

# Using Multiple Imputation, Survival Analysis, And Propensity Score Analysis In Cancer Data With Missingness

Master's Thesis

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# In an Ideal World

When exploring the effect of something

- We would have a large dataset that...
  - Was obtained from a Randomized Controlled Trial (RCT)
  - Would help answer a clearly defined question
  - Had all the covariates of scientific interest
  - Contained no missing data
- But this almost never happens!

# YES!

- Without an RCT, we can't be sure if differences in outcome is due to the treatment or something else
- Omitting important factors may bias our results
- With missing data, we will be throwing away data and biasing our results

# The Solution

This thesis aims to fix these problems by...

- Filling in missing data via multiple imputation
- Creating meaningful analytical models via survival analysis
- Getting a causal interpretation from observational data via propensity scores

Goal: To be able to apply methods to cancer data

# Plan For This Presentation



Data obtained from  
MDACC

Multiple  
Imputation:  
m=50  
40 iterations

Survival Analysis:  
Kaplan-Meier  
Log-Rank test  
Cox model

Causal Analysis:  
IPTW weighted Cox  
model

# Data Explanation

- 1514 MD Anderson patients who had brain mets from breast cancer between October 2009 and December 2012
- 1242 usable cases
- 90 covariates
  - Missingness from 0 to 65%

Type	Example
Subject data	Age range, race, date of birth
Breast Cancer data	TNM staging, type, receptor status
Pre brain mets data	Treatment types
Post brain mets clinical observations	Seizures, headache, nausea
Post brain mets data	Treatment type, type of brain mets
Survival data	Survival time after brain mets, censoring indicator

# Questions of Interest

Want to explore...

- 1 Chemotherapeutic drugs: Capecitabine vs other chemotherapeutic agents
- 2 HER2 directed therapies (Lapatinib, Trastuzumab) in HER2+ subjects

Note: treatment not determined at time of diagnosis

- landmark (2 months)

# A Few Important Covariates

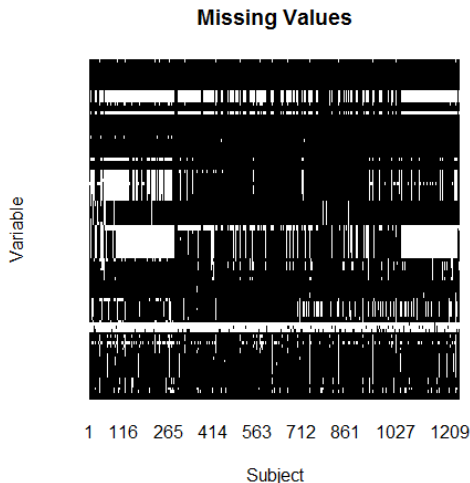
Name	Percent Missing	Meaning
capeothno	18%	Indicator: Capecitabine, other, or no chemotherapeutic treatment. Treatment variable 1
lapatrasno	18%	Indicator: Lapatinib, Trastuzumab, or no HER2 treatment. Treatment variable 2
controlled	12%	Indicator: Extracranial progression of brain mets
hrher2	5%	Categorical variable: The hormonal receptor and HER2 receptor status of the subject
braintype	4%	Categorical: Single, multiple, Leptomenigeal disease
timedx	1%	Indicator: Time (years) from breast cancer diagnosis to brain mets diagnosis greater or less than 6 years
site5	1%	Indicator: First metastasis was to brain
race2	0%	Categorical: White, Black, Hispanic, other
priorn	0%	Indicator: Number of prior treatments in metastatic setting before brain mets
os	0%	Overall survival (months)
dead	0%	Indicator: death indicator
agebrainmet	0%	Indicator: Age greater or less than 60 at time of brain mets

Table 2: Table of important covariates to be used in the analysis

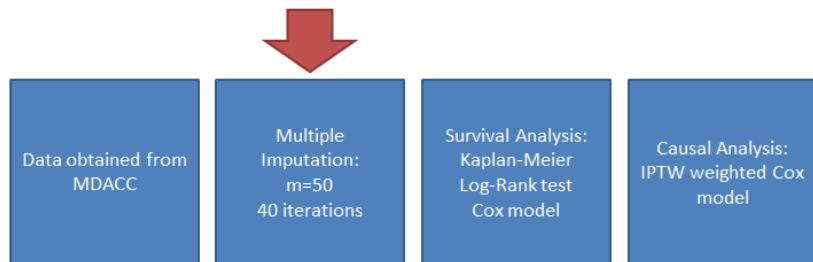


# Visualization of Missingness

Figure 1: Visualization of missingness in the cancer dataset



# Plan For This Presentation



# Missing Data and Historical Approaches

- Missing data happens when we intend to collect a piece of data but don't actually get it
- Historical approaches:
  - Complete Case (CC) analysis: Throw away any record that is not complete
  - Available Case (AC) analysis: Use records so long as they are complete for the specific analysis in question
  - Single Imputation (SI): Fill in the missing value, deduct degrees of freedom to account for it

# Multiple Imputation

- Theorized in the 70's and 80's by Donald Rubin to improve SI
- Instead of imputing one value, impute  $m \geq 2$  values
- Draw the values from the missing data's posterior distribution given the observed data and the process that generated the missing data
- It is the gold standard method for missing data today

# How Does MI work?

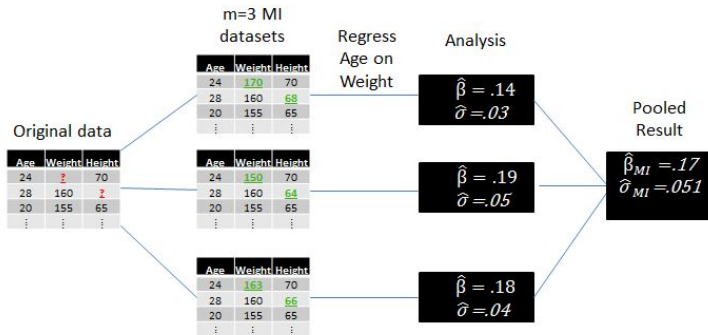


Figure 2: Visualization of MI data

Missingness is displayed by ?'s and the imputed data is shown as #'s. We then regress age on weight, get the results from the individual datasets, and then pool them together.

