Graph Theory Fall 2022

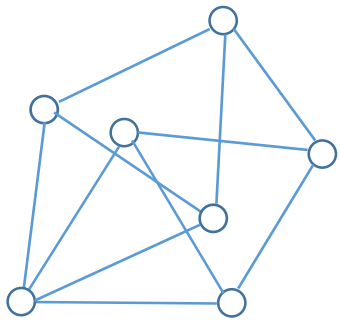
Assignment 6

1. Consider rooted trees with exactly 10^{12} non-parent vertices. Recall that we denote by H the height of such a tree.
   A. Find a tight lower bound for H among such trees.
   B. Find a tight upper bound for H among such trees.
   C. Find a tight lower bound for H among binary rooted trees with exactly 10^{12} non-parent vertices.
   D. Find a tight lower bound for H among n-ary rooted trees with exactly 10^{12} non-parent vertices. Your formula should involve n.

2. Consider the binary rooted tree (T, r) each of whose vertices is a finite string over $\{0,1\}$. We construct our tree by the following rules:
   - The root is the vertex 0.
   - If v is a string that ends in a 0, then the left child of v is "$v0$" and the right child is "$v1$".
   - If v is a string that ends in a 1, then the only child of v is "$v0$".

   A. Draw Levels 0 through 4 of this tree, with the vertices labeled appropriately.
   B. Make a conjecture of how to compute the number of vertices at a given level.

3. Show that the graph below cannot be 3-colored:



4. Find the chromatic polynomial of the graph below: