Noah Kasmanoff
nsk367

# Homework 4

Computational Physics
November 8, 2019

## 8.18

link to code

Please see below for the resulting plot on the Brussellator, solved with the adaptive Bulirsch–Stoer method.

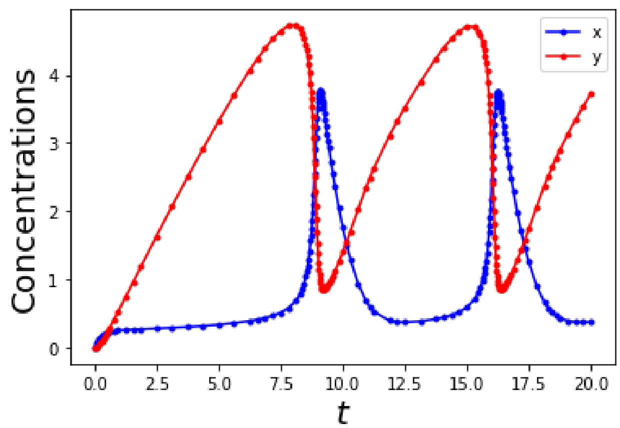


Figure 1: The resulting concentrations as a function of time for $x(t)$ and $y(t)$.

As you can see, the point are very close together when the variables are changing most rapidly (i.e. the position in time where $x$ and $y$ have the largest slopes.)

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# Homework 4

## Black Hole Binaries

link to code

a) To solve for the proper value of $\delta$, we what this system would look like with no dynamical friction, but the black hole still manages to get very close to it's companion black hole. In other words, this will be an a highly elliptical orbit, with a peri-centric distance of $10^{-7}$ units $/10^{-5}$ parsecs.

To begin this problem, we start with the black hole we are tracking to be at it's apo-centric distance, or 1 / 100 parsecs. Our first step is to determine what the velocity of this black hole should be at aphelion.

This value is given by the formula

$$v = \sqrt{\frac{GM}{4}[\frac{2}{r} - \frac{1}{a}]}$$

we can further elaborate on this by saying that the semi-major axis $a = \frac{r_{apo}+r_{peri}}{2} = \frac{1+10^{-7}}{2}$ in the unit domain, or

$$\frac{1+10^{-7}}{2} = \frac{100+10^{-5}}{2}$$

parsecs.

Next, we plug in for the other quantities we know, $G = M = 1$, and that for the apo-centric velocity $v_{apo}$ we are looking at the value of $r$ at the apo-center, preset to 1.

From here, we recover that $v_{apo} = 0.000223$ units per unit time. From here it is important to learn what the unit of time is, and we can do so by solving for the velocity at apo-center while in SI units, and after converting all units to the proper ones and create a ratio between the SI version and coding unit version and solve for the number of years as opposed to seconds, I recover this to be approximately 1.427 million years.

Now that all these factors are derived, our next step is to determine the optimal value of delta, such that the decay of the orbital radius $r$ at aphelion, remains constant for at least 10 orbits.

To do so, I integrated over an array of different delta parameters, and recover the following graph.

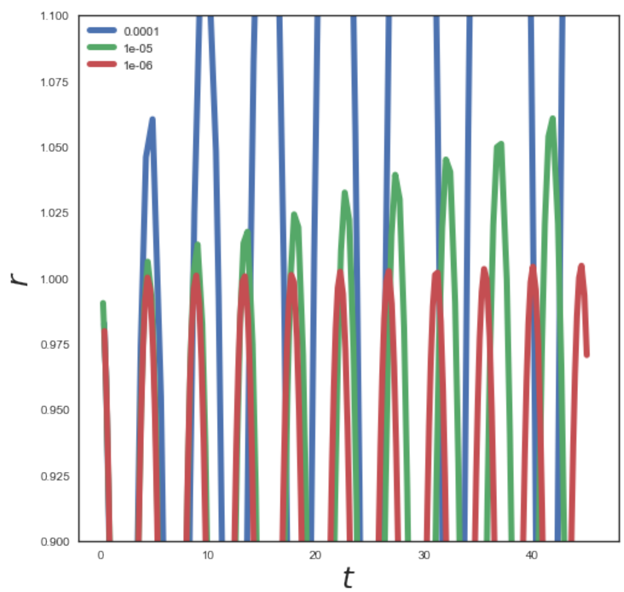


Figure 2: The testing of different error tolerance $\delta$ parameters to use throughout the remainder of the experiment.

In the end, this graph demonstrated to me that for the most stable orbital integration, a parameter of $\delta = 10^{-6}$ was selected.

b) Now that we have found an acceptable value for error tolerance, we now include the formula for dynamical friction to the differential equation and observe how this affects orbital decay.

The result is shown below, and acts under the assumption that the initial velocity of this orbit is at 80 percent of it's circular velocity. So while the previous part of the question acted as if this was already a highly eccentric orbit, we begin as if it were nearly circular, and observe how the dynamical friction causes it to decay. We recover the following diagrams.

