Discrete Choice with Endogenous Peer Selection

Please, do not circulate.

Nail Kashaev
Western University
nkashaev@uwo.ca

Natalia Lazzati
UC Santa Cruz
nlazzati@ucsc.edu

July 15, 2025

Abstract   We develop a continuous-time peer-effect discrete choice model where peers that affect the preferences of a given agent are randomly selected based on their previous choices. We characterize the equilibrium behavior and study the empirical content of the model. In the model, changes in the choices of peers affect both the set of peers the agent pays attention to and her preferences over the alternatives. We exploit variation in choices coupled with variation in the size of the set of potential peers to recover agents' preferences and the peer selection mechanism. These nonparametric identification results do not rely on exogenous variation of covariates.

1. Introduction

Social interactions are a cornerstone of many economic and social outcomes (Durlauf and Young, 2001). Consumers purchase products after seeing their friends buy them, and firms open new stores in response to the actions of their competitors. A large empirical literature measures these peer effects in discrete choice settings under the assumption that agents observe or pay attention to all their peers. This means that, for instance, consumers take into account the purchasing decisions of all their friends and firms consider the actions of all their competitors at the moment of making a decision. In many settings, however, different constraints or limitations might lead agents to pay attention to the choices of a smaller subset of peers. Ignoring this peer selection mechanism can lead to incorrect parameter estimates and misleading counterfactual and policy recommendations.

Similar to Kashaev and Lazzati (2019) and Kashaev, Lazzati and Xiao (2025), we study a continuous-time discrete choice model with social interactions. A finite set of agents is connected by links that identify potential peers. These links are known by the agents but not by the researcher. At random times governed by independent Poisson alarm clocks, an agent wakes up and selects an alternative from a finite menu. When the clock rings, the agent first randomly draws a subset of peers, called the active set, from the fixed set of her connected peers. The probability of selecting any given peer depends on the current choices of the agent and her peers, allowing agents to pay more attention to peers that behave similarly. Conditional on the active set, the agent evaluates the alternatives under the influence of the active peers and then selects an option. The resulting profile of choices evolves according to a continuous-time Markov process. We show that if the researcher observes the choices of all agents over a long period of time, then she can recover the social network (that is, the set of potential peers for each agent), the peer selection mechanism, and the random preferences of each agent captured by the distribution over the choice set conditional on realization of the choices of the active peers.

We offer conditions under which there is a unique equilibrium. In our model, this is an invariant distribution over the set of choice configurations that specifies the frequency of each profile of choices in the long run. For the identification of the model, the important feature is that the

invariant distribution has full support. Since the probability of selecting a peer is affected by previous choices of connected agents, which are determined by the equilibrium distribution, the realized equilibrium distribution over random active sets is endogenous.

We illustrate some features of the model via a simple example with closed form solution. Among others, this example illustrates the idea that allowing agents to pay attention model often to agents that select similar alternatives increases the correlation of choices among similar agents. In specific applications, this can be interpreted as polarization!

Our contribution is threefold. First, we connect bounded rationality with peer effects by embedding endogenous peer selection into a discrete choice peer effect model. Second, similar to the use of menu variation in consideration set models, we exploit variation in the number of potential peers to identify the network, the peer-selection mechanism, and preference parameters from a single long panel of choices while imposing only mild exclusion and dimension-reducing conditions. Finally, in a simple two-agent example we show how endogenous attention can qualitatively change comparative statics: paying attention to peers that make similar choice can either reinforce or dampen coordination relative to a standard model depending on whether peer effects are positive or negative.

The starting point of the identification strategy is the set of Conditional Choice Probabilities (CCPs). Each CCP specifies the frequency of choices made by an agent given the choice configuration at the time of making the decision. We assume the analyst can consistently estimate these CCPs from the data. We first show that, under a set of mild restrictions, variation in the choices of potential peers generates variation in the frequency of choices of a given agent. Thus, the set of potential peers can be recovered from the data. We then assume that there is some variability in the size of the set of potential peers across agents. (Since links can be recovered from the data, this restriction is testable.) If agents are people, this is the same as saying that some people have more friends than others. The differences in the number of potential peers allow us to recover the rest of the model via a recursive strategy. We also show that if the variation in the number of connections is rather limited, then the model can still be recovered assuming some extra shape restrictions.

We finally connect our work to the existing literature. First, the ideas are related to the

consideration set models where agents do not pay attention to all available alternatives (see, for instance, Aguiar, Boccardi, Kashaev and Kim, 2023). While this literature focuses on consideration sets of alternatives, we study "consideration sets" of peers in an interactive framework. In doing so, our work also contributes to the growing econometric literature on network formation (see Graham, 2015, De Paula, 2017, Chandrasekhar, 2016, for an overview). Within this literature, the studies by Leung (2015), Menzel (2015), Miyauchi (2016), Boucher and Mourifie (2017), Mele (2017), de Paula, Richards-Shubik and Tamer (2018), Thirkettle (2019), Ridder and Sheng (2020), Sheng (2020), Badev (2021) and Gualdani (2021) have analyzed game-theoretic models of network formation. While the sets of potential peers in our work remain stable over time, we allow agents to redefine the set of peers they pay attention to each time they make a decision. By allowing this attention to depend on the current choices of the agents, we offer a mechanism for effective network formation. As we mentioned earlier, this mechanism allows for homophily and can be used to explain polarization.

The remainder of the paper is organized as follows. Section 2 presents the model and its assumptions. Section 3 elaborates on equilibrium behavior. We establish identification of the model parameters in Section 4 and conclude in Section 5. All proofs can be find in Appendix A.

2. Model

This section describes the model and the main assumptions we use in the paper.

2.1. Network, Peer Selection, and Preferences

Network. There is a finite set of agents $\mathcal{A} = \{1, 2, \ldots, A\}$, $A \geq 2$, choosing from a finite set of alternatives $\mathcal{Y} = \{0, 1, \ldots, Y\}$, $Y \geq 1$. We refer to a vector $\mathbf{y} = (y_a)_{a \in \mathcal{A}} \in \mathcal{Y}^A$ as a choice configuration.

