

## Inflection points for $\sigma > 0$

# Scale-Based Description and Recognition of Planar Curves and Two-Dimensional Shapes

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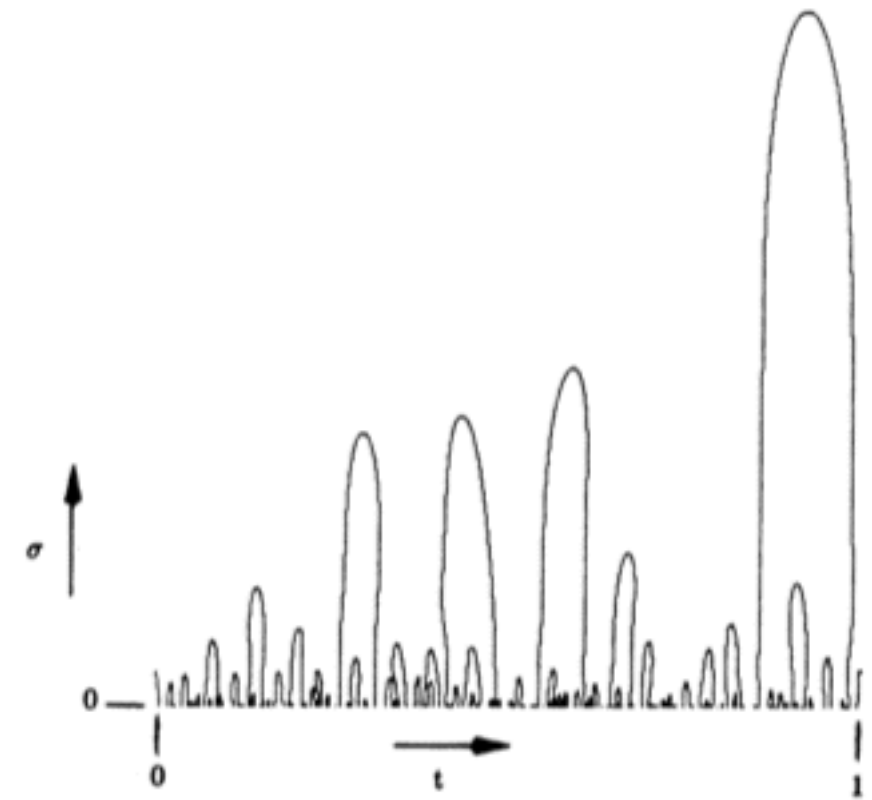
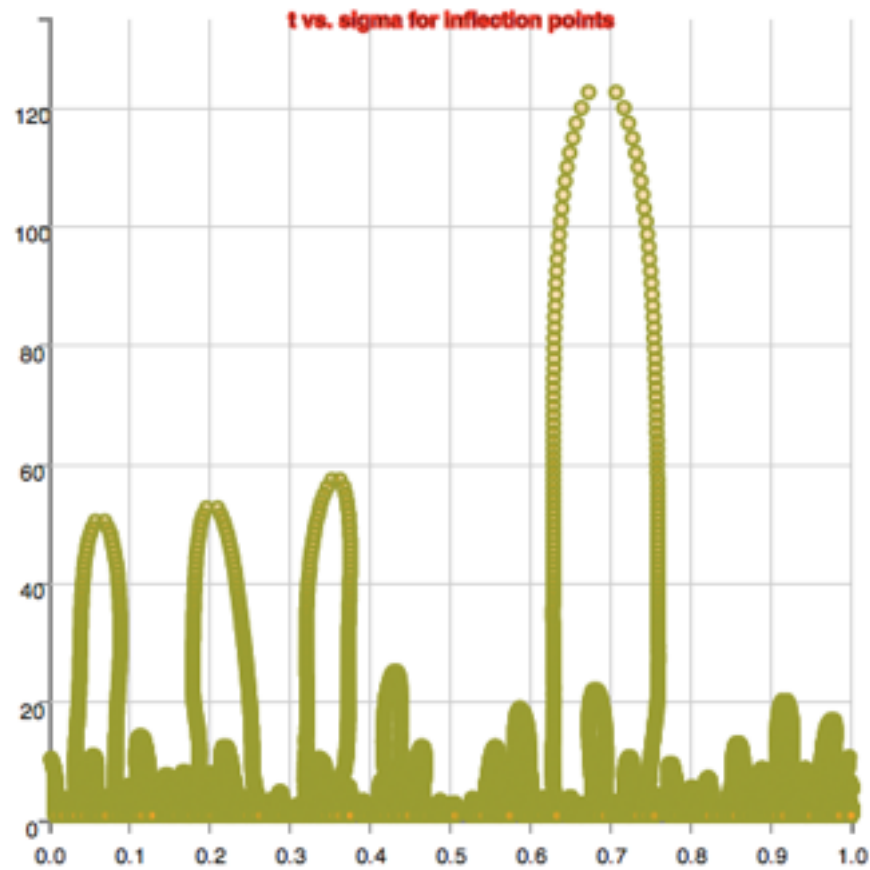
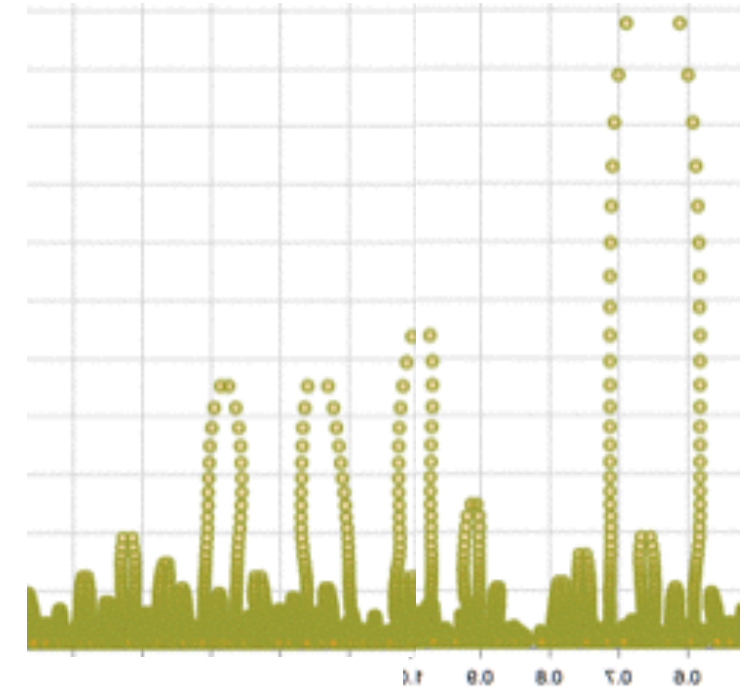


Fig. 3. Generalized scale space image of Africa.

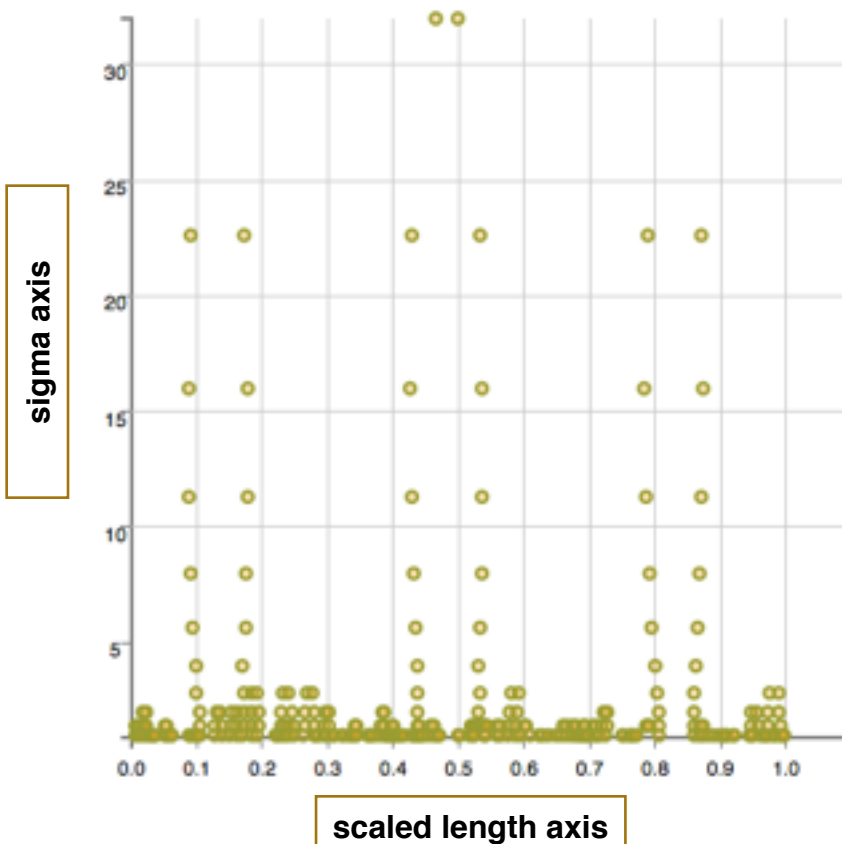
my scale space image agrees with theirs (my implementation follows their paper)