- Missing data model:  $p(R|Y_{obs}, Y_{mis}, \psi)$ 
  - $R$  is the response missingness indicator
  - $Y_{obs}, Y_{mis}$  are observed and missings of  $Y$  (the set of covariates with missingness)
  - $\psi$  parameterizes the missing data model

# Missing Data Mechanisms

- MCAR: Missing completely at random:

$$P(R = 0 | Y_{obs}, Y_{mis}, \psi) = P(R = 0 | \psi)$$

- The missingness in the data is not at all related to any of the data that we do or don't have
- MAR: Missing at random:

$$p(R = 0 | Y_{obs}, Y_{mis}, \psi) = p(R = 0 | Y_{obs}, \psi)$$

- The missingness we have is related to something in the data
- MNAR: Missing not at random:

$$p(R = 0 | Y_{obs}, Y_{mis}, \psi)$$

does not simplify

- and the missingness depends on data that we have as well as have not collected

# Missing Data Mechanisms Example

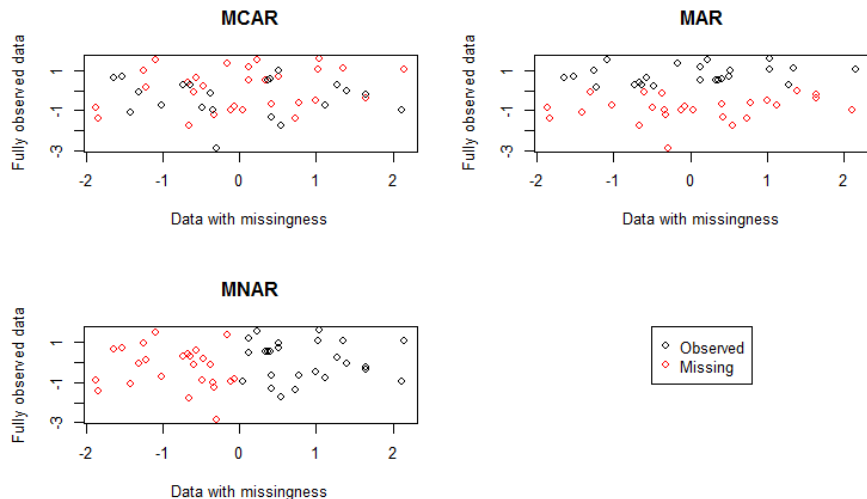


Figure 3: Visualization of Missing Data Mechanisms



# Implementing MI: Full Conditional Specification (FCS)

- Assume MAR missing data mechanism
- Missing data is imputed iteratively on a variable by variable basis
- Drawing from  $p(Y, X, R|\theta)$  through the full conditionals  $p(Y_j|X, Y_{-j}, R, \theta_j)$ 
  - $X$ : Fully observed data
  - $Y_{-j}$  is the missing components without column  $j$
  - $\theta$  parameterizes full data model
- Generalization of univariate imputation
- Idea: Specify  $k$  one dimensional models to impute on the missing data columns

# FCS Algorithm - MICE

- ① Specify an imputation/posterior predictive model  
 $P(Y_j^{mis} | Y_j^{obs}, Y_{-j}, X, R)$  for variable  $Y_j$  with  $j = 1, \dots, p$
- ② For each  $j$ , fill in starting imputations  $\dot{Y}_j^0$  by random draws from  $Y_j^{obs}$
- ③ Repeat for  $t = 1, \dots, T$  (number of iterations):
- ④ Repeat for  $j = 1, \dots, p$  (number of covariates with missingness):
  - Define  $\dot{Y}_{-j}^t = (\dot{Y}_1^t, \dots, \dot{Y}_{j-1}^t, \dot{Y}_{j+1}^{t-1}, \dots, \dot{Y}_p^{t-1})$  as the currently complete data except  $Y_j$
  - Draw  $\dot{\phi}_j^t \sim P(\phi_j^t | Y_{obs}, \dot{Y}_{-j}^t, R)$
  - Draw imputation  $\dot{Y}_j^t \sim P(Y_j^{mis} | Y_j^{obs}, \dot{Y}_{-j}^t, R, X, \dot{\phi}_j^t)$
- ⑤ End repeat  $j$
- ⑥ End repeat  $t$

FCS imputation pseudocode, taken from [2]

## Pros

- Flexible
- Easy to specify models
- Handles mixed continuous categorical data
- Yields unbiased estimates with appropriate coverage

## Cons

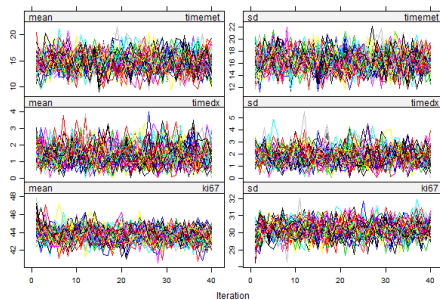
- No guarantee that full conditionals are compatible
- Takes time to set up
- Gets much harder as sample size increases to specify models

# Imputation with the Cancer Data

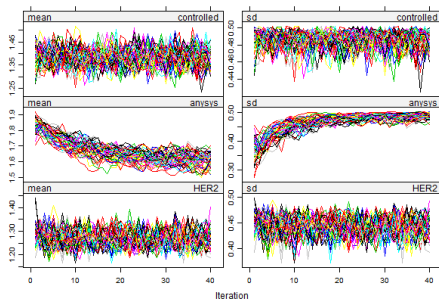
- MAR assumption seems reasonable
- $m = 50$  datasets
- 40 iterations

