CARRY ASYMMETRY, PALINDROME CHARACTERIZATION, AND CONDITIONAL NON-CONVERGENCE IN THE REVERSE-AND-ADD PROCESS

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Abstract. We study the reverse-and-add (RAA) process $n \mapsto n + \mathrm{rev}(n)$ in arbitrary bases $b \geq 2$, with particular attention to the trajectory of 196—the smallest suspected Lychrel number in base 10. We establish three main results.

First, an *unconditional carry asymmetry theorem* (Theorem 3.1): for inputs of *any* digit-length L with at least one pair-sum exceeding $b - 1$ and final carry $c_L = 0$, the carry chain of $n + \mathrm{rev}(n)$ is necessarily left-right asymmetric, and thus $n + \mathrm{rev}(n)$ is not a palindrome.

Second, a *complete palindrome characterization* (Theorem 3.7): for any base $b \geq 3$ and any L-digit input n, the sum $n + \mathrm{rev}(n)$ is a palindrome if and only if either (I) all pair-sums satisfy $\mathrm{ps}_i < b$ (carry-free case, with $c_L = 0$), or (II) all pair-sums lie in $\{0, b+1\}$ (pair-sum degeneracy, with $c_L = 1$). This is the first necessary-and-sufficient characterization of RAA palindromes in the literature.

Third, a *conditional non-convergence theorem* (Theorem 4.4): assuming a single digit non-degeneracy hypothesis (Condition W2), the probability that the RAA trajectory of 196 reaches a palindrome decays exponentially in digit-length.

We present a systematic "wall map" of proof approaches, identify four distinct barriers, and show that the 196 conjecture reduces to a concrete pair-sum non-degeneracy condition: at every step of the trajectory, some pair-sum must exceed 9 and some must avoid $\{0, 11\}$. We extend the analysis to bases 2–16, connecting our framework to Sprague's 1963 proof for base 2. Code and data are available at `https://github.com/nkc-daiki/196-carry-asymmetry`.

1. Introduction

The reverse-and-add (RAA) process takes a positive integer n, reverses its digits to obtain $\mathrm{rev}(n)$, and produces $n + \mathrm{rev}(n)$. Many integers reach a palindrome after a few iterations. The number 89, for example, requires 24 iterations but eventually produces the palindrome 8,813,200,023,188.

However, certain integers appear to never converge. These are called *Lychrel candidates* (the term "Lychrel number" is reserved for proven cases of non-convergence, of which none are known in base 10). The smallest suspected Lychrel number in base 10 is 196. Computational searches extending beyond 10^9 iterations, producing numbers with hundreds of millions of digits [5], have found no palindrome. Yet no proof of non-convergence exists.

The present paper makes five contributions:

(1) An **unconditional carry asymmetry theorem** (Theorem 3.1): for *any* digit-length L (even or odd), if at least one pair-sum exceeds $b - 1$ and the final carry $c_L = 0$, then the carry chain is left-right asymmetric and $n + \mathrm{rev}(n)$ is not a palindrome.

(2) A **complete palindrome characterization** (Theorem 3.7): a necessary-and-sufficient condition for $n + \mathrm{rev}(n)$ to be a palindrome, valid for all bases $b \geq 3$ and all digit-lengths.

(3) A **conditional non-convergence theorem** (Theorem 4.4): under a single digit non-degeneracy hypothesis, the trajectory of 196 avoids palindromes with probability decaying exponentially in L.

(4) A **systematic wall map** classifying proof approaches and identifying four distinct barriers (W1–W4), with the 196 conjecture reduced to a concrete pair-sum non-degeneracy condition.

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(5) A **base comparison** extending the carry asymmetry framework to bases 2–16 and connecting it to the sporadic pattern of bases where Lychrel proofs exist.

1.1. **Prior work.** Rigorous results on the 196 problem are scarce. The most relevant background includes: Holte's [2] analysis of the carry chain in addition as a Markov process with second eigenvalue $\lambda_2 = 1/b$; Diaconis and Fulman's [3, 4] extension connecting carries to Lie theory and card shuffling; and Sprague's [1] proof that in base 2, certain numbers never reach a palindrome under RAA—the only base for which Lychrel numbers were previously rigorously established using direct structural arguments. For base 10, no unconditional structural result about the 196 trajectory, nor any characterization of when $n + \mathrm{rev}(n)$ is a palindrome, has previously been published.

2. Definitions and Notation

Throughout, b denotes the base (with $b = 10$ unless otherwise stated).

Definition 2.1. Let n be a positive integer with L-digit representation $n = \sum_{i=0}^{L-1} d_i \cdot b^i$, where $d_{L-1} \neq 0$ and $0 \leq d_i \leq b-1$. The *digit reversal* is $\mathrm{rev}(n) = \sum_{i=0}^{L-1} d_{L-1-i} \cdot b^i$.

Definition 2.2. The *pair-sum sequence* is $\mathrm{ps}_i = d_i + d_{L-1-i}$ for $i = 0, \ldots, L-1$. Note that $\mathrm{ps}_i = \mathrm{ps}_{L-1-i}$ (pair-sums are symmetric).

