

Random Utility and Limited Consideration^{*†}

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Abstract The random utility model (RUM, [McFadden and Richter, 1990](#)) has been the standard tool to describe the behavior of a population of decision makers. RUM assumes that decision makers behave as if they maximize a rational preference over a choice set. This assumption may fail when consideration of all alternatives is costly. We provide a theoretical and statistical framework that unifies well-known models of random (limited) consideration and generalizes them to allow for preference heterogeneity. We apply this methodology in a novel stochastic dataset that we collected in a large-scale online experiment. Our dataset is unique since it exhibits both choice set and (attention) frame variation. We run a statistical survival race between competing models of random consideration and RUM. We find that RUM cannot explain the population behavior. In contrast, we cannot reject the hypothesis that decision makers behave according to the logit attention model ([Brady and Rehbeck, 2016](#)).

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1. Introduction

A fundamental question in social science is how to describe the behavior of a population of decision makers (DMs). The random utility model (RUM, [McFadden and Richter, 1990](#)) is the standard tool to *describe* behavior.¹ RUM assumes that DMs behave *as if* they maximize their preferences over their choice set. However, RUM may fail at describing behavior if DMs do not consider all available alternatives. For instance, DMs may behave as if they use a two-stage procedure: first simplifying choice by using a consideration set, and only then choosing the best alternative among those considered. Thus, DMs may choose dominated alternatives.² A large literature, pioneered by [Masatlioglu et al. \(2012\)](#) and [Manzini and Mariotti \(2014\)](#), has proposed theories of consideration-mediated choice. These theories accommodate departures from RUM caused by inattention, feasibility, categorization, and search.³ In contrast to RUM,⁴ little is known about the empirical validity of these models. Our work aims to fill this important gap in the literature.

Methodologically, we provide a unifying theoretical framework that generalizes well-known theories of random consideration. We unify these theories with a new concept called attention-index. The attention-index is a net measure of how enticing a collection of alternatives is, or how costly it is to pay attention to it. We show how to test these theories statistically, and how to recover the preference distribution and consideration rules. Our framework extends many theories of consideration-mediated choice to allow for preference heterogeneity. This allows us to take these theories of individual behavior to the population level, thus permitting the use of cross-sectional datasets to test them. Following [McFadden and Richter \(1990\)](#) and [Kitamura and Stoye \(2018\)](#), we take seriously the fact that all theories of stochastic choice have as their primitive the unobserved distribution over choices that can only be estimated in finite samples by sample frequencies of choice. Thus, to test these models in finite samples, we need to account for sampling variability.

Empirically, we design a large experiment⁵ with two independent sources of exogenous variation:

¹RUM was first proposed by [Block and Marschak \(1960\)](#) and [Falmagne \(1978\)](#) in an environment similar to ours.

²For evidence of choice of dominated alternatives see [De Los Santos et al. \(2012\)](#), [Honka \(2014\)](#), [Heiss et al. \(2016\)](#), [Ho et al. \(2017\)](#), [Honka et al. \(2017\)](#), [Hortaçsu et al. \(2017\)](#) and [Barseghyan et al. \(2019a\)](#).

³See, for instance, [Aguiar et al. \(2016\)](#), [Brady and Rehbeck \(2016\)](#), [Aguiar \(2017\)](#), [Lleras et al. \(2017\)](#), [Caplin et al. \(2019\)](#), [Horan \(2019\)](#), [Kovach and Ülkü \(2020\)](#), and [Cattaneo et al. \(2020\)](#).

⁴For evidence for RUM see [Kitamura and Stoye \(2018\)](#) and [McCausland et al. \(2018\)](#).

⁵Other experiments that we are aware of that have collected stochastic choice data focusing on choice set variation are [Apesteguia et al. \(2018\)](#) (87 individuals) and [McCausland et al. \(2018\)](#) (141 participants). In contrast to our

(i) full variation in choice sets (menus), and (ii) variation in frames. We conducted this experiment online in Amazon Mechanical Turk (MTurk), collecting 12297 independent choice observations from 2135 individuals. A *frame* consists of observable information that is irrelevant in the rational assessment of the alternatives (Salant and Rubinstein, 2008).⁶ Full variation in choice sets means that all possible choice sets are observed by the researcher. It allows us to test consideration-mediated choice theories in a large cross-section of heterogeneous individuals. Frame variation means that we vary the complexity of the description of alternatives without affecting the relevant payoffs. This is equivalent to varying the cost of consideration. In this sense, we induce an attention frame. This variation in frames allows us to differentiate between RUM and models of limited consideration since consideration could change with frames but preferences must remain stable.

In general, without frame variation, many models of consideration are empirically indistinguishable from RUM. For instance, if a dataset, without frame variation, is consistent with the classical consideration model of Manzini and Mariotti (2014), then it is also consistent with RUM. However, this model and RUM will typically recover a distinct distribution of preferences. Varying frames, we can test whether the distribution of preferences remains the same across frames. This will imply that the model in Manzini and Mariotti (2014) and RUM may have different empirical implications in our experimental setup. The same reasoning applies to any model of limited consideration.

In our experimental design, we introduce three attention-cost/complexity frames for every choice set. Each DM faces all three frames. These treatments require the DM to solve a simple cognitive task to understand the alternatives. The consideration cost is progressively reduced across frames while we keep the choice set fixed. The fact that the choice remains the same is not explicitly stated in the experiment instructions. Within this design we can understand how changing consideration costs across frames may affect choices while we keep (the distribution of) preferences fixed. Under our incentives protocol (pay-at-random across tasks), and under consequentialism, the distribution of preferences must remain constant regardless of the frame. By exploiting this feature of our design, we show that, in our sample, RUM fails to describe the population behavior, while the logit attention model of Brady and Rehbeck (2016) describes it well.

Our paper generalizes the methodology of Cattaneo *et al.* (2020) by allowing heterogeneity in preferences when the choice sets include a default alternative. We show that in our setting, work, both focus mainly on binary choice sets and goodness-of-fit measures (including the computation of Bayes factors).

⁶We interpret the notion of rational assessment to be an assessment compatible with consequentialism and RUM.

if one assumes independence of preferences and consideration, at the population level, testing a consideration-mediated choice theory with heterogeneous preferences and a dominated default is equivalent to testing whether this hypothetical full-consideration distribution over choices is consistent with RUM. We, therefore, use this result to test these models using the framework of Kitamura and Stoye (2018).

To exploit all possible implications of the limited-consideration models of interest, we need full variation in choice sets. A limited consideration model may describe behavior well for a nonexhaustive dataset, but it may fail to do so for an extended one. That is, one may have false positives when observing choices from a nonexhaustive set of menus, as discussed at length in De Clippel and Rozen (2018) and Cattaneo *et al.* (2020). Nonetheless, full exogenous variation in choice sets is an important data feature that is usually not satisfied in field data. Our testing procedure exploits rich experimental variation to increase the statistical power of our tests.

In addition, our experiment introduces a dominated default alternative that works as an opportunity cost of paying attention. For any choice set and frame, the default is always present and shown first. Moreover, it is pre-selected as the default choice—if the subject decides to skip the task, she is informed that the default alternative will be chosen for her. This design allows us to use the default alternative as the opportunity cost of incurring in the cost of consideration and understanding the other alternatives in the choice set. The set of alternatives in our experiment consists of lotteries. Hence, we use a degenerate lottery as the default due to its simplicity. In this sense, we believe the default alternative in our design has effectively zero cost of consideration. This dominated default is key to disentangle the distribution of preferences from random consideration.⁷

We use these theoretical and experimental innovations to test two well-known models of random consideration: (i) the logit attention model of Brady and Rehbeck (2016) (LA), and (ii) the version of elimination-by-aspects model of Tversky (1972) characterized by Aguiar (2017) (EBA).⁸ There is increasing interest in incorporating limited consideration in discrete choice.⁹ In particular, the influential and tractable model of Manzini and Mariotti (2014) (MM) has become an important tool for the analysis of limited attention in empirical work (e.g., Dardanoni *et al.*, 2020, Abaluck and

⁷We formulate a sensitivity analysis when the default is not dominated in Appendix B.

⁸We refer to EBA as the version of the original model in Tversky (1972) by Cattaneo *et al.* (2020) (Example 6). The model in Aguiar (2017) coincides with EBA in the special case where there is a dominated default with a restriction that any category that does not contain the default has zero mass. The EBA model characterization for the case without a default remains an open question.

⁹See, for instance, Goeree (2008), Barseghyan *et al.* (2019a,b), Dardanoni *et al.* (2020), and Abaluck and Adams (2021).

Adams, 2021, and Kashaev and Lazzati, 2021). However, MM is highly stylized and assumes that consideration is driven by an item-dependent parameter (i.e., independence in consideration). We investigate from an experimental perspective whether this strong assumption is effective in explaining choice when it is hard. To do so we consider two extensions of MM that allow for substitution and complementarity in consideration, the LA and EBA models. These two generalizations have the property that their intersection is exactly the MM model (Suleymanov, 2018). This implies that MM explains the population behavior if and only if both the LA and EBA models explain it.

We test these models and the benchmark RUM conditioning on the frame. Crucially, we require the underlying preference relation to be stable among frames while allowing the consideration rules to vary with the frame. Our main findings are: (i) We reject the hypothesis that RUM provides a good description of population behavior. (ii) In contrast, the LA model with heterogeneous preferences cannot be rejected at the 5 percent significance level. (iii) However, we reject the hypothesis that EBA, and hence MM, describe the population behavior.

Our work contributes to the recent experimental literature on stochastic choice, limited consideration, and departures from RUM. Even though by now, limited attention in many environments is well documented, it is less clear what structural models of limited attention should the practitioner use. We see our contribution as providing answers to this second issue.¹⁰ We hope that our findings of which models of limited consideration are successful empirically will inform future empirical work in the field. For instance, our findings have already been used to motivate the choice of the parametric specification of limited consideration in the recent work of Abaluck and Adams (2021).¹¹

Outline

The paper proceeds as follows. Section 2 presents our model. Section 3 details frame variation and our testing procedure. Section 4 presents our experiment. Section 5 presents the testing results. Finally, Section 6 concludes. All proofs and additional results are in the appendix.

¹⁰There is a vast literature documenting departures from fully rational behavior, but it is not focused on limited consideration. See Rieskamp *et al.* (2006) for a survey.

¹¹Abaluck and Adams (2021) structurally estimate a model of discrete choice with limited consideration under the LA and MM model.

2. Environment – Model

We consider a finite choice set X and we denote the outside alternative or default as $o \notin X$. We let the set of all possible choice sets be $\mathcal{A} = 2^X \setminus \{\emptyset\}$, where 2^X denotes the set of all subsets of X . A probabilistic choice rule is a mapping $p : X \cup \{o\} \times \mathcal{A} \mapsto [0, 1]$. The probabilistic choice rules for a given choice set add up to 1, $\sum_{a \in A} p(a, A) + p(o, A) = 1$. Moreover, $p(a, A) = 0$ if $a \notin A$. We fix $p(o, \emptyset) = 1$. A complete stochastic choice rule is a vector $P = (p(a, A))_{A \in \mathcal{A}, a \in A \cup \{o\}}$. For identification purposes, we treat P as a known object. In practice, we do not observe P , but can consistently estimate it by the collection of sample frequencies \hat{P} (see Section 3.2 for further details).

2.1. Random Behavioral Model

We consider an environment where DMs, faced with a choice set $A \in \mathcal{A}$, first pick $D \subseteq A$ (consideration set) and then choose the alternative in D that maximizes their preferences. With probability $\pi(\succ)$, DMs are endowed with preferences $\succ \in X \times X$ drawn from the set of all linear orders (strict preferences) on X , $R(X)$.¹² Note that since $o \notin X$, following [Manzini and Mariotti \(2014\)](#), we implicitly assume that the default is picked if and only if nothing else is considered. A typical interpretation of this situation is the sleeping agent behavior (see, for instance, [Abaluck and Adams, 2021](#)). When the agent is sleeping (i.e., she considers the empty set) she chooses the default alternative. Otherwise, the agent wakes up and considers some nonempty set and maximizes her preferences in her consideration set. In the second case, the default is assumed to be dominated by the rest of alternatives. We show how to relax this assumption in Appendix B.

The distribution $\pi \in \Delta(R(X))$ fully captures preference heterogeneity.¹³ The distribution over random consideration sets given the menu A is fully characterized by $m_A : 2^A \rightarrow [0, 1]$, $\sum_{D \subseteq A} m_A(D) = 1$. In other words, m_A is an element of the simplex $\Delta(2^A)$. Let m denote the complete collection of those distributions for all possible menus. That is, $m = (m_A(D))_{A \in \mathcal{A}, D \in 2^A}$. We assume that the random consideration sets and random preferences are *independent*.

¹²Linear orders are complete, transitive, and antisymmetric orders.

¹³For any set C , $\Delta(C)$ denotes the set of all probability distributions (simplex) on C . $\mathbf{1}(B)$ denotes indicator of the statement B and is equal to 1 if B is true and is equal to zero otherwise.

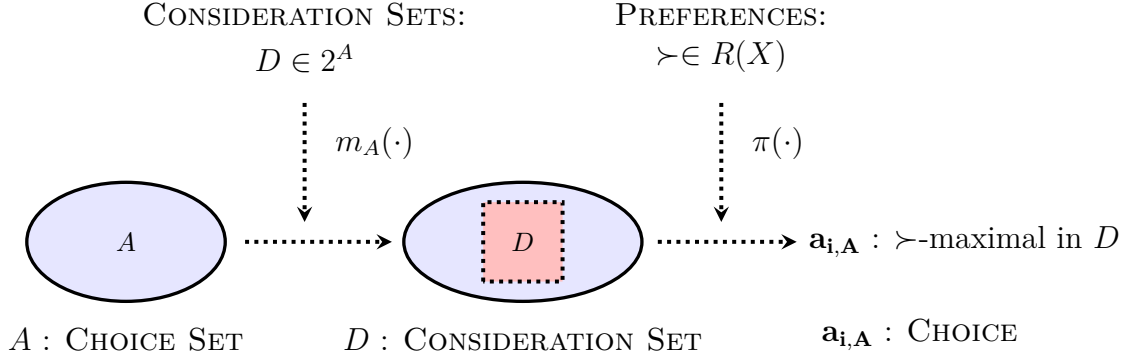


Figure 1 – CONSIDERATION MEDIATED CHOICES Choices are the result of a two stage process, first pick a consideration set and then pick the best alternative in that set. We observe choices (\mathbf{a}_i) and menus (A). We do not observe and we aim to identify the theory objects: distribution of preferences in the population (π) and stochastic choice rule (m_A).

Definition 1 (Random Behavioral Model, B-rule). A complete stochastic choice rule P is a B-rule if there exists a pair (m, π) such that

$$p(a, A) = \sum_{\succ \in R(X)} \pi(\succ) \sum_{D \subseteq A} m_A(D) \mathbb{1}(a \succ b, \forall b \in D)$$

for all $a \in X$ and $A \in \mathcal{A}$.

This choice rule is illustrated in Figure 1. Definition 1 implicitly assumes that the random consideration set rule and the heterogeneous preferences are independent. Independence is a good starting assumption in the sterile environment of our experiment, as we want to achieve a decomposition of any observed probabilistic choice rule into its consideration (captured by m) and preference (captured by π) components. Independence has been assumed successfully in the structural work of [Abaluck and Adams \(2021\)](#). Also, we are interested in modeling decision-making in two-stages where DMs simplify a hard choice task using fast-and-frugal heuristics (consideration) that are independent of preferences, and then choose rationally from the simplified choice set. If the researcher observes additional information (e.g. age, gender, education, and income levels of individuals), then random consideration rule and random preferences need to be independent only conditionally on those observables. ¹⁴

Independence holds trivially for the cases of homogeneous preferences, such as all models covered by the Random Attention Model (RAM) of [Cattaneo et al. \(2020\)](#). Moreover, as the following lemma demonstrates, the B-rule does not have empirical content even under the independence

¹⁴See [Kashaev and Aguiar \(2021\)](#) for a study of the correlation between preferences and consideration.

assumption.

Lemma 1. *Every complete stochastic choice rule P is a B-rule.*

Without additional restriction on π and m the model is not falsifiable. That is, any P can be generated by some independent π and m . We will impose constraints on m that will allow us to test an important class of random consideration sets models without restricting heterogeneity in preferences.

2.2. Attention-Index Consideration Set Rule

We restrict m by considering a family of consideration set rules that are governed by an *attention-index*. The attention-index $\eta \in \Delta(2^X)$ is a distribution over the power set. The value $\eta(D)$ captures the unconditional attention that DMs pay to the set $D \in 2^X$. The attention-index of a set is a net measure of its attractiveness with respect to how hard it is to consider it. The attention-index measures how enticing a consideration set is, and how complex it is to understand. Therefore, $\eta(C) > \eta(D)$ means that C , in net terms, attracts more attention than D .

