ML-PW-05 Magnin-Kocher

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0.0.1 MSE - T-MachLe

1 PW 05

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1.1 Exercice 1 Confusion Matrix

```
In [326]: import pandas as pd
          import numpy as np
          import matplotlib.pyplot as plt
          %matplotlib inline
          from numpy.linalg import inv

          classes = ['p0','p1','p2','p3','p4','p5','p6','p7','p8','p9','y']

          dataset = pd.read_csv('data/ex1-system-a.csv', sep=';',
                                dtype={'y': np.uint32},
                                usecols=[0,1,2,3,4,5,6,7,8,9,10],
                                header=-1, names=classes)

          N = len(dataset['p0'])
          N_features = 10

          print(dataset.loc[0:0].values[0][-1])
```

```
7.0
```

1.1.1 a) Write a function to take classification decisions on such outputs according to Bayes'rule.

```
In [327]: test = [0,1,20,3]

          def classBayes(post_priors):
              return np.argmax(post_priors)
```

```
#print(classBayes(test))
```

1.1.2 b) What is the overall error rate of the system ?

```
In [331]: true_guess = 0

          for i in range(0, N):
              row = dataset.loc[i:i].values[0] # get the row
              guess = classBayes(row[0:-1])
              if guess == row[-1]:
                  true_guess = true_guess + 1

          print("True guesses: ", true_guess)
          print("Error rate: ", (N - true_guess)/N)
          print("Performance: ", true_guess/N)
```

```
True guesses:  8927
Error rate:  0.1073
Performance:  0.8927
```

1.1.3 c) Compute and report the confusion matrix of the system.

```
In [334]: from collections import defaultdict
          cmatrix = defaultdict(lambda: [0] * N_features)


          for i in range(N):
              row = dataset.loc[i:i].values[0] # get the row
              guess = classBayes(row[0:-1])
              yi = np.int(row[-1])
              cmatrix[classes[yi]][guess] = cmatrix[classes[yi]][guess] + 1


          print(pd.DataFrame(cmatrix, columns=classes[:-1], index=classes[:-1]))
```

```
     p0    p1   p2   p3   p4   p5   p6   p7   p8   p9
p0  944     0   10    1    2   12   14    0   12   10
p1    0  1112    6    1    3    3    3   14   16    4
p2   11     2  921   31    6    6   21   30   18    6
p3    0     3   12  862    2   29    2    9   26   22
p4    0     1   15    2  910   19   22    7   24   53
p5    2     4    3   72    1  768   28    2   46   18
p6   10     3   19    5   12   19  865    1   22    0
p7    7     1   15   14    6    9    0  929   19   48
p8    5     9   26   12    4   21    3    3  772    4
p9    1     0    5   10   36    6    0   33   19  844
```

1.1.4 d) What are the worst and best classes in terms of sensitivity (recall) ?

In [336]:
```
csensitivity = {}
crecall = {}

for i in range(N_features):
    # le hit
    TP = cmatrix[classes[i]][i]
    # sommer la ligne
    FN = sum(cmatrix[classes[j]][i] for j in range(N_features) ) - TP
    # sommer la colonne
    FP = sum(x for x in cmatrix[classes[i]]) - TP

    recall = TP / (TP + FN)
    precision = TP / (TP + FP)
    csensitivity[classes[i]] = precision
    crecall[classes[i]] = recall

print("Sensitivity: ", pd.DataFrame(csensitivity, index=[0]).transpose())
print("Best sensitivity: ", max(csensitivity, key=csensitivity.get))

print("")

print("Recall: ", pd.DataFrame(crecall, index=[0]).transpose())
print("Best recall: ", max(crecall, key=crecall.get))
```

```
Sensitivity:             0
p0  0.963265
p1  0.979736
p2  0.892442
p3  0.853465
p4  0.926680
p5  0.860987
p6  0.902923
p7  0.903696
p8  0.792608
p9  0.836472
Best sensitivity:  p1

Recall:             0
p0  0.939303
p1  0.956971
p2  0.875475
p3  0.891417
p4  0.864198
p5  0.813559
p6  0.904812
p7  0.886450
p8  0.898719
```

```
p9  0.884696
Best recall:  p1
```

