

# **Advance Vehicle Location**

**Senior Design Project** 

Swarthmore College Engineering 90

Nicole Kato Advisor: Dr. Matthew Zucker For my senior design project, I created a system to determine a vehicle's location along a known route and derived additional information such as estimated time of arrival which is displayed on a website with the use of Google Maps for the student body. This system uses GPS to measure the position of the vehicle. A Kalman Filter is then used to integrate the noisy unreliable GPS measurements along with the predefined route to find a combined position estimate. A centralized server then runs the Kalman Filter and presents data in real-time to users through a web interface. While this Advance Vehicle Locator (AVL) applications can only be simulated with previously collected GPS data, it could be easily modified so it can be implemented for several student run shuttles at Swarthmore College.

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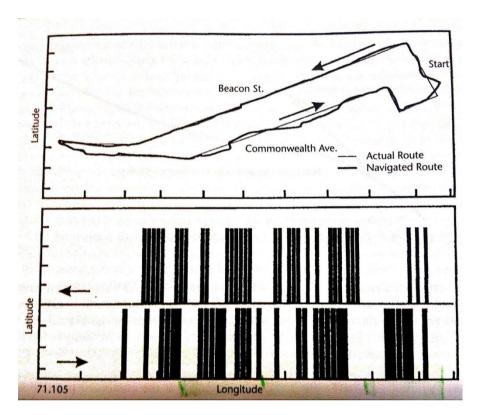
## Introduction

Director of the Technical Services of the American Public Transportation Association (APTA), Louis Sanders has stated, "The fundamental element of these transit Intelligent Transportation Systems (ITS) systems is Automatic Vehicle Location, which is enabled by GPS technology" (Sanders, Louis). The most common uses of AVL for public transportation are with buses. From 2001 to 2009 the growth of AVL and/or GPS in buses has increased by 42 percent (2010 Public Transportation Fact Book). Using AVL for buses, bus companies are able to keep track of real-time traffic, bus bunching, driver safety, passenger safety and other useful information.

Recently real-time passenger information has been implemented at bus stations and the internet. This information is able to provide accurate information on where buses are located and when they will reach different stations. ITS found that 80 percent of visitors to Acadia National Park in Maine found it easier to travel with real-time bus departure signs and on-board next-stop announcements (benefits). This allows passengers to plan accordingly and live more efficient and happy lives.

Several years ago, Swarthmore College implemented the Target shuttle. On Tuesdays and Sundays from 7:00 PM to 9:00 PM, students drive a shuttle from the College to Target and back. Currently the Target shuttle is not considered to be reliable. I personally have spent several nights in freezing temperatures waiting for the shuttle to arrive. By using AVL, people could monitor the shuttle's location inside and go outside only when the shuttle has arrived. This would significantly improve the utility of the student body.

While it would appear logical to just place a GPS on top of the Target shuttle and to just transmit its signal wirelessly, this option is very problematic, due to signal interference from trees, buildings, and possibly weather. Below in figure 1 is the route of a vehicle with just a GPS receiver tracking it.



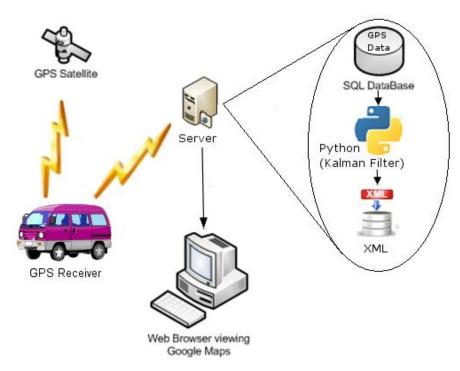
(Figure 1: Effects of GPS Receiver Interference)

The top graph of figure 1 displays the actual route and the navigated route. The bottom graph displays where there is signal interference. The black spikes represent when there is signal interference. While a GPS receiver is able to provide a good estimate of the automobile's positioning, it needs to be integrated with other technologies to get a more accurate position.

By integrating a GPS receiver with a Kalman Filter, we can make predictions if there are any inaccuracies or interference that occurs with the GPS receiver. A Kalman Filter takes into account the noise of the GPS receiver in addition to where the object should be to determine a more accurate position of the automobile.

This paper explains the technical and theoretical background of how this AVL was implemented in addition to how accurate the estimated time of arrival is compared to the measured times along the route.

Figure 2 is an overview of the implemented AVL system that I created for my Senior Design Project.



(Figure 2: Basic Advance Vehicle Location Architecture)

#### **GPS** Receiver

GPS is a system that allows a receiver to determine a user's position and velocity. It is composed of space, ground, and user segments.

# Space Segment

The space segment is composed of a satellite constellation with six orbital planes and nominally four satellites in each plane at an altitude of 20,200 kilometers above the surface of the earth. These satellites encrypt three dimensional position and velocity information, which are different for civil and military use. Civil users receive the L1 frequency broadcast and military users receive both the L1 and L2 frequency broadcast. Currently GPS is composed of 24 satellites and

seven additional satellites due to the actual lifespan of these satellites exceeding the predicted (Borre, Kai).

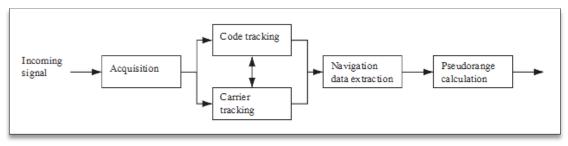
#### Ground Segment

The ground segment is first composed of a Master Control Station at Schriever Air Force Base in Colorado Springs, Colorado. There are also six Air Force, and 11 National Geospatial-Intelligence Agency monitoring stations, plus four ground antennas located around the world. The monitoring stations are used to track and collect information from all GPS satellites that are in range of each station. The antennas are used to upload information to the GPS satellites (Borre, Kai).

#### User Segment

The user segment is composed thousands of different types of receivers; however they can be divided into two different categories, military and civilian. Both types of receivers are able to determine the position and velocity of a specific location. The main difference between military and civil type receivers is the accuracy of the position and velocity that is received, as discussed above in the satellite segment. These receivers range in price from just a few dollars to thousands of dollars (Borre, Kai).

Because I will manually use the GPS receiver the rest of the GPS receiver section will be dedicated to how it operates. Below is the basic architecture of a basic GPS receiver channel (Borre, Kai).



(Figure 3: Overview of GPS Receiver Channel)

#### Acquisition

First the GPS receiver receives the incoming signal through its antenna and performs acquisition. Acquisition is used to identify the number of satellites, frequency and code phase of the signal. For the receiver to perform proper trilateration it must be able to identify four satellites. I will be reading off of the L1 frequency for civil users as discussed in the space segment above. The code phase is the position in the data block where the C/A code starts. C/A code is the spreading sequence is used to send information to the receiver (Borre, Kai).

#### Code Tracking and Carrier Tracking

After acquisition, the receiver simultaneously performs code tracking and carrier tracking. It performs these two tracking techniques to track the changes in the C/A code over time. This helps determine the accuracy of the receiver. Code tracking is a delay lock loop (DLL). It takes three codes and correlates them to the new incoming signal from the antenna. Carrier tracking tracks the phase or frequency of the signal (Borre, Kai).

#### Navigation Data Extraction

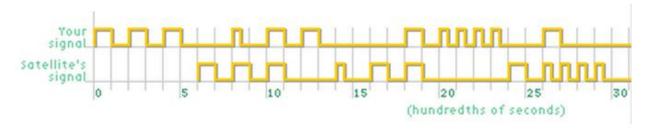
After the tracking has been properly implemented, the navigation data bits are found by removing the C/A code from the carrier wave. The navigation data bits are found by integrating the signal over the navigation bit period. After 30 seconds of data extraction, the time of data transmission from the satellite is found (Borre, Kai).

## Computation of Position

The time found from the navigation data extraction is used to determine the position of the receiver through trilateration. Trilateration is the process of finding one's location by measuring the distances of at least three satellites from the receiver (Borre, Kai).

Every satellite transmits a ranging signal that travels at the speed of light and is controlled by an onboard clock. When the receiver gets the signal, the receiver is able to determine when the satellite sent the signal. The GPS receiver knows what each signal each satellite should send at what time Suppose the receiver knows that a satellite should send a signal at "Your Signal" as

seen in the graph below. However, because the "Satellite's Signal," as seen below, is received at six hundredth of a second late because of satellite are positioned 20,000 km above the earth's surface (GPS: The New Navigation).