# Convergence

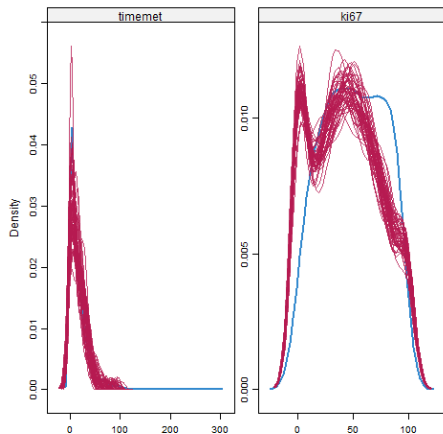
Traceplots, continuous



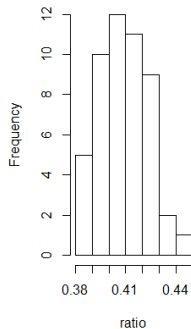
Traceplots, binary



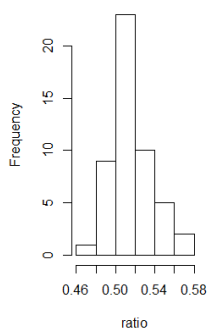
# Validity Checks



**Ratio of Cape/Other**  
**AC ratio = 0.415**



**Ratio of lapat/trastuz**  
**AC ratio = 0.561**



# MI Data Breakdown

	Sys. therapy AC	Sys. therapy MI	No Sys. therapy AC	No Sys. therapy MI
Age (mean,sd)	51.4(10.8)	51.2(10.9)	52.7(11.9)	52.9(11.4)
Breast Cancer subtype				
HR+/HER2-	27%	31%	28%	33%
HR+/HER2+	19%	18%	12%	13%
HR-/HER2+	22%	20%	15%	12%
Triple negative	32%	32%	45%	42%
Prior therapies for stage 4	1(0-3)	2(0-4)	2(0-4)	2(0-4)
Single brain lesion	25%	23%	23%	20%
Controlled extra-cranial	40%	40%	35%	36%
ECOG 0-1	84%	70%	53%	40%
Local Therapy				
Resection Alone	5%	5%	9%	7%
SBRT alone	13%	12%	9%	8%
WBRT	60%	59%	52%	53%
Resection/SBRT+WBRT	12%	14%	10%	8%
no local therapy	10%	10%	20%	23%

Table 3: Characteristics of available case data versus MI data

# Rubin's Rules

Let

- $\hat{Q}_i$  be the scientific estimand from the  $i^{th}$  MI dataset
- $U_i$  be the variance-covariance matrix of the  $i^{th}$  MI estimand

Then

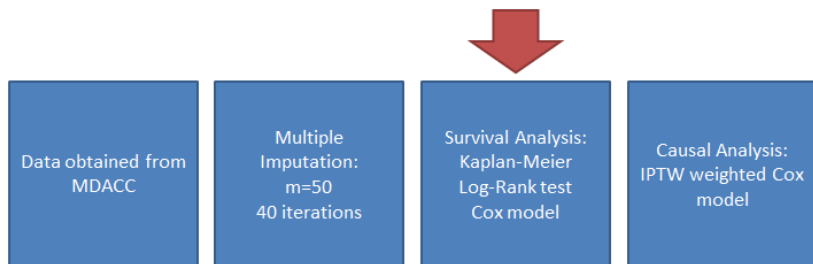
- The MI estimate is given by  $\bar{Q} = \frac{1}{m} \sum_{i=1}^m \hat{Q}_i$
- The MI “within” variance is given by  $\bar{U} = \frac{1}{m} \sum_{i=1}^m U_i$
- the MI “between” variance is given by
$$B = \frac{1}{m-1} \sum_{i=1}^m (\hat{Q}_i - \bar{Q})(\hat{Q}_i - \bar{Q})'$$
- Total variance given by [1]

$$T = \bar{U} + B + \frac{B}{m}$$

- Theory developed under normality assumption



# Plan For This Presentation



## Survival Analysis

Survival analysis is a field of statistics concerned with analyzing time to event data, often in the face of censoring or truncation.

Example:

- The survival of patients after brain mets from breast cancer
- Censoring/Truncation:
  - study ending and no death
  - subject dies before study starts
  - subject moves away and can't contact them
  - exact death time only known in an interval

# Kaplan-Meier Estimator

- The survival function  $S(t) = P(T > t) = \int_t^\infty f(u)du$  is estimated by the nonparametric Kaplan-Meier Estimator

$$\hat{S}(t) = \prod_{t_i < t} \frac{n_i - d_i}{n_i}$$

- $n_i$  is the number of subject in the risk set at time  $t_i$
- $d_i$  is the number of deaths at time  $t_i$

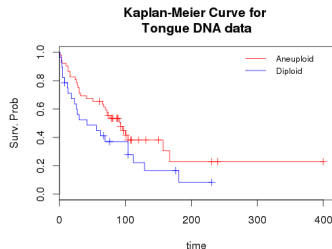


Figure 4: Tongue Cancer data from [3]

# Kaplan-Meier in the MI Setting

- Ensure that we have non-informative censoring
- Algorithm: Pool the complimentary log-log of the Kaplan-Meier curve via Rubin's Rules at each unique event time, get estimates, back transform [4]

# Median Survival Time

- Want a measure of central tendency
  - Survival distributions often skewed, so mean is poor choice
- Median: smallest time such that  $\hat{S}(t) \leq .5$
- Algorithm: Take ML Kaplan-Meier curve, observe first time it goes below 50%
- Confidence interval at median: the median of the upper and lower confidence bands

# Log Rank test

$H_0$ : No difference between the survival curves of the two populations

$$\frac{\sum_{j=1}^J (O_{1j} - E_{1j})}{\sqrt{\sum_{j=1}^J V_j}} \sim N(0, 1)$$

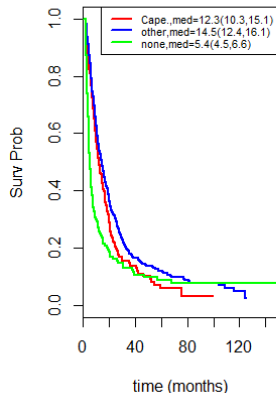
- $N_j = N_{1j} + N_{2j}$  is the number at risk at time  $j$  (composed from deaths in each group)
- $O_j = O_{1j} + O_{2j}$  is the observed number of deaths at time  $j$  (composed from the observed deaths in each group)
- $E_{1j} = \frac{O_j N_{1j}}{N_j}$
- $V_j = \frac{O_j(N_{1j}/N_j)(1-N_{1j}/N_j)(N_j-O_j)}{N_j-1}$