1.1.5 e) In file `ex1-system-b.csv` you find the output of a second system B. What is the best system between (a) and (b) in terms of error rate and F1.

```
In [350]: dataset2 = pd.read_csv('data/ex1-system-b.csv', sep=';',
                                  dtype={'y': np.uint32},
                                  usecols=[0,1,2,3,4,5,6,7,8,9,10],
                                  header=-1, names=classes)

          N2 = len(dataset2['p0'])

          true_guess = 0

          for i in range(0, N2):
              row = dataset2.loc[i:i].values[0] # get the row
              guess = classBayes(row[0:-1])
              if guess == row[-1]:
                  true_guess = true_guess + 1

          print("True guesses2: ", true_guess)
          print("Error rate2: ", (N2 - true_guess)/N2)
          print("Performance2: ", true_guess/N2)
```

```
True guesses2:  9613
Error rate2:  0.0387
Performance2:  0.9613
```

L'error rate est meilleurs avec ce dataset!

```
In [353]: cmatrix2 = defaultdict(lambda: [0] * N_features)


          for i in range(N2):
              row = dataset2.loc[i:i].values[0] # get the row
              guess = classBayes(row[0:-1])
              yi = np.int(row[-1])
              cmatrix2[classes[yi]][guess] = cmatrix2[classes[yi]][guess] + 1

          csensitivity2 = {}
          crecall2 = {}

          for i in range(N_features):
              # le hit
              TP = cmatrix2[classes[i]][i]
              # sommer la ligne
```

```
            FN = sum(cmatrix2[classes[j]][i] for j in range(N_features) ) - TP
            # sommer la colonne
            FP = sum(x for x in cmatrix2[classes[i]]) - TP

            recall = TP / (TP + FN)
            precision = TP / (TP + FP)
            csensitivity2[classes[i]] = precision
            crecall2[classes[i]] = recall

        print("Sensitivity2: ", pd.DataFrame(csensitivity2, index=[0]).transpose())
        print("Best sensitivity2: ", max(csensitivity2, key=csensitivity.get))

        print("")

        print("Recall2: ", pd.DataFrame(crecall2, index=[0]).transpose())
        print("Best recall2: ", max(crecall2, key=crecall2.get))
```

```
Sensitivity2:              0
p0  0.982653
p1  0.989427
p2  0.965116
p3  0.975248
p4  0.955193
p5  0.930493
p6  0.970772
p7  0.954280
p8  0.936345
p9  0.946482
Best sensitivity2:  p1

Recall2:              0
p0  0.969789
p1  0.984224
p2  0.950382
p3  0.924883
p4  0.968008
p5  0.971897
p6  0.955807
p7  0.967456
p8  0.963041
p9  0.958835
Best recall2:  p1
```

Je calcule le F1 de la meilleurs classification pour le dataset a et b et le compare:

```
In [165]: F1a = 2 * (csensitivity['p1']*crecall['p1']) / \
                (csensitivity['p1']+crecall['p1'])
```

```
print("F1a: ", F1a)

F1b = 2 * (csensitivity2['p1']*crecall2['p1']) / \
      (csensitivity2['p1']+crecall2['p1'])

print("F1b: ", F1b)
```

```
F1a:  0.9682194166303876
F1b:  0.9868189806678382
```

Le dataset b est donc bien meilleurs.