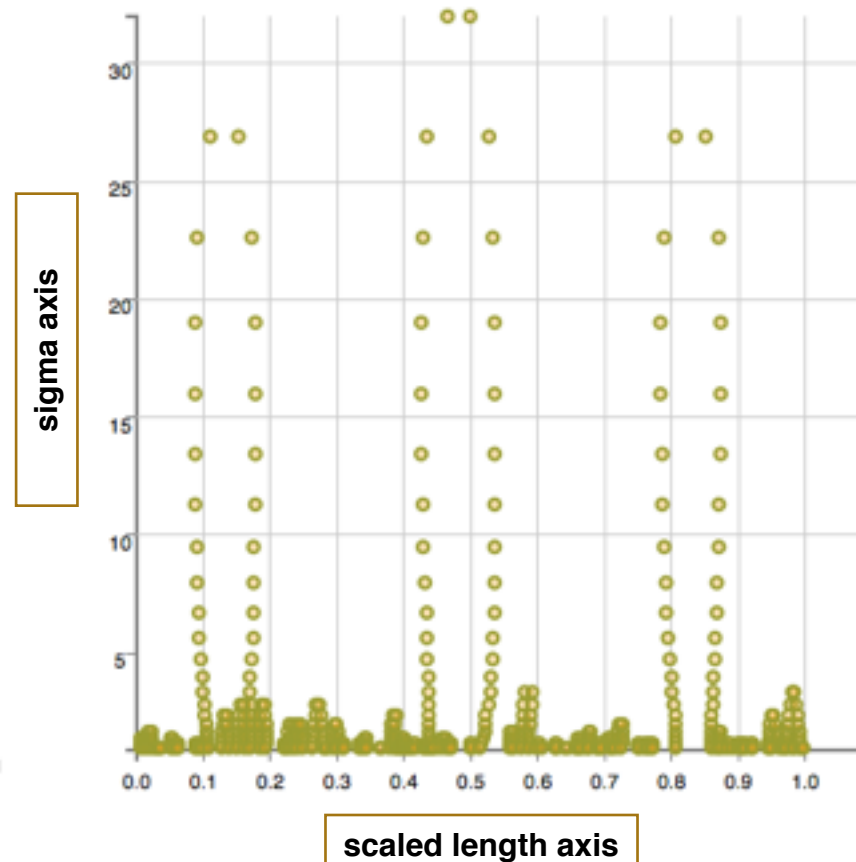
# contour finder



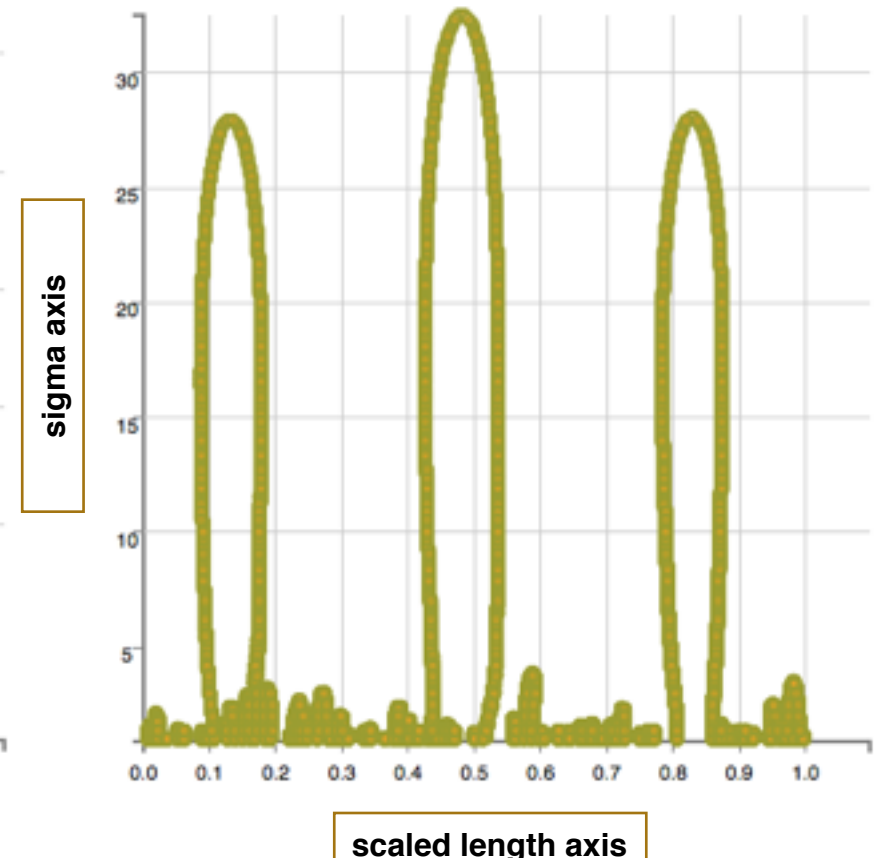
scale space image produced  
for sigma factors of  $\sqrt{2}$



scale space image produced  
for sigma factors of  $2^{1/8}$



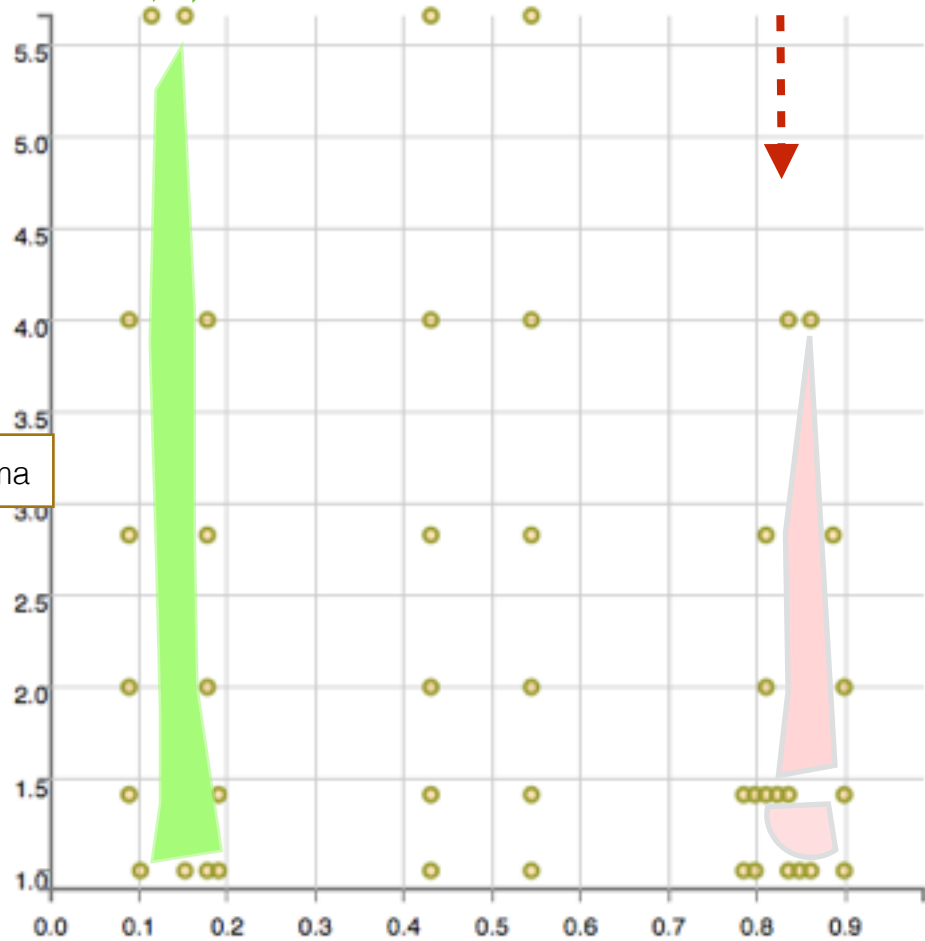
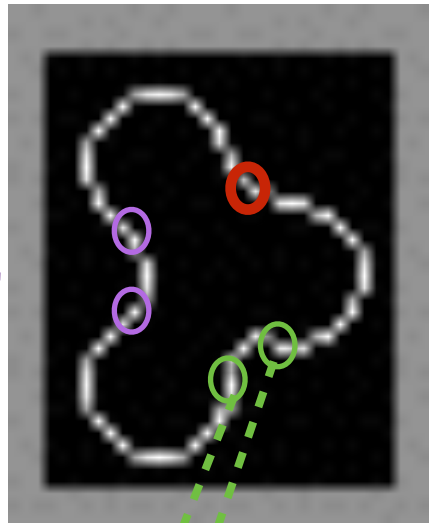
scale space image produced  
for sigma factors of  $2^{1/128}$



There is an error in estimating the peak of a contour for fastest creation of scale space images ( $\leq \sqrt{2}$ ). That error can be reduced overall, by having more contours in the final solution. For an error  $< 10\%$  in determining a contour's peak height, one should choose a sigma factor of  $2^{1/8}$  or smaller. It takes  $2^3$  more convolutions if the smaller sigma factor of  $2^{1/8}$  is used instead of  $2^{1/2}$ .