Agents are connected through a social network. The network is described by a set of edges in $\mathcal{A}$,

Γ. Each edge identifies two connected agents and the direction of the connection. For each Agent $a \in \mathcal{A}$ her reference group is defined as follows:

$$\mathcal{N}_a = \{a' \in \mathcal{A} : a' \neq a \text{ and there is an edge from } a \text{ to } a' \text{ in } \Gamma\}.$$

Example 1. Suppose that there are four agents and three alternatives. That is, $\mathcal{A} = \{1, 2, 3, 4\}$ and $\mathcal{Y} = \{0, 1, 2\}$. The reference groups are

$$\mathcal{N}_1 = \{2, 3\}, \quad \mathcal{N}_2 = \{1\}, \quad \mathcal{NC}_3 = \{2\}, \quad \mathcal{N}_4 = \emptyset.$$

This means that, for instance, Agents 2 and 3 may affect the choices of Agent 1. $\qquad \square$

Peer Selection and Preferences. The revision of choices follows a standard continuous-time Markov process. Agents have independent Poisson alarm clocks with rates $\lambda = (\lambda_a)_{a \in \mathcal{A}}$. The alarm of Agent a is triggered at exponentially distributed moments with mean $1/\lambda_a$. When this happens, the agent first selects a subset of peers $\mathcal{N} \subseteq \mathcal{N}_a$ to whom she will pay attention to and then makes a choice under their influence (i.e., at the moment of making a decision only the subset of peers that the agent selects affect her current choice). We do not model the cognitive process that lead to peer selection. We take it as given and will identify it nonparametrically.

For any agent, this introduces a two-stage decision process that depends on the current configuration of choices $\mathbf{y}$:

Step 1 Agent a picks $\mathcal{N} \subseteq \mathcal{N}_a$. The ex-ante probability that the set $\mathcal{N}$ is picked is

$$\mathrm{S}^a\left(\mathcal{N} \mid \mathbf{y}, \mathcal{N}_a\right)$$

with $\mathrm{S}^a\left(\mathcal{N} \mid \mathbf{y}, \mathcal{N}_a\right) \geq 0$ and $\sum_{\mathcal{N} \subseteq \mathcal{N}_a} \mathrm{S}^a\left(\mathcal{N} \mid \mathbf{y}, \mathcal{N}_a\right) = 1$.

Step 2 After selecting peers $\mathcal{N} \subseteq \mathcal{N}_a$, the agent selects an alternative according to a choice rule $\mathrm{R}^a\left(\cdot \mid \mathbf{y}, \mathcal{N}\right)$ that satisfies $\mathrm{R}^a\left(v \mid \mathbf{y}, \mathcal{N}\right) \geq 0$ for all v and $\sum_{v \in \mathcal{Y}} \mathrm{R}^a\left(v \mid \mathbf{y}, \mathcal{N}\right) = 1$. The choice rule summarizes the decision process after the peer selection is completed. When it is degenerate, it leads to a deterministic choice process. We associate choice rules with preferences since, in many

instances, the preference parameters can be recovered from the choice rules.

Example 1 (continued)**.** Suppose that, at the moment of choice, Agent 1 only selects Agent 2 from her reference group $\mathcal{N}_a = \{2, 3\}$. That is $\mathcal{N} = \{2\}$. Hence, her probability of selecting alternative 1 is $\mathrm{R}^1 (1 \mid \mathbf{y}, \{2\})$. Assume Agent a's indirect utility from v given the set of selected peers $\mathcal{N}$ is given by $u_{a,v} (\mathbf{y}, \mathcal{N}) + \xi_{a,v,\mathcal{N}}$, where $u_{a,v}$ captures the mean utility from the alternative. The vector of agent- and peer-group-specific taste shocks $\xi_{a,\mathcal{N}} = (\xi_{a,v,\mathcal{N}})_{v \in \mathcal{Y}}$ is continuously distributed with conditional cumulative distribution function (c.d.f.) $F_{a,\xi}(\cdot \mid \mathbf{y}, \mathcal{N})$. Then, for $v \in \mathcal{Y}$

$$\mathrm{R}^1 1 (v \mid \mathbf{y}, \{2\}) = \int \mathbb{1} \left(v = \arg\max_{v' \in \mathcal{Y}} \{ u_{a,v'} (\mathbf{y}, \{2\}) + \xi_{v',a,\{2\}} \} \right) dF_{a,\xi}(\xi_{a,\{2\}} \mid \mathbf{y}, \{2\}).$$

If $\xi_{v,a,\mathcal{N}}$s are independent and identically distributed (i.i.d.) shocks, distributed according to the standard Type I extreme value distribution, then the above expressions simplifies to

$$\mathrm{R}^1 (v \mid \mathbf{y}, \{2\}) = \frac{\exp (u_{1,v} (\mathbf{y}, \{2\}))}{\sum_{v' \in \mathcal{Y}} \exp (u_{1,v'} (\mathbf{y}, \{2\}))}. \qquad \square$$

As the above example demonstrates, our framework allows dependence between preferences (the mean utility and the latent preference heterogeneity captured by $\xi_{a,\mathcal{N}}$) and the random set of selected peers.

Altogether, at the moment of making a choice, the ex-ante probability for Agent a of selecting each alternative is a finite mixture and is given by

$$\mathrm{P}_a (v \mid \mathbf{y}) = \sum_{\mathcal{N} \subseteq \mathcal{N}_a} \mathrm{R}^a (v \mid \mathbf{y}, \mathcal{N}) \, \mathrm{S}^a (\mathcal{N} \mid \mathbf{y}, \mathcal{N}_a) .$$

2.2. Assumptions

This section adds more structure to the peer selection process and preferences. Regarding the selection process, we assume that each peer is selected independently from the others.

Assumption 1 (Independent Selection)**.** *The probability of considering a subset of peers $\mathcal{N} \subseteq \mathcal{N}_a$*

is given by

$$\mathrm{S}^a\left(\mathcal{N} \mid \mathbf{y}, \mathcal{N}_a\right) \equiv \prod_{a' \in \mathcal{N}} \mathrm{Q}^a(a' \mid \mathbf{y}) \prod_{a' \in \mathcal{N}_a \setminus \mathcal{N}} (1 - \mathrm{Q}^a(a' \mid \mathbf{y})),$$

where $\mathrm{Q}^a(a' \mid \mathbf{y})$ is the probability that Agent a' is selected by Agent a given $\mathbf{y}$ such that $\mathrm{Q}^a(a' \mid \mathbf{y}) = 0$ for $a' \notin \mathcal{N}_a$ and $0 < \mathrm{Q}^a(a' \mid \mathbf{y}) < 1$ for $a' \in \mathcal{N}_a$.

Assumption 1 substantially reduces the dimensionality of the problem. Without it, one would need to learn S^a over $2^{|\mathcal{N}|}$ points. Assumption 1 reduces this number to $|\mathcal{N}_a|$. It may be restrictive in some settings. For instance, if selecting Agent a' increases or decreases the likelihood of Agent a'' being selected, then Assumption 1 is violated.

