# Peer Effects in Consideration and Preferences*

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**Abstract** We develop a general model of discrete choice that incorporates peer effects in preferences and consideration sets. We characterize the equilibrium behavior and establish conditions under which all parts of the model can be recovered from a sequence of choices. We allow peers to affect only preferences, only consideration, or both. We exploit different types of variations to separate the peer effects in preferences and consideration sets. This allows us to recover the set (and type) of connections between the agents in the network. We then use this information to recover the random preferences and the attention mechanisms of each agent. These nonparametric identification results allow unrestricted heterogeneity across agents and do not rely on the variation of either covariates or the set of available options (or menus). We apply our results to model expansion decisions by coffee chains and find evidence of limited consideration. We simulate counterfactual predictions and show how limited consideration slows down competition.

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## 1. Introduction

It has long been agreed that agents are subject to social influence when making decisions.[1] These interactions have been shown important for people in areas such as health and education and for firms in a variety of decisions such us opening a new store. It has also been argued that agents might affect the decisions of others in different ways.[2] A comprehensive social influence approach is needed to understand the mechanisms by which the interactions operate in practice. This is in turn required to inform private and public policies. We offer a model of social influence where the choices of related agents or peers can affect different aspects of the decision process: First, the choices of peers might affect the subset of options that the agent ends up considering.[3] Second, these choices can affect the preferences of the agent over the set of alternatives. We show that all parts of the model can be recovered from a sequence of choices and illustrate our ideas with an empirical application of expansion decisions of two largest coffee chains in China.

In our model, agents are linked through a social network. An important aspect of our approach is that different agents might have different roles in the decision process of related agents. Thus, the network defines not just the direction of the connection but also the way in which agents affect each other. In particular, a link specifies whether a peer affects the consideration of alternatives, the ranking of preferences, or both. (Note that along the paper we use a behavioral definition of peers. For a given agent, her peers are defined as all other agents that have a direct impact on her choices.) At a randomly given time, an agent gets the opportunity to select a new option out of a finite set of alternatives. The agent sticks to her new option until the revision opportunity arises again. As in the consideration set models, the agent does not include all the available options when revising her selection. Instead, she first forms a consideration set and then picks her most preferred option from it. The distinctive feature of our model is that the probability that a given alternative enters the consideration set depends on the number of peers currently adopting that option. As

[1]See Durlauf and Young (2001) references therein for examples.

[2]See Manski (2000) for a discussion of channels through which agents can affect each other.

[3]This possibility has been (explicitly or implicitly) discussed by other researchers in specific contexts —e.g., the choices of peers may help us discover a new television show (Godes and Mayzlin, 2004), a new welfare program (Caeyers, 2014), a new retirement plan (Duflo and Saez, 2003), a new restaurant (Qiu, Shi and Whinston, 2018), or an opportunity to protest (Enikolopov, Makarin and Petrova, 2020).

in the canonical peer effects models, the choices of some peers can also affect the preferences of the agent regarding the alternatives that are being considered. This model leads to a sequence of choices that evolves through time according to a Markov random process.

The model we build might fit in a large number of applications. Before describing our main results, let us offer two applications for our framework. As a first one, let us consider an online platform that offers video games to a set of users.[4] (These games could include Super Mario Bros, Castlevania, or Sonic the Hedgehog.) A user can enter the platform and decide which game to get, at a given price, if any. The number of games offered by the platforms are often quite large. Some platforms create a reference group for each user and share with her the last purchasing decisions of the other group members. It is often argued that this information helps the user to circumscript the subset of games she ends up considering. It is also the case that some of these games are played in groups. In these cases, the choices of friends can also directly affect the ranking of preferences of the user for the different options. Our model can help the platform to personalize each user's reference group to maximize profits or the probability of making a sale.

As a second example, we can rethink our model as one of endogenous social norms or rules within a group of people. Let us consider the case of drink driving. The choices of some friends might determine whether a person even considers the possibility of driving under the influence of alcohol. Also, since alcohol consumption frequently occurs in social situations, the choices of friends might also affect the preferences for drinking in events that precede driving. We could use our model to evaluate targeted policies. (See Remark 4 for an application to COVID-19 vaccines.)

After we show equilibrium existence and characterize equilibrium behavior, we consider a researcher who observes a long sequence of choices made by the network members. We show that all primitives of the model can be uniquely recovered. These primitives include the network structure, the attention mechanism (or consideration probabilities), and the random preferences. There are three aspects of our nonparametric identification results that deserve special attention. First, we allow for unrestricted heterogeneity across agents with respect to all parts of the model. Second, in contrast to most other works on consideration sets, we do not rely on the variation of either covariates or the set of available options (or menus) to recover these components. Third, regarding

[4]See, Lee (2015) for an example.

the network structure, we recover not just the set of peers for each agent, but whether each of them affects consideration, preferences, or both. That is, we identify the full network structure.

In our model, the observed choices of the network members are generated by a system of conditional choice probabilities (CCPs): each CCP specifies the frequency of choices of a given agent conditional on the choices of others (at the moment of revising her selection). The identification strategy we offer is a two-step procedure. First, we show how to identify the primitives of the model by using these CCPs and the variation in the choices of peers. Second, we study identification of the CCPs from observed data.

The identification strategy we propose for the primitives of the model is novel, simple, and constructive. We exploit the fact that, in our framework, changes in the choices of peers induce stochastic variation in the CCPs of a given agent. We use this variation to recover the set of connections between agents in the network. We separate the peer effect in consideration from the peer effect in preferences by showing that these effects have different behavioral implications. We finally use this information to recover the consideration mechanism of each agent, i.e., the probability of including a specific option in the consideration set as a function of the number of friends currently choosing it. We then recover the peer effect in random preferences. As we just mentioned, this identification result allows for full heterogeneity across agents regarding preferences and consideration.

To identify the CCPs, we consider two datasets: continuous-time data and discrete-time data with arbitrary time intervals. These two datasets coincide in that they provide a long sequence of choices from people in the network. They differ in the timing at which the researcher observes these choices. In continuous-time datasets, the researcher observes people's choices in real-time. We can think of this dataset as the "ideal dataset." With the proliferation of online platforms and scanners, this sort of data might be available in many applications. In this dataset, the researcher directly recovers the CCPs. In discrete-time datasets, the researcher observes the joint vector of choices at fixed time intervals (e.g., the choices of agents are observed every Monday). In this case, the CCPs are not directly observed, they need to be inferred from the data. Adding an extra mild condition, we show that the CCPs are also uniquely identified. For this last result, we invoke insights from

Blevins (2017, 2018).

We provide several empirically relevant extensions of our baseline model. In particular, we show that our results extend to finite histories; we explain how to proceed when one of the choices (e.g., "do nothing") is not observed in the data; and we revise the framework to accommodate the case where multiple agents make choices simultaneously. We also build a random consideration model of bundles and show that all aspects of that model can be identified. Finally, we provide a new identification argument for a general model of random consideration that uses payoff-specific unbounded covariates. To the best of our knowledge, the latter two results are new to the literature on random consideration sets and are of independent interest.

To showcase our methodology, we examine how possible limited consideration may have affected the expansion decision of China's top two coffee chains, Starbucks and Luckin. Given a large set of markets (152 markets), we argue that the firms may be boundedly rational by not considering all of them due to limited knowledge or ability of "administrative man" (Simon, 1945, 1955).[5] We show that overlooking limited consideration can lead to misunderstandings about market profitability and, consequently, firm behavior. Specifically, a full consideration model would explain the slow expansion of Luckin in some markets by low profitability. However, our estimation results indicate that these markets are often profitable, but Luckin does not expand in them because they are not considered. We also measure the impact of limited consideration and peer effect in consideration in shaping market structure. This is done by carrying out two counterfactual scenarios where we eliminate one of these factors. We find that limited consideration and peer effect in it work in opposite directions–while peer effects increase the rate at which markets are served by both firms, limited consideration slows down the emergence of duopolies.

We finally relate our results with the existing literature. From a modeling perspective, our setup combines the dynamic model of social interactions of Blume (1993, 1995) with the (single-agent) model of random consideration sets of Manski (1977) and Manzini and Mariotti (2014). By adding the peer effect in consideration sets, we can use variation in the choices of others as the main tool to recover random preferences. The literature on the identification of single-agent consideration

[5]See also the discussion of boundedly rational firm behavior in Armstrong and Huck (2010) and Heidhues and Kőszegi (2018).

set models has mainly relied on the variation of the set of available options (menus). The latter includes Aguiar (2017), Aguiar, Boccardi and Dean (2016), Brady and Rehbeck (2016), Caplin, Dean and Leahy (2019), Cattaneo, Ma, Masatlioglu and Suleymanov (2020), Horan (2019), Kashaev ⓡ Aguiar (2022), Lleras, Masatlioglu, Nakajima and Ozbay (2017), Manzini and Mariotti (2014), and Masatlioglu, Nakajima and Ozbay (2012). (See Aguiar, Boccardi, Kashaev and Kim, 2023 for a comparison of several consideration set models in an experiment.) Other papers have relied on exogenous covariates that shift preferences or consideration sets. The latter include Barseghyan, Molinari and Thirkettle (2021b), Barseghyan, Coughlin, Molinari and Teitelbaum (2021a), Crawford, Griffith and Iaria (2021), Conlon and Mortimer (2013), Draganska and Klapper (2011), Gaynor, Propper and Seiler (2016), Goeree (2008), Mehta, Rajiv and Srinivasan (2003), and Roberts and Lattin (1991). Variation of exogenous covariates has also been used by Abaluck and Adams-Prassl (2021) via an approach that exploits symmetry breaks with respect to the full consideration set model. Aguiar ⓡ Kashaev (2021), Crawford et al. (2021), and Dardanoni, Manzini, Mariotti and Tyson (2020) use repeated choices but do not allow for peer effects (i.e., they work with panel data).

There is a vast econometric literature on the identification of models of social interactions where choices of peers affect preferences but not the choice sets (see Blume, Brock, Durlauf and Ioannides, 2011, Bramoullé, Djebbari and Fortin, 2020, De Paula, 2017, and Graham, 2015 for comprehensive reviews of this literature). We depart from this literature in that in our framework, the direct interdependence between choices (endogenous effects) is captured by consideration sets in addition to preferences. Interestingly, we show that the two mechanisms can be set apart in practice. We view our work as complementing the existing results on the peer effect in preferences by adding an additional mechanism for the social interaction effects that might be particularly important in specific applications.

As we mentioned earlier, we can recover from the data the set of connections between the agents in the network. (We can also recover the type of interdependence.) In the context of linear models, a few recent papers have made progress in the same direction. Among them, Blume, Brock, Durlauf and Jayaraman (2015), Bonaldi, Hortaçsu and Kastl (2015), De Paula, Rasul and Souza (2018), Lewbel, Qu and Tang (2023), and Manresa (2013). In the context of discrete choice, Chambers, Cuhadaroglu and Masatlioglu (2019) also identifies the network structure. However, in the latter, peers do not affect consideration sets but directly change preferences (among other differences).

Let us finally mention two other papers that incorporate peer effects in the formation of consideration sets. Borah and Kops (2018) do so in a static framework and rely on the variation of menus for identification. Lazzati (2020) considers a dynamic model, but the time is discrete, and she focuses on two binary options that can be acquired together.

The rest of the paper is organized as follows. Section 2 presents the model, the main assumptions, and some key insights of our approach. Section 3 studies the empirical content of the model. Section 4 extends the initial model in several dimensions. Section 5 applies our model to entry decisions by firms in the coffee market. Section 6 concludes, and all the proofs are collected in Appendix B.

# 2. The Model

This section describes the model and the main assumptions we invoke in the paper. It also establishes equilibrium existence.

## 2.1. Social Network, Consideration Sets and Preferences

**Network and Choice Configuration** There is a finite set of agents $\mathcal{A} = \{1, 2, \ldots, A\}$, $A \geq 2$, and a finite set (menu) of alternatives $\mathcal{Y} = \{0, 1, 2, \ldots, Y\}$, $Y \geq 1$, from which the agents might choose. Alternative 0 is called the default alternative. (In the online platform, the default alternative might be not to play any game.) We refer to $\mathbf{y} = (y_a)_{a \in \mathcal{A}} \in \mathcal{Y}^A$ as a choice configuration.[6]

The agents are connected through a social network. We allow agents to interact with others in different ways. Specifically, the choices of peers can affect the set of alternatives the agent ends up considering, the preferences over the alternatives, or both. That is, the network is described by two

[6]The model easily extends to settings where menus are agent-specific if, for every pair of agents, there is a one-to-one mapping between their choice sets.

sets of edges between agents in $\mathcal{A}$, $\Gamma = (\Gamma_C, \Gamma_R)$, where $\Gamma_C$ and $\Gamma_R$ are the sets of consideration and preference edges, respectively. Each edge identifies two connected agents and the direction of the connection. Hence, the reference group of Agent $a$ consists of reference groups for consideration of alternatives, $\mathcal{NC}_a$, and for preferences, $\mathcal{NR}_a$. Formally, for each Agent $a \in \mathcal{A}$

$$\mathcal{NC}_a = \{a' \in \mathcal{A} : \exists \text{ edge from } a \text{ to } a' \text{ in } \Gamma_C\} \text{ and } \mathcal{NR}_a = \{a' \in \mathcal{A} : \exists \text{ edge from } a \text{ to } a' \text{ in } \Gamma_R\}.$$

The full reference group is $\mathcal{N}_a = \mathcal{NC}_a \bigcup \mathcal{NR}_a$. We follow the convention and assume that $a \notin \mathcal{N}_a$. We show in Section 4.2 that all results hold if the past choices of Agent $a$ affect either her consideration sets or preferences. Since we allow for the possibility that some peers affect both considerations and preferences, $\mathcal{NCR}_a = \mathcal{NC}_a \bigcap \mathcal{NR}_a$ can be non-empty.

**Choice Revision Process** We model the revision process of alternatives as a standard continuous-time Markov process on the space of choice configurations. We assume that agents are endowed with independent Poisson "alarm clocks" with rates $(\lambda_a)_{a\in\mathcal{A}}$. At randomly given moments (exponentially distributed with mean $1/\lambda_a$) the alarm of Agent $a$ goes off.[7] When this happens, the agent forms a consideration set and then selects an alternative among the ones she is actually considering. Since the probability of any two alarm clocks going off simultaneously is zero, the probability that two agents make choices simultaneously is also zero. This observation has useful implications for identification. (We discuss a model with perfectly correlated clocks in Section 4.5.)

**Peer Effect in Consideration Sets** The probability that Agent $a$ pays attention to and, thereby, includes a particular alternative in her consideration set depends on the choice configuration at the moment of revising decisions. We indicate by $\mathrm{Q}_a(v \mid \mathbf{y}, \mathcal{NC}_a)$ the probability that Agent $a$ considers alternative $v$ given a choice configuration $\mathbf{y}$ and her consideration reference group $\mathcal{NC}_a$. Assuming that each alternative is added to the consideration set independently from other alternatives, the probability of facing consideration set $\mathcal{C}$, which is a subset of menu $\mathcal{Y}$, is

$$\mathrm{C}_a(\mathcal{C} \mid \mathbf{y}, \mathcal{NC}_a, \mathcal{Y}) = \prod\nolimits_{v\in\mathcal{C}} \mathrm{Q}_a(v \mid \mathbf{y}, \mathcal{NC}_a) \prod\nolimits_{v\in\mathcal{Y}\setminus\mathcal{C}} (1 - \mathrm{Q}_a(v \mid \mathbf{y}, \mathcal{NC}_a)).$$

[7] That is, each Agent $a$ is endowed with a collection of random variables $\{\tau_n^a\}_{n=1}^{\infty}$ such that each difference $\tau_n^a - \tau_{n-1}^a$ is exponentially distributed with mean $1/\lambda_a$. These differences are independent across people and time.

By definition, the default alternative is always considered, so the consideration set is never empty. That is, $\mathrm{Q}_a\left(0 \mid \mathbf{y}, \mathcal{NC}_a\right) = 1$. Leaving aside peer effects, this process of formation of consideration sets is analogous to the one studied by Manski (1977) and Manzini and Mariotti (2014). We next offer a simple example.

**Example 1.** Assume that the attention given to alternative $v$ is determined by its popularity among peers. In particular, its inclusion into the consideration set could be modeled as an indicator function $\mathbb{1}\left(c_{v,a}\left(\mathbf{y}, \mathcal{NC}_a\right) \geq \varepsilon_v\right)$, where $c_{v,a}$ measures the mean attention of Agent $a$ to alternative $v$ as a function of the choices of her peers and $\varepsilon_v$ is an independent of $\mathbf{y}$ random attention shock. That is, $v$ is considered if and only if $c_{v,a}\left(\mathbf{y}, \mathcal{NC}_a\right) \geq \varepsilon_v$. Then, the probability of considering $v$ is

$$\mathrm{Q}_a\left(v \mid \mathbf{y}, \mathcal{NC}_a\right) = F_\varepsilon\left(c_{v,a}\left(\mathbf{y}, \mathcal{NC}_a\right)\right)$$

where $F_\varepsilon$ denotes the cumulative distribution function (cdf) of $\varepsilon$. $\square$

**Peer Effect in Preferences** After the consideration set is formed, the agent selects an alternative among the ones she is actually considering according to some choice rule. We allow preferences to be random. We do not need to specify a particular form of utility functions since our object of interest is the choice rule —the distribution over elements of a given consideration set. In practice, one can identify and estimate the underlying preferences from the choice rule. Therefore, we focus on the choice rule and leave the association between the choice rule and preferences to be flexible. Formally, given consideration set $\mathcal{C}$, we let the choice rule $\mathrm{R}_a\left(\cdot \mid \mathbf{y}, \mathcal{NR}_a, \mathcal{C}\right)$ be a distribution function supported on $\mathcal{C}$. That is, $\mathrm{R}_a\left(v \mid \mathbf{y}, \mathcal{NR}_a, \mathcal{C}\right) \geq 0$ for all $v$ and

$$\textstyle\sum_{v \in \mathcal{C}} \mathrm{R}_a\left(v \mid \mathbf{y}, \mathcal{NR}_a, \mathcal{C}\right) = 1.$$

The choice rule tells us what the probability of picking an alternative is in a given consideration set. Choice rules summarize the decision process after the consideration set is formed. The choices of agents may be random from the researcher's perspective if there are latent preference shocks (see Example 2) or if agents randomize when indifferent.

Importantly, we incorporate the peer effect in preferences by allowing the choice rule of each agent to depend on the configuration of choices and her preference reference group.

We next offer a simple example.

**Example 2.** Suppose the utility that Agent $a$ gets from alternative $v$ if it is considered in set $\mathcal{C}$ is given by $u_{a,v,\mathcal{C}}(\mathbf{y}, \mathcal{NR}_a) + \xi_v$, where $u_{a,v,\mathcal{C}}$ captures the mean utility from the alternative when it is in the given consideration set and $\xi_v$s are independent and identically distributed (i.i.d.) taste shocks that are distributed according to the standard Type I extreme value distribution. As a result, for $v \in \mathcal{C}$,

$$\mathrm{R}_a(v \mid \mathbf{y}, \mathcal{NR}_a, \mathcal{C}) = \frac{\exp\left(u_{a,v,\mathcal{C}}(\mathbf{y}, \mathcal{NR}_a)\right)}{\sum_{v' \in \mathcal{C}} \exp\left(u_{a,v',\mathcal{C}}(\mathbf{y}, \mathcal{NR}_a)\right)}.$$

Note that in this example, we allow consideration sets to directly affect utilities from alternatives. In particular, we allow for violations of the independence of irrelevant alternatives since, for $v \neq v'$,

$$\frac{\mathrm{R}_a(v \mid \mathbf{y}, \mathcal{NR}_a, \mathcal{C})}{\mathrm{R}_a(v' \mid \mathbf{y}, \mathcal{NR}_a, \mathcal{C})} = \frac{\exp\left(u_{a,v,\mathcal{C}}(\mathbf{y}, \mathcal{NR}_a)\right)}{\exp\left(u_{a,v',\mathcal{C}}(\mathbf{y}, \mathcal{NR}_a)\right)}$$

can vary with $\mathcal{C}$. □

By combining preferences and random consideration sets, the probability that Agent $a$ selects (at the moment of choosing) alternative $v$ given choice configuration $\mathbf{y}$, $\mathrm{P}_a(v \mid \mathbf{y})$, has to satisfy

$$\mathrm{P}_a(v \mid \mathbf{y}) = \sum_{\mathcal{C} \subseteq \mathcal{Y}} \mathrm{R}_a(v \mid \mathbf{y}, \mathcal{NR}_a, \mathcal{C}) \prod_{v' \in \mathcal{C}} \mathrm{Q}_a(v' \mid \mathbf{y}, \mathcal{NC}_a) \prod_{v' \in \mathcal{Y} \setminus \mathcal{C}} \left(1 - \mathrm{Q}_a(v' \mid \mathbf{y}, \mathcal{NC}_a)\right). \quad (1)$$

The continuous-time Markov process leads the timing for the revision process of alternatives. Altogether, the elements just described characterize our initial model of peer effects in choices. This model leads to a sequence of decisions of every agent that evolve through time according to a Markov random process. We aim to identify $\mathcal{NR}_a$, $\mathcal{NC}_a$, $\mathrm{R}_a$, and $\mathrm{Q}_a$ from observing a sequence of choices of agents over time.

*Remark* 1. Our identification arguments use *only* variation in the choices of peers. That is, we do not use exogenous variation in observable characteristics (i.e., covariates) or menus. Thus, to simplify the exposition, at this stage, we do not include observable covariates in the model. We can interpret our setting as if we were conditioned on them. We show in our application that covariates in the data can be easily incorporated into the model for estimation purposes.