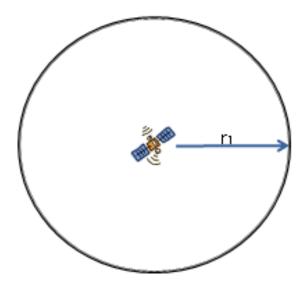


(Figure 4: Delayed Satellite Signal)

By knowing the change in time and the speed of the signal, the receiver can use the formula 1 below to determine the satellite-to-user range,  $r_1$ .

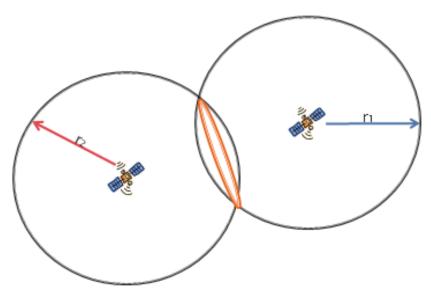
$$r_1 = speed \ of \ light * \Delta time$$
 (1)

By knowing that the speed of light is 300,000 km/s and the  $\Delta \text{time}$  is 0.06 s, we can find  $r_1$  to be 18,000 km. This  $r_1$  value creates a sphere of all of the possible positions the receiver could be with respect to the satellite as seen below.



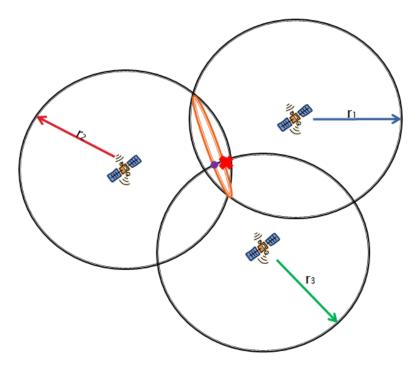
(Figure 5: One Satellite)

If a receiver is able to see two satellites, the receiver can find the satellite-to-user range of the additional satellite. The receiver can determine where the two ranges intersect to form a circle of intersection. This is a circle of all possible locations the receiver could be with respect to the two satellites, as seen orange in the figure below.



(Figure 6: Two Satellite Intersection)

If a receiver is able to see three satellites, the receiver is able to use each satellite's satellite-touser ranges to determine two possible positions the receiver could be. Because it is assumed the receiver is on earth, we can eliminate one of the positions because it is above the earth's surface as seen in the figure below.



(Figure 7: Three Satellite Intersection)

If a receiver is able to see four satellites we can find one position and not have to determine one position is impossible.

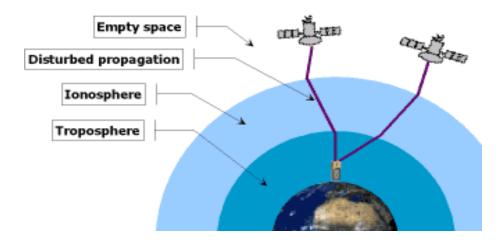
After the GPS receiver is able to determine its position, it outputs a series of messages that can be parsed with a computer programming language. Appendix A shows the signal parsed from the GPS receiver in order to get latitude, longitude, and timing information. While the receiver sends several different messages I choose to parse the Recommended Minimum Specific GNSS Data Message. I pared this message because in addition to sending time and position information, it also sends the ground speed of the receiver, which I found useful to determine the speed of the vehicle at certain positions along the route.

# **GPS Errors**

Even though GPS is considered to be one of the most accurate and reliable positioning systems in the world, there are several areas where errors occur. GPS is susceptible to two types of errors, visibility and measurement.

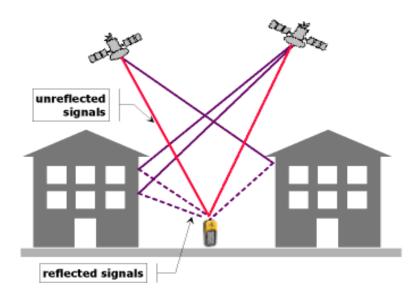
Visibility errors occur if the GPS satellites are not able to be seen by the GPS receiver. This can occur if there is an object such as a tree, building, or mountain blocking a satellite's signal. Measurement errors occur if the GPS signal from the satellite becomes distorted so the position the receiver finds is inaccurate.

The most consistent source of errors comes from the violation of the theory of the linearity of the signal. It is assumed that the signal travels linearly, through the ionosphere and the troposphere. Because of free floating ions, the signals can be rotated as seen in the figure below. While the GPS receiver, used for this AVL system, did not account for ionosphere and troposphere errors, other GPS receivers are able to take into account this error. Several experts have stated that no matter how advance GPS receivers become, they will probably never to have a measurement error better than  $\pm$  10 centimeters due to the ionosphere. While these errors normally cause only a few meters of error, strong solar winds and solar flares can increase the inaccuracies significantly (The GPS System).



(Figure 8: Atmospheric Effects on GPS)

In addition to ionosphere and troposphere errors, another large source of error comes from the multipath effects. Multipath occurs when satellite signals are reflected off of objects such as buildings and trees. This most often occurs in areas with tall buildings such as Philadelphia. This error can cause a measurement error of a few meters (The GPS System).



(Figure 9: Multipath Effects on GPS)

Below is a table showing the most common sources of GPS error measurement and their average measurement error (The GPS System).

Source of GPS Error	Average Measurement Error
Ionosphere effects	± 5m
<b>Troposphere effects</b>	± 0.5m
Multipath effect	± 1m
Clock errors of clocks	± 2m
Shifts in the satellite orbits	± 2.5m
Calculation errors	± 1m

(Table 1: Source of GPS Errors)

In addition to ionosphere, troposphere, and multipath effects, there are also clock errors, shifts in satellite orbits and calculation errors.

All GPS satellites run off of the same clock system, which is reset every Sunday. If two satellites do not have the same time or if the GPS receiver's clock is off, the trilateration calculations described above can be off by several meters. Not only can clock errors be caused by the satellites in orbit but also shifts in satellite orbits (The GPS System).

If the US Air Force believes that they need to move a satellite because of a solar flare or military purpose they will move that satellite, which can cause errors. Also, because GPS satellites are constantly orbiting the earth at a speed of 14,000 km/hour there can also be measurement errors just from the satellite moving.

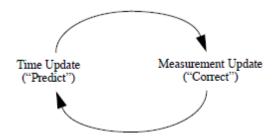
# Kalman Filter

Suppose we are diving in the green car in the image below along the blue path to Target, but the GPS receiver says the car is in the position shown by the satellite. For a user this GPS measured position is not logical, because the car must be on the road. A Kalman Filter would be optimal to determine the position of the car, because it takes into account both the planned route and the GPS measured position to create a position estimate.



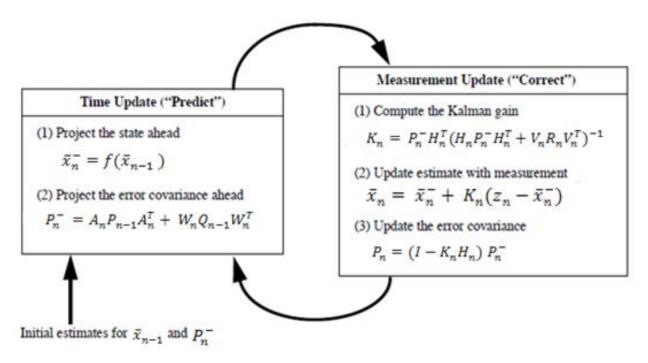
(Figure 10: Purpose of Kalman Filter)

While there are several different types of Kalman Filters, all Kalman filters can be decomposed into two basic steps, the Predict Step, and the Correct Step as seen in the diagram below (Welch, Greg).



(Figure 11: Basic Theory behind Kalman Filter)

The Predict State determines the next step of the vehicle along the route, and the Correct State adjusts the Predict State based on the GPS measured state of the vehicle and where the vehicle should be. While the basic Kalman Filter is not able to take into account variations in speed, the Extended Kalman Filter or EKF can. Therefore I decided to use an EKF to find estimates for a vehicle's position. Below is a figure showing that describes the overall equations involved in a basic EKF (Welch, Greg).



(Figure 12: Operations of an Extended Kalman Filter)

While this chart of operations appears to be complicated the basic concept is simple. First the previous position estimate and error covariance uses the planned route to predict the next position without any measurement or processing noise. An error covariance is basically a two dimensional standard deviation array. The GPS measurement and processing noise are then added to the updated position along the route in order to get the updated position estimate and the updated error covariance.