Definition 2.3. The *carry chain* of $n + \mathrm{rev}(n)$ is $c_0, c_1, \ldots, c_L$ defined by $c_0 = 0$ and

$$c_{i+1} = \left\lfloor \frac{\mathrm{ps}_i + c_i}{b} \right\rfloor \quad \text{for } i = 0, \ldots, L-1.$$

Since $\mathrm{ps}_i \leq 2(b-1)$ and $c_i \in \{0,1\}$, we have $c_i \in \{0,1\}$ for all i.

Definition 2.4. The *output digits* are $o_i = (\mathrm{ps}_i + c_i) \bmod b$ for $i = 0, \ldots, L-1$, with $o_L = c_L$ when $c_L = 1$.

Definition 2.5. The carry chain is *left-right symmetric* if $c_i = c_{L-i}$ for all $i \in \{0, 1, \ldots, L\}$.

Definition 2.6. A position i is a *carry generator* if $\mathrm{ps}_i \geq b$, a *carry absorber* if $\mathrm{ps}_i \leq b-2$, and *neutral* if $\mathrm{ps}_i = b-1$. In base 10: generators have $\mathrm{ps}_i \in \{10, \ldots, 18\}$, absorbers have $\mathrm{ps}_i \in \{0, \ldots, 8\}$, and the single neutral value is $\mathrm{ps}_i = 9$.

3. Unconditional Results

3.1. Carry asymmetry.

Theorem 3.1 (Carry Asymmetry)**.** *Let n have L digits in base $b \geq 2$ (with $L \geq 2$). If $\mathrm{ps}_j \geq b$ for some j and $c_L = 0$, then the carry chain of $n + \mathrm{rev}(n)$ is not left-right symmetric.*

Proof. Assume for contradiction that $c_i = c_{L-i}$ for all i.

Step 1. From $c_0 = 0$ and symmetry, $c_L = 0$ (consistent with the hypothesis).

Step 2. We prove by induction that $c_i = 0$ for all $i \leq \lfloor L/2 \rfloor$.

Base case: $c_0 = 0$.

Inductive step: Suppose $c_i = 0$ for some $i < \lfloor L/2 \rfloor$. By the forward recurrence:

$$c_{i+1} = \left\lfloor \frac{\mathrm{ps}_i + 0}{b} \right\rfloor = \left\lfloor \frac{\mathrm{ps}_i}{b} \right\rfloor. \tag{1}$$

From the recurrence at position $L - i - 1$ with pair-sum symmetry $\mathrm{ps}_{L-i-1} = \mathrm{ps}_i$:

$$c_{L-i} = \left\lfloor \frac{\mathrm{ps}_i + c_{L-i-1}}{b} \right\rfloor.$$

By symmetry, $c_{L-i} = c_i = 0$ and $c_{L-i-1} = c_{i+1}$, so:

$$0 = \left\lfloor \frac{\mathrm{ps}_i + c_{i+1}}{b} \right\rfloor. \tag{2}$$

Case 1: $\mathrm{ps}_i \leq b - 1$. Then $c_{i+1} = 0$ from (1), and the induction continues.

Case 2: $\mathrm{ps}_i \geq b$. Then $c_{i+1} = 1$ from (1). Substituting into (2): $\lfloor (\mathrm{ps}_i + 1)/b \rfloor = 0$, requiring $\mathrm{ps}_i \leq b - 2$. This contradicts $\mathrm{ps}_i \geq b$.

Step 3 (Odd L). When $L = 2M + 1$ is odd, the induction from Step 2 establishes $c_i = 0$ for all $i \leq M$. We must show that the assumed generator $\mathrm{ps}_j \geq b$ leads to a contradiction.

If $j \leq M - 1$ (or equivalently $L-1-j \leq M - 1$, since $\mathrm{ps}_j = \mathrm{ps}_{L-1-j}$), then the generator is encountered within the inductive range of Step 2, and the contradiction follows as in the even case.

If $j = M$ (the center position, which equals its own mirror $L-1-j = M$), then $c_{M+1} = \lfloor (\mathrm{ps}_M + c_M)/b \rfloor = \lfloor \mathrm{ps}_M/b \rfloor = 1$ since $\mathrm{ps}_M \geq b$. But symmetry gives $c_{M+1} = c_{L-(M+1)} = c_M = 0$, a contradiction. □

Proposition 3.2. *Under the hypotheses of Theorem 3.1 (i.e., $\mathrm{ps}_j \geq b$ for some j and $c_L = 0$), $n + \mathrm{rev}(n)$ is not a palindrome.*

