

CPTS 453 Graph Theory -- Assessment

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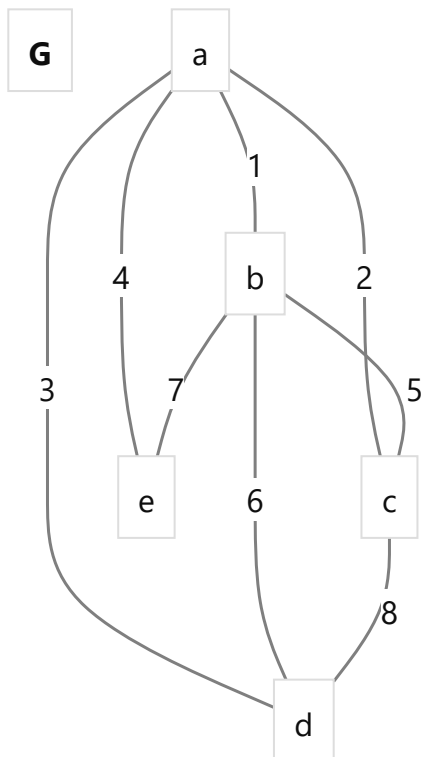
1. Incidence Matrices

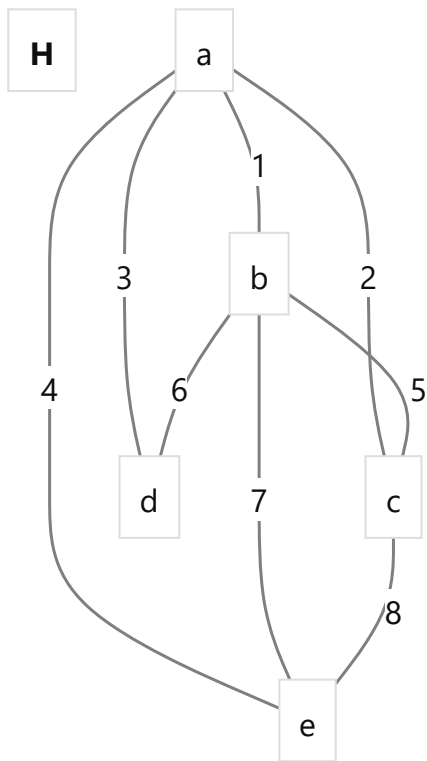
A. Draw graphs from incidence matrix G and H .

Given the matrices:

$$M_G = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}; \quad M_H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Their graphs:

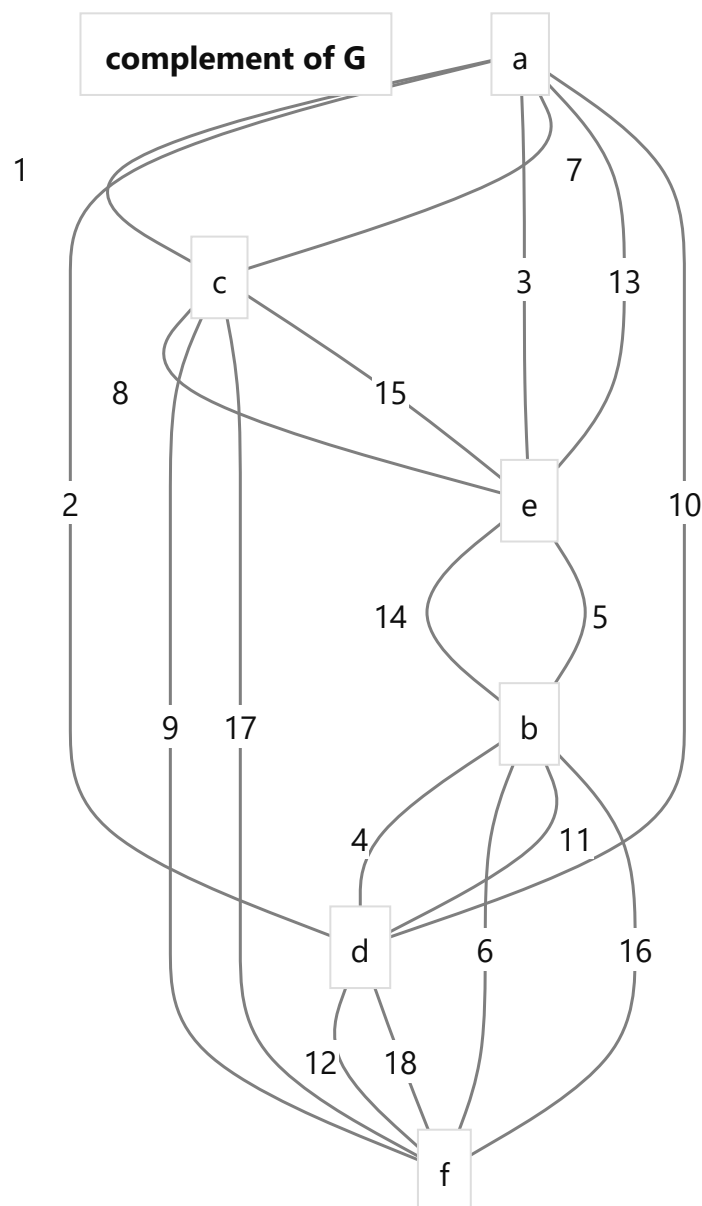
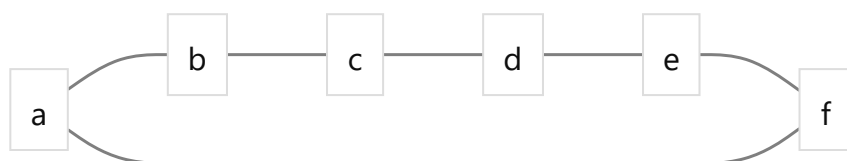




B. The two graph G and H are not isomorphic because there is no bijection between the vertex set of G and vertex set of H . Counter evident is the number of edges incident on G . $d = 3$ while H . $d = 2$.