Definition 2 (Attention-index representation). A consideration set rule m admits an attention-index representation if there exists an attention-index η , a link function ψ , and an index correspondence g such that $g(D, A) \subseteq 2^X \setminus D$ and

$$m_A(D) = \psi \left(\eta(D), \sum_{C \in g(D, A)} \eta(C) \right)$$

for all $A \in \mathcal{A}$ and $D \subseteq A$.

In what follows, we assume that the link function ψ and the correspondence g are known. A given link function captures the particular way in which the attention-index shapes consideration given a choice set or menu. In other words, the link function transforms unconditional attention into conditional (on the choice set) attention. The index set $g(D, A)$ captures the collection of sets that are used in the attention aggregator $\sum_{C \in g(D, A)} \eta(C)$. However, we do not assume that η is known and do not impose any restrictions on it, so the setting is still semiparametric.

Next, we define several important models of limited consideration that admit the attention-index representation. They only differ in how they use the attention-index to form the consideration set

at a given menu.

Definition 3. The logit attention (LA, [Brady and Rehbeck, 2016](#)), the choice set independent (MM, [Manzini and Mariotti, 2014](#)), the random consideration (EBA, [Tversky, 1972](#), [Aguiar, 2017¹⁵](#)), and the full consideration (FC) models admit an attention-index representation such that:

- $m \in \mathcal{M}^{\text{LA}}$ if and only if there exists $\eta \in \Delta(2^X)$ such that for

$$m_A(D) = \frac{\eta(D)}{\sum_{C \subseteq A} \eta(C)} > 0$$

for all $A \in \mathcal{A}$ and $D \in 2^A$;

- $m \in \mathcal{M}^{\text{MM}}$ if and only if $m \in \mathcal{M}^{\text{LA}}$ with

$$\eta(A) = \prod_{a \in X \setminus A} (1 - \gamma(a)) \prod_{b \in A} \gamma(b)$$

for a given $\gamma : X \rightarrow (0, 1)$ and for all $A \in 2^X$;

- $m \in \mathcal{M}^{\text{EBA}}$ if and only if there exists $\eta \in \Delta(2^X)$ such that

$$m_A(D) = \sum_{C: C \cap A = D} \eta(C)$$

for all $A \in \mathcal{A}$ and $D \in 2^A$;

- $m \in \mathcal{M}^{\text{FC}}$ if and only if $m \in \mathcal{M}^{\text{EBA}}$ with

$$\eta(A) = \mathbb{1}(A = X).$$

The MM-rule imposes independence in consideration across items, making the model highly stylized and tractable. Both the LA- and the EBA-rules generalize this item-independence to allow for substitution and complementarity of attention between different items. The LA-rule predicts that the probability of considering D is proportional to the attention-index value $\eta(D)$. The EBA-rule predicts that the probability of considering a subset D given menu A is equal to the probability that D is the intersection of the subset of alternatives (considered randomly using the

¹⁵The model in [Aguiar \(2017\)](#) is a special case of the model in [Tversky \(1972\)](#) and coincides with it when the attention-index of only sets that contain the default is nonzero.

attention-index) and the choice set.¹⁶ In our application, we focus on these four particular models because they allow us to learn about the true data generating process governing our experimental application systematically.¹⁷ However, our approach extends to any model of consideration that admits an attention-index representation.¹⁸

Given the definition of the attention-index representation, we can define a restricted version of the B-rule (L-B-rule). Let \mathcal{M}^L be the set of all consideration rules induced by a given link function ψ^L , the correspondence g^L , and all attention-indices η

$$\mathcal{M}^L = \left\{ m : m_A(D) = \psi^L \left(\eta(D), \sum_{C \in g^L(D,A)} \eta(C) \right) \text{ for some } \eta \in \Delta(2^X) \right\}.$$

Definition 4 (L-B-rule). A complete stochastic choice rule P is L-B-rule if P is a B-rule with $m \in \mathcal{M}^L$.

We believe that random consideration rules that admit an attention-index representation have several theoretical advantages over other generalizations in the literature. First, they cover, as special cases, well-known models of consideration and allow us to introduce a new class of semiparametric models of limited consideration with heterogeneous preferences. In other words, the L-B-rule unifies existing models into a common structure. This structure helps us to understand the common traits of these models, to classify them, and to provide an identification and testing framework to possibly new models covered in this structure.

Second, they allow for unique identification of the consideration rule and the underlying stochastic choice full consideration probability from a cross-section of choices with menu variation and heterogeneous preference (see Theorem 3). We are not aware of any other work that achieves the same and is more general than ours. Allowing heterogeneous preferences is important in analysis of any datasets, and is unavoidable in cross-sections. An important alternative generalization is RAM, which imposes only a shape constraint on random consideration. However, it does not allow heterogeneous preferences, does not obtain point identification of neither consideration nor the underlying full consideration choice rule. Moreover, RAM is completely uninformative about

¹⁶Note that for the LA-model $\psi^{\text{LA}}(x, y) = x/(x + y)$ and $g^{\text{LA}}(D, A) = \{C \in 2^X \setminus D : C \cap A = C\}$, and for the EBA-model $\psi^{\text{EBA}}(x, y) = x + y$ and $g^{\text{EBA}}(D, A) = \{C \in 2^X \setminus D : C \cap A = D\}$.

¹⁷LA and EBA are completely distinct generalizations of MM. Thus, rejecting or accepting either of these models is very informative.

¹⁸For instance, one can generate a continuum of such models by taking all possible convex combinations of consideration rules m^{LA} and m^{EBA} that are generated by the same attention-index η .

preferences, when the stochastic choice rule is regular,¹⁹ as in the random utility models.²⁰ Also, the L-B-rule allows cycles of probabilities that can violate regularity while RAM cannot (see Appendix C.2).

Third, the concept of attention-index is intuitive, simple, and of behavioral interest, as it provides a useful (unconditional) index of attention for any subset of alternatives. The link function and the attention-index follow the tradition of the classical stochastic choice theory of simple scalability where the probability of choice of an alternative in a menu is a nonlinear function (a link function) of some scale/index (Krantz, 1965, Tversky, 1972). In the simple scalability tradition, the scale/index captures the intensity of the stimuli associated with a particular alternative. A classical example is the logit model of choice. Here, we apply the same intuition to consideration—the attention-index captures the net attractiveness of a given category of all alternatives.

Fourth, consideration rules that admit an attention-index representation are compatible with optimal random consideration of a representative DM. Here, we show that a L-B-rule can be obtained as the result of allocating attention optimally (see Example 2).

In addition, consideration rules that admit an attention-index representation are compatible with a flexible interpretation of randomness. In our framework, the randomness due to limited consideration can arise both at the individual and population level. Indeed, consideration can be random at the individual level and independent and identically distributed (i.i.d.) at the population level; and consideration can be deterministic at the individual level but heterogeneous at the population level. The next example shows how the latter case can be described by a L-B-rule.

Example 1 (Heterogeneous Categorization). Consider a population of DMs with two types of agents endowed with different deterministic attention rules. Assume that half of the DMs are fully attentive, while the other half follows a rule of thumb: they pay full attention to option b if it is present in a given choice set, else the consideration set is empty. The DMs pick the best alternative, according to the (heterogeneous) preference realization governed by some $\pi \in \Delta(R(X))$. If the consideration set is empty, then the outside option is selected. This population has heterogeneous (deterministic) consideration, however, the population behavior can be fully captured by a random consideration rule with the EBA restriction. Namely, let $\eta(X) = \frac{1}{2}$ and $\eta(\{b\}) = \frac{1}{2}$. Then the

¹⁹Regular random choice means that $p(a, A) \geq p(a, B)$ for any two menus $A \subseteq B$ and any $a \in A$.

²⁰Kashaev and Aguiar (2021) extend RAM to allow for heterogeneous preferences and show that when the stochastic choice rule is regular nothing can be said as well about random consideration and preferences even under independence among them.

EBA-B-rule can describe this population behavior.

The next example demonstrates that the consideration rules that admit an attention-index representation can be derived as a solution to the problem where representative DMs optimally allocate their attention.

Example 2 (Costly Attention Allocation). Consider a (representative) DM whose preferences are given by a (mean) utility $u : X \rightarrow \mathbb{R}$ and additive random vector $\boldsymbol{\xi} = (\xi_x)_{x \in X}$ such that the random utility of a given item x is given by $u(x) + \xi_x$. The taste shocks $\boldsymbol{\xi}$ are distributed with respect to some continuous distribution.²¹ When the DM is faced with a menu A , she needs to allocate her attention, measured by $m_A \in \Delta(2^A)$, over all possible consideration sets in A (including the empty set). The attractiveness of a set D is captured by the McFadden's surplus of a given set defined by $\alpha(D) = \mathbb{E}[\max_{x \in D} (u(x) + \xi_x)]$ for all $D \subseteq 2^A \setminus \{\emptyset\}$, where the expectation is taken with respect to $\boldsymbol{\xi}$. The attractiveness of the empty set is normalized to be 0, $\alpha(\emptyset) = 0$. The surplus $\alpha(D)$ is a measure of average attractiveness, capturing how enticing a consideration set is for the representative DM. The difficulty of picking a consideration set, or the cost of attention, is captured by a cognitive cost function $K : [0, 1] \rightarrow \mathbb{R} \cup \{\infty\}$. If D is considered with probability $m(D)$, then the cognitive cost is $K(m(D))$. The cost function is menu independent, but depends on the allocated attention $m(D)$. Following [Fudenberg et al. \(2015\)](#), we assume that K is convex. In this case, DM's problem is to find $m_A \in \Delta(2^A)$ that maximizes the expected attractiveness of the menu, given the cognitive cost of processing it. Formally,

$$m_A = \arg \max_{m \in \Delta(2^A)} \sum_{D \subseteq A} [m(D)\alpha(D) - K(m(D))].$$

When $K(t) = 0$ for all $t \in [0, 1]$, the solution is such that $m_A(A) = 1$. That is, the DM is consistent with full consideration (FC). When $K(t) = -t \log(t)/\theta$, where θ is the cost parameter, we get that

$$m_A(D) = \frac{\exp(\theta\alpha(D))}{\sum_{C \subseteq A} \exp(\theta\alpha(C))}.$$

That is, optimal consideration in this case is consistent with LA with the attention-index $\eta(D) = \exp(\theta\alpha(D))$. Note that in this definition of η , the independence assumption is still satisfied since $\eta(D)$ is an aggregate quantity and only depends on the mean utilities $(u(x))_{x \in X}$ and the

²¹This guarantees that the implied random utility rule π exists because ties have zero probability.

distribution of ξ but not on the realizations of ξ (e.g., when ξ follows the Gumbel distribution, then $\alpha(D) = \log \sum_{x \in D} e^{u(x)}$). Of course, we can replace $\alpha(D)$ with any other measure of attractiveness and all our derivations will go through. This means we can generate any model consistent with LA this way. Finally, when $K(t) = \frac{1}{2}t^2$, we can get a special case of the EBA model.²²

Finally, the attention-index representation also has several econometric advantages. It allows for a significant reduction of the dimensionality of the consideration set rules, making them tractable. In general, the number of parameters controlling the random consideration is $\sum_{A \subseteq X} 2^{|A|} - 1$. The single-attention index and the link function reduce the number of unknown parameters to $2^{|X|} - 1$. It also leads to statistical testing (i.e. we can take into account sample variability) of known models of random consideration in a cross-section dataset of choices (see Section 5). Thus, one can confront existing models to experimental datasets in a competitive fashion to guide the exploration of models of consideration sets and to inform which models are more successful.

2.3. Characterization and Identification of the L-B-model

In this section, we answer the following questions: (i) When can we recover different consideration rules from the data? (ii) What are their observable implications? We answer these questions by *decomposing* the observed probabilities of choice P into an attention rule m and a distribution of preferences π . In other words, we recover from the dataset P the primitives of the B-rule that generated it, and provide necessary and sufficient conditions that guarantee that a dataset P can be generated by a L-B-rule.

Our starting point is to exploit the fact that if a consideration rule m admits an attention-index representation, then the probability of choosing the default alternative is completely determined by the attention-index η . In particular, the probability of choosing the outside option is independent of the distribution of preferences due to the independence assumption that we have imposed between preferences and consideration. In addition, recall that in our model the outside option is only chosen when nothing else in the menu is considered.²³ If we denote $p_o = (p(o, A))_{A \in \mathcal{A}}$ and

²²The EBA model then requires that the DM does not re-optimize in smaller menus $A \subset X$. Instead, she uses the heuristic that whatever category she draws at X it is intersected with the given menu to obtain the consideration set.

²³For an extension that relaxes this assumption see Appendix B.

$\psi_\emptyset(\eta) = \left(\psi \left(\eta(\emptyset), \sum_{C \in g(\emptyset, A)} \eta(C) \right) \right)_{A \in \mathcal{A}}$, abusing notation we can write the system of equations

$$p_o = \psi_\emptyset(\eta).$$

When ψ_\emptyset is *invertible*, we can uniquely recover the random consideration rule from the probability of choosing the outside alternative from all different menus. Since our objective is the identification of the consideration rule, we provide a natural restriction on the attention-index rule that guarantees invertibility of ψ . This restriction is satisfied by the models of interest in this paper (but not restricted to them).

Definition 5 (Totally monotone consideration). A consideration rule m admitting an attention-index representation is *totally monotone* if, we can write, for all $A \in \mathcal{A}$

$$m_A(\emptyset) = \varphi \left(\eta(\emptyset), \sum_{C \subseteq A} \eta(C) \right),$$

where $\varphi : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is a strictly monotone in each argument function.

The probability of not considering any object, conditional on a given choice set, is assumed to be a monotonic function of the cumulative probability of paying attention to at least some alternative in the menu (according to η), and of the probability of not considering anything (unconditionally). Crucially, the dependence on the correspondence g disappears in a totally monotone attention-index representation, with respect to the general attention-index representation. Note, this happens only for the case of not considering anything (i.e., the consideration set is \emptyset).

Totally monotone attention-index rules are such that the random consideration is monotone as in Cattaneo *et al.* (2020), namely $m_A(\emptyset) \leq m_B(\emptyset)$ if $B \subseteq A$, when φ is strictly increasing in the first entry. However, in this case, they imply more. Since, the mapping $m_{(\cdot)}(\emptyset) : \mathcal{A} \rightarrow [0, 1]$ is a function of the cumulative probability of considering at least one item in any given menu (i.e., a function of $\sum_{C \subseteq A} \eta(C)$), the behavior of the probability of choosing the outside option will be restricted. For instance, for the case of EBA it will satisfy a form of marginal decreasing propensity of choice (Aguilar, 2017).

We highlight that total monotonicity is testable. Observe that the marginal probability of choosing the outside option, when a new set of alternatives is added to a menu, is weakly decreasing

(e.g., $\Delta_{CP}(o, A) = p(o, A \cup C) - p(o, A) \leq 0$ and $\Delta_D(\Delta_{CP}(o, A) \leq 0)$).²⁴ Alternatively, φ can be strictly decreasing in the first entry, in which case it provides an antithetic behavior to that of RAM (yet testable). In this sense, this restriction on attention is neither weaker nor stronger than the monotonicity restriction in Cattaneo *et al.* (2020).

Strict monotonicity of φ in each of its entries implies the invertibility of ψ .²⁵ This sufficient condition for invertibility of the link function ψ is mild. It holds in all the examples of interest in this work. Importantly, it is a testable restriction. Note that since the inverse of ψ is known under the model of interest, we can compute a candidate η from the data P . If the computed η is not an element of the simplex $\Delta(2^X)$ then ψ is not invertible.

The next lemma shows that the models we consider admit a totally monotone attention-index representation.

Lemma 2. *Any $m \in \mathcal{M}^L$, $L \in \{\text{LA}, \text{MM}, \text{EBA}, \text{FC}\}$, admits a totally monotone attention-index representation with*

- $\varphi^{\text{LA}}(\eta_o, t) = \frac{\eta_o}{t}$;
- $\varphi^{\text{EBA}}(\eta_o, t) = 1 - t + \eta_o$;
- $\varphi^{\text{MM}}(\eta_o, t) = \varphi^{\text{LA}}(\eta_o, t)$ and $\varphi^{\text{MM}}(\eta_o, t) = \varphi^{\text{EBA}}(\eta_o, t)$;
- $\varphi^{\text{FC}}(\eta_o, t) = \varphi^{\text{EBA}}(\eta_o, t)$.

The proof is omitted because of its simplicity for the cases of LA, EBA, and FC. For the case of MM the statement follows from Brady and Rehbeck (2016) and Aguiar (2017).²⁶

The key assumption in this section has been that the default that is always present is always dominated. We formulate a sensitivity analysis when the default is not dominated in Appendix B.