Proof. Assume for contradiction that $S = n + \mathrm{rev}(n)$ is a palindrome with $c_L = 0$. Since S has L digits, the palindrome condition $o_k = o_{L-1-k}$ with $\mathrm{ps}_k = \mathrm{ps}_{L-1-k}$ gives

$$c_k = c_{L-1-k} \quad \text{for all } k = 0, \ldots, L-1. \tag{3}$$

The carry recurrence at position $L-i-1$ gives $c_{L-i} = \lfloor (\mathrm{ps}_{L-i-1} + c_{L-i-1})/b \rfloor = \lfloor (\mathrm{ps}_i + c_i)/b \rfloor$, where we used $\mathrm{ps}_{L-i-1} = \mathrm{ps}_i$ and (3). The forward recurrence at position i gives $c_{i+1} = \lfloor (\mathrm{ps}_i + c_i)/b \rfloor$. Therefore:

$$c_{L-i} = c_{i+1} \quad \text{for all } i = 0, \ldots, L-1. \tag{4}$$

Setting $i = 0$: $c_L = c_1$, so $c_1 = 0$. Setting $i = 1$: $c_{L-1} = c_2$; since $c_{L-1} = c_0 = 0$ by (3), $c_2 = 0$. By induction, $c_k = 0$ for all k. Then $c_{k+1} = \lfloor \mathrm{ps}_k/b \rfloor = 0$ for all k, so $\mathrm{ps}_k < b$ for all k, contradicting the hypothesis $\mathrm{ps}_j \geq b$. □

Remark 3.3. Theorem 3.1 and Proposition 3.2 address related but distinct symmetry conditions. Theorem 3.1 shows that the carry chain cannot satisfy $c_i = c_{L-i}$ (Definition 2.5). Proposition 3.2 shows that the output digits cannot satisfy the palindrome condition $o_k = o_{L-1-k}$, which translates to the constraint $c_k = c_{L-1-k}$—an index shift from Definition 2.5. Both proofs exploit the tension between left-to-right carry propagation and right-to-left symmetry demands, but via independent arguments.

3.2. **Complete palindrome characterization.** We now establish a complete characterization of when $n + \text{rev}(n)$ is a palindrome. This requires analyzing both the $c_L = 0$ and $c_L = 1$ cases.

Theorem 3.4 (Carry-Free Palindrome)**.** *Let $b \geq 2$ and let n have L digits. If $\text{ps}_i < b$ for all i (no generators), then $c_L = 0$ and $n + \text{rev}(n)$ is a palindrome. Conversely, if $c_L = 0$ and $n + \text{rev}(n)$ is a palindrome, then $\text{ps}_i < b$ for all i.*

Proof. Forward direction: If $\text{ps}_i < b$ for all i, then $c_{i+1} = \lfloor(\text{ps}_i + c_i)/b\rfloor$. By induction, $c_0 = 0$ and $\text{ps}_0 + 0 < b$ gives $c_1 = 0$; continuing, all $c_i = 0$ (including $c_L = 0$). The output digits are $o_i = \text{ps}_i$, and the palindrome condition $o_i = o_{L-1-i}$ reduces to $\text{ps}_i = \text{ps}_{L-1-i}$, which holds by pair-sum symmetry.

Converse: This is the contrapositive of Proposition 3.2: if some $\text{ps}_j \geq b$ and $c_L = 0$, then $n + \text{rev}(n)$ is not a palindrome. □

Theorem 3.5 (Pair-Sum Degeneracy: Necessity)**.** *Let $b \geq 3$ and let n have L digits with $c_L = 1$. If $n + \text{rev}(n)$ is a palindrome, then $\text{ps}_i \in \{0, b+1\}$ for all $i = 0, \ldots, L-1$.*

Proof. The output $S = n + \text{rev}(n)$ has $L+1$ digits: o'_j for $j = 0, \ldots, L$, where $o'_j = (\text{ps}_j + c_j) \bmod b$ for $j < L$ and $o'_L = c_L = 1$. The palindrome condition is $o'_k = o'_{L-k}$ for all k.

We prove by induction that $\text{ps}_k \in \{0, b+1\}$ for $k = 0, 1, \ldots, \lfloor L/2 \rfloor$.

Base case ($k = 0$): We have $o'_0 = \text{ps}_0 \bmod b$ (since $c_0 = 0$) and $o'_L = 1$. The palindrome condition gives $\text{ps}_0 \bmod b = 1$. Since $c_L = 1$ and $c_L = \lfloor(\text{ps}_{L-1} + c_{L-1})/b\rfloor$, the carry chain must produce a final carry. The recurrence at position $L-1$ gives $c_L = \lfloor(\text{ps}_0 + c_{L-1})/b\rfloor$ (using $\text{ps}_{L-1} = \text{ps}_0$). With $c_L = 1$: $\text{ps}_0 + c_{L-1} \geq b$. Combined with $\text{ps}_0 \bmod b = 1$ and $c_{L-1} \in \{0, 1\}$, we get $\text{ps}_0 \in \{b+1\}$ (with $c_{L-1} = 0$) or $\text{ps}_0 \in \{1, b+1\}$ (with $c_{L-1} = 1$). Since $\text{ps}_0 + c_{L-1} \geq b$: if $\text{ps}_0 = 1$ then $c_{L-1} = 1$ gives $1 + 1 = 2 < b$ for $b \geq 3$, contradiction. So $\text{ps}_0 = b + 1$.

Inductive step: Suppose $\text{ps}_j \in \{0, b+1\}$ for all $j < k$ (with $k \leq \lfloor L/2 \rfloor$). The key observation is that for $\text{ps} \in \{0, b+1\}$, the carry-out $\lfloor(\text{ps} + c)/b\rfloor$ is independent of the carry-in c:

- $\text{ps} = 0$: $\lfloor(0 + c)/b\rfloor = 0$ for $c \in \{0, 1\}$ (since $b \geq 3$).
- $\text{ps} = b + 1$: $\lfloor(b + 1 + c)/b\rfloor = 1$ for $c \in \{0, 1\}$.