2. Complement Graph

A. Suppose G is the six-cycle drawn below. Draw its complement \overline{G} .

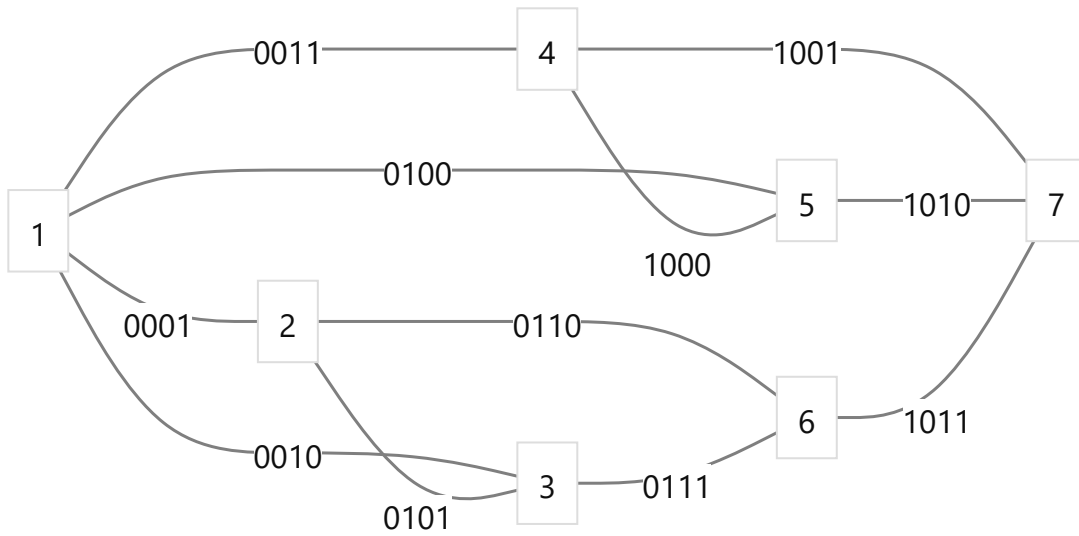


B. Suppose G is an r -regular simple graph of order n . Explain why \overline{G} is an s -regular simple graph of order n and determine the value of s in terms of r .

C. If $G = K_{p,q}$ where p and q are positive integers, describe \overline{G} .

3.

Given G :



A. Write incidence matrix and adjacency matrix for G .

$$M_G = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

The adjacency matrix of G is:

$$M_G \cdot M_G^T = D + A$$

$$A = M_G \cdot M_G^T - D$$

where,

D : diagonal matrix

A : adjacency matrix

Thus,

$$M_G \cdot M_G^T = \begin{bmatrix} 4 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 3 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 3 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 4 & & & & & & \\ & 3 & & & & & \\ & & 3 & & & & \\ & & & 3 & & & \\ & & & & 3 & & \\ & & & & & 3 & \\ & & & & & & 3 \end{bmatrix}; A = \begin{bmatrix} & 1 & 1 & 1 & 1 & & \\ 1 & & 1 & & & 1 & \\ 1 & 1 & & & & 1 & \\ 1 & & & 1 & & 1 & \\ & 1 & 1 & & & & 1 \\ & & & 1 & 1 & 1 & \end{bmatrix}$$

B. G has diameter 6 (max eccentricity between 1 and 2: 1-4-5-7-6-3-2) and radius 4 (min of max eccentricities between 1 and 7: 1-2-3-6-7).

C. This is the graph from question 3 in homework 6. Explain why $(x - k)$ is a factor of its chromatic polynomial for every $k \in \{0, 1, 2, 3\}$. You are not required to find the chromatic polynomial of G .

4. Circuitry

Recall that a **circuit** is a closed walk (one in which the starting and ending vertices are the same) that does not repeat an edge. Explain why if G has a nontrivial circuit, then it must have a nontrivial cycle.

5. Boundaries of n-ary tree

Let T be a full ternary (3-ary) tree of height 7.

- A. Determine, with justification, a tight upper bound on the number of vertices T can have.
- B. Determine, with justification, a tight upper bound on the number of edges T can have.
- C. Determine, with justification, a tight upper bound on the number of leaves T can have.
- D. Determine, with justification, whether it is possible for T to have exactly 100 leaves.

6.

- A. Determine, in terms of q and r , the number of vertices in $S_{q,r}$
- B. Determine, in terms of q and r , the number of edges in $S_{q,r}$
- C. Show that r is even if and only if $S_{q,r}$ is bipartite.