2.4. Characterization of the L-B-model

As a preliminary step for characterizing the L-B-model, we construct a candidate calibrated attention-index η^L from the data P . Informally, this calibrated (revealed) attention-index is the

²⁴Note, however, that this restriction does not mean that m satisfies the monotonicity property for other menus different from the empty set.

²⁵This is a consequence of Mobius invertibility of the mapping $v(\cdot) = \sum_{C \subseteq \cdot} \eta(C)$ (Chateauneuf and Jaffray, 1989).

²⁶Note that since MM and FC are special cases of LA and EBA respectively, they share the same link function. However, empirically we will be able to differentiate among them because of the additional restrictions they pose on η .

result of inverting the link function φ with respect to the probability of choosing the default alternative. The link function invertibility is a consequence of the monotonicity assumptions and the existence of a unique Mobius inverse of the cumulative attention-index $v(\cdot) = \sum_{C \subseteq \cdot} \eta(C)$ (Chateauneuf and Jaffray, 1989). We do this recursively. For a given φ^L , let $\varphi_1^{-1,L}$ and $\varphi_2^{-1,L}$ be the inverses of φ^L with respect to the first and the second argument, respectively. Let $|A|$ denote the cardinality of a finite set A .

Definition 6 (Calibrated attention-index). For given P , $\eta^L : 2^X \rightarrow \mathbb{R}$ is such that (i) $\eta^L(\emptyset) = \varphi_1^{-1,L}(p(o, X), 1)$, and (ii) for all $A \in 2^X \setminus X$

$$\eta^L(A) = \sum_{B \subseteq D} (-1)^{|D \setminus B|} \varphi_2^{-1,L}(\eta^L(\emptyset), p(o, D)).$$

The calibrated attention-index depends only on the dataset P and the model L . If the calibrated attention-index of a set is negative, then P could not have been generated by model L . This testable implication is analogous to the Block and Marschak (1960) inequalities.

Now, we construct an object, m^L , that is a distribution over consideration sets if the model L is correctly specified.

Definition 7. For a given P , let $m^L = (m_A^L(D))_{A \in \mathcal{A}, D \in 2^A}$, where $m_A^L : 2^A \rightarrow \mathbb{R}$ is such that for all $A \in \mathcal{A}$ and $D \in 2^A$

$$m_A^L(D) = \psi^L \left(\eta^L(D), \sum_{C \in g^L(D, A)} \eta^L(C) \right).$$

We can apply this generic formula for totally monotone attention-index rules to the specific models of interest.

- Example 3.**
- $m_A^{\text{LA}}(D) = \frac{\eta^{\text{LA}}(D)}{\sum_{C \subseteq A} \eta^{\text{LA}}(C)}$, where $\eta^{\text{LA}}(D) = \sum_{B \subseteq D} (-1)^{|D \setminus B|} \frac{p(o, X)}{p(o, B)}$;
 - $m_A^{\text{MM}}(D) = \frac{\eta^{\text{MM}}(D)}{\sum_{C \subseteq A} \eta^{\text{MM}}(C)}$, where $\eta^{\text{MM}}(D) = \prod_{a \in X \setminus D} (1 - \gamma^{\text{MM}}(a)) \prod_{b \in D} \gamma^{\text{MM}}(b)$, and $\gamma^{\text{MM}} : X \rightarrow \mathbb{R}$ such that $\gamma^{\text{MM}}(a) = 1 - \frac{p(o, A)}{p(o, A \setminus \{a\})}$ for some $A \in \mathcal{A}$ that contains a ;
 - $m_A^{\text{EBA}}(D) = \sum_{C: C \cap A = D} \eta^{\text{EBA}}(C)$, where $\eta^{\text{EBA}}(D) = \sum_{A \subseteq D: D \in \mathcal{A}} (-1)^{|D \setminus A|} p(o, X \setminus A)$;
 - $m_A^{\text{FC}}(D) = \mathbb{1}(A = D)$.

In general, m^L may not be a distribution (some components may be negative or greater than 1) since m^L is calibrated from observed frequencies. Moreover, m^{LA} or m^{MM} may not be well-defined if probabilities of choosing the outside option for some choice sets are zero.

To be able to estimate m^L from the data with probability approaching 1, we need the following definition that formalizes the above discussion.

Definition 8 (Well-defined m^L). m^L is well-defined if $m_A^L \in \Delta(2^A)$ for all $A \in \mathcal{A}$.

We are ready to state our main result.

Theorem 1. *For every link function L , the following are equivalent:*

- (i) P is a L-B-rule;
- (ii) m^L is a well-defined and P is a B-rule described by (m^L, π) .

Theorem 1 provides a testable implication of the model. If P is a L-B-rule, then m^L has to be well-defined. Theorem 1 implies that to test a given model one does not need to consider all possible distributions over considerations sets. It suffices to check the unique distribution that is calibrated from observed P according to Definition 7.

Initially, we had to find two objects (the distribution over preferences π and the distribution over consideration sets m) to make the data consistent with the model. Now we just need to find π . In other words, we achieved dimensionality reduction. Unfortunately, the testing problem is still not tractable since the set of all possible distributions over preferences $\Delta(R(X))$ is big. To solve this problem we introduce another fictitious object.

Definition 9. For given model L and P , let $P_\pi^L = (p_\pi^L(a, A))_{A \in \mathcal{A}, a \in X}$, where $p_\pi^L : X \times \mathcal{A} \rightarrow \mathbb{R}$ is such that for all $A \in \mathcal{A}$ and $a \in A$

$$p_\pi^L(a, A) = \frac{p(a, A) - \sum_{C \subset A} m_A^L(C) p_\pi^L(a, C)}{m_A^L(A)}.$$

Note that when P has been generated by a L-B-rule, P_π^L corresponds to the underlying full-consideration random utility rule. In fact, we can write a L-B model equivalently as:

$$p(a, A) = \sum_{D \subseteq A} m_A(D) p_\pi(a, D),$$

where $p_\pi(a, A) = \sum_{\succ \in R(X)} \pi(\succ) \mathbb{1}(a \succ b, \forall b \in A)$ is the underlying FC distribution over (nondefault) choices that is weighted by the random consideration rule m to produce the observed behavior. When P has been generated by this L-B-rule, it follows that $p_\pi^L = p_\pi$. That is why we call P_π^L the *calibrated full consideration rule*.

Similar to m^L , P_π^L has interpretation when the L-B-rule is consistent with the data. The next theorem provides the last missing piece of our characterization before testing.

Theorem 2. *Suppose that for given model L and stochastic choice rule P (i) m^L is well-defined, (ii) $m_A^L(A) > 0$ for all $A \in \mathcal{A}$. Then the following are equivalent:*

(i) P is a L-B-rule;

(ii) P_π^L is a FC-B-rule.

Note that both $m^L \geq 0$ and P_π^L can be computed from P . Thus, Theorem 2 implies that to test a given model L it is necessary and sufficient to test whether m^L is well defined, and whether calibrated P_π^L is a full consideration rule. Theorems 1 and 2 provide a generalization of the characterization results in Manzini and Mariotti (2014), Brady and Rehbeck (2016), and Aguiar (2017). Moreover, it provides a unified result for all models that admit a (totally monotone) attention-index representation.

In practice, we do not observe P , but can consistently estimate it by the collection of sample frequencies \hat{P} . In Section 3.2 we discuss how to test the L-B-rule accounting for sampling variability in \hat{P} .

2.5. Identification

Assuming independence between the distribution of preferences and the random consideration set rule, we uniquely identify the consideration set rule from P if it is a L-B-rule, for all models with totally monotone link functions. Moreover, if there is a positive mass of individuals that consider all alternatives in the choice set, the recoverability of preferences is as good as in the case of full consideration.

Theorem 3 (Identification). *Suppose that for a given model L (i) P is a L-B-rule and (ii) $m_A^L(A) > 0$ for all $A \in \mathcal{A}$. If P is described by (m, π) and (m', π') , then $m = m'$ and $p_\pi = p_{\pi'}$.*

We underline that we achieve a unique decomposition of the dataset P into its attention and preference components. Identification of preferences and consideration rules is not a trivial task. Even for simple datasets where stochastic behavior arises from only one channel (for example limited consideration), models that only allow for stochastic behavior because of preference heterogeneity

(e.g. RUM), or because of random consideration/attention without additional assumptions (e.g. RAM) may fail to identify underlying preferences even when they perfectly describe observed choices. Our framework shows that the recoverability of preferences is as good as the RUM benchmark in stark contrast with RAM, where nothing can be learned about preferences for regular random choice.

3. Frames and Testing

3.1. Frames

In this section, we introduce another source of variation in the data–attention frames. This additional source of variation will allow us to differentiate between behavior consistent with RUM and behavior consistent with L-B. Following [Salant and Rubinstein \(2008\)](#), we define the extended choice set as a pair of a choice set $A \in \mathcal{A}$ and an attention frame $f \in F$. In this extended environment, for a given frame $f \in F$, we can define a probabilistic choice rule p_f . Similarly, a complete stochastic choice rule with frame $f \in F$ is $P_f = (p_f(a, A))_{A \in \mathcal{A}, a \in A \cup \{o\}}$.

The elements of F contain descriptions of physical items that only vary in presentation but not in the information they contain. Our interpretation of attention frames is the same as [Bhattacharya et al. \(2021\)](#). These descriptions are available to DMs and should not affect their preferences, but may influence their attention.

Example 4 (Stochastic choice rule with frames). Let $X = \{a, b, c\}$ and consider two frames: (i) f that describes $a = 1$ token, $b = 2$ tokens and $c = 3$ tokens; (ii) and f' that describes $a = 3 - 2$ tokens, $b = 10 - 8$ tokens, and $c = 2 + 1$ tokens. An example of an (incomplete) stochastic choice rule with frame f is $P_f = (p_f(a, \{a, b\}) = 0, p_f(b, \{a, b\}) = 1, p_f(b, \{b, c\}) = 0, p_f(c, \{b, c\}) = 1)$. An example of an (incomplete) stochastic choice rule with frame f' is $P_{f'} = (p_{f'}(a, \{a, b\}) = 1/2, p_{f'}(b, \{a, b\}) = 1/2, p_{f'}(b, \{b, c\}) = 1/2, p_{f'}(c, \{b, c\}) = 1/2)$.

In [Example 4](#), f and f' present the same information about the same alternatives in two distinct ways. In particular, different frames correspond to different numbers of arithmetic operations used to describe the value of the option. The same value is expressed in each frame with a different

sequence of arithmetic operations.

The B-rule with frame f can be defined analogously to the definition of the B-rule in Section 2. A complete stochastic choice rule with frame f , P_f , is a B-rule if there exists a frame dependent distribution over random consideration sets, m_f and a frame dependent distribution over strict linear orders π_f such that

$$p_f(a, A) = \sum_{\succ \in R(X)} \pi_f(\succ) \sum_{D \subseteq A} m_{f,A}(D) \mathbf{1}(a \succ b, \forall b \in D).$$

We take the stand that preferences should not depend on the way the information is presented. Only attention may change due to frames. We formalize these ideas by the assumption of consequentialism, which is a common implicit assumption in the RUM framework of [McFadden and Richter \(1990\)](#).

Definition 10 (Consequentialism). A collection of B-rules $(P_f)_{f \in F}$ described by $((m_f, \pi_f))_{f \in F}$ is said to satisfy consequentialism if there exists π such that $\pi_f = \pi$ for all $f \in F$.

Consequentialism means that an attention frame does not alter the payoffs that a DM obtains from choosing a given alternative. In the same way that the standard rational choice framework imposes frame independence ([Salant and Rubinstein, 2008](#)), the classical RUM imposes consequentialism ([McFadden and Richter, 1990](#)).

At this point, it is useful to formally define the Random Utility Model (RUM) over the whole choice set $X \cup \{o\}$. RUM treats the default alternative as just another item with no special status. That is, it is not assumed to be a dominated alternative. Let $R(X \cup \{o\})$ be a set of linear orders over the extended choice set $X \cup \{o\}$.

Definition 11 (Random Utility Model, RUM). A collection of complete stochastic choice rules with frame $(P_f)_{f \in F}$ is consistent with random utility if there exists $\pi_o \in \Delta(R(X \cup \{o\}))$ such that

$$p_f(a, A) = \sum_{\succ \in R(X \cup \{o\})} \pi_o(\succ) \mathbf{1}(a \succ b, \forall b \in A),$$

for all $a \in A$, $A \in \mathcal{A}$, and all $f \in F$.

Note that RUM satisfies consequentialism and π_o does not depend on the frame. The L-B-rule extends the RUM framework to allow for frame dependence only through the random consideration rule.

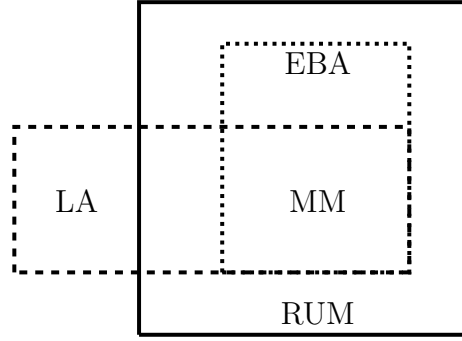


Figure 2 – RELATION AMONG CONSIDERATION SET RULES FOR A FIXED FRAME: RUM, LA, MM, AND EBA

By assuming that preferences satisfy consequentialism, we are essentially making two assumptions: (i) we assume that the distribution of preferences does not depend on how the alternatives are presented; (ii) consideration in turn can depend on payoff irrelevant information. These assumptions are important, since our experiment has three treatments where a subject must perform a different number of arithmetic operations to evaluate the monetary value of a prize.

These assumptions are auxiliary and are not necessary for either RUM or L-B-rules. However, these assumptions are a reasonable baseline to compare RUM and random consideration models. The reason these assumptions are not immediately implied is because RUM is often viewed as descriptive. Taking this one step further, one could assume that the distribution of preferences varies with the difficulty of evaluating a task in our experiment. This would violate our assumption of consequentialism. While frame-dependent preference could be a reasonable assumption in some settings, it would limit the ability to predict counterfactual choices. For example, one would need to identify preferences in each setting to get a prediction on choices.

Specifically, frame variation allows us to differentiate between L-B-rules and RUM as follows. Without it LA is not nested by nor nests RUM. For example, LA allows for attraction effect which violates regularity and therefore is inconsistent with RUM. Also, their intersection is nonempty because MM is both consistent with RUM and LA (Manzini and Mariotti, 2014, Brady and Rehbeck, 2016). Moreover, for a fixed frame EBA is nested in RUM and nests MM, therefore its intersection with LA is nonempty (Aguiar, 2017) (see Figure 2).²⁷ However, with frame variation this is no longer true: RUM and EBA intersect, but are not nested within each other.²⁸

²⁷Suleymanov (2018) shows that MM is the intersection of LA and EBA.

²⁸All these relationships are preserved when allowing for heterogeneous preferences under the independence assumption of preferences and attention. The reason is that the outside probability does not depend on the distribution of preferences. For more details see Appendix C.

3.2. Testing Procedure

Theorem 2 allows us to test whether for a given frame f , a given stochastic choice rule P_f is a L-B-rule: it is necessary and sufficient to test whether m_f^L is well-defined (satisfies a set of linear inequalities) and $P_{f,\pi}^L$ is consistent with the full consideration model. Note that the full consideration model is equivalent to the random utility model without outside option. Testing for RUM is a well-understood problem and amounts to solving a quadratic optimization with cone constraints (see [McFadden and Richter, 1990](#) and [Kitamura and Stoye, 2018](#)). The approach proposed by [Kitamura and Stoye \(2018\)](#) allows us to test these conditions while accounting for sampling variability induced by using \hat{P}_f instead of unknown P_f .

We, however, need to slightly modify the testing procedure in [Kitamura and Stoye \(2018\)](#) to take into account the frame variation. First we describe the testing procedure for the fixed frame and then extended it to environments with frame variation.

To introduce the testing procedure, we need to define several objects. Note that, for a fixed frame f , the calibrated full consideration rule, $P_{f,\pi}^L$, is a vector of length $d_p = \sum_{k=1}^{|X|} k \binom{|X|}{k}$.²⁹ The k -th element of $P_{f,\pi}^L$ corresponds to some pair (a, A) such that $a \in A \cup \{o\}$.