1.2 Exercice 2 Classification system

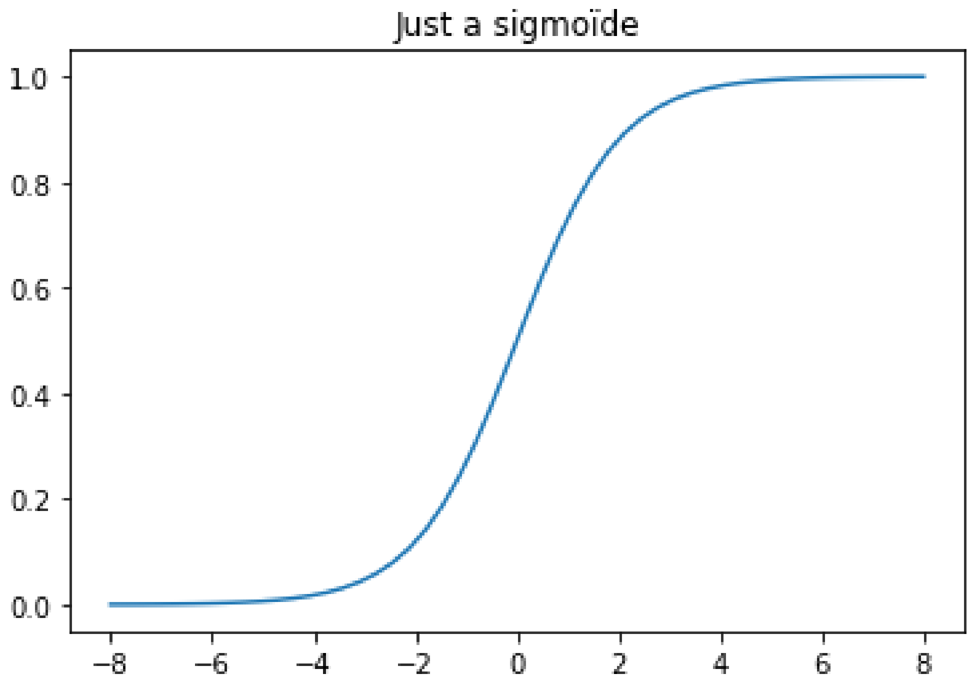
1.2.1 a. Getting the training data

```
In [192]: dataset = pd.read_csv('data/ex2-data-train.csv', names=['x1', 'x2', 'y'])

          N = len(dataset['x1'])

          x1 = dataset['x1'].values
          x2 = dataset['x2'].values
          y = dataset['y'].values
```

1.2.2 b. Logistic regression classifier with linear decision boundary

a) Implement a sigmoid function. Use numpy to compute the exp so that your function can take numpy arrays as input. Plot the sigmoid function.

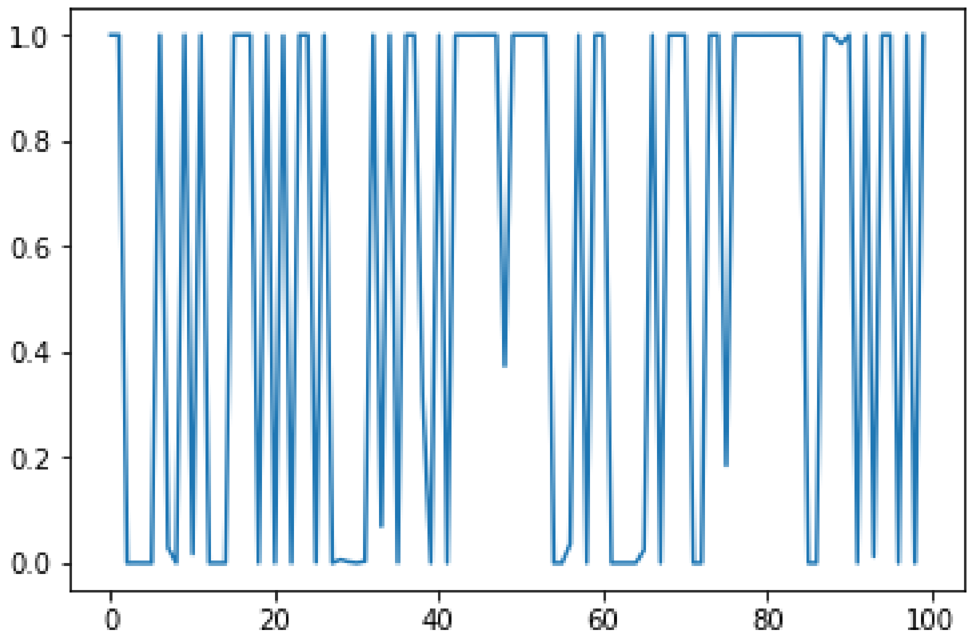
```
In [478]: def sig(X):
              return 1.0 / (1.0 + np.exp(-X))

          xs = np.linspace(-8, 8)

          plt.plot(xs, sig(xs))
          plt.title("Just a sigmoïde")
          plt.show()

          print(sig(200))
```




```
1.0
```