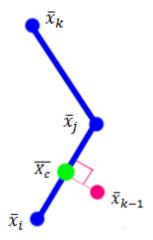
First I will discuss the time update and then the measurement update processes. Each position vector,  $x_i$ ,  $x_j$ ,  $x_k$ , is composed of a horizontal position, x, vertical position, y, and a time value, t.

The time value is 0 seconds at  $x_0$ . Let's suppose we are driving along the route in the figure below, and our previous position is at  $\bar{x}_{n-1}$ .



(Figure 13: Initial Map)

We now need to find the closest point to the previous estimated position along the planned path, which is on the true path. This will end up to be position,  $\bar{x}_c$ , which has to be perpendicular to the estimated position, as seen below.



(Figure 14: Closest Position)

In order to solve for  $\bar{x}_c$ , we need to find it in terms of  $\bar{x}_i$ , and  $\bar{x}_j$  and a proportion constant  $u_{n-1}$  that ranges between 0 and 1 as seen below.

$$\overline{x_c} = \overline{x_l} + u_{n-1}(\overline{x_l} - \overline{x_l}) \tag{2}$$

If  $u_{n-1}$  is 0, then  $\bar{x}_{n-1}$  is closest to  $\bar{x}_i$ . If  $u_{n-1}$  is 1, then  $\bar{x}_{n-1}$  is at position to  $\bar{x}_j$ . If  $u_{n-1}$  is between 0 and 1,  $\bar{x}_{n-1}$  is somewhere on the blue route between  $\bar{x}_i$ , and  $\bar{x}_j$ . Because we know the closest point is where the lines between  $\bar{x}_i$ , and  $\bar{x}_j$  and  $\bar{x}_c$ , and  $\bar{x}_{n-1}$ , are perpendicular so, we can dot product the two displacements and set them to zero get the formula below.

$$(\bar{x}_{n-1} - \bar{x}_c) \cdot (\bar{x}_i - \bar{x}_i) = 0 \tag{3}$$

By substituting equation 2 into equation 3 we get the formula below.

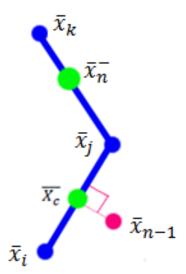
$$(\bar{x}_{n-1} - \bar{x}_i - u_{n-1}(\bar{x}_j - \bar{x}_i)) \cdot (\bar{x}_j - \bar{x}_i) = 0$$
 (4)

We can then simplify the equation above and solve for  $u_{n-1}$  to get the formula below.

$$u_{n-1} = \frac{(x_c - x_i)(x_j - x_i) + (y_c - y_i)(y_j - y_i)}{\|\bar{x}_i - \bar{x}_i\|}$$
 (5)

By knowing our current position, the speed of the vehicle, and the change in time until the next measurement is taken,  $\Delta t$ , we can determine the projected state ahead  $\bar{x}_n^-$  along the route. From the operations of the EKF figure above we know that  $\bar{x}_n^-$  is determined by the previous state and its corresponding  $u_{n-1}$  value, is found with the formula and figure below.

$$\bar{x}_n^- = f(\bar{x}_{n-1}) \tag{6}$$



(Figure 15: Next Position)

We can assume the magnitude of the displacement between  $\bar{x}_i$ , and  $\bar{x}_j$  and  $\bar{x}_j$ , and  $\bar{x}_k$  to be the two equations respectfully.

$$||d_{ji}|| = \sqrt{(x_j - x_i)^2 + (x_j - x_i)^2}$$
 (7)

$$||d_{kj}|| = \sqrt{(x_k - x_j)^2 + (x_k - x_j)^2}$$
 (8)

In addition to  $u_{n-1}$  equaling equation 5, above, it is also equal to the equation below.

$$u_{n-1} = \frac{t_{n-1} - t_i}{t_j - t_i} \tag{9}$$

By rearranging this equation we can get  $t_{n-1}$ , as seen in the equation below.

$$t_{n-1} = t_i + u_{n-1}(t_j - t_i) (10)$$

After we solve for  $t_{n-1}$ , we add the change in time,  $\Delta t$ , which represents the time it takes for the GPS receiver to take another measurement, as seen in the formula below to get  $t_n$ .

$$t_n = t_{n-1} + \Delta t \tag{11}$$

By using equation 9, but solving for  $u_n$ , we get the equation below.

$$u_n = \frac{t_n - t_i}{t_i - t_i} \tag{12}$$

If  $u_n$  is greater than 1 we know that our next position must be between  $\bar{x}_j$  and  $\bar{x}_k$ . In order to adjust for this change we much set  $u_{n-1}$  to 0, and adjust  $t_{n-1}$ , with the following equation below.

$$t_{n-1} = t_i + u_{n-1}(t_i - t_i) - (t_k - t_{n-1})$$
(13)

With this new  $t_{n-1}$  value, we must recalculate  $t_n$  with the new  $t_{n-1}$  value in addition to calculate the new  $u_n$  with the new equation below.

$$u_n = \frac{t_n - t_j}{t_k - t_j} \tag{14}$$

If  $u_n$  is once again larger than 1 we know that we are at the end of position  $\bar{x}_k$ . Hypothetically, if we had more positions after  $\bar{x}_k$ , we can do the same process again until  $u_n$  is less than 1.

By finding  $u_n$  we can find the projected state ahead,  $\bar{x}_n^-$ , with the formula below.

$$\bar{x}_n^- = \bar{x}_i + u_n * (\bar{x}_k - \bar{x}_l) \tag{15}$$

In addition to this value we can also find our new  $A_n$  value, where  $A_n$  is equal to the formula below.

$$A_n = \frac{d\bar{x}_n^-}{d\bar{x}_{n-1}} \tag{16}$$

Using the chain rule we can equate the formula above to the formula below.

$$A_n = \frac{d\bar{x}_n^-}{du_n} * \frac{du_n}{dt_n} * \frac{dt_{n-1}}{du_{n-1}} * \frac{du_{n-1}}{d\bar{x}_{n-1}}$$
(17)

Using formula 15 above, we can solve for  $\frac{d\bar{x}_n^-}{du_n}$  to be

$$\frac{d\bar{x}_n^-}{du_n} = \begin{bmatrix} d_{kjx} \\ d_{kjy} \end{bmatrix} \tag{18}$$

Using formula 14 above, we can solve for  $\frac{du_n}{dt_n}$  to be

$$\frac{du_n}{dt_n} = \frac{1}{t_k - t_j} \tag{19}$$

Using formula 10 above, we can solve for  $\frac{dt_{n-1}}{du_{n-1}}$  to be

$$\frac{dt_{n-1}}{du_{n-1}} = t_j - t_i {20}$$

Using formula 2 above, we can solve for  $\frac{du_{n-1}}{d\bar{x}_{n-1}}$  to be

$$\frac{du_{n-1}}{d\bar{x}_{n-1}} = \begin{bmatrix} d_{jix} & d_{jiy} \end{bmatrix} \tag{21}$$

Therefore A<sub>n</sub> is equal to

$$A_{n} = \frac{t_{j} - t_{i}}{t_{k} - t_{j}} \begin{bmatrix} d_{kjx} \\ d_{kjy} \end{bmatrix} [d_{jix} \quad d_{jiy}]$$
 (22)

After finding the projected state ahead, we can find the projected error covariance,  $P_n^-$ .  $P_n^-$  that represents the covariance error of the projected state in the x and y direction. This value will tend to be higher along the route and smaller perpendicular to the route, due to the fact that you are more likely to be along the route than left or right of the route.  $P_n^-$  is equal to the formula below

$$P_n^- = A_n P_{n-1} A_n^T + W_n Q_{n-1} W_n^T$$
 (23)

Where  $P_{n-1}$  is the previous error covariance, W is a 2x2 identity matrix, and Q is equal to a square diagonal matrix representing the processing noise as seen below.

$$Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \tag{24}$$

To find the updated measurement, we need to first find the Kalman gain,  $K_n$ . The Kalman gain's purpose is to propagate the important components of the GPS receiver. Below is the formula for the Kalman gain.

$$K_n = P_n^- H_n^T (H_n P_n^- H_n^T + V_n R_n V_n^T)^{-1}$$
(25)

Where H and V in our case are equal to the identity matrix, and R is equal to the measurement noise as seen in the formula below.

$$R = \begin{bmatrix} 15 & 0\\ 0 & 15 \end{bmatrix} \tag{26}$$

The value of 15 was chosen because I believe there is around  $\pm$  15 meters of error with our GPS receiver.

After the Kalman gain is found we can find the updated estimate with our GPS measurement to be the formula below

$$\bar{x}_n = \bar{x}_n^- + K_n(z_n - \bar{x}_n^-) \tag{27}$$

where  $z_n$  is a GPS position measurement.