# Log Rank Test in MI Setting

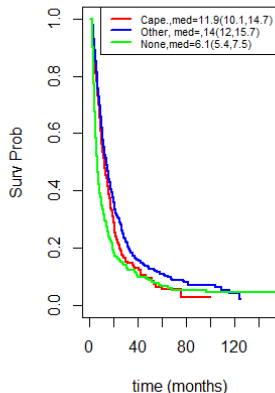
- Under no tied times, the score test on Cox Regression on treatment only is equivalent to the log rank test
  - And very similar under tied times
- Idea: Derive log rank test from Cox regression
  - Pooling LRT and Score test is unstable [4]
  - Wald test is asymptotically equivalent
- Final Solution: Run the Wald test on Cox regression as an approximation

# Chemo KM and Log Rank Test

**Available case OS for chemo  
2 month landmark**



**MI OS for chemo  
2 month landmark**

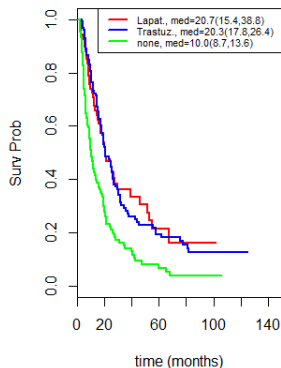


	Chemo	
	AC	MI
cape/other/none	<.0001	<.0001
cape/other	0.0321	0.033
cape/none	0.00039	.0016
other/none	<.0001	<.0001

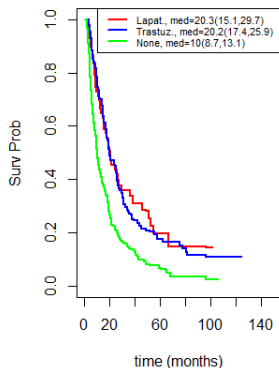


# HER2 Directed KM and Log Rank Test

**AC - OS for HER2 therapy  
2 month landmark**



**MI - OS for HER2 therapy  
2 month landmark**



	HER2	
	AC	MI
Lapat/Trastuz/none	<.0001	<.0001
Lapat/Trastuz	.87	.81
Lapat/none	.00017	.00018
Trastuz/none	<.0001	<.0001

# Cox Regression: Hazard Function

- Hazard is the instantaneous rate of event given that you have survived until time  $t$ , given by

$$h(t) = \lim_{\Delta t \rightarrow 0+} \frac{P[t \leq T < t + \Delta t | T \geq t]}{\Delta t}$$

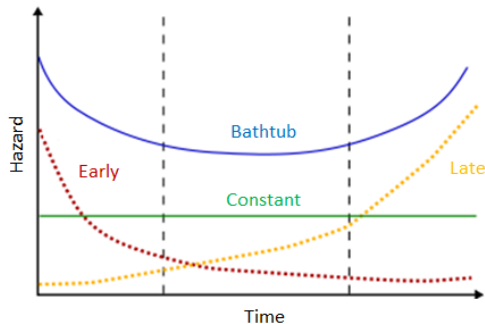


Figure 5: A few different hazard function shapes [5]

# Cox Regression

- Cox regression models hazard by

$$h(t|Z) = \underbrace{h_0(t)}_{\text{time}} * \underbrace{\exp(\sum_{k=1}^p \beta_k Z_k)}_{\text{covariates}}$$

- Where  $h_0(t)$  is the baseline hazard
- $Z_k$  is the  $k^{th}$  covariate
- $\beta_k$ 's are found by maximizing the partial likelihood function
- The covariates act to multiply the hazard function.

Quantity of interest: Hazard ratio  $\frac{h(t|Z)}{h(t|Z^*)} = \exp(\sum_{k=1}^p \beta_k (Z_k - Z_k^*))$

# Cox Regression in the MI Setting

- Goal: To get a “baseline” Cox regression, then add treatment variables
- Need to check for proportional hazards assumption
  - Problem: MI Cox regression doesn't have residuals
  - Solution: Check assumptions (Schoenfeld residuals) on each MI dataset individually
- Cox regression is normally distributed, use Rubin's Rules to pool
- Add treatment covariates, rerun models, pool