# contour finder

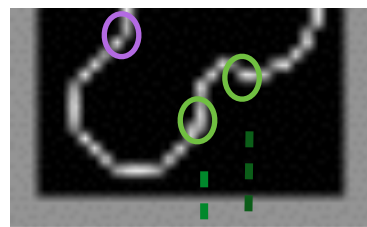
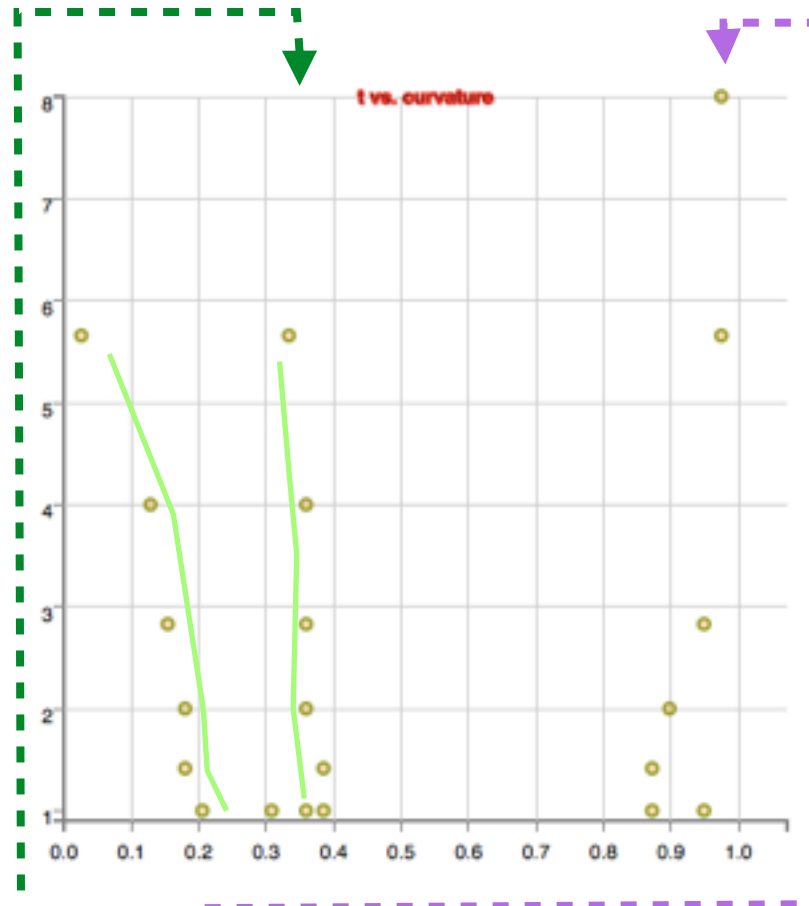
single closed curve's scale space image:



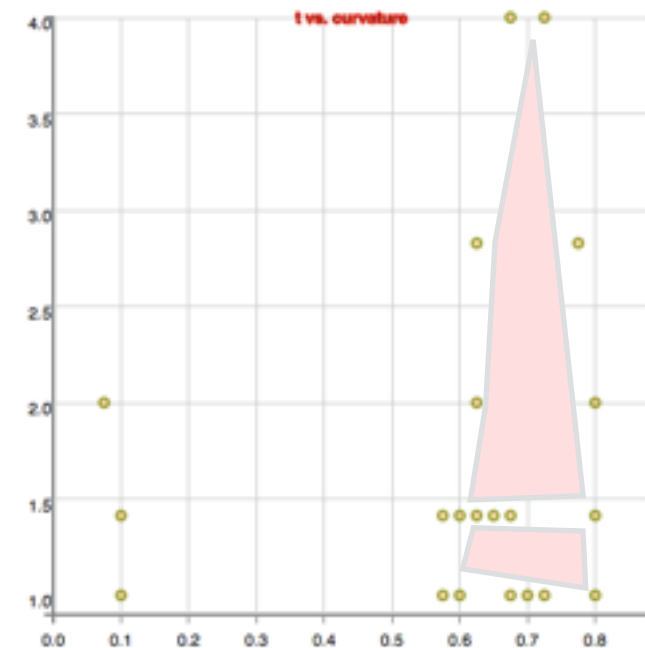
sigma

t (fraction of edge scale length)

divided into 2 curves manually, then made into scale images (that is, 2 open curves possibly create open contours):



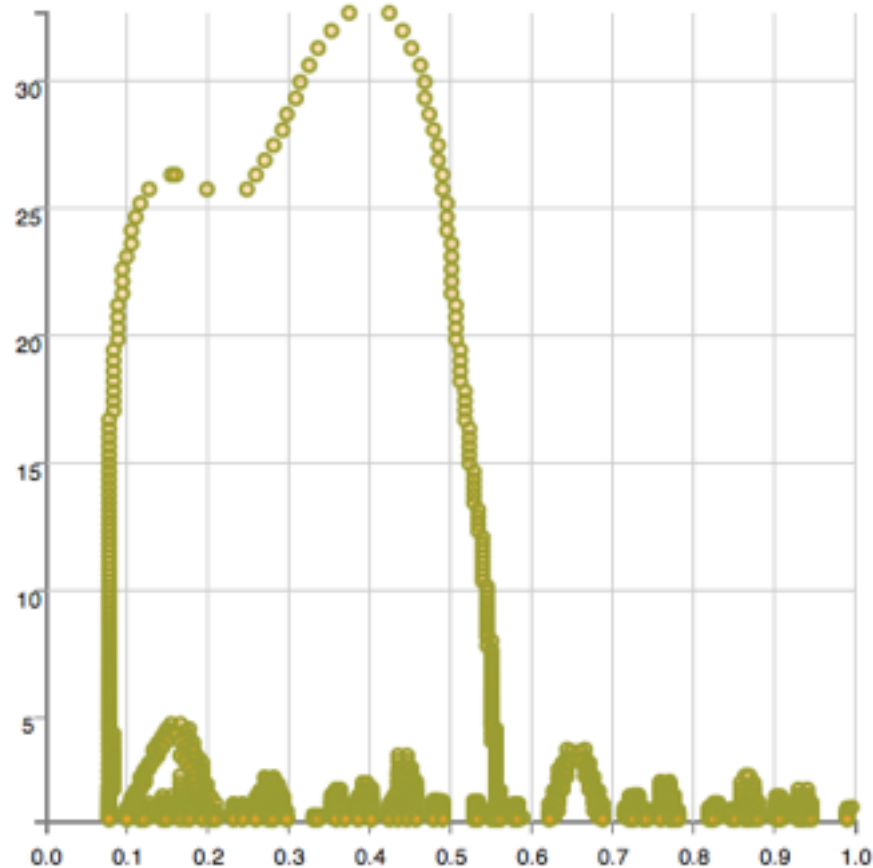
∴ open contours are hard to match in another image's scale images



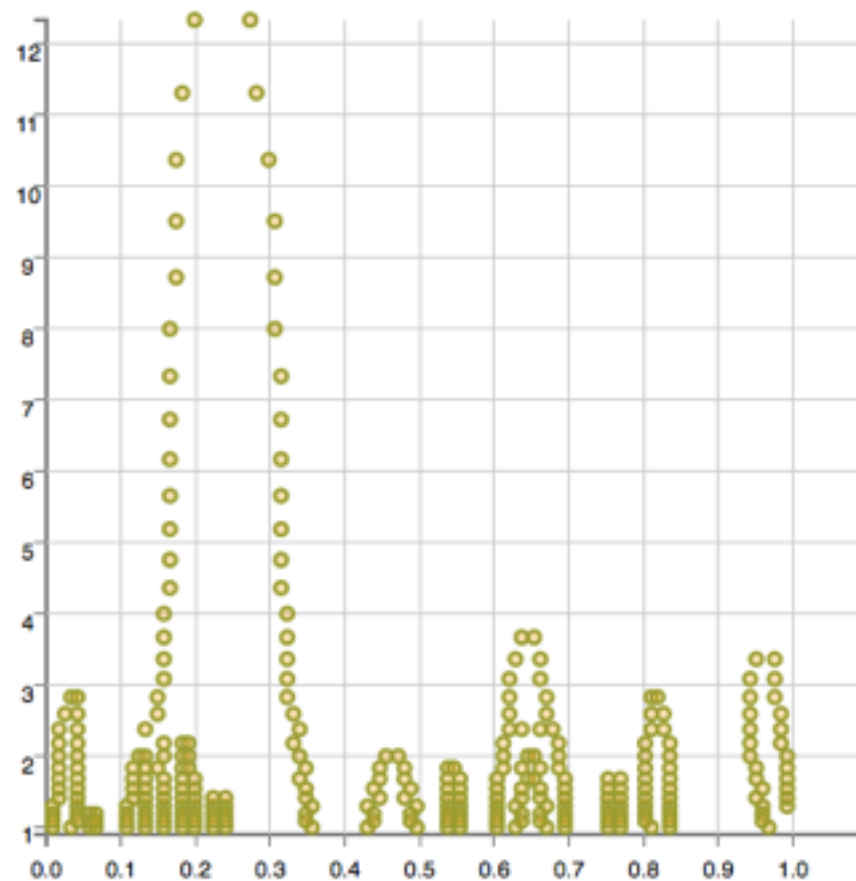
sigma

t (fraction of edge scale length)

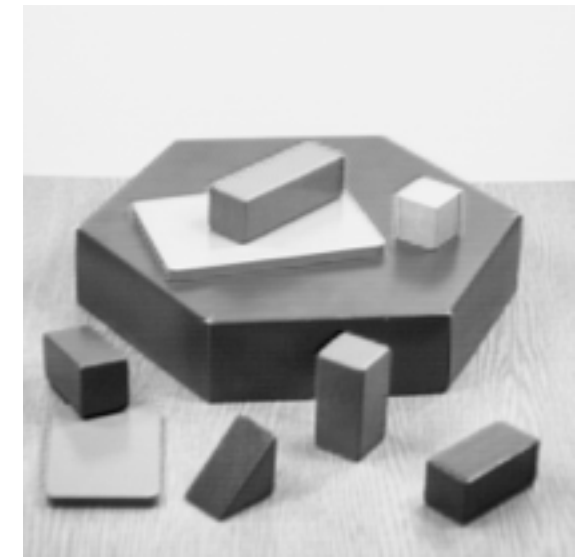
## contour finder



Latest results  
with improved line  
thinner.



