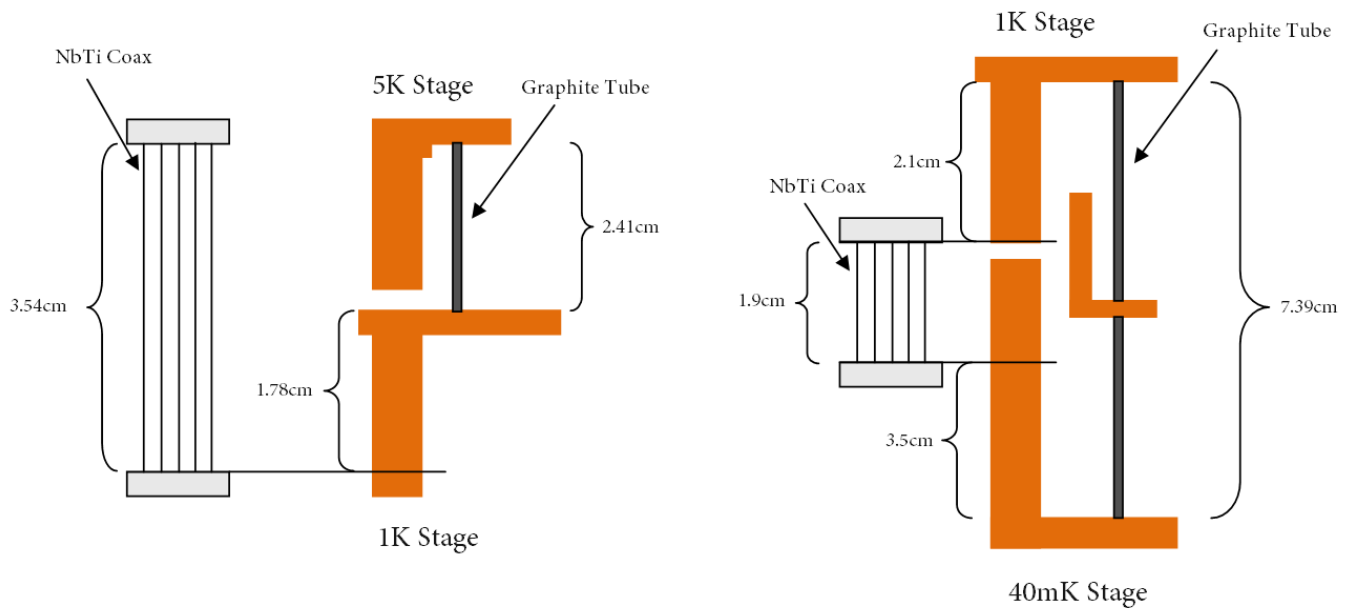


# NbTi Side Coax Stresses

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October 13, 2012

## 1 Tower Model for Contraction Lengths



**Figure 1:** The following tower model was used in considering the relative contractions of the tower components.

The above tower model was used to determine the tensile stress on the side coaxes. These models consider the contiguous structure supporting the NbTi side coaxes, and its length change. This is compared to the length change of the NbTi wires.

## 2 Stress on 5K - 1K Span Coax

To calculate the stress on the NbTi wires, the strain was first calculated. Referring to Figure 1, we see that the contiguous structure supporting the side coax consists of 2.41cm of graphite tube and 1.78cm of copper. The contraction of these will bring the ends of the wire closer together, decreasing strain. Conversely, the 3.54cm of NbTi wire will contract, acting to increase the strain. A third factor which comes into play is the tension created in the wire during the construction of the side coax. Combining all these effects, we calculate the wire length at 4K and compare it to the equilibrium length. This gives a strain which can be translated into a stress.

### 3 Coefficients of Thermal Expansion

Using NIST's database, the temperature-dependent CTE of OFHC Copper was found. This is shown in figure 2 to the right.

The equation for the thermal contraction coefficient could then be integrated from 300K to 4K (the lowest data range for the equation) to get total fractional contraction.

The contraction of NbTi was found differently from copper. An equation for integrated thermal contraction was used, rather than one for just the coefficient. This equation reads,

$$\frac{L_T - L_{293}}{L_{293}} = (a + bT + cT^2 + dT^3 + eT^4) \cdot 10^{-5}$$

where a,b,c,d,e are a measured set of coefficients. This gives the fractional contraction from 293K to some  $T \geq 4K$ . For the sake of comparison to the CTE of copper, this formula was plotted for T from 4K to 293K. The slope between all points was then found, which gave the temperature-dependent CTE. This is shown in Figure 3 below copper.

The CTE for UF-4S graphite was obtained from a data sheet provided by Mersen, and is not given as a function of temperature. The CTE for UF-4S graphite is low, and depends on the grain orientation:

$$CTE_{\parallel} = 1.8 \cdot 10^{-6}/C$$

$$CTE_{\perp} = 2.9 \cdot 10^{-6}/C$$

where  $CTE_{\parallel}$  is measured with the grain, and  $CTE_{\perp}$  against the grain. The lower value of CTE is used in the calculations that follow, as a safety buffer.