b) Implement the hypothesis function h(x)

```
In [531]: X = np.matrix( [np.ones(len(x1)), x1, x2] ).T

          thethas = [0, -1, 1]

          def h(Xn, thethas):
              result = []
              # Xn is like
              # [
              #    [1 x10 x20],
              #    [1 x11 x22]
              # ]
              # thethas is like: [thetha0, thetha1, thetha2]
              return sig(np.matrix(Xn).dot(np.matrix(thethas).T))


          plt.plot(np.arange(0, len(x1), 1), h(X, thethas))
          plt.show()
```



c) Implement the objective function J() :

```
In [641]: thethas = [1, 1, 1]

          def j(X, y, thethas):

              sum = 0
              for i in range(len(X)):
                  sum = sum + y[i] * np.log(h(X[i], thethas).item(0)) + (1 - y[i]) * \
                                                            np.log(1 - h(X[i], thethas))
              sum = sum * 1.0 / N
              return sum.item(0)
```

d) In a similar way as in PW02 and PW03, implement the gradient ascent with the update rule :

```
In [568]: N = len(x1)
          X = np.matrix( [np.ones(len(x1)), x1, x2] ).T

          thethas = [1, 1, 1]

          def updateThetha(thethas, X, y, alpha, thethai=-1):
              thetharange = range(len(thethas))
              if thethai != -1:
                  thetharange = [thethai]
              # for each thetha
              for l in thetharange:
```

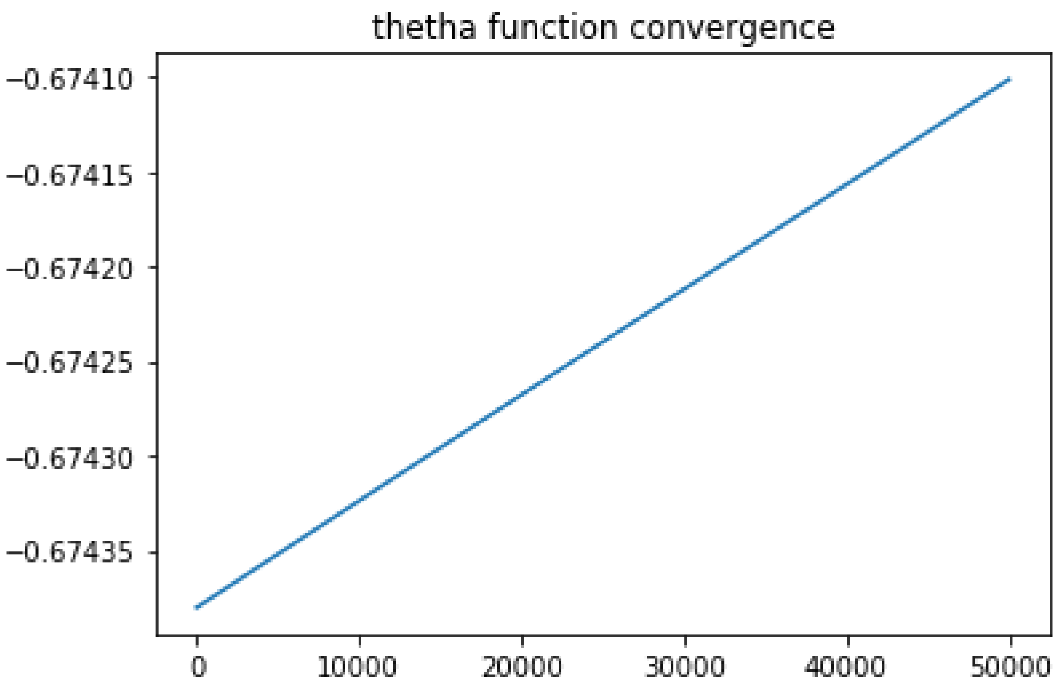
```
            sum = 0
            # do the sum
            for i in range(N):
                sum = sum + (y[i] - h(X[i], thethas).item(0)) * X[i,l]
            sum = sum * alpha * 1.0 / N
            # update the thetha
            thethas[l] = thethas[l] + sum

        return thethas

    for i in range(10):
        thethas = updateThetha(thethas, X, y, 0.000001)

    print(thethas)
```