We further assume that the population of agents can be decomposed into a finite set of types $\mathcal{H} = \{1, 2, \ldots, H\}$, $H \leq A$. Formally, there is a known mapping $h : \mathcal{A} \to \mathcal{H}$ such that $h(a)$ encodes the type of Agent a. Types might relate to covariates or other individual characteristics that are observed by the researcher. They allow differences between agents beyond the network structure. Each type in our model incorporates two dimensions: a random choice that captures the preferences of the person over the set of alternatives; and a limited attention to peers that connects the preferences of each person to the choices of the subset of peers the person is paying attention to. These types do not impose any restrictions on the reference group of peers $\mathcal{N}_a$, $a \in \mathcal{A}$: an agent might not be connected to some agents of the same type and she can be connected to agents that are of different types.

Example 1 (continued)**.** Suppose that Agents 1 and 2 have a college degree while Agents 3 and 4 do not. Thus, we can define h as $h(1) = h(2) = 1$ and $h(3) = h(4) = 2$ with $\mathcal{H} = \{1, 2\}$. $\qquad\square$

We also impose restrictions on R^a to make the problem more tractable. Let $\bar{N}_a^v(\mathbf{y}, \mathcal{N})$ be the average number of peers of Agent a in the nonempty set $\mathcal{N}$ who pick v in $\mathbf{y}$. That is,

$$\bar{N}_a^v(\mathbf{y}, \mathcal{N}) = \frac{|\{a' \in \mathcal{N} \ : \ y_{a'} = v\}|}{|\mathcal{N}|}.$$

We let $\bar{N}_a(\mathbf{y}, \mathcal{N}) = (\bar{N}_a^v(\mathbf{y}, \mathcal{N}))_{v \in \mathcal{Y}}$ be the vector of such averages. We follow the convention that $\bar{N}_a(\mathbf{y}, \emptyset) = \mathbf{0}$ since, for any nonempty $\mathcal{N}$, $\bar{N}_a(\mathbf{y}, \mathcal{N})$ has at least a non-zero component.

Assumption 2. *For each a, $\mathbf{y}$, $a' \in \mathcal{N}_a$, $\mathcal{N} \subseteq \mathcal{N}_a$, $\mathcal{N} \neq \emptyset$, and v*

(i) $Q^a(a' \mid \mathbf{y}, \mathcal{N}_a) = Q_{h(a)}(y_a, y_{a'})$;

(ii) $R^a(v \mid \mathbf{y}, \mathcal{N}) = R_{h(a)}(v \mid y_a, \bar{N}_a(\mathbf{y}, \mathcal{N}))$; *and*

(iii) $R^a(v \mid \mathbf{y}, \mathcal{N}) \neq R^a(v \mid \mathbf{y}, \emptyset)$.

Assumption 2(i) states that the selection probability of a specific peer depends on the current choice of the agent and the one of the target peer, but not on the identity of the target peer. This process allows agents, for example, to consider more often agents that have selected similar alternatives in the recent past. Choices of the other peers do not affect this selection probability.

Example 1 (continued)**.** Assume that Agent a follows a threshold rule when selecting peers to pay attention to: Agent a' is selected at the moment of making a choice if and only if $c_{h(a)}(y_a, y_{a'}) \geq \varepsilon$, where $c_{h(a)}(y_a, y_{a'})$ is an attention index that depends on the previous choices of the agents and ε is a random attention shock independent of $\mathbf{y}$. Then, the probability of considering Agent a' is $Q_{h(a)}(y_a, y_{a'}) = F_\varepsilon\left(c_{h(a)}(y_a, y_{a'})\right)$, where F_ε is the c.d.f. of ε. $\qquad\square$

Assumption 2(ii) imposes a type-homogeneity restriction on preferences. Moreover, it requires the choice rules to depend on the average choices of the group of selected peers. Assumption 2(iii) means that selecting at least one peer always has an effect on choice rules and if there are no peers selected, then only own previous choice matters.

Example 1 (continued)**.** Suppose that indirect utilities of alternatives are linear-in-means: $u_v^a(\mathbf{y}, \mathcal{N}) = \alpha_{h(a),v}(y_a) + \beta_{h(a),v}(y_a)\bar{N}_a^v(\mathbf{y}, \mathcal{N})$, where $\alpha_{h(a),v}(y_a)$ and $\beta_{h(a),v}(y_a) \neq 0$ are unknown parameters that depend on the current choice and the type of Agent a. $\qquad\square$

The last assumption is a mild regularity condition that rules out some ties. This restriction makes it sure that, when we switch agents choice, the effect of this choice on selection does no cancel the effect of this choice on preferences.

Assumption 3 (Regularity)**.** *For all $a \in \mathcal{A}$, $a' \in \mathcal{N}_a$, there exists v such that*

$$\frac{Q_{h(a)}(0, v)}{Q_{h(a)}(0, 0)} \neq \frac{\sum_{\mathcal{N} \subseteq \mathcal{N}_a \backslash \{a'\}} \left[R^a(v \mid \mathbf{0}, \mathcal{N} \cup \{a'\}) - R^a(v \mid \mathbf{0}, \mathcal{N})\right] S^a(\mathcal{N} \mid \mathbf{0}, \mathcal{N}_a \backslash \{a'\})}{\sum_{\mathcal{N} \subseteq \mathcal{N}_a \backslash \{a'\}} \left[R^a(v \mid \mathbf{0}_{a'}^v, \mathcal{N} \cup \{a'\}) - R^a(v \mid \mathbf{0}, \mathcal{N})\right] S^a(\mathcal{N} \mid \mathbf{0}, \mathcal{N}_a \backslash \{a'\})}.$$

3. Equilibrium Behavior

The independent Poisson processes of the alarm clocks of the agents lead the selection process of alternatives through time. They guarantee that the transition rate from choice configuration $\mathbf{y}$ to any different one $\mathbf{y}'$ is as follows

$$\mathrm{m}\left(\mathbf{y}' \mid \mathbf{y}\right) = \begin{cases} 0 & \text{if} \quad \sum_{a \in \mathcal{A}} \mathbb{1}\left(y_a' \neq y_a\right) > 1 \\ \sum_{a \in \mathcal{A}} \lambda_a \, \mathrm{P}_a\left(y_a' \mid \mathbf{y}\right) \mathbb{1}\left(y_a' \neq y_a\right) & \text{if} \quad \sum_{a \in \mathcal{A}} \mathbb{1}\left(y_a' \neq y_a\right) = 1 \end{cases}. \tag{1}$$

These transition rates are the out of diagonal terms of the *transition rate matrix*.[1] The diagonal terms are constructed from them in a simple way

$$\mathrm{m}\left(\mathbf{y} \mid \mathbf{y}\right) = -\sum_{\mathbf{y}' \in \overline{\mathcal{Y}}^A \setminus \{\mathbf{y}\}} \mathrm{m}\left(\mathbf{y}' \mid \mathbf{y}\right).$$

We will indicate by $\mathcal{M}$ the transition rate matrix. As the number of choice configurations is $(Y+1)^A$, it follows that $\mathcal{M}$ is a $(Y+1)^A \times (Y+1)^A$ matrix. An equilibrium in this model is an invariant distribution $\mu : \overline{\mathcal{Y}}^A \to [0,1]$, with $\sum_{\mathbf{y} \in \overline{\mathcal{Y}}^A} \mu\left(\mathbf{y}\right) = 1$, of the dynamic process with transition rate matrix $\mathcal{M}$. It indicates the likelihood of each choice configuration $\mathbf{y}$ in the long run. This equilibrium behavior relates to the transition rate matrix in a linear fashion

$$\mu \mathcal{M} = \mathbf{0}.$$

The assumptions in the paper guarantee the existence and uniqueness of an invariant distribution. Importantly, this invariant distribution has full support (i.e., any choice configuration $\mathbf{y}$ realizes with positive probability).

