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Homework 1

Computational Physics
September 27, 2019

Problem 1

a) My code for question 1 on my github

b) The log-log plot of the absolute error vs step size is shown below.

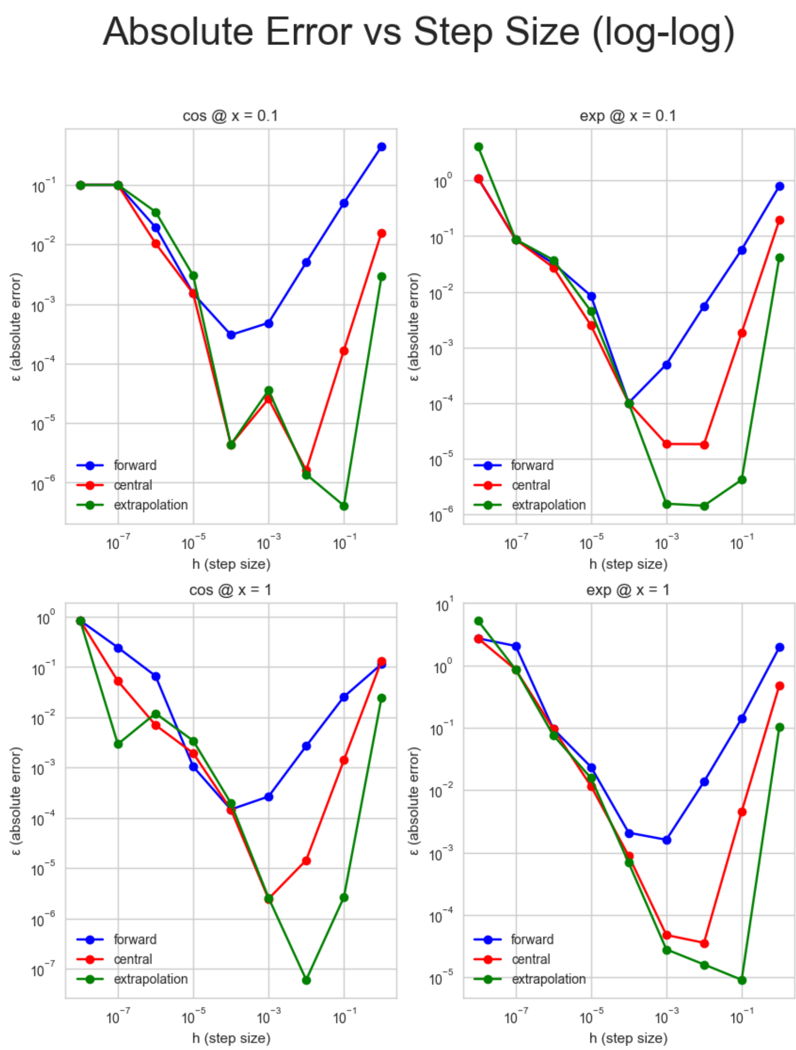

Figure 1: How the absolute error varies with step size for calculations of the derivative.

This basic trends we see here are in agreement with their simple estimates. Fom the Taylor series approximation of $f'(x)$ about x for the three methods used, we expect the forward difference method to scale with $\approx O(h)$, the central difference method to scale with $\approx O(h^2)$, and the extrapolation difference method to perform best with a scaling of $\approx O(h^4)$. In the regime where h is sufficiently large, this appears to hold true under all circumstances, given the different functions and values at which we are evaluating.

c) Using linear, central, and extrapolation difference techniques for numerical differentiation, we observe the transition between round-off and truncation error manifest around $1e-3$. This is a consequence of single precision, the computer can allocate up to 7 digits to a value. When this is the case, once the step size h has reached the limit designated by single precision, all of the gains acquired by the previous steps of differentiation are gone. We observe that approximately midway through this process, the decrease in error (due to decreasing the step size h) is now offset by round-off error, and the absolute error begins to rise. As such, the battle between step-size and round-off is a losing one for overwhelms the previous decrease in error mainly due to truncation, and once the limit of single precision is reached, the machine can no longer discern the difference between $f(x)$ and $f(x+h)$ and error is nearly the same as if no technique was even used, which is known as cancellation.

Problem 2

a) My code for question 2 on my github

b) The log-log plot of the absolute error vs step size is shown below.