# Schoenfeld Residual Splines

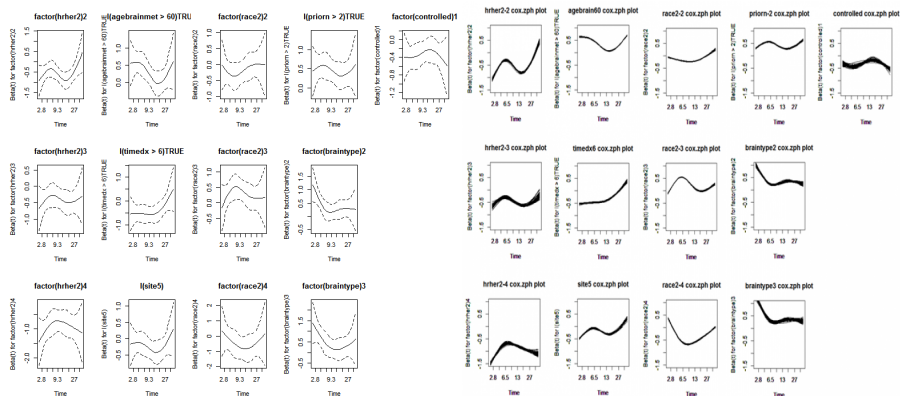


Figure 6: AC and MI Schoenfeld splines

# Base model

			AC n= 845			MI		
Variable	Contrast	HR	95% CI	pvalue		HR	95% CI	pvalue (t test)
HR/HER2	-/+ vs. -/-	0.57	(0.46,0.71)	<0.0001		0.59	(0.48,0.72)	<0.0001
	+/- vs. -/-	0.66	(0.54,0.81)	<0.0001		0.63	(0.52,0.76)	<0.0001
	+/+ vs. -/-	0.4	(0.31,0.50)	<0.0001		0.4	(0.32,0.50)	<0.0001
Age	>60 vs. <60	1.37	(1.13,1.65)	0.0011		1.45	(1.22,1.72)	<0.0001
Dx to BM	>6 vs. <6	0.66	(0.54,0.82)	0.00013		0.71	(0.59,0.86)	0.0002
First DM	Brain vs. Oth	0.8	(0.66,0.97)	0.026		0.83	(0.70,0.99)	0.02
Race	Hisp. Vs. White	0.85	(0.68,1.07)	0.17		0.88	(0.71,1.08)	0.11
	Black vs. White	1.31	(1.06,1.63)	0.014		1.25	(1.02,1.52)	0.015
	Other vs. White	0.65	(0.40,1.04)	0.075		0.7	(0.45,1.07)	0.05
# prior Rx	>2 vs. 0-2	1.58	(1.31,1.91)	<0.0001		1.53	(1.29,1.82)	<0.0001
BM type	Mult. Vs. Single	1.45	(1.20,1.76)	<0.0001		1.48	(1.24,1.76)	<0.0001
	LMD vs. Single	1.6	(1.21,2.13)	0.001		1.58	(1.25,2.00)	<0.0001
Sys. Cont.	Yes vs. No	0.71	(0.61,0.83)	<0.0001		0.73	(0.63,0.85)	<0.0001

Table 4: AC and MI baseline Cox regression

# MI Cox Regression, Chemo

			AC n= 745			MI		
Variable	Contrast	HR	95% CI	p-value		HR	95% CI	p-value (t test)
HR/HER2	-/+ vs. -/-	0.62	(0.49,0.79)	<.0001		0.63	(0.51,0.77)	<.0001
	+/- vs. -/-	0.65	(0.53,0.81)	0.00011		0.64	(0.53,0.78)	<.0001
	+/+ vs. -/-	0.41	(0.31,0.53)	<.0001		0.42	(0.34,0.53)	<.0001
Age	>60 vs. <60	1.34	(1.10,1.64)	0.0041		1.44	(1.21,1.72)	<.0001
Dx to BM	>6 vs. <6	0.72	(0.58,0.90)	0.0032		0.71	(0.58,0.86)	0.00039
First DM	Brain vs. Oth	0.77	(0.63,0.95)	0.014		0.81	(0.68,0.96)	0.016
Race	Hisp. Vs. White	0.77	(0.61,0.98)	0.034		0.86	(0.69,1.06)	0.15
	Black vs. White	1.29	(1.02,1.63)	0.032		1.23	(1.01,1.51)	0.043
	Other vs. White	0.76	(0.47,1.25)	0.28		0.7	(0.45,1.08)	0.11
# prior Rx	>2 vs. 0-2	1.61	(1.32,1.98)	<.0001		1.53	(1.28,1.82)	<.0001
BM type	Mult. Vs. Single	1.46	(1.20,1.78)	0.00017		1.51	(1.27,1.81)	<.0001
	LMD vs. Single	1.45	(1.04,2.03)	0.029		1.41	(1.11,1.80)	0.0049
Sys. Cont.	Yes vs. No	0.57	(0.48,0.68)	<.0001		0.69	(0.59,0.80)	<.0001
Chemo	Cape. vs. none	0.69	(0.53,0.89)	0.0046		0.75	(0.60,0.95)	0.018
	other vs. none	0.52	(0.42,0.65)	<.0001		0.58	(0.47,0.71)	<.0001

Table 5: AC and MI Cox regression with Chemo Treatment

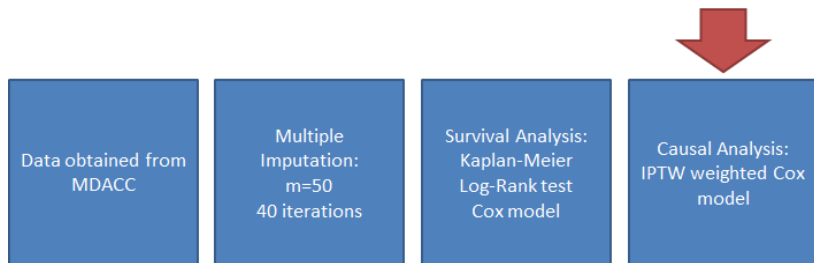
# MI Cox Regression, HER2-Directed

			AC n=292			MI n between 391 and 415	
Variable	Contrast	HR	95% CI	p-value	HR	95% CI	p-value (t test)
HR/HER2	+/+ vs. -/+	0.65	(0.49,0.87)	0.0036	0.66	(0.51,0.85)	0.0015
Age	>60 vs. <60	1.38	(0.95,2.01)	0.092	1.58	(1.15,2.18)	0.0054
Dx to BM	>6 vs. <6	0.64	(0.43,0.97)	0.033	0.69	(0.49,0.99)	0.041
First DM	Brain vs. Oth	0.84	(0.58,1.20)	0.34	0.86	(0.62,1.17)	0.34
Race	Hisp. Vs. White	0.69	(0.46,1.02)	0.064	0.76	(0.53,1.09)	0.14
	Black vs. White	1.41	(0.94,2.11)	0.1	1.43	(1.00,2.04)	0.047
	Other vs. White	0.7	(0.32,1.53)	0.38	0.83	(0.46,1.52)	0.55
# prior Rx	>2 vs. 0-2	1.88	(1.34,2.63)	0.00028	1.71	(1.28,2.28)	0.00028
BM type	Mult. Vs. Single	1.3	(0.92,1.86)	0.14	1.25	(0.91,1.70)	0.16
	LMD vs. Single	2.15	(1.20,3.88)	0.011	1.77	(1.10,2.83)	0.018
Sys. Cont.	Yes vs. No	0.73	(0.55,0.97)	0.029	0.78	(0.60,1.01)	0.063
HER2 therapy	Lapat vs. none	0.47	(0.32,0.69)	0.00015	0.52	(0.37,0.75)	0.00036
	Trastuz vs. none	0.45	(0.33,0.61)	<.0001	0.51	(0.38,0.68)	<.0001