Let B_1 be the matrix of the size $d_p \times |X|!$ such that (k, l) element of it is equal to

$$B_{1,k,l} = \mathbb{1}(a \in A) \mathbb{1}(a \succ_l c, \forall c \in A),$$

where k corresponds to a pair (a, A) such that $a \in A$, and \succ_l is l -th linear order on X . We define G as the matrix of size $(d_p + d_m) \times d_1$, where $d_1 = |X|! + d_m$ and $d_m = \sum_{A \subseteq X} 2^{|A|}$ is the dimension of m_f^L , such that

$$G_1 = \begin{bmatrix} B_1 & 0_{d_p \times d_m} \\ 0_{d_m \times |X|!} & I_{d_m} \end{bmatrix},$$

where $0_{d_p \times d_m}$ denotes the zero matrix of size $d_p \times d_m$, and I_{d_m} denotes the identity matrix of size $d_m \times d_m$. The next result establishes an equivalent characterization of the L-B-rule via m_f^L and $P_{f,\pi}^L$. Let $\mathbb{R}_+^{d_1}$ denote component wise nonnegative elements of the d_1 -dimensional Euclidean space \mathbb{R}^{d_1} .

Theorem 4. *For a fixed frame f the following are equivalent:*

- (i) $P_{f,\pi}^L$ is a FC-B-rule and m_f^L is well-defined;

²⁹ $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ and $n! = 1 \cdot 2 \cdot \dots \cdot n$.

$$(ii) \inf_{v \in \mathbb{R}_+^{d_1}} \|g_f^L - G_1 v\| = 0, \text{ where } g_f^L = (P_{f,\pi}^{L'}, m_f^{L'})'.$$

Proof. See [McFadden and Richter \(1990\)](#) and [Kitamura and Stoye \(2018\)](#). ■

Theorem 4 implies that we can test the null hypothesis that $\inf_{v \in \mathbb{R}_+^d} \|g_f^L - Gv\| = 0$. Fortunately, this testing problem can be directly cast to the testing problem in [Kitamura and Stoye \(2018\)](#).

To take into account the frame variation and consequentialism, we need to modify the matrix G . Let $d_f = |F|$ and B be a matrix of size $d_f \cdot d_p \times |X|!$ that consists of d_f matrices B_1 stacked together. That is, $B = (B_1' B_1' \dots B_1')'$. Let $d = |X|! + d_f \cdot d_m$. Similarly to G_1 define,

$$G = \begin{bmatrix} B & 0_{d_f \cdot d_p \times d_f \cdot d_m} \\ 0_{d_f \cdot d_m \times |X|!} & I_{d_f \cdot d_m} \end{bmatrix}.$$

Note that when $d_f = 1$ (i.e., the frame is fixed), then $G = G_1$.

Corollary 1. *The following are equivalent:*

- (i) $(P_f)_{f \in F}$ satisfies consequentialism; $P_{f,\pi}^L$ is a FC-B-rule and m_f^L is well-defined for all $f \in F$.
- (ii) $\inf_{v \in \mathbb{R}_+^d} \|g^L - Gv\| = 0$, where $g^L = ((P_{f,\pi}^L)'_{f \in F}, m_f^L'_{f \in F})'$.

4. The Experiment

Our testing approach does not have requirements in terms of repeated individual choice data. Exploiting this feature, our experiment was designed to study the performance of different theories of random consideration sets with few observations per individual. In particular, we conducted the experiment in Amazon MTurk for a large cross-section with at most two (disjoint) choice sets per individual (see Section 4.1). The large sample size of the dataset generated by our experiment is fundamental for ensuring high statistical power using the tools in [Kitamura and Stoye \(2018\)](#).

All sessions were run between August 25, 2018 and September 17, 2018 on the MTurk platform with surveys designed in Qualtrics.³⁰ We surveyed 2135 individuals. They were paid on average

³⁰By clicking the link on the MTurk page, subjects were randomly directed to one of the treatments implemented by Qualtrics. After completing their task, subjects were also asked to complete a short survey regarding their demographic information. Subjects were not allowed to participate in the experiment more than once. Only subjects living in the USA were recruited.

\$1.09 as a result of \$0.25 for participation fee and the outcome of a randomly selected task that pays a minimum of \$0 and a maximum of \$2. All payments were made in USD. The average duration of the session was 251.68 seconds (slightly over 4 minutes).³¹ This means that our average payment per hour is roughly \$15.

The payment in our experiment is comparable to other well-known experiments conducted in MTurk. To name a few, [Horton *et al.* \(2011\)](#) studied behavior in MTurk using games with the payment range between \$0.40 and \$1.60. They find that behavior in MTurk is consistent with behavior in the lab, where the stakes of games are ten times bigger. They also estimate the minimum wage in MTurk as \$0.14 per hour. [Dean and McNeill \(2014\)](#) conducted experiments of decision-making. The average payment for completing 15-min long tasks was between \$1.35 and \$1.55 including the show-up fee of \$0.25. [Kim \(2016\)](#) conducted an experiment in MTurk for several weeks with one 10-min task each week. The average earnings from each week’s task were below \$1.00. [Rand *et al.* \(2012\)](#) also conducted a public good game with MTurkers and the payment range was between \$0.90 and \$1.50 including the show-up fee of \$0.50.

4.1. Experimental Design

In our experiment, we have two independent sources of exogenous variation: full variation in choice sets, and variation in frames. Recall that full variation in choice sets means that all possible choice sets are observed, and variation in frames means that we vary observable information without affecting the rational assessment of the alternatives. These two sources of variation allow us to test consideration-mediated choice theories in a large cross-section of heterogeneous individuals, and differentiate these theories from RUM. The latter is possible since consideration is allowed to change with frames, but preferences must remain stable due to consequentialism. We vary the frame through changes in the cost of consideration.

³¹The average duration of each task is about 23 seconds, and the duration is significantly correlated with the length of the choice set and the frame.

Choice set design

To induce preference heterogeneity, we consider lottery alternatives with different expected values and variances. Table 1 shows the alternatives and implied preference rankings if DMs are expected utility maximizers with CRRA Bernoulli utility function. The outside option is dominated for moderate levels of risk aversion (e.g. Holt and Laury, 2002).³²

Table 1 – LOTTERIES MEASURED IN TOKENS, EXPECTED VALUES, AND VARIANCE

LOTTERY	EXPECTATION	VARIANCE	PREFERENCE RANK $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ WITH σ					
			-2	0	0.25	0.30	0.50	0.75
(1) $\frac{1}{2}50 + \frac{1}{2}0$	25.000	625.00	1	1	2	5	5	6
(2) $\frac{1}{2}30 + \frac{1}{2}10$	20.000	100.00	5	5	5	2	1	1
(3) $\frac{1}{4}50 + \frac{1}{4}30 + \frac{1}{4}10 + \frac{1}{4}0$	22.500	368.75	3	3	4	4	3	4
(4) $\frac{1}{4}50 + \frac{1}{5}48 + \frac{3}{20}14 + \frac{2}{5}0$	24.125	511.73	2	2	1	3	4	5
(5) $\frac{1}{5}48 + \frac{1}{4}30 + \frac{3}{20}14 + \frac{1}{4}10 + \frac{3}{20}0$	21.625	251.11	4	4	3	1	2	3
(o) 12 with probability 1	12.000	0.00	6	6	6	6	6	2

Let $X = \{l_1, l_2, l_3, l_4, l_5\}$ be the set of all nondefault alternatives, and let o be the default/outside option. All menus $A \in \mathcal{A}$ are observed in the sample. The outside option is always present and is shown first, while the order of other alternatives is randomized. Menus can be thought as different treatments.

Our primitive to test L-B is $\hat{P} = (\hat{p}(a, A))_{a \in A \cup \{o\}, A \in \mathcal{A}}$, therefore we proceeded with stratified sampling, setting the minimal number of observations per choice set to be proportional to its cardinality, i.e. $n_A = \lambda(|A| + 1)$ with $\lambda \geq 30$. This design requires a minimum of $\sum_{A \in \mathcal{A}} |A| = 3330$ tasks.

For each menu, the DM faced three consideration frames or cost-treatments: High (H), Medium (M), and Low (L). These frames/cost-treatments were induced by introducing a k -length two digit addition/subtraction to compute each prize in the lottery. The length k was set equal to 5, 3, and 1, for the high, the medium, and the low cost, respectively. Since in our experiment attention frames only change the complexity of the description of lotteries, we assume that preferences of DMs do not depend on the way alternatives are described, thus are consistent with consequentialism.

³²Recall that the assumption that the default is dominated is a testable assumption in our framework. In addition, without cost of consideration treatments, the outside alternative is easier to understand than the rest because of its simplicity. Hence, it works as a consideration-reference point in the sense of Suleymanov (2018).

The numbers for the cognitive task were randomly generated. Examples can be seen in Figure 3. The default alternative o was presented as is, and there was no need to solve an arithmetic problem to understand it across the different levels of cost.

To prevent possible learning that could attenuate consideration costs, subjects were faced with disjoint choice sets. That is, subjects were either presented with the full choice set and the outside option ($X \cup \{o\}$); or a partition of X (presented at random order), i.e. $A_j \cup \{o\}, A_k \cup \{o\}$ with $A_j \cup A_k = X$ and $A_j \cap A_k = \emptyset$. Our experimental design is summarized in Figure 4.

The default alternative Our design allows us to use o as the opportunity cost of incurring in the cost of consideration and understanding the other lotteries in the choice set. We use a degenerate lottery as the default due to its simplicity. In this sense, we believe the alternative o in our design has effectively zero cost of consideration. Recall, that for any choice set/frame the outside option is always present and shown first. Moreover, it is pre-selected as the default alternative. If the subject decides to skip the task, she is informed that o will be chosen for her.

4.2. Sample

The sample consists of 2135 individuals that selected alternatives from one or two choice sets for all costs of attention, as shown in Figure 4, for a total of 12297 observations. The number of observations per alternative/choice set are shown in Table 4. Based on these observations the primitive for our analysis is the collection of observed frequencies $(\hat{p}(a, A))_{a \in A, A \in \mathcal{A}}$. We compute these frequencies for all costs. Unless otherwise stated $\hat{p}(a, A)$ refers to observed frequency in the data pooled across attention costs.

Figure 5 summarizes the distribution of gender, age, education, ethnicity, labor, and income in our sample. Our subjects are a diverse sample of US individuals. By design, demographics are balanced across consideration cost treatments and choice sets (that can also be thought of as treatments).

Select (only) one of the following alternatives. Click over the selected option and hit next [->].

Get 12 tokens for sure.

Prize	Probability
(3-8+22+4+9) tokens	50%
(5-7+41+10-39) tokens	50%

Prize	Probability
(27-10-2+11+24) tokens	25%
(25+6-1+3+15) tokens	20%
(1-18+3+17+11) tokens	15%
(9-6-7-4+8) tokens	40%

(a) High Cost

Select (only) one of the following alternatives. Click over the selected option and hit next [->].

Get 12 tokens for sure.

Prize	Probability
(18+2+10) tokens	50%
(4+5+1) tokens	50%

Prize	Probability
(66+5-21) tokens	25%
(28-24+44) tokens	20%
(10-3+7) tokens	15%
(2-5+3) tokens	40%

(b) Medium Cost

Select (only) one of the following alternatives. Click over the selected option and hit next [->].

Get 12 tokens for sure.

Prize	Probability
50 tokens	25%
48 tokens	20%
14 tokens	15%
0 tokens	40%

Prize	Probability
30 tokens	50%
10 tokens	50%

(c) Low Cost

Figure 3 – CONSIDERATION COST TREATMENTS Different induced costs for Choice set $\{o, l_2, l_4\}$

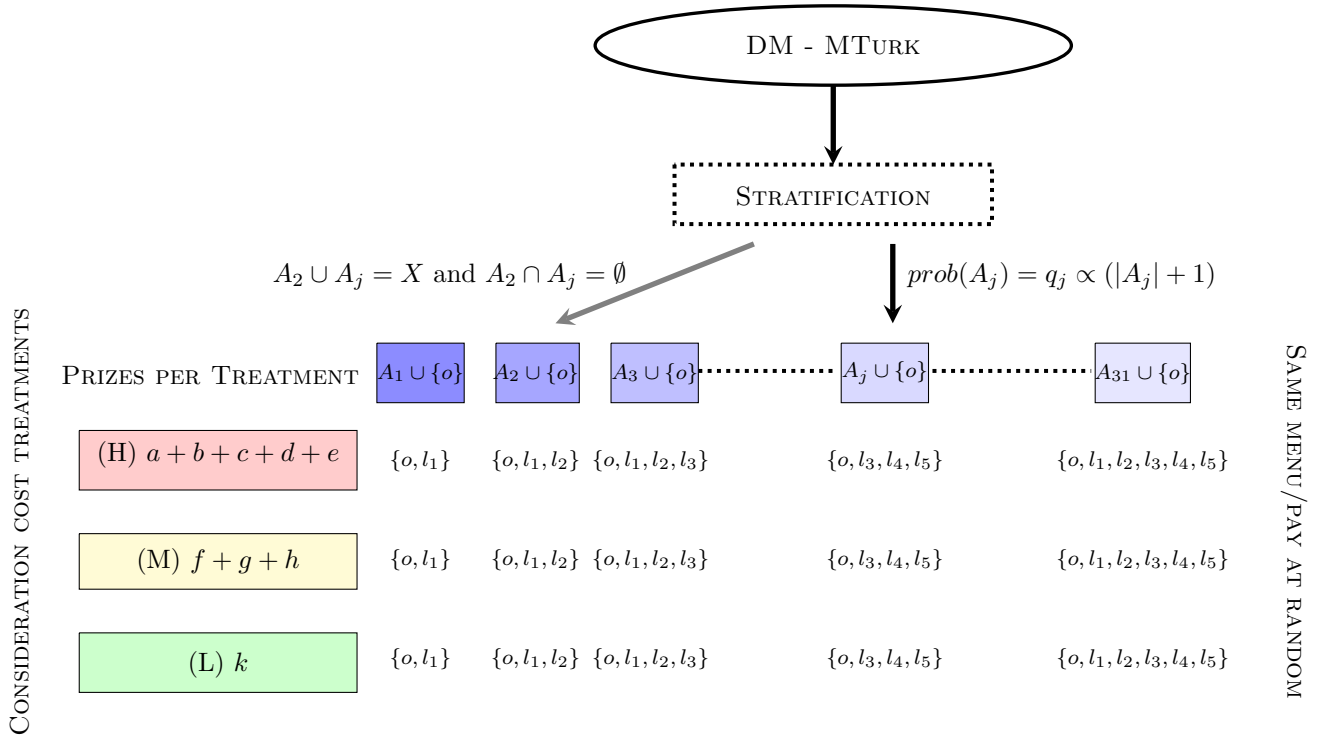


Figure 4 – EXPERIMENTAL DESIGN: DM i draws with probability $p(A_j)$ menu A_j with $|A_j| \in \{3, 4, 5\}$. In the picture, $A_j = \{l_3, l_4, l_5\}$. Therefore she is asked to choose from menus $A_j \cup \{o\}$ and $A_2 \cup \{o\}$, since $A_2 \cup A_j = X$ and $A_2 \cap A_j = \emptyset$. She is faced with one of these menus first (randomly selected) and asked to choose when the cost is H, M and L. Then, she faced the other menu for the three cost treatments.

4.3. Descriptive Analysis: Evidence for Costly Consideration and Frame Effects

In this section, we describe the behavior of individuals in our sample and present suggestive evidence that our cost treatments effectively induce costly consideration or frame effects. In particular, we observe that the consideration cost treatments/frames: (i) affect the choice frequency of the outside option; (ii) have a heterogeneous effect on the choice frequencies of all other alternatives; (iii) and affect the patterns of choice with respect to the size of the menu. Moreover, all these effects depend monotonically on the level of difficulty of choice induced by each treatment.

Under the null hypothesis of full consideration and consequentialism, the observed frequency of choice of the outside option should remain the same across frames. The reason is that the choice menu remains the same across cost treatments, and payment is at random. However, the outside option is chosen more often as the cost increases (see Figure 6). This is evidence against full consideration and in favor of frame-dependent choice.