#### 3.1 Tension from Fabrication

The NbTi wires are soldered under tension to prevent loss of tension (and shorts) at operation temperature. This is done with two 30g weights - one hung at either end - for a total of 60g. Dividing by the cross-section of the wires, we get a stress of  $805MPa$ . The Modulus of Elasticity,  $E$ , for our Nb-47Ti (47% Ti by weight) is  $75.15GPa$ . Using this, we get a strain of,

$$Strain = \frac{Stress}{E} = \frac{75.15e9}{805e6} = 0.0107 \text{ or } 1.07\%$$

so the wires are tensioned 1.07% beyond their equilibrium length.

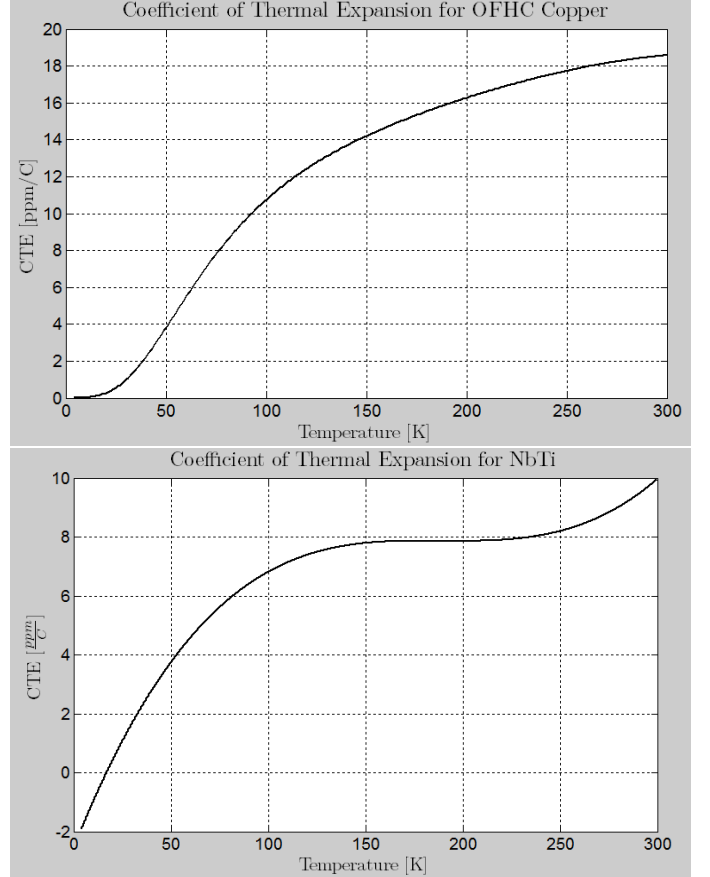
#### 3.2 Total Stress and Strain at 4K

The strain is calculated from

$$Strain = \frac{L - L_{eq}}{L_{eq}}$$

where  $L$  is the actual length of the wires and  $L_{eq}$  is the equilibrium length at 4K. We can see that

$$L = L_{NbTi} - |\Delta L_{Cu}| - |\Delta L_{Tube}|$$



**Figure 2:** Temperature-dependent Coefficient of thermal expansion for OFHC Copper and NbTi. The NbTi coefficient of thermal expansion was inferred from the formula for total contraction of NbTi, which explains its inaccuracy at very low temperatures. At higher temperatures, however, the curve is believable.

while,

$$L_{eq} = L_{NbTi} \cdot (1 - 0.0107 - |\Delta L_{NbTi}|)$$

where  $\Delta L_{Cu}$  is the total contraction of the copper whose length is given in figure 1,  $\Delta L_{Tube}$ , is the graphite tube contraction,  $\Delta L_{NbTi}$  is the NbTi contraction, and  $L_{NbTi}$  is the original length of the wires at 300K (3.54cm in this case). Multiplying by  $1 - 0.0107$  gives the length at 300K without the fabrication tension. Putting these into the strain equation gives 1.05% strain. From this we find,

$$Stress = E \cdot Strain = 75.15e9 \cdot 0.0105 = 791MPa$$

## 4 Stress on 1K - 40mK Span Coax

The analysis carried out for the 1K - 40mK cable is similar to that for the 5K - 1K cable. For this cable, the contiguous support structure is more complicated. First, we have 7.39cm of graphite tube which will act to bring the wire ends closer together, decreasing strain. Second, we have the contracting copper. Due to the design of the tower (coax solder attachment points, etc.) we actually have  $2.1cm + 3.5cm = 5.6cm$  of copper whose contraction will act to increase strain. Third, we have the contraction of the NbTi wires themselves, acting to increase the strain. Finally, we have the same applied tension at before, which increases strain by the same 1.07%.