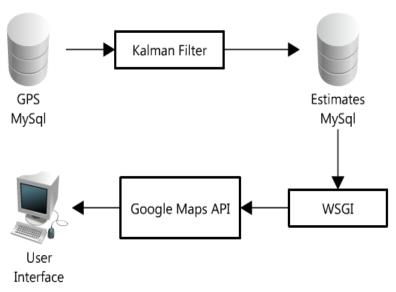
We can also calculate our updated error covariance to be the formula below

$$P_n = (I - K_n H_n) P_n^- (28)$$

By continually updating theses values along a known path, we are able to get better estimates than just the GPS measurements alone.

### **Centralized Server**

Because this vehicle tracker is meant to be used in real time by the student body, it is most optimal to place this information on an internet server to be placed on a website. Internet servers are computers that are always on and are connected to each other to form the internet. While the computer I used to program my code was not a server it can connect to a server through a Secure Shell or SSH. Below is a flow chart demonstrating how the GPS measurements are sent to the server and how the server updates the position estimates and displays the information on a website.



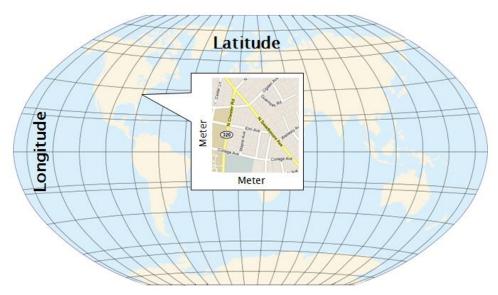
(Figure 16: Centralized Server)

After the GPS receiver gets finds the vehicle's position, the latitude, longitude, and time information are sent to the server to MySql. MySql is a relational database management system similar to Microsoft Access or Excel where data can be stored and extracted for later use. The most current positioning information is extracted and run through an EKF written in Python. Python is a high level object oriented programming language similar to Java or C++. The main reasons why I chose to use Python because the server is able to run it in addition to being able to do matrix arithmetic by importing numPy. NumPy is a package that is imported to Python to allow for matrix arithmetic. After the position and time remaining estimates are found, they are sent to another MySql database. These position and time estimates are then extracted by mod\_WSGI an apache module, which can run Python and output Extensible Markup Language (XML). XML allows information to be tagged so it can be easily extracted. JavaScript is a

scripting language that is normally implemented on a website. JavaScript gets the position estimates and places them onto a Google Map with the use of Google Maps API. Google Maps API is an application programming interface that allows a user to program a Google Map to be placed on their website. JavaScript also places the estimated time remaining on the website as well. The Google Maps and the estimated time remaining is then all placed on a website for the use of the student body.

# Results and Discussion

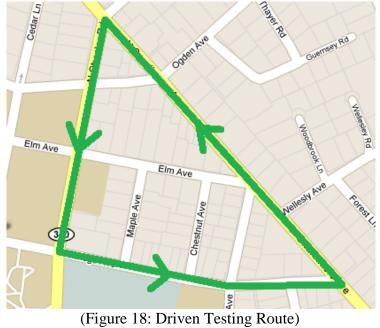
Figure 17 below shows that Latitude and Longitude coordinates do not map linearly to meters.



(Figure 17: Latitude/Longitude to Meters)

Because we are dealing with such a small location with respect to the rest of the earth, we can use a conversion factor to convert latitude and longitude degrees to meters as seen in the plots done in MATLAB in the results section.

A GPS receiver was placed on a car which was driven seven times around a triangular route as seen in figure 18, below.



After the GPS receiver collected positioning and timing data, the data was sent to the server to be analyzed.

Figure 19 shows a plot of the GPS measured position, the estimated EKF positions, the EKF error covariance ellipses and the planned route. While this plot if very cluttered, we can see that all of the EKF positions are in close proximity to the GPS measured positions and the planned route.

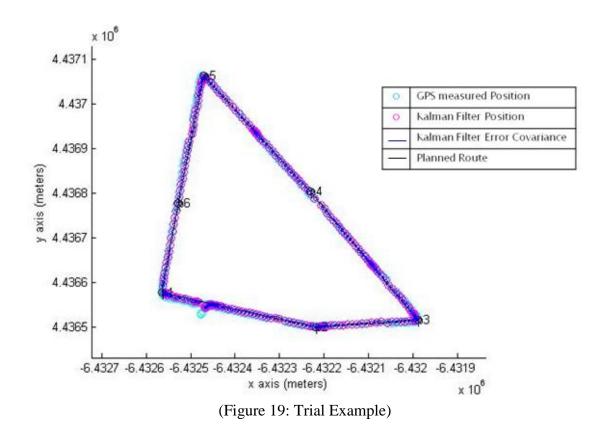
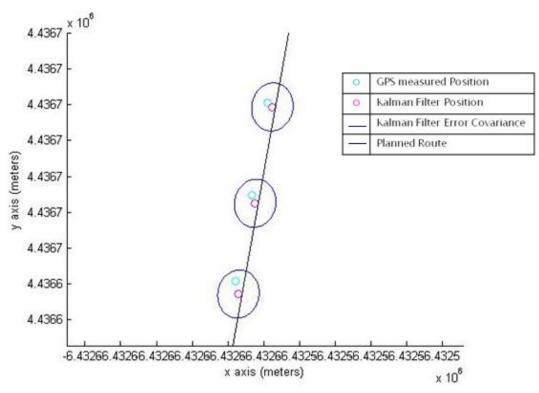
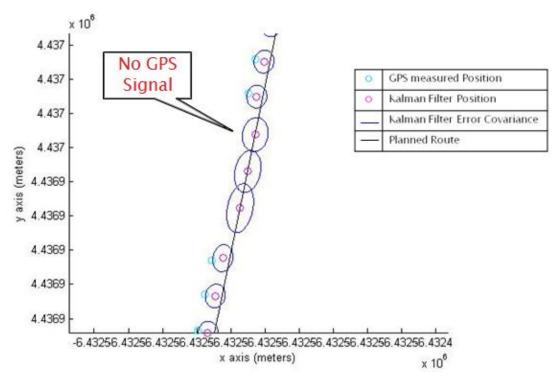


Figure 20 shows a zoomed in location along the route in Figure 19, where the GPS receiver is not cutting out. We can see that the EKF positions are located between the GPS measured positions and the planned route. We can also see that the error covariance ellipses are longer along the planned route than left or right of the route. This is because the vehicle is more likely to be along the planned route than left or right of the route. We can also see that the error covariance ellipses become more elongated the further away the GPS measured signal is from the EKF positions.



(Figure 20: Zoomed in GPS Signal)

Figure 21 shows a zoomed in location along the route in Figure 19, where the GPS receiver working probably due to a tree or building. We can see that when the GPS receiver does not get positioning information, the EKF positions are placed along the planned route. In addition to the EKF positions being placed along the planned route, we can see the error covariance ellipses changing shape as well. The error covariance ellipses become more elongated along the route the longer there is no GPS signal.

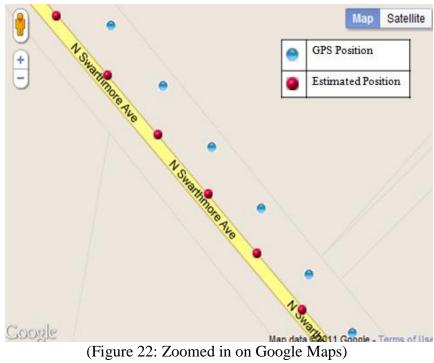


(Figure 21: Zoomed in No GPS Signal)

Figure 22 shows a zoomed in location along the route on Google Maps in Figure 19, where the GPS positions are not on a road. Because the planned path is on the road, the EKF positions correct for the GPS positions not being placed on the road.

After plotting the GPS and EKF positions on Google Maps, we can see an inconsistency with the GPS positions and the planned route in figure 23. Because the GPS positions are always on the outside of the planned route, there is an error with how Google Maps plot latitude and longitude positions and not the receiver. This is believed because the GPS positions appear to be shifted by the same amount off the planned route that was driven.

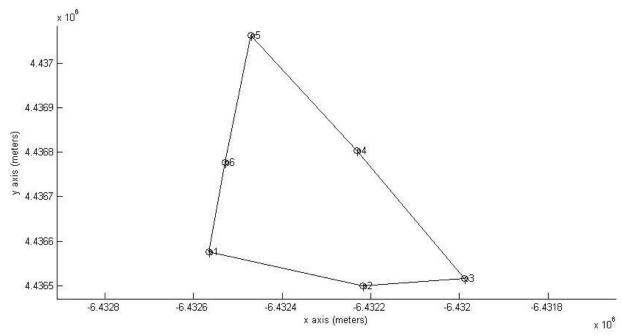
All of these signs show that the EKF works properly when determining how to estimate the vehicle's position along the known route.