We remind the reader here that low, medium, and high cost corresponds to 1, 3, and 5 arithmetic

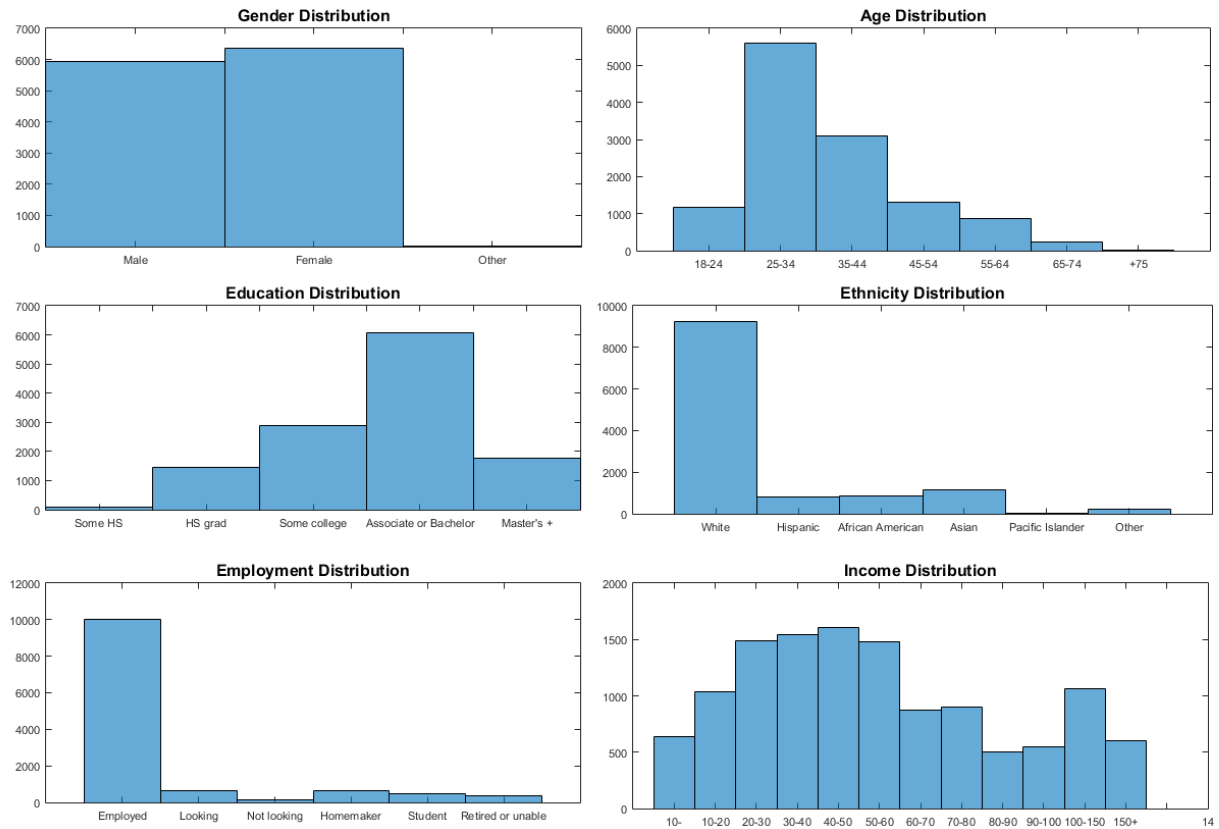


Figure 5 – DISTRIBUTION OF DEMOGRAPHICS IN SAMPLE

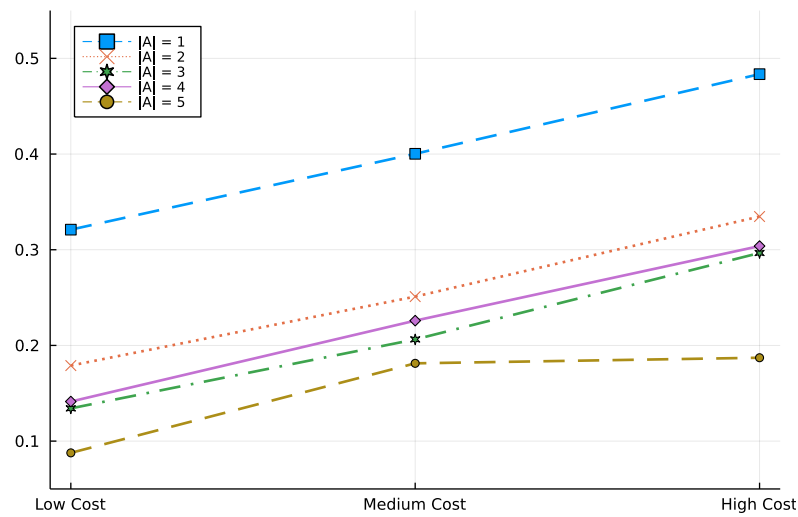


Figure 6 – FREQUENCY OF THE OUTSIDE OPTION CONDITIONAL ON THE MENU SIZE AND FRAME.

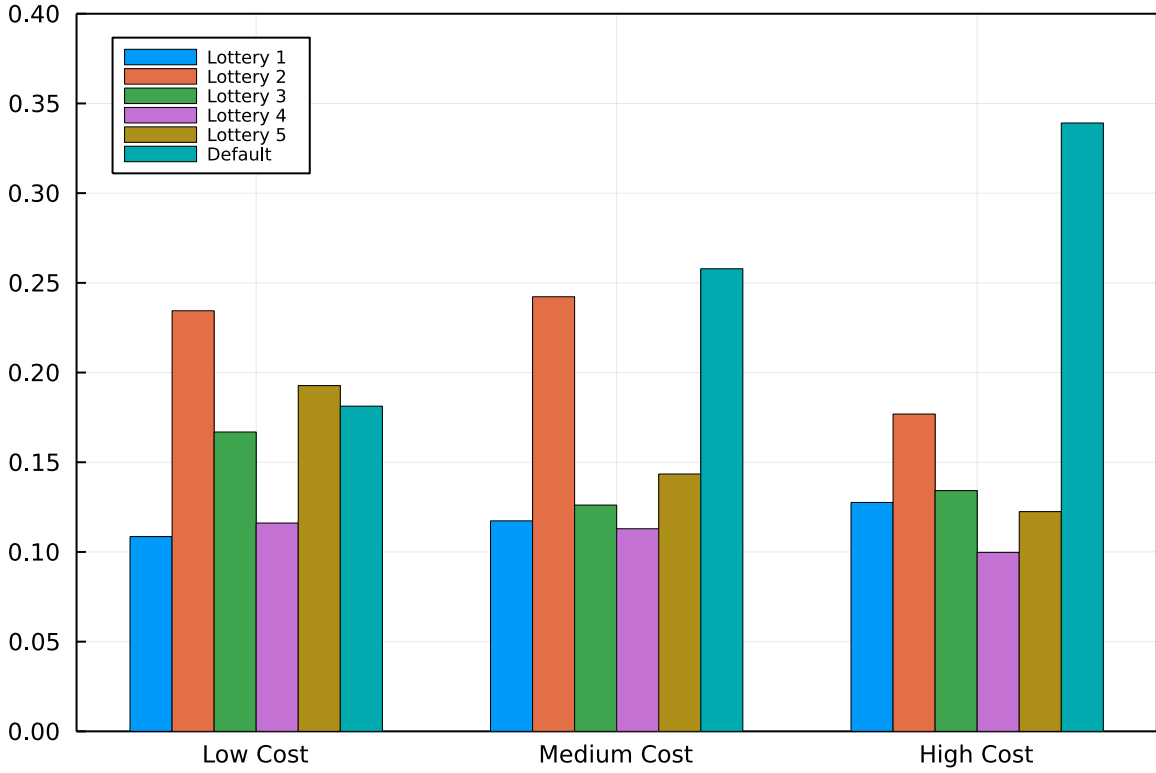


Figure 7 – FREQUENCIES OF CHOICES FOR DIFFERENT LOTTERIES AND FRAMES

operations required to understand the monetary prize of each lottery, respectively. The monotone relation shows that our treatments were effective, and that the frequency of choice of the default is in fact ordered in the way it was expected.

Figure 7 shows the effect of the frames/cost-treatments on the choice frequencies. The harder it is to understand the lotteries,³³ the more likely subjects opt to not consider them and instead choose the outside option. These results support the effectiveness of the induced treatments. Figure 7 also shows that the effect of the cost treatment is not homogeneous across alternatives. The choice frequency of lottery 1 (the simplest to understand after the default) increases with the cost treatment; the cost does not have a significant impact on the probability of selecting lottery 4; while it has a negative impact on the other lotteries.

Overall, the probability of selecting any given lottery does not necessarily decrease with the size of the menu, suggesting that regularity is violated in our sample, as shown in Figure 8. Indeed, we test RUM later and confirm that these violations are significant, since we reject RUM at the 5

³³Here simplicity comes in the form of how easy (number of arithmetic operations) it is to compute expectation, variance, and expected utility of the lottery in terms of the number of prizes and whether the probabilities are uniform on the support of the lottery or not.

percent significance level.

Notice that, for a fixed frame/attention cost, some lotteries are harder to compute than others. For instance, lottery 2 can pay 30 or 10 tokens, while lottery 3 can pay 50, 30, 10, or 0. So for every frame, lottery 3 is harder to compute than lottery 2. Our attention-index framework allows for alternatives or lotteries with heterogeneous complexity (e.g., the simplest model MM has a lottery-specific attention parameter). For instance, the simplest lottery 1 is picked more often as the cost rises, while the hardest to compute lottery 5 displays the opposite pattern. These observations confirm the presence of heterogeneous attention patterns.

4.4. Evidence for Total Monotonic Attention

Recall that, under the null hypothesis, the dataset is generated by an attention-index model with a link function that is totally monotone. Hence, the frequency of choosing the outside option is equal to a monotone transformation of cumulative attention associated with the attention-index $\sum_{C \subseteq A} \eta(C)$:

$$p(o, A) = m_A(\emptyset) = \varphi \left(\sum_{C \subseteq A} \eta(C), \eta(\emptyset) \right).$$

In Figure 8, we observe that the frequency of choice of the outside option *is not* a decreasing function of the cardinality of the choice set. The latter is inconsistent with total monotonicity, and, thus, we may naively conclude that neither RUM, LA, nor EBA can explain our dataset. However, as we confirm in the next section, this slight violation of monotonicity is an artifact of sample variability, and it is not statistically significant.

Choice overload refers to the case when the propensity of not choosing any alternative (i.e., the probability of picking the default alternative) increases with the size of the choice sets (Iyengar and Lepper, 2000). Our findings are informative on whether this effect, which may be present at the individual level, still matters at the population level. Note that neither RUM nor any model of limited consideration that we study (including Cattaneo *et al.*, 2020) can rationalize choice overload.³⁴ Existing models of limited consideration are fundamentally at odds with choice overload, since one of the important reasons to form a consideration set is to simplify choice.³⁵ We find no

³⁴Chernev *et al.* (2015) provides a meta-data analysis of the determinants of choice overload.

³⁵Other models of stochastic choice that are not models of limited consideration usually can accommodate choice overload. See, Fudenberg *et al.* (2015), Echenique *et al.* (2018), Kovach and Tserenjigmid (2018), and Natenzon (2018).

statistical support for choice overload in our dataset. However, our choice set is of moderate size.

4.5. Differentiating Between LA and EBA

In Figure 8, we also plot the frequencies of choice of the nondefault alternatives and find that there are violations of total monotonicity (e.g., lottery 3). This evidence suggests that EBA cannot describe this dataset because total monotonicity should hold for all alternatives (Block and Marschak, 1960, Aguiar, 2017). However, this is not enough to conclude that EBA cannot describe the dataset because of sample variability. Nonetheless, we reject the null hypothesis that EBA can describe this dataset. In addition, we must highlight that LA is the only candidate that can accommodate the observed nonmonotonicity of the nondefault alternatives observed in this dataset. We confirm this insight in our formal testing section by showing that LA does a good job at describing this dataset.

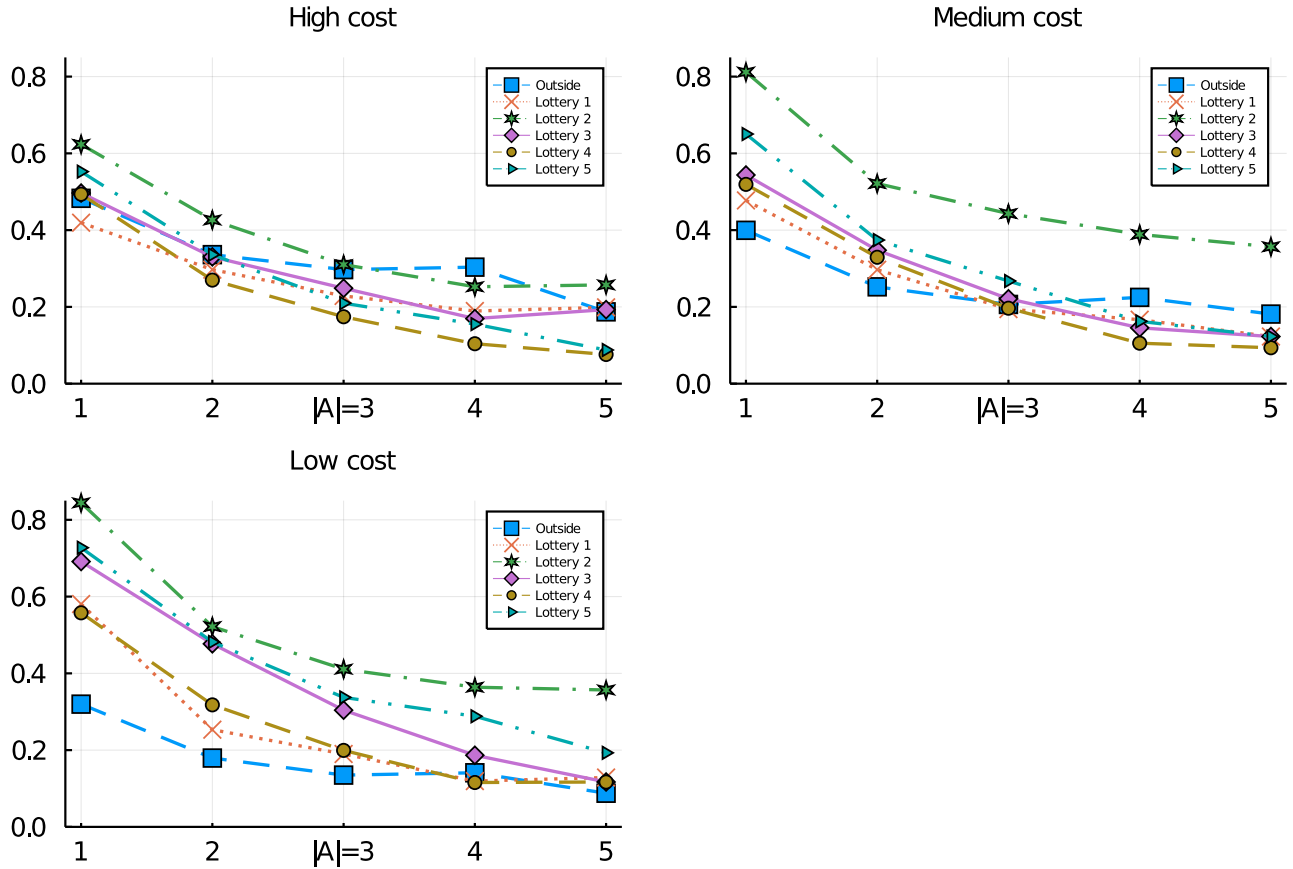


Figure 8 – FREQUENCY OF CHOICE FOR ALL FRAMES AS A FUNCTION OF THE MENU SIZE

The nonmonotonicity that we observe in Figure 8 is usually called *attraction effect* (Huber

et al., 1982). The attraction effect refers to a phenomenon when, as a new alternative is added to the choice set, the probability of the existing items is boosted. Our findings support the presence of the attraction effect in our experimental sample.

5. Testing Random Consideration Models

In this section, we report the results of testing the ability of RUM, LA, and EBA to describe our experimental data. We test these models without imposing any restrictions on preferences except stability over frames. Unless otherwise stated, the tested hypothesis is that, for a particular specification of our model (consideration set stochastic rule), there exists (m, π) that is a L-B representation for behavior.

5.1. Econometric Testing

For every frame f , although P_f is not observed, the realized choice frequencies \hat{P}_f are. For every $A \in \mathcal{A}$ and $f \in F$, let $n_{f,A}$ denote the number of individuals in the sample that faced choice set A and frame f , and let $\mathbf{a}_{i,f,A}$, $i = 1, \dots, n_A$ be the observed choice of individual i from choice set $A \cup \{o\}$ and frame f . We assume that the researcher observes a cross-section of observations (i.e., i.i.d. observations) for every menu and frame.³⁶ Then we define the estimated stochastic choice rule as

$$\hat{P}_f = (\hat{p}_f(a, A))_{A \in \mathcal{A}, a \in A \cup \{o\}}.$$

with $\hat{p}_f(a, A) = n_{f,A}^{-1} \sum_{i=1}^{n_{f,A}} \mathbf{1}(\mathbf{a}_{i,f,A} = a)$ for any $a \in A \cup \{o\}$.

Given the model of interest L and the estimator of P_f , \hat{P}_f , we can compute the estimators of m_f^L and $P_{f,\pi}^L$, \hat{m}_f^L and $\hat{P}_{f,\pi}^L$, using Definitions 7 and 9.³⁷ Given the results of Corollary 1, a natural

³⁶For a given frame, this is a standard stochastic choice dataset in the literature on limited consideration.

