Problem Set 1

Due by 2/28, 11:59PM

You will use NumPy module to answer the following questions. Import `numpy` *as* `np`.

(0) Create the following as arrays and as matrices:

$$a = [1, 2, 3, 1], \quad b = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 5 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 7 & 9 & 11 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

(1) Create the following arrays:

$$x = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}, \quad y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad z = \begin{pmatrix} x & x \\ y & y \end{pmatrix}.$$

How would you extract x from z?

(2) Consider the arrays/matrices in (0). Calculate $a \times a^{'}$, $b + a^{'}$, $A \times A^{'}$, A^3 and $A \times B$ (where the multiplication follows the algebra of matrices). Note that $a^{'}$ is the transpose of a and $A^{'}$ is the transpose of A.

(3) Generate a 10×5 and a 5×10 array of matrices by randomly drawing observations from the standard normal distribution. Denote them A and B, respectively. Calculate $C = A \times B$ and replace diagonal elements of C to ones.

(4) Generate a 20×15 array D using A in (3), and a 15×20 array E using B in (3). Calculate $F = D \times E$. Replace nonpositive (≤ 0) elements of F to 0.5.

(5) Reshape D and E in (4) as $3 \times 10 \times 10$ arrays.

(6) Set `x = np.arange(100.0)`. Use `shape` attribute to reshape `x` into 10×10, 20×5, $10 \times 10 \times 1$ arrays.

(7) Set `x = np.reshape(np.arange(100.0),(5,20))`. Use `ravel`, `flatten` and `flat` to extract elements $\{1, 3, \ldots, 99\}$ from `x`.

(8) Let $x = \begin{pmatrix} 16 & 10 \\ 13 & 11 \end{pmatrix}$, $y = 5$, and $z = \begin{pmatrix} 16 & 10 \\ 13 & 11 \\ 12 & 33 \end{pmatrix}$. Use `hstack`, `vstack` and `tile` to generate $m = \left(\begin{array}{c|ccc} x & y & y & y \\ & y & y & y \\ \hline z & \multicolumn{3}{c}{z^{'}} \\ \hline & y & y & y \end{array}\right)$ where $z^{'}$ is the transpose of z. Find the shape and dimension of m. Extract $z^{'}$ from m.

(9) Using the diagonal elements of m in (8) construct a diagonal array. Also, compute eigenvalues, rank, determinant, inverse, trace of m in (8).