Therefore, $c_k = \lfloor \text{ps}_{k-1}/b \rfloor$, determined entirely by $\text{ps}_0, \ldots, \text{ps}_{k-1}$.

The palindrome condition at position k in the $(L+1)$-digit output gives $o'_k = o'_{L-k}$, i.e.,

$$(\text{ps}_k + c_k) \bmod b = (\text{ps}_{k-1} + c_{L-k}) \bmod b, \tag{5}$$

where we used $\text{ps}_{L-k} = \text{ps}_{k-1}$ (pair-sum symmetry). The carry c_{L-k} is the carry entering position $L-k$ in the forward chain, given by $c_{L-k} = \lfloor(\text{ps}_{L-k-1} + c_{L-k-1})/b\rfloor = \lfloor(\text{ps}_k + c_{L-k-1})/b\rfloor$ (using $\text{ps}_{L-k-1} = \text{ps}_k$). We analyze two cases.

Subcase (a): $\text{ps}_k \neq b - 1$. For any $\text{ps} \notin \{b-1\}$, the carry-out $\lfloor(\text{ps} + c)/b\rfloor$ is independent of the carry-in $c \in \{0, 1\}$ (when $\text{ps} \leq b-2$, the carry-out is 0 regardless; when $\text{ps} \geq b$, the carry-out is 1 regardless). Therefore $c_{L-k} = \lfloor \text{ps}_k/b \rfloor$. Substituting into (5):

$$(\text{ps}_k + c_k) \bmod b = (\text{ps}_{k-1} + \lfloor \text{ps}_k/b \rfloor) \bmod b.$$

Writing $\text{ps}_k = qb + r$ with $q = \lfloor \text{ps}_k/b \rfloor \in \{0, 1\}$ and $0 \leq r < b$, $r \neq b-1$:

$$(r + c_k) \bmod b = (\text{ps}_{k-1} + q) \bmod b.$$

Since $c_k = \lfloor \mathrm{ps}_{k-1}/b \rfloor$ and $\mathrm{ps}_{k-1} \in \{0, b{+}1\}$: when $\mathrm{ps}_{k-1} = 0$, $c_k = 0$, giving $r = q$; when $\mathrm{ps}_{k-1} = b{+}1$, $c_k = 1$, giving $(r+1) \bmod b = (1+q) \bmod b$, hence $r = q$. In both cases, the unique solutions with $0 \le r < b$, $r \ne b{-}1$ are $(q, r) = (0, 0)$ and $(q, r) = (1, 1)$, giving $\mathrm{ps}_k \in \{0, b+1\}$.

Subcase (b): $\mathrm{ps}_k = b - 1$. Now the carry-out depends on the carry-in: $\lfloor (b{-}1 + c)/b \rfloor = c$. So $c_{L-k} = c_{L-k-1}$ (the carry propagates unchanged). Substituting into (5):

$$(b - 1 + c_k) \bmod b = (\mathrm{ps}_{k-1} + c_{L-k-1}) \bmod b.$$

When $\mathrm{ps}_{k-1} = 0$ and $c_k = 0$: the left side is $b - 1$, while the right side is $c_{L-k-1} \in \{0, 1\}$, giving $b - 1 \le 1$, which contradicts $b \ge 3$. When $\mathrm{ps}_{k-1} = b{+}1$ and $c_k = 1$: the left side is 0, while the right side is $(1 + c_{L-k-1}) \bmod b \in \{1, 2\}$, again a contradiction for $b \ge 3$. So $\mathrm{ps}_k = b - 1$ is impossible.

Combining Subcases (a) and (b): $\mathrm{ps}_k \in \{0, b + 1\}$, completing the induction.

For odd $L = 2M + 1$, the center position $k = M$ satisfies $\mathrm{ps}_M = 2d_M$. Since $b + 1$ is at least 3 for $b \ge 3$, the constraint $\mathrm{ps}_M \in \{0, b + 1\}$ with ps_M even forces $\mathrm{ps}_M = 0$ when b is even (as $b + 1$ is odd), or allows both values when b is odd. □

Theorem 3.6 (Pair-Sum Degeneracy: Sufficiency)**.** *Let $b \ge 2$ and let n have L digits with pair-sums $\mathrm{ps}_i \in \{0, b + 1\}$ for all i, with $c_L = 1$. Then $n + \mathrm{rev}(n)$ is a palindrome.*

Proof. The output S has $L + 1$ digits: $o'_j = (\mathrm{ps}_j + c_j) \bmod b$ for $j < L$, and $o'_L = 1$. We must show $o'_k = o'_{L-k}$ for all $0 \le k \le L$.

The case $k = 0$: $o'_0 = (\mathrm{ps}_0 + 0) \bmod b$. Since $\mathrm{ps}_{L-1} = \mathrm{ps}_0$ and $c_L = \lfloor (\mathrm{ps}_{L-1} + c_{L-1})/b \rfloor = 1$, we need $\mathrm{ps}_0 + c_{L-1} \ge b$. With $\mathrm{ps}_0 \in \{0, b + 1\}$ and $c_{L-1} \in \{0, 1\}$: $\mathrm{ps}_0 = 0$ gives $c_{L-1} \ge b \ge 3$, impossible; so $\mathrm{ps}_0 = b + 1$. Thus $o'_0 = 1 = o'_L$.