Relatedly, though, as we just said, we only use endogenous variation in the choice of peers for identification, our results can be extended to the case of exogenous variation in observed covariates. In particular, alternative-specific covariates can serve as exclusion restrictions for the consideration or preference. For example, the product-specific level of advertisement might only affect attention to such a specific product (Goeree, 2008).

### 2.2. Main Assumptions

Our results for equilibrium existence and identification build on the three assumptions we discuss next. (We will add later a few extra restrictions needed for the recoverability of specific elements of our model.) To present them, we need to add a little bit of notation. Let $\mathrm{NC}_a^v(\mathbf{y})$ and $\mathrm{NR}_a^v(\mathbf{y})$ be the number of agents in the consideration and preference reference groups of Agent $a$ who select option $v$ in choice configuration $\mathbf{y}$. Formally,

$$\mathrm{NC}_a^v(\mathbf{y}) = |\{a' \in \mathcal{NC}_a \ : \ y_{a'} = v\}| \ \text{ and } \ \mathrm{NR}_a^v(\mathbf{y}) = |\{a' \in \mathcal{NR}_a \ : \ y_{a'} = v\}|,$$

where $|A|$ is the cardinality of $A$. Define, $\mathrm{NR}_a^{\mathcal{C}}(\mathbf{y}) = (\mathrm{NR}_a^v(\mathbf{y}))_{v\in\mathcal{C}\setminus\{0\}}$ and $\mathrm{NC}_a^{\mathcal{C}}(\mathbf{y}) = (\mathrm{NC}_a^v(\mathbf{y}))_{v\in\mathcal{C}\setminus\{0\}}$. We write $\mathrm{nc}^v$ for a possible value of $\mathrm{NC}_a^v(\mathbf{y})$. The first assumption is as follows.

**Assumption 1** (Consideration)**.** For each $a \in \mathcal{A}$, $\mathbf{y} \in \mathcal{Y}^A$, and $v \neq 0$, we have that

(i) $\mathrm{Q}_a(v \mid \mathbf{y}, \mathcal{NC}_a) > 0$;

(ii) $\mathrm{Q}_a(v \mid \mathbf{y}, \mathcal{NC}_a) \equiv \mathrm{Q}_a(v \mid \mathrm{NC}_a^v(\mathbf{y}))$; and

(iii) $\mathrm{Q}_a(v \mid n+1) / \mathrm{Q}_a(v \mid n)$ is different from 1 and is also different from $\mathrm{Q}_a(v \mid n+2) / \mathrm{Q}_a(v \mid n+1)$ for $n = 0$.

Assumption 1(i) states that the probability of considering each option is strictly positive regardless of how many peers have selected that option. This assumption captures the idea that people can eventually pay attention to an alternative for various reasons that are outside the control of our model (e.g., watching an ad on television or receiving a coupon). Moreover, we allow alternatives to be considered with probability 1. Assumption 1(ii) says that the probability of considering a specific option depends on the number (but not the identity) of the consideration peers that currently selected it. Assumption 1(iii) is a mild shape restriction that is satisfied if the consideration probabilities are not constant or exponential functions (i.e., $\ln \mathrm{Q}_a(v \mid \cdot)$ is nonlinear) of the number of peers. Instead of $n = 0$, this restriction can be imposed at any point in the support. This assumption allows for different levels of satiation —e.g. consideration changes only when the number of peers picking the option achieves a given threshold (e.g., 10 agents, 20 agents, etc.). Assumption 1(iii) is a variability requirement stating that choices of consideration peers effectively have an effect on consideration probabilities. We do not need the full power of Assumption 1(iii) for all our results. For instance, identification of $\mathcal{N}_a$ and $\mathcal{NR}_a$ would only require the consideration probabilities to vary with the choices of peers. However, the identification of $\mathcal{NC}_a$ requires non-exponential consideration probabilities.

**Example 1** (continued)**.** Suppose that the mean attention of Agent $a$, $c_{v,a}$, is such that

$$c_{v,a}(\mathbf{y}, \mathcal{NC}_a) = \mathbb{1}\left( \mathrm{NC}_a^v(\mathbf{y}) > 0 \right).$$

As a result,

$$\mathrm{Q}_a(v|\mathbf{y}, \mathcal{NC}_a) = F_\varepsilon \left( \mathbb{1}\left( \mathrm{NC}_a^v(\mathbf{y}) > 0 \right) \right),$$

and, if $F_\varepsilon(1) > F_\varepsilon(0) > 0$, then Assumption 1 is satisfied. □

The second assumption restricts the preference part of the decision process.

**Assumption 2** (Preferences)**.** For each $a \in \mathcal{A}$, $\mathbf{y} \in \mathcal{Y}^A$, $\mathcal{C} \subseteq \mathcal{Y}$, and $v \in \mathcal{C} \setminus \{0\}$, we have that

(i) $\mathrm{R}_a(v \mid \mathbf{y}, \mathcal{NR}_a, \mathcal{C}^*) > 0$ for some $\mathcal{C}^*$ such that $\mathrm{C}_a(\mathcal{C}^*|\mathbf{y}, \mathcal{NC}_a) > 0$;

(ii) $\mathrm{R}_a(v \mid \mathbf{y}, \mathcal{NR}_a, \mathcal{C}) \equiv \mathrm{R}_a\left(v \mid \mathrm{NR}_a^{\mathcal{C}}(\mathbf{y}), \mathcal{C}\right)$; and

(iii) $\mathrm{R}_a\left(v \mid \mathrm{nr}^{\mathcal{C}}, \mathcal{C}\right)$, where $nr^{\mathcal{C}} = (nr^w)_{w \in \mathcal{C}\setminus\{0\}}$, is injective in $\mathrm{nr}^w$ for $\mathrm{nr}^w \in \{0, 1\}$ and $\mathrm{nr}^{\mathcal{C}\setminus\{w\}} = (0, 0, \ldots, 0)'$ for all $w \in \mathcal{C}$.

Assumption 2(i) requires each alternative to be picked with a positive probability at least in one consideration set that is observed with a positive probability. Together with Assumption 1(i), it implies that every alternative can be picked with a positive probability. This assumption allows for both random and deterministic choice rules. Assumption 2(ii) states that the choice rule depends on the number (but not the identity) of preference peers that selected each of the alternatives in the consideration set. Assumption 2(iii) assumes that the random preferences over the alternatives are changing —either increasing or decreasing— on the number of preference peers that select each option in the consideration set. The effect is required to be strict only around the origin, that is, when all other peers select the default. Similarly to Assumption 1(iii), the direction of the peer effect in preferences does not need to be known and can be different for different agents and alternatives. (The signs of all these effects can be recovered from the data.)

**Example 2** (continued)**.** Suppose that the mean utility of Agent $a$ from alternative $v$ given the consideration set $\mathcal{C}$, $u_{a,v,\mathcal{C}}$, is such that

$$u_{a,v,\mathcal{C}}\left(\mathbf{y},\mathcal{NR}_a\right)=\bar{u}_{a,v,\mathcal{C}}\left(\mathrm{NR}_a^v\left(\mathbf{y}\right)\right),$$

where $\bar{u}_{a,v,\mathcal{C}}(\cdot)$ is a function that satisfies $\bar{u}_{a,v,\mathcal{C}}(0)\neq\bar{u}_{a,v,\mathcal{C}}(1)$. Then Assumptions 2 is satisfied. □

We extend the model in Section 4.2 to allow the dependence of $\mathrm{Q}_a$ and $\mathrm{R}_a$ on the current or past choices of Agent $a$ (e.g., a Markov process with memory), thus capturing environments with inertia by setting the probability of considering the current choice with probability 1. We write the assumption in a stricter way here only to simplify the exposition.

The third assumption imposes some restrictions on the network of each person.

**Assumption 3** (Network)**.** For each $a\in\mathcal{A}$, if $|\mathcal{NCR}_a|\geq 1$, then $|\mathcal{NC}_a\setminus\mathcal{NR}_a|+|\mathcal{NR}_a\setminus\mathcal{NC}_a|\geq 1$.

Assumption 3 is an exclusion restriction. It simply states that if the agent has a peer that simultaneously affects consideration and preferences, then the agent also has at least another peer that affects either only consideration or only preferences. The choice of such peer only enters either consideration probability or choice rule, and so provides an exclusion restriction. Note that we do not need the two sets of agents to be nonempty. Indeed, if only one of the peer-effect mechanisms operates in practice, our results would allow us to state whether the interdependencies in choices of agents are due to the peer effect in preferences or in consideration. Moreover, this assumption does not rule out agents with no peers at all. (This happens in our empirical application. We will state there how to proceed when some agents have no links.)

### 2.3. Equilibrium Behavior

In this subsection, we define a notion of equilibrium and establish its existence and uniqueness.

The i.i.d. Poisson alarm clocks, which lead the revision process, guarantee that at each moment of time, at most, one agent revises her selection almost surely. Thus, the transition rates between choice configurations that differ in more than one agent changing the current selection are zero. The advantage of this fact for model identification is that there are fewer terms to recover. Blevins (2017, 2018) offer a nice discussion of this feature and its advantage over discrete time models. (In Section 4.5, we extend the idea to coordinated clocks.) Formally, the transition rate from choice configuration $\mathbf{y}$ to any different one $\mathbf{y}'$ is as follows

$$\mathrm{m}\left(\mathbf{y}' \mid \mathbf{y}\right) = \begin{cases} 0 & \text{if } \sum_{a\in\mathcal{A}} \mathbb{1}\left(y'_a \neq y_a\right) > 1 \\ \sum_{a\in\mathcal{A}} \lambda_a \, \mathrm{P}_a\left(v \mid \mathbf{y}\right) \mathbb{1}\left(y'_a \neq y_a\right) & \text{if } \sum_{a\in\mathcal{A}} \mathbb{1}\left(y'_a \neq y_a\right) = 1 \end{cases}. \tag{2}$$

In the statistical literature on continuous-time Markov processes, these transition rates are the out of diagonal terms of the *transition rate matrix* (also known as the *infinitesimal generator matrix*). The diagonal terms simply build from these other values as follows

$$\mathrm{m}\left(\mathbf{y} \mid \mathbf{y}\right) = -\sum\nolimits_{\mathbf{y}'\in\mathcal{Y}^A\setminus\{\mathbf{y}\}} \mathrm{m}\left(\mathbf{y}' \mid \mathbf{y}\right).$$

We indicate by $\mathcal{M}$ the transition rate matrix. In our model, the number of possible choice configurations is $(Y+1)^A$. Thus, $\mathcal{M}$ is a $(Y+1)^A \times (Y+1)^A$ matrix. There are many different ways of ordering the choice configurations and thereby writing the transition rate matrix. To avoid any sort of ambiguity in the exposition, we let the choice configurations be ordered according to the lexicographic order. Constructed in this way the first element of $\mathcal{M}$ is $\mathcal{M}_{11} = \mathrm{m}\Big((0,0,\ldots,0)' \mid (0,0,\ldots,0)'\Big)$.

Formally, let $\iota(\mathbf{y}) \in \left\{1, 2, \ldots, (Y+1)^A\right\}$ be the position of $\mathbf{y}$ according to the lexicographic order. Then,

$$\mathcal{M}_{\iota(\mathbf{y})\iota(\mathbf{y}')} = \mathrm{m}(\mathbf{y}' \mid \mathbf{y}).$$

An equilibrium in this model is an invariant distribution $\mu : \mathcal{Y}^A \to (0,1)$, with $\sum_{\mathbf{y}\in\mathcal{Y}^A} \mu(\mathbf{y}) = 1$, of the dynamic process with transition rate matrix $\mathcal{M}$. It indicates the likelihood of each choice configuration $\mathbf{y}$ in the long run. This equilibrium behavior relates to the transition rate matrix in a linear fashion

$$\mu\mathcal{M} = \mathbf{0}.$$

The next proposition establishes equilibrium existence and uniqueness for our model.

**Proposition 2.1.** *If Assumptions 1(i) and 2(i) hold, then there exists a unique equilibrium $\mu$.*

*Proof.* Given the simplicity of the result, we prove it here. For an irreducible, finite-state, continuous Markov chain, the equilibrium $\nu$ exists, and it is unique. Thus, we only need to prove that Assumptions 1(i) and 2(i) imply that the Markov chain induced by our model is irreducible. Note that

$$\mathrm{P}_a(v \mid \mathbf{y}) = \sum\nolimits_{\mathcal{C}\subseteq\mathcal{Y}} \mathrm{R}_a(v \mid \mathbf{y}, \mathcal{NR}_a, \mathcal{C})\, \mathrm{C}_a(\mathcal{C} \mid \mathbf{y}, \mathcal{NC}_a, \mathcal{Y}).$$

Assumption 1(i) implies that given any $\mathbf{y}$, any $v$ is always considered with a positive probability by any Agent $a$. Assumption 2(i) then implies that any option is picked with a positive probability if considered. Thus, $0 < \mathrm{P}_a(v \mid \mathbf{y}) < 1$ for all $a$ and $\mathbf{y}$, and we can go from one configuration to the other one in less than $A$ steps with a positive probability. ■

It is interesting to remark that (under our restrictions) the equilibrium of the model is an invariant distribution with full support on $\mathcal{Y}^A$. The full support feature allows the identification of all parts of the model. If this result fails, then identification of the model can still be achieved, but it requires some extra restrictions. Section 4.2 extends the analysis to cover that case.

# 3. Empirical Content of the Model

This section shows that under specific modelling restrictions, the researcher can uniquely recover the set of connections, $\mathcal{NC}_a$ and $\mathcal{NR}_a$, the attention mechanism, $\mathrm{Q}_a$, the choice rule, $\mathrm{R}_a$, and the Poisson alarm clock, $\lambda_a$, for every Agent $a$.

We separate the identification analysis in two parts. Let $\mathrm{P} = (\mathrm{P}_a)_{a \in \mathcal{A}}$ be the profile of choice probabilities of agents in the network. Each $\mathrm{P}_a(v \mid \mathbf{y}) : \mathcal{Y} \times \mathcal{Y}^A \to (0,1)$ specifies the (ex-ante) probability that Agent $a$ selects option $v$ when the choice configuration is $\mathbf{y}$. Under our main assumptions, we have that

$$\mathrm{P}_a(v \mid \mathbf{y}) = \sum\nolimits_{\mathcal{C} \subseteq \mathcal{Y}} \mathrm{R}_a\left(v \mid \mathrm{NR}_a^{\mathcal{C}}(\mathbf{y}), \mathcal{C}\right) \prod\nolimits_{v \in \mathcal{C}} \mathrm{Q}_a\left(v \mid \mathrm{NC}_a^v(\mathbf{y})\right) \prod\nolimits_{v \in \mathcal{Y} \setminus \mathcal{C}} \left(1 - \mathrm{Q}_a\left(v \mid \mathrm{NC}_a^v(\mathbf{y})\right)\right).$$

First, we show that each set of CCPs P maps into a different set of connections, choice rules, and consideration mechanisms. Thus, knowledge of CCPs allows us to uniquely recover all the elements of the model. Second, we build identification of the CCPs P from a long sequence of observed choices.

## 3.1. Identification of the Model from P

The identification strategy we offer is constructive. We start by recovering the network structure, which is achieved in three stages. First, we recover the reference group of Agent $a$ by checking whether Agent $a'$ affects her choices. Second, we can recover whether peer $a'$ affects Agent $a$'s consideration only by checking a double difference of the log CCP using another agent in the reference group of Agent $a$. Lastly, we show how to distinguish between a peer that affects preferences only (preference-only peer) and a peer that affects consideration and preferences (consideration-preference peer). We then use this information to recover the attention mechanisms and the random preferences.

**Network** Let us first note that, under Assumptions 1 and 2, changes in the choices of peers induce variation in the CCPs. To see this, let us first note that

$$\mathrm{P}_a\left(v \mid \mathbf{y}\right) = \mathrm{Q}_a\left(v \mid \mathrm{NC}_a^v\left(\mathbf{y}\right)\right) \times \sum\nolimits_{\mathcal{C}\subseteq\mathcal{Y}\setminus\{v\}} \mathrm{R}_a\left(v \mid \mathrm{NR}_a^{\mathcal{C}\cup\{v\}}\left(\mathbf{y}\right), \mathcal{C}\cup\{v\}\right) \mathrm{C}_a\left(\mathcal{C} \mid \mathrm{NC}_a^{\mathcal{Y}\setminus\{v\}}(\mathbf{y}), \mathcal{Y}\setminus\{v\}\right).$$

In words, the observed probability that $v$ is picked is equal to the product of the probability that it is considered and the probability that it is picked when considered. Moreover, the first term $\mathrm{Q}_a\left(v \mid \mathrm{NC}_a^v\left(\mathbf{y}\right)\right)$ only depends on $\mathrm{NC}_a^v\left(\mathbf{y}\right)$, and $\mathrm{NC}_a^v\left(\mathbf{y}\right)$ does not affect the second term. This multiplicative structure allows us to take advantage of certain changes in logarithms of $\mathrm{P}_a$.

Let $\Delta_{a'}^v$ be a linear operator that indicates the increment of a given function when the action of Agent $a$ changes to $v$ in the action configuration $\mathbf{y}$. Formally, given $f: \mathcal{Y}^A \to \mathbb{R}$, let

$$\Delta_{a'}^v f(\mathbf{y}) = f(\mathbf{y}_{a'}^v) - f(\mathbf{y}),$$

where $\mathbf{y}_{a'}^v$ denotes the vector in which the $a'$-th component of $\mathbf{y}$ is replaced by $v$.

We first identify the reference group of Agent $a$ by using changes in her CCPs. Intuitively, Agent $a'$ is in the reference group of Agent $a$ if changing her choice in the choice configuration would affect the decision of Agent $a$. Specifically, by computing the implied difference in the logarithms of $\mathrm{P}_a$, we get that

$$\begin{aligned}\Delta_{a'}^v \ln \mathrm{P}_a\left(v \mid \mathbf{0}\right) &\equiv \ln \mathrm{P}_a\left(v \mid \mathbf{0}_{a'}^v\right) - \ln \mathrm{P}_a\left(v \mid \mathbf{0}\right)\\ &= \Delta_{a'}^v \ln \mathrm{Q}_a\left(v \mid \mathrm{NC}_a^v\left(\mathbf{0}\right)\right) +\\ &\Delta_{a'}^v \ln \sum\nolimits_{\mathcal{C}\subseteq\mathcal{Y}\setminus\{v\}} \mathrm{R}_a\left(v \mid \mathrm{NR}_a^{\mathcal{C}\cup\{v\}}\left(\mathbf{0}\right), \mathcal{C}\cup\{v\}\right) \mathrm{C}_a\left(\mathcal{C} \mid \mathrm{NC}_a^{\mathcal{Y}\setminus\{v\}}(\mathbf{0}), \mathcal{Y}\setminus\{v\}\right), \quad (3)\end{aligned}$$

where $\mathbf{0} = (0, 0, \ldots, 0)'$ denotes the configuration where everyone picks the default. Each term in Equation (3) relates to one (and only one) mechanism of peer effects: the first term reflects (if present) the peer effect in consideration. The second term captures the peer effect in preferences.

When peer effects in consideration and preferences are of the same sign, then, under Assumptions 1 and 2, $\Delta_{a'}^v \ln \mathrm{P}_a\left(v \mid \mathbf{0}\right) \neq 0$ if and only if Agent $a'$ is in the reference group of Agent $a$ (i.e. $a' \in \mathcal{N}_a$). When the interaction effects are of different signs, the "if" part of this result requires a "regularity condition" (Assumption 5) that we discuss in details in Appendix A. This extra condition rules out the possibility that peer effects in consideration and preferences be of opposite signs *and* of equal magnitude. (Though we formally need to rule out this possibility, we note that it represents an extremely unlikely event under any thinkable model.) Under these conditions, it follows that the reference groups (even if they are empty) can be recovered from the CCPs.