Older results  
with older line  
thinner.



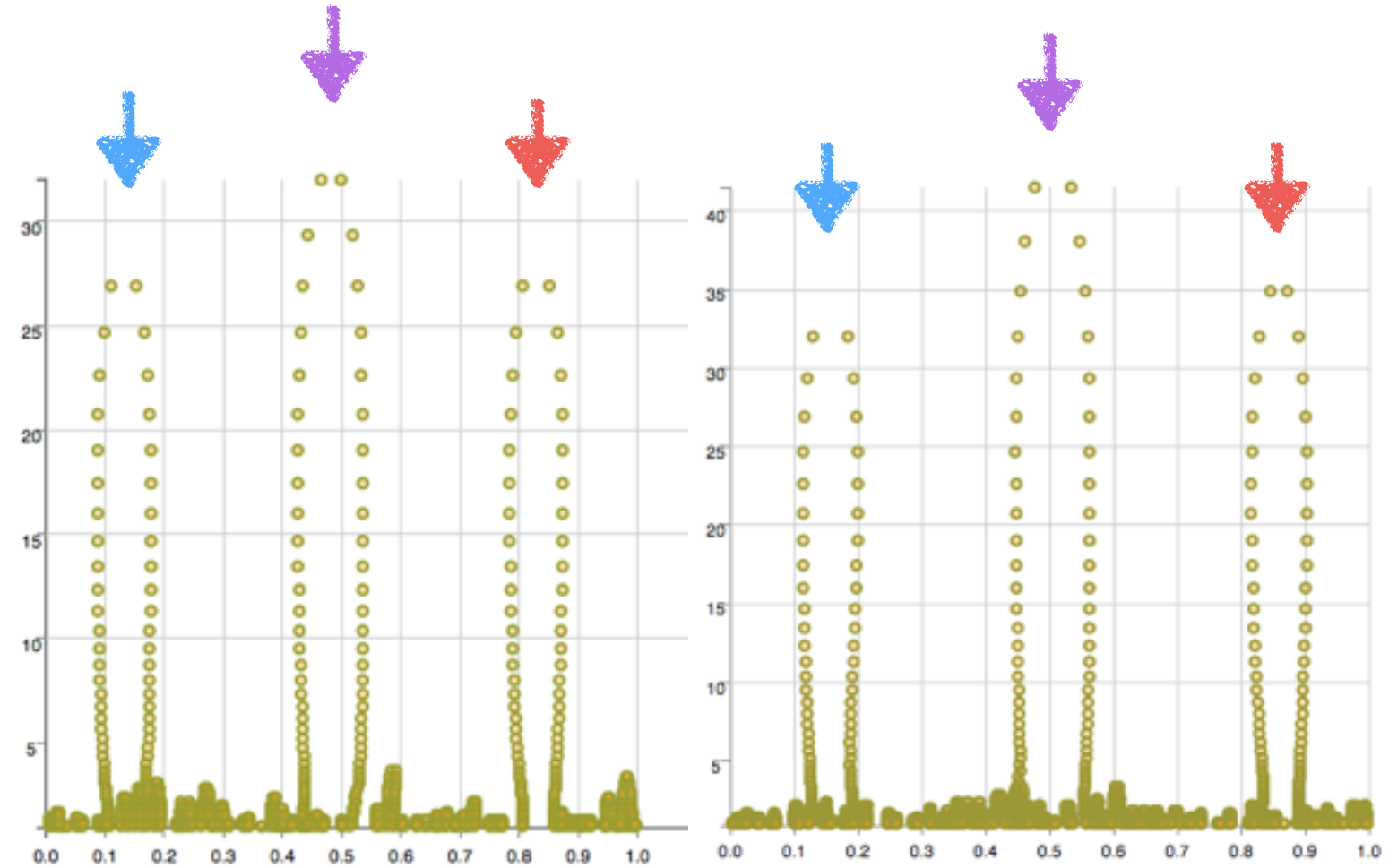
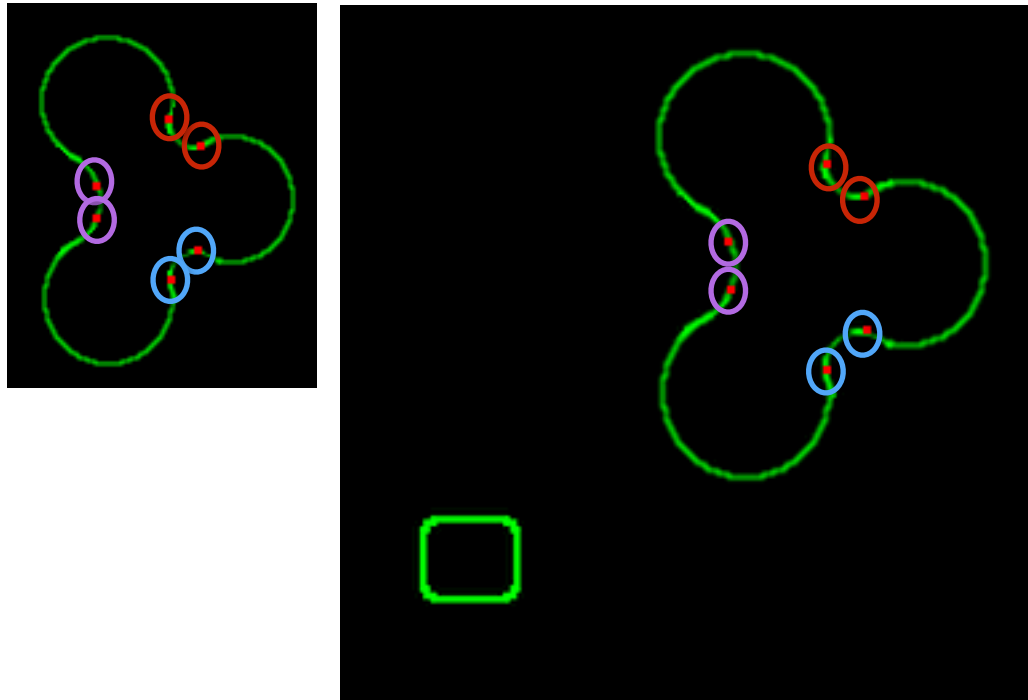
The contour finder looks for the peaks at the highest sigma and then follows the left and right branches down, subtracting that contour from the scale space image. each contour is found that way and subtracted to a lower threshold.

Then contours from one image are matched with the contours of another image (an image having the same content, that is).

Then euclidean transformation parameters rotation, scale, and translation are calculated from the matched contour peak coordinates.

Then, the parameters are refined with small changes and applied to the edges in image 1. The transformed closed curve edges from image 1 are compared to the closest matches in image 2 to find the best fitting transformation parameters.