Proposition 3.1. *If Assumptions 1 and 2 hold, there exists a unique, full support equilibrium μ.*

Example 2. Suppose that the network consists of two identical agents that select between two alternatives, option 1 and the default option 0. Let us also assume that the random preferences

---

[1] This transition rate matrix has many zeros in known locations. Blevins (2017, 2018) offers a nice discuss of this feature and its advantage for identification over discrete time models.

only depend on the choice of the other agent. As there is only one agent type, we will drop $h(a)$ from the notation. Thus, for $a = 1, 2$, the CCPs can be written in a rather succinct form

$$P_a (v \mid y_1, y_2) = Q(y_a, y_{-a}) R(v \mid 1 - y_{-a}, y_{-a}) + (1 - Q(y_a, y_{-a})) R(v \mid 0, 0).$$

The rates for their Poisson alarm clocks are 1. The transition rate matrix $\mathcal{M}$ is as follows.

	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	$*$	$P_2(1 \mid 0,0)$	$P_1(1 \mid 0,0)$	0
$(0,1)$	$P_2(0 \mid 0,1)$	$*$	0	$P_1(1 \mid 0,1)$
$(1,0)$	$P_1(0 \mid 1,0)$	0	$*$	$P_2(1 \mid 1,0)$
$(1,1)$	0	$P_1(0 \mid 1,1)$	$P_2(0 \mid 1,1)$	$*$

The diagonal terms, $*$, are such that the elements in each line add up 0. Note that, by symmetry, we have $P_1 (1 \mid 1,1) = P_2 (1 \mid 1,1)$, $P_1 (0 \mid 0,1) = P_2 (0 \mid 1,0)$, $P_1 (0 \mid 0,1) = P_2 (0 \mid 1,0)$, and $P_1 (0 \mid 0,0) = P_2 (0 \mid 0,0)$. Thus, the invariant distribution of choices, or steady-state equilibrium, can be expressed as follows:

$$\mu_{(0,0)} = \frac{P_1 (0 \mid 1,0) P_1 (0 \mid 1,1)}{P_1 (1 \mid 0,0) P_1 (1 \mid 0,1) + P_1 (0 \mid 1,0) P_1 (0 \mid 1,1) + 2 P_1 (1 \mid 0,0) P_1 (0 \mid 1,1)},$$

$$\mu_{(1,0)} = \mu_{(0,1)} = \frac{P_1 (1 \mid 0,0) P_1 (0 \mid 1,1)}{P_1 (1 \mid 0,0) P_1 (1 \mid 0,1) + P_1 (0 \mid 1,0) P_1 (0 \mid 1,1) + 2 P_1 (1 \mid 0,0) P_1 (0 \mid 1,1)},$$

$$\mu_{(1,1)} = \frac{P_1 (1 \mid 0,0) P_1 (1 \mid 0,1)}{P_1 (1 \mid 0,0) P_1 (1 \mid 0,1) + P_1 (0 \mid 1,0) P_1 (0 \mid 1,1) + 2 P_1 (1 \mid 0,0) P_1 (0 \mid 1,1)}.$$

Note that, in equilibrium, the probability that agents pick the same alternatives (i.e., coordinate) is

$$\mu_{(1,1)} + \mu_{(0,0)} = \frac{1}{1 + \dfrac{2}{\dfrac{P_1 (1 \mid 0,1)}{P_1 (0 \mid 1,1)} + \dfrac{P_1 (0 \mid 1,0)}{P_1 (1 \mid 0,0)}}}.$$

Next suppose that agents always select their peer if their choices coincide and ignore her otherwise. That is, $Q(0,0) = Q(1,1) = 1$ and $Q(0,1) = Q(1,0) = 0$. Then the probability that agents pick

10

the same alternatives simplifies to

$$\text{Pr}_{\text{same}} = \frac{1}{1 + \dfrac{2}{\dfrac{R(1 \mid 0,0)}{R(0 \mid 0,1)} + \dfrac{R(0 \mid 0,0)}{R(1 \mid 1,0)}}}.$$

If, in contrast, agent select each other only when their choices are different (i.e., $Q(0,0) = Q(1,1) = 0$ and $Q(0,1) = Q(1,0) = 1$), then

$$\text{Pr}_{\text{diff}} = \frac{1}{1 + \dfrac{2}{\dfrac{R(1 \mid 0,1)}{R(0 \mid 0,0)} + \dfrac{R(0 \mid 1,0)}{R(1 \mid 0,0)}}}.$$

Suppose that agents are indifferent between alternatives if the peer is no selected (i.e., $R(1 \mid 0,0) = 0.5$). Then, since $f(x) = x(1-x)$ achieves its maximum at 0.5,

$$R(1 \mid 0,0) R(0 \mid 0,0) = R(1 \mid 0,0)(1 - R(1 \mid 0,0))$$

$$\geq \max\{R(1 \mid 0,1)(1 - R(1 \mid 0,1)), R(1 \mid 1,0)(1 - R(1 \mid 1,0))\}$$

$$= \max\{R(1 \mid 0,1) R(0 \mid 0,1), R(1 \mid 1,0) R(0 \mid 1,0)\}.$$

Thus,

$$\frac{R(1 \mid 0,0)}{R(0 \mid 0,1)} > \frac{R(1 \mid 0,1)}{R(0 \mid 0,0)} \text{ and } \frac{R(0 \mid 0,0)}{R(1 \mid 1,0)} > \frac{R(0 \mid 1,0)}{R(1 \mid 0,0)}$$

and

$$\text{Pr}_{\text{same}} > \text{Pr}_{\text{diff}}.$$

We can interpret this result by saying that when the peer selection is based on choice similarity, then agents with similar preferences select each other more often.