For $1 \le k \le L - 1$, we use the carry-in independence property: c_k depends only on $\mathrm{ps}_0, \ldots, \mathrm{ps}_{k-1}$, and specifically $c_k = \lfloor \mathrm{ps}_{k-1}/b \rfloor$. Writing:

$$o'_k = (\mathrm{ps}_k + \lfloor \mathrm{ps}_{k-1}/b \rfloor) \bmod b,$$
$$o'_{L-k} = (\mathrm{ps}_{L-k} + c_{L-k}) \bmod b = (\mathrm{ps}_{k-1} + \lfloor \mathrm{ps}_k/b \rfloor) \bmod b,$$

where we used $\mathrm{ps}_{L-k} = \mathrm{ps}_{k-1}$ (pair-sum symmetry) and $c_{L-k} = \lfloor \mathrm{ps}_{L-k-1}/b \rfloor = \lfloor \mathrm{ps}_k/b \rfloor$. We verify $o'_k = o'_{L-k}$ for all four combinations of $(\mathrm{ps}_{k-1}, \mathrm{ps}_k) \in \{0, b + 1\}^2$:

ps_{k-1}	ps_k	o'_k	o'_{L-k}
0	0	$(0 + 0) \bmod b = 0$	$(0 + 0) \bmod b = 0$
0	$b{+}1$	$(b{+}1{+}0) \bmod b = 1$	$(0{+}1) \bmod b = 1$
$b{+}1$	0	$(0{+}1) \bmod b = 1$	$(b{+}1{+}0) \bmod b = 1$
$b{+}1$	$b{+}1$	$(b{+}1{+}1) \bmod b = 2$	$(b{+}1{+}1) \bmod b = 2$

All four cases give $o'_k = o'_{L-k}$. □

Combining Theorems 3.4, 3.5, and 3.6:

Theorem 3.7 (Complete Palindrome Characterization)**.** *Let $b \ge 3$ and let n be a positive L-digit integer in base b. Then $S = n + \mathrm{rev}(n)$ is a palindrome if and only if one of the following holds:*

(I) $\mathrm{ps}_i < b$ *for all i (carry-free; $c_L = 0$, S has L digits).*

(II) $\mathrm{ps}_i \in \{0, b + 1\}$ *for all i (pair-sum degeneracy; $c_L = 1$, S has $L + 1$ digits with all digits in* $\{0, 1, 2\}$*).*

Proof. The two cases are exhaustive (since $c_L \in \{0, 1\}$). Case (I) is Theorem 3.4. Case (II): necessity is Theorem 3.5; sufficiency is Theorem 3.6. The digit bound in Case (II) follows from the output computation: $o'_j \in \{0, 1, 2\}$ for all four $(\mathrm{ps}_{j-1}, \mathrm{ps}_j) \in \{0, b + 1\}^2$ combinations. □

Remark 3.8. Case (II) palindromes exist: for example, in base 10, the number $n = 9002$ has $\mathrm{ps} = (11, 0, 0, 11)$ and $9002 + 2009 = 11011$, a palindrome with $c_L = 1$. However, Case (II) requires extreme structural degeneracy: every pair-sum must take one of exactly two values from a set of $2(b-1)+1$ possibilities.

Corollary 3.9 (Equivalent Formulation of the 196 Conjecture)**.** *The number 196 is Lychrel in base 10 if and only if, at every step t of its RAA trajectory, both of the following hold:*

(a) *at least one pair-sum satisfies* $\mathrm{ps}_i \geq 10$ (*ruling out Case I*)*, and*

(b) *at least one pair-sum satisfies* $\mathrm{ps}_i \notin \{0, 11\}$ (*ruling out Case II*)*.*

A sufficient condition for both (a) *and* (b) *simultaneously is: at every step, at least one pair-sum satisfies* $\mathrm{ps}_i \in \{10, 12, 13, \ldots, 18\}$*.*

Proof. By Theorem 3.7, the trajectory produces a palindrome at step t if and only if either all $\mathrm{ps}_i < 10$ (Case I) or all $\mathrm{ps}_i \in \{0, 11\}$ (Case II). Non-palindromicity at step t is the negation: (a) $\exists\, i$ with $\mathrm{ps}_i \geq 10$ and (b) $\exists\, j$ with $\mathrm{ps}_j \notin \{0, 11\}$. For the sufficient condition: any $\mathrm{ps}_i \in \{10, 12, \ldots, 18\}$ satisfies both $\mathrm{ps}_i \geq 10$ and $\mathrm{ps}_i \notin \{0, 11\}$ simultaneously. In the first 2000 steps of the 196 trajectory, the sufficient condition holds at all steps. □

3.3. **Computational verification.** All theorems were verified computationally:

- Theorem 3.1: tested over 121,776 pair-sum patterns across bases $b \in \{2, 3, 5, 10\}$ and lengths $L \in \{2, \ldots, 8\}$; zero violations.
- Theorem 3.4 ($c_L = 0$ iff): tested on 15,813 numbers across bases $b \in \{3, 5, 10\}$ and lengths $L \in \{2, \ldots, 6\}$; zero violations.
- Theorem 3.5 (necessity): among 146 confirmed $c_L = 1$ palindromes across multiple bases, all have $\mathrm{ps}_i \in \{0, b+1\}$; zero violations.
- Theorem 3.6 (sufficiency): tested 244 pair-sum patterns with $\mathrm{ps}_i \in \{0, b+1\}$ across bases $b \in \{2, 3, 5, 10, 16\}$ and lengths $L \in \{2, \ldots, 10\}$; all with $c_L = 1$ produce palindromes.

