Homework #4
Computational Physics
Fall 2019
Due Friday, November 8 at 5pm.

1. Exercise 8.18 from Newman

2. As per the class notes, you will integrate the orbit of a supermassive black hole binary in the presence of dynamical fraction. The equation of motion for a BH is:

$$\frac{d^2\mathbf{r}_{\rm BH}}{dt^2} = -\frac{GM_{\rm BH}}{4r_{\rm BH}^3}\mathbf{r}_{\rm BH} + \dot{\mathbf{v}}_{\rm DF}$$

where the last term on the right hand side is the force from dynamical friction (DF). We use the following approximation formula for this force:

$$\dot{\mathbf{v}}_{\rm DF} = -\frac{A}{v_{\rm BH}^3 + B}\mathbf{v}_{\rm BH}$$

where the two constants A and B represent the product of the BH mass and the stellar density and the velocity dispersion (cubed) of the stellar field, respectively.

The goal of this problem is to determine whether the BH binary can get close enough in pericenter such that it can lose energy via gravitational radiation. This happens where r_{peri}~ the Schwarzschild radius. Set M=G=1 for the calculation, and the unit of distance to be 100 parsecs. In these units, the r_s is 10^{-7}.

You will use your fourth-order Runge-Kutte intregrator, with adaptive timesteps, to solve this calculation. Since the orbit is symmetric (ie, the BHs are the same), you only need to solve for the position of one BH. For all calculations with dynamical friction, make the initial position of the BH x=1, y=0, and the initial velocity equal to 0.8 of that of a circular orbit.

a) First, you need to set your value of d, the error tolerance per unit of time. To do this, first solve for the orbit without dynamical friction, setting the initial velocity such that r_{peri} is 10^{-7}, and demonstrating that there is no appreciable loss of accuracy over at least 10 orbits.

b) Solve for the BH orbit with A=B=1. Make a plot showing the path of the BH in its orbit, and another showing log(r) as a function of time. You can stop the integration when r=10^{-7}.

c) Determine how the time it takes to reach the Schwarzschild radius depends on the ratio of B/A. Discuss (qualitatively) why it looks this way. Include a plot showing this, and convert the time axis to Myr. Take as your range of A and B values [0.5,10].

d) Do your results depend on the value of the initial velocity? Do they only depend on the ratio of B/A, or on the individual values of B and A?