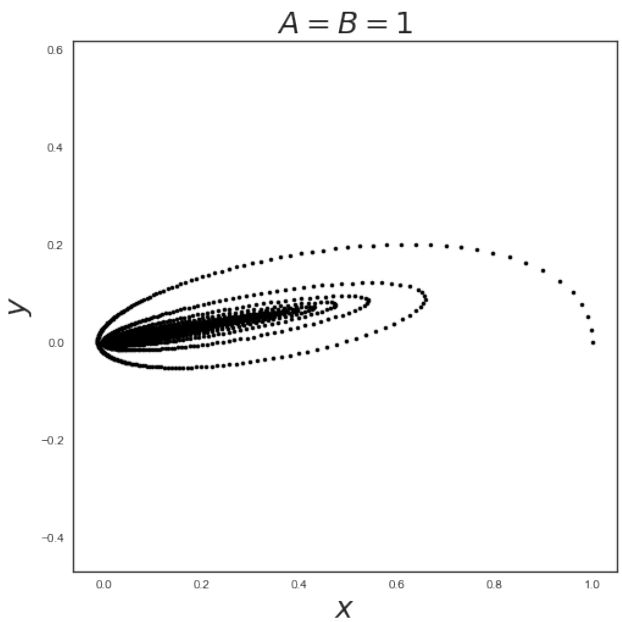


Figure 3: How the radius of the orbit decays over time.

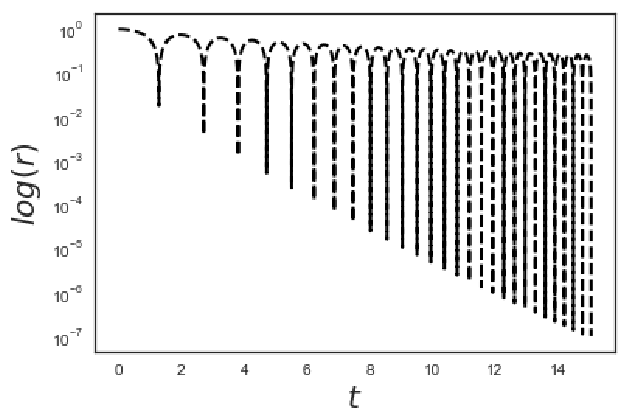


Figure 4: The orbital decay of this system in the context of the orbital radius r versus time.

So under the condition of $A = B = 1$, one such case for the parameters of dynamical friction, we see what we would expect, the orbit of the binary black holes to decay under the presence of dynamical friction due to the surrounding bodies in the system.

c) Next, we explore how varying the ratio $\frac{B}{A}$ affects the time it takes to reach the Schwarzschild radius. When comparing these quantites, we recover the following relationship (in log space).

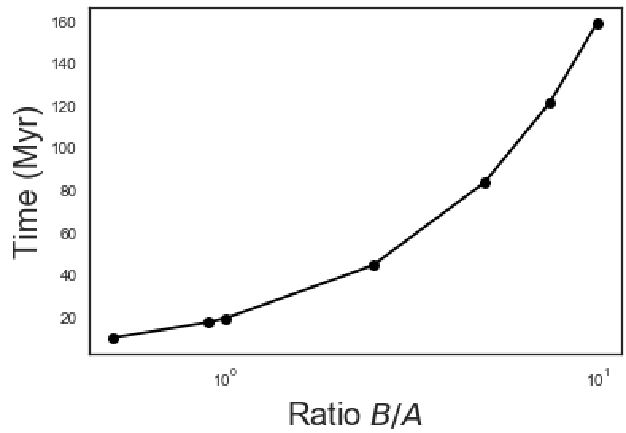


Figure 5: How the decay time varies with the ratio of parameters B and A.

$A$ and $B$ are parameters associated with the strength of the dynamical friction. As such, we would expect that for this ratio that as it increases $B > A$, the amount of time taken until the system reaches the Schwarzschild radius to be longer. After testing different permutations of the ratio, we found exactly that.

Another interesting result from this graph is that we are now also looking at the time it takes in millions of years (Myr) and see that even under the largest ratio, all of these values fall well under the age of the universe (1300 Myr). The reason why this may be an important result is that it demonstrates how all of these systems would have been able to decay within the lifespan of the universe, and thus serve as some reason as to why such systems are not well understood and observed.

d) Beyond the ratio of B and A, the results of our study depend not only on them, but on the other parameters which make up the formula for dynamical friction, such as the initial velocity of the black hole, as well as the individual values for A and B.

   To understand why initial velocity matters, we need to think about this problem in a physical sense. Depending on how fast the black hole we are tracking is going, there will more or less drag affecting it. In turn, this affects the amount of time it takes for it to reach a peri-center at which gravitational radiation has an effect.

   To demonstrate such a result, I have included below a plot of the different convergence times (for a constant ratio of 1) where the initial velocity is allowed to vary.

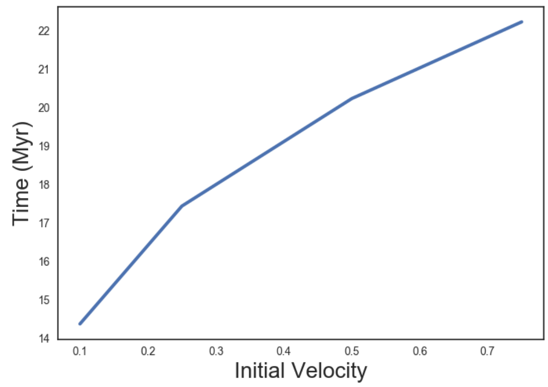


Figure 6: How the initial velocity of the system determines how long it takes for the orbit to decay to the designated distance.

From here, we clearly see what was expected! If the black hole is moving faster to start, it will take longer to slow down.

Additionally, the individual values for A and B matter. While the ratio is essential for determining the decay time, the individual importance of each is due to the second term in the denominator, the total velocity of the black hole cubed. Even if the ratio of A and B is extremely in the favor of A (in the event B is extremely small), this does not necessarily mean that it will take a very long time for the black hole to decay. Since the denominator is $B + V_{BH}^3 = V_{BH}^3$, the individual affect of B is negligible when $B << V_{BH}^3$ In this circumstance, the individual value of B matters since it determines how important the ratio between B and A truly is.