Table 6: AC and MI Cox regression with HER2 Treatment



# Plan For This Presentation



Example: Examining weight loss between new weight loss drug or placebo

- We would like to be able to say “The drug leads to more weight loss”
  - But need an RCT to say this
    - Randomization minimizes differences between groups at baseline
  - We only have observational data
  - Thus differences could be attributed to the drug or confounding
    - e.g. healthier people were much more likely to take the drug at baseline

Idea: Try to balance the covariates to reduce the effects of confounding so the two groups seem identical at baseline

# Counterfactuals

- Suppose that for or every person, there are two potential outcomes
  - $Y_i(0)$  - The outcome if they had taken the control,  $T = 0$
  - $Y_i(1)$  - The outcome if they had taken the treatment,  $T = 1$
- The observed value for subject  $i$ :  $Y_i = Y_i(1)T + Y_i(0)(1 - T)$

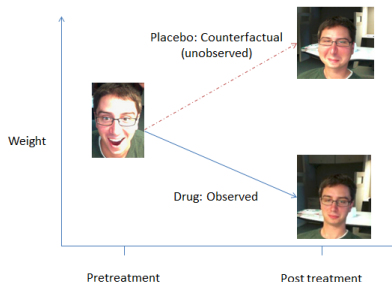


Figure 7: Example of a counterfactual

# Fundamental Problem of Causal Inference

- Obviously we only observe one. *The fundamental problem of causal inference*
- If we could observe both, then we could observe the causal effects for each person

- Stable Unit Treatment Value Assumption (SUTVA): Treatment status of another subject does not affect outcome of other units.
- Ignorability/No Unmeasured Confounders:  $(Y(0), Y(1)) \perp T|X$ , [6]

# Estimands of Interest

- Individual Treatment Effect:  $Y_i(1) - Y_i(0)$
- Average Treatment Effect (ATE):  $E[Y(1) - Y(0)]$ . The effect of moving entire population from treated to untreated
- Average treatment effect for the treated (ATT):  $E[Y(1) - Y(0) | T = 1]$ . The average treatment effect for those actually treated
- Note:  $E[Y(1) | T = 1] \neq E[Y(1)]$ , because  $E[Y | T = 1] = E[Y_1 T + Y_0(1 - T) | T = 1] = E[Y_1 | T = 1] \neq E[Y(1)]$
- If assumptions hold, ATE is unbiased estimator of true treatment effect

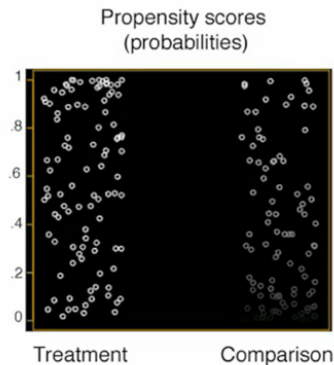
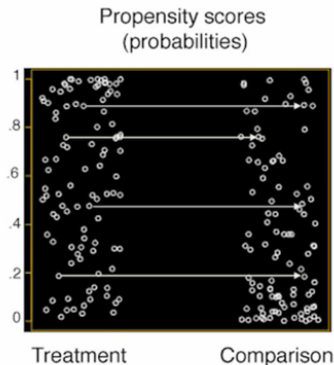
## Definition

The propensity score is the probability that the subject received the treatment given the subjects *pretreatment* covariates. It is computed using the patient's baseline (pretreatment) information [6]

- Defined as  $e_i(x) = P(T_i = 1|X_i)$
- Assume that the covariates play a role in how the subject chose treatment
- If we assume that  $(Y(0), Y(1)) \perp T|X \implies (Y(0), Y(1)) \perp T|e(X)$ , [6]
- Controlling for propensity score will make groups seem indistinguishable
- Thus, we may treat it as if it were an RCT

# Common Propensity Score Methods

- Matching: Match treatment and controls on their propensity score, calculate ATE
- Weighting: Weight each observation by the inverse of its propensity score, and then calculate ATE





- IPTW: Inverse probability of treatment weights
- Idea: Weight sample by propensity score so that we get a sample where there is no confounding
- Weights:  $1/e(X)$  for treatment,  $1/(1 - e(X))$  for control
- Can be shown that
  - $E[\frac{TY(1)}{e(X)} | T = 1] = E[Y(1)]$
  - $E[\frac{(1-T)Y(0)}{1-e(X)} | T = 0] = E[Y(0)]$

# Propensity Score in MI Setting

- Mitra and Reiter propose two methods [8]
- Within: Work with propensity score on each of the  $m$  MI datasets
- Across: Average propensity scores across the  $m$  datasets and then analyze with the averaged propensities
- Which to use: Dependent on the data

# Propensity Score: Questions To Ask

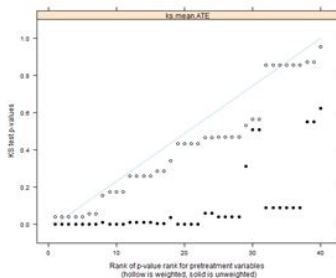
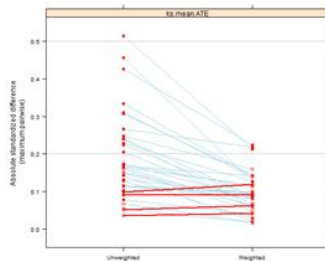
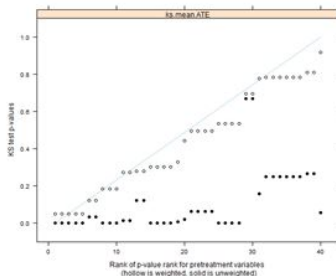
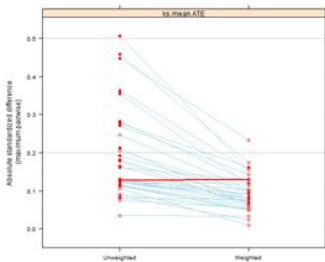
- Generating the propensity score: GBM
  - What confounders to put in propensity score model
  - Stage, race, IDC, breast cancer surgery, HR/HER2 status, breast cancer radiation, first met site, number of prior treatments, ECOG score, localized brain mets treatment, age at brain met, type of brain mets, brain met controlled
- What estimand do we care about? ATE

# Verifying Balance

For each IPTW MI dataset...