³⁷To compute $\hat{P}_{f,\pi}^L$, in our empirical application, we minimized the Euclidean distance between \hat{P}_f and $\left(\sum_{D \subseteq A} \hat{m}_f^L p_{f,\pi}(a, D)\right)_{a \in A, A \in \mathcal{A}}$ subject to $p_{f,\pi}(a, A) \geq 0$, $\sum_{a \in D} p_{f,\pi}(a, D) = 1$ for all a and A , and $p_{f,\pi}(a, D) = 0$ for all D and $a \notin D$.

test statistic is

$$T_n = n \min_{v \in \mathbb{R}_+^d} \left\| \hat{g}^L - Gv \right\|^2,$$

where $n = \min_f (\sum_A n_{f,A})$ is the smallest sample size across frames and $\hat{g}^L = \left(\left(\hat{P}_{f\pi}^L \right)'_{f \in F}, \left(\hat{m}_f^L \right)'_{f \in F} \right)'$.

Let $\hat{g}_l^{L,*}$, $l = 1, \dots, L$, be bootstrap replications of \hat{g}^L . Let $\tau_n \geq 0$ be a tuning parameter and ι be a vector of ones of dimension d .³⁸ To compute the critical values of T_n we follow the bootstrap procedure proposed in Kitamura and Stoye (2018):

- (i) Compute $\hat{\eta}_{\tau_n} = Gv_{\tau_n}$, where v_{τ_n} solves

$$n \min_{[v - \tau_n \iota / d] \in \mathbb{R}_+^d} \left\| \hat{g}^L - Gv \right\|^2;$$

- (ii) Compute the bootstrap test statistic

$$T_{n,l}^* = n \min_{[v - \tau_n \iota / d] \in \mathbb{R}_+^d} \left\| \hat{g}_l^{L,*} - \hat{g}^L + \hat{\eta}_{\tau_n} - Gv \right\|^2, \quad l = 1, \dots, L;$$

- (iii) Use the empirical distribution of the bootstrap statistic to compute critical values of T_n .

For a given significance level $\alpha \in (0, 1/2)$, the decision rule for the test is “reject the null hypothesis if $T_n > \hat{c}_{1-\alpha}$ ”, where $\hat{c}_{1-\alpha}$ is an $(1 - \alpha)$ -quantile of the empirical distribution of the bootstrap statistic.

We would like to conclude this section by observing that we can test the model conditional on additional observables (e.g., gender, income brackets, and education level). For discrete (or discretized) covariates one just needs to perform the test for a subgroup of the population.

5.2. Survival Race the L-B-rule v.s. RUM: Stability of Preferences

Without frame variation, many models of consideration are empirically indistinguishable from RUM. For instance, if a dataset is consistent with EBA or MM, for a fixed frame, then it is also

³⁸In our empirical application we conducted tests for different values of τ_n (e.g., $\tau_n = \sqrt{\frac{\log(\min_{f,A} n_{f,A})}{\min_{f,A} n_{f,A}}}$ following Kitamura and Stoye (2018), and $\tau_n = 0$). The results are qualitatively the same.

consistent with RUM. However, these models and RUM will typically recover a distinct distribution of preferences. Varying frames, we can test whether the distribution of preferences remains the same across frames.

L-B-model assumes that the distribution of preferences in the population is independent of the consideration rule. In our experiment, the choice sets faced by any subject are exactly the same for the three consideration cost treatments. Given our pay-at-random incentives scheme, choices from each choice set can be considered as i.i.d. draws from the underlying random utility distribution under the null hypothesis of stochastic rationality. Therefore, the independence assumption together with our experimental design imply that if one of the L-B theories describes the behavior of the high-cost treatment, it must also describe the behavior of the low-cost treatment. That is, if the independence assumption holds, then the distribution of preferences, π , should be invariant to changes in consideration costs for theories that we cannot reject. We check the validity of the different models of interest under this preference stability restriction.

We apply the procedure described in Section 5.1 to test whether the RUM, LA, and EBA models can explain the data with the restriction that the distribution of preferences remains frame-independent (i.e., consequentialism). The results of testing are presented in Table 2. In this table, we report the values of the test statistic and the corresponding p-values coming from the bootstrap distribution (1000 bootstrap replications for every test statistic were conducted) for different models.³⁹ First, we strongly reject RUM at any reasonable significance level. In other words, for RUM we reject the hypothesis that the same distribution of preferences can rationalize behavior across consideration cost treatments. In contrast, we cannot reject the LA model at any standard significance level. In addition, we reject the hypothesis that EBA explains the population behavior under preference stability. Note that we can discriminate between RUM and EBA because of variation in frames—EBA is more general than RUM because it allows for flexible attention per frame. So the rejection of EBA with stable preferences does not follow from the rejection of RUM. Taken together, our results show that our experimental subjects behave as if they are maximizing their preferences given a consideration set that follows the LA restriction.

³⁹The p-value is interpreted as the probability of observing a realization of the test statistic that is above the one that is actually observed due to sample variability, if the null hypothesis is indeed correct. Then, the smaller the p-value is, the more evidence the researcher has to reject the hypothesis of the validity of a given model.

Table 2 – TESTING RESULTS UNDER PREFERENCE STABILITY

MODEL	T_n	P-VALUE
RUM	3231.59	<0.001
LA	24959.06	0.524
EBA	24840.23	0.001

Notes: Number of bootstrap replications=1000.

5.3. Discussion

Our findings strongly support the hypothesis that the population behaves as if it is consistent with the LA model of limited consideration and has a stable distribution of preferences across frames.⁴⁰ All frame effects observed in our descriptive analysis are fully captured by the variation in the random consideration rule that changes conditional on the frame. In contrast, the traditional RUM fails to describe the population behavior. To confirm that our testing procedure has power against LA, in Appendix E, we assess the performance of our procedure using Monte Carlo simulation. In particular, we show that our test can reject the false null hypothesis of data being consistent with the LA model with high frequency in finite samples that are comparable to our experiment.

We highlight that our analysis cannot exclude the possibility that other models of behavior could also explain the population behavior in our experiment. We have only established that the population behavior is *as if* it is consistent with a L-B-rule.

Although we do not impose any restrictions on preferences, e.g. expected utility, our results relate to the work of Freeman *et al.* (2018). They provide an alternative mechanism for the selection of a riskless lottery (default) over dominant risky choices from pairwise comparisons, when binary choice sets are presented as lists. They propose a theoretical explanation of the choice of the riskless choice with a model of reference dependence. The class of reference dependence models used by these authors is a special case of utility maximization. Recall that we find evidence against RUM in our experiment, thus, ruling out Freeman *et al.* (2018) mechanism for our environment with

⁴⁰We also rejected the hypothesis of whether the LA model with homogeneous preferences can explain the data. See Appendix F.

costly consideration. In addition, in our experimental design, subjects are not required to choose from lists nor are restricted to pairwise comparisons. For an extended discussion of the role of misperception see Appendix C.3.

We have maintained the assumption that the default alternative is also the worst alternative for both the LA and EBA models. However, RUM allows the default to be ranked arbitrarily. Hence, the main findings that LA explains the dataset and RUM fails to do the same are robust to this assumption. We leave it as an open question whether EBA can explain this dataset if this assumption is relaxed. Nevertheless, we believe that this assumption is reasonable in our experimental setup.

We have done our empirical analysis without conditioning on observable heterogeneity (e.g., age or gender). Attention and preferences may differ across different demographic groups. Methodologically, our tools can be applied after conditioning on observable heterogeneity, as explained in Kitamura and Stoye (2018). The study of consideration set rules and their relation to demographics is beyond the scope of this paper.

It is noteworthy that if individuals have convex risk preferences to mix between lotteries, repetition of discretized choice tasks may make it hard to estimate risk preferences. Feldman and Rehbeck (2020) show that subjects who mix between lotteries in convex budgets sets are more likely to randomize choices in a repeated discretized task. Our experiment that uses different cost-treatments and randomizes the order of lotteries in presentation may alleviate such concerns compared to previous experiments in which the same set of lotteries in a fixed order was repeatedly presented to subjects.⁴¹

We finish this section by discussing our model and our findings in relation to Rational Inattention (RI) models. Caplin *et al.* (2019) shows that rational inattentive DMs form (deterministic) consideration sets. Generally, RI primitives cannot be point-identified with standard stochastic choice datasets. Nonetheless, RI models may still have testable implications in standard stochastic choice datasets. In Appendix C.4, we show that a representative RI DM is compatible with deterministic consideration sets (i.e, the presence of zero probability of choice), which is not supported in our data. The case of a population of heterogeneous rational inattentive DMs and the aggregation of such behavior in the population is left for future research.

⁴¹We thank an anonymous referee for pointing out this issue.

6. Conclusion

We have designed and implemented a novel experiment with a large sample that allowed us to statistically discern among competing models of population behavior. By exogenously varying choice sets and the frames induced by the cost of considering alternatives, we can disentangle two sources of stochastic behavior: limited consideration and preference heterogeneity. We use this novel dataset to test RUM and two models of limited consideration, LA and EBA.

These models provide testable implications on choices that uniquely identify the stochastic consideration set rule from data. By calibrating consideration given the theory, we show that testing the L-B-model can be cast into [Kitamura and Stoye \(2018\)](#) framework for testing RUM. That is, we show that there exists a stochastic rule (computed from data) that is RUM if and only if observed choices are generated by a population of individuals consistent with the L-B-model.

We provide evidence against classical RUM, since consideration costs are binding for some individuals in the population. In contrast, we find support for the LA model with heterogeneous preferences. Crucially, we cannot reject that the distribution of preferences implied by LA is the same across all attention frames. This means that once we disentangled attention and preferences under LA, the recovered distribution of preferences does not change with the frame.

A. Proofs

A.1. Proof of Lemma 1

We define $m_A(\{a\}) = p(a, A)$, and $m_A(D) = 0$ for all $D \subseteq A$, $D \neq \{a\}$. Let $\tilde{\pi} \in \Delta(R(X))$ be the uniform distribution. The pair $((m_A)_{A \in \mathcal{M}}, \tilde{\pi})$ is a B-rule. We now prove that it generates any data P . By definition if P can be generated by a B-rule, then we have that

$$p(a, A) = \sum_{D \subseteq A} m_A(D) \sum_{\succ \in R(X)} \tilde{\pi}(\succ) \mathbf{1}(a \succ b, \forall b \in D)$$

for all A and $a \in A$. Rearranging and replacing the choice of $\tilde{\pi}$ in the above equation we get that

$$\sum_{D \subseteq A} m_A(D) \sum_{\succ \in R} \tilde{\pi}(\succ) \mathbb{1}(a \succ b, \forall b \in D) = \frac{1}{|R(X)|} \sum_{\succ \in R(X)} \left[\sum_{D \subseteq A} m_A(D) \mathbb{1}(a \succ b, \forall b \in D) \right].$$

For given \succ and $m_A(\{a\}) = p(a, A)$, we have

$$\sum_{D \subseteq A} m_A(D) \mathbb{1}(a \succ b, \forall b \in D) = p(a, A) \mathbb{1}(a \succ a) = p(a, A)$$

because \succ includes the diagonal $a \succ a$ for all $a \in X$.

This implies that

$$\frac{1}{|R(X)|} \sum_{\succ \in R(X)} \left[\sum_{D \subseteq A} m_A(D) \mathbb{1}(a \succ b, \forall b \in D) \right] = p(a, A)$$

given that

$$\frac{1}{|R(X)|} \sum_{\succ \in R(X)} \left[\sum_{D \subseteq A} m_A(D) \mathbb{1}(a \succ b, \forall b \in D) \right] = \frac{1}{|R(X)|} \sum_{\succ \in R(X)} p(a, A) = p(a, A).$$

A.2. Proof of Theorem 1

(i) implies (ii). A complete stochastic choice rule P is a B-rule if there exists a pair (m, π) such that

$$p(a, A) = \sum_{D \subseteq A} m_A(D) \sum_{\succ \in R(X)} \pi(\succ) \mathbb{1}(a \succ b, \forall b \in D),$$

for all $a \in X$ and $A \in \mathcal{A}$, where we exchanged the summation operator with respect to the consideration sets and the linear orders exploiting independence.

Note that we can write the probability of the default alternative as $p(o, A) = 1 - \sum_{a \in A} p(a, A)$.

This implies that

$$1 - p(o, A) = \sum_{D \subseteq A} m_A(D) \left[\sum_{a \in A} \sum_{\succ \in R(X)} \pi(\succ) \mathbb{1}(a \succ b, \forall b \in D) \right],$$

where the summation operator with respect to the items $a \in A$ can be exchanged with the summation over consideration sets. This is possible because the latter summation does not depend

on the items $a \in A$.

Now, we notice that $\sum_{a \in A} \sum_{\succ \in R(X)} \pi(\succ) \mathbb{1}(a \succ b, \forall b \in D) = 1$ for all $D \subseteq A$. This implies that the default probability does not depend on the distribution of preferences and can be written in terms of the cumulative distribution of the consideration set distribution:

$$p(o, A) = 1 - \sum_{D \subseteq A, D \neq \emptyset} m_A(D).$$

We let the capacity $\varphi^* : 2^X \rightarrow [0, 1]$ be defined by $\varphi^*(A) = p(o, A)$.

The fact that $\eta = \eta^L$ under the correct specification of the link function follows from our monotonicity assumptions and the existence of a unique Mobius inverse of the mapping $v(\cdot) = \sum_{C \subseteq \cdot} \eta(C)$ (Shafer, 1976, Chateauneuf and Jaffray, 1989). We provide specific derivations for each of the models of interest in this paper, to connect them with the existing literature, but they follow directly from the general η^L formula.

For given $L \in \{\text{LA}, \text{MM}, \text{EBA}, \text{FC}\}$ and P :

- If $m \in \mathcal{M}^{\text{LA}}$, then $m_A(D) = \frac{\eta(D)}{\sum_{C \subseteq A} \eta(C)}$ for some $\eta \in \Delta(2^X) \cap \mathbb{R}_{++}$.

This means that $\frac{\varphi^*(X)}{\varphi^*(A)} = \sum_{D \subseteq A} \eta(D)$. Then by Shafer (1976) it must be that

$$\eta(D) = \sum_{B \subseteq D} (-1)^{|D \setminus B|} \frac{\varphi^*(X)}{\varphi^*(B)} = \sum_{B \subseteq D} (-1)^{|D \setminus B|} \frac{p(o, X)}{p(o, B)};$$

- If $m \in \mathcal{M}^{\text{MM}}$, then $m_A(D) = \frac{\eta(D)}{\sum_{C \subseteq A} \eta(C)}$ for some $\eta \in \Delta(2^X) \cap \mathbb{R}_{++}$ with,

$$\eta(D) = \prod_{a \in X \setminus D} (1 - \gamma(a)) \prod_{b \in D} \gamma(b),$$

and $\gamma : X \rightarrow (0, 1)$. This implies by simple computation that

$$\gamma(a) = 1 - \frac{\varphi^*(A)}{\varphi^*(A \setminus \{a\})} = 1 - \frac{p(o, A)}{p(o, A \setminus \{a\})}$$

for some $A \in \mathcal{A}$ that contains a ;

- If $m \in \mathcal{M}^{\text{EBA}}$, then $m_A(D) = \sum_{C: C \cap A = D} \eta(C)$ for some $\eta \in \Delta(2^X)$. Then

$$\varphi^*(A) = \sum_{D \cap A \neq \emptyset} \eta(D).$$

Using [Shafer \(1976\)](#) and [Chateauneuf and Jaffray \(1989\)](#) we conclude that

$$\eta(D) = \sum_{A \subseteq D: D \in \mathcal{A}} (-1)^{|D \setminus A|} (1 - \varphi^*(X \setminus A)) = \sum_{A \subseteq D: D \in \mathcal{A}} (-1)^{|D \setminus A|} (p(o, X \setminus A));$$

- If m is FC, then obviously $m_A(D) = 1$ ($A = D$).

To establish that $m = m^L$ for given $L \in \{\text{LA}, \text{MM}, \text{EBA}, \text{FC}\}$ and P , we exploit the uniqueness of m , which is a consequence of the invertibility of the Mobius transform and the completeness of P . In particular, if (m, π) and (m', π) represent the same P then it must be that $m' = m$ for the cases of $L \in \{\text{LA}, \text{MM}, \text{EBA}, \text{FC}\}$. To see that this is true recall that if P is a L-B-rule with (m, π) , then $1 - \sum_{D \subseteq A, D \neq \emptyset} m_A(D) = \varphi^*(A)$. This is exactly the same for the case where there is homogeneity in the preferences such that there is a linear order $\succ \in R(X)$ such that $\pi(\succ) = 1$. Since this equivalence does not depend on the distribution of preferences and due to the completeness of the dataset, we can use this fact to apply known results from the consideration set literature regarding the uniqueness of m .