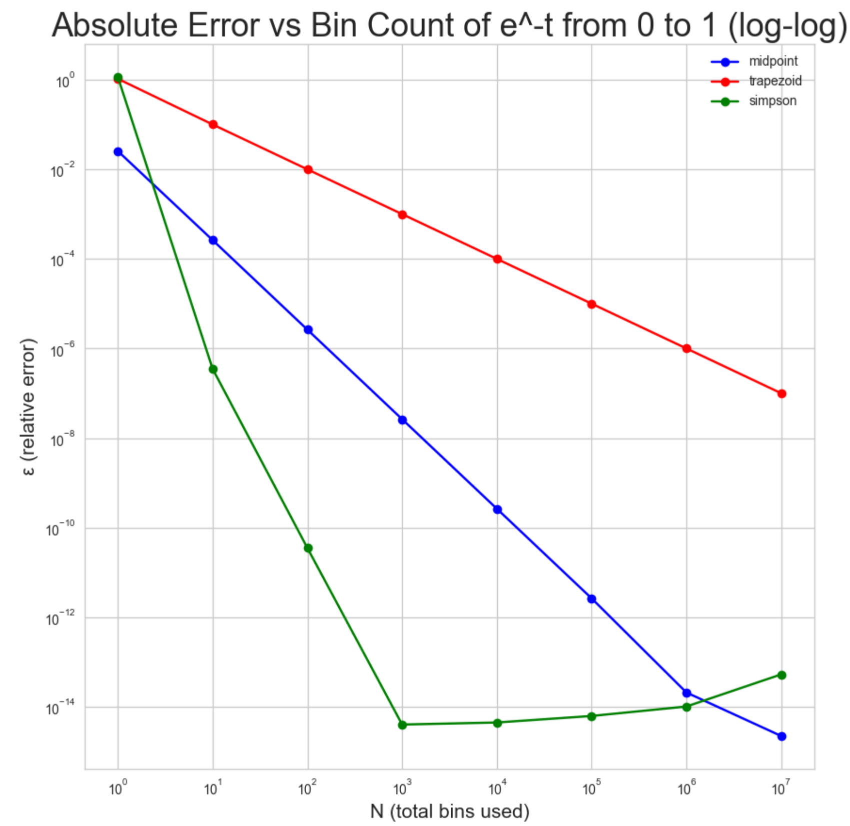

Figure 2: How the absolute error varies with bin count for calculations of the integral.

c) Again, the numerical integration techniques plotted on this graph seem to obey the expected trends derived for the midpoint method, trapezoidal method, and Simpson's. For the function $f(x) = e^{-t}$ and in general, we found that the midpoint and trapezoid method error obeys an $O(N^{-2})$ error, and Simpson's is $O(N^{-4})$. Upon examination of this graph, we observe this is the case. However, as was just mentioned in question 1, there comes a point at which round-off error will start to cancel out the gains attained by increasing the number of bins. Given this problem now employs double precision, the number of digits allocated to each variable is 17. As a result, we see round-off error gradually force the error to increase. Due to time constraints I did not go all the way up to the point at which cancellation in double precision totally offsets the approximation method. Furthermore, we see an interesting result, which is that the midpoint method proves to be a better numerical integration technique than trapezoidal. This is a surprising outcome, since the trapezoidal method intuitively seemed like a better approach since an actual approximation of the function is made, whereas only the midpoint between two points is used. The reason for why this is the case was already discussed in class, but the reasoning here is that the midpoint method will both over and underestimate the area underneath the curve, and trapezoid just underestimates. This over and under estimation in turn cancel out somewhat, producing a closer result than trapezoid's. Both scale with error of $O(N^2)$, but midpoint has the advantage by over and under guessing.