- Need the distribution of the groups to be similar
  - Standardized bias:  $|\bar{X}_{k1} - \bar{X}_{k0}|/\hat{\sigma}_k$
  - Kolmogorov-Smirnov (KS) test
- Need to be sure that propensity scores are between 0 and 1

# Balance Checks



# Results of IPTW: Chemotherapeutic

	AC Unweighted		AC IPTW		MI unweighted		MI IPTW	
	HR	95% CI	HR	95% CI	HR	95% CI	HR	95% CI
Cape. vs none	0.396	(0.325,0.482)	0.655	(0.481,0.894)	0.484	(0.400,0.585)	0.702	(0.543,0.906)
Other vs none	0.336	(0.287,0.394)	0.567	(0.46,0.754)	0.413	(0.354,0.481)	0.593	(0.470,0.748)
Cape. vs other	1.179	(0.983,1.416)	1.156	(0.966,1.383)	1.173	(0.981,1.402)	1.183	(0.998,1.404)

**Table 7:** Chemotherapeutic ATE with IPTW weights, AC and MI

	AC Unweighted		AC IPTW		MI unweighted		MI IPTW	
	HR	95% CI	HR	95% CI	HR	95% CI	HR	95% CI
Cape. vs none	0.687	(0.530,0.891)	0.603	(0.443,0.820)	0.752	(0.595,0.952)	0.701	(0.536,0.917)
Other vs none	0.521	(0.416,0.653)	0.452	(0.340,0.602)	0.579	(0.474,0.707)	0.532	(0.416,0.681)
Cape. vs other	1.318	(1.078,1.612)	1.334	(1.109,1.604)	1.300	(1.076,1.570)	1.317	(1.100,1.579)

**Table 8:** Chemotherapeutic ATE, Doubly Robust, AC, MI

# Results of IPTW: HER2 Directed

	AC Unweighted		AC IPTW		MI Unweighted		MI IPTW	
	HR	95% CI	HR	95% CI	HR	95% CI	HR	95% CI
Lapat. vs none	0.467	(0.355,0.616)	0.571	(0.381,0.855)	0.474	(0.362,0.622)	0.485	(0.304,0.775)
Trastuz. vs none	0.488	(0.398,0.597)	0.566	(0.421,0.759)	0.506	(0.417,0.614)	0.480	(0.313,0.735)
Lapat. vs Trastuz.	0.958	(0.693,1.324)	1.009	(0.680,1.496)	0.927	(0.673,1.28)	1.011	(0.763,1.338)

Table 9: HER2 directed ATE with IPTW weights, AC and MI

	AC Unweighted		AC IPTW		MI Unweighted		MI IPTW	
	HR	95% CI	HR	95% CI	HR	95% CI	HR	95% CI
Lapat. vs none	0.468	(0.316,0.692)	0.514	(0.331,0.798)	0.524	(0.367,0.747)	0.410	(0.257,0.652)
Trastuz. vs none	0.447	(0.328,0.6089)	0.456	(0.328,0.632)	0.511	(0.381,0.685)	0.388	(0.249,0.602)
Lapat. vs Trastuz.	1.048	(0.704,1.560)	1.128	(0.726,1.754)	1.026	(0.713,1.477)	1.057	(0.788,1.417)

Table 10: HER2 directed ATE with IPTW weights, double robust

- Applied survival and causal analysis on MI cancer data
- Found overall, all treatments better than none
- Other chemotherapeutics better than Capecitabine
- Lapatinib and Trastuzumab are about the same



- MI skeptics and method critiques
- Use of propensity scores
- Assumptions made throughout

# Further Research and Extensions

- Exploring the “other chemotherapeutics”
- Competing risks
- AFT models
- Differing propensity score methods and instrumental variables
- Estimating counterfactuals as an MI problem in MI setting

# Acknowledgments

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- Dr. Hess - For advising me
- Dr. Ibrahim and Dr. Bugano - For letting me work with their data and advising on project matters
- Margaret Poon - For being there to help out
- My family - For always supporting me
- My friends - For always believing in me and helping critique my thesis

- Discuss in depth topics should time allow

# Propensity Score Issues

- Unmeasured confounders
- Choice of pretreatment covariates in the propensity score model
- Different models and methods may lead to different conclusions

# Joint Modelling (JM)

- Assume ignorable MAR missing data mechanism
- Missing data imputed by sampling from a user specified distribution
- A lot of theory developed for Normal, not much else
  - Normal imputation has been shown to perform well, even under non normality [9]
- Idea: pull imputations by missing data row pattern

1. Sort the rows of  $Y$  into  $S$  missing data patterns  $Y_{[s]}, s = 1, \dots, S$ .
2. Initialize  $\theta^0 = (\mu^0, \Sigma^0)$  by a reasonable starting value.
3. Repeat for  $t = 1, \dots, T$ :
4. Repeat for  $s = 1, \dots, S$ :
5. Calculate parameters  $\dot{\phi}_s = \text{SWP}(\hat{\theta}^{t-1}, s)$  by sweeping the predictors of pattern  $s$  out of  $\hat{\theta}^{t-1}$ .
6. Calculate  $p_s$  as the number missing data in pattern  $s$ . Calculate  $o_s = p - p_s$ .
7. Calculate the Choleski decomposition  $C_s$  of the  $p_s \times p_s$  submatrix of  $\dot{\phi}_s$  corresponding to the missing data in pattern  $s$ .
8. Draw a random vector  $z \sim N(0, 1)$  of length  $p_s$ .
9. Take  $\dot{\beta}_s$  as the  $o_s \times p_s$  submatrix of  $\dot{\phi}_s$  of regression weights.
10. Calculate imputations  $\dot{Y}_{[s]}^t = Y_{[s]}^{\text{obs}} \dot{\beta}_s + C_s' z$ , where  $Y_{[s]}^{\text{obs}}$  is the observed data in pattern  $s$ .
11. End repeat  $s$ .
12. Draw  $\hat{\theta}^t = (\dot{\mu}, \dot{\Sigma})$  from the normal inverted-Wishart distribution according to Schafer (1997, p. 184).
13. End repeat  $t$ .

