1 Theory

The purpose of this program (entitled $nkeiru\_ubadike\_hw8.f08$) is to solve the Poisson equation given by

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 4\pi q \tag{1}$$

Where U is the potential and q is the charge.

Equation 1 is solved numerically using the finite difference method for a grounded metal box. Thus the boundary conditions are U = 0 on the four edges of the box. This method produces a discretized approximation of equation 1 given by

$$\frac{U_{i+1,j} + U_{i-1,j} + U_{i,j+1} U_{i,j-1} - 4U_{i,j}}{h^2} = 4\pi q_{i,j} \tag{2}$$

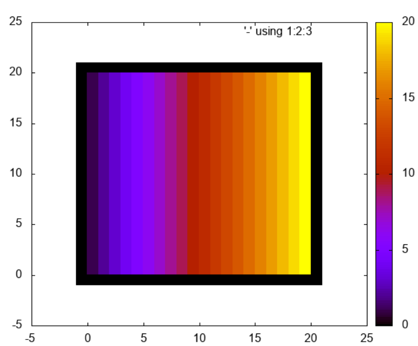
Equation 2 is solved for $U_{i,j}$ and an initial guess $U^0_{i,j}$ is used to obtain a new guess $U_{i,j}$

$$U^1_{i,j} = \frac{U^0_{i+1,j} + U^0_{i-1,j} + U^0_{i,j+1} + U^0_{i,j-1} - 4\pi h^2 q_{i,j}}{4} \tag{3}$$

This process is repeated until the solution converges. This method is known as the relaxation method. The convergence criteria is $max(|(U^0_{i,j} - U^1_{i,j}|) < accuracy$

2 Input Parameters

The number of cells in the i and j direction is denoted as $\mathbf{n_i}$ and $\mathbf{n_j}$ respectively. In this case, $n_i = n_j = 20$ Each cell is a square with $\mathbf{h_{size}} = 1.0$. The charge density $\mathbf{q_{den}}$, is a gradient of electric charge that is increasing the in the positive i direction and is constant in the j direction. The accuracy = 1.0 $\times 10^-5$ determines when our solution converges.



Charge density plot

Our charge density is pictured in figure 2.

3 Results

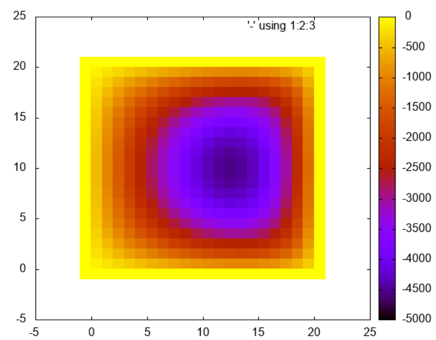


Figure 1: Potential density plot

The greatest region of potential is negative and is centered to the left of the box. The boundary conditions, namely U = 0 on all four edges, are clearly pictured in Figure 1.