Now, by the Mobius inverse, it follows that

$$\eta^{\text{LA}}(D) = \sum_{B \subseteq D} (-1)^{|D \setminus B|} \frac{p(o, X)}{p(o, B)},$$

for all $D \in 2^X$. In particular,

- By Theorem 3.1 in [Brady and Rehbeck \(2016\)](#), it must be that m is uniquely identified by

$$m_A(D) = \frac{\eta(D)}{\sum_{C \subseteq D} \eta(D)},$$

where $\eta \in \Delta(2^X) \cap \mathbb{R}_{++}$ follows from the requirement that $\sum_{B \subseteq D} (-1)^{|D \setminus B|} \frac{p(o, X)}{p(o, B)} > 0$ for all $D \in 2^X$.

- Given $\gamma^{\text{MM}}(a) = 1 - p(o, a) \in (0, 1)$ for all $a \in X$ (which is well-defined by the completeness of P) and $\eta^{\text{MM}}(D) = \prod_{a \in X \setminus D} (1 - \gamma^{\text{MM}}(a)) \prod_{b \in D} \gamma^{\text{MM}}(b)$, it follows that m is uniquely identified by

$$m_A(D) = \prod_{a \in D} \gamma^{\text{MM}}(a) \prod_{b \in A \setminus D} (1 - \gamma^{\text{MM}}(b)),$$

for all $A \subseteq D$. Note that $\prod_{b \in \emptyset} \gamma^{\text{MM}}(b) = 1$ by convention. Also observe that uniqueness

follows from Theorem 3.3 in [Brady and Rehbeck \(2016\)](#) since the MM restriction is a special case of the LA restriction.

- Given $\eta^{\text{EBA}}(D) = \sum_{A \subseteq D: D \in \mathcal{A}} (-1)^{|D \setminus A|} (1 - p(o, X \setminus A)) \geq 0$ it follows by Theorem 1 in [Aguilar \(2017\)](#) that m is uniquely identified by

$$m_A^{\text{EBA}}(D) = \sum_{C: C \cap A = D} \eta^{\text{EBA}}(C),$$

for all $D \subseteq A$, where $D \neq \emptyset$ and $m_A(\emptyset) = 1 - \sum_{D \subseteq A, D \neq \emptyset} m_A(D)$.

- The case of FC is trivial.

A.3. Proof of Theorem 2

(i) implies (ii). If P is a L-B-rule then by Theorem 1, under conditions (i) and (ii), it must be that

$$p_\pi^{\text{L}}(a, A) = \frac{p_{m, \pi}(a, A) - \sum_{C \subseteq A} m_A^{\text{L}}(C) p_\pi^{\text{L}}(a, C)}{m_A^{\text{L}}(A)},$$

where $p_{m, \pi}(a, A) = \sum_{D \subseteq A} m_A^{\text{L}}(D) [\sum_{\succ \in R(X)} \pi(\succ) \mathbb{1}(a \succ b \forall b \in D)]$. Following the recursive formula, we can show that

$$p_\pi^{\text{L}}(a, A) = \frac{m_A^{\text{L}}(A) [\sum_{\succ \in R(X)} \pi(\succ) \mathbb{1}(a \succ b \forall b \in A)]}{m_A^{\text{L}}(A)} = \sum_{\succ \in R(X)} \pi(\succ) \mathbb{1}(a \succ b \forall b \in A).$$

This implies that P^{L} is a FC-B-rule.

- (ii) implies (i). Under conditions (i) and (ii), the fact that

$$p_\pi^{\text{L}}(a, A) = \frac{p_{m, \pi}(a, A) - \sum_{C \subseteq A} m_A^{\text{L}}(C) p_\pi^{\text{L}}(a, C)}{m_A^{\text{L}}(A)}$$

implies that for all $A \in \mathcal{A}$ and all $a \in A$,

$$p(a, A) = \sum_{D \subseteq A} m_A^{\text{L}}(D) p_\pi^{\text{L}}(a, D).$$

If P^L is a FC-B-rule, then it implies that there exists $\pi \in \Delta(R(X))$ such that

$$p_\pi^L(a, A) = \sum_{\succ \in R(X)} \pi(\succ) \mathbb{1}(a \succ b \forall b \in A) .$$

Hence, P is a L-B-rule. In fact, for all $A \in \mathcal{A}$ and all $a \in A$, it must be that the pair (m^L, π) generates the dataset P :

$$p(a, A) = \sum_{D \subseteq A} m_A^L(D) \sum_{\succ \in R(X)} \pi(\succ) \mathbb{1}(a \succ b \forall b \in A) , .$$

A.4. Proof of Theorem 3

We first prove that if P is described by (m, π) and (m', π') , then it must be that $m = m'$. This follows from Chateaufneuf and Jaffray (1989). In particular, Brady and Rehbeck (2016) shows the identification results for $L = LA$, while Aguiar (2017) provides identification results for $L = EBA$. For $L = MM$ the result holds trivially.

Fixing m , if P is described by both (m, π) and (m, π') , then

$$p_\pi^L(a, A) = \frac{p(a, A) - \sum_{C \subseteq A} m_A^L(C) p_\pi^L(a, C)}{m_A^L(A)} ,$$

and

$$p_{\pi'}^L(a, A) = \frac{p(a, A) - \sum_{C \subseteq A} m_A^L(C) p_{\pi'}^L(a, C)}{m_A^L(A)} ,$$

for all $a \in A$ and nonempty $A \subseteq X$, which follows from Definition 9. By condition (ii), $m_A^L(A) > 0$ and using the recursive definitions above for binary sets, we can see that $p_\pi^L(a, \{a, b\}) = p_{\pi'}^L(a, \{a, b\})$ for any $a, b \in X$. For a fixed m the recursive formula leads to the equivalence $p_{\pi'}^L = p_\pi^L$.

B. Sensitivity Analysis for the Default

One key assumption in our setup is that the outside option is only picked under full consideration if the choice set only contains the outside option. In our notation, we write this as $p_\pi(o, \emptyset) = 1$ and

$p_\pi(o, A) = 0$ for all $A \neq \emptyset$. In this section, we relax this assumption to allow the probability of choosing the default to satisfy $p_\pi(o, \emptyset) = 1$ and $p_\pi(o, A) = \frac{e}{|A|}$ with $e \in [0, 1)$ for all $A \neq \emptyset$. The sensitivity parameter e is interpreted as the fraction of DMs that choose the dominated default even when there are other available alternatives. This is a violation of the assumption that the default is the worst alternative. Then this implies that under the null of consistency with the B-rule the probability of choosing the default is:

$$p(o, A) = m_A(\emptyset) + \frac{e}{|A|}(1 - m_A(\emptyset)).$$

This assumption is compatible with RUM and implies that the probability of o being the best alternative in all linear orders is constant across menus under full consideration.⁴² Notice, that under this assumption we can calibrate $m_A(\emptyset)$ for all $A \in \mathcal{A}$:

$$m_A(\emptyset) = \frac{p(o, A) - \frac{e}{|A|}}{\left(1 - \frac{e}{|A|}\right)}.$$

Then for a given sensitivity parameter $e \in [0, 1)$, we can calibrate the empirical attention-index η^L without any changes. Next, we can compute p_π given the calibrated m^L , for $A \cup \{o\}$, and we can test RUM here using the tools we have developed. In our current empirical results, since the LA model passes for $e = 0$, it is unnecessary to do this sensitivity analysis.

C. Comparison with Models of Stochastic Choice

In this appendix we analyze the connection between the three consideration-mediated choice theories discussed in this paper and models that allow for stochastic behavior exclusively in preferences or in consideration.

⁴²Indeed, this assumption allows the default to be chosen when compared to other alternatives. This is achieved by putting a mass (equal to $e/|A|$) on the event that o is the first among all alternatives in a given menu. This is a restriction on the distribution of preferences.

C.1. Comparison with RUM

As explained previously, randomness arising from limited consideration as in EBA and MM can be rationalized under the umbrella of random utility for a fixed frame. However, LA allows for behavior that is inconsistent with regularity. Therefore LA is not nested in RUM. By construction our model L-B generalizes FC by allowing for independent variation in choices due to limited consideration. In particular, L-B is RUM defined over X (what we call, equivalently, FC) when the stochastic choice rule is such that $m_A(D) = \mathbb{1}(D = A)$. We call this model FC.

Moreover, the L-B model is more general than RUM. This follows from the analysis in previous sections. In particular, fixing preferences, $\pi(\succ_i) = \mathbb{1}(\succ_i = \succ)$ for $\succ_i \in R(X)$, L-B with $L = LA$ reduces to original LA model by [Brady and Rehbeck \(2016\)](#), and therefore potentially inconsistent with RUM. Of course, when we add frame variation we can distinguish between EBA and RUM.

C.2. Comparison to RAM

[Cattaneo et al. \(2020\)](#) extends many theories of consideration by proposing the Random Attention Model (RAM). The authors allow for random consideration maps in the context of limited attention models. RAM abstracts away from the particular consideration-set-formation rule by considering a class of nonparametric random attention rules. The authors acknowledge that *RAM is best suited for eliciting information about preference ordering of a single decision-making unit when her choices are observed repeatedly*, which justifies the preference homogeneity assumption in their setting.

Many of the canonical models of limited attention proposed in the literature satisfy the Monotonic Attention property of [Cattaneo et al. \(2020\)](#). For instance, RAM nests LA, MM and EBA without preference heterogeneity among other salient models of consideration sets. Additionally, RAM is a strict generalization of RUM. However, our L-B is not nested in RAM, see [Cattaneo et al. \(2020\)](#) for a complete description of its relationship to the literature.

Here we show that, in the presence of preference heterogeneity RAM may fail to rationalize behavior that can be explained by L-B. First, we formally define the restrictions imposed by RAM.

RAM imposes a *monotonic attention* restriction on consideration rules: the probability of paying attention to a particular subset does not decrease when the total number of possible

consideration sets decreases. Formally,

Definition 12 (Monotonic Attention). For any $a \in A \setminus D$, $m_A(D) \leq m_{A \setminus \{a\}}(D)$

Moreover, [Cattaneo *et al.* \(2020\)](#) provides a characterization of the model in terms of the revealed preference information inferred from data. Formally,

Definition 13 (Revealed Preference, RAM). Let p be a RAM. Define P_R as the transitive closure of P defined as

$$aPb \text{ if there exists } A \in \mathcal{A} \text{ with } a, b \in A \text{ such that } p(a, A) > p(a, A \setminus \{b\}).$$

Then a is revealed preferred to b if and only if $a P_R b$.

Then, a choice rule has a RAM representation if and only if P_R has no cycles. The following example of a L-B-rule, which results from a $m \in \mathcal{M}^{LA}$ for two linear orders \succ_1 and \succ_2 with $\pi(\succ_i) = 0.5$ with $i = 1, 2$, cannot be generated by RAM.

Example 5 (RAM violation). Let $X = \{a, b, c\}$ and consider a LA model for the random consideration set probability measure with $\eta(D)$ given as in [Table 3](#). Moreover, consider two preference relations \succ_1 such that $a \succ_1 b \succ_1 c$, and \succ_2 such that $c \succ_2 b \succ_2 a$.⁴³ The resulting probabilistic choice rule is generated by a LA-B by construction. However, it cannot be rationalized by RAM since both aPb and bPa (i.e., $p(a, \{a, b, c\}) > p(a, \{a, c\})$ and $p(b, \{a, b, c\}) > p(b, \{b, c\})$). This means that a LA-B-rule allows cycles of the revealed preference relation P , which is ruled out by RAM.

C.3. Consideration Cost and Imperfect Perception

One possible concern with our design is that DMs consider an alternative but misperceives the attributes (i.e., computes the wrong utility). We must point out that this concern applies broadly to any experimental design in which subjects have a nontrivial cognitive task. The following analysis assumes that the consideration cost is fixed. First, we need some preliminaries. For a given

⁴³In our experiment, this preference heterogeneity can be explained by heterogeneity in risk aversion. For example, let $a \equiv l_4$, $b \equiv l_3$, $c \equiv l_2$, and assume that DMs are expected-utility-maximizers with CRRA Bernoulli utilities. Then $a \succ b \succ c$ for individuals that are risk-neutrals ($\sigma = 0$), while $c \succ b \succ a$ for risk averse individuals ($\sigma > 0.5$). [Holt and Laury \(2002\)](#) finds that these types are common in their experiment across payment schemes.

Table 3 – EXAMPLE 5 Stochastic choice rule and random consideration set probability. p is consistent with LA-B but cannot be generated by RAM.

	$\{a, b, c\}$	$\{a, b\}$	$\{a, c\}$	$\{b, c\}$	$\{a\}$	$\{b\}$	$\{c\}$	\emptyset
a	0.305	0.339	0.157		0.208			
b	0.250	0.339		0.227		0.208		
c	0.255		0.300	0.341			0.345	
o	0.190	0.322	0.543	0.432	0.792	0.792	0.655	1
$\eta(D)$	0.20	0.30	0.01	0.10	0.05	0.05	0.10	0.19

distribution of preferences $\pi \in \Delta(R(X))$, with perfect perception, there exists a random utility array $\mathbf{u} = (\mathbf{u}_a)_{a \in A}$ supported on $\mathbb{R}^{|A|}$ such that for a given menu of alternatives $A \in \mathcal{A}$:

$$\mathbb{P}(\mathbf{u}_a > \mathbf{u}_b, \forall b \in A \setminus \{a\}) = \pi(\succ : a \succ b, \forall b \in A \setminus \{a\}).$$

Now, miss-perception of any alternative $a \in A$ can be represented by another (possibly wrong) random utility variable \mathbf{w}_a supported on the reals. We let $\mathbf{w} = (\mathbf{w}_a)_{a \in A}$ be the array of such random variables. This random variable represents the subjective value that the DMs assigns to alternative a given her own perception of the item. Hence, \mathbf{w} and \mathbf{u} may be different (even when they may be correlated). Without loss of generality, we can define miss-perception as:

$$\mathbf{e}_a = \mathbf{w}_a - \mathbf{u}_a,$$

for all $a \in A$ (and $\mathbf{e} = (\mathbf{e}_a)_{a \in A}$).⁴⁴

Under the assumption of independence of preference and attention. The only implication of miss-perception is that subjects' behavior will be governed not by π but rather by a different distribution of preferences π_e such that:

$$\pi_e(\succ : a \succ b \forall b \in A \setminus \{a\}) = \mathbb{P}(\mathbf{u}_a + \mathbf{e}_a > \mathbf{u}_b + \mathbf{e}_b \forall b \in A \setminus \{a\}).⁴⁵$$

⁴⁴Note that in our experimental design, menus are randomly assigned to a DM. In addition, the presentation of each alternative remains the same across menu, conditional on the cost of consideration. Then, it must be that the distribution of miss-perception (by-design) is the same across menus.

⁴⁵Where $\pi_e(\succ : \text{property})$ denotes the cumulative probability of preferences that have a certain property.

In other words, the population of DMs behavior captured by P will still be represented by a L-B model with (π_e, m) instead of the true (π, m) . This means that our design is robust to any arbitrary miss-perception error, in terms of the validity of our conclusions about how good are the different models to describe the population.

The only possible problem induced by miss-perception of alternatives is that we may lose the capacity to identify the true distribution of preferences. This possibility again is unavoidable in any experimental design that has a cognitive task. Nonetheless, this possibility is testable in our framework. In particular, if miss-perception exists, it must depend on the cognitive cost. Hence, we have the triple $(\mathbf{e}_H, \mathbf{e}_M, \mathbf{e}_L)$ that represents the miss-perception random array for the high, medium and low cost, respectively. Then, the distribution of preferences for any L-B model must not be stable across attention treatments with corresponding $(\pi_{e_H}, \pi_{e_M}, \pi_{e_L})$ distribution of preferences (that differ among costs).

However, we cannot reject the null hypothesis that LA has a stable distribution of preferences among the different cost distributions (i.e., $\pi_{e_H} = \pi_{e_M} = \pi_{e_L} = \pi$). In that sense, there is no evidence that miss-perception is important in our design.

C.4. Relation with Rational Inattention Models

Rational inattention (RI) models have recently gained a lot of interest to model situations when choice is hard. However, RI models usually need very rich datasets to be indentified/tested. That is, generally they cannot be identified with standard stochastic choice datasets. In that sense, we cannot do a full comparison between RI models and our approach, since the dataset requirements are different. However, we can derive some implications of RI behavior for our dataset.