Let us finally compare these results with the standard model where agents always pay attention to their reference groups. In this case, we have $P_1(1 \mid 1,1) = P_1(1 \mid 0,1) = R(1 \mid 0,1)$ and $P_1(1 \mid 0,0) = P_1(1 \mid 1,0) = R(1 \mid 1,0)$. Thus, the probability that agents pick the same alternatives

11

is

$$\mathrm{Pr}_{\mathrm{std}} = \cfrac{1}{1 + \cfrac{2}{\cfrac{\mathrm{R}\,(1 \mid 0,1)}{\mathrm{R}\,(0 \mid 0,1)} + \cfrac{\mathrm{R}\,(0 \mid 1,0)}{\mathrm{R}\,(1 \mid 1,0)}}}.$$

Interestingly, if the peer effect on preferences is positive (i.e, $\mathrm{R}\,(1 \mid 0,1) > \mathrm{R}\,(1 \mid 0,0) > \mathrm{R}\,(1 \mid 1,0)$), then

$$\frac{\mathrm{R}\,(1 \mid 0,1)}{\mathrm{R}\,(0 \mid 0,1)} > \frac{\mathrm{R}\,(1 \mid 0,0)}{\mathrm{R}\,(0 \mid 0,1)} \text{ and } \frac{\mathrm{R}\,(0 \mid 1,0)}{\mathrm{R}\,(1 \mid 1,0)} > \frac{\mathrm{R}\,(0 \mid 0,0)}{\mathrm{R}\,(1 \mid 1,0)}$$

and

$$\mathrm{Pr}_{\mathrm{std}} > \mathrm{Pr}_{\mathrm{same}} > \mathrm{Pr}_{\mathrm{diff}}.$$

However, if the peer effect is negative (i.e, $\mathrm{R}\,(1 \mid 0,1) < \mathrm{R}\,(1 \mid 0,0) < \mathrm{R}\,(1 \mid 1,0)$), then

$$\frac{\mathrm{R}\,(1 \mid 0,1)}{\mathrm{R}\,(0 \mid 0,1)} < \frac{\mathrm{R}\,(1 \mid 0,1)}{\mathrm{R}\,(0 \mid 0,0)} \text{ and } \frac{\mathrm{R}\,(0 \mid 1,0)}{\mathrm{R}\,(1 \mid 1,0)} < \frac{\mathrm{R}\,(0 \mid 1,0)}{\mathrm{R}\,(1 \mid 0,0)}$$

and

$$\mathrm{Pr}_{\mathrm{same}} > \mathrm{Pr}_{\mathrm{diff}} > \mathrm{Pr}_{\mathrm{std}}.$$

$\square$

4. Identification of the Model

4.1. Identification of the Model from the CCPs

This section shows that all parts of the model can be recovered from a long sequence of choices. These parts include the network structure, the random preferences and the selection mechanism. That is, we will recover

$$(\mathcal{N}_a)_{a \in \mathcal{A}}, (\mathrm{R}_a)_{a \in \mathcal{A}}, (\mathrm{Q}_a)_{a \in \mathcal{A}}.$$

The starting point of our identification argument is the set of CCPs $P = (P_a)_{a \in \mathcal{A}}$ that can be calculated from the long sequence of choices made by the group members. Recall that, for each $v \in \mathcal{Y}$ and $a \in \mathcal{A}$

$$P_a(v \mid \mathbf{y}) = \sum_{\mathcal{N} \subseteq \mathcal{N}_a} R^a(v \mid \mathbf{y}, \mathcal{N}) S^a(\mathcal{N} \mid \mathbf{y}, \mathcal{N}_a).$$

Proposition 4.1. *Under Assumptions 1, 2, and 3, $\mathcal{N}_a$ is identified from P_a for every a.*

Proposition 4.1 states that we can recover the reference groups of peers from the CCPs. We next use a recursive argument to show that we can also recover the random preferences and consideration probabilities of each agent in the group. These results require variation in the number of peers across agents within each type.

Assumption 4. *For every type $t \in \mathcal{H}$, $|\{|\mathcal{N}_a| : a \in \mathcal{A}, h(a) = t\}| \geq 3$*

Having at least 3 agent of the same type with different number of potential peers provides enough variation to identify the selection probabilities and choice rules when no peer or just one peer is selected.

Proposition 4.2. *Under Assumptions 1, 2, 3, and 4, Q^a, $R^a(\cdot|\cdot, \emptyset)$ and $R^a(\cdot|\cdot, \{a'\})$, $a' \in \mathcal{N}_a$, are identified from P_a for every a.*

Although we may not identify the choice rule for all possible active peer groups nonparametrically, knowing it for the empty set and for singletons is enough for the standard linear-in-means multinomial logit model.

Proposition 4.3. *If the choice rule satisfies*

$$R^a(v \mid \mathbf{y}, \mathcal{N}) = \frac{e^{\alpha_{h(a),v}(y_a) + \beta_{h(a),v}(y_a)\bar{N}_a^v(\mathbf{y},\mathcal{N})}}{\sum_{v' \in \mathcal{Y}} e^{\alpha_{h(a),v'}(y_a) + \beta_{h(a),v'}(y_a)\bar{N}_a^{v'}(\mathbf{y},\mathcal{N})}}$$

with the normalization $\alpha_{h(a),0}(y_a) = 0$, then R^a is identified from $R^a(\cdot|\cdot, \emptyset)$ and $R^a(\cdot|\cdot, \{a'\})$, $a' \in \mathcal{N}_a$.

Once the selection mechanism and the choice rule are recovered for the cases in which none and only one peer is selected, we can recursively identify the choice rule for all other subsets of peers if

we have full variation in the size of connected agents within the type. For example, suppose agent a has 2 peers, a_1 and a_2 (i.e., $\mathcal{N}_a = \{a_1, a_2\}$). Then

$$\mathrm{P}_a(v \mid \mathbf{y}) = \mathrm{S}^a(\emptyset \mid \mathbf{y}, \mathcal{N}_a)\,\mathrm{R}^a(v \mid \mathbf{y}, \emptyset) + \mathrm{S}^a(\{a_1\} \mid \mathbf{y}, \mathcal{N}_a)\,\mathrm{R}^a(v \mid \mathbf{y}, \{a_1\})$$
$$+ \mathrm{S}^a(\{a_2\} \mid \mathbf{y}, \mathcal{N}_a)\,\mathrm{R}^a(v \mid \mathbf{y}, \{a_2\}) + \mathrm{S}^a(\{a_1, a_2\} \mid \mathbf{y}, \mathcal{N}_a)\,\mathrm{R}^a(v \mid \mathbf{y}, \{a_1, a_2\}).$$

Hence, since the selection mechanism and the choices rule when noone is selected and when only one peer is selected is identified, we can recover the choice rule when 2 peers are selected, $\mathrm{R}^a(v \mid \mathbf{y}, \{a_1, a_2\})$. Repeating the same argument for someone who has 3 peers, we can identify the choice rule when 3 peers are selected. Thus, with enough variation in the number of potential peers for each type, we can identify R^a without any parametric restrictions.