**Proposition 3.1.** *Suppose Assumptions 1, 2, and 5 hold. Then, for any $a \in \mathcal{A}$, $\mathcal{N}_a$ is identified.*

We now proceed to identify whether Agent $a'$ in $\mathcal{N}_a$ affects Agent $a$'s consideration only. Note that by analyzing the differences of logarithms of $\mathrm{P}_a$, we can identify the reference group of each agent, but these differences are silent about the mechanism by which the interactions happen. To see why, note that (for instance) a nonzero $\Delta_{a'}^{v} \ln \mathrm{P}_a\left(v \mid \mathbf{0}\right)$ could be generated from the first summand in Equation (3) via $\mathrm{Q}_a$ and/or from the second summand via $\mathrm{R}_a$. But these two terms differ in that the second summand varies with the number of peers that select alternatives that are *different* from $v$, while the first term does not. Thus, the two mechanisms can be set apart (at least partially) by a second shift in $\ln \mathrm{P}_a\left(v \mid \mathbf{0}\right)$. Let $a', a'' \in \mathcal{N}_a$ and $w \in \mathcal{Y} \setminus \{0\}$ with $w \neq v$. Since $\Delta_a^v$ is a linear operator, we can define the double difference as follows

$$\begin{aligned}
\Delta_{a''}^{w} \Delta_{a'}^{v} \ln \mathrm{P}_a\left(v \mid \mathbf{0}\right) &= \Delta_{a''}^{w}\left[\ln \mathrm{P}_a\left(v \mid \mathbf{0}_{a'}^{v}\right) - \ln \mathrm{P}_a\left(v \mid \mathbf{0}\right)\right] \\
&= \Delta_{a''}^{w} \ln \mathrm{P}_a\left(v \mid \mathbf{0}_{a'}^{v}\right) - \Delta_{a''}^{w} \ln \mathrm{P}_a\left(v \mid \mathbf{0}\right) \\
&= \left[\ln \mathrm{P}_a\left(v \mid (\mathbf{0}_{a'}^{v})_{a''}^{w}\right) - \ln \mathrm{P}_a\left(v \mid \mathbf{0}_{a'}^{v}\right)\right] - \left[\ln \mathrm{P}_a\left(v \mid \mathbf{0}_{a''}^{w}\right) - \ln \mathrm{P}_a\left(v \mid \mathbf{0}\right)\right].
\end{aligned}$$

Specifically, we have that

$$\begin{aligned}
\Delta_{a''}^{w} \Delta_{a'}^{v} \ln \mathrm{P}_a\left(v \mid \mathbf{0}\right) = & \Delta_{a''}^{w} \Delta_{a'}^{v} \ln \mathrm{Q}_a\left(v \mid \mathrm{NC}_a^{v}\left(\mathbf{0}\right)\right) + \\
& \Delta_{a''}^{w} \Delta_{a'}^{v} \ln \textstyle\sum_{\mathcal{C} \subseteq \mathcal{Y} \setminus \{v\}} \mathrm{R}_a\left(v \mid \mathrm{NR}_a^{\mathcal{C} \cup \{v\}}\left(\mathbf{0}\right), \mathcal{C} \cup \{v\}\right) \mathrm{C}_a\left(\mathcal{C} \mid \mathrm{NC}_a^{\mathcal{Y} \setminus \{v\}}(\mathbf{0}), \mathcal{Y} \setminus \{v\}\right).
\end{aligned}$$

Note that if Agent $a'$ is a consideration-only peer (i.e. $a' \in \mathcal{NC}_a \setminus \mathcal{NR}_a$), then

$$\Delta_{a'}^{v} \ln \textstyle\sum_{\mathcal{C} \subseteq \mathcal{Y} \setminus \{v\}} \mathrm{R}_a\left(v \mid \mathrm{NR}_a^{\mathcal{C} \cup \{v\}}\left(\mathbf{0}\right), \mathcal{C} \cup \{v\}\right) \mathrm{C}_a\left(\mathcal{C} \mid \mathrm{NC}_a^{\mathcal{Y} \setminus \{v\}}(\mathbf{0}), \mathcal{Y} \setminus \{v\}\right) = 0.$$

As a result,

$$\Delta_{a''}^{w}\Delta_{a'}^{v}\ln \mathrm{P}_{a}\left(v \mid \mathbf{0}\right) = \Delta_{a''}^{w}\Delta_{a'}^{v}\ln \mathrm{Q}_{a}\left(v \mid \mathrm{NC}_{a}^{v}\left(\mathbf{0}\right)\right) = \Delta_{a''}^{w}\left[\ln \mathrm{Q}_{a}\left(v \mid 1\right) - \ln \mathrm{Q}_{a}\left(v \mid 0\right)\right] = 0,$$

since $\mathrm{Q}_{a}\left(v \mid \mathrm{NC}_{a}^{v}\left(\mathbf{0}\right)\right)$ does not depend on the number of peers who picked $w$. However, if Agent $a'$ affects preferences, then the second summand in Equation (3) will not disappear after switching Agent $a''$ from $v$ to $w$. (For this result, we invoke again the regularity condition we mentioned above.) Overall, we show that $\Delta_{a''}^{w}\Delta_{a'}^{v}\ln \mathrm{P}_{a}\left(v \mid \mathbf{0}\right) \neq 0$ if and only if Agent $a'$ is a preference peer of Agent $a$ (i.e. $a' \in \mathcal{NR}_{a}$). Thus, by checking the double difference for each agent in the reference group of Agent $a$, we can divide her reference group into consideration-only peers and preference peers (which may or may not affect consideration).

Finally, the group of preference peers can be further separated into two sets of agents by the magnitude of the changes in CCPs. Specifically, for $a' \in \mathcal{NCR}_{a}$ and $a'' \in \mathcal{NR}_{a} \setminus \mathcal{NC}_{a}$, we have that

$$\Delta_{a'}^{v}\ln \mathrm{P}_{a}\left(v \mid \mathbf{0}\right) \neq \Delta_{a''}^{v}\ln \mathrm{P}_{a}\left(v \mid \mathbf{0}\right).$$

This allows us to separate the preference peers into two different groups. Let us refer to these sets as $\mathcal{M}'$ and $\mathcal{M}''$ (each of these sets is allowed to be empty). While we know that one of these sets is $\mathcal{NCR}_{a}$ (without further restrictions), we cannot tell which. (We address this last step after the next proposition.)

**Proposition 3.2.** *Suppose Assumptions 1, 2, and 5 hold. For any $a \in \mathcal{A}$, if $Y \geq 2$ and $|\mathcal{N}_{a}| \geq 2$, then $\mathcal{NC}_{a} \setminus \mathcal{NR}_{a}$ and $\mathcal{NR}_{a} = \mathcal{M}' \cup \mathcal{M}''$ are identified. Moreover, $\mathcal{NCR}_{a} \in \{\mathcal{M}', \mathcal{M}''\}$.*

Our last step in recovering the network structure is to identify the set of consideration-preference peers (i.e. $\mathcal{NCR}_{a}$) from the group of peers that affect preferences. We discuss the identification with and without consideration-only peers separately. By Assumption 3, if this group is non-empty, then there exists a peer that is either consideration-only or preferences-only. Let us assume that we have already identified two peers such that Agent $a'$ is a consideration-only peer (i.e., $a' \in \mathcal{NC}_{a} \setminus \mathcal{NR}_{a}$)

and Agent $a''$ affects preferences (i.e., $a'' \in \mathcal{NR}_a$). Note that

$$\Delta^v_{a''}\Delta^v_{a'} \ln \mathrm{P}_a (v \mid \mathbf{0}) = \Delta^v_{a''}\Delta^v_{a'} \ln \mathrm{Q}_a (v \mid \mathrm{NC}^v_a (\mathbf{0})).$$

This is so because, since Agent $a'$ only affects consideration, the second term in Equation (3) is zero. Thus, if Assumption 1 holds, $a'' \in \mathcal{NCR}_a$ if and only if $\Delta^v_{a''}\Delta^v_{a'} \ln \mathrm{P}_a (v \mid \mathbf{0}) \neq 0$.

Suppose next that there is no consideration-only peer. We can still implement a similar idea by replicating the consideration-only peer behavior with two peers, one that affects consideration and preferences and the other one that affects only preferences. Notice that these two peers can be identified by Proposition 3.2. Let us pick some Agent $a' \in \mathcal{M}'$ and Agent $a'' \in \mathcal{M}''$. Next, take $\mathbf{y}$ such that $y_a = 0$ for all $a \neq a'$ and $y_{a'} = v$. Next note that

$$\ln \mathrm{P}_a (v \mid \mathbf{y}) - \ln \mathrm{P}_a \left(v \mid \left(\mathbf{y}^0_{a'}\right)^v_{a''}\right) = (-1)^{\mathbb{1}(a' \notin \mathcal{NCR}_a)} (\ln \mathrm{Q}_a (v \mid 1) - \ln \mathrm{Q}_a (v \mid 0)).$$

Thus, this operation with two peers, one from $\mathcal{NCR}_a$ and the other one from $\mathcal{NR}_a \setminus \mathcal{NC}_a$, is equivalent (up to sign) to switching one peer from $\mathcal{NC}_a \setminus \mathcal{NR}_a$. Then, let us take another Agent $a'''$ in either $\mathcal{M}'$ or $\mathcal{M}''$ and proceed as we did before —when there was a peer in the consideration-only group. In doing so, we identify whether Agent $a'''$ is a consideration-preference or preference-only peer. (Note that this procedure requires to have at least three peers in $\mathcal{N}_a$.) This information allows us to know whether $\mathcal{NCR}_a = \mathcal{M}'$ or $\mathcal{NCR}_a = \mathcal{M}''$.

**Proposition 3.3.** *Suppose Assumptions 1, 2, 3, and 5 hold. Suppose also that $\mathcal{NC}_a \setminus \mathcal{NR}_a$ is identified (or known) and $|\mathcal{N}_a| \geq 3 - |\mathcal{NC}_a \setminus \mathcal{NR}_a|$. Then, $\mathcal{NC}_a$ and $\mathcal{NR}_a$ are identified.*

The last proposition offers final conditions for all the parts of the network structure to be identified. It takes as an input the consideration-only set $\mathcal{NC}_a \setminus \mathcal{NR}_a$, which is allowed to be empty. It states that if we know or identify the consideration-only peers, the full network structure is identified when there are enough peers. Moreover, the result holds even for binary settings (i.e. $Y = 1$). However, we still need $Y \geq 2$ to identify $\mathcal{NC}_a \setminus \mathcal{NR}_a$ using Proposition 3.2. We discuss the binary case in detail in Section 4.1.

To sum up, the reference group of Agent $a$, $\mathcal{N}_a$, is identified by checking the variation in $\ln \mathrm{P}_a$

as we switch other agents from the default alternative to a specific one $v$. If, in doing so, we identify that the agent has two or more friends, then we can recover the consideration-only peers by using the additive separability of $\ln \mathrm{P}_{\mathrm{a}}(v \mid \mathbf{y})$ in $\ln \mathrm{Q}_a(v \mid \mathbf{y})$. Finally, if we identify at least one consideration-only peer, then we can use her as a baseline to identify all other types of peers. Otherwise, we *create* such a peer by mixing the behavior of a consideration-preference peer with the behavior of a preference-only peer, and use the behavior of the *constructed* peer as a baseline to complete the network identification. We need to have at least three peers in this scenario.

**Attention Mechanisms and Random Preferences** We first state that if we know the network structure, and each agent has at least one consideration-only peer —or such peer can be constructed from consideration-preference and preference-only peers, as we do above— then we can recover ratios of probabilities of considering alternatives. To show this claim, let $a' \in \mathcal{NC}_a \setminus \mathcal{NR}_a$. Then, it is easy to see that since Agent $a'$ only affects consideration, we can shift Agent $a'$'s choice from the default to $v$ and recover some information about the peer effect in consideration. Specifically, we have

$$\Delta_{a'}^{v} \ln \mathrm{P}_a(v \mid \mathbf{0}) = \ln \mathrm{Q}_a(v \mid 1) - \ln \mathrm{Q}_a(v \mid 0).$$

Thus, we can identify

$$\mathrm{Q}_a(v \mid 1) / \mathrm{Q}_a(v \mid 0).$$

If $\mathcal{NC}_a \setminus \mathcal{NR}_a$ is empty, but $\mathcal{NR}_a \setminus \mathcal{NC}_a$ is not, we can use preference-only peers in a similar way. In particular, suppose $a' \in \mathcal{NR}_a \setminus \mathcal{NC}_a$ and $a'' \in \mathcal{NCR}_a$. Then, if we switch $a'$ from $v$ to the default and $a''$ from the default to $v$, then the changes in $\mathrm{P}_a$ are driven by changes in consideration probabilities only. That is, for $\mathbf{y}$ such that $y_{\tilde{a}} = 0$ for all $\tilde{a} \neq a'$ and $y_{a'} = v$,

$$\ln \mathrm{P}_a\left(v \mid (\mathbf{y}_{a''}^{v})_{a'}^{0}\right) - \ln \mathrm{P}_a(v \mid \mathbf{y}) = \ln \mathrm{Q}_a(v \mid 1) - \ln \mathrm{Q}_a(v \mid 0),$$

so the ratio of consideration probabilities is identified. By applying the same ideas to different initial configurations, we can identify ratios of consideration probabilities (as we state next).

**Proposition 3.4.** *Suppose $\mathcal{NC}_a$ and $\mathcal{NR}_a$ are known and Assumptions 1, 2, and 3 hold. Then*

$$\mathrm{Q}_a\left(v \mid n+1\right) / \, \mathrm{Q}_a\left(v \mid n\right)$$

*is identified from $\mathrm{P}_a$ for each $n$ from 0 to $|\mathcal{NC}_a| - 1$. (We use the convention that if $|\mathcal{NC}_a| = 0$, then the set "from 0 to -1" is empty.)*

*Remark* 2. Proposition 3.4 is valid for a substantially more general consideration set model. For example, the assumption that each alternative is added to the consideration sets independently from other alternatives can be completely dropped. Indeed, by definition, we have that

$$\mathrm{P}_a(v \mid \mathbf{y}) = \mathrm{Q}_a(v \mid \mathrm{NC}_a^v(\mathbf{y})) \, \mathrm{Pr}_a(v \mid \mathbf{y}, v \text{ is considered}),$$

where the second term is the conditional probability that $v$ is picked by Agent $a$ conditional on being considered. Thus, the variation in choices of $a' \in \mathcal{NC}_a \setminus \mathcal{NR}_a$ would identify $\mathrm{Q}_a$ up to scale. Note, however, that in this case, knowing $\mathrm{Q}_a$ does not identify $\mathrm{C}_a$ since $\mathrm{Q}_a$ in general does not convey information about a probability of several items being considered simultaneously.

We next show that we can also recover some counterfactual objects of interest. Adding some restrictions, these counterfactuals will allow us to recover the choice rules. Let us define

$$\mathrm{P}_a^*\left(v \mid \mathbf{y}, \mathcal{Y} \setminus \mathcal{Z}\right) = \textstyle\sum_{\mathcal{C} \subseteq \mathcal{Y} \setminus \mathcal{Z}} \mathrm{R}_a \left(v \mid \mathrm{NR}_a^{\mathcal{C}}\left(\mathbf{y}\right), \mathcal{C}\right) \mathrm{C}_a \left(\mathcal{C} \mid \mathrm{NC}_a^{\mathcal{Y} \setminus \mathcal{Z}}(\mathbf{y}), \mathcal{Y} \setminus \mathcal{Z}\right)$$

for each $\mathcal{Z} \subseteq \mathcal{Y} \setminus \{0\}$. That is, $\mathrm{P}_a^*\left(v \mid \mathbf{y}, \mathcal{Y} \setminus \mathcal{Z}\right)$ is the counterfactual probability of selecting alternative $v$ under choice configuration $\mathbf{y}$ when we restrict the set of available options or the menu from $\mathcal{Y}$ to $\mathcal{Y} \setminus \mathcal{Z}$. That is, $\mathrm{P}_a^*$ tells us what happens to CCPs when we remove set $\mathcal{Z}$ from the original menu. Note that, by definition, $\mathrm{P}_a^*\left(v \mid \mathbf{y}, \mathcal{Y}\right) = \mathrm{P}_a\left(v \mid \mathbf{y}\right)$.

To fix the ideas behind the next result, consider the setting with $\mathcal{A} = \{a, a'\}$, $\mathcal{Y} = \{0, v, v'\}$, $\mathcal{NR}_a = \emptyset$, and $\mathcal{NC}_a = \{a'\}$. Take $\mathbf{y}$ such that $y_{a'} = 0$ ($y_a$ can be arbitrary). Recall that $\mathbf{y}_{a'}^{v'}$ denotes a configuration where the $a'$-th component of $\mathbf{y}$ is replaced by $v'$. Since

$$\mathrm{P}_a^*(v \mid \mathbf{y}, \mathcal{Y} \setminus \{v'\}) = \mathrm{Q}_a\,(v \mid 0)\,\mathrm{R}_a\,(v \mid 0, \{0, v\})\,,$$

we have that

$$\mathrm{P}_a\,(v \mid \mathbf{y}) = \mathrm{Q}_a\,(v' \mid 0)\,\mathrm{Q}_a\,(v \mid 0)\,\mathrm{R}_a\,(v \mid (0,0), \{0, v, v'\}) + \Big[1 - \mathrm{Q}_a\,(v' \mid 0)\Big]\,\mathrm{P}_a^*(v \mid \mathbf{y}, \mathcal{Y} \setminus \{v'\}).$$

This probability is the observed probability of Agent $a$ choosing alternative $v$ given that her peer $a'$ previously chose the default. Moreover, by switching $a'$'s choice from the default to $v'$, we have

$$\mathrm{P}_a\left(v \mid \mathbf{y}_{a'}^{v'}\right) = \mathrm{Q}_a\,(v' \mid 1)\,\mathrm{Q}_a\,(v \mid 0)\,\mathrm{R}_a\,(v \mid (0,0), \{0, v, v'\}) + (1 - \mathrm{Q}_a\,(v' \mid 1))\,\mathrm{P}_a^*(v \mid \mathbf{y}, \mathcal{Y} \setminus \{v'\}).$$

Note that we used the fact that since Agent $a'$ only affects Agent $a$'s consideration probability, but not the preference, the variation of Agent $a'$'s choice in the configuration provides variation in the consideration probability but not the choice rule. That is, $\mathrm{R}_a\,(v \mid (0,0), \{0, v, v'\})$ does not vary when $a'$ switches from the default to a different alternative. Moreover, we also used the fact that

$$\mathrm{P}_a^*(v \mid \mathbf{y}, \mathcal{Y} \setminus \{v'\}) = \mathrm{P}_a^*(v \mid \mathbf{y}_{a'}^{v'}, \mathcal{Y} \setminus \{v'\}),$$

which follows from $v'$ being excluded from the menu and, thus, switches to it do not change the probability of picking $v$.

Therefore, solving this system of two equations with respect to $\mathrm{P}_a^*(v \mid \mathbf{y}, \mathcal{Y} \setminus \{v'\})$, we obtain that

$$\mathrm{P}_a^*(v \mid \mathbf{y}, \mathcal{Y} \setminus \{v'\}) = \frac{\mathrm{P}_a\left(v \mid \mathbf{y}_{a'}^{v'}\right) - t_{v'}\,\mathrm{P}_a\,(v \mid \mathbf{y})}{1 - t_{v'}},$$

where $t_{v'} = \mathrm{Q}_a\,(v' \mid 1)\,/\,\mathrm{Q}_a\,(v' \mid 0) \neq 1$ can be identified using Proposition 3.4. Hence, we can recover the counterfactual CCP $\mathrm{P}_a^*(v \mid \mathbf{y}, \mathcal{Y} \setminus \{v'\})$ for any $\mathbf{y}$ such that one component that corresponds to a consideration-only peer is equal to 0 (i.e., $y_{a'} = 0$). Essentially, we just used a consideration-only peer to exclude one alternative from the menu. Applying the same argument to these new counterfactual

CCPs, we can exclude two alternatives as long as we have two consideration-only peers. Again, we can use any initial $\mathbf{y}$ as long as the components that correspond to any two consideration-only peers are set to 0. In other words, we can exclude any set of nondefault alternatives as long as its cardinality is less than or equal to the cardinality of $\mathcal{NC}_a \setminus \mathcal{NR}_a$.

The next result formalizes this argument.

**Proposition 3.5.** *Suppose $\mathcal{NC}_a$ and $\mathcal{NR}_a$ are known, and Assumptions 1 and 2 are satisfied. Then $\mathrm{P}^*_a(v \mid \mathbf{y}, \mathcal{Y} \setminus \mathcal{Z})$ is identified from $\mathrm{P}_a$ for every $\mathcal{Z} \subseteq \mathcal{Y} \setminus \{0\}$ such that $|\mathcal{Z}| \leq |\mathcal{NC}_a \setminus \mathcal{NR}_a|$ and each $\mathbf{y}$ for which at least $|\mathcal{Z}|$ of its components corresponding to any peers in $\mathcal{NC}_a \setminus \mathcal{NR}_a$ are 0.*

Proposition 3.5 allows one to answer a fundamental question of what would happen if some alternatives are removed or become not available. Notice that the identification of these counterfactual CCPs does not require identifying either $\mathrm{Q}_a$ or $\mathrm{R}_a$. Only the identification of the relative $\mathrm{Q}_a$ is required.

In our setting, the variation in the choices of consideration-only peers is equivalent to menu variation in the stochastic choice literature. In particular, if one has enough of those peers, then one can identify the counterfactual CCPs for binary menus

$$\mathrm{P}^*_a(v \mid \mathbf{y}, \{0, v\}) = \mathrm{Q}_a\left(v \mid \mathcal{NC}^v_a(\mathbf{y})\right) \mathrm{R}_a\left(v \mid \mathrm{NR}^v_a(\mathbf{y}), \{0, v\}\right).$$

Hence, if either $\mathrm{Q}_a\left(v \mid \mathcal{NC}^v_a(\mathbf{y})\right)$ or $\mathrm{R}_a\left(v \mid \mathrm{NR}^v_a(\mathbf{y}), \{0, v\}\right)$ is known, we can recover $\mathrm{Q}_a(v \mid \cdot)$ (by using Proposition 3.4) and then $\mathrm{R}_a\left(v \mid \mathrm{NR}^v_a(\mathbf{y}), \{0, v\}\right)$ from our recent ideas. Applying the same argument for menus of size three, we can identify $\mathrm{R}_a$ for sets of size three, and so on.

**Proposition 3.6.** *Suppose that the assumptions of Proposition 3.5 are satisfied. If, in addition, we have that $|\mathcal{NC}_a \setminus \mathcal{NR}_a| \geq Y - 1$ and, for each $v \neq 0$, either $\mathrm{Q}_a(v \mid n_1)$ or $\mathrm{R}_a(v \mid n_2, \{0, v\})$ is known for some $n_1$ and $n_2$ in the support, then $\mathrm{Q}_a$ and $\mathrm{R}_a$ are identified from $\mathrm{P}_a$.*

### 3.2. Identification of $\mathrm{P}$

This section studies the identification of the CCPs, P, and the rates of the Poisson alarm clocks from two different datasets. These two datasets coincide in that they contain long sequences of choices from agents in the network. They differ in the timing at which the researcher observes these choices. In Dataset 1, agents' choices are observed in real-time. This allows the researcher to record the precise moment at which an agent revises her strategy and the configuration of choices at that time. In Dataset 2, the researcher simply observes the joint configuration of choices at fixed time intervals.

