CptS 453 — Homework-03

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Problem 1:

The bijection is $\psi : V_i \to U_i$, where:

$$V = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$U = \{a, d, j, e, b, c, i, g, h, f\}$$

are both ordered sets.

Problem 2:

For:

- $P_a : a - 1$

- $C_a : a - 1$

- $K_a : a - 1$

- $K_{a,b} : 2a - 1, \text{ where } a \leq b$

- $Q_a : a - 1$

Problem 3:

A. For $0 \leq d(u, v)$, $d = 0$ when $u = v$, otherwise $d > 0$.

B. As explained in A, when $u = v$ then $d = 0$.

C. Due to reflexivity, $d(u, v) = d(v, u)$.

D. Given the premises, we know that there is a shortest path from $u \to v$ and similarly from $v \to w$. Given that $u \neq v$ and $v \neq w$, due to transitivity, there must be a shortest path from $u \to w$ where $u \neq w$. Thus, $d(u, w)$ must be finite, or $d(u, w) < \infty$.

E. It could be the case that $u = w$, or $u = v$, or $v = w$ which cause cycles in the walk.

F. Given D, we we can see that:

- $d(u, v)$ = number of edges between u and v where neither is repeating.

- $d(v, w)$ = number of edges between v and u where neither is repeating.

Thus, $d(u, w)$ is the number of edges between u and w. Since both $d(u, v)$ and $d(v, w)$ denote *paths* and $u \neq w$, $d(u, w)$ must also be a path. Thus, $d(u, w) < \infty$.

G. $d(u, w) = \infty$.

H. $d(u, w) = \infty$.

Problem 4:

A. In terms of $|V_G| = n_1$ and $|V_H| = n_2$, $|V_{G\times H}| = n_1 \cdot n_2$.

B. In terms of $|V_G| = n_1$ and $|V_H| = n_2$, and $E_G = m_1$ and $E_H = m_2$, then $|E_{G\times H}| = n_1 \cdot m_2 + n_2 \cdot m_1$.

C. Using the definition of the Cartesian product, let $Z = V_{G\times H}$. For a vertex $(v_1, w_1) \in Z$ there are b neighbors (v_2, w_2) such that $v_1 = v_2$ and $w_1 w_2 \in E_H$. Similarly, there are a neighbors (v_1, w_1) such that $w_1 = w_2$ and $v_1 v_2 \in E_G$. Since, both neighbors sets are in distinct edge sets, we count each neighbor of (v_1, w_1) exactly once. Thus, Z is $(a + b)$-regular.