Assumption 5. *For every type* $t \in \mathcal{H}$,

$$\{2, 3, \ldots, \max_{\{a \in \mathcal{A},\, h(a) = t\}} |\mathcal{N}_a|\} \subseteq \{|\mathcal{N}_a| \;:\; a \in \mathcal{A},\, h(a) = t\}.$$

Assumption 4.4 means that within the type there is some one with 2 peers, someone with 3 peers, etc. This assumption is similar to the full menu variation in the stochastic choice literature (Aguiar et al., 2023).

The proof of the following results directly follows from Proposition 4.2 and the above discussion.

Proposition 4.4. *Suppose the assumptions of Proposition 4.2 are satisfied. If, moreover, Assumption 5 is satisfied, then* Q^a *and* R^a *are identified from* P_a *for every* a.

4.2. Identification of Conditional Choice Probabilities

This section shows that the CCPs, P, and the rates of the Poisson alarm clocks, λ, can be recovered from a long sequence of choices. We assume the researcher observes the choices of the agents at time intervals of length Δ and can consistently estimate $\mathrm{Pr}\left(\mathbf{y}^{t+\Delta} = \mathbf{y}' \mid \mathbf{y}^t = \mathbf{y}\right)$ for each pair

$\mathbf{y}', \mathbf{y} \in \mathcal{Y}^A$, to construct a matrix $\mathcal{P}(\Delta)$.[2] Matrix $\mathcal{P}(\Delta)$ relates to the transition rate matrix $\mathcal{M}$ by $\mathcal{P}(\Delta) = e^{(\Delta \mathcal{M})}$. (Here $e^{(\Delta \mathcal{M})}$ is the matrix exponential of $\Delta \mathcal{M}$.) Often, the researcher observes the precise moment at which an agent revises her strategy and the configuration of choices at that time. In other cases, the researcher simply observes the configuration of choices at fixed time intervals —e.g., every Monday. Kashaev et al. (2025) refer to these two cases as Dataset 1 and 2, respectively. Formally, in Dataset 1, the researcher can consistently estimate $\lim_{\Delta \to 0} \mathcal{P}(\Delta)$, and in Dataset 2, the researcher can consistently estimate $\mathcal{P}(\Delta)$. In the two cases, the identification question is whether (or under what extra restrictions) we can uniquely recover $\mathcal{M}$ from $\mathcal{P}(\Delta)$. The identification problem in Dataset 1 is straightforward. Kashaev et al. (2025), using Theorem 1 in Blevins (2018), offer a mild condition under which the transition rate matrix can be identified from Dataset 2.[3]

Proposition 4.5. *If Assumptions 1 and 2 hold, then the CCPs* P *and the rates of the Poisson alarm clocks* $(\lambda_a)_{a \in \mathcal{A}}$ *are identified from Dataset 1. If, moreover,* $\mathcal{M}$ *has distinct eigenvalues that do not differ by an integer multiple of* $2\pi i / \Delta$*, where i denotes the imaginary unit, then* P *and* $(\lambda_a)_{a \in \mathcal{A}}$ *are generically identified from Dataset 2.*

The restriction on eigenvalues of $\mathcal{M}$ is a regularity condition that is generically satisfied.[4] The key element in proving the second statement in Proposition 4.5 is that the transition rate matrix in our model is rather parsimonious. Since, at each time, at most one person revises her selection with a nonzero probability, $\mathcal{M}$ has many zeros in known locations.

5. Concluding Remarks

This paper studies dynamic interactions among agents that are connected through a social network. In the model, each agent is linked to a finite set of agents and selects a choice from a finite set of

---

[2]Here again, we assume that the choice configurations are ordered according to the lexicographic order when we construct $\mathcal{P}(\Delta)$.

[3]It is also known that if the researcher to observe the dynamic system at two different intervals Δ_1 and Δ_2 that are not multiples of each other (see, for example, Blevins, 2017 and the literature therein).

[4]See Blevins (2017) for a discussion of this assumption.

alternatives. The timing of the decision making of each group member follows a simple Poisson process that is independent across the agents. The distinctive feature of the model is that, at the moment of making a decision, the agent does not pay attention to all her linked agents. Instead, she first form a reference group and then makes a decision under their influence. The previous choices of the linked agents affect both the probability that different agents are included in her reference group and the preferences of the agent over the alternatives once the group is formed. This model can lead to choice-based homophily in decision-making. We exploit variation in the choices of all agents through time and variation in the size of their reference groups to recover all parts of the model. These parts include random preferences and the probability of paying attention to different agents.

References

Aguiar, Victor H, Maria Jose Boccardi, Nail Kashaev, and Jeongbin Kim (2023) "Random utility and limited consideration," *Quantitative Economics*, 14 (1), 71–116.

Badev, Anton I. (2021) "Nash equilibria on (un)stable networks," *Econometrica*, 89 (3), 1263–1294.

Blevins, Jason R (2017) "Identifying restrictions for finite parameter continuous time models with discrete time data," *Econometric Theory*, 33 (3), 739–754.

——— (2018) "Identification and estimation of continuous time dynamic discrete choice games,"Technical report, Mimeo, Ohio State University.

Boucher, Vincent and Ismael Mourifie (2017) "My friend far, far away: a random field approach to exponential random graph models," *The econometrics journal*, 20 (3), S14–S46.

Chandrasekhar, Arun G (2016) "Econometrics of network formation."

De Paula, Aureo (2017) "Econometrics of network models," in *Advances in Economics and Econo-*

metrics: Theory and Applications, Eleventh World Congress, 268–323, Cambridge University Press Cambridge.

Durlauf, Steven N and H Peyton Young (2001) *Social dynamics*, 4: Mit Press.

Graham, Bryan S (2015) "Methods of identification in social networks," *Annu. Rev. Econ.*, 7 (1), 465–485.

Gualdani, Chiara (2021) "An econometric model of network formation with an application to board interlocks between firms," *Journal of Econometrics*, 224 (2), 345–370.

Kashaev, Nail and Natalia Lazzati (2019) "Peer effects in random consideration sets," *arXiv preprint arXiv:1904.06742*.

Kashaev, Nail, Natalia Lazzati, and Ruli Xiao (2025) "Peer effects in consideration and preferences," *Working paper*.

Leung, Michael P (2015) *Econometric Methods for Network Data*: Stanford University.

Mele, Angelo (2017) "A structural model of dense network formation," *Econometrica*, 85 (3), 825–850.

Menzel, Konrad (2015) "Strategic network formation with many agents,"Technical report, Working papers, NYU.