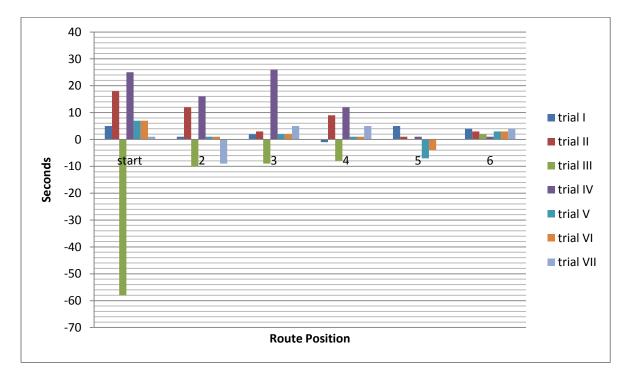
(Figure 23: GPS Consistently off Road)

Figure 24 shows 6 different positions along the planned route. The vehicle's start varies between position 1 and 2, and then the vehicle traveled around path in a counter clockwise manner until the vehicle was considered to have arrived at its destination at position 1.



(Figure 24: Positions on Route)

The actual time of arrival were found with the GPS receiver's clock at the start position, and positions 2 through 6. The estimated time of arrival were also found with the EKF. These times were then subtracted from each other in order to create figure 25 below.



(Figure 25: Difference of Timing Prediction Errors)

	trial I	trial II	trial III	trial IV	trial V	trial VI	trial VII
start (sec)	5	18	-58	25	7	7	1
position 2 (sec)	1	12	-10	16	1	1	-9
position 3(sec)	2	3	-9	26	2	2	5
position 4 (sec)	-1	9	-8	12	1	1	5
position 5 (sec)	5	1	0	1	-7	-4	0
position 6 (sec)	4	3	2	1	3	3	4

(Table 2: Difference of Timing Prediction Errors)

It appears that there is a larger variance the further away the vehicle is from the end point or position 1. This is further supported by looking at table 3 below. These both show the mean and standard deviation of the different positions along the route. The standard deviation in decreasing the further along the path the vehicle is driving, with exception of position 2 and 3.

	start	position 2	position 3	position 4	position 5	position 6
mean (sec)	0.71	1.71	4.43	2.71	-0.57	2.86
standard deviation (sec)	27.18	9.69	10.53	6.65	3.87	1.07

(Table 3: Mean and Standard Deviation of Timing Prediction Errors for Positions)

Table 4, below, shows the mean and standard deviation of the time differences on the seven trials that were conducted. We can there that there is a correlation between the higher mean and standard deviations of the difference between the measured and estimated time left.

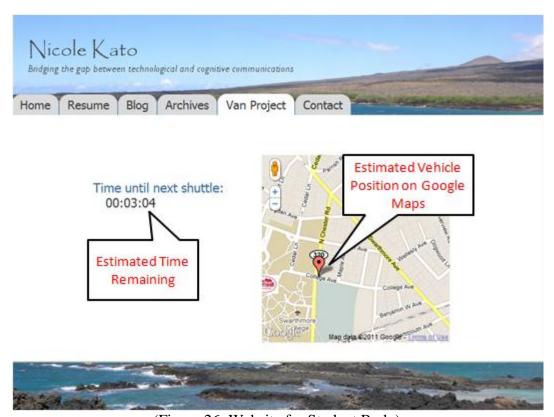
	Trial I	Trial II	Trial III	Trial IV	Trial V	Trial VI	Trial VII
mean (sec)	2.67	7.67	-13.83	13.50	1.17	1.67	1.00
standard deviation (sec)	2.42	6.56	22.20	11.04	4.58	3.56	4.86

(Table 4: Mean and Standard Deviation of Timing Prediction Errors for Trials)

From a usability standpoint, it will not make a significant difference if the estimated time of arrival is off by 30 seconds. However, the mean and the standard deviations are larger than they

could be due to manually inputting the speed of the vehicle along the path. We could significantly reduce the difference mean and standard deviation values by taking into account the speed of the vehicle from the GPS receiver. While it would appear easy to just add the GPS speed to the EKF, but because there is also a measurement error for with the speed, the speed have to be calculated into the  $A_n$  value before it is added to the position state and the error covariance in the EKF.

Figure 26 below is a screen shot of the website created by the centralized server. This website shows the estimated time left on the left side of the screen and the position of the vehicle on a Google Map on the right hand of the screen. Eye tracking studies done by Janice Redish shows that people place more priority to the left side of a website than the right. Because I believe the estimated time of arrival is more usually important for users than the position of the vehicle on Google Maps, I placed the estimated time of arrival on the left side of the website and the map on the right side of the website.



(Figure 26: Website for Student Body)

## Conclusion

Two main areas that need to be modified in order to have this vehicle tracker to be implemented for the student body. The GPS data needs to be sent in real time to MySql and the GPS receiver needs to be more robust from a circuit and power source standpoint.

Currently the GPS positioning data is stored by a computer and then sent to MySql after all of the data is collected. Due to the lack of transportation, I first collected all of the GPS positioning data. This could easily be done by using a USB wireless internet router. This will allow for real time information to be sent directly to the server.

In addition to the USB wireless internet router, the GPS receiver needs to become more robust. Because the GPS receiver is wired in on a breadboard, the wires can be easily become undone which will causes the GPS receiver to turn off and takes at least 30 seconds for the receiver to warm up again. This could be fixed by creating a printed circuit board.

Currently for the GPS receiver to be mobile it is powered by batteries. The batteries I used to power the receiver in the vehicle only last three hours. This could be fixed by used to power the GPS receiver. I am hopeful that these small changes would allow this AVL to be implemented for the Swarthmore Target Shuttle.

I was able to create an AVL system by extracting GPS measurement data in a moving vehicle and send it to a server's MySql database. These measurements where used to determine position estimates and estimated time of arrival along the planned route with an EKF. A website was created to display this information for the student body. I was able to create a complete system that was not only interesting but significantly improve the quality of Swarthmore College. It allowed me not just consider mathematical equations, but also usability and efficiently. Overall this was been a very successful Senior Design Project.

# Acknowledgements

First and foremost I would like to thank Professor Matthew Zucker for whom this project would not have been possible. Thank you for being my advisor and even though this was his first year as a professor, he advised as though he has been here for years. Thanks for the weekday (and sometimes weekends) meetings where you were willing to explain to the same concept over several times. I am confident the engineering department made the correct choice in hiring you and I hope you will continue to prosper on a campus that has given me so much joy.

Second, I would like to thank Janet Zarate, Cecilia Jou, and Jing Yi Ng. Thanks Janet and Cecilia and helping with collecting GPS data by driving 10 times in the circular path. Thanks Jing for the car and keeping me company the night before this paper is due, even though you came for Janet. Thanks Cecilia for allowing me to write part of my report in your lab even though I believe that the rug is assisting in my allergies.

Third, I would like to thank Joyce and Professor Lynne Molter. Thank you Joyce for helping Joyce for getting my my receipts for the 2 GPS components I purchased. Thank you Professor Molter for taking care of ABET. I know it has been a really tough year and I hope you both get a long deserved break.

### Appendix A: RMC – Recommended Minimum Specific GNSS Data

#### RMC - Recommended Minimum Specific GNSS Data

Time, date, position, course and speed data provided by a GNSS navigation receiver.