# JM Pros and Cons

## Pros

- Fast
- Easy to derive posteriors with common distributions

## Cons

- Inflexible
- Limited to known distributions
- How to deal with mixed categorical and continuous missing data
- Poor with derived variables
- Can give impossible combinations



# Inference with Rubin's Rules

- Assume that with complete data, inference on the estimand  $Q$  would be based on the statement  $(Q - \hat{Q}) \sim N(0, U)$ 
  - $\hat{Q}$  is the statistic estimating  $Q$
  - $U$  is the variance-covariance of  $(Q - \hat{Q})$
- Since true  $T$  is not known, then

$$\frac{Q - \hat{Q}}{\sqrt{T}} \sim t_\nu$$

- $\nu$  is given by [10]

$$\nu = \frac{\nu_{old}\nu_{obs}}{\nu_{old} + \nu_{obs}}$$

- Where  $\nu_{obs} = \frac{\nu_{com}+1}{\nu_{com}+3}\nu_{com}(1 - \frac{B+B/m}{T})$
- $\nu_{com}$  is the hypothetical complete sample degrees of freedom
- $\nu_{old} = \frac{m-1}{(\frac{B+B/m}{T})^2}$

# The Stack Method

- Rubin's Rules work well, but not always
  - Ex: partitioning the MI data on an imputed variable
  - Taking the average is not a good idea
- Solution: Stack the MI datasets on top of each other to get one huge dataset
  - Will get unbiased results
  - But sample size is falsely inflated, thus cannot trust variance



# KM issues in the MI setting

- Issue: Kaplan-Meier is not normally distributed
  - Solution: Complimentary log log transformation, pool [4]
- Issue: Imputations leave one KM curve much shorter than the rest
  - Solution 1: Truncate all curves at the lowest time
  - Solution 2: Extend the curves out to the longest time
  - Solution 3: Use the stacked method

# Log Rank Issues in MI setting

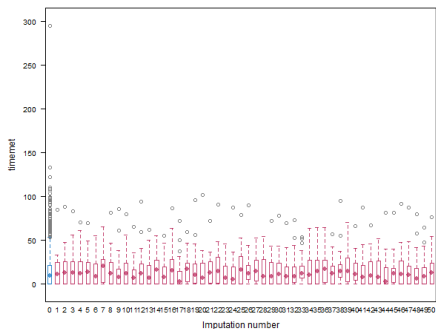
- Idea: Combine log rank tests from each MI dataset
  - Problem: Wastes information and is unstable [4]
  - Idea: Calculate log rank from the MI Kaplan-Meier curve
  - Problem: Risk set and deaths no longer meaningful

# Setting up the model- Issues

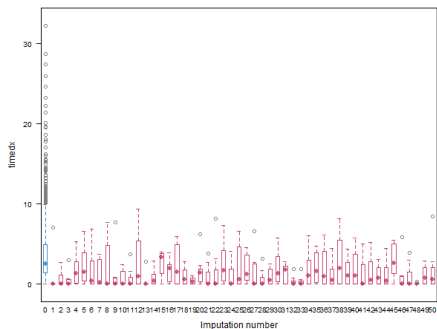
- Many categorical variables
- Collinearity between predictors
- Variables with poor influx/outflux [2]
- How many iterations and imputations to draw?

# Validity Checks

bw plot, timemet by imputation



bw plot, timedx by imputation



# Tabluar Checks

AC	os>10	
hrher2	FALSE	TRUE
-/-	0.246	0.0967
-/+	0.0916	0.0916
+/-	0.1917	0.1154
+/+	0.0712	0.0958
[[1]]	os>10	
hrher2	FALSE	TRUE
-/-	0.2432	0.0974
-/+	0.091	0.0878
+/-	0.2005	0.1184
+/+	0.0692	0.0926
[[2]]	os>10	
hrher2	FALSE	TRUE
-/-	0.2448	0.0966
-/+	0.0894	0.0886
+/-	0.1989	0.1192
+/+	0.0709	0.0918
[[3]]	os>10	
hrher2	FALSE	TRUE
-/-	0.244	0.095
-/+	0.0894	0.0894
+/-	0.1937	0.1208
+/+	0.0709	0.091

AC	controlled	
0	1	
0.6048	0.3952	
[[1]]	controlled	
0	1	
0.602254	0.3977	
[[2]]	controlled	
0	1	
0.605475	0.3945	
[[3]]	controlled	
0	1	
0.603865	0.3961	

AC	lapatrasno	
1	2	3
0.080709	0.1437	0.7756
[[1]]	lapatrasno	
1	2	3
0.0668	0.132	0.8011
[[2]]	lapatrasno	
1	2	3
0.0684	0.1337	0.7979
[[3]]	lapatrasno	
1	2	3
0.0741	0.1449	0.781

AC	capeothno	
1	2	3
0.206693	0.498	0.2953
[[1]]	capeothno	
1	2	3
0.190821	0.4686	0.3406
[[2]]	capeothno	
1	2	3
0.196457	0.4646	0.339
[[3]]	capeothno	
1	2	3
0.191626	0.471	0.3374

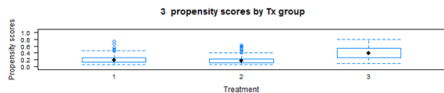
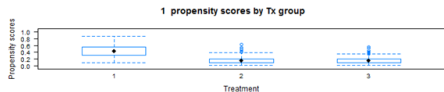
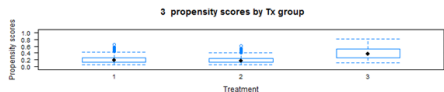
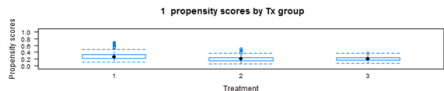
Figure 10: Selected tabluar checks

# Issues with Propensity Score in our Setting

- Problem: Theory was developed for binary treatments, we have ternary
  - Solution: Run each treatment as binary, then compare groups
- Propensity score model specification
  - Solution: Boosting, subject to KS statistic minimization



# Propensity Score Histogram Checks



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