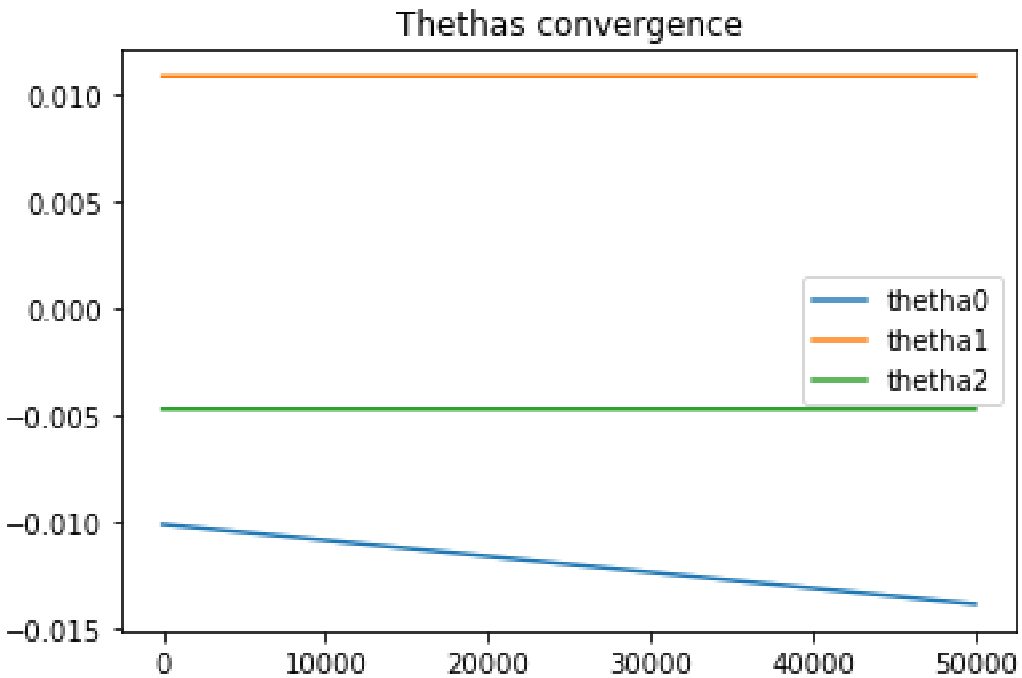
```
[0.99999500000000041, 0.99976484199182392, 0.99971816233712674]
```

e) Test your implementation by observing the evolution of the objective function `J(thetha)` during the gradient ascent

```
In [570]: results = []
          steps = 50000

          thethas = [0, 0, 0]
          # après 15000
          thethas = [-0.0010939110746739354, 0.0087227663757557319, -0.0030173491474521884]
          # après 10000
          thethas = [-0.0018466122230984753, 0.0099659937736634435, -0.0040992829381749658]
          # après 10000
          thethas = [-0.002598695947319265, 0.010452841979434073, -0.0045155857644742633]
          # après 100'000
          thethas = [-0.010112392859010402, 0.010812747166791963, -0.0047208550101313 51]
          # À ce stade, je remarque que thetha0 ne converge pas encore. Je relance donc
          # juste pour thetha0
          # après 50000 juste pour thetha0
          thethas = [-0.013844793527077574, 0.010812747166791963, -0.004720855010131351]

          thethas0 = []
          thethas1 = []
          thethas2 = []

          for i in range(steps):
              thethas = updateThetha(thethas, X, y, 0.000001, thethai=0)
              results.append(j(X, y, thethas))
              thethas0.append(thethas[0])
              thethas1.append(thethas[1])
              thethas2.append(thethas[2])
```

```
plt.plot(np.arange(0, steps, 1), results)
plt.title('thetha function convergence')
plt.show()

xss = np.arange(0, len(thethas0), 1)
plt.plot(xss, thethas0, label="thetha0")
plt.plot(xss, thethas1, label="thetha1")
plt.plot(xss, thethas2, label="thetha2")
plt.legend()
plt.title("Thethas convergence")
plt.show()

print("Théthas: ", thethas)
```






```
Théthas:  [-0.013844793527077574, 0.010812747166791963, -0.004720855010131351]
```