Let us assume the researcher observes agents' choices at time intervals of length $\Delta$ and can consistently estimate $\Pr\left(\mathbf{y}^{t+\Delta} = \mathbf{y}' \mid \mathbf{y}^{t} = \mathbf{y}\right)$ for each pair $\mathbf{y}', \mathbf{y} \in \mathcal{Y}^{A}$. We capture these transition probabilities by a matrix $\mathcal{P}(\Delta)$. (Here again, we assume that the choice configurations are ordered according to the lexicographic order when we construct $\mathcal{P}(\Delta)$.) The connection between $\mathcal{P}(\Delta)$ and the transition rate matrix $\mathcal{M}$ described in Equation (2) is given by

$$\mathcal{P}(\Delta) = e^{(\Delta\mathcal{M})},$$

where $e^{(\Delta\mathcal{M})}$ is the matrix exponential of $\Delta\mathcal{M}$. The two datasets we consider differ regarding $\Delta$: in Dataset 1, we let the time interval be very small. This is an ideal dataset that registers agents' choices at the exact time at which any given agent revises her choice. As we mentioned earlier, with the proliferation of online platforms and scanners this sort of data might indeed be available for some applications. In Dataset 2, we allow the time interval to be of arbitrary size. The next table formally describes Datasets 1 and 2

| | |
|---|---|
| **Dataset 1** | The researcher knows $\lim_{\Delta\to 0}\mathcal{P}(\Delta)$, |
| **Dataset 2** | The researcher knows $\mathcal{P}(\Delta)$. |

In both cases, the identification question is whether (or under what extra restrictions) it is possible to uniquely recover $\mathcal{M}$ from $\mathcal{P}(\Delta)$. The first result in this section is as follows.

**Proposition 3.7** (Dataset 1)**.** *The CCPs* $\mathrm{P}$ *and the rates of the Poisson alarm clocks* $\lambda_a$ *are* *identified from Dataset 1 for all $a \in \mathcal{A}$.*

The proof of Proposition 3.7 relies on the fact that when the time interval between the observations goes to zero, then we can recover $\mathcal{M}$. There are at least two well-known cases that produce the same outcome without assuming $\Delta \to 0$. One of them requires the length of the interval $\Delta$ to be below a threshold $\overline{\Delta}$. The main difficulty of this identification approach is that the value of the threshold depends on the details of the model that are unknown to the researcher. The second case requires the researcher to observe the dynamic system at two different intervals $\Delta_1$ and $\Delta_2$ that are not multiples of each other. (See, e.g., Blevins, 2017, and the literature therein.)

The next proposition states that, by adding an extra restriction, the transition rate matrix can be identified from agents' choices even if these choices are observed at the endpoints of discrete time intervals. In this case, the researcher needs to know the rates of the Poisson alarm clocks or normalize them in empirical work.

**Proposition 3.8** (Dataset 2)**.** *If Assumptions 1 and 2 are satisfied, the researcher knows $\lambda$, and $\mathcal{M}$ has distinct eigenvalues that do not differ by an integer multiple of $2\pi i/\Delta$, where $i$ here denotes the imaginary unit, then the CCPs $\mathrm{P}$ are generically identified from Dataset 2.*

The key element in proving Proposition 3.8 is that the transition rate matrix in our model is rather parsimonious. To see why, recall that, at any given time, only one person revises her selection with a nonzero probability. This feature of the model translates into a transition rate matrix $\mathcal{M}$ that has many zeros in known locations.

## 4. Extensions

In this section, we provide several extensions of our baseline model.

### 4.1. Binary Choice

Identification of consideration-only peers —Proposition 3.2— requires the existence of at least two non-default alternatives (i.e., $Y \geq 2$). This condition cannot be relaxed without extra assumptions. Indeed, if $\mathcal{Y} = \{0, 1\}$, the choices of Agent $a$ are completely described by one equation:

$$\mathrm{P}_a(1 \mid \mathbf{y}) = \mathrm{Q}_a(1 \mid \mathrm{NC}_a^v(\mathbf{y}))\, \mathrm{R}_a(1 \mid \mathrm{NC}_a^1(\mathbf{y}), \{0, 1\}).$$

Thus, although we still can recover the set $\mathcal{N}_a$, we cannot implement double changes across alternatives to distinguish whether a peer affects only consideration or preferences. To identify the different types of peers we need to add some extra restrictions.

The main result is based on a stronger version of Assumption 3.

**Assumption 3′.** For each $a \in \mathcal{A}$, if $|\mathcal{NCR}_a| \geq 1$, then $|\mathcal{NC}_a \setminus \mathcal{NR}_a|\, |\mathcal{NR}_a \setminus \mathcal{NC}_a| \geq 1$.

This assumption requires that if there are consideration-preference peers, then there must be consideration-only and preference-only peers. Note that as with Assumption 3, if $\mathcal{NCR}_a$ is empty, then no restrictions are imposed.

**Proposition 4.1.** *Suppose that $Y = 1$ and Assumptions 1, 2, 3′, and 5 hold. If $\mathcal{NC}_a$ or $\mathcal{NR}_a$ is known, then the network structure is identified.*

*Remark* 3. The same result can be obtained if, instead of restricting the network structure, we add a fourth condition to the assumption on preferences —i.e., Assumption 2: (iv) For $\mathcal{Y} = \{0, 1\}$, $\mathrm{R}(1 \mid 1, \{0, 1\}) / \mathrm{R}(1 \mid 0, \{0, 1\})$ is different from $\mathrm{R}(1 \mid 2, \{0, 1\}) / \mathrm{R}(1 \mid 1, \{0, 1\})$. The extra condition is the analog of Assumption 1(iii) —on consideration— to the choice rule.

*Remark* 4. Interestingly, assumption that the researcher knows either $\mathcal{NC}_a$ or $\mathcal{NR}_a$ in Proposition 4.1 can be substituted by a sign restriction. Throughout the paper, we sustained the assumption that the researcher does not know the signs of the peer effects in preferences and consideration. Indeed, we offered conditions under which these signs can be recovered from the data. But, in some cases, it might be reasonable to think that the signs of these effects are known. If this were the case, and peer effects in considerations and preferences were of opposite signs, then we could dispense with the assumption that the researcher knows either $\mathcal{NC}_a$ or $\mathcal{NR}_a$ in Proposition 4.1. An interesting example, where this sign restriction could be used, is the case of vaccines. One could argue that people become aware of a particular vaccine if more of their friends are getting shots. Moreover, it has long been argued that the peer effect in preferences would be negative. The reason is that if more of her peers get vaccinated, then the chances of getting sick reduce, and this reduces the payoff of getting the vaccine. Indeed, in a very different model, a similar idea has been used by Agranov, Elliott and Ortoleva (2021) to explain some data on COVID-19 vaccine uptake.

After the network structure is identified, the identification of the attention mechanisms and preferences follows directly from Proposition 3.6.

### 4.2. Longer Histories and Own Previous Choice

Along the paper, we assume that (at the moment of choosing) agents take into account only aggregate information about the choices of peers and ignore their own past choices. In this section, we relax these modelling restrictions.

Let us suppose that both the consideration and preferences of a given agent depend on the history of her own choices and those of her peers. Formally, let $\{t_k\}_{k=1}^{+\infty}$ be a random (increasing) sequence of time periods at the moments in which the different clocks went off. Let $\mathbf{y}_{t_k}$ denote the configuration of choices in the network at time $k$-th (note that, at this moment, the alarm clock of some agent went off). Given the history of choice configurations $h_t = (\mathbf{y}_{t_k})_{t_k<t}$, the probability that alternative $v$ is picked by Agent $a$ at time $t$ would be

$$\begin{aligned} \mathrm{P}_a\left(v \mid \mathbf{y}_t, h_t\right) = & \sum\nolimits_{\mathcal{C}\subseteq\mathcal{Y}} \mathrm{R}_a\left(v \mid \mathbf{y}_t, h_t, \mathcal{NR}_a, \mathcal{C}\right) \\ & \prod\nolimits_{v'\in\mathcal{C}} \mathrm{Q}_a\left(v' \mid \mathbf{y}_t, h_t, \mathcal{NC}_a\right) \prod\nolimits_{v'\in\mathcal{Y}\setminus\mathcal{C}} \left(1-\mathrm{Q}_a\left(v' \mid \mathbf{y}_t, h_t, \mathcal{NC}_a\right)\right). \end{aligned}$$

Note that these CCPs only depend on past choices but not on the exact times at which these choices were made. Also, note that none of our previous results use variation beyond the choices of peers at the moment of making a decision. Hence, if we condition on the choice of Agent $a$, $y_{at}$, and the history $h_t$ of choices, then we can establish the identification of all parts of the model from

$\mathrm{P}_a$ by using our previous ideas —thus, we omit the proof of the next result.

**Proposition 4.2.** *Suppose that Assumptions 1,2, 3, and 5 are satisfied conditional on $y_{at}$ and the history $h_t$ for all possible $y_{at}$ and $h_t$. Also, extend the definition of $\mathrm{P}_a^*$ to allow for dependence on $y_{at}$ and the history $h_t$. Then, conclusions of all propositions from Section 3.1 are still valid.*

Proposition 4.2 takes as input the CCPs that (now) depend on the histories of choices of everyone in the network. That is, it is assumed that $\mathrm{P}_a$ is identified. Since we only observe choices of agents from one network, it is impossible to identify the CCPs conditional on *all* histories without further assumptions. Thus, we restrict the length of the history that affects $\mathrm{P}_a$.

**Proposition 4.3.** *Suppose that Assumptions 1,2, 3, and 5 are satisfied conditional $y_{at}$ and the history $h_t$ for all possible $y_{at}$ and $h_t$. Assume also that there exists finite $K \geq 1$ such that for any $k > K$, $(\mathbf{y}_{t_{k'}})_{k'=1,\ldots,k}$, $v$, and $a$*

$$\mathrm{P}_a\left(v \mid \mathbf{y}_{t_k}, (\mathbf{y}_{t_{k'}})_{k'=1,\ldots,k-1}\right) = \mathrm{P}_a\left(v \mid \mathbf{y}_{t_k}, (\mathbf{y}_{t_{k'}})_{k'=k-K,\ldots,k-1}\right).$$

*Then* $\mathrm{P}$ *is identified from Dataset 1.*

We conclude this section by noting that under additional restrictions on how choices made many periods ago affect current choices, the CCPs can be identified even if consideration probabilities and choice rules depend on the whole history of choices (i.e., $k$ in Proposition 4.3 goes to infinity). These restrictions usually imply that the impact of the remote past is decaying sufficiently fast with time (see, Härdle, Lütkepohl and Chen, 1997 for examples). Moreover, the past history often enters via an index. For example, in our empirical application, we assume that history enters only via the total number of times an agent picked the alternative in the past. In cases like this, P can be identified using the results in Bierens (1996).

### 4.3. Bundles

Introducing small modifications, our framework can be extended to cover some bundles models.[8] Suppose that when agents face a consideration set $\mathcal{C}$, they are allowed to pick more that one option from it. Define

$$\mathcal{B}(\mathcal{C}) = \{b \subseteq \mathcal{C} \setminus \{0\} \,:\, b \neq \emptyset\} \cup \{0\}.$$

That is, $\mathcal{B}(\mathcal{C})$ is the collection of all possible bundles from consideration set $\mathcal{C}$. Note that the default cannot be bundled with other options. In this case, we only need to extend the definition of $\mathrm{R}_a$ from $\mathcal{C}$ to $\mathcal{B}(\mathcal{C})$. A bundle (i.e., a representative elements of $\mathcal{B}(\mathcal{C})$) will be denoted by $b$. Let $\mathrm{R}_a\left(b \mid \mathbf{y}, \mathcal{NR}_a, \mathcal{B}(\mathcal{C})\right)$ denote the probability that bundle $b$ is picked from $\mathcal{B}(\mathcal{C})$. We also need to slightly modify Assumption 2 and the regularity condition (Assumption 5 in Appendix A).

**Assumption 2′.** For each $a \in \mathcal{A}$, $\mathbf{y} \in \mathcal{B}(\mathcal{Y})^A$, $\mathcal{C} \subseteq \mathcal{Y}$, and $b \in \mathcal{B}(\mathcal{C}) \setminus \{0\}$, we have that

(i) $\mathrm{R}_a\left(b \mid \mathbf{y}, \mathcal{NR}_a, \mathcal{B}(\mathcal{C}^*)\right) > 0$ for some $\mathcal{C}^*$ such that $\mathrm{C}_a(\mathcal{C}^*|\mathbf{y}, \mathcal{NC}_a) > 0$;

(ii) $\mathrm{R}_a\left(b \mid \mathbf{y}, \mathcal{NR}_a, \mathcal{B}(\mathcal{C})\right) \equiv \mathrm{R}_a\left(b \mid \mathrm{NR}_a^{\mathcal{B}(\mathcal{C})}(\mathbf{y}), \mathcal{B}(\mathcal{C})\right)$; and

(iii) $\mathrm{R}_a\left(b \mid \mathrm{nr}^{\mathcal{B}(\mathcal{C})}, \mathcal{B}(\mathcal{C})\right)$ is injective in $\mathrm{nr}^w$ for $\mathrm{nr}^w \in \{0,1\}$ and $\mathrm{nr}^{\mathcal{B}(\mathcal{C})\setminus\{w\}} = (0, 0, \ldots, 0)'$.

Assumption 2′ and the modified regularity condition (Assumption 5′) only differ from the original conditions in terms of the domain: the new conditions are defined on the set of all possible bundles $\mathcal{B}(\mathcal{Y})$. The meaning of each restriction is exactly as before.

The following proposition establishes the validity of some of the previous results under the above modifications. Its proof is omitted since it directly follows from the proofs of Propositions 2.1-3.4.

**Proposition 4.4.** *The conclusions of Propositions 2.1-3.4 are valid for the bundles model if Assumptions 2 and 5 are replaced by Assumptions 2′ and 5′.*

The analogs of Propositions 3.5 and 3.6 for bundles can be established with a small modification

[8] See, for instance, Gentzkow (2007), Dunker, Hoderlein and Kaido (2017), Fox and Lazzati (2017), Iaria and Wang (2020), Allen and Rehbeck (2022), Kashaev (2023), and Wang (2023).

of the definition of $\mathrm{P}_a^*$. Let us define

$$\mathrm{P}_a^*\left(b \mid \mathbf{y}, \mathcal{B}(\mathcal{Y} \setminus \mathcal{Z})\right) = \textstyle\sum_{\mathcal{C} \subseteq \mathcal{Y} \setminus \mathcal{Z}} \mathrm{R}_a\left(b \mid \mathrm{NR}_a^{\mathcal{B}(\mathcal{C})}\left(\mathbf{y}\right), \mathcal{B}(\mathcal{C})\right) \mathrm{C}_a\left(\mathcal{C} \mid \mathrm{NC}_a^{\mathcal{Y} \setminus \mathcal{Z}}(\mathbf{y}), \mathcal{Y} \setminus \mathcal{Z}\right)$$

for each $\mathcal{Z} \subseteq \mathcal{Y} \setminus \{0\}$. That is, $\mathrm{P}_a^*\left(b \mid \mathbf{y}, \mathcal{B}(\mathcal{Y} \setminus \mathcal{Z})\right)$ is the counterfactual probability of selecting bundle $b$ under choice configuration $\mathbf{y} \in \mathcal{B}(\mathcal{Y})^A$ when we restrict the set of available options or the menu from $\mathcal{Y}$ to $\mathcal{Y} \setminus \mathcal{Z}$. (The set of available bundles changes from $\mathcal{B}(\mathcal{Y})$ to $\mathcal{B}(\mathcal{Y} \setminus \mathcal{C})$.)

**Proposition 4.5.** *Suppose $\mathcal{NC}_a$ and $\mathcal{NR}_a$ are known, and Assumptions 1 and 2 are satisfied. Then $\mathrm{P}_a^*\left(b \mid \mathbf{y}, \mathcal{B}(\mathcal{Y} \setminus \mathcal{Z})\right)$ is identified from $\mathrm{P}_a$ for each $\mathcal{Z} \subseteq \mathcal{Y} \setminus \{0\}$ such that $|\mathcal{Z}| \leq |\mathcal{NC}_a \setminus \mathcal{NR}_a|$ and each $\mathbf{y}$ for which at least $|\mathcal{Z}|$ of its components corresponding to peers in $\mathcal{NC}_a \setminus \mathcal{NR}_a$ are* 0.

**Proposition 4.6.** *Suppose that the assumptions of Proposition 4.5 are satisfied. If, in addition, we have that $|\mathcal{NC}_a \setminus \mathcal{NR}_a| \geq Y - 1$ and either $\mathrm{Q}_a\left(v \mid n_1\right)$ or $\mathrm{R}_a\left(v \mid n_2, \{0, v\}\right)$ is known for some $n_1$ and $n_2$ in the support and for each $v \in \mathcal{Y} \setminus \{0\}$, then $\mathrm{Q}_a$ and $\mathrm{R}_a$ are identified from $\mathrm{P}_a$.*

These two results follow directly from the proofs of Propositions 3.5 and 3.6 and are thereby omitted.

### 4.4. Non-observable Default

In many settings, the decision to choose the default alternative often is not observed. For example, if the default is "do nothing," then at any point in time that there is no change in the behavior of a given agent, we do not know whether she woke up and decided to do nothing or she did not have an opportunity to make a new decision. When this happens, even in continuous time data setting (Dataset 1), there is no hope to separately identify $\lambda_a$ and $\mathrm{P}_a$. Therefore, some form of normalization is required. In the empirical application at the end of the paper, we find it convenient to assume that $\lambda_a = 1$. This implies that, on average, agents have an opportunity to make a choice once per unit of time (in our empirical application, on average, firms make a decision every day). Once $\lambda_a$ is normalized, we can identify the CCPs $\mathrm{P}_a$ from the data directly, with which we can follow the identification results for network structure, consideration probabilities, and choice rules.

## 4.5. Perfectly Correlated Clocks

Sometimes, one may need to model a situation where two or more connected agents have perfectly synchronized clocks. In this case, they make decisions simultaneously. Given the boundedly rational nature of limited consideration, we assume that in these cases, the agents are unaware that their clocks are perfectly correlated and make a decision without taking into account the strategic consideration. This allows us to avoid complications caused by a potential multiplicity of equilibria. As a result, one would need to treat synchronized agents as one agent and redefine their choice set to consist of all pairs of alternatives from the original choice set.

We assume that each agent behaves as if she is unaware of the synchronized clocks. Therefore, the identification of the CCPs, the network structure, the consideration probabilities, and the choice rules are the same as the baseline result. If the choice of default is not observed in the data, one then needs to normalize the arrival rate for all agents, i.e., $\lambda_a = 1$.

## 4.6. More General Consideration Mechanism

The identification results we presented do not use any exogenous variation in observed covariates. These results only relied on the variation of choices of different peers —mainly the ones that affect only consideration— to recover the different parts of the model. In this section, we show that when covariates (with large support) that affect only preferences are available in the data, then (under minimal restrictions on $\mathrm{R}_a$) we can use them to identify a very general model of consideration.

Let us assume that, in addition to the variation of choice of peers, we observe a vector of covariates, $w$, that only affects preferences. Let us indicate its support by $W$. In particular, assume that given a consideration set, the choices of agents are consistent with the additive random utility model. That is, for all $v$ and $\mathcal{C} \neq \emptyset$ such that $v \in \mathcal{C}$

$$\mathrm{R}_a\left(v \mid w, \mathbf{y}, \mathcal{NR}_a, \mathcal{C}\right) = \int \prod_{v' \in \mathcal{C} \setminus \{v\}} \mathbb{1}\left(\, U_a(v|w, \mathbf{y}, \mathcal{NR}_a) + \varepsilon_v \geq U_a(v'|w, \mathbf{y}, \mathcal{NR}_a) + \varepsilon_{v'}\,\right) dF_{a,\varepsilon}(\varepsilon \mid \mathbf{y}),$$

where $F_{a,\varepsilon}$ is the agent specific distribution of shocks $\varepsilon = (\varepsilon_v)_{v \in \mathcal{Y}}$ and $U_a$ is the mean utility function.

**Assumption 4.** For every $a \in \mathcal{A}$ and $v \in \mathcal{Y} \setminus \{0\}$, there exist a known covariate $w_{a,v}$ and a function of it $f_{a,v}(\cdot)$ such that (i) $\mathrm{U}_a(v'|\mathbf{y}, w)$ does not depend on $w_{a,v}$ for all $v' \neq v$; and (ii) the closure of the conditional support of $w_{a,v}$ conditional on all other covariates contains a point $\overline{w}_{a,v}$ such that

$$\lim_{w_{a,v} \to \overline{w}_{a,v}} \mathrm{U}_a(v|w, \mathbf{y}, \mathcal{NR}_a) = +\infty,$$
$$\lim_{w_{a,v} \to \overline{w}_{a,v}} \frac{\mathrm{U}_a(v|w, \mathbf{y}, \mathcal{NR}_a)}{f_{a,v}(w_{a,v})} = O(1).$$

This assumption requires the existence of an alternative specific covariates that can make this non-default alternative to be picked every time it is considered. (This covariate could or could not be agent-specific as well.) The presence of such covariates essentially serves as exclusion restrictions in the utility function. Moreover, the local behavior of the utility function in the neighborhood of the extreme point $\overline{w}^p_{a,v}$ is known and is captured by $f_{a,v}(\cdot)$. Assumption 4 is satisfied if, for instance,

$$\mathrm{U}_a(v|w, \mathbf{y}, \mathcal{NR}_a) = f_{a,v}(w_{a,v}) + g_{a,v}(\mathbf{y}, \mathcal{NR}_a)$$

with $\{(f_{a,v}(w_{a,v}))_{v \in \mathcal{Y} \setminus \{0\}} \, : \, w \in W\}$ is equal to $\mathbb{R}^Y_+$. Note that $f_{a,v}$ does not have to vary over the whole Euclidean space.

The next result establishes a nonparametric identification of $\mathrm{C}_a$ without assuming that consideration sets are formed by independent draws from $\mathcal{Y}$.