Structure:

\$GPRMC,hhmmss.sss,A,dddmm.mmmm,a,dddmm.mmmm,a,x.x,x.x,ddmmyy,,,a\*hh<CR><LF>

1 2 3 4 5 678 9 1011

Example:

\$GPRMC,111636.932,A,2447.0949,N,12100.5223,E,000.0,000.0,030407,,,A\*61<CR><LF>

Field	Name	Example	Description
1	UTC time	0111636.932	UTC time in hhmmss.sss format (000000.00 ~
			235959.999)
2	Status	A	Status
			'V' = Navigation receiver warning
			'A' = Data Valid
3	Latitude	2447.0949	Latitude in dddmm.mmmm format
			Leading zeros transmitted
4	N/S indicator	N	Latitude hemisphere indicator
			'N' = North
			'S' = South
5	Longitude	12100.5223	Longitude in dddmm.mmmm format
			Leading zeros transmitted
6	E/W Indicator	E	Longitude hemisphere indicator
			'E' = East
			'W' = West
7	Speed over ground	000.0	Speed over ground in knots (000.0 ~ 999.9)
8	Course over ground	000.0	Course over ground in degrees (000.0 ~ 359.9)
9	UTC Date	030407	UTC date of position fix, ddmmyy format
10	Mode indicator	A	Mode indicator
			'N' = Data not valid
			'A' = Autonomous mode
			'D' = Differential mode
			'E' = Estimated (dead reckoning) mode
			'M' = Manual input mode
			'S' = Simulator mode
11	checksum	61	

# Appendix B: Planned Route data

latitude	longitude	speed (m/s)
39.90633883	-75.3502289	18
39.905649	-75.3461689	18
39.905737	-75.343916	1.5
39.90580233	-75.34348849	1
39.906049	-75.343637	17
39.908387	-75.34632	17
39.910559	-75.349056	1
39.910725	-75.349119	3
39.910279	-75.349238	16
39.90815	-75.349804	17
39.906848	-75.350107	17
39.906263	-75.350214	15

Appendix C: Sample Data Sent From GPS Receiver to MySql

UTC Time	Time	latitude	longitude
(hmmss.ss)	Difference	20.00042604	75.2402
15606.504	1	39.90613684	-75.3492
15607.504	1	39.906136	-75.3492
15608.504	1	39.90613516	-75.3492
15608.504	0	39.90613516	-75.3492
15608.504	0	39.90613516	-75.3492
15608.504	0	39.90613516	-75.3492
15613.504	5	39.9061325	-75.3492
15614.504	1	39.90613184	-75.3492
15615.504	1	39.90613166	-75.3492
15616.504	1	39.90613033	-75.3492
15617.504	1	39.90612983	-75.3492
15618.504	1	39.90612817	-75.3492
15619.504	1	39.90611816	-75.3492
15620.504	1	39.90610567	-75.3491
15621.504	1	39.90608633	-75.349
15622.504	1	39.90607017	-75.3489
15623.504	1	39.906055	-75.3488
15624.504	1	39.90603167	-75.3486
15625.504	1	39.90600883	-75.3485
15626.504	1	39.90599	-75.3484
15627.504	1	39.90596883	-75.3483
15628.504	1	39.90594866	-75.3482
15629.504	1	39.90592867	-75.348
15630.504	1	39.90591116	-75.3479
15631.504	1	39.90589317	-75.3478
15632.504	1	39.905876	-75.3477
15633.504	1	39.90585733	-75.3476
15634.504	1	39.90583884	-75.3475
15635.504	1	39.90582233	-75.3474
15636.504	1	39.905803	-75.3472
15637.504	1	39.90578233	-75.3471
15638.504	1	39.9057605	-75.347
15639.504	1	39.90574333	-75.3469
15639.504	0	39.90574333	-75.3469
15639.504	0	39.90574333	-75.3469
15643.504	4	39.90566684	-75.3464
15644.504	1	39.90565567	-75.3463
15645.504	1	39.90565066	-75.3462
15646.504	1	39.90564883	-75.3462

15647.504	1	39.9056475	-75.3461
15648.504	1	39.90564734	-75.3461
15649.504	1	39.9056495	-75.3461
15650.504	1	39.90565267	-75.3461
15651.504	1	39.9056575	-75.346
15652.504	1	39.90566317	-75.346
15653.504	1	39.90566633	-75.3459
15654.504	1	39.90567083	-75.3458
15655.504	1	39.90567284	-75.3457
15656.504	1	39.905677	-75.3456
15657.504	1	39.90568234	-75.3455
15658.504	1	39.90568867	-75.3454
15659.504	1	39.90569267	-75.3452
15700.504	1	39.90569717	-75.3451
15701.504	1	39.9057025	-75.345
15702.504	1	39.90570667	-75.3448
15703.504	1	39.90570967	-75.3447
15704.504	1	39.905713	-75.3446
15705.504	1	39.90571384	-75.3444
15706.504	1	39.90571533	-75.3443
15707.504	1	39.90571733	-75.3442
15708.504	1	39.90571917	-75.3441
15708.504	0	39.90571917	-75.3441
15708.504	0	39.90571917	-75.3441
15712.504	4	39.90573334	-75.3437
15713.504	1	39.90573733	-75.3436
15714.504	1	39.905742	-75.3436
15715.504	1	39.90574833	-75.3435
15716.504	1	39.90575783	-75.3435
15717.504	1	39.9057635	-75.3435
15718.504	1	39.90576816	-75.3435
15719.504	1	39.90577584	-75.3435
15720.504	1	39.90579917	-75.3435
15721.504	1	39.9058365	-75.3435
15722.504	1	39.9058825	-75.3435
15723.504	1	39.90593533	-75.3435
15724.504	1	39.90599867	-75.3436
15725.504	1	39.90606683	-75.3437
15726.504	1	39.90613516	-75.3438
15727.504	1	39.9062025	-75.3438
15728.504	1	39.9062705	-75.3439
15729.504	1	39.90633233	-75.344
15730.504	1	39.90639433	-75.344

15731.504	1	39.90645616	-75.3441
15732.504	1	39.90651384	-75.3442
15733.504	1	39.90657366	-75.3443
15734.504	1	39.906635	-75.3443
15735.504	1	39.90669516	-75.3444
15736.504	1	39.90674683	-75.3445
15737.504	1	39.90678583	-75.3445
15737.504	0	39.90678583	-75.3445
15737.504	0	39.90678583	-75.3445
15737.504	0	39.90678583	-75.3445
15742.504	5	39.90688017	-75.3447
15743.504	1	39.90691783	-75.3447
15744.504	1	39.90696684	-75.3448
15745.504	1	39.9070185	-75.3448
15746.504	1	39.90707983	-75.3449
15747.504	1	39.90714484	-75.345
15748.504	1	39.9072155	-75.345
15749.504	1	39.90728516	-75.3451
15750.504	1	39.90735084	-75.3452
15751.504	1	39.90741433	-75.3453
15752.504	1	39.9074765	-75.3454
15753.504	1	39.90754216	-75.3454
15754.504	1	39.907614	-75.3455
15755.504	1	39.90768767	-75.3456
15756.504	1	39.90776167	-75.3457
15757.504	1	39.90783467	-75.3458
15758.504	1	39.90791434	-75.3459
15759.504	1	39.907997	-75.3459
15800.504	1	39.908077	-75.346
15801.504	1	39.9081545	-75.3461
15802.504	1	39.90823233	-75.3462
15803.504	1	39.90831117	-75.3463
15804.504	1	39.90838817	-75.3464
15805.504	1	39.908462	-75.3465
15806.504	1	39.90853433	-75.3466
15807.504	1	39.908609	-75.3467
15807.504	0	39.908609	-75.3467
15807.504	0	39.908609	-75.3467
15811.504	4	39.9089045	-75.347
15812.504	1	39.90897033	-75.3471
15813.504	1	39.90903333	-75.3471
15814.504	1	39.90910133	-75.3472
15815.504	1	39.9091715	-75.3473

15816.504	1	39.90923567	-75.3474
15817.504	1	39.90930033	-75.3475
15818.504	1	39.90935733	-75.3475
15819.504	1	39.90940734	-75.3476
15820.504	1	39.909442	-75.3476
15821.504	1	39.90946366	-75.3477
15822.504	1	39.90947167	-75.3477
15823.504	1	39.90947716	-75.3477
15824.504	1	39.90948683	-75.3477
15825.504	1	39.9095035	-75.3477
15826.504	1	39.90953217	-75.3477
15827.504	1	39.90957434	-75.3478
15828.504	1	39.90962884	-75.3478
15829.504	1	39.909686	-75.3479
15830.504	1	39.90974917	-75.348
15831.504	1	39.90981417	-75.348
15832.504	1	39.90987767	-75.3481
15833.504	1	39.9099485	-75.3482
15834.504	1	39.91001367	-75.3483
15835.504	1	39.91008	-75.3483
15836.504	1	39.91014483	-75.3484
15836.504	0	39.91014483	-75.3484
15836.504	0	39.91014483	-75.3484
15836.504	0	39.91014483	-75.3484
15841.504	5	39.91049817	-75.3488
15842.504	1	39.91056283	-75.3489
15843.504	1	39.91061784	-75.349
15844.504	1	39.910656	-75.349
15845.504	1	39.91066917	-75.349
15846.504	1	39.91067117	-75.3491
15847.504	1	39.91065767	-75.3491
15848.504	1	39.91062667	-75.3492
15849.504	1	39.91057567	-75.3492
15850.504	1	39.910507	-75.3492
15851.504	1	39.91042433	-75.3492
15852.504	1	39.91033	-75.3493
15853.504	1	39.9102305	-75.3493
15854.504	1	39.91013417	-75.3493
15855.504	1	39.91003583	-75.3493
15856.504	1	39.9099325	-75.3494
15857.504	1	39.909821	-75.3494
15858.504	1	39.90971417	-75.3494
15859.504	1	39.90961017	-75.3495