f) Compute the correct classification rate on ex2-data-test.csv after convergence assu- ming you have an estimator of the posterior probabilities with `P(yn = 1|xn;) = h(xn)` `P(yn =0|xn;)=1h(xn)`

```
In [588]: dataset = pd.read_csv('data/ex2-data-test.csv', names=['x1', 'x2', 'y'])

          N = len(dataset['x1'])

          x1 = dataset['x1'].values
          x2 = dataset['x2'].values
          y = dataset['y'].values

          ones_x1 = [x1[i] for i in range(0, N) if y[i] == 1]
          ones_x2 = [x2[i] for i in range(0, N) if y[i] == 1]

          zero_x1 = [x1[i] for i in range(0, N) if y[i] == 0]
          zero_x2 = [x2[i] for i in range(0, N) if y[i] == 0]

          thethas = [-0.013844793527077574, 0.010812747166791963, -0.004720855010131351]
          N = len(x1)
```

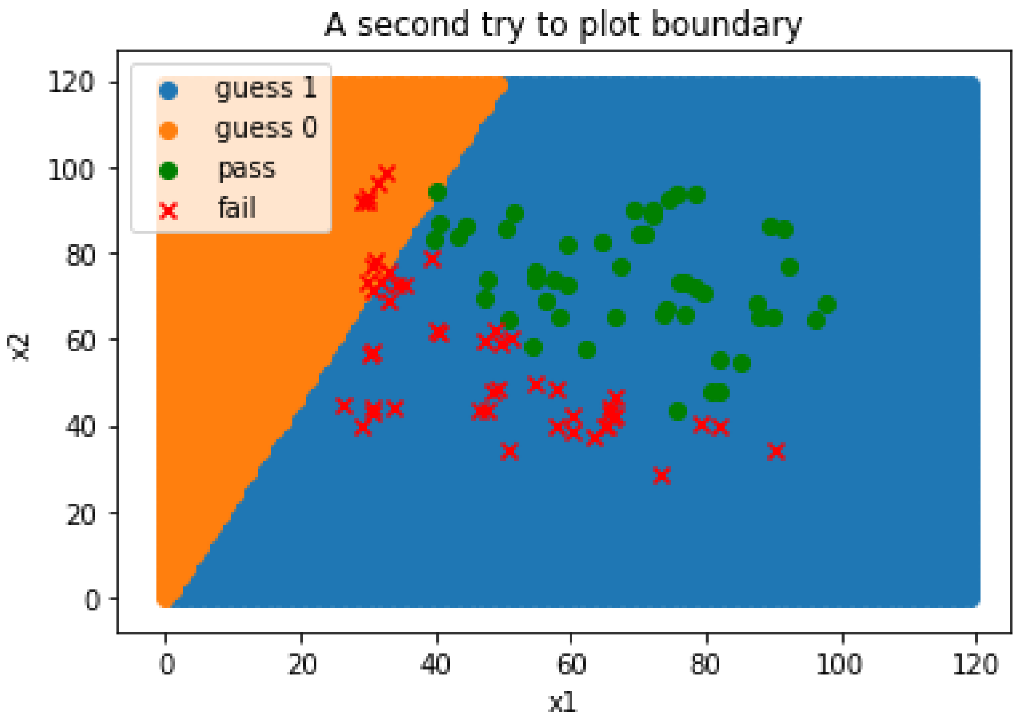
```
true_guess = 0
for i in range(len(x1)):
    prob1 = h([1, x1[i], x2[i]], thethas)
    prob0 = 1 - prob1
    guess = 1
    if prob0 > prob1:
        guess = 0
    if y[i] == guess:
        true_guess = true_guess + 1

print("Performance: ", true_guess / N)
print("Error rate: ", (N - true_guess) / N)
```

```
Performance:  0.6
Error rate:  0.4
```

g) Draw the decision boundary of your system on top of the scatter plot of the testing data.