**Proposition 4.7.** *If Assumption 4 is satisfied and $\mathrm{C}_a$ does not depend on $w$, then $\mathrm{C}_a$ is identified for all $a \in \mathcal{A}$.*

The proof of Proposition 4.7 is based on the idea that we can send the mean utilities of alternatives to $+\infty$ at different rates. As a result, the observed distribution over choices would correspond to choices of an agent with deterministic preferences (i.e, $\mathrm{R}_a(v \mid w, \mathbf{y}, \mathcal{NR}_a, \mathcal{C}) \in \{0, 1\}$). In other words, particular limits of observed $\mathrm{P}_a$ correspond to a model with random consideration only. For instance, if given deterministic preferences are such that alternative $v$ is picked only when nothing else (except the default) is considered (i.e., $v$ is the worst alternative after the default), then

$$\mathrm{P}_a(v \mid w, \mathbf{y}) = \mathrm{C}_a(\{0, v\} \mid \mathbf{y}),$$

and we identify $\mathrm{C}_a(\{0, v\} \mid \mathbf{y})$. Since we can use different rates for different alternatives, we can identify $\mathrm{C}_a(0, v)$ for all $v$. Repeating the above argument for consideration sets of bigger sizes, we can recursively identify $\mathrm{C}_a(\mathcal{C})$ for all $\mathcal{C}$.

Next, note that

$$\mathrm{P}_a\left(v \mid w, \mathbf{y}\right) = \sum\nolimits_{\mathcal{C} \subseteq 2^{\mathcal{Y}}} \mathrm{R}_a\left(v \mid w, \mathbf{y}, \mathcal{NR}_a, \mathcal{C}\right) \mathrm{C}_a\left(\mathcal{C} \mid \mathbf{y}, \mathcal{NC}_a\right),$$

where $\mathrm{C}_a\left(\mathcal{C} \mid \mathbf{y}, \mathcal{NC}_a\right)$ has been already identified. Thus, if there are peers that only affect consideration, we can use the known variation in consideration probabilities $\mathrm{C}_a$ to build a system of linear equations in which the unknown parameters are $\{\mathrm{R}_a\left(v \mid w, \mathbf{y}, \mathcal{NR}_a, \mathcal{C}\right)\}_{\mathcal{C} \subseteq \mathcal{Y}, v \in \mathcal{C}}$. Hence, if Agent $a$ has enough peers in $\mathcal{NC}_a \setminus \mathcal{NR}_a$ to generate variation, then $\{\mathrm{R}_a\left(v \mid \mathbf{y}, \mathcal{NR}_a, \mathcal{C}\right)\}_{\mathcal{C} \subseteq \mathcal{Y}, v \in \mathcal{C}}$ can be identified as a solution to a system of linear equations. Formally, for a given $v \in \mathcal{Y} \setminus \{0\}$, let $\{\mathcal{C}_k\}_{k=1}^{2^{Y-1}}$ be the collection of all subsets of $\mathcal{Y}$ that contain $v$ and 0. Also, let $\{n^l\}$ be a set of all nonnegative integer-valued vectors of length $Y$ such that $\sum_{v' \in \mathcal{Y} \setminus \{0, v\}} n^l_{v'} \leq |\mathcal{NC}_a \setminus \mathcal{NR}_a|$ for each $l$. Then, under Assumption 1, for fixed $a \in \mathcal{A}$, $y_a \in \mathcal{Y}$, and $(\mathrm{NR}^v_a)_{v' \in \mathcal{Y}}$, let matrix $B$ be such that the $(l, k)$-th element of it is

$$B_{l,k} = \mathrm{C}_a(\mathcal{C}_k \mid n^l).$$

If $B$ has full column rank, then $\mathrm{R}_a$ is identified.

## 5. Application

In our empirical application we investigate the effect of limited consideration on the analysis of the two dominant coffee chains' strategic competition in China. The goal of this application is three-fold. First, we showcase our identification strategy. Second, we show that ignoring the presence of limited consideration might mislead our understanding regarding the profitability of different markets and, thus, firm behavior. In particular, under limited consideration, some stores would not be opened not because of the lack of profitability but rather inattention. Third, we quantify the effects of limited consideration and peer effect in consideration on market structure by conducting two counterfactual exercises where we shut one of these channels down. This type of counterfactual analysis is crucial for consumer welfare and, thus, informed policy recommendations.

### 5.1. Coffee industry background and data

China is quickly turning into one of the fastest-growing coffee markets worldwide, with sales increasing from 47 billion yuan in 2015 to 82 billion yuan in 2020 and projected to be 219 billion yuan in 2025.[9] Starbucks, the dominant coffee chain in China, opened its first store in Beijing in 1999 and its $6,000$th store in September 2022, also the $1,000$th store in Shanghai, the first city in the world to pass the milestone.

To avoid complications related to the COVID-19 pandemic, we focus our sample on the end of 2019. By then, Starbucks had expanded into 176 markets with 4042 stores. The second key player in the Chinese market is Luckin. It registered its first store in Beijing in September 2017 but quickly expanded to 54 markets with 3588 stores by the end of 2019. This allowed Luckin to become comparable in size to Starbucks and its main competitor. Costa and Pacific, two other coffee chains, expand much slower and operate on a much lower scale in the Chinese market.[10] (We plot the nationwide number of stores for the four coffee chains in Figure 1.) Therefore, in our analysis, we focus on the entry and expansion decisions of Starbucks and Luckin only.

Starbucks has not accepted franchisee relationships in mainland China starting from September 2017, so all Starbucks stores in China are directly owned by the company. Luckin focuses mainly on the self-operating model. It only started a partnership operation in September 2019, but not for its coffee stores. The franchised stores are only allowed for its tea brand, which was separated from

[9] https://www.statista.com/statistics/1171765/china-coffee-market-size/.

[10] Specifically, Costa and Pacific registered, respectively, their first stores in Shanghai in September 2006 and January 2011. Costa had a presence in 36 markets with 458 stores, while Pacific had expanded into 29 markets with 238 stores by the end of 2019.

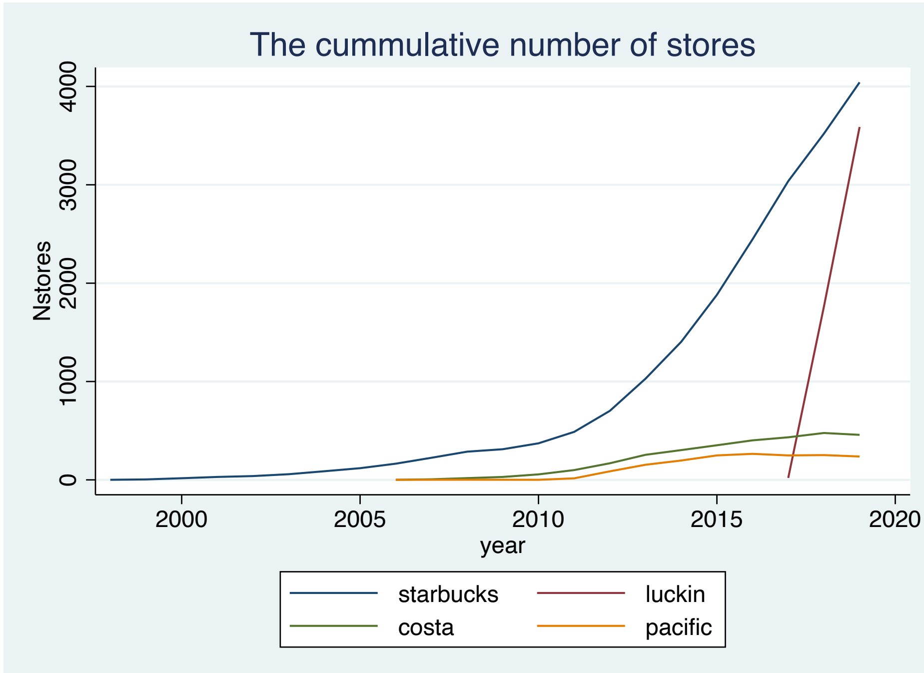



**Figure 1** – Total number of stores of each coffee chain over time.

the operation of its coffee stores. Therefore, all stores in our data period are operated by Luckin.[11]

### 5.2. Model

In this subsection, we describe the model of firm expansion within and across markets and introduce the specifications for the consideration and payoff. All unknown parameters of the model below will be collected to a vector $\theta$. After we set the model, we formally list all of them.

**Choice Set, Agents, and Peers** There is a finite set of firms $\mathcal{F}$ and a finite set of markets to expand $\mathcal{M}$. Every firm $f$ decides whether to open a store ($v = 1$) in a market $m$ or not ($v = 0$). We call every pair $(f, m) \in \mathcal{F} \times \mathcal{M}$ an agent. Thus, $\mathcal{A} = \mathcal{F} \times \mathcal{M}$ and $\mathcal{Y} = \{0, 1\}$. When firm $f$ decides whether to open a new store in market $m$, she looks at her own and her competitor's past behavior in "neighboring markets." Formally, $\mathcal{N}_a$ is the set of pairs $(f', m')$ that influence decisions of firm $f$ in market $m$ ($a = (f, m)$). Similarly, $\mathcal{NC}_a$ and $\mathcal{NR}_a$ are the sets of pairs of firms and markets that affect consideration and preferences, respectively, of firm $f$ in market $m$.

Since $Y = 1$ in this setting, we cannot identify all the components of the network structure

[11] https://www.hehuoren.cn/news/dongtai/1/160.html.

without additional assumptions (see Section 4.1 for a detailed discussion). We rely on the competition feature to first identify the peers only affecting payoffs in the market. Specifically, we follow the literature (e.g., Arcidiacono, Bayer, Blevins and Ellickson, 2016) and assume that the marginal profit of firm $f$ in market $m$ from opening a new store is only affected by her own and her competitors choices in market $m$. That is, only the competitors in the same markets are the peers affecting payoffs (or preferences). Formally,

$$(f', m') \in \mathcal{NR}_{(f,m)} \iff m = m'.$$

Therefore, we focus on recovering the consideration set network or the scope of neighborhood markets. No restrictions on $\mathcal{NC}_a$ (except that it is nonempty) are needed for identification. We will only impose assumptions on $\mathcal{NC}_a$ to facilitate the estimation. Assuming that $\mathcal{NR}_a$ is known allows us to use Proposition 4.1 to identify all the rest of the components of the network structure.

**Observable Characteristics** Every market $m$, which is part of $a = (f, m)$ for some given $f$, at every moment of time $t$, is characterized by observed market characteristics $S_{mt}$ (e.g., GDP and population density) that include a constant. Let $N_{at}$ denote the number of stores of agent $a$ (i.e. the number of stores of firm $f$ in market $m$). Let us also define $S_t = (S_{mt})_{m\in\mathcal{M}}$ and $N_t = (N_{at})_{a\in\mathcal{A}}$.

**Attention to Markets** In our application, there are more than 150 markets where firms can open a new store. Given how complex the decision of opening a new store is, we allow firms to consider only a subset of markets at the moment of making a decision. The consideration set is formed based on an attention index of a market at a given time period for a given firm:

$$\tilde{\pi}_{at} = \bar{\tilde{\pi}}_{at}(S_t, N_t; \theta) - \tilde{\varepsilon}_{at},$$

where the $\tilde{\varepsilon}_{at}$s are i.i.d. across $a$ and $t$ with a known c.d.f. $F_{\tilde{\varepsilon}}$ (in our application $F_{\tilde{\varepsilon}}$ is the Logistic c.d.f.). We allow market $m$'s current market features (including the market characteristics and all firms' number of stores) to affect Agent $a$'s attention index. Moreover, we allow the market structure of Agent $a$'s neighborhood markets to affect her attention to market $m$. Specifically, we parameterize the mean attention index for market $m$ for Agent $a$ as

$$\begin{aligned}\bar{\tilde{\pi}}_{at}(S_t, N_t; \theta) =& S'_{mt}\tilde{\beta}_f + \sum_{f'} \left[ \ln\left(1 + N_{(f',m)t}\right) \tilde{\alpha}_{f,f'} + \ln^2\left(1 + N_{(f',m)t}\right) \tilde{\gamma}_{f,f'} \right] + \\ &+ \sum_{f'} \left[ \ln\left(1 + \sum_{a'' \in \mathcal{NC}_a : f''=f'} N_{a''t}\right) \tilde{\delta}_{f,f'} + \ln^2\left(1 + \sum_{a'' \in \mathcal{NC}_a : f''=f'} N_{a't}\right) \tilde{\eta}_{f,f'} \right]\end{aligned}$$

is the mean attention of market $m$.[12] The mean attention consists of three parts. The first one captures the observable market characteristics. The second one captures the history of previous choices of all firms in market $m$. Finally, the third part captures the peer effect in consideration from markets different from $m$, where we allow the peer effect to be firm-specific. That is, for the peer effect in consideration, we allow firm $f$'s number of stores in her neighborhood market to affect her mean attention differently from her competitor's number of stores in her neighborhood market.

The firm pays attention to a market if its attention index is above 0, i.e. $\tilde{\pi}_{at} \geq 0$. As a result, the probability that firm $f$ considers opening a new store in market $m$ at time $t$ is

$$\mathrm{Q}_a(1 \mid (\mathbf{y}_{t'})_{t' \leq t}, S_t, \mathcal{NC}_a) = F_{\tilde{\varepsilon}}\left(\bar{\tilde{\pi}}_{at}(S_t, N_t; \theta)\right).$$

The vector of parameters $\theta$ contains the consideration parameter $\tilde{\alpha}_{f,f'}$, $\tilde{\beta}_{f,f'}$, $\tilde{\gamma}_{f,f'}$, $\tilde{\delta}_{f,f'}$, and $\tilde{\eta}_{f,f'}$, where$f, f' \in \mathcal{F}$, and the network structure $\mathcal{NC}_a$, $a \in \mathcal{A}$.

**Payoff from a New Store** Conditional on a market being considered, the firm decides whether to open at least one new store in that market based on its marginal profit $\pi_{at}$. We assume that the marginal profit of opening an extra store in market $m$ at time $t$ by firm $f$ is

$$\pi_{at} = \bar{\pi}_{at}(S_t, N_t; \theta) - \varepsilon_{at},$$

where $\varepsilon_{at}$s are i.i.d. across $a$ and $t$ with a known c.d.f. $F_\varepsilon$ (in our application $F_\varepsilon$ is the Logistic

[12]We take logarithms of the number of stores to offset a rapid increase in the number of stores in some markets. The results for $N_{a't}$ are qualitatively the same.

c.d.f.); and

$$\bar{\pi}_{at}(S_t, N_t; \theta) = S'_{mt}\beta_{f'} + \Big[\ln(1+N_{at})\alpha_f + \ln^2(1+N_{at})\gamma_f\Big] + \\ + \left[\ln\left(1 + \sum_{a' \in \mathcal{NR}_a} N_{a't}\right)\alpha_{f'} + \ln^2\left(1 + \sum_{a' \in \mathcal{NR}_a} N_{a't}\right)\gamma_{f'}\right]$$

is the mean marginal profit from one extra store. Hence, the probability of opening a new store in market $m$ by firm $f$ at time $t$ conditional on it being considered is

$$\mathrm{R}_a(1 \mid (\mathbf{y}_{t'})_{t' \leq t}, S_t, \mathcal{NR}_a, \{0,1\}) = F_{\varepsilon}\left(\bar{\pi}_{at}(S_t, N_t; \theta)\right).$$

Thus, the probability that a new store is opened in market $m$ by firm $f$ at time $t$ is

$$\mathrm{P}_a(1 \mid (\mathbf{y}_{t'})_{t' \leq t}, S_t) = F_{\tilde{\varepsilon}}\left(\bar{\tilde{\pi}}_{at}(S_t, N_t; \theta)\right) F_{\varepsilon}\left(\bar{\pi}_{at}(S_t, N_t; \theta)\right), \tag{4}$$

which completely characterizes the probability of observing a new store in a given market by a given store conditional on the past history and the market characteristics.

As a result, the vector of parameters $\theta$ contains the consideration parameters $\tilde{\alpha}_{f,f'}$, $\tilde{\beta}_{f,f'}$, $\tilde{\gamma}_{f,f'}$, where $f, f' \in \mathcal{F}$, the consideration network structure $\mathcal{NC}_a$, $a \in \mathcal{A}$, and the preference parameters $\alpha_{f'}$, $\beta_{f'}$, and $\gamma_{f'}$, $f' \in \mathcal{F}$. Note that $\mathcal{NR}_a = \{(f', m') \,:\, f' \neq f, m = m'\}$ is assumed to be known and, thus, is not the part of $\theta$.

### 5.3. Estimation and Inference

The data we have consist of three objects: (i) the exact times of new store openings $\{t_k\}_{k=1}^{K}$; (ii) the data on the state of the network $\{N_{at_k}\}_{a \in \mathcal{A}, k=1,\ldots,K}$ sampled from a continuous time over interval $[0, t_K]$, where $N_{at_k}$ is the number of stores owned by firm $f$ in market $m$ immediately prior to $k$-th change at time $t_k$ —last date of measurements coincides with the last day when any action was observed; and (iii) the data on observable market characteristics $\{S_{m,t_k}\}_{a \in \mathcal{A}, k=1,\ldots,K}$. Given the data on the number of stores, we can construct the state vector $r_{t_k} = (r_{at_k})_{a \in \mathcal{A}}$ that captures whether there was a change in the number of stores of firm $f$ in market $m$ at time $t_k$. That is,

$$r_{at_k} = \mathbb{1}\left( N_{at_{k+1}} > N_{at_k} \right).$$

Moreover, the probability of observing $r_{t_k}$, given the data and $\theta$ conditional on an alarm clock going off is

$$\begin{aligned} p(r_{t_k}, S_{t_k}, N_{t_k}; \theta) = & \prod_{a: r_{at_k}=1} F_{\tilde{\varepsilon}}\left(\bar{\tilde{\pi}}_{at}(S_{t_k}, N_{t_k}; \theta)\right) F_{\varepsilon}\left(\bar{\pi}_{at_k}(S_{t_k}, N_{t_k}; \theta)\right) \\ & \times \prod_{a: r_{at_k}=0} \left[1 - F_{\tilde{\varepsilon}}\left(\bar{\tilde{\pi}}_{at}(S_{t_k}, N_{t_k}; \theta)\right) F_{\varepsilon}\left(\bar{\pi}_{at_k}(S_{t_k}, N_{t_k}; \theta)\right)\right]. \end{aligned}$$

Hence, the probability that no new stores are opened in any market by any firm (i.e. the probability of picking the default) is

$$p_0(S_{t_k}, N_{t_k}; \theta) = \prod_{a \in \mathcal{A}} \left[1 - F_{\tilde{\varepsilon}}\left(\bar{\tilde{\pi}}_{at}(S_{t_k}, N_{t_k}; \theta)\right) F_{\varepsilon}\left(\bar{\pi}_{at_k}(S_{t_k}, N_{t_k}; \theta)\right)\right].$$

Finally, given that the arrival process is exponential, the log-likelihood of observing the data given $\theta$ and with the alarm sounding off rate of $\lambda = 1$ (see Section 4.4 for details on this normalization) is

$$\hat{L}(\theta) = \sum_{k=1}^{K} -(t_{k+1} - t_k)\lambda(1 - p_0(S_{t_k}, N_{t_k}; \theta)) + \ln(\lambda p(r_{t_k}, S_{t_k}, N_{t_k}; \theta)),$$

and we can define the maximum likelihood estimator of $\theta$, $\hat{\theta}$, as the maximizer of $\hat{L}$ over a parameter space $\Theta$.

**Parameter Space Restrictions** The vector of parameters $\theta$ consists of two very different parts: the parameters corresponding to the network structure (i.e., $\mathcal{NC}_a$, $a \in \mathcal{A}$), and the parameters corresponding to the attention index and marginal profits. The second set of parameters is standard and does not pose any challenge in estimation. Searching $\theta$ in its parameter space, in theory, could be done in an inner and outer loop fashion. Specifically, in the inner loop, fixing the network structure, one can maximize over the consideration and payoff parameters using the profiled likelihood estimation. The outer loop would search for the network structure that leads to the highest likelihood. Unfortunately, checking all possible network structures is often computationally prohibitive without restrictions. For example, in our application, without any restrictions, the parameter space for $\mathcal{NC}_a$, $a \in \mathcal{A}$, consists of $2^{2\times 152\times(152-1)} = 2^{45904} > 10^{10000}$ elements (i.e., there are $2 \times 152 \times (152 - 1)$ binary variables). If we assume that every firm has the same reference groups in every market and that links are not directed, then the size of the parameter space drops to $2^{152\times(152-1)/2} = 2^{11476} > 10^{1000}$. To simplify estimation a bit more, we will use spatial information about markets. In particular, we assume that if market $m'$ is in the neighborhood of market $m$, then at least one of the following three conditions holds.

(i) $m'$ and $m$ are in the same province;

(ii) the prefectures where $m'$ and $m$ are located share a border;

(iii) $m'$ is at least 5-th closest (in terms of geographical distance) market to market $m$.

With these additional constraints, the number of binary parameters describing the network structure is 1582. Searching through all possible network structures given such a simpler initial one is still computationally infeasible, i.e., $2^{1582}$. To further facilitate the estimation, instead of searching every possible network, we start the search from the initial/largest possible network and then do some local search by shutting down different collections of links. This method is only guaranteed to converge to a local optimum, which may not be global. However, we believe it provides an informative approximation of the solution.

**Estimation of Full Consideration Model** We also estimate the preference parameters, assuming that every market is considered by every firm. This allows us to compare the probability of opening a new store conditional on considering under the full consideration and limited consideration models. We expect that the full consideration model would predict smaller probabilities of opening a new store. This is due to the fact that the full consideration model would attribute negative marginal profits to a situation in which the market is not considered.

**Inference** To construct confidence sets for the payoffs and consideration parameters (or their functions), we assume that the estimated consideration network is the true network. In this case, the asymptotic variance matrix of the profits and consideration parameters is a standard inverse of the Fisher information matrix. We suspect that estimation error in the network structure should not affect the asymptotic behavior of the profit and consideration parameters since the cardinality of the parameter space for the network is finite.

