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Logic, Sets, Functions and Relations

- Analyze the meanings of predicates
- Convert logical formulas to and from English sentences
- Analyze logical formulas
- Describe and explain sets and set-elements
- Identify and analyze functions and relations

Q1

Given the following predicates and their meanings

- $P(x, y) : x > y$
- $Q(x, y) : x \leq y$
- $R(x) : x - 7 = 2$
- $S(x) : x > 9$

> If the universe of discourse is the real numbers, give the truth value (true or false) of each of the following propositions:

1. $(\exists x)R(x)$
2. $(\forall y)[\neg S(y)]$
3. $(\forall x)(\exists y)P(x, y)$
4. $(\exists y)(\forall x)Q(x, y)$
5. $(\forall x)(\forall y)[P(x, y) \vee Q(x, y)]$
6. $(\exists x)S(x) \wedge \neg(\forall x)R(x)$
7. $(\exists y \forall x)[S(y) \wedge Q(x, y)]$
8. $(\forall x)(\forall y)[R(x) \wedge S(y) \rightarrow Q(x, y)]$

Your answer:

1. Interpretation: There exists some value x (in designated universe) for which x-7 = 2.

Answer: True. (e.g. x = 9)

1. Interpretation: For every value of y (in designated universe), the predicate does not hold that y > 9.

Answer: False. (e.g. y = 8)

1. Interpretation: For every value of x, there exists a y in which x > y.

Answer: True. (e.g. x = -9999, y = -9999.1)

1. Interpretation: There exists a y for which every x value <= y.

Answer: False. (e.g. y = 9999, x = 9999.1)

1. Interpretation: For every value of x, every value of y holds the condition that x > y or x <= y.

Answer: True. (e.g. x=1, y=1)

1. Interpretation: There exists a value x for which x > 9 holds -- AND -- there are no values of x for which x - 7 = 2.

Answer: False. (e.g. x = 10 for first statement (True), but there are values within specified domain (like 9) for which x-7=2 holds).

1. Interpretation: There exists a y value, for which every x value holds y > 9 and x <= y.

Answer: False. (e.g. y = 10, x = 11).

1. Interpretation: For all values of x, every value of y holds the following condition -- if x - 7 = 2 and x > 9, then x <= y.

Answer: True. There will never be a combination of x and y in which the first proposition is true, so the resulting conditional statement can never meet the singular false condition of T -> F.

Q2

> Which of the following sentences has the logical form $(p \wedge q) \rightarrow r$?

1. If you don’t attend the wedding, then Sam will be angry with you
2. Matt is happy and so are Sam and Fae
3. If it rains and it snows then flooding will result
4. Students will play football or students will play soccer; but they will not attend classes
5. Gene is smart and strong, additionally he is a good swimmer

Your answer:

Interpretation: If p and q, then r.

Answer: The closest correct proposition appears to be number 3, because it suggests a contract/conditional relationship.

Q3

- p means There are no fruit in the market
- q means Farmers didn't plant fruit trees
- r means Farmers didn't water the trees

> Which of the following formulas represents the sentence, "If there are no fruit in the market then the farmers didn't plant fruit trees or the farmers didn't water the trees"

1. $\neg p \rightarrow q$

1. $p \rightarrow q \vee r$

1. $(p \rightarrow q) \vee \neg r$

1. $p \rightarrow q \vee \neg r$

1. $p \vee q \rightarrow \neg r$

Your answer: The answer is number 2.

Q4

> Show $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology.

Your answer:

Scenario 1: p = T, q = T

1. [T∧(T→T)]→T
2. [T∧T]→T
3. [T]→T == T

Scenario 2: p = T, q = F

1. [T∧(T→F)]→F
2. [T∧F]→F
3. [F]→F == T

Scenario 3: p = F, q = T

1. [F∧(F→T)]→T
2. [F∧T]→T
3. [F]→T == T

Scenario 4: p = F, q = F

1. [F∧(F→F)]→F
2. [F∧T]→F
3. [F]→F == T

Propositional Logic

The following exercises give you a chance to refresh your knowledge and test your understanding of some of the basic concepts of Propositional Logic.

Q1

Which of the following propositional formula(s) represent(s) the sentence,

“Tim will go to Charlottesville or Tim will go to Fairfax; if the former, he will visit our classroom”.