# coordinate transformation, after matching contours



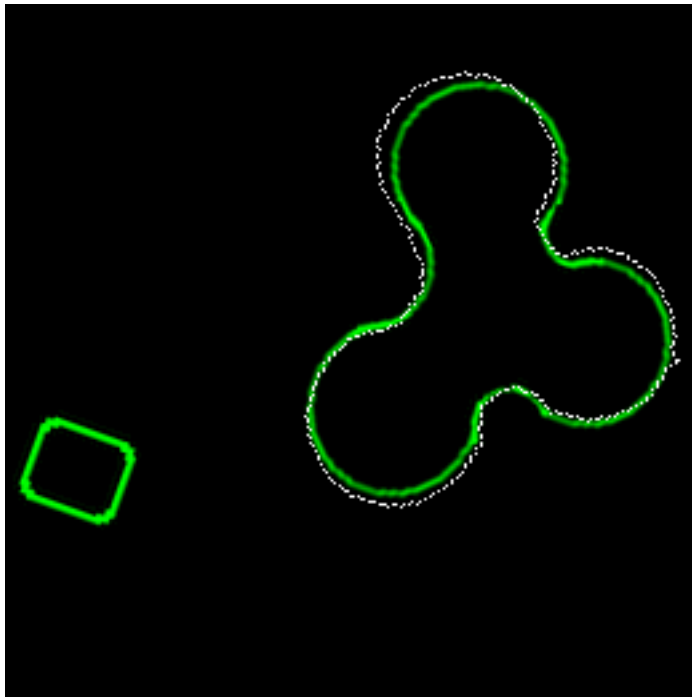
scale should be 1.3  
rotation should be 360 - 20

Contour matcher solution scale=1.354256510734558  
Contour matcher solution shift=-0.1688411384820938  
CONTOUR PEAK1: (32.000237, 0.497126) (34, 78) (35, 72)  
CONTOUR PEAK1: (26.908875, 0.146552) (70, 93) (61, 99)  
CONTOUR PEAK1: (26.908875, 0.846264) (60, 45) (69, 54)  
offsetImgX1=10 offsetImgY1=10  
offsetImgX2=1 offsetImgY2=26  
rotationInRadians=6.0030236  
rotationInDegrees=343.94791799660214  
scale=1.3542565  
translationX=108.1361  
translationY=15.72716

CONTOUR PEAK2: (43.336529, 0.504396) (157, 108) (159, 101)  
CONTOUR PEAK2: (34.148750, 0.157143) (190, 143) (177, 150)  
CONTOUR PEAK2: (34.896511, 0.859341) (200, 85) (209, 97)



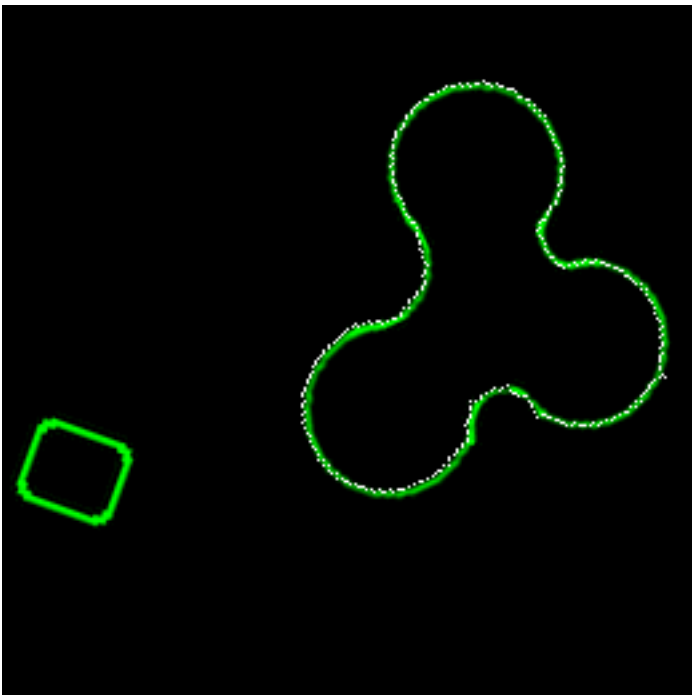
apply coordinate transformation



rotationInRadians=6.0030236  
rotationInDegrees=343.94791799660214  
scale=1.3542565  
translationX=108.1361  
translationY=15.72716

scale should be 1.3  
rotation should be 360-20

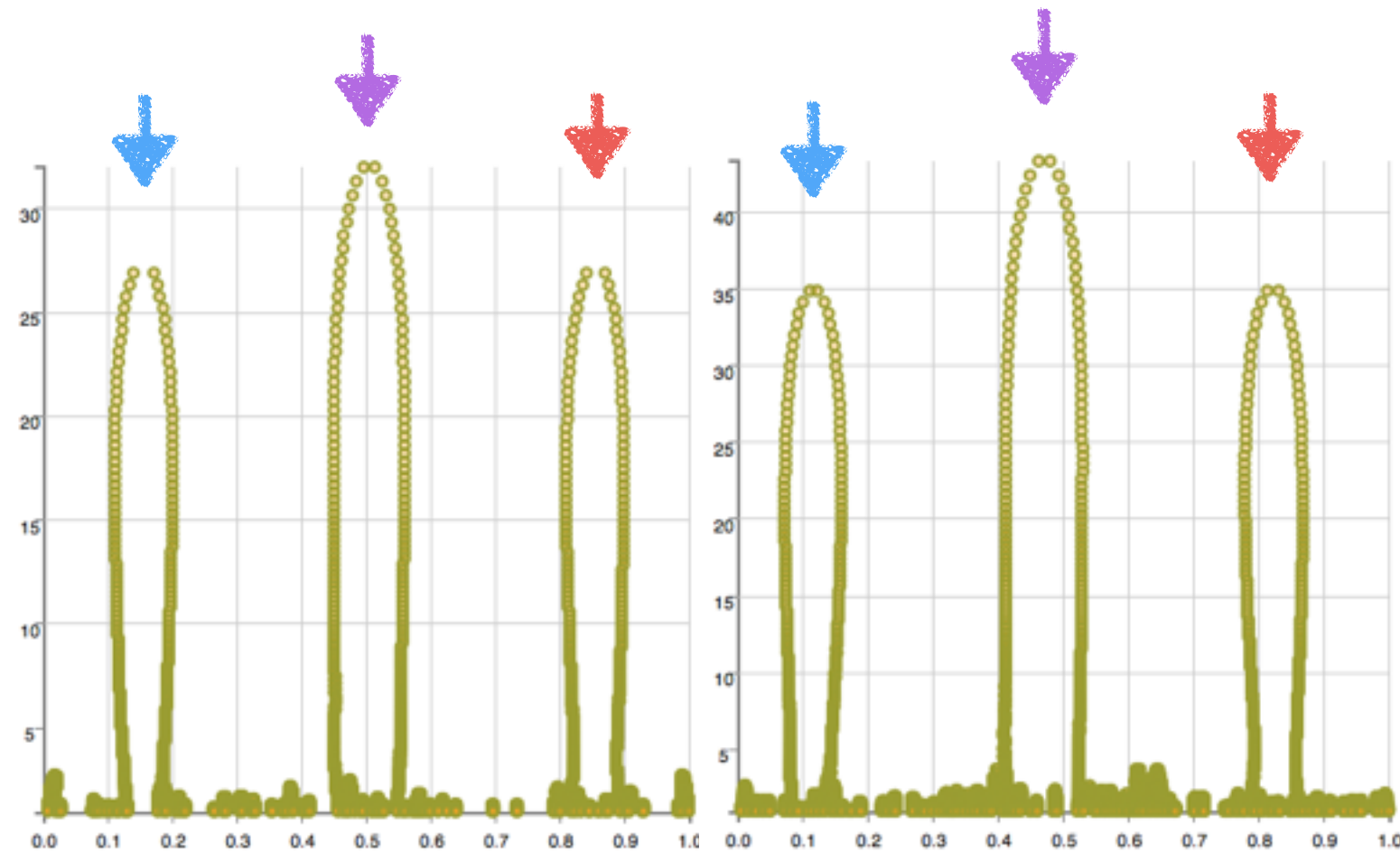
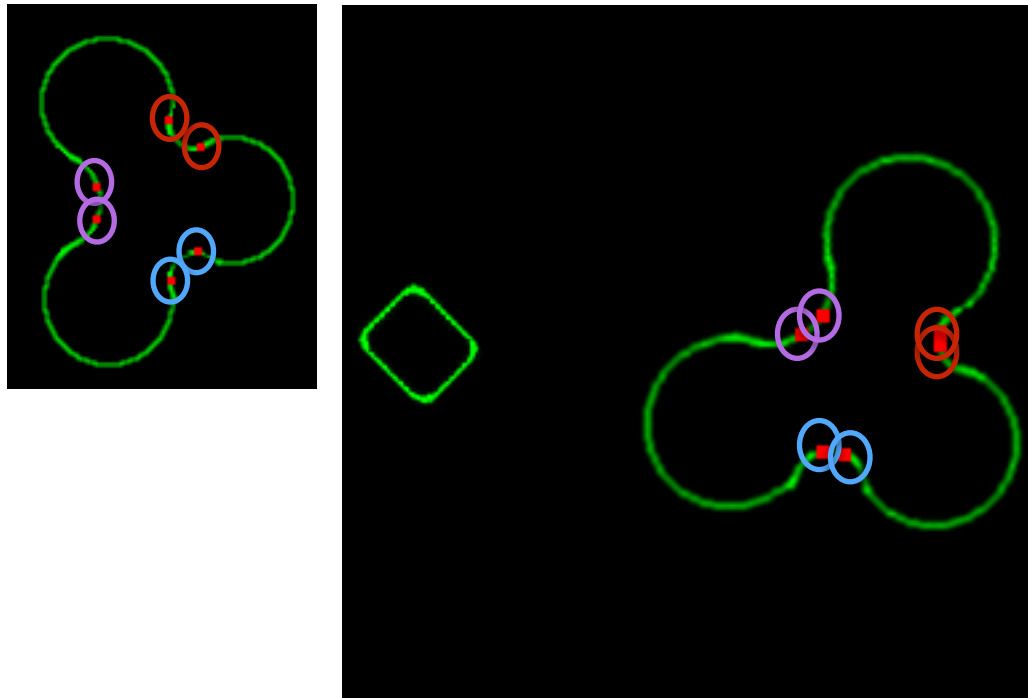
This shows that it's difficult to estimate scale unless some of the inflection points are further from the center of the shape



