph is maximally connected, i.e. the number of edges is maximum and is given by,

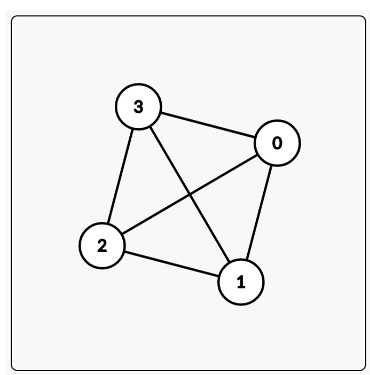
$$\binom{n}{2} = \frac{n(n-1)}{2}$$

where $k = n - 1$, $n = k + 1$.

Thus, it becomes $\frac{nk}{2}$. This quantity has to yield an integer and therefore nk must be even. Recall that for a cubic graph the degree $k = 3$, thus n must be an even value.

b.

Given the proof above, in order for a cubic graph of order $n = 4$ to exist, we can see right away that the degree $k = n - 1 = 3$. Thus the number of edges in the graph will be $\frac{nk}{2} = 6$. The following image shows such a graph:



c.

Given input V of order $2n$, we can decompose V into V_1 of order n and V_2 of the same order n. This allows us to construct a *biparte* graph of degree $k = n$.