We leave a formal analysis of this problem for future research.

### 5.4. Estimation Results

**Network Structure** The estimated consideration network has 960 directed links (i.e., the adjacency matrix is not symmetric and has 960 non-zero elements). (The restrictions we impose would allow up to 1582 links.) Figure 2 displays the histogram of the number of peer markets in the network. Though around 30% of markets have no peer markets affecting the consideration, more than 70% of the markets have at least 1 peer market affecting the consideration probability only. The average and median number of peer markets are 6.3 and 4.5, respectively. Note that even though a large fraction of markets are not connected to other markets, we can still identify and estimate the preference and consideration parameters since we assume they do not vary across markets.

**Consideration** We display the estimation results for Shanghai, which, by the last date of measurements, is the biggest in terms of number of stores market (740 and 419 Starbucks and Luckin stores, respectively). From the network structure estimation, the markets that are included in Shanghai's peer groups are the five closest markets to Shanghai in distance. The overall number of stores in Shanghai's neighborhood markets for Starbucks and Luckins are 458 and 336, respectively. To illustrate the impacts of each different factor on consideration, we keep the fixed variables (e.g., market characteristics) the one taken from the last time period.

Figure 3 depicts the probability of considering Shanghai as a function of the number of stores that belong to the firm/competitor in the market/neighborhood for both firms together with pointwise 95% confidence bands. There is substantial heterogeneity in how firms change their consideration with increasing their own presence in the market–they move in the opposite direction as the number of stores increases (except the behaviour around 0). It is interesting to note that

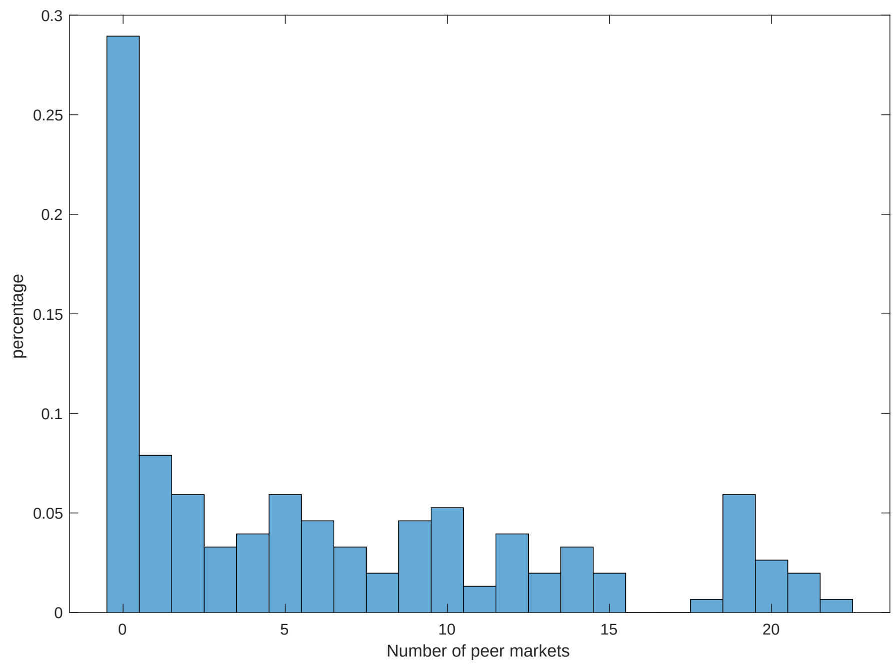
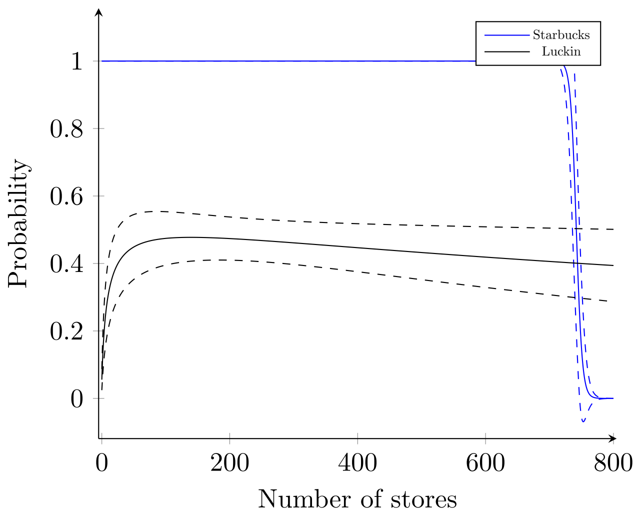



**Figure 2** – Histogram of Network

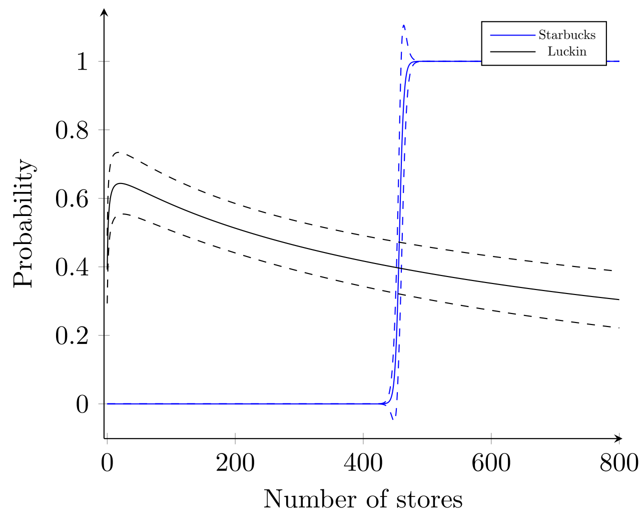
the Starbucks consideration probabilities as the function of the number of own stores (both in the market and in the neighborhood) resembles a step function. Specifically, when the number of own stores in the market is sufficiently large, then the market is essentially not considered. Similarly, the market is considered only when the number of own stores in the neighbouring markets exceeds a threshold. The consideration probabilities of Luckin responding to own stores are increasing around 0 and then slowly decreasing (Figures 3a and 3b).

Competitors' stores affect the consideration of Starbucks and Luckin in opposite directions. Specifically, Starbucks' attention reduces with Luckin's number of stores in the market but increases with that in the neighborhood markets. Luckin's attention increases with Starbucks' number of stores in the market but reduces with that in the neighborhood markets.

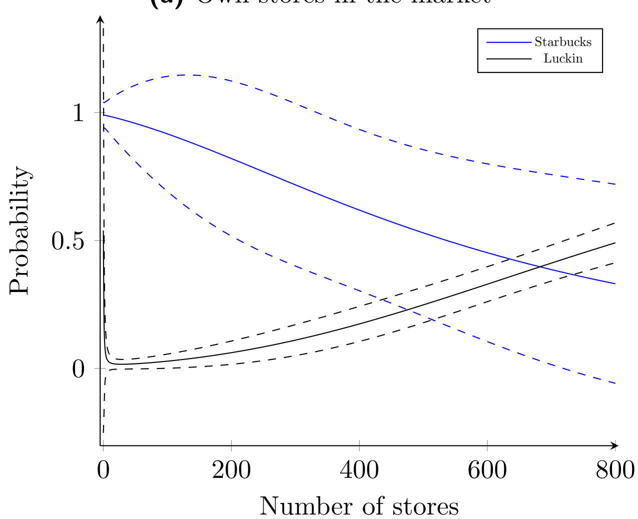
Overall, there is a robust increase in attention to markets with a small number of stores in and around the market for Luckins. In contrast, Starbucks' attention is not affected by the number of stores in and around the market when that number is small, consistent with the fact that Starbucks has been the leader in the market for quite a while before Luckin entered.



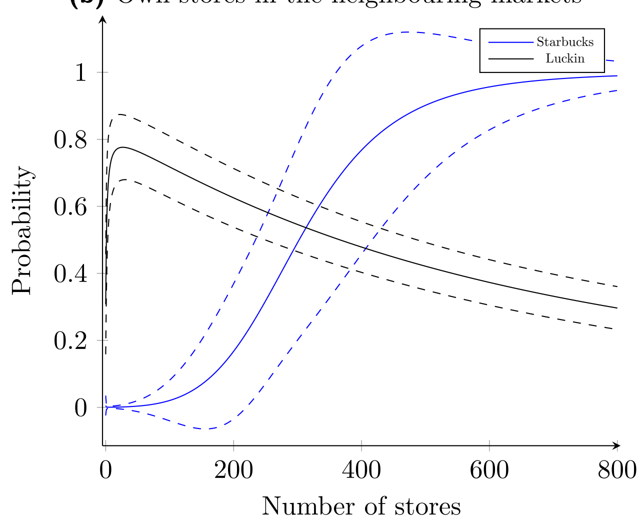

**(a)** Own stores in the market




**(b)** Own stores in the neighbouring markets




**(c)** Competitor's stores in the market




**(d)** Competitor's stores in the neighbouring markets

**Figure 3** – Consideration probabilities for both coffee chains as a function of the number of own or competitor's stores in Shanghai or its neighborhood. Blue and black lines correspond to Sturbacks and Luckin, respectively. Dashed lines depict pointwise 95% confidence bands. The fixed variables (e.g., market characteristics) are taken from the last time period.

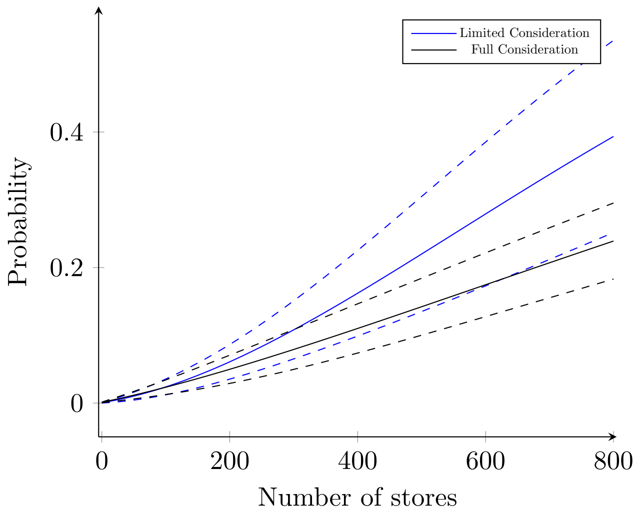
**Marginal Profits** Next, we analyze the probability of opening a new store in Shanghai conditional on Shanghai being considered as a function of the number of stores in and around the market. To quantify the effect of adding limited consideration to the expansion model, we also estimated the marginal profit parameters assuming that all markets are considered (i.e., we assume that $F_{\tilde{\varepsilon}}(\tilde{\varepsilon}) = 1$ for all $\tilde{\varepsilon}$). The results of the estimation are presented in Figure 4. We note that the estimated choice probabilities under limited consideration are uniformly bigger than those with full consideration. Importantly, the difference between these probabilities is substantial for Luckin. Thus, ignoring limited consideration leads to completely misleading estimates about the profitability of different markets. Qualitatively, this difference is explained by the fact that the full consideration model attributes "not-opening" a new store to negative marginal profits instead of limited consideration (a new store is not opened because the market is not considered). The Starbucks limited consideration expansion probabilities are not that different from the full consideration ones. This result is unsurprising given the estimated consideration probabilities–Starbucks considers almost all markets.

### 5.5. Counterfactuals

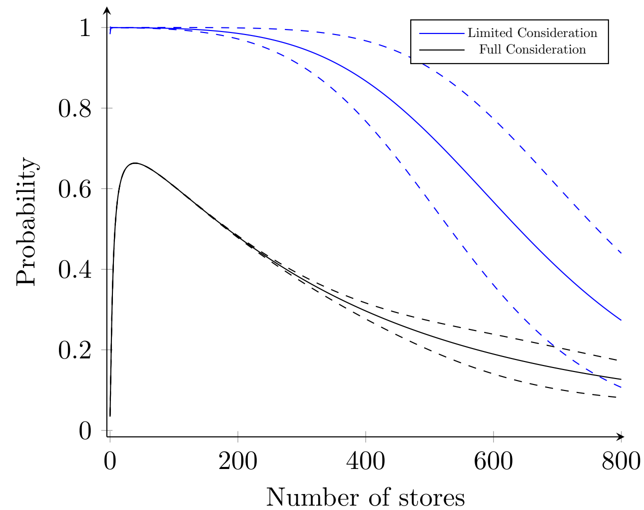
We aim to evaluate the effect of limited consideration and spillover effects across markets on the market structure by looking at changes in the fraction of monopolistic and duoplolistic markets across time under different counterfactual scenarios. Scenario 1: we force both firms to consider all 152 markets. That is, we assume that $F_{\tilde{\varepsilon}}(\tilde{\varepsilon}) = 1$ for all $\tilde{\varepsilon}$ (but we do not re-estimate the preference parameters). Scenario 2: we remove connections across markets. In particular, we assume that the consideration probabilities are affected by the number of own and opponent stores in the market but are not influenced by the number of stores in neighbouring markets. In both counterfactual exercises, we start from the first time period with the number of Starbucks in the market being the one observed in the data, and Luckin has not made its entry, and then simulate the expansion actions for the length of actual data (about 900 days).

**Effects of Limited Consideration** Figure 5 depicts the fraction of duopolistic and monopolistic markets as a function of time for full consideration and limited consideration models. Limited consideration has a negative effect on competition. In particular, it substantially slows down competition. For example, the full consideration model achieves 44% of duopoly markets about 17 months before the limited consideration model. Similarly, the fraction of 48% of monopolistic markets is achieved by the full consideration model about 29 months faster.

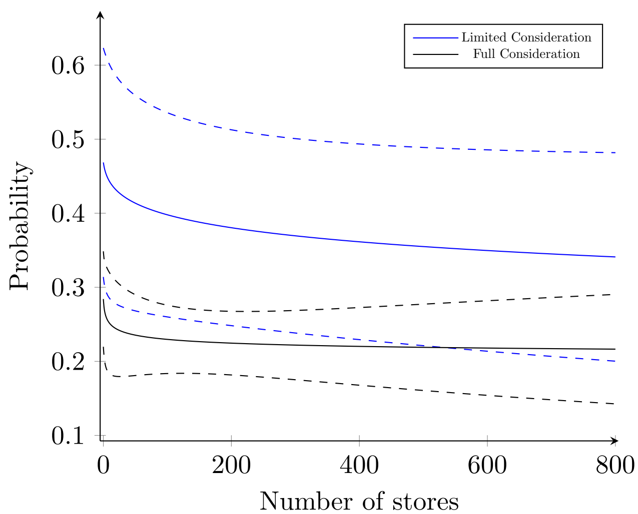
**Peer Effect in Consideration** Figure 6 depicts the fraction of duopolistic and monopolistic markets as a function of time for models with and without peer effects. The spillover effects across markets have a positive effect on competition. The final level of the fraction of duopolies and



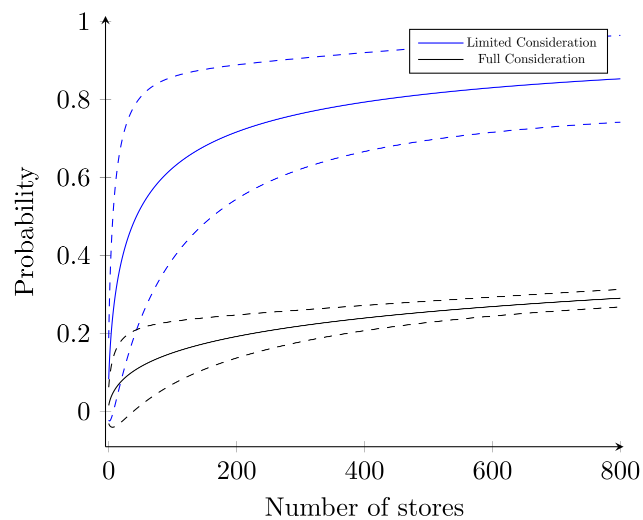

**(a)** Starbucks. Own stores in the market.




**(b)** Luckin. Own stores in the market.

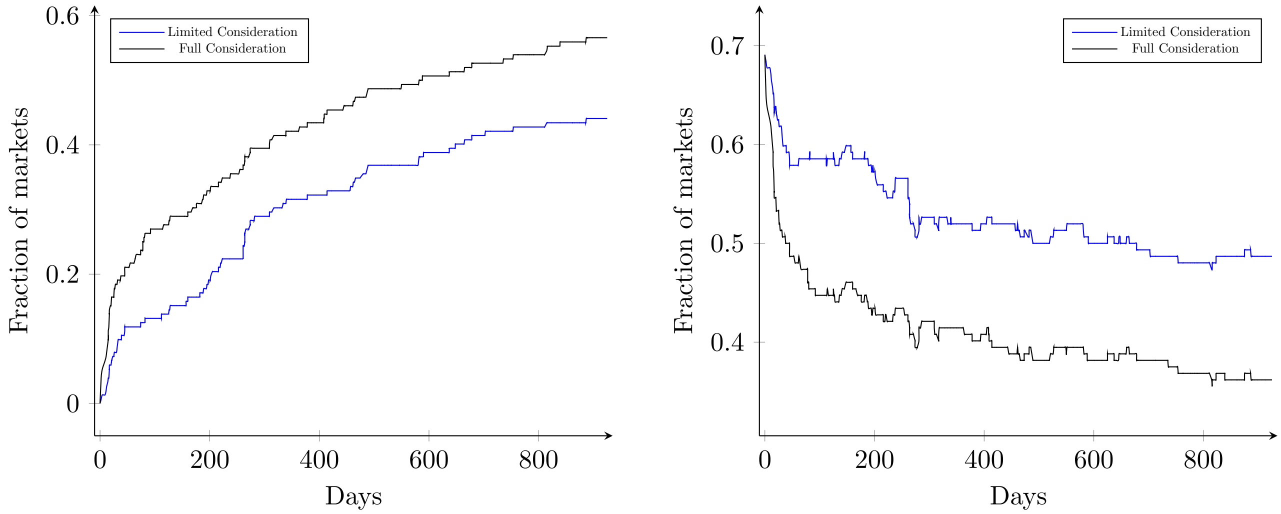



**(c)** Starbucks. Competitor's stores in the market




**(d)** Luckin. Competitor's stores in the market

**Figure 4** – Expansion probabilities for both coffee chains conditional on considering Shanghai as a function of the number of own or competitor's stores in Shanghai. Blue and black lines correspond to our model and to the model estimated assuming full consideration, respectively. Dashed lines depict pointwise 95% confidence bands. The fixed variables (e.g., market characteristics) are taken from the last time period.

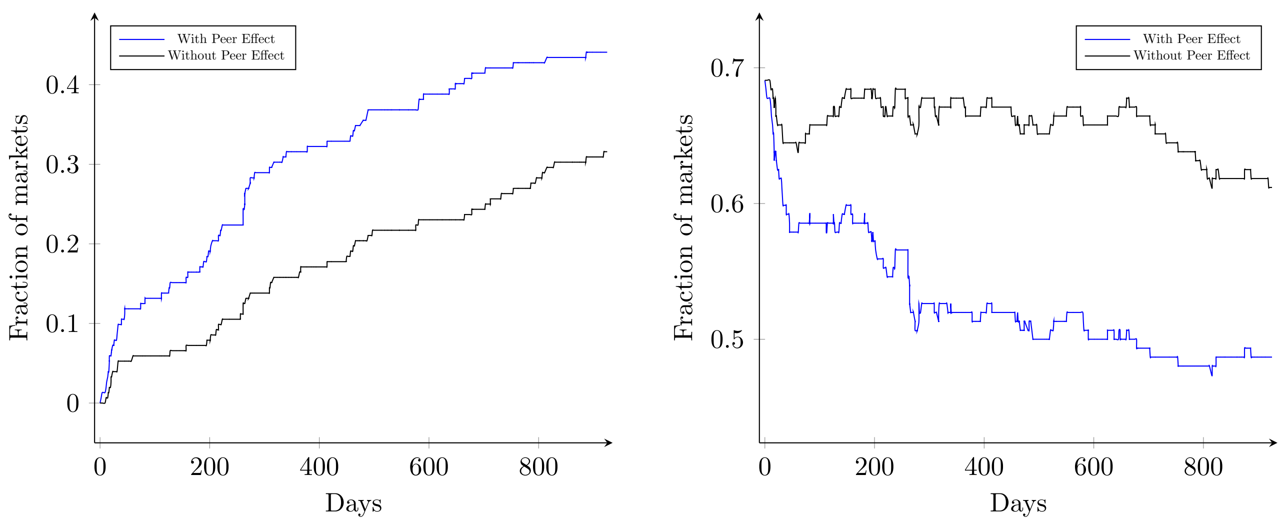



**(a)** Duopolies.

**(b)** Monopolies.

**Figure 5** – Fraction of duopolistic or monopolistic markets over time. Blue and black lines correspond to the cases with and without limited consideration, respectively.

monopolies under Scenario 2 is achieved by our model in about 18 and 30 months, respectively.




**(a)** Duopolies.

**(b)** Monopolies.

**Figure 6** – Fraction of duopolistic or monopolistic markets over time. Blue and black lines correspond to the cases with and without peer effects, respectively.

# 6. Final Remarks

This paper offers a rich social interaction model that builds on two basic ideas. First, the choices of friends can affect the decision of a given person in different ways. In particular, the choices of friends might affect the set of options that the person ends up considering, the preferences over these options, or both. Second, different friends can have different roles in each part. That is, while some of them might impact consideration probabilities only, others might only affect preferences, and a third group might impact both. From a methodological perspective, the model we offer combines the dynamic model of social interactions of Blume (1993, 1995) with the (single-agent) model of random consideration sets of Manski (1977) and Manzini and Mariotti (2014).