**After Refinement**

rotationInRadians=5.915757  
rotationInDegrees=338.94791899582935  
scale=1.3042566  
translationX=111.0  
translationY=20.0

# coordinate transformation, after matching contours

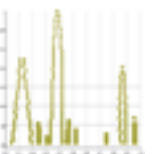


reversed to have CCW ordering

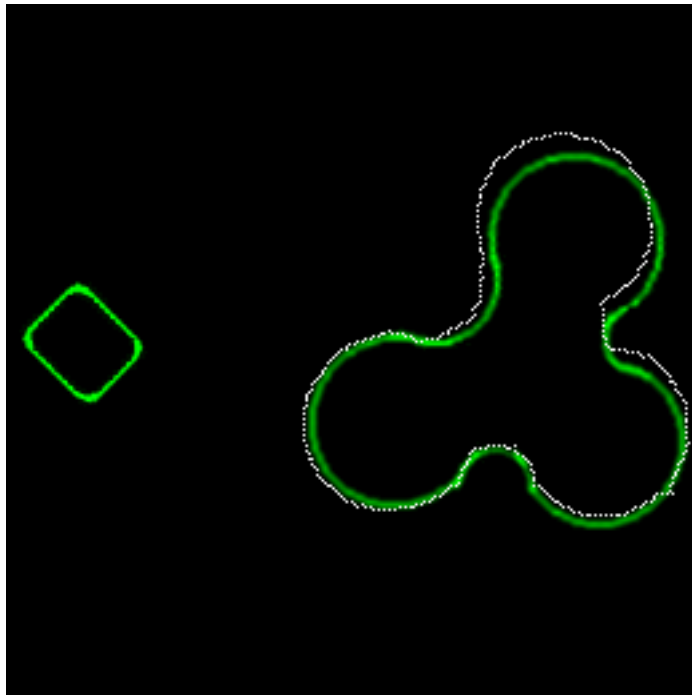
reversed to have CCW ordering

scale should be 1.3  
rotation should be 360 - 45

Contour matcher solution scale=1.354256510734558  
Contour matcher solution shift=-0.1425349861383438  
CONTOUR PEAK1: (32.000237, 0.497126) (34, 78) (35, 72) CONTOUR PEAK2: (43.336529, 0.530702) (172, 123) (180, 116)  
CONTOUR PEAK1: (26.908875, 0.146552) (70, 93) (61, 99) CONTOUR PEAK2: (34.896511, 0.179825) (188, 168) (180, 167)  
CONTOUR PEAK1: (26.908875, 0.846264) (60, 45) (69, 54) CONTOUR PEAK2: (34.896511, 0.884868) (224, 122) (224, 127)  
offsetImgX1=10 offsetImgY1=10  
offsetImgX2=3 offsetImgY2=52  
rotationInRadians=5.5627766  
rotationInDegrees=318.7236195803948  
scale=1.3542565  
translationX=119.984764  
translationY=39.13688

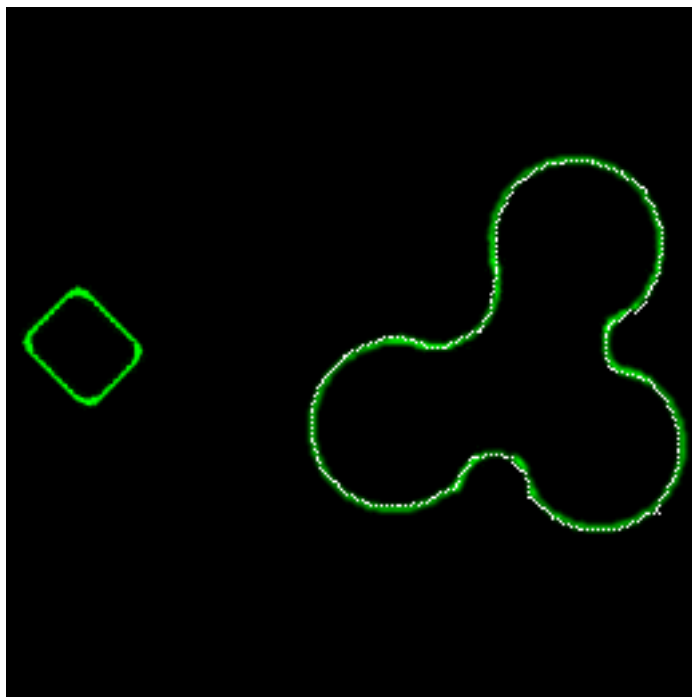


## apply coordinate transformation



offsetImgX1=10 offsetImgY1=10  
offsetImgX2=3 offsetImgY2=52  
rotationInRadians=5.5627766  
rotationInDegrees=318.7236195803948  
scale=1.3542565  
translationX=119.984764  
translationY=39.13688

scale should be 1.3  
rotation should be 360-45

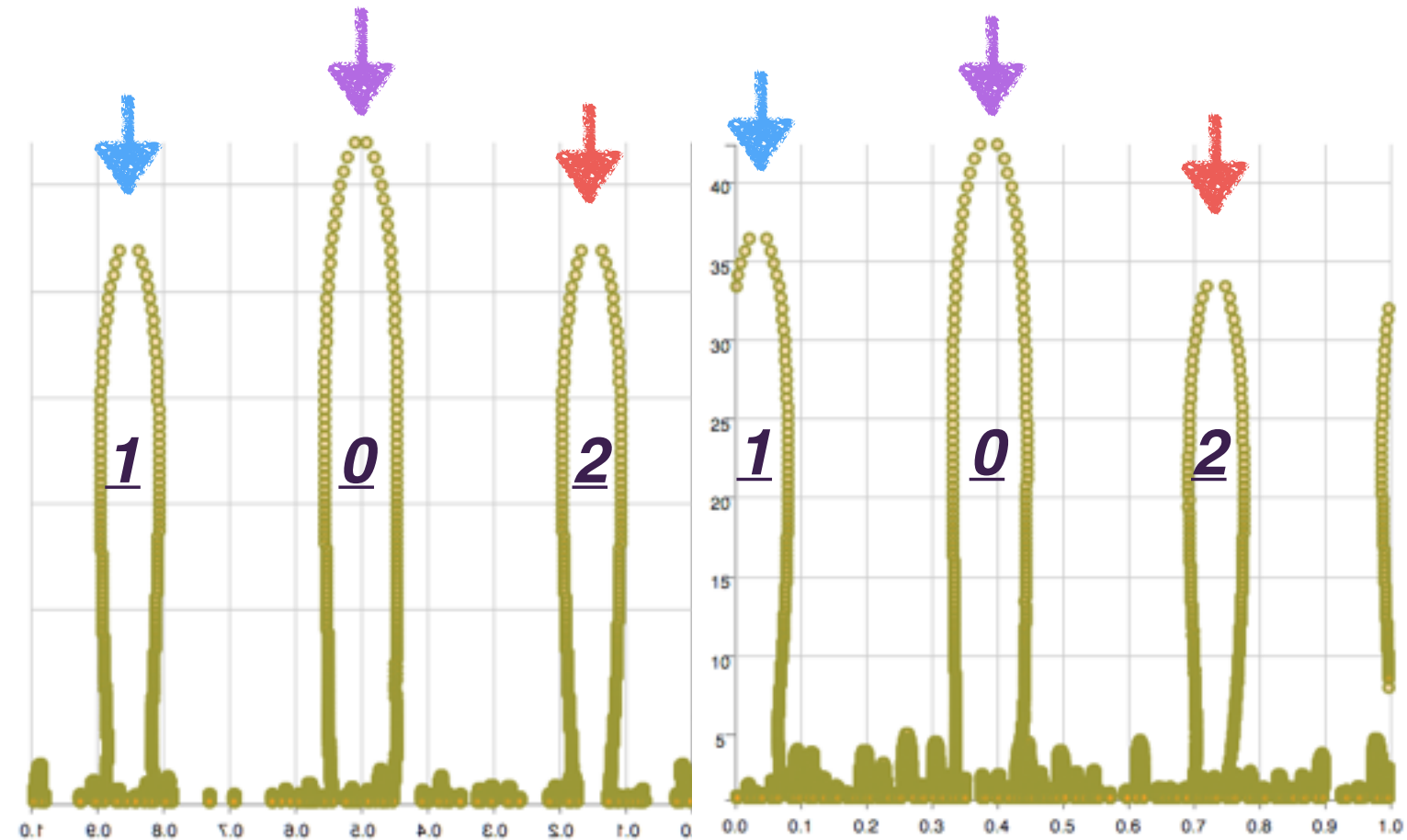
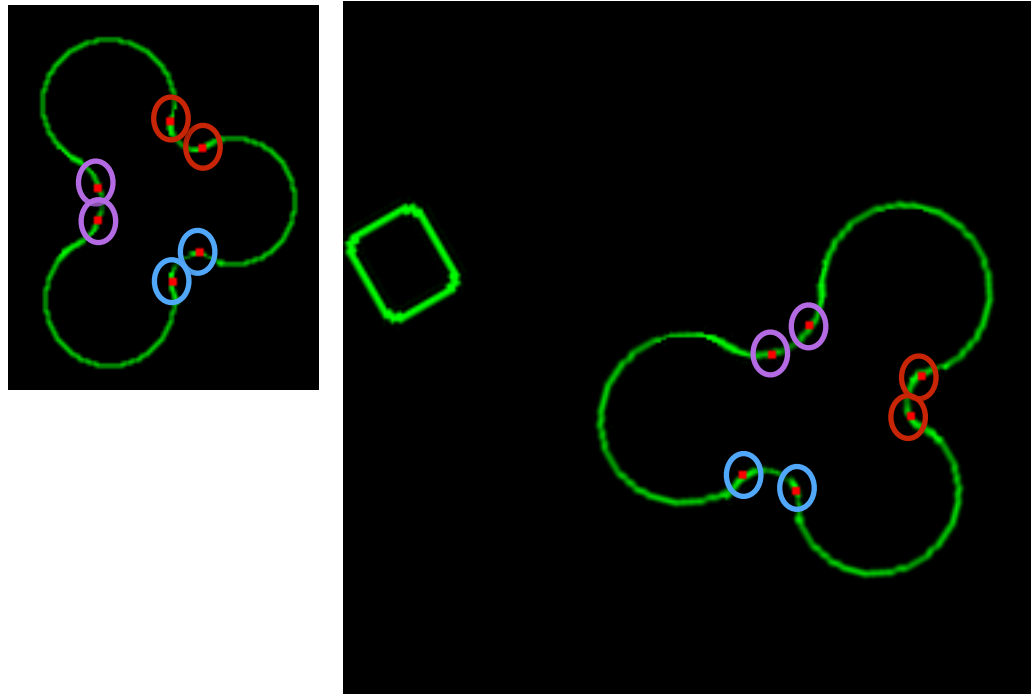