> Let the three atomic propositions above be p, q, & r

1. $p \to q \land r$
2. $p \land q \to r$
3. $(p \to q) \land (p \land r)$
4. $p \lor \neg q \to r$
5. $(p \lor q) \land (p \to r)$.

Your answer: Answer 4.

Q2

> Which of the following sentences has the logical form $(p \wedge q) \rightarrow r$

1. If the baby is crying then you can't sleep
2. My students are smart and hardworking
3. Jane will go by bus but Tom will walk
4. If the sky is clear or the moon is full then night driving will be safe
5. If our team wins and all the students get A+ grades then everyone will be happy.

Your answer: Answer 5.

Q3

A = The field is flooded

B = It rained 22 inches

C = The plants will die

> Make propositions from A, B and C to match the following:

1. $A \vee B$: *``The field is flooded or it rained 22 inches (or both)"*.
2. $A \wedge B$: *Your answer* ``The field is flooded and it rained 22 inches."
3. $B \rightarrow A$: *Your answer* ``If it rained 22 inches, then the plant will die."
4. $(A \wedge B) \rightarrow C$: *Your answer* `` If the field is flooded and it rained 22 inches, then the plants will die."
5. $(A \vee B) \rightarrow C$: *Your answer* ``If the field is flooded or it rained 22 inches, then the plants will die"
6. $(\neg A \wedge \neg B) \rightarrow \neg C$: *Your answer* `` If the field is not flooded and it did not rain 22 inches, then the plants will not die."

Q4:

> Why an atomic proposition can't be a tautology?

Your answer: An atomic proposition has a truth value of true or false, by definition. It is the most basic structure of propositional logic. A tautology is a result in which any interpretation of a given sentence will yield true. However, because atomic propositions are either true or false at their essence, they are not really "interpretable" in the way that assessing a tautology requires.

Q5

Given:

A= Kat met Clayton

B= Kat and Clayton had coffee together

C= Kat and Clayton went swimming

> Present the following as symbolic formulas.

1. Kat and Clayton had a cup of coffee then went swimming
2. Kat met Clayton then they had a cup of coffee and went swimming

Your answer:

1. B ∧ C
2. A ∧ (B ∧ C)

Q5

> Is this proposition a tautology: $B \to \neg(\neg B)$

Your answer: Yes, this proposition is a tautology. The only way for a conditional to be assessed as "false" is if the value to the left of the implication goes from T to F. This will never happen, because whenever B is T, the implication will be T -> T, which yields T.

Relations and Functions

Q1

Using the sets below of students and test scores (highest grade = 100), write out the expression, domain, range, and codomain.

title

Your answer: S = {(Joe, 90), (Daniel, 99)}; domain = {Joe, Daniel}; range = {90,99}; codomain = {x|0<= x <= 100, x is a whole number}

Q2

Let S be the set of students at UVA, let M be the set of sporting events at UVA, and let $V(s; m)$ be "student s has been to m." Rewrite each of the following statements without using the symbol $\forall$, and $\exists$, or variables.

1. $\exists s|S$ such that $V(s;$ Basketball)
2. $\forall s|S$, $V(s; Football)$
3. $\forall s|S$, $\forall m \in M$ such that $V(s, m)$
4. $\exists m \in M$, such that $\forall s \in S$, $V(s, m)$

Your answer:

1. Answer: There exists a student within the UVA student set that has been to a basketball sporting event.
2. Answer: Every student within the UVA student set has been to a football sporting event.
3. Answer: Every student has attended a sporting event.
4. Answer: There exists a sporting event for which all students have attended that sporting event.

Q3

For each of the following relations on the set {5, 6, 7, 8} decided whether it is reflexive, symmetric, and/or transitive. Which of these are equivalence relations?

1. {(6,6), (6,7), (6,8), (7,6), (7,7), (7,8)}
2. {(5,5), (5,6), (6,5), (6,6), (7,7), (8,8)}
3. {(6,8), (8,6)}

Your answer:

1. Transitive.
2. Symmetric, Transitive, and Reflexive. This is an equivalence relation.
3. Symmetric.

Q4

Calculate the composition $(g(f(x)))$ of the following two functions, $f(x)$ and $g(x)$:

$$f(x) = x + 5$$
$$g(x) = x^2 + 2x + 10$$

Your answer:

g(f(x)) = (x+5)^2 + 2(x+5) + 10

= x^2 + 10x + 25 + 2x + 10 + 10

g(f(x)) = x^2 + 12x + 45

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In [ ]:
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