In [635]:
```
thethas = [-0.013844793527077574, 0.010812747166791963, -0.004720855010131351]

# on a donc thetha0, thetha1, thetha2
# notre droite est donc thetha2*x2 = -thetha1*x1 - thetha0

def bound(x1):
    return (-thethas[1]*x1 - thethas[0]) * 1.0 / thethas[2]

plt.plot(np.linspace(0,100), bound(np.linspace(0,100)), label="best")

thethas = [-0.0018466122230984753, 0.0099659937736634435, -0.0040992829381749658]
plt.plot(np.linspace(0,100), bound(np.linspace(0,100)), label="less good")
plt.scatter(ones_x1, ones_x2, marker="o", label="pass", color="green")
plt.scatter(zero_x1, zero_x2, marker="x", label="fail", color="red")
plt.xlabel('x1')
plt.ylabel('x2')
plt.legend()
plt.title("A first try to plot boundary")
plt.show()

square = 120

x1s = np.arange(0, square, 1)
x2s = np.arange(0, square, 1)

onepx = []
onepy = []
zeropx = []
zeropy = []
```

```
for ix in range(0, square):
    for iy in range(0, square):
        prob1 = h( np.matrix([1, ix, iy]), thethas )
        prob0 = 1 - prob1
        guess = 1
        if prob0 > prob1:
            guess = 0
        if guess == 1:
            onepx.append(ix)
            onepy.append(iy)
        else:
            zeropx.append(ix)
            zeropy.append(iy)

plt.scatter(onepx, onepy, label="guess 1")
plt.scatter(zeropx, zeropy, label="guess 0")
plt.scatter(ones_x1, ones_x2, marker="o", label="pass", color="green")
plt.scatter(zero_x1, zero_x2, marker="x", label="fail", color="red")
plt.xlabel('x1')
plt.ylabel('x2')
plt.legend()
plt.title("A second try to plot boundary")
plt.show()
```



The decision boundary is quite wrong. We suppose that either our process is wrong or we neeed mor iterations. The iteration process took a long time, about an hour!

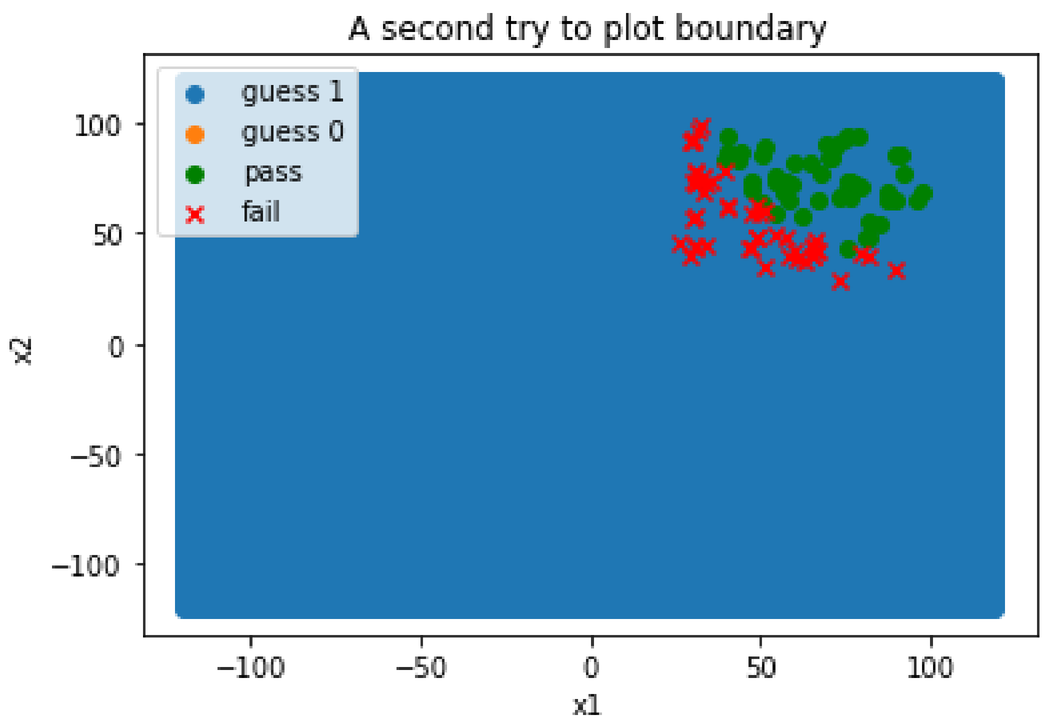
h) Compare the performance of the logistic regression system with the ones of previous's week. Here we have a performance of 60%. In the other work, we had a performance of about 91%. But we think that more iterations would bring better results. ### c. Logistic regression classifier with non-linear decision boundary