## After Refinement

rotationInRadians=5.47551  
rotationInDegrees=313.72362057962204  
scale=1.3042566  
translationX=124.0  
translationY=47.0



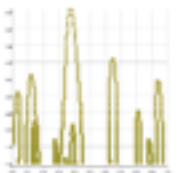
# coordinate transformation, after matching contours



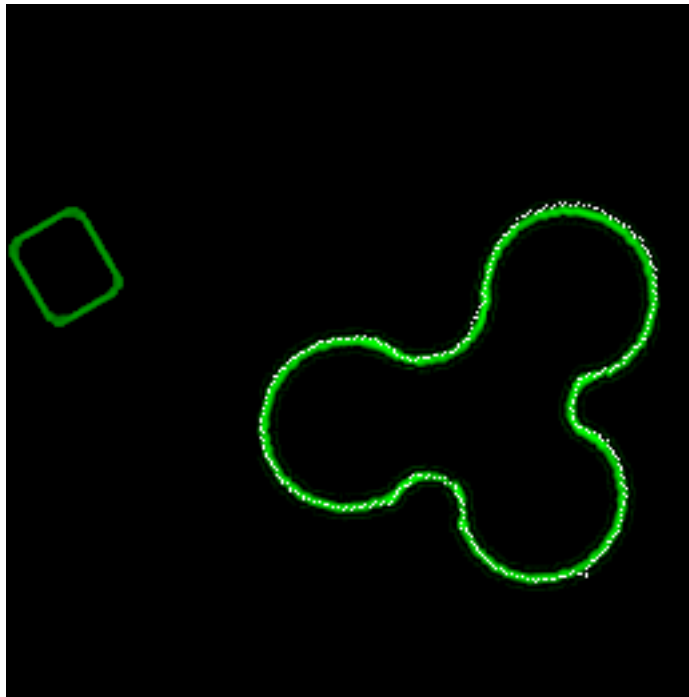
reversed to have CCW ordering

scale should be 1.3  
rotation should be 360 - 60

Contour matcher solution scale=1.325237512588501  
Contour matcher solution shift=-0.2715021073818207  
CONTOUR PEAK1: (32.000237, 0.497126) (34, 78) (35, 72) CONTOUR PEAK2: (42.407913, 0.387309) (161, 133) (172, 126)  
CONTOUR PEAK1: (26.908875, 0.146552) (70, 93) (61, 99) CONTOUR PEAK2: (36.441517, 0.035011) (168, 182) (157, 176)  
CONTOUR PEAK1: (26.908875, 0.846264) (60, 45) (69, 54) CONTOUR PEAK2: (33.417011, 0.734136) (215, 143) (213, 156)  
offsetImgX1=10 offsetImgY1=10  
offsetImgX2=0 offsetImgY2=71  
rotationInRadians=5.2398615  
rotationInDegrees=300.2219485151509  
scale=1.3252375  
translationX=107.96625  
translationY=59.87517

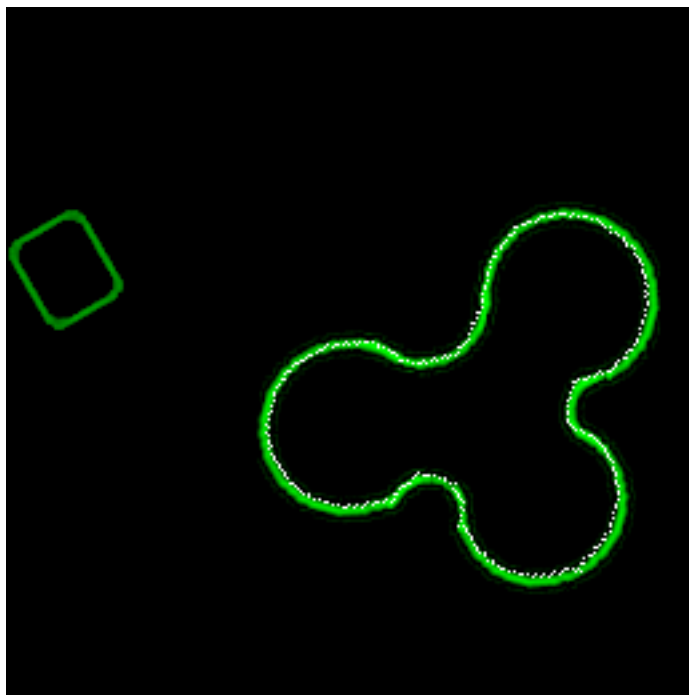


## apply coordinate transformation



```
Contour matcher solution scale=1.325237512588501
Contour matcher solution shift=-0.2715021073818207
CONTOUR PEAK1: (32.000237, 0.497126) (34, 78) (35, 72)
  CONTOUR PEAK2: (42.407913, 0.387309) (161, 133) (172, 126)
CONTOUR PEAK1: (26.908875, 0.146552) (70, 93) (61, 99)
  CONTOUR PEAK2: (36.441517, 0.035011) (168, 182) (157, 176)
CONTOUR PEAK1: (26.908875, 0.846264) (60, 45) (69, 54)
  CONTOUR PEAK2: (33.417011, 0.734136) (215, 143) (213, 156)
offsetImgX1=10 offsetImgY1=10
offsetImgX2=0 offsetImgY2=71
rotationInRadians=5.2398615
rotationInDegrees=300.2219485151509
scale=1.3252375
translationX=107.96625
translationY=59.87517
```