Miyauchi, Yuhei (2016) "Structural estimation of pairwise stable networks with nonnegative externality," *Journal of econometrics*, 195 (2), 224–235.

de Paula, Aureo, Seth Richards-Shubik, and Elie Tamer (2018) "Identifying preferences in networks with bounded degree," *Econometrica*, 86 (2), 643–667.

Ridder, Geert and Shuyang Sheng (2020) "Two-step estimation of a strategic network formation model with clustering," *arXiv preprint arXiv:2001.03838*.

Sheng, Shuyang (2020) "Econometric Analysis of Large Network Formation Models," Working paper.

Thirkettle, Matthew (2019) "Identification and Estimation of Network Statistics with Missing Link Data," Working paper.

A. Proofs

A.1. Proof of Proposition 4.1

Fix any $a, a' \in \mathcal{A}$ with a' different from a. Let $\mathbf{y}_{a'}^v$ be the choice configuration obtained from $\mathbf{y}$ by replacing the a' component, $y_{a'}$, by v. First, note that if $a' \notin \mathcal{N}_a$ then

$$\mathrm{P}_a(v \mid \mathbf{0}_{a'}^v) - \mathrm{P}_a(v \mid \mathbf{0}) = 0,$$

where $\mathbf{0} = (0, \ldots, 0)'$ is the choice configuration where everyone picks 0. We next show that if the previous difference in CPPs is different from zero, then $a' \in \mathcal{N}_a$. Note that if $a' \in \mathcal{N}_a$

$$\mathrm{P}_a(v \mid \mathbf{y}) = \mathrm{Q}^a(a' \mid \mathbf{y}) \sum_{\mathcal{N} \subseteq \mathcal{N}_a \backslash \{a'\}} \mathrm{R}_a(v \mid \mathbf{y}, \mathcal{N} \cup \{a'\}) \, \mathrm{S}_a(\mathcal{N} \mid \mathbf{y}, \mathcal{N}_a \backslash \{a'\}) +$$

$$(1 - \mathrm{Q}_a(a' \mid \mathbf{y})) \sum_{\mathcal{N} \subseteq \mathcal{N}_a \backslash \{a'\}} \mathrm{R}_a(v \mid \mathbf{y}, \mathcal{N}) \, \mathrm{S}_a(\mathcal{N} \mid \mathbf{y}, \mathcal{N}_a \backslash \{a'\}) =$$

$$\mathrm{Q}_a(a' \mid \mathbf{y}) \sum_{\mathcal{N} \subseteq \mathcal{N}_a \backslash \{a'\}} [\mathrm{R}_a(v \mid \mathbf{y}, \mathcal{N} \cup \{a'\}) - \mathrm{R}_a(v \mid \mathbf{y}, \mathcal{N})] \, \mathrm{S}_a(\mathcal{N} \mid \mathbf{y}, \mathcal{N}_a \backslash \{a'\}) +$$

$$\sum_{\mathcal{N} \subseteq \mathcal{N}_a \backslash \{a'\}} \mathrm{R}_a(v \mid \mathbf{y}, \mathcal{N}) \, \mathrm{S}_a(\mathcal{N} \mid \mathbf{y}, \mathcal{N}_a \backslash \{a'\}) \, .$$

Moreover, $\mathrm{R}_a(v \mid \mathbf{y}, \mathcal{N})$ does not depend on $y_{a'}$ for any $\mathcal{N}$ that does not contain a' by Assumption 2(ii). Similarly, by Assumption 2(i), $\mathrm{S}_a(\mathcal{N} \mid \mathbf{y}, \mathcal{N}_a \backslash \{a'\})$ does not depend on $y_{a'}$. Hence,

$$\mathrm{P}_a(v \mid \mathbf{0}_{a'}^v) - \mathrm{P}_a(v \mid \mathbf{0}) =$$

$$\mathrm{Q}_a(a' \mid \mathbf{0}_{a'}^v) \sum_{\mathcal{N} \subseteq \mathcal{N}_a \backslash \{a'\}} [\mathrm{R}_a(v \mid \mathbf{0}_{a'}^v, \mathcal{N} \cup \{a'\}) - \mathrm{R}_a(v \mid \mathbf{0}_{a'}^v, \mathcal{N})] \, \mathrm{S}_a(\mathcal{N} \mid \mathbf{0}_{a'}^v, \mathcal{N}_a \backslash \{a'\})$$

$$-\,\mathrm{Q}_a(a' \mid \mathbf{0}) \sum_{\mathcal{N} \subseteq \mathcal{N}_a \setminus \{a'\}} [\mathrm{R}_a\,(v \mid \mathbf{0}, \mathcal{N} \cup \{a'\}) - \mathrm{R}_a\,(v \mid \mathbf{0}, \mathcal{N})]\,\mathrm{S}_a\,(\mathcal{N} \mid \mathbf{0}, \mathcal{N}_a \setminus \{a'\}) =$$

$$\mathrm{Q}_a(a' \mid \mathbf{0}_{a'}^v) \sum_{\mathcal{N} \subseteq \mathcal{N}_a \setminus \{a'\}} [\mathrm{R}_a\,(v \mid \mathbf{0}_{a'}^v, \mathcal{N} \cup \{a'\}) - \mathrm{R}_a\,(v \mid \mathbf{0}, \mathcal{N})]\,\mathrm{S}_a\,(\mathcal{N} \mid \mathbf{0}, \mathcal{N}_a \setminus \{a'\})$$

$$-\,\mathrm{Q}_a(a' \mid \mathbf{0}) \sum_{\mathcal{N} \subseteq \mathcal{N}_a \setminus \{a'\}} [\mathrm{R}_a\,(v \mid \mathbf{0}, \mathcal{N} \cup \{a'\}) - \mathrm{R}_a\,(v \mid \mathbf{0}, \mathcal{N})]\,\mathrm{S}_a\,(\mathcal{N} \mid \mathbf{0}, \mathcal{N}_a \setminus \{a'\}) \neq 0,$$

where the last inequality follows from Assumption 3.

A.2. Proof of Proposition 4.2

Fix some type $t \in \mathcal{H}$. By Assumption 4 there are at least 3 agents of type t with different number of peers. Let these agents be a_1, a_2, and a_3 and let $N_1 < N_2 < N_3$ be the number of agents in their corresponding reference groups of peers. Take any three configurations y^{*j}, $j = 1, 2, 3$, such that $y_{a_1}^{*1} = y_{a_2}^{*2} = y_{a_3}^{*3} = v^*$ for some $v^* \in \mathcal{Y}$ and all other components are set to v', which may coincide with or be different from v^*. Note that

$$\mathrm{Q}^{a_1}(a' \mid \mathbf{y}^{*1}, \mathcal{N}_{a_1}) = \mathrm{Q}^{a_2}(a'' \mid \mathbf{y}^{*2}, \mathcal{N}_{a_2}) = \mathrm{Q}^{a_3}(a''' \mid \mathbf{y}^{*3}, \mathcal{N}_{a_3}) = \mathrm{Q}_t(v^*, v'),$$

for any $a' \in \mathcal{N}_{a_1}$, $a'' \in \mathcal{N}_{a_2}$, and $a''' \in \mathcal{N}_{a_1}$.