Redo the experiments of 2.b by increasing the complexity of the model in order to have a non-linear decision boundary.

```
In [639]: thethas = [0, 0, 0, 0, 0, 0]

          def h(Xn, thethas):
              result = []
              # Xn is like
              # [
              #   [1 x10 x20 x10^2 x20^2 x10x20],
              #   [1 x11 x21 x11^2 x21^2 x11x21],
              # ]
              # thethas is like: [thetha0, thetha1, thetha2, ... thetha5]
              return sig(np.matrix(Xn).dot(np.matrix(thethas).T))

          def updateThetha(thethas, X, y, alpha, thethai=-1):
              thetharange = range(len(thethas))
```

```
    if thethai != -1:
        thetharange = [thethai]
    # for each thetha
    for l in thetharange:
        sum = 0
        # do the sum
        for i in range(N):
            sum = sum + (y[i] - h(X[i], thethas).item(0)) * X[i,l]
        sum = sum * alpha * 1.0 / N
        # update the thetha
        thethas[l] = thethas[l] + sum

    return thethas

steps = 10000

thethas = [0, 0, 0, 0, 0, 0]

dataset = pd.read_csv('data/ex2-data-train.csv', names=['x1', 'x2', 'y'])

N = len(dataset['x1'])

x1 = dataset['x1'].values
x2 = dataset['x2'].values
y = dataset['y'].values

X = np.matrix( [np.ones(len(x1)), x1, x2, x1*x1, x2*x2, x1*x2] ).T

for i in range(steps):
    thethas = updateThetha(thethas, X, y, 0.00001, thethai=0)


# ------------- Plot
square = 120

x1s = np.arange(-square, square, 1)
x2s = np.arange(-square, square, 1)

onepx = []
onepy = []
zeropx = []
zeropy = []

for ix in range(-square, square):
    for iy in range(-square, square):
        prob1 = h( np.matrix([1, ix, iy, ix*ix, iy*iy, ix*iy]), thethas )
        prob0 = 1 - prob1
        guess = 1
```

```
            if prob0 > prob1:
                guess = 0
            if guess == 1:
                onepx.append(ix)
                onepy.append(iy)
            else:
                zeropx.append(ix)
                zeropy.append(iy)

    plt.scatter(onepx, onepy, label="guess 1")
    plt.scatter(zeropx, zeropy, label="guess 0")
    plt.scatter(ones_x1, ones_x2, marker="o", label="pass", color="green")
    plt.scatter(zero_x1, zero_x2, marker="x", label="fail", color="red")
    plt.xlabel('x1')
    plt.ylabel('x2')
    plt.legend()
    plt.title("A second try to plot boundary")
    plt.show()
```



Our more complex model is not better!

```
In [ ]:
```