RI is a model for individual behavior. To the best of our knowledge the aggregate implications of this model are unknown. Hence we will focus on comparing our approach to a case of a representative RI behavior. The problem of the representative RI DM is to choose the best possible alternative from a choice set. She has a prior μ over the true value of alternatives, $V = (v_k)_{k \in X \cup \{o\}}$, with $\mu \in \Delta(V)$. In response to the information structure, the RI DM chooses her optimal information to acquire and optimal action. We focus here on the subclass of RI problems with an additive cost of perception. The result of this problem is a true-value or state dependent stochastic choice rule

$p_v(\cdot, A) \in \Delta(A \cup \{o\})$, defined as:

$$p_v(\cdot, A) = \arg \max_p \sum_{a \in A \cup \{o\}} p_v(a, A) v_a \mu(v_a) - \kappa(p_v(\cdot, A), \mu).$$

For the specification of κ , we focus on the generalized entropy proposed in Fosgerau *et al.* (2017), which generalizes widely used entropic cost. Fosgerau *et al.* (2017) shows that this state-dependent stochastic choice is observationally equivalent to an additive random utility choice rule conditional on the support. That is, if $p_v(\cdot, A) \in \Delta(A \cup \{o\})$ (positive probability of choice), then $p_v(\cdot, A)$ is a random utility rule. Even when the underlying utility is fixed (and equal to v without loss of generality), there is randomness in choice due to costly information acquisition. The state-dependent stochastic choice only differs from RUM when there are items in the choice set that are never chosen. Therefore, the RI DM is compatible with **deterministic** consideration sets. However, in our experiment we do not observe any element chosen with zero probability. In fact, the lowest probability of choice is 6 percent across all alternatives in $X \cup \{o\}$ and across all choice sets.

We have to aggregate across states to derive testable implications for the representative RI DM for our dataset. This is because in our setup, the experimenter does not know ex-ante the true value of alternatives. Preferences over lotteries (when there is not first-order stochastic dominance ordering among them) is unknown before choice. This is an important difference between our experiment and RI experimental literature, since they generally rely on enhanced datasets. We focus on collecting datasets that replicate standard stochastic choice data.

Using the fact that the sum of random utility rules is also a random utility rule, we notice that the marginal probability of choosing across different states is just the sum over the likelihood of this states (or the distribution of the true preferences). Then, if P admits a representative RI DM:

$$p(a, A) = \sum_{v \in V} p_v(a, A) \rho(v),$$

where $\rho \in \Delta(V)$ is the objective probability of the unobserved states.

Lemma 3. *If $p_v(a, A) > 0$ for all $a \in A \cup \{o\}$, and all $v \in V$, it follows that if P admits a representative RI DM then, P also admits a RUM representation.*

The proof of this lemma follows from Fosgerau *et al.* (2017) and Aguiar *et al.* (2016) that showed that the weighted sum of RUM is also RUM. The case in which one allows heterogeneity

in discrete consideration sets, induced by RI, is difficult and left for future research.

D. Experiment

D.1. Sample

The primitive for the considered models is the estimated stochastic choice rule $\hat{p}_f(a, A)$ for $f \in \{H, M, L\}$. Therefore, for a fixed level of the cost f , the minimal required sample size was calculated to be proportional to the cardinality of the choice set. To maximize the number of observations for a given set of individuals, some individuals faced two decision tasks. To prevent possible learning, these subjects faced disjoint choice sets. That is, every subject faced either the full choice set $X \cup \{o\}$ or two choice sets that only had the outside option in common. Therefore, because of random assignment, in our experiment

- (i) 171 subjects faced only the whole choice set (the targeted number is 180);
- (ii) 757 subjects faced pairs of disjoint choice sets: the set of size 4 and the set of size 1 (the targeted number is 750);
- (iii) 1207 subjects faced pairs of disjoint choice sets: the set of size 3 and the set of size 2 (the targeted number is 1200).

This implies a total number of 2135 (the targeted number is 2130) observations for 4099 tasks (the targeted number of tasks is 4080). Additionally, demographic data and preferences over binary comparison of lotteries were asked and incentivized. The effective number of observations per alternative/choice set/cost is summarized in Table 4.

E. Performance of the Test

In this section we study the performance of our test in terms of statistical power. We are going to test the null hypothesis of LA-B when the true choice process presents choice overload.

Table 4 – AVERAGE NUMBER OF OBSERVATIONS PER ALTERNATIVE/CHOICE SET

CHOICE SET	N	$N/ A $	CHOICE SET	N	$N/ A $
o12345	171	28.50	o12	131	43.67
o2345	155	31.00	o13	118	39.33
o1345	154	30.80	o14	125	41.67
o1245	149	29.80	o15	116	38.67
o1235	156	31.20	o23	112	37.33
o1234	143	28.60	o24	123	41.00
o345	131	32.75	o25	120	40.00
o245	118	29.50	o34	121	40.33
o235	125	31.25	o35	122	40.67
o234	116	29.00	o45	119	39.67
o145	112	28.00	o1	155	77.50
o135	123	30.75	o2	154	77.00
o134	120	30.00	o3	149	74.50
o125	121	30.25	o4	156	78.00
o124	122	30.50	o5	143	71.50
o123	119	29.75			

We consider behavior arising from a mixed population. A fraction $\lambda \in [0, 1]$ of the population is consistent with MM-B with $\gamma(x) = 1/2$ for all $x \in X$ and preferences consistent with expected utility maximization. The remaining fraction, λ , follows simple heuristics such the DM chooses outside option with probability proportional to the cardinality of the set. If she decides to pay attention to the menu, then she chooses uniformly at random from it. The process is then consistent with the following stochastic choice rule

$$p(a, A) = \lambda p^{\text{MM-B}}(a, A) + (1 - \lambda) p^{\text{CO}}(a, A) \quad (1)$$

where $p^{\text{MM-B}}(a, A)$ is consistent with MM-B with (m^{MM}, π) and $\pi(\succ) = 1/120$ for all $\succ \in R(X)$; and

$$p^{\text{CO}}(o, A) = \frac{|A| + 1}{|X| + 1} \text{ and } p^{\text{CO}}(a, A) = \frac{1 - p^{\text{CO}}(o, A)}{|A|}.$$

The assumed process implies that a fraction $1 - \lambda$ of the population exhibits choice overload.⁴⁶

⁴⁶This process may intuitively arise when a DM that faced with a choice set only knows the size of the choice set and the alternatives in the grand set X . Knowing about the alternatives implies paying a cost c per alternative. Assume that preferences over information are modelled by a willingness to pay attention variable, w , that is distributed

Table 5 – The table displays the proportion of rejections at the 10 percent and 5 percent significance levels for LA-B. Sample size=4000. Number of MC replications=500. Number of bootstrap replications=500

PROCESS	SIGNIFICANCE LEVEL	
	10%	5%
$\lambda = 0.25$	1	1
$\lambda = 0.50$	0.464	0.73

As the proportion of the population that exhibits choice overload increases so should increase the probability of rejecting the null that population behavior is generated by L-B. On the other extreme, when $\lambda = 1$ we should not reject the model. In particular, for any $\lambda < 32/39$ the process defined by equation (1) exhibits choice overload. However, for high values of λ the magnitude of this effect may not be significant to reject L-B.

Table 5 presents the results for power simulations for sample size 4000 and $\lambda \in \{0.25, 0.50\}$. For 500 replications, the table displays the proportion of simulations that are rejected at the 10 percent and 5 percent significance levels. As expected, the fraction of rejections is bigger for smaller values of λ . For $\lambda = 0.25$ the rejection rate is 100 percent. We observe that at comparable sample size to our experiment the mixed process is rejected with power close to 1 when the choice overload fraction of DMs is moderate.

F. The L-model with Homogeneous Preferences

In our setup, given an attention rule L (e.g., LA or MM) and a given frame, we can recover the underlying full consideration probabilistic choice rule p_π^L . Under the null hypothesis that our experimental dataset can be generated by a L-rule with a strict preference relation over $X \cup \{o\}$ (such that o is the worst alternative), it must be that p_π^L is degenerate (i.e., $p_\pi^L(a, A) \in \{0, 1\}$ for *all* A and $a \in A$). In particular, if we reject that $p_\pi^L(a_1, \{a_1, a_2\}) \in \{0, 1\}$ for *some* binary menu $\{a_1, a_2\}$

uniformly in $[0, 1]$. Then, given a choice set realization, after knowing $|A|$ DM i decides to pay attention to choice set A if $w_i > |A| \times c$. This implies that the DM pays attention and decide in the interior of the set with probability $\sum_{a \in A} p(a, A) = 1 - c|A|$; and $p(o, A) = c|A|$.

and for *some* attention cost, then we have to reject the L model with homogeneous preferences.

Given that the only model with heterogeneous preferences that was not rejected in our experiment is LA, to show the importance of preference heterogeneity, we tested whether the LA model with homogeneous preferences can explain the data. We took menu $\{1, 3\}$ and the high cost frame and computed the implied by the LA model the full consideration probability $\hat{p}_\pi^{\text{LA}}(1, \{1, 3\})$. If the data can be explained by the LA model with homogeneous preferences, then $\hat{p}_\pi^{\text{LA}}(1, \{1, 3\})$ should converge in probability to either 0 ($3 \succ 1$ with probability 1) or 1 ($1 \succ 3$ with probability 1). We tested two hypotheses: (i) $p_\pi(1, \{1, 3\}) = 0$ and (ii) $p_\pi(1, \{1, 3\}) = 1$. Both were rejected at the 5 percent significance level (p-value $< 10^{-3}$). As a result we can conservatively claim that the null hypothesis that our experimental dataset can be explained by the LA-rule with a single preference relation is rejected at least at the 5 percent significance level.

G. Experiment Instructions

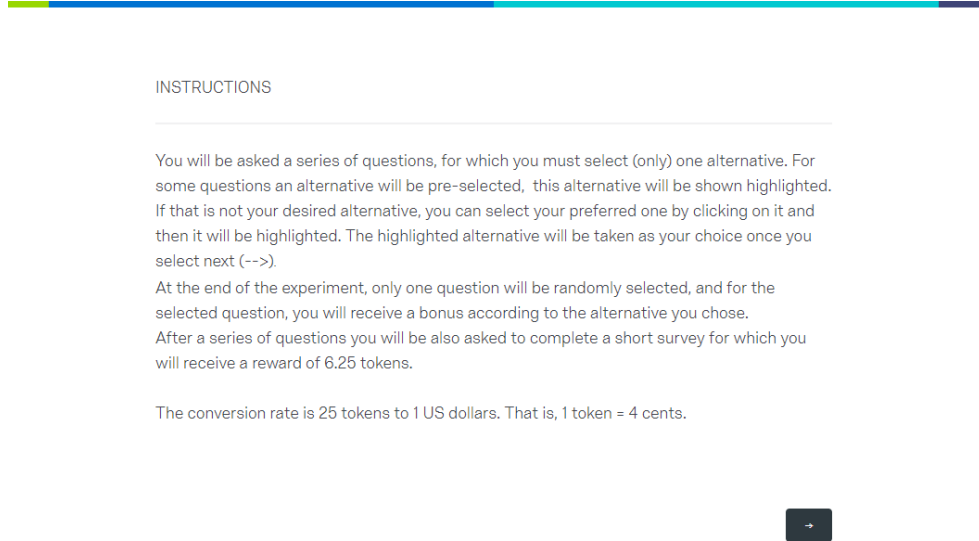


Figure 9 – Instructions page 1

For example, if you face the following question

Select (only) one of the following alternatives. Click over the selected option and hit next [->].

Get 5 tokens for sure.

Prize	Probability
8 tokens	50%
2 tokens	50%

Prize	Probability
28 tokens	25%
0 tokens	75%

Figure 10 – Instructions page 2

For example, if you face the following question, you must select an answer. Imagine you have selected alternative 3.

Select (only) one of the following alternatives. Click over the selected option and hit next [->].

Get 5 tokens for sure.

Prize	Probability
8 tokens	50%
2 tokens	50%

Prize	Probability
28 tokens	25%
0 tokens	75%

Figure 11 – Instructions page 3

For example, if you face the following question, you must select an answer. Imagine you have selected alternative 3. Then you must click on it, and it will be highlighted.

Select (only) one of the following alternatives. Click over the selected option and hit next [->].

Get 5 tokens for sure.

Prize	Probability
8 tokens	50%
2 tokens	50%

Prize	Probability
28 tokens	25%
0 tokens	75%

Figure 12 – Instructions page 4

For example, if you face the following question, you must select an answer. Imagine you have selected alternative 3. Then you must click on it, and it will be highlighted. Once you have chosen your desired alternative, hit next (-->)

Select (only) one of the following alternatives. Click over the selected option and hit next [->].

Get 5 tokens for sure.

Prize	Probability
8 tokens	50%
2 tokens	50%

Prize	Probability
28 tokens	25%
0 tokens	75%

Figure 13 – Instructions page 5

You may also get questions in a slightly modified format. You must choose your preferred alternative among the ones displayed on the screen. For example:

Select (only) one of the following alternatives. Click over the selected option and hit next [->].

Get 5 tokens for sure.

Prize	Probability
$(8+4-3-5-2)$ tokens	25%
$(11-9+5+3-2)$ tokens	25%
$(2+3+1-5+11)$ tokens	50%

Prize	Probability
$(2+5+9+7-3)$ tokens	25%
$(8+9-7-5-3)$ tokens	75%

→

Figure 14 – Instructions page 6

INSTRUCTIONS

You will be paid your show up fee of 10 tokens and a variable amount of tokens that depends on your choices. The variable amount will be the outcome of a randomly selected question among the answered ones. Given the selected question, your chosen alternative will be executed and you will be paid accordingly.

For example, assume that you were only asked the two questions above. Namely:

QUESTION 1

Select (only) one of the following alternatives. Click over the selected option and hit next [->].

Get 5 tokens for sure.

Prize	Probability
8 tokens	50%
2 tokens	50%

Prize	Probability
28 tokens	25%
0 tokens	75%

Figure 15 – Instructions page 7

INSTRUCTIONS

You will be paid a bonus of tokens that depends on your choices in addition to the reward of 25 cents. The bonus will be the outcome of a randomly selected question among the answered ones. Given the selected question, your chosen alternative will be executed and you will be paid accordingly.

For example, assume that you were only asked the two questions above. Namely:

QUESTION 2

Select (only) one of the following alternatives. Click over the selected option and hit next [->].

Get 5 tokens for sure.

Prize	Probability
(8+4-3-5-2) tokens	25%
(11-9+5+3-2) tokens	25%
(2+3+1-5+1) tokens	50%

Prize	Probability
(2+5+9+7-3) tokens	25%
(8+9-7-5-3) tokens	75%

Figure 16 – Instructions page 8

INSTRUCTIONS

You will be paid a bonus of tokens that depends on your choices in addition to the reward of 25 cents. The bonus will be the outcome of a randomly selected question among the answered ones. Given the selected question, your chosen alternative will be executed and you will be paid accordingly.

For example, assume that you were only asked the two questions above. Assume that these were your choices.

CHOICE IN QUESTION 1

Select (only) one of the following alternatives. Click over the selected option and hit next [->].

Get 5 tokens for sure.

Prize	Probability
8 tokens	50%
2 tokens	50%

Prize	Probability
28 tokens	25%
0 tokens	75%

Figure 17 – Instructions page 9

INSTRUCTIONS

You will be paid a bonus of tokens that depends on your choices in addition to the reward of 25 cents. The bonus will be the outcome of a randomly selected question among the answered ones. Given the selected question, your chosen alternative will be executed and you will be paid accordingly.

For example, assume that you were only asked the two questions above. Assume that these were your choices.

CHOICE IN QUESTION 2

Select (only) one of the following alternatives. Click over the selected option and hit next [->].

Get 5 tokens for sure.

Prize	Probability
$(8+4-3-5-2)$ tokens	25%
$(11-9+5+3-2)$ tokens	25%
$(2+3+1-5+11)$ tokens	50%

Prize	Probability
$(2+5+9+7-3)$ tokens	25%
$(8+9-7-5-3)$ tokens	75%

Figure 18 – Instructions page 10

INSTRUCTIONS

You will be paid a bonus of tokens that depends on your choices in addition to the reward of 25 cents. The bonus will be the outcome of a random selected question among the answered ones. Given the selected question, your chosen alternative will be executed and you will be paid accordingly.

Since you answered two questions, each will be selected with probability $1/2$. You can think that a coin is being flipped and if it shows heads then you will be paid according to your choice in Question 1; while if it shows tails you will be paid according to your choice in Question 2. If you were to answer 4 questions, then each of these will be selected with probability $1/4$.

For the example this implies that:

With probability $1/2$ you will get the outcome of Question 1. That is, with probability $1/4$ you get 28 tokens, with probability $3/4$ you get 0.

With probability $1/2$ you will get the outcome of Question 2. That is you would get 5 tokens for sure.



Figure 19 – Instructions page 11

Before you start making your choices here is an example.

Select (only) one of the following alternatives or opt out of this decision. Click over the selected option and hit next [-->]

Get 5 tokens for sure.

Prize	Probability
8 tokens	50%
2 tokens	50%

Prize	Probability
28 tokens	25%
0 tokens	75%



Figure 20 – Instructions page 12

With probability 1/2 you get 8 tokens, with probability 1/2 you get 2 tokens.



Figure 21 – Instructions page 13



Figure 22 – Instructions page 14

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