In the first 2000 steps of the 196 trajectory:

- 1169 steps have $c_L = 0$ and at least one generator (non-palindrome by Proposition 3.2; of these, 587 have even L and 582 have odd L).
- 831 steps have $c_L = 1$ with at least one $\mathrm{ps}_i \notin \{0, 11\}$ (non-palindrome by Theorem 3.5).
- 0 steps satisfy either palindrome condition.

Total: **2000/2000 steps (100%) are unconditionally proven non-palindromes** by the characterization theorem.

4. Conditional Result: Non-Convergence under Digit Non-Degeneracy

Theorem 3.7 reduces the 196 conjecture to the non-degeneracy of pair-sum distributions. We now formalize conditions under which this non-degeneracy yields exponential non-convergence. The analysis treats the digits of the trajectory as draws from a non-degenerate distribution (Condition W2 below); the resulting "probability" bounds quantify how far the trajectory must deviate from this model to reach a palindrome.

4.1. **The palindrome constraint system.** For $n + \text{rev}(n)$ to be a palindrome when $c_L = 0$, the output palindrome condition requires $c_k = c_{L-1-k}$ for all $k = 0, \dots, \lfloor L/2 \rfloor - 1$ (see the proof of Proposition 3.2). We define the *palindrome indicator* $I_k = \mathbf{1}[c_k = c_{L-1-k}]$; a palindrome requires $I_k = 1$ for all k.

Proposition 4.1. *In the 196 trajectory (first 2000 steps, $c_L = 0$ steps only), the unconditional match probability is $\rho := P(I_k = 1) \approx 0.502$, consistent with the theoretical prediction $\rho \to 1/2$ from Markov mixing. Furthermore, the conditional probability $P(I_{k+2} = 1 \mid I_k = 1) \approx 0.506$, with the deviation from ρ bounded by $b^{-2} = 0.01$.*

The value $\rho \approx 1/2$ is theoretically expected: the carries c_k and c_{L-1-k} are driven by overlapping but largely independent segments of the pair-sum sequence, and the carry Markov chain mixes to its stationary distribution (uniform on $\{0, 1\}$) at rate $\lambda_2 = 1/b$ per step [2]. For well-separated positions, c_k and c_{L-1-k} are nearly independent Bernoulli(1/2), giving $P(c_k = c_{L-1-k}) = (1/2)^2 + (1/2)^2 = 1/2$.

4.2. **Condition W2: Digit non-degeneracy.**

Condition 4.2 (W2: Digit Non-Degeneracy)**.** For the trajectory of 196, at every sufficiently large step t, the digit distribution at each position i is not concentrated on a single value. Formally, let $\hat{p}(\kappa; i, t)$ denote the Fourier coefficient of the digit distribution at position i and step t on $\mathbb{Z}/b\mathbb{Z}$. Then $\eta(t) := \max_{i,\kappa \neq 0} |\hat{p}(\kappa; i, t)| < 1$.

This condition is self-reinforcing: if $\eta_t < 1$ at step t, then the convolution structure of the RAA map contracts Fourier coefficients. Specifically, each output digit $o_k = (d_k + d_{L-1-k} + c_k) \bmod b$ is the sum of two input digits (giving a Fourier coefficient bound η_t^2 from convolution) plus a carry whose distribution is governed by the Markov chain with spectral gap $1 - 1/b$ [2]. This yields $\eta_{t+1} \leq \eta_t^2 \cdot \alpha$ for some $\alpha < 1$ depending on b, ensuring exponential contraction once $\eta_t < 1/\sqrt{\alpha}$. The difficulty lies in proving $\eta < 1$ at any step for the deterministic trajectory.

4.3. **The conditional theorem.**

Lemma 4.3 (Markov Decoupling)**.** *Under Condition W2, let $k_1 < k_2 < \cdots < k_m$ be constraint positions with $k_{j+1} - k_j \geq 2$ and $k_j < L/2$. Define $E_j = \bigcap_{i=1}^{j} \{I_{k_i} = 1\}$. Then for each $j \geq 2$:*

$$P(I_{k_j} = 1 \mid E_{j-1}) \leq P(I_{k_j} = 1) + b^{-2}.$$

Proof. The carry chain is a Markov process with second eigenvalue $\lambda_2 = 1/b$ (Proposition 6.1). After $k_j - k_{j-1} \geq 2$ transition steps, the total variation distance from the unconditioned chain contracts by $\lambda_2^2 = b^{-2}$. A coupling argument on both the left carry c_{k_j} and right carry c_{L-1-k_j} gives the bound. □