### 4.1 Total Stress and Strain

The strain is still calculated from

$$Strain = \frac{L - L_{eq}}{L_{eq}}$$

For this span, however, we will have,

$$L = L_{NbTi} + |\Delta L_{Cu}| - |\Delta L_{Tube}|$$

where the sign change in front of  $|\Delta L_{Cu}|$  reflects the tendency to increase separation in this case. As before, we have,

$$L_{eq} = L_{NbTi} \cdot (1 - 0.0107 - |\Delta L_{NbTi}|)$$

with the same designations as before. Putting these into the strain equation gives  $\approx 2.13\%$  strain. This will be slightly less (about 2%) if the low CTE for graphite is used. From this we find,

$$Stress = E \cdot Strain = 75.15e9 \cdot 0.0213 = 1604MPa$$

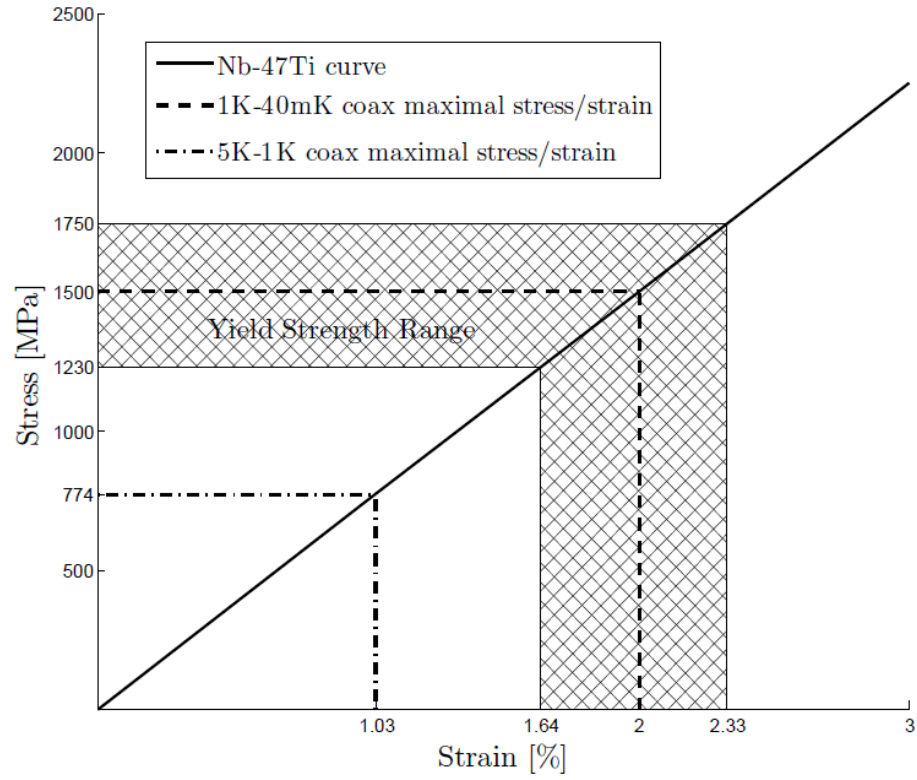
This stress is much higher than the 5K - 1K coax stress. This is expected due to the short length of the wires for this span, as well as the large length of copper which is contracting. Though there is also a large length of graphite, its low CTE means it won't affect the total stress significantly.

## 5 Acceptability of Stresses

To see whether these loads are acceptable or not, we must know the yield strength for our NbTi alloy at base temperature. Our alloy, provided by Tekdata, is 47% by weight Ti, which corresponds to 36.75% *Niobium* in atomic percentage. Yield strengths for similar alloys were found in the literature to range from 1230 MPa (34% atm Nb, cold-worked) to 1750 MPa (39% atm Nb, cold-worked and heat-treated) at 4.2K.

The room temperature stress v. strain curve for our specific NbTi alloy is shown in Figure 3. The range of yield strengths is shown in the hatched region. We see that the 5K - 1K coax is well below either yield strength; however, the 1K - 40mK coax stress falls well into the range for yield strengths of similar compositions.

To know for sure whether this is a real issue in our tower, we would need to 1), obtain the yield strength for our specific alloy/thermo-mechanical preparation and 2), find the modulus of elasticity at low temperatures for our alloy. However, the first order approximation for yield limits is definitely worrisome.



**Figure 3:** Stress v. Strain curve for Tekdata's Nb-47Ti (wt %) alloy at room temperature. The maximal stresses/strains at base temperature are shown for the 5K-1K coax and 1K-40mK coax by a dot-dashed and a dashed line respectively. The range of yield strengths for similar alloys (and their corresponding strains) are given by the hatched region in the plot. We see that the 1K-40mK line may be exceeding its yield strength at base.