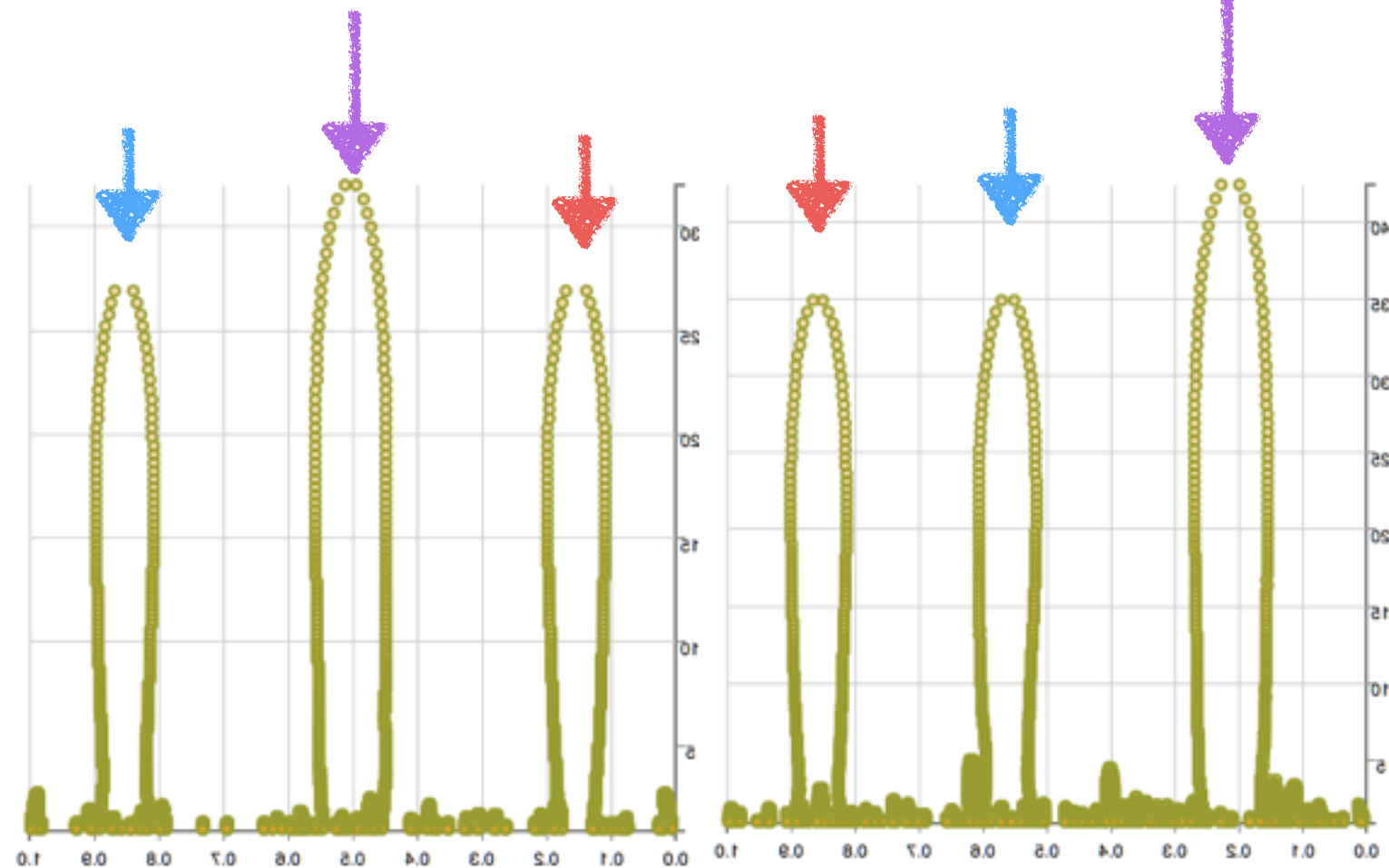
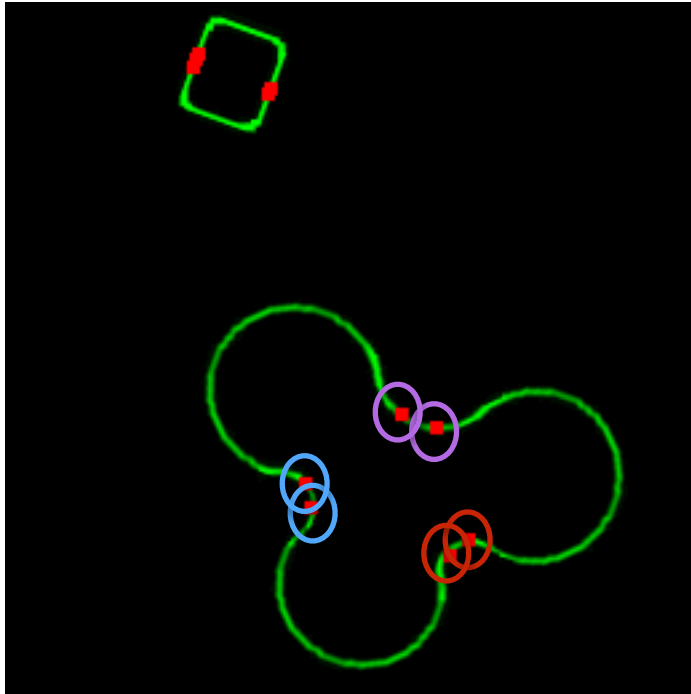
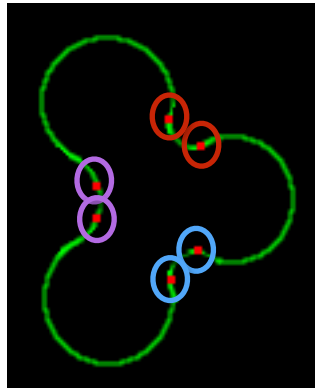
scale should be 1.3  
rotation should be 360 - 60



### After Refinement

```
rotationInRadians=5.2398615
rotationInDegrees=300.2219485151509
scale=1.2752376
translationX=110.0
translationY=63.0
```

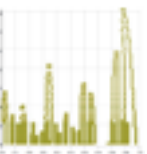
# coordinate transformation, after matching contours



reversed to have CCW ordering

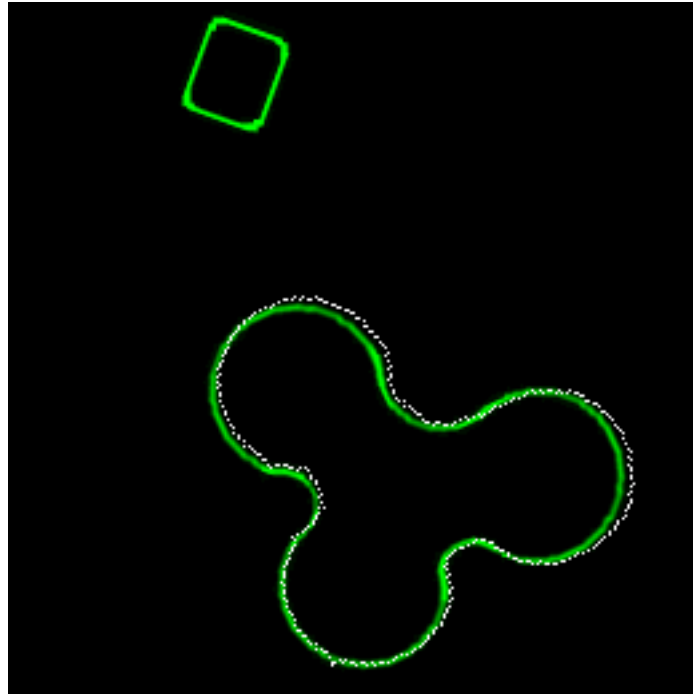
reversed to have CCW ordering

scale should be 1.3  
rotation should be 360 -110



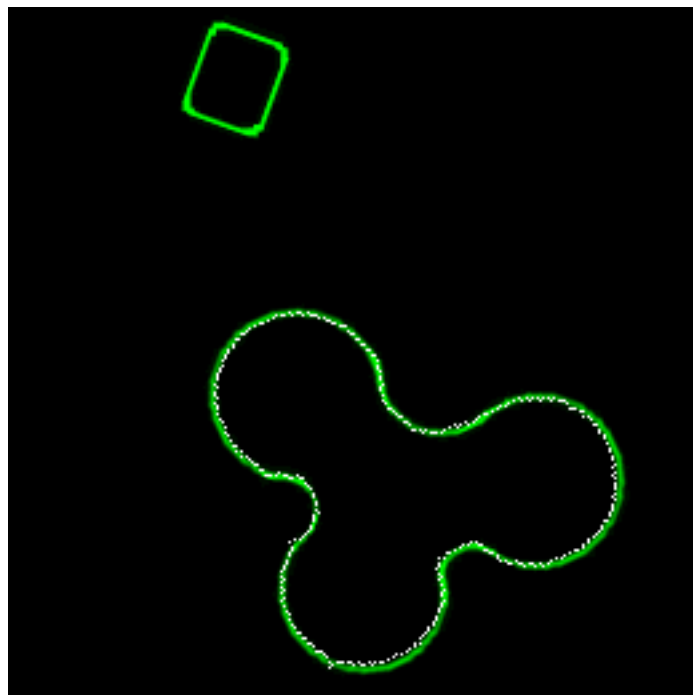
CONTOUR PEAK1: (32.000237, 0.497126) (34, 78) (35, 72) CONTOUR PEAK2: (42.407913, 0.787445) (148, 154) (161, 159)  
CONTOUR PEAK1: (26.908875, 0.146552) (70, 93) (61, 99) CONTOUR PEAK2: (34.896511, 0.439427) (114, 189) (112, 180)  
CONTOUR PEAK1: (26.908875, 0.846264) (60, 45) (69, 54) CONTOUR PEAK2: (34.896511, 0.142070) (173, 201) (166, 207)  
offsetImgX1=10 offsetImgY1=10  
offsetImgX2=62 offsetImgY2=1  
rotationInRadians=4.336784  
rotationInDegrees=248.47941332561248  
scale=1.3252375  
translationX=70.99721  
translationY=91.054344

## apply coordinate transformation



```
CONTOUR PEAK1: (32.000237, 0.497126) (34, 78) (35, 72)
CONTOUR PEAK2: (42.407913, 0.787445) (148, 154) (161, 159)
CONTOUR PEAK1: (26.908875, 0.146552) (70, 93) (61, 99)
CONTOUR PEAK2: (34.896511, 0.439427) (114, 189) (112, 180)
CONTOUR PEAK1: (26.908875, 0.846264) (60, 45) (69, 54)
CONTOUR PEAK2: (34.896511, 0.142070) (173, 201) (166, 207)
offsetImgX1=10 offsetImgY1=10
offsetImgX2=62 offsetImgY2=1
rotationInRadians=4.336784
rotationInDegrees=248.47941332561248
scale=1.3252375
translationX=70.99721
translationY=91.054344
```