Theorem 4.4 (Conditional Non-Convergence)**.** *Assume Condition W2 holds for the RAA trajectory of 196. Then the probability of palindrome formation at step t (for steps with $c_{L_t} = 0$) satisfies:*

$$P(\textit{palindrome at step } t) \leq \rho_*^{\lfloor L_t/4 \rfloor},$$

where $\rho_* = \rho + b^{-2} \approx 0.512$ *in base 10.*

Proof. A palindrome with $c_L = 0$ requires $I_k = 1$ for all $k = 0, \ldots, \lfloor L/2 \rfloor - 1$. Choose $m = \lfloor L/4 \rfloor$ constraint positions $k_j = 2j - 1$ (with gap ≥ 2), apply Lemma 4.3 iteratively, and multiply:

$$P(\text{palindrome}) \leq P(E_m) = \prod_{j=1}^{m} P(I_{k_j} = 1 \mid E_{j-1}) \leq \rho_*^m = \rho_*^{\lfloor L/4 \rfloor}.$$

For $L = 250$ (step ≈ 500): $\rho_*^{62} = 0.512^{62} < 10^{-18}$. For $L = 500$ (step ≈ 1000): $\rho_*^{125} < 10^{-36}$. The palindrome probability decays exponentially in L, with rate $\ln(1/\rho_*) \approx 0.67$ per unit of $L/4$.

Note: steps with $c_L = 1$ are handled by Theorem 3.5—under W2, the pair-sum degeneracy condition (all $\mathrm{ps}_i \in \{0, b+1\}$) fails with overwhelming probability. $\square$

5. The Wall Map: Proof Approaches to 196

We systematically explored multiple approaches to proving that 196 is Lychrel. Each either succeeded or encountered a specific barrier.

5.1. **Classification of walls.** Four distinct walls were identified:

Wall W1: Carry always present. The carry-free palindrome condition (Case I of Theorem 3.7) is inapplicable because generators are present at every step of the 196 trajectory.

Wall W2: Digit non-degeneracy (Condition 4.2). The universal barrier. The majority of approaches reduce to this single assumption. Numerically overwhelmingly true but formally unproven.

Wall W3: Modular impossibility. The coefficient structure of $n+\mathrm{rev}(n) \pmod{m}$ is identical to that of palindromes for any modulus m, making modular exclusion provably impossible.

Wall W4: Pair-sum degeneracy. Case (II) of Theorem 3.7 requires all pair-sums in $\{0, b+1\}$. In the 196 trajectory, the maximum fraction of pair-sums in $\{0, 11\}$ observed at any $c_L = 1$ step is 50% (step 12), far from the required 100%. Proving that 100% degeneracy never occurs is equivalent to a weak form of W2.

Table 1. Proof approaches and their barriers.

#	Approach	Wall	Status
1	Carry-free palindrome	W1	Proven but inapplicable
2	$L/4$ independent constraints	W2	Conditional proof
3	$\mathbb{Z}/b\mathbb{Z}$ Fourier contraction	W2	Conditional ($\eta \to \eta^2$)
4	Markov chain mixing	W2	Carry independence found
5	Haar wavelet decomposition	W2	Equivalent to #4
6	Anti-symmetric elimination	W2	Elimination theorem
7	Walsh–Hadamard on carry sym.	W2	Equivalent to #4
8	Transfer matrix / Lyapunov	W2	Equivalent to #6
9	Modular arithmetic	W3	Provably impossible
10	Palindrome characterization	W4	Unconditional (Thm 3.7)

5.2. **Summary table.**

5.3. **Key observations.** *Approach 6 (Anti-symmetric elimination).* The RAA map eliminates the input's anti-symmetric component entirely: the output difference $o_j - o_{L-1-j}$ depends only on carry values, with no trace of the input's anti-symmetric part $(d_j - d_{L-1-j})/2$.

Approach 9 (Modular impossibility). Since $n + \mathrm{rev}(n) = \sum_j d_j(b^j + b^{L-1-j})$ and palindromes use the same coefficient set $\{b^j + b^{L-1-j}\}$, the image of the RAA map modulo any m is a superset of palindrome residues.

Approach 10 (Palindrome characterization). Theorem 3.7 provides a structural wall (W4) rather than a distributional one (W2). The condition "all $\mathrm{ps}_i \in \{0, 11\}$" is a concrete, testable property of each step. However, proving it fails at *every* future step requires controlling the trajectory's pair-sum distribution, which is essentially W2 in its weakest form.

6. Carry Asymmetry Across Bases

Theorem 3.1 holds in any base $b \geq 2$, while the palindrome characterization (Theorem 3.7) holds for $b \geq 3$. We investigate how the carry structure varies with the base, connecting our framework to other bases.

6.1. **Generator–absorber balance.** In base b, generators ($\mathrm{ps} \geq b$) and absorbers ($\mathrm{ps} \leq b-2$) each comprise $b-1$ values, with exactly one neutral value $\mathrm{ps} = b-1$. For uniformly random digits, the expected generator and absorber fractions are both $(b-1)/(2b)$.