Problem 3

link to code

At what best can be called a trial and error process, I recovered the following plots:

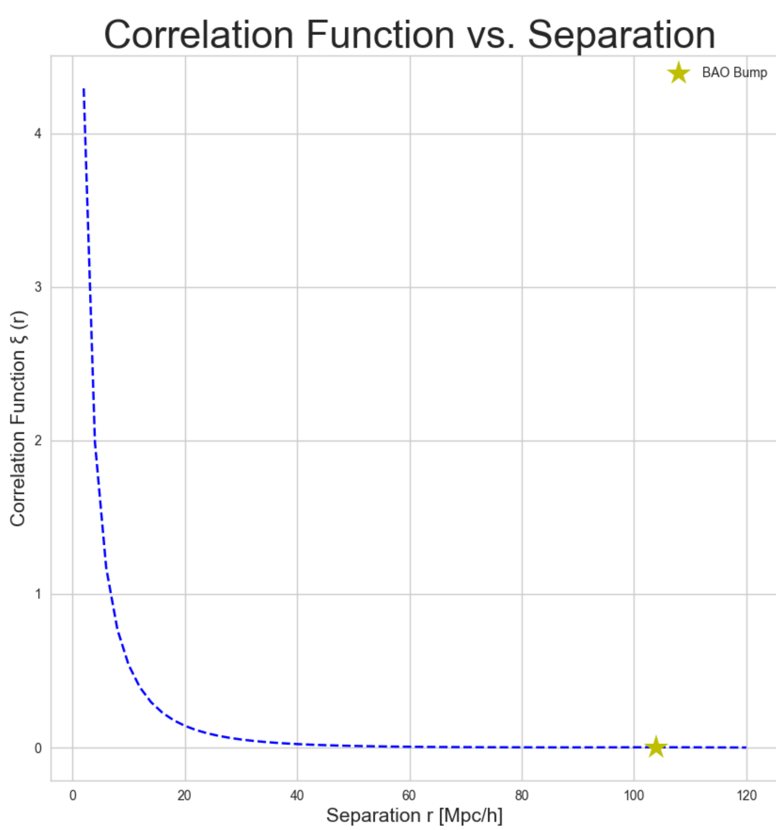

Figure 3: The correlation function plotted versus separation r

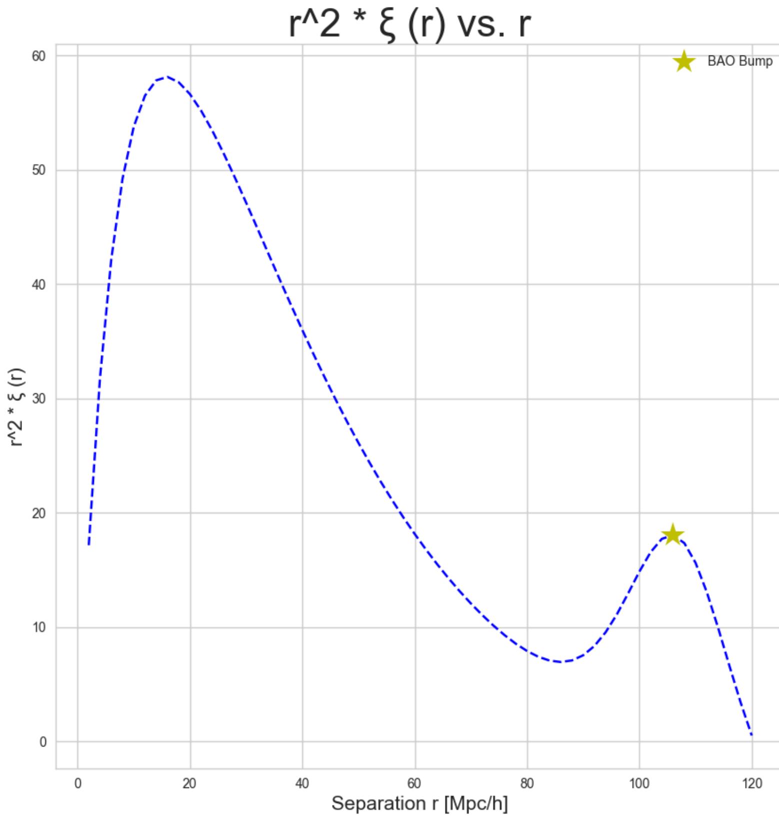

Figure 4: The correlation adjusted by factor r^2 function plotted versus separation r

From figure 4 we recover the same result from figure 3, but with more noticeable effects. In particular, what we observe is an anomalous bump in correlation, (known as the BAO Peak) at approximately:

$$r_{peak} \approx 105\frac{Mpc}{h}$$

This peak is denoted and labelled as the yellow star on both graphs, providing more motivation as to why the r^2 manipulation was used to better visualize this peculiarity.