Noah Kasmanoff
nsk367

Homework 3

7.2

link to code

a) First, we download and load in the sunspots.txt file, and plot it below.

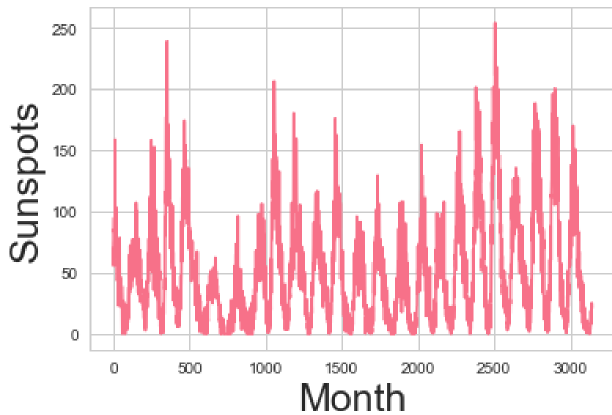

Figure 1: The recorded number of sunspots versus month.

Eyeballing the figure, the inferred length of a sunspot cycle is approximately every 130 months/12.5 years.

b) Calculating the discrete Fourier transform, we recover the following power spectrum.

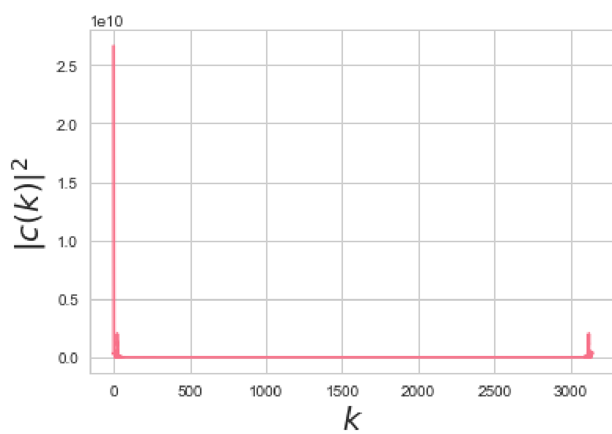

Figure 2: The power spectrum of the DFT of the sunspot dataset.

Immediately from this spectrum we observe the Nyquist frequency at large k, and can throw out results for any $k > N/2$. Additionally, there is an extremely large value for the power at extremely small values of k. We can also ignore this artifact of the power spectrum, by recognizing that physically, this small k corresponds to a large wavelength λ, and this manifestation in power spectrum is related to the mean value of the signal, which is not relevant to the true periodicity observed in the sunspot cycle.

Moving past these artifacts, we zoom in to better capture this k.

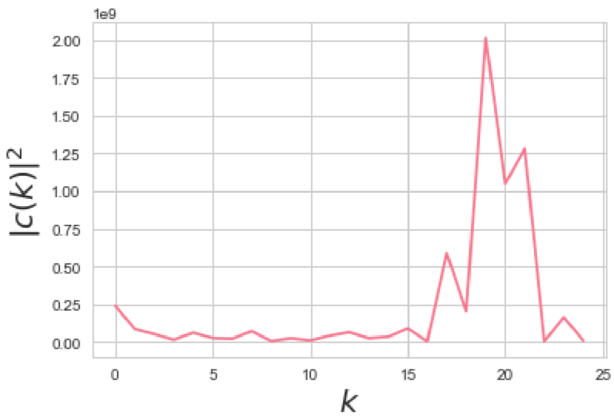

Figure 3: Zoomed in version of power spectrum, here, we clearly see that the peak wavenumber is approximately 24.

From here, we now calculate the k_max. Our original guess for the wavelength was 130 months, and if we apply $\lambda = \frac{N}{k_{max}} = 130.95$, months which is almost exactly what we guessed!

This dataset of sunspot data was relatively clean and it was easy for us to infer the periodicity upon inspection, but this is not always going to be the case. Fourier analysis via the discrete Fourier transform proves to be an invaluable (albeit computationally expensive) tool. To remedy this issue, we will demonstrate the power of the FFT in the following problem.

7.9

link to code

a) First, we load in and display the blurred image.