15900.504	1	39.90951317	-75.3495
15901.504	1	39.90941783	-75.3495
15902.504	1	39.90932283	-75.3495
15903.504	1	39.909223	-75.3495
15904.504	1	39.90912734	-75.3496
15905.504	1	39.90902866	-75.3496
15906.504	1	39.90892816	-75.3496
15906.504	0	39.90892816	-75.3496
15906.504	0	39.90892816	-75.3496
15910.504	4	39.90848634	-75.3497
15911.504	1	39.90837216	-75.3498
15912.504	1	39.90825433	-75.3498
15913.504	1	39.90813984	-75.3498
15914.504	1	39.9080215	-75.3498
15915.504	1	39.90790316	-75.3499
15916.504	1	39.90777617	-75.3499
15917.504	1	39.9076475	-75.3499
15918.504	1	39.90751766	-75.35
15919.504	1	39.90738566	-75.35
15920.504	1	39.90725366	-75.35
15921.504	1	39.90711967	-75.3501
15922.504	1	39.90698867	-75.3501
15923.504	1	39.90686333	-75.3501
15924.504	1	39.906746	-75.3501
15925.504	1	39.90663333	-75.3502
15926.504	1	39.906528	-75.3502
15927.504	1	39.9064385	-75.3502
15928.504	1	39.90636983	-75.3502
15929.504	1	39.90631034	-75.3502
15930.504	1	39.90626983	-75.3502
15931.504	1	39.90624	-75.3501
15932.504	1	39.90621817	-75.3501
15933.504	1	39.90620484	-75.35
15934.504	1	39.90619316	-75.3499
15935.504	1	39.9061865	-75.3498
15935.504	0	39.9061865	-75.3498
15935.504	0	39.9061865	-75.3498
15939.504	4	39.9061245	-75.3495
15940.504	1	39.90611384	-75.3494
15941.504	1	39.9061075	-75.3493
15942.504	1	39.90610433	-75.3493
15943.504	1	39.90610417	-75.3492
15944.504	1	39.90610284	-75.3492