Let $q = \mathrm{Q}_t(v^*, v')$ and $t(N) = 1 - (1 - q)^N$ be the probability that at least one peer out of N peers is considered. Since all 3 agents pick the same option and all their peers are picking the same alternative (i.e., the average choice does not depend on the number of selected peers) we have that

$$\mathrm{P}_{a_j}(v \mid \mathbf{y}^{*j}) = \mathrm{R}_t(v \mid v^*, \mathbf{0})(1 - t(N_j)) + \mathrm{R}_t(v \mid v^*, \mathbf{1}_{v'})t(N_j)$$

$$= \mathrm{R}_t(v \mid v^*, \mathbf{0}) + [\mathrm{R}_t(v \mid v^*, \mathbf{1}_{v'}) - \mathrm{R}_t(v \mid v^*, \mathbf{0})]t(N),$$

where $\mathbf{1}_{v'}$ is a vector of length $|\mathcal{Y}|$ with all elements equal to zero except for the one that corresponds to option v' (since every peer is picking v', the fraction of selected peers picking it is always 1).

19

Thus, given that $R_t(v \mid v^*, \mathbf{1}_{v'}) - R_t(v \mid v^*, \mathbf{0}) \neq 0$, we deduce that

$$\frac{P_{a_3}(v \mid \mathbf{y}^{*3}) - P_{a_1}(v \mid \mathbf{y}^{*1})}{P_{a_2}(v \mid \mathbf{y}^{*2}) - P_{a_1}(v \mid \mathbf{y}^{*1})} = \frac{t(N_3) - t(N_1)}{t(N_2) - t(N_1)}.$$

Since the left-hand-side of the last expression is observed, if we show that the right-hand-side is a known strictly monotone function of q, then we prove that q can be identified from the data. Let $x = 1 - q$, $n_2 = N_2 - N_1$, and $n_3 = N_3 - N_1$ and note that

$$f(x) = \frac{t(N_3) - t(N_1)}{t(N_2) - t(N_1)} = \frac{1 - x^{n_3}}{1 - x^{n_2}}.$$

We next show that $f'(x) > 0$ for all $x \in (0, 1)$ and $n_3 > n_2$. After some manipulation of the terms

$$f'(x) = \frac{n_2 x^{n_2-1} - n_3 x^{n_3-1} + (n_3 - n_2) x^{n_3+n_2-1}}{(1 - x^{n_2})^2}.$$

Note that the denominator is strictly positive and the numerator can be written as

$$x^{n_2-1}(n_2 - n_3 x^{n_3-n_2} + (n_3 - n_2) x^{n_3}).$$

Note that

$$\tilde{f}(x) = n_2 - n_3 x^{n_3-n_2} + (n_3 - n_2) x^{n_3}$$

is such

$$\tilde{f}'(x) = -n_3(n_3 - n_2) x^{n_3-n_2-1} + (n_3 - n_2) n_3 x^{n_3-1} = (n_3 - n_2) n_3 x^{n_3-n_2-1}[1 - x^{n_2}] < 0$$

for all $x \in (0, 1)$. Hence, $\tilde{f}$ is strictly decreasing on $(0, 1)$ and thus $\tilde{f}(x) \geq \tilde{f}(1) = 1 > 0$ for all x. Thus, $f'(x) > 0$ for all $x \in (0, 1)$ and f is strictly increasing. Since, $x = 1 - q$, then q is identified from observed CCPs.

Next, we identify $t(N)$ from q and

$$R_t(v \mid v^*, \mathbf{1}_{v'}) - R_t(v \mid v^*, \mathbf{0}) = \frac{P_{a_3}(v \mid \mathbf{y}^{*3}) - P_{a_1}(v \mid \mathbf{y}^{*1})}{t(N_3) - t(N_1)}.$$

Finally, we identify

$$\mathrm{R}^a(v \mid \mathbf{y}, \emptyset) = \mathrm{R}_t(v \mid v^*, \mathbf{0}) = \mathrm{P}_{a_3}(v \mid \mathbf{y}^{*3}) - \frac{\mathrm{P}_{a_3}(v \mid \mathbf{y}^{*3}) - \mathrm{P}_{a_1}(v \mid \mathbf{y}^{*1})}{t(N_3) - t(N_1)} t(N_3)$$

and

$$\mathrm{R}^a(v \mid \mathbf{y}, \{a'\}) = \mathrm{R}_t(v \mid v^*, \mathbf{1}_{v'}) = \mathrm{P}_{a_3}(v \mid \mathbf{y}^{*3}) + \frac{\mathrm{P}_{a_3}(v \mid \mathbf{y}^{*3}) - \mathrm{P}_{a_1}(v \mid \mathbf{y}^{*1})}{t(N_3) - t(N_1)} (1 - t(N_3))$$

for any $a' \in \mathcal{N}_a$ and $\mathbf{y}$ such that $y_a = v^*$ and $y_{a'} = v'$. The fact that the choice of v^*, v', and t was arbitrary completes the proof.

A.3. Proof of Proposition 4.3

Note that
$$\mathrm{R}^a(v \mid \mathbf{y}, \emptyset) = \frac{e^{\alpha_{h(a),v}(y_a)}}{1 + \sum_{v' \in \mathcal{Y} \setminus \{0\}} e^{\alpha_{h(a),v'}(y_a)}}.$$

Hence, $\alpha_{h(a),v}(y_a) = \log(\mathrm{R}^a(v \mid \mathbf{y}, \emptyset)) - \log(\mathrm{R}^a(0 \mid \mathbf{y}, \emptyset))$. To identify $\beta_{h(a),v}(y_a)$, note that

$$\mathrm{R}^a(v \mid \mathbf{y}, \{a'\}) = \frac{e^{\alpha_{h(a),v}(y_a) + \beta_{h(a),v}(y_a) \mathbb{1}(y_{a'} = v)}}{\sum_{v' \in \mathcal{Y}} e^{\alpha_{h(a),v'}(y_a) + \beta_{h(a),v'}(y_a) \mathbb{1}(y_{a'} = v')}}.$$

As a result, $\beta_{h(a),v}(y_a) = \log(\mathrm{R}^a(v \mid \mathbf{y}^*, \{a'\})) - \log(\mathrm{R}^a(0 \mid \mathbf{y}^*, \{a'\})) - \alpha_{h(a),v}(y_a)$ for any $\mathbf{y}^*$ such that $y^*_{a'} = v$.

21