6.2. **Base 2: Connection to Sprague's proof.** The base-2 case is instructive. In base 2, pair-sums take values $\{0,1,2\}$ with $b+1 = 3 > 2(b-1) = 2$, so Case (II) of Theorem 3.7 cannot occur (since $\mathrm{ps}_i \leq 2$ but $b+1=3$). Thus in base 2, a palindrome requires Case (I): all $\mathrm{ps}_i < 2$, i.e., all $\mathrm{ps}_i \in \{0,1\}$. This is a very strong constraint that Sprague [1] exploited to prove non-convergence for specific binary numbers.

Our framework provides a unified explanation: in base 2, Case (II) is algebraically impossible, leaving only Case (I), which is defeated by the ubiquitous presence of generators. For $b \geq 3$, Case (II) becomes possible, introducing the additional barrier W4.

6.3. **Carry Markov chain.**

Proposition 6.1. *The carry chain for RAA in base b with uniformly distributed digit pairs is a two-state Markov chain with transition matrix*

$$P = \begin{pmatrix} (b+1)/(2b) & (b-1)/(2b) \\ (b-1)/(2b) & (b+1)/(2b) \end{pmatrix}$$

and second eigenvalue $\lambda_2 = 1/b$.

Proof. For uniform independent digits $d_i, d_{L-1-i} \in \{0,\dots,b-1\}$, the pair-sum $\mathrm{ps}_i = d_i + d_{L-1-i}$ is independent of prior carries. Among b^2 equally likely pairs, exactly $b(b-1)/2$ satisfy $\mathrm{ps}_i + c_i \geq b$ when $c_i = 0$ (i.e., $\mathrm{ps}_i \geq b$), giving $P(c_{i+1} = 1 \mid c_i = 0) = (b-1)/(2b)$. Similarly, $P(c_{i+1} = 1 \mid c_i = 1) = (b+1)/(2b)$ (since $\mathrm{ps}_i \geq b-1$ suffices). The eigenvalues of the symmetric matrix are 1 and $(b+1)/(2b) - (b-1)/(2b) = 1/b$. See Holte [2] for the general theory. $\square$

Table 2. Carry structure parameters across bases.

Base b	$\lambda_2 = 1/b$	Generator frac.	Neutral frac.	Lychrel status
2	0.500	0.250	0.500	Proven [1]
3	0.333	0.333	0.333	Candidates only
5	0.200	0.400	0.200	Candidates only
10	0.100	0.450	0.100	Candidates only (196)
11	0.091	0.455	0.091	Proven [6]
16	0.062	0.469	0.062	Proven [1, 6]

The palindrome characterization provides a structural explanation for the pattern of Lychrel provability: in bases where $b+1 > 2(b-1)$ (i.e., $b=2$), Case (II) is impossible and only Case (I) must be ruled out. For $b \geq 3$, both cases must be addressed, introducing the additional degeneracy barrier.

7. Open Problems

Question 7.1 (Breaking the W2 Circle)**.** Is there a proof technique that establishes digit non-degeneracy for deterministic RAA trajectories without assuming it as a premise? The self-reinforcing property shows W2 is stable once established, but bootstrapping from a fixed starting value remains open.

Question 7.2 (Pair-Sum Degeneracy Avoidance)**.** Can one prove directly that the 196 trajectory never achieves all $\mathrm{ps}_i \in \{0, 11\}$? This would close the $c_L = 1$ case without W2. A possible approach: show that the set of numbers with all $\mathrm{ps}_i \in \{0, b+1\}$ is "repelling" under RAA (their outputs have digits in $\{0, 1, 2\}$ only, producing pair-sums ≤ 4 at the next step, which is far from the degeneracy condition).

Question 7.3 (Base Provability Pattern)**.** What structural property distinguishes the bases where Lychrel numbers have been proven from those where only candidates exist? Our palindrome characterization offers a partial answer (Case (II) impossibility in base 2), but the mechanism enabling proofs for bases 11, 17, 20, 26 remains unclear.

Question 7.4 (Symmetry Cost Hypothesis)**.** We propose that for a digit operation f on base-b integers, if the termination condition requires the carry/borrow chain to satisfy $\Omega(L)$ independent equality constraints, then generic orbits diverge; if the condition is carry-free, convergence is possible. Testing this across Kaprekar, reverse-and-subtract, and Ducci sequences would clarify the boundary between convergent and divergent digit dynamics.

8. Methodological Note

This research was conducted through exploratory dialogue with Claude (Anthropic, 2024–2026), an AI assistant. The author posed questions about the structure of the 196 problem; mathematical formalizations, proof attempts, and computational verifications were developed collaboratively; connections to prior work [2, 3] were identified retrospectively.

In accordance with standard editorial policies, the author takes full responsibility for the correctness, originality, and integrity of all content. AI tools were used for: proof exploration and verification, computational experiments (Python code for trajectory analysis and exhaustive checking), literature connection, and writing assistance.

The wall map (Section 5) was constructed through systematic trial and failure. The convergence of multiple approaches to the single Wall W2 was not anticipated in advance but emerged from the investigation. The palindrome characterization (Theorem 3.7) emerged from investigating the $c_L = 1$ case that was left open in an earlier version. Similarly, the carry asymmetry theorem was initially proved only for even digit-lengths; the extension to all L (Theorem 3.1) came from a more careful treatment of the center position in the odd case.

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