15945.504	1	39.9061025	-75.3492

# Appendix D: Position Estimate Data From Extended Kalman Filter

39.90613       -75.3491         39.90612       -75.3491         39.90612       -75.349         39.90611       -75.3489         39.90609       -75.3488         39.90606       -75.3487         39.90606       -75.3486         39.90602       -75.3484         39.90596       -75.3481         39.90598       -75.3481         39.90599       -75.3478         39.90590       -75.3478         39.90591       -75.3478         39.90592       -75.3478         39.90584       -75.3475         39.90585       -75.3473         39.90586       -75.3473         39.90587       -75.3469         39.90579       -75.3468         39.90575       -75.3466         39.90567       -75.3461         39.90565       -75.3461         39.90565       -75.3461         39.90566       -75.3459         39.90566       -75.3459         39.90566       -75.3458         39.90567       -75.3458         39.90566       -75.3458         39.90567       -75.3458         39.90566       -75.3458         39.905	latitude	longitude
39.90612       -75.3491         39.90611       -75.349         39.90609       -75.3488         39.90608       -75.3487         39.90606       -75.3486         39.90602       -75.3484         39.90598       -75.3482         39.90596       -75.3481         39.90597       -75.3481         39.90598       -75.3478         39.90599       -75.3477         39.90580       -75.3476         39.90581       -75.3474         39.90583       -75.3472         39.90584       -75.3472         39.90579       -75.3469         39.90570       -75.3468         39.90572       -75.3466         39.90565       -75.3461         39.90565       -75.3461         39.90566       -75.3461         39.90566       -75.3469         39.90566       -75.3459         39.90566       -75.3459         39.90566       -75.3458         39.90566       -75.3458         39.90567       -75.3458         39.90566       -75.3458         39.90567       -75.3458         39.90566       -75.3458         39.905	39.90613	-75.3491
39.90612       -75.349         39.90611       -75.3489         39.90608       -75.3488         39.90606       -75.3487         39.90604       -75.3486         39.90602       -75.3484         39.90596       -75.3483         39.90598       -75.3481         39.90594       -75.348         39.90595       -75.3478         39.90596       -75.3478         39.90597       -75.3477         39.90588       -75.3476         39.90584       -75.3473         39.90585       -75.3473         39.90586       -75.3472         39.90579       -75.3463         39.90576       -75.3468         39.90577       -75.3466         39.90569       -75.3461         39.90565       -75.3461         39.90565       -75.3461         39.90566       -75.3459         39.90566       -75.3459         39.90566       -75.3458         39.90566       -75.3458         39.90567       -75.3458         39.90566       -75.3458         39.90567       -75.3458         39.90566       -75.3458         39.9056	39.90613	-75.3491
39.90611       -75.349         39.90609       -75.3488         39.90606       -75.3487         39.90604       -75.3486         39.90602       -75.3484         39.90598       -75.3483         39.90596       -75.3481         39.90597       -75.3481         39.90598       -75.3478         39.90599       -75.3477         39.90580       -75.3476         39.90581       -75.3474         39.90583       -75.3472         39.90584       -75.3472         39.90579       -75.3469         39.90575       -75.3468         39.90576       -75.3466         39.90567       -75.3461         39.90565       -75.3461         39.90566       -75.3461         39.90566       -75.3459         39.90566       -75.3459         39.90566       -75.3458         39.90566       -75.3458         39.90567       -75.3458         39.90566       -75.3458         39.90567       -75.3458         39.90566       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.905	39.90612	-75.3491
39.90609       -75.3489         39.90608       -75.3487         39.90604       -75.3486         39.90602       -75.3484         39.90602       -75.3483         39.90598       -75.3481         39.90599       -75.3481         39.90594       -75.3478         39.90595       -75.3477         39.90586       -75.3476         39.90586       -75.3475         39.90584       -75.3473         39.90585       -75.3473         39.90576       -75.3472         39.90576       -75.3468         39.90575       -75.3468         39.90569       -75.3464         39.90565       -75.3461         39.90565       -75.3461         39.90566       -75.3459         39.90566       -75.3459         39.90566       -75.3458         39.90566       -75.3458         39.90567       -75.3458         39.90566       -75.3458         39.90567       -75.3458         39.90566       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90	39.90612	-75.349
39.90608       -75.3488         39.90606       -75.3486         39.90602       -75.3484         39.90602       -75.3483         39.90598       -75.3481         39.90596       -75.3481         39.90597       -75.3478         39.90598       -75.3477         39.90589       -75.3477         39.90580       -75.3475         39.90581       -75.3472         39.90579       -75.3472         39.90576       -75.3468         39.90575       -75.3468         39.90576       -75.3466         39.90567       -75.3461         39.90565       -75.3461         39.90566       -75.346         39.90566       -75.3459         39.90566       -75.3459         39.90567       -75.3458         39.90566       -75.3458         39.90566       -75.3458         39.90567       -75.3458         39.90566       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458	39.90611	-75.349
39.90606       -75.3487         39.90604       -75.3484         39.90602       -75.3484         39.90596       -75.3481         39.90596       -75.3481         39.90597       -75.3478         39.90598       -75.3478         39.90599       -75.3477         39.90580       -75.3475         39.90584       -75.3474         39.90583       -75.3473         39.90579       -75.3473         39.90576       -75.3469         39.90575       -75.3468         39.90569       -75.3464         39.90567       -75.3463         39.90565       -75.3461         39.90565       -75.3461         39.90566       -75.3459         39.90566       -75.3459         39.90566       -75.3458         39.90567       -75.3458         39.90566       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458	39.90609	-75.3489
39.90604       -75.3486         39.90602       -75.3484         39.90598       -75.3482         39.90596       -75.3481         39.90594       -75.348         39.90592       -75.3477         39.90588       -75.3476         39.90586       -75.3475         39.90587       -75.3473         39.90588       -75.3473         39.90581       -75.3472         39.90579       -75.3469         39.90575       -75.3468         39.90572       -75.3466         39.90567       -75.3463         39.90565       -75.3461         39.90565       -75.3461         39.90566       -75.346         39.90566       -75.3459         39.90566       -75.3459         39.90566       -75.3458         39.90567       -75.3458         39.90566       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458	39.90608	-75.3488
39.90602       -75.3484         39.906       -75.3483         39.90598       -75.3481         39.90594       -75.348         39.90592       -75.3478         39.90599       -75.3477         39.90588       -75.3476         39.90586       -75.3475         39.90587       -75.3474         39.90581       -75.3472         39.90579       -75.3469         39.90575       -75.3468         39.90567       -75.3464         39.90567       -75.3463         39.90565       -75.3461         39.90565       -75.3461         39.90566       -75.346         39.90566       -75.3459         39.90566       -75.3459         39.90566       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458	39.90606	-75.3487
39.906       -75.3483         39.90598       -75.3481         39.90594       -75.3481         39.90592       -75.3478         39.90599       -75.3477         39.90588       -75.3476         39.90586       -75.3475         39.90584       -75.3474         39.90581       -75.3472         39.90579       -75.3469         39.90575       -75.3468         39.90572       -75.3466         39.90567       -75.3463         39.90565       -75.3461         39.90565       -75.3461         39.90566       -75.346         39.90566       -75.3459         39.90566       -75.3459         39.90566       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458	39.90604	-75.3486
39.90598       -75.3482         39.90596       -75.3481         39.90592       -75.3478         39.90599       -75.3477         39.90588       -75.3476         39.90586       -75.3475         39.90584       -75.3474         39.90583       -75.3472         39.90579       -75.3469         39.90575       -75.3468         39.90572       -75.3463         39.90565       -75.3461         39.90565       -75.3461         39.90565       -75.346         39.90566       -75.346         39.90566       -75.346         39.90566       -75.3459         39.90566       -75.3459         39.90566       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458	39.90602	-75.3484
39.90596       -75.3481         39.90594       -75.348         39.90592       -75.3478         39.90599       -75.3477         39.90588       -75.3476         39.90586       -75.3475         39.90584       -75.3474         39.90581       -75.3472         39.90579       -75.3469         39.90575       -75.3468         39.90572       -75.3466         39.90567       -75.3463         39.90565       -75.3461         39.90565       -75.3461         39.90565       -75.346         39.90566       -75.3459         39.90566       -75.3459         39.90566       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3458	39.906	-75.3483
39.90594       -75.348         39.90592       -75.3478         39.90599       -75.3477         39.90588       -75.3475         39.90586       -75.3474         39.90581       -75.3472         39.90579       -75.3472         39.90576       -75.3469         39.90575       -75.3468         39.90569       -75.3464         39.90567       -75.3463         39.90565       -75.3461         39.90565       -75.3461         39.90566       -75.346         39.90566       -75.346         39.90566       -75.3459         39.90566       -75.3459         39.90566       -75.3458         39.90567       -75.3458         39.90567       -75.3458         39.90567       -75.3457	39.90598	-75.3482
39.90592       -75.3478         39.9059       -75.3477         39.90586       -75.3475         39.90584       -75.3474         39.90583       -75.3472         39.90579       -75.347         39.90576       -75.3468         39.90572       -75.3464         39.90569       -75.3463         39.90565       -75.3461         39.90565       -75.3461         39.90565       -75.346         39.90565       -75.346         39.90566       -75.346         39.90566       -75.3459         39.90566       -75.3459         39.90566       -75.3458         39.90567       -75.3458         39.90567       -75.3457	39.90596	-75.3481
39.9059       -75.3477         39.90588       -75.3476         39.90586       -75.3475         39.90584       -75.3474         39.90581       -75.3472         39.90579       -75.3469         39.90575       -75.3468         39.90572       -75.3463         39.90567       -75.3463         39.90565       -75.3461         39.90565       -75.3461         39.90565       -75.346         39.90566       -75.346         39.90566       -75.345         39.90566       -75.3459         39.90566       -75.3458         39.90567       -75.3458         39.90567       -75.3457	39.90594	-75.348
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39.90586       -75.3475         39.90584       -75.3474         39.90581       -75.3472         39.90579       -75.347         39.90576       -75.3468         39.90575       -75.3466         39.90569       -75.3463         39.90565       -75.3461         39.90565       -75.3461         39.90565       -75.346         39.90565       -75.346         39.90566       -75.346         39.90566       -75.3459         39.90566       -75.3459         39.90566       -75.3458         39.90567       -75.3457	39.9059	-75.3477
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39.90575       -75.3468         39.90572       -75.3466         39.90569       -75.3464         39.90567       -75.3463         39.90565       -75.3461         39.90565       -75.3461         39.90565       -75.346         39.90566       -75.3459         39.90566       -75.3459         39.90566       -75.3458         39.90567       -75.3458         39.90567       -75.3457	39.90579	-75.347
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39.90567 -75.3463 39.90565 -75.3462 39.90565 -75.3461 39.90565 -75.3461 39.90566 -75.346 39.90566 -75.3459 39.90566 -75.3459 39.90566 -75.3459 39.90566 -75.3458 39.90567 -75.3458	39.90572	-75.3466
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39.90565 -75.346 39.90566 -75.3459 39.90566 -75.3459 39.90566 -75.3459 39.90566 -75.3458 39.90567 -75.3458 39.90567 -75.3457	39.90565	-75.3461
39.90566 -75.346 39.90566 -75.3459 39.90566 -75.3459 39.90566 -75.3459 39.90566 -75.3458 39.90567 -75.3458 39.90567 -75.3457	39.90565	-75.3461
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39.9063	-75.3439
39.90637	-75.344
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39.90656	-75.3442
39.90662	-75.3443
39.90669	-75.3444
39.90675	-75.3445
39.9068	-75.3445
39.90685	-75.3446
39.90697	-75.3447
39.90709	-75.3448

39.9072	-75.345
39.90699	-75.3447
39.90701	-75.3448
39.90704	-75.3448
39.90709	-75.3449
39.90714	-75.3449
39.9072	-75.345
39.90727	-75.3451
39.90734	-75.3451
39.90741	-75.3452
39.90747	-75.3453
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39.90672	-75.3501
39.9066	-75.3502
39.90649	-75.3502
39.90639	-75.3502
39.90632	-75.3502

#### Appendix E: List of Worked Cited

- "2010 Public Transportation Fact Book." <u>American Public Transportation Association</u>. Accessed 6 July 2010
  - <a href="http://apta.com/resources/statistics/Documents/FactBook/APTA\_2010\_Fact\_Book.pdf">http://apta.com/resources/statistics/Documents/FactBook/APTA\_2010\_Fact\_Book.pdf</a>
- "Benefits." Research and Innovative Technology Administration. Accessed 7 July 7, 2010 < <a href="http://www.benefitcost.its.dot.gov/its/benecost.nsf/0/37DF0E8A110B7E898525733A006">http://www.benefitcost.its.dot.gov/its/benecost.nsf/0/37DF0E8A110B7E898525733A006</a> D580C?OpenDocument&Query=Home>.
- Borre, Kai, et al. *A Software-Defined GPS and Galileo Receiver: A Single-Frequency Approach*. New York: Birkhauser Boston, 2007. Print.
- "GPS: The New Navigation." *Public Broadcasting Station*. Web. Accessed 6 May 2011. <a href="http://www.pbs.org/wgbh/nova/longitude/gps/gps03.html">http://www.pbs.org/wgbh/nova/longitude/gps/gps03.html</a> >.
- Kaplan, Elliott, and Christopher Hegarty, ed. *Understanding GPS: Principles and Applications Second Edition*. Norwood: Artech House, Inc., 2006. Print.
- Sanders, Louis. APEC GIT/14 Conference. American Public Transportation Association. Renaissance Hotel, Seattle WA. 21 June 2010. GPS and GIS in Public Transport.
- "The GPS System." *Kowoma.de*. Web. Accessed 6 May 2011. < <a href="http://www.kowoma.de/en/gps/errors.htm">http://www.kowoma.de/en/gps/errors.htm</a> >.
- Welch, Greg and Gary Bishop. "An Introduction to the Kalman Filter." *University of North Caronia at Chapel Hill*. July 24, 2006. Web. Accessed 6 May 2011. < <a href="http://74.125.155.132/scholar?q=cache:rVwQujgtvesJ:scholar.google.com/&hl=en&as\_s">http://74.125.155.132/scholar?q=cache:rVwQujgtvesJ:scholar.google.com/&hl=en&as\_s</a> dt=0,39

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