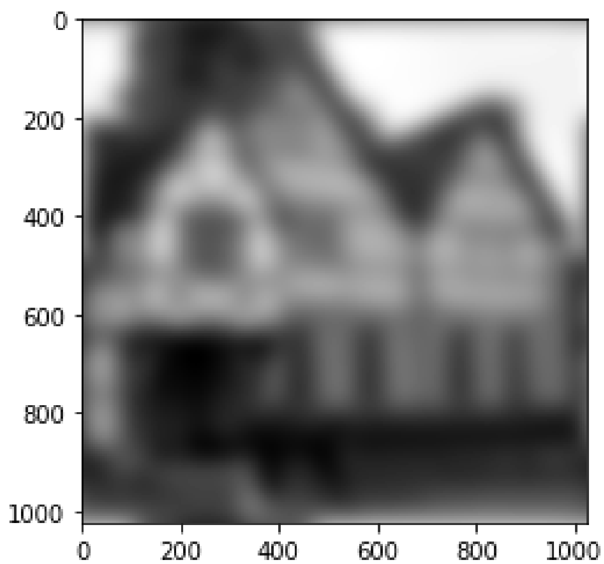

Figure 4: Some blurry image to de-convolve with FFTs.

b) Next, we construct a point spread function of the same size as this image, with a given $\sigma = 25$, and assume that this point spread is the error convolved with the true image which we wish to remove from the blurry one.

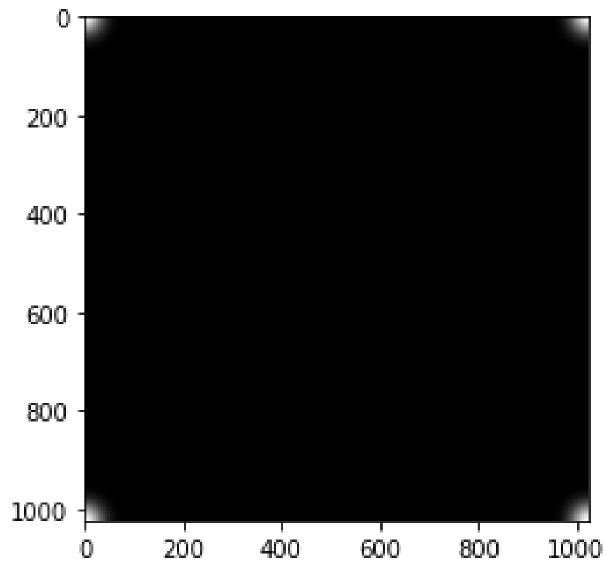

Figure 5: The point spread function convolved with our true image.

c) Exploiting the properties of a convolution of functions undergoing a Fourier transform, we can recover our "sharp" image S as a result of exploiting

$$\tilde{b} = \tilde{p}\tilde{S}$$

Where $\tilde{b}$ is the blurred image's Fourer transform, $\tilde{p}$ is the Fourier transform of the point spread function, and $\tilde{S}$ is the Fourier transform of the cleaned image we wish to recover.

From here it is straightforward to solve for S as the inverse Fourier transform of $\tilde{S} = \frac{\tilde{b}}{\tilde{p}}$, and both b and p are known quantities. Going through this process, we recover the figure below.

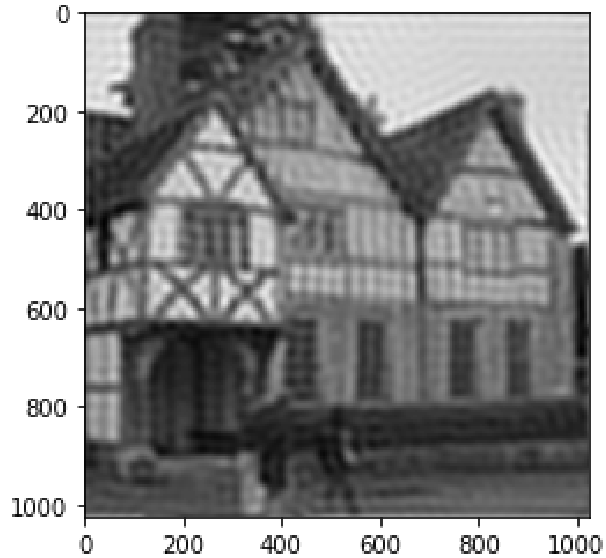

Figure 6: The de-convolved image of the house.

d) Although this process produces an impressive result, it is not without it's flaws. For instance, one issue that arises is for values of k where $\tilde{p} = 0$. In these scenarios we are dividing by 0, and for other values near 0 the value tends to infinity. The solution for this was to set these terms equal to 1, and while this still yields an impressive result, demonstrates that for small enough values of the point spread function (or any convolved kernel with an image), the de-convolution method cannot effectively characterize for small values of k, and therefore is unable to ever fully de-blend a true image with whatever kernel is blurring it.