From an applied perspective, changes in the choices of friends induce stochastic variation of the choices of a given person. We show that this variation can be used to recover the main parts of the model. We nonparametrically identify not just the set of connections between the people in the network, but the type of interactions between them. We then use variation in the choices of peers that affect consideration sets as the main tool to recover the consideration probabilities and the random preferences. The identification strategy allows unrestricted heterogeneity across people. We propose a consistent estimator of model parameters and apply it to data on coffee chains expansions in China.

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## A. The Regularity Condition

In this appendix, we formally state and discuss the regularity condition needed for identification of the network in Section 3.

For a given $a \in \mathcal{A}$, define the set of all possible values that $\mathrm{NR}_a^{\mathcal{Y}}(\mathbf{y})$ and $\mathrm{NC}_a^{\mathcal{Y}}(\mathbf{y})$ can take:

$$\mathrm{Nrc}_a = \left\{ \left( \mathrm{NR}_a^{\mathcal{Y}}(\mathbf{y}), \mathrm{NC}_a^{\mathcal{Y}}(\mathbf{y}) \right) \, : \, \mathbf{y} \in \mathcal{Y}^A \right\}.$$

Let us also define $\bar{\mathrm{P}}_a \left( v \mid \mathbf{nr}, \mathbf{nc} \right)$ as the probability that Agent $a$ picks option $v \neq 0$ conditional on $(\mathbf{nr}, \mathbf{nc}) \in \mathrm{Nrc}_a$, where $nr^{v'}$ and $nc^{v'}$ denote the number of peers that affect preference only and consideration only, respectively, picking alternative $v'$, $v' \in \mathcal{Y} \setminus \{0\}$. That is,

$$\bar{\mathrm{P}}_a \left( v \mid \mathbf{nr}, \mathbf{nc} \right) = \sum_{\mathcal{C} \subseteq \mathcal{Y}} \mathrm{R}_a \left( v \mid nr^{\mathcal{C}}, \mathcal{C} \right) \mathrm{C}_a \left( \mathcal{C} \mid \mathbf{nc}, \mathcal{Y} \right),$$

where $nr^{\mathcal{C}} = \left( nr^{v'} \right)_{v' \in \mathcal{C} \setminus \{0\}}$ and

$$\mathrm{C}_a \left( \mathcal{C} \mid \mathbf{nc}, \mathcal{Y} \right) = \prod_{v' \in \mathcal{C}} \mathrm{Q}_a(v \mid nc^{v'}) \prod_{v' \in \mathcal{Y} \setminus \mathcal{C}} (1 - \mathrm{Q}_a(v \mid nc^{v'})).$$

Let $\Delta_{v,v'} f(\mathbf{x}, \mathbf{y})$ denote an operator that computes the increment of a given function when the $v$-th component of $\mathbf{x}$ and the $v'$-th component of $\mathbf{y}$ are increased by 1, respectively. We use a convention that if $v = 0$ ($v' = 0$), then $\mathbf{x}$ ($\mathbf{y}$) remains unchanged.

**Assumption 5.** For any $a \in \mathcal{A}$,

(i) there exist an alternative $v \in \mathcal{Y} \setminus \{0\}$ and a vector of aggregate peers' choices $(\mathbf{nr}, \mathbf{nc}) \in \mathrm{Nrc}_a$ such that

$$\Delta_{v,v} \ln \bar{\mathrm{P}}_a(v \mid \mathbf{nr}, \mathbf{nc}) \neq 0;$$

(ii) there exist three sets of alternative pairs and aggregate peers' choices, i.e., $\{v_i, w_i, \mathbf{nr}_i, \mathbf{nc}_i\}$, where $v_i, w_i \in \mathcal{Y} \setminus \{0\}$, $v_i \neq w_i$, $(\mathbf{nr}_i, \mathbf{nc}_i) \in \mathrm{Nrc}_a$, and $i = 1, 2, 3$, such that

$$\Delta_{w_1,0}\Delta_{v_1,0} \ln \bar{\mathrm{P}}_a(v_1 \mid \mathbf{nr}_1, \mathbf{nc}_1) \neq 0,$$

$$\Delta_{0,w_2}\Delta_{v_2,0} \ln \bar{\mathrm{P}}_a(v_2 \mid \mathbf{nr}_2, \mathbf{nc}_2) \neq 0,$$

$$\Delta_{w_3,w_3}\Delta_{v_3,0} \ln \bar{\mathrm{P}}_a(v_3 \mid \mathbf{nr}_3, \mathbf{nc}_3) \neq 0.$$

Assumption 5(i) guarantees that the peer effects in consideration and preferences do not cancel each other. That is, peers that affect both consideration and preferences are distinguishable from those who are not in one's reference group. Assumption 5(ii) is needed for distinguishing peers who affect consideration only from those who affect preference. Specifically, for the peers who affects consideration only, the double shift described above always equal zero, while the double shift in the observed CCPs is guaranteed to be nonzero for peers who affect preference for at least three scenarios by Assumption 5(ii).

It is worth emphasizing that the above inequality is only required to hold for one selection of the actions and peers' configuration. Additionally, Assumption 5(ii) allows $v_1 = v_2 = v_3$, $w_1 = w_2 = w_3$, and $(\mathbf{nr}_1, \mathbf{nc}_1) = (\mathbf{nr}_2, \mathbf{nc}_2) = (\mathbf{nr}_3, \mathbf{nc}_3)$. Furthermore, as the number of peers and/or the size of the menu is growing, it gets harder to violate Assumption 5. Therefore, Assumption 5 is a mild functional form restriction that is usually generically satisfied.

The following example clarifies the scope of Assumption 5.

**Example 3.** Suppose that

$$\mathrm{R}_a\left(v \mid t, \mathcal{C}\right) = \frac{u_v(t_v)}{\sum_{v' \in \mathcal{C}} u_{v'}(t_{v'})},$$

where $u_0(t_0) = 1$ and $u_v(\cdot)$, $v \in \mathcal{Y} \setminus \{0\}$, are strictly monotone positive functions. That is, after the consideration set is formed, Agent $a$ picks alternatives according to a logit-type rule.

Then, for the binary choice case, i.e., $Y = 1$ and $v = 1$, we have that

$$\bar{\mathrm{P}}_a(v \mid nr^v, nc^v) = \mathrm{Q}_a\left(v \mid nc^v\right) \frac{u_1(nr^v)}{1 + u_1(nr^v)}.$$

Note that Assumption 5(i) is violated if, the following equality holds:

$$\frac{\mathrm{Q}_a\left(v \mid nc^v + 1\right)}{\mathrm{Q}_a\left(v \mid nc^v\right)} = \frac{u_1(nr^v)}{1 + u_1(nr^v)} \frac{1 + u_1(nr^v + 1)}{u_1(nr^v + 1)},$$

for all admissible values of $nr^v$ and $nc^v$. Evidently, the larger the peer group is, the harder this equality to hold for all values.

We also illustrate that the same conclusion holds with a larger menu choice. Specifically, if we add one more alternative $v' = 2$, then

$$\begin{aligned}
\bar{\mathrm{P}}_a(v \mid nr^v, nr^{v'}, nc^v, nc^{v'}) &= \mathrm{Q}_a\left(v \mid nc^v\right) \mathrm{Q}_a\left(v' \mid nc^{v'}\right) \frac{u_1(nr^v)}{1 + u_1(nr^v) + u_2(nr^{v'})} \\
&\quad + \mathrm{Q}_a\left(v \mid nc^v\right) (1 - \mathrm{Q}_a\left(v' \mid nc^{v'}\right)) \frac{u_1(nr^v)}{1 + u_1(nr^v)} \\
&= \mathrm{Q}_a\left(v \mid nc^v\right) \frac{u_1(nr^v)}{1 + u_1(nr^v)} \frac{1 + u_1(nr^v) + \left[1 - \mathrm{Q}_a\left(v' \mid nc^{v'}\right)\right] u_2(nr^{v'})}{1 + u_1(nr^v) + u_2(nr^{v'})}.
\end{aligned}$$

So Assumption 5(i) is violated only if for $v \in \{1, 2\}$, $\Delta_{v,v} \ln \bar{\mathrm{P}}_a(v \mid \mathbf{nr}, \mathbf{nc}) = 0$ for all admissible values of $\mathbf{nr}$ and $\mathbf{nc}$. Specifically, it is violated at $\mathbf{nr} = \mathbf{nc} = \mathbf{0}$ indicates that

$$\begin{aligned}
&\ln \frac{\mathrm{Q}_a\left(v \mid 1\right)}{\mathrm{Q}_a\left(v \mid 0\right)} + \ln \left[\frac{u_1(1)}{1 + u_1(1)} \frac{1 + u_1(0)}{u_1(0)}\right] \\
&+ \ln \frac{1 + u_1(0) + u_2(0)}{1 + u_1(1) + u_2(0)} - \ln \frac{1 + u_1(0) + \left[1 - \mathrm{Q}_a\left(v' \mid 0\right)\right] u_2(0)}{1 + u_1(1) + \left[1 - \mathrm{Q}_a\left(v' \mid 0\right)\right] u_2(0)} = 0
\end{aligned}$$

for $v \in \{1, 2\}$. Note that if $\mathrm{Nrc}_a$ is rich enough to allow for switch from $\mathrm{Q}_a\left(v' \mid 0\right)$ to $\mathrm{Q}_a\left(v' \mid 1\right)$ without changing other parameters, then if Assumption 5(i) is violated at any point of the support, then

$$\ln \frac{\mathrm{Q}_a\left(v \mid 1\right)}{\mathrm{Q}_a\left(v \mid 0\right)} + \ln \left[\frac{u_1(1)}{1 + u_1(1)} \frac{1 + u_1(0)}{u_1(0)}\right]$$

$$+\ln\frac{1+u_1(0)+u_2(0)}{1+u_1(1)+u_2(0)}-\ln\frac{1+u_1(0)+[1-\mathrm{Q}_a(v'\mid 1)]\,u_2(0)}{1+u_1(1)+[1-\mathrm{Q}_a(v'\mid 1)]\,u_2(0)}=0.$$

Thus, it has to be the case that

$$\frac{1+u_1(0)+[1-\mathrm{Q}_a(v'\mid 1)]\,u_2(0)}{1+u_1(1)+[1-\mathrm{Q}_a(v'\mid 1)]\,u_2(0)}=\frac{1+u_1(0)+[1-\mathrm{Q}_a(v'\mid 0)]\,u_2(0)}{1+u_1(1)+[1-\mathrm{Q}_a(v'\mid 0)]\,u_2(0)}.$$

The latter is possible if and only if $\mathrm{Q}_a(v'\mid 0)=\mathrm{Q}_a(v'\mid 1)$ which violates Assumption 1(iii).

Note that Assumption 1(iii) also establishes that

$$\Delta_{0,v'}\Delta_{v,0}\ln\bar{\mathrm{P}}_a(v\mid \mathbf{0},\mathbf{0})\neq 0,$$

which guarantees Assumption 5(ii) with $i=2$. However, Assumption 5(ii) is more general. It only requires the nonzero to hold for one configuration and one action. We illustrate the restrictions of Assumption 5(ii) below. Specifically, violations of Assumption 5(iii) for case $i=1$ means that the following equations hold

$$\Delta_{v',0}\Delta_{v,0}\ln\bar{\mathrm{P}}_a(v\mid \mathbf{nr},\mathbf{nc})=0,$$
$$\Delta_{v,0}\Delta_{v',0}\ln\bar{\mathrm{P}}_a(v'\mid \mathbf{nr},\mathbf{nc})=0,$$

for all combinations of the choice configuration $\mathbf{nr}$ and $\mathbf{nc}$, including $\mathbf{nr}=\mathbf{0}$ and $\mathbf{nc}=\mathbf{0}$. It is harder for all equations to hold when the peer group size is getting larger or if the size of the alternatives becomes larger. Similar logic carries to cases $i=2$ and $i=3$.

### A.1. The case of Bundles

In this appendix, we discuss how the regularity condition needs to be modified to handle the extension of our model to bundles in Section 4.3. Note that with bundles the menu becomes $\mathcal{B}(\mathcal{Y})$.

Thus, we need to redefine $\mathrm{Nrc}_a$ and $\bar{\mathrm{P}}_a$. Let

$$\mathrm{Nrc}_a^{\mathcal{B}(\mathcal{Y})} = \left\{ \left( \mathrm{NR}_a^{\mathcal{B}(\mathcal{Y})}(\mathbf{y}), \mathrm{NC}_a^{\mathcal{B}(\mathcal{Y})}(\mathbf{y}) \right) \,:\, \mathbf{y} \in \mathcal{B}(\mathcal{Y})^A \right\}.$$

Let us also define $\bar{\mathrm{P}}_a\left(b \mid \mathbf{nr}, \mathbf{nc}\right)$, with $(\mathbf{nr}, \mathbf{nc}) \in \mathrm{Nrc}_a^{\mathcal{B}(\mathcal{Y})}$, as the probability that Agent $a$ picks bundle $b \neq 0$ conditional on $nr^{v'}$ and $nc^{v'}$ peers that affect preference only and consideration only, respectively, picking bundles $v'$, $v' \in \mathcal{B}(\mathcal{Y}) \setminus \{0\}$. That is,

$$\bar{\mathrm{P}}_a\left(b \mid \mathbf{nr}, \mathbf{nc}\right) = \sum_{\mathcal{C} \subseteq \mathcal{Y}} \mathrm{R}_a\left(v \mid nr^{\mathcal{B}(\mathcal{C})}, \mathcal{B}(\mathcal{C})\right) \mathrm{C}_a\left(\mathcal{C} \mid \mathbf{nc}, \mathcal{Y}\right),$$

where $nr^{\mathcal{C}} = \left(nr^{b'}\right)_{b' \in \mathcal{B}(\mathcal{Y}) \setminus \{0\}}$ and

$$\mathrm{C}_a\left(\mathcal{C} \mid \mathbf{nc}, \mathcal{Y}\right) = \prod_{v' \in \mathcal{C}} \mathrm{Q}_a(v \mid nc^{v'}) \prod_{v' \in \mathcal{Y} \setminus \mathcal{C}} (1 - \mathrm{Q}_a(v \mid nc^{v'})).$$

The modified version of Assumption 5 then becomes

**Assumption 5′.** For any $a \in \mathcal{A}$

(i) there exist $b \in \mathcal{B}(\mathcal{Y}) \setminus \{0\}$ and $(\mathbf{nr}, \mathbf{nc}) \in \mathrm{Nrc}_a^{\mathcal{B}(\mathcal{Y})}$ such that

$$\Delta_{v,v} \ln \bar{\mathrm{P}}_a(b \mid \mathbf{nr}, \mathbf{nc}) \neq 0;$$

(ii) there exist $b_i, d_i \in \mathcal{B}(\mathcal{Y}) \setminus \{0\}$, and $(\mathbf{nr}_i, \mathbf{nc}_i) \in \mathrm{Nrc}_a$, $i = 1, 2, 3$, such that $b_i \neq d_i$, $i = 1, 2, 3$, and

$$\Delta_{d_1,0} \Delta_{b_1,0} \ln \bar{\mathrm{P}}_a(b_1 \mid \mathbf{nr}_1, \mathbf{nc}_1) \neq 0,$$

$$\Delta_{0,d_2} \Delta_{b_2,0} \ln \bar{\mathrm{P}}_a(b_2 \mid \mathbf{nr}_2, \mathbf{nc}_2) \neq 0,$$

$$\Delta_{d_3,d_3} \Delta_{b_3,0} \ln \bar{\mathrm{P}}_a(b_3 \mid \mathbf{nr}_3, \mathbf{nc}_3) \neq 0.$$

# B. Proofs

## B.1. Proof of Proposition 3.1

Fix some $a \in \mathcal{A}$. We will prove that

$$a' \notin \mathcal{N}_a \iff \frac{\mathrm{P}_a(v \mid \mathbf{y})}{\mathrm{P}_a(v \mid \mathbf{y}')} = 1 \text{ for all } v, \text{ and } \mathbf{y}, \mathbf{y}' \text{ that are different in the } a'\text{th component only.}$$

The "only if" part is straightforward. To show the "if" part, assume, towards a contradiction, that

$$\frac{\mathrm{P}_a(v \mid \mathbf{y})}{\mathrm{P}_a(v \mid \mathbf{y}')} = 1 \text{ for all } \mathbf{y}, \mathbf{y}' \text{ that are different in the } a'\text{th component only,}$$

but $a' \in \mathcal{N}_a$. Let $\mathbf{y}_z^v$ denote the vector in which the $z$-th component of $\mathbf{y}$ is replaced by $v$. If $a' \in \mathcal{NC}_a \setminus \mathcal{NR}_a$, then

$$\frac{\mathrm{P}_a(v \mid \mathbf{0}_{a'}^v)}{\mathrm{P}_a(v \mid \mathbf{0})} = \frac{\mathrm{Q}_a(v \mid 1)}{\mathrm{Q}_a(v \mid 0)} \neq 1,$$

where the last inequality follows from Assumption 1(iii). Similarly, by Assumption 2(iii), if $a' \in \mathcal{NR}_a \setminus \mathcal{NC}_a$, then

$$\frac{\mathrm{P}_a(v \mid \mathbf{0}_{a'}^v)}{\mathrm{P}_a(v \mid \mathbf{0})} \neq 1.$$

Hence, $a' \in \mathcal{NCR}_a$. But the latter contradicts Assumption 5(i), since $a' \in \mathcal{NCR}_a$ would imply that the consideration peer effect offsets the preference peer effect *everywhere* over the support. The contradiction completes the proof.

## B.2. Proof of Proposition 3.2

Note that $\mathcal{N}_a$ is identified by Proposition 3.1. Take any two distinct agents $a', a'' \in \mathcal{N}_a$. We will show that $a' \in \mathcal{NC}_a \setminus \mathcal{NR}_a$ if and only if

$$\frac{\mathrm{P}_a(v \mid \mathbf{y}_{a''}^w)}{\mathrm{P}_a(v \mid \mathbf{y})} = \frac{\mathrm{P}_a(v \mid (\mathbf{y}_{a'}^v)_{a''}^w)}{\mathrm{P}_a(v \mid \mathbf{y}_{a'}^v)}, \tag{5}$$

for all $v \in \mathcal{Y} \setminus \{0\}$, all $w \neq v$, and $\mathbf{y}$ with $y_{a''} \neq v$. Thus, $\mathcal{NC}_a \setminus \mathcal{NR}_a$ is identified from $\mathrm{P}_a$.

The "only if" part of the statement follows, because switching $a''$ from another to $w$ does not change the probability that $v$ is considered, therefore, it does not change the relative probability $v$ is being selected of switching $a'$'s choice from other alternative to $v$.

To prove the "if" part, note that it is equivalent to the statement that if $a' \in \mathcal{NR}_a$, then there exist $a'' \in \mathcal{N}_a$, $v$, $w$, and $\mathbf{y}$ with $y_{a''} \neq v$ such that

$$\frac{\mathrm{P}_a\left(v \mid \mathbf{y}_{a''}^{w}\right)}{\mathrm{P}_a\left(v \mid \mathbf{y}\right)} \neq \frac{\mathrm{P}_a\left(v \mid (\mathbf{y}_{a'}^{v})_{a''}^{w}\right)}{\mathrm{P}_a\left(v \mid \mathbf{y}_{a'}^{v}\right)}.$$

or equivalently

$$\Delta_{a'}^{v}\Delta_{a''}^{w} \log \mathrm{P}_a\left(v \mid \mathbf{y}\right) \neq 0.$$

If $a'' \in \mathcal{NR}_a \setminus \mathcal{NC}_a$, then let $i = 1$. If $a'' \in \mathcal{NC}_a \setminus \mathcal{NR}_a$, then let $i = 2$. Finally, if $a'' \in \mathcal{NCR}_a$, then let $i = 3$. Take $v = v_i$, $w = w_i$, and $\mathbf{y}$ such that $y_{a'} = y_{a''} = 0$, and $\mathrm{NR}_a^{\mathcal{Y}}(\mathbf{y}) = \mathbf{nr}_i$ and $\mathrm{NC}_a^{\mathcal{Y}}(\mathbf{y}) = \mathbf{nc}_i$ from Assumption 5(ii). Then

$$\Delta_{a'}^{v}\Delta_{a''}^{w} \ln \mathrm{P}_a\left(v \mid \mathbf{y}\right) = \Delta_{a''}^{w}\Delta_{a'}^{v} \log \mathrm{P}_a\left(v \mid \mathbf{y}\right) = \begin{cases} \Delta_{w_1,0}\Delta_{v_1,0} \ln \bar{\mathrm{P}}_a(v_1 \mid \mathbf{nr}_1, \mathbf{nc}_1) \neq 0 \text{ if } i = 1, \\ \Delta_{0,w_2}\Delta_{v_2,0} \ln \bar{\mathrm{P}}_a(v_2 \mid \mathbf{nr}_2, \mathbf{nc}_2) \neq 0 \text{ if } i = 2, \\ \Delta_{w_3,w_3}\Delta_{v_3,0} \ln \bar{\mathrm{P}}_a(v_3 \mid \mathbf{nr}_3, \mathbf{nc}_3) \neq 0 \text{ if } i = 3, \end{cases}$$

where the first equality follows from exchangeability of the difference operator, the second equality follows from the definition of $\bar{\mathrm{P}}_a$, and the last inequality follows from Assumption 5(ii). So in all possible cases, Assumption 5(ii) implies that if $a' \in \mathcal{NR}_a$, then there exist $v$, $w$, and $\mathbf{y}$ with $y_{a''} \neq v$ such that

$$\frac{\mathrm{P}_a\left(v \mid \mathbf{y}_{a''}^{w}\right)}{\mathrm{P}_a\left(v \mid \mathbf{y}\right)} \neq \frac{\mathrm{P}_a\left(v \mid (\mathbf{y}_{a'}^{v})_{a''}^{w}\right)}{\mathrm{P}_a\left(v \mid \mathbf{y}_{a'}^{v}\right)}.$$

### B.3. Proof of Proposition 3.3

Note that we identified (knew) $\mathcal{N}_a$ and $\mathcal{NR}_a$ (or $\mathcal{NC}_a \setminus \mathcal{NR}_a$). To identify the rest of the network structure ($\mathcal{NR}_a \setminus \mathcal{NC}_a$ and $\mathcal{NCR}_a$), suppose that $\mathcal{NC}_a \setminus \mathcal{NR}_a \neq \emptyset$. Take $a' \in \mathcal{NC}_a \setminus \mathcal{NR}_a$. First, note that

$$\Delta^v_{a'} \ln \mathrm{P}_a(v \mid \mathbf{0}) = \ln \mathrm{Q}_a(v \mid 1) - \ln \mathrm{Q}_a(v \mid 0).$$

Thus, for any $a'' \in \mathcal{NR}_a$,

$$\Delta^v_{a''}\Delta^v_{a'} \ln \mathrm{P}_a(v \mid \mathbf{0}) \neq 0 \iff a'' \in \mathcal{NCR}_a.$$

Hence, $\mathcal{NCR}_a$ is identified from $\mathrm{P}_a$.

Next, suppose that $\mathcal{NC}_a \setminus \mathcal{NR}_a = \emptyset$. Then by Assumption 3 either $\mathcal{N}_a = \mathcal{NR}_a \setminus \mathcal{NC}_a$ or both $\mathcal{NR}_a \setminus \mathcal{NC}_a$ and $\mathcal{NCR}_a$ are nonempty. Since the consideration effect is nonzero, the effects of preference-only peers and preference-and-consideration peers have to be different. As a result we can identify the partition of $\mathcal{NR}_a$, $\mathcal{M}'$ and $\mathcal{M}''$, such that one of its elements is $\mathcal{NCR}_a$. Since $|\mathcal{N}_a| \geq 3 - |\mathcal{NC}_a \setminus \mathcal{NR}_a| = 3$, we can take $a' \in \mathcal{M}'$ and $a'' \in \mathcal{M}''$. Next, take $\mathbf{y}$ such that $y_a = 0$ for all $a \neq a'$ and $y_{a'} = v$. Next note that

$$\ln \mathrm{P}_a (v \mid \mathbf{y}) - \ln \mathrm{P}_a \left(v \mid \left(\mathbf{y}^0_{a'}\right)^v_{a''}\right) = (-1)^{\mathbb{1}(a' \notin \mathcal{NCR}_a)}(\ln \mathrm{Q}_a (v \mid 1) - \ln \mathrm{Q}_a (v \mid 0)).$$

Finally, take another $a''' \notin \{a', a''\}$ in either $\mathcal{M}'$ or $\mathcal{M}''$. Without loss of generality assume that $a''' \in \mathcal{M}'$. Note that by Assumption 1

$$\Delta^v_{a'''} \ln \mathrm{P}_a (v \mid \mathbf{y}) - \Delta^v_{a'''} \ln \mathrm{P}_a \left(v \mid \left(\mathbf{y}^0_{a'}\right)^v_{a''}\right) = 0 \iff a''' \in \mathcal{NR}_a \setminus \mathcal{NC}_a.$$

Thus, we identify $\mathcal{NR}_a \setminus \mathcal{NC}_a$ and $\mathcal{NCR}_a$.

## B.4. Proof of Proposition 3.4

Fix $a \in \mathcal{A}$ and $v \in \mathcal{Y} \setminus \{0\}$. Assume first that $|\mathcal{NC}_a \setminus \mathcal{NR}_a| \geq 1$. Under this situation, the relative consideration probability is identified via switching the choice of just one consideration-only peer from alternative $v$ to the default while keeping the configuration of others fixed. Specifically, take $a' \in \mathcal{NC}_a \setminus \mathcal{NR}_a$ and $\mathbf{y}$ such that every peer in $\mathcal{NC}_a$ picks $v$. Then

$$\frac{\mathrm{P}_a(v|\mathbf{y})}{\mathrm{P}_a(v|\mathbf{y}^0_{a'})} = \frac{\mathrm{Q}_a(v|\,|\mathcal{NC}_a|)}{\mathrm{Q}_a(v|\,|\mathcal{NC}_a|-1)}.$$

Next, redefine $\mathbf{y}$ as before except that we let one of the peers from $\mathcal{NCR}_a$ to pick 0. As a result,

$$\frac{\mathrm{P}_a(v|\mathbf{y})}{\mathrm{P}_a(v|\mathbf{y}^0_{a'})} = \frac{\mathrm{Q}_a(v|\,|\mathcal{NC}_a|-1)}{\mathrm{Q}_a(v|\,|\mathcal{NC}_a|-2)}.$$

Repeating this procedure, we identify

$$\mathrm{Q}_a(v \mid n_1)/\,\mathrm{Q}_a(v \mid n_1 - 1) \text{ for all } n_1 \in \{|\mathcal{NC}_a| - |\mathcal{NCR}_a|\,, \ldots, |\mathcal{NC}_a|\}.$$

Next, we define $\mathbf{y}$ as before except that all peers in $\mathcal{NCR}_a$ and one of the peers in $\mathcal{NC}_a \setminus \mathcal{NR}_a$ different from $a'$ are picking 0. Switching one by one all peers in $\mathcal{NC}_a \setminus \mathcal{NR}_a$ we identify $\mathrm{Q}_a(v \mid n_1)/\,\mathrm{Q}_a(v \mid n_1 - 1)$ for all $n_1$.

We next show that the relative consideration probability can be identified even if the consideration-only group is empty. Specifically, assume that $|\mathcal{NC}_a \setminus \mathcal{NR}_a| = 0$, so we have $|\mathcal{NR}_a \setminus \mathcal{NC}_a| \geq 1$ by Assumption 3. Then the relative consideration probability can be identified by switching one preference-only peer from $v$ to the default and one preference-and-consideration peer from the default to alternative $v$. Specifically, take $a' \in \mathcal{NR}_a \setminus \mathcal{NC}_a$, $a'' \in \mathcal{NCR}_a$, and $\mathbf{y}$ such that every peer in $\mathcal{NCR}_a$ picks $v$ and $a'$ picks 0. Then, comparing Agent $a$'s probability of choosing alternative $v$ between configuration $\mathbf{y}$ and a configuration of switching Agent $a'$ from 0 to alternative $v$ and Agent $a''$ from alternative $v$ to 0, which does not change the choice probability given consideration because the number of peers affecting preference is the same in both scenario, so we have

$$\frac{\mathrm{P}_a(v|\mathbf{y})}{\mathrm{P}_a\left(v|\,(\mathbf{y}^v_{a'})^0_{a''}\right)} = \frac{\mathrm{Q}_a(v|\,|\mathcal{NC}_a|)}{\mathrm{Q}_a(v|\,|\mathcal{NC}_a|-1)}.$$

Next, redefine $\mathbf{y}$ as before except that we let one of the peers from $\mathcal{NCR}_a$ different from $a''$ to pick 0. As a result,

$$\frac{\mathrm{P}_a(v|\mathbf{y})}{\mathrm{P}_a\left(v|\,(\mathbf{y}^v_{a'})^0_{a''}\right)} = \frac{\mathrm{Q}_a(v|\,|\mathcal{NC}_a|-1)}{\mathrm{Q}_a(v|\,|\mathcal{NC}_a|-2)}.$$

Repeating this procedure finitely many times we identify $\mathrm{Q}_a(v \mid n_1)/\,\mathrm{Q}_a(v \mid n_1 - 1)$ for all $n_1 \in \{1, \ldots, |\mathcal{NC}_a|\}$.

### B.5. Proof of Proposition 3.5

Fix some $a \in \mathcal{A}$ and $a' \in \mathcal{NC}_a \setminus \mathcal{NR}_a$. Moreover, take any $v, v' \in \mathcal{Y} \setminus \{0\}$, $v \neq v'$. Take any $\mathbf{y}$ such that no one picks $v'$. Since we will only use the variation in choices of Agent $a'$, we drop the choices of everyone else from the notation. For example, $\mathrm{P}_a(v|v')$ is equal to $\mathrm{P}_a(v|\mathbf{y})$, where $y_{a'} = v'$. We use $t_{v'}$ to denote the ratio between the probability that Agent $a$ picks $v'$ conditional on Agent $a'$ choosing $v'$ and the default 0:

$$t_{v'} \equiv \frac{\mathrm{P}_a(v'|v')}{\mathrm{P}_a(v'|0)} = \frac{\mathrm{Q}_a(v'|1)}{\mathrm{Q}_a(v'|0)} \neq 1,$$

where the second equality holds because we can cancel out the choice probability conditional on considering $v'$, and the last equality holds by Assumption 1(iii). Note that $t_{v'}$ is identified from the data.

Moreover,

$$\mathrm{P}_a(v|0) = \mathrm{Q}_a(v'|0)\left\{\mathrm{R}^*_a(v|v') - \mathrm{P}^*_a\left(v \mid \mathcal{Y} \setminus \{v'\}\right)\right\} + \mathrm{P}^*_a(v \mid \mathcal{Y} \setminus \{v'\}),$$
$$\mathrm{P}_a(v|v') = \mathrm{Q}_a(v'|1)\left\{\mathrm{R}^*_a(v|v') - \mathrm{P}^*_a(v \mid \mathcal{Y} \setminus \{v'\})\right\} + \mathrm{P}^*_a(v \mid \mathcal{Y} \setminus \{v'\}),$$

where $\mathrm{R}^*_a(v|v')$ denotes the probabilities that Agent $a$ picks $v$ conditional on considering $v'$. Since,

$Q_a(v'|0)t_{v'} = Q_a(v'|1)$, we obtain from the above two equations that

$$\mathrm{P}^*_a(v \mid \mathcal{Y} \setminus \{v'\}) = \frac{\mathrm{P}_a(v|v') - t_{v'}\,\mathrm{P}_a(v|0)}{1 - t_{v'}}.$$

Since the choice of $v$, $v'$, $a$, $a'$, and choices of everyone else was arbitrary, we can identify $\mathrm{P}^*_a(v \mid \mathbf{y}, \mathcal{Y} \setminus \{v'\})$ for all $a \in \mathcal{A}$, $v' \neq v$, $v' \neq 0$, and $\mathbf{y}$ such that (i) $y_{a'} \neq v'$ for all $a' \in \mathcal{N}_a$ and (ii) $y_{a'} = 0$ for some $a' \in \mathcal{NC}_a \setminus \mathcal{NR}_a$.

Applying the above argument to $\mathrm{P}^*_a(\cdot|\cdot, \mathcal{Y} \setminus \{v'\})$, we can identify $\mathrm{P}^*_a(v \mid \mathbf{y}, \mathcal{Y} \setminus \{v', v''\})$ for all $a \in \mathcal{A}$, $v'' \neq v$, $v'' \neq v'$, $v'' \neq 0$, and $\mathbf{y}$ such that (i) $y_{a'} \notin \{v', v''\}$ for all $a' \in \mathcal{N}_a$ and $y_{a'} = y_{a''} = 0$ for some $a', a'' \in \mathcal{NC}_a \setminus \mathcal{NR}_a$, $a' \neq a''$.

Repeating the above argument $|\mathcal{NC}_a \setminus \mathcal{NR}_a|$ times, we can identify $\mathrm{P}^*_a(\cdot \mid \mathbf{y}, \mathcal{Y} \setminus \mathcal{Z})$ for all $\mathcal{Z} \subseteq Y \setminus \{0\}$ and $\mathbf{y}$ such that (i) $y_{a'} \notin \mathcal{Z}$ for all $a' \in \mathcal{N}_a$, (ii) any different $|\mathcal{Z}|$ components of $\mathbf{y}$ that corresponds to peers from $\mathcal{NC}_a \setminus \mathcal{NR}_a$ are equal to 0.

### B.6. Proof of Proposition 3.6

Fix some $v \neq 0$. If $Q_a(v \mid n_1)$ is known for some $n_1$ in the support, by Proposition 3.4, we identify $Q_a(v \mid \cdot)$. If, instead, we know $\mathrm{R}_a(v \mid n_2, \{0, v\})$, then, since $|\mathcal{NC}_a \setminus \mathcal{NR}_a| \geq Y$, by Proposition 3.5, we identify

$$\mathrm{P}^*_a(v \mid \mathbf{y}, \{0, v\}) = Q_a(v \mid \mathcal{NC}^v_a(\mathbf{y}))\,\mathrm{R}_a(v \mid \mathrm{NR}^v_a(\mathbf{y}), \{0, v\}).$$

for some $\mathbf{y}$ such that $\mathrm{NR}^v_a(\mathbf{y}) = n_2$. Hence, we identify $Q_a(v \mid \mathcal{NC}^v_a(\mathbf{y}))$ and, by Proposition 3.4, we also identify $Q_a(v \mid \cdot)$. Since, the choice of $v$ was arbitrary, we identify $Q_a$.

By Proposition 3.5, we now can identify $\mathrm{R}_a(v \mid n_2, \{0, v\})$ for all $v \neq 0$ and $n_2$ in the support. Next, consider

$$\begin{aligned}\mathrm{P}^*_a(v \mid \mathbf{y}, \{0, v, v'\}) = {}& Q_a(v \mid \mathcal{NC}^v_a(\mathbf{y}))\,Q_a\left(v' \mid \mathcal{NC}^{v'}_a(\mathbf{y})\right)\mathrm{R}_a(v \mid \mathrm{NR}^v_a(\mathbf{y}), \{0, v\}) + \\ & + Q_a(v \mid \mathcal{NC}^v_a(\mathbf{y}))\,(1 - Q_a\left(v' \mid \mathcal{NC}^{v'}_a(\mathbf{y})\right))\,\mathrm{R}_a\left(v \mid \mathrm{NR}^v_a(\mathbf{y}), \mathrm{NR}^{v'}_a(\mathbf{y}), \{0, v, v'\}\right).\end{aligned}$$

Since $Q_a$ and $\mathrm{R}_a$ for binary consideration sets are identified, we identify $\mathrm{R}_a$ for all possible sets of size 3. Repeating the above argument, we identify $\mathrm{R}_a$ for all possible sets of size 4. Applying this argument finitely many times, we can identify $\mathrm{R}_a$ for all possible sets.

### B.7. Proof of Proposition 3.7

Since $\lim_{\Delta\to 0}\mathcal{P}(\Delta)=\mathcal{M}$, we can recover transition rate matrix from the data. Recall that

$$\mathrm{m}(\mathbf{y}'\mid\mathbf{y})=\begin{cases}0 & \text{if } \sum_{a\in\mathcal{A}}\mathbb{1}(y'_a\neq y_a)>1\\ \sum_{a\in\mathcal{A}}\lambda_a\,\mathrm{P}_a(y'_a\mid\mathbf{y})\,\mathbb{1}(y'_a\neq y_a) & \text{if } \sum_{a\in\mathcal{A}}\mathbb{1}(y'_a\neq y_a)=1\end{cases}.$$

Thus, $\lambda_a\,\mathrm{P}_a(y'_a\mid\mathbf{y})=\mathrm{m}(y'_a,\mathbf{y}_{-a}\mid\mathbf{y})$. It follows that we can recover $\lambda_a\,\mathrm{P}_a(v\mid\mathbf{y})$ for each $v\in\overline{\mathcal{Y}}$, $\mathbf{y}\in\overline{\mathcal{Y}}^A$, and $a\in\mathcal{A}$. Note that, for each $\mathbf{y}\in\overline{\mathcal{Y}}^A$,

$$\sum_{v\in\overline{\mathcal{Y}}}\lambda_a\,\mathrm{P}_a(v\mid\mathbf{y})=\lambda_a\sum_{v\in\overline{\mathcal{Y}}}\mathrm{P}_a(v\mid\mathbf{y})=\lambda_a.$$

Then we can also recover $\lambda_a$ for each $a\in\mathcal{A}$.

### B.8. Proof of Proposition 3.8

This proof builds on Theorem 1 of Blevins (2017) and Theorem 3 of Blevins (2018). For the present case, it follows from these two theorems, that the transition rate matrix $\mathcal{M}$ is generically identified if, in addition to the conditions in Proposition 3.8, we have that

$$(Y+1)^A-AY-1\geq\frac{1}{2}.$$

This condition is always satisfied if $A>1$. Thus, identification of $\mathcal{M}$ follows because $A\geq 2$. We can then uniquely recover $(\mathrm{P}_a)_{a\in\mathcal{A}}$ from $\mathcal{M}$ as in the proof of Proposition 3.7.

### B.9. Proof of Proposition 4.1

Under the assumptions of the propositions, by Proposition 3.2, we identify $\mathcal{N}_a$.

Suppose we know $\mathcal{NR}_a$. If it is empty, then $\mathcal{NR}_a \setminus \mathcal{NC}_a$ is empty. Hence, by Assumption $3'$, $\mathcal{NCR}_a$ is empty and we identify the network. Suppose $\mathcal{NR}_a$ is nonempty and is equal to $\mathcal{N}_a$. Then $\mathcal{NC}_a \setminus \mathcal{NR}_a$ is empty and Assumption $3'$ implies that $\mathcal{NCR}_a$ is empty and we identify the network. Finally, suppose $\mathcal{NR}_a$ is nonempty and is not equal to $\mathcal{N}_a$. Hence, we identify a consideration-only peer and can use her to separate $\mathcal{NCR}_a$ from $\mathcal{NR}_a \setminus \mathcal{NC}_a$ by using double differences of $\ln \mathrm{P}_a$.

Suppose we know $\mathcal{NC}_a$. Similarly, to the above, the cases when $\mathcal{NC}_a$ is empty or is equal to $\mathcal{N}_a$, Assumption $3'$ imply identification of the network. Suppose $\mathcal{NC}_a$ is nonempty and is different from $\mathcal{N}_a$. Hence, we identify $\mathcal{NR}_a \setminus \mathcal{NC}_a$ and the sign of the peer effect in preferences. Next, we look at the set

$$\{\Delta^1_{a'} \ln \mathrm{P}_a(1 \mid \mathbf{0}) \ : \ a' \in \mathcal{NC}_a\}.$$

If this set has cardinality 2, then since we know the sign of the preference effect we can identify the sign of the consideration effect and thus identify $\mathcal{NCR}_a$. If the cardinality of the set is 1,then $\mathcal{NC}_a = \mathcal{NC}_a \setminus \mathcal{NR}_a$ by Assumption $3'$ and the network structure is identified.

### B.10. Proof of Remark 3

Suppose we know $\mathcal{NR}_a$. Then $\mathcal{NC}_a \setminus \mathcal{NR}_a$ is identified. If it is nonempty, then using the double difference involving a consideration-only peer and any peer from $\mathcal{NR}_a$, would identify if that peer from $\mathcal{NR}_a$ is in $\mathcal{NCR}_a$ or not. Similarly, if we know $\mathcal{NC}_a$, we identify the network structure if $\mathcal{NR}_a \setminus \mathcal{NC}_a$ is not empty (we use Assumption 2(iv) to show that the double difference involving one preference-only peer and one peer from $\mathcal{NCR}_a$ is different from 0). Hence, the only two cases that are left are $\mathcal{NR}_a = \mathcal{N}_a$ and $\mathcal{NC}_a = \mathcal{N}_a$. But they are excluded by assumption of the proposition. The equivalent condition is that either $\mathcal{NR}_a \setminus \mathcal{NC}_a$ or $\mathcal{NC}_a \setminus \mathcal{NR}_a$ is known and nonempty.

### B.11. Proof of Proposition 4.7

We fix any $a$ and $\mathbf{y}$ and drop them from the notation to simplify the exposition. First note that for any linear order $\succ$ on $\mathcal{Y}$ that ranks the default the worst, by manipulating how fast the excluded covariates converge to the extreme point, we can always construct a sequence

$$\left\{w_{\succ,k} = \left(w_{a,v,k}\right)_{v\in\mathcal{Y}\setminus\{0\}}\right\}_{k=1}^{\infty}$$

with $\lim_{k\to+\infty} w_{a,v,k} = \overline{w}_{a,v}$ such that

$$\lim_{k\to\infty}\frac{\mathrm{U}_a(v'|w_{\succ,k})}{\mathrm{U}_a(v''|,w_{\succ,k})} = \lim_{k\to\infty}\frac{f_{a,v'}(w_{a,v'})}{f_{a,v''}(w_{a,v''})}\begin{cases}0, \text{ if } v''\succ v'\\ +\infty, \text{ otherwise.}\end{cases}$$

Next take $\succ$ such that $v$ is the second worst according to $\succ$ and default is the worst. Then

$$\lim_{k\to\infty}\mathrm{P}_a\left(0\mid w_{\succ,k}\right) = \mathrm{C}_a\left(\{0\}\right)$$

identifies the probability of considering the default only and

$$\lim_{k\to\infty}\mathrm{P}_a\left(v\mid w_{\succ,k}\right) = \mathrm{C}_a\left(\{0,v\}\right)$$

identifies $\mathrm{C}_a\left(\{0,v\}\right)$. Since we can do it for any $v$ we identify the consideration probabilities for all sets of cardinality 2. Next let $\succ$ be such that $v'\succ v\succ 0$ and $v'$ is the third worst. Then

$$\lim_{k\to\infty}\mathrm{P}_a\left(v'\mid w_{\succ,k}\right) = \mathrm{C}_a\left(\{0,v'\}\right) + \mathrm{C}_a\left(\{0,v,v'\}\right).$$

Since $\mathrm{C}_a\left(\{0,v'\}\right)$ was identified in the previous step and the choice of $v$ and $v'$ was arbitrary, we can identify the consideration probabilities for all sets of cardinality 3. Recursively, we can identify $\mathrm{C}_a$ for sets of all sizes. The fact that the choice of $a$ and $\mathbf{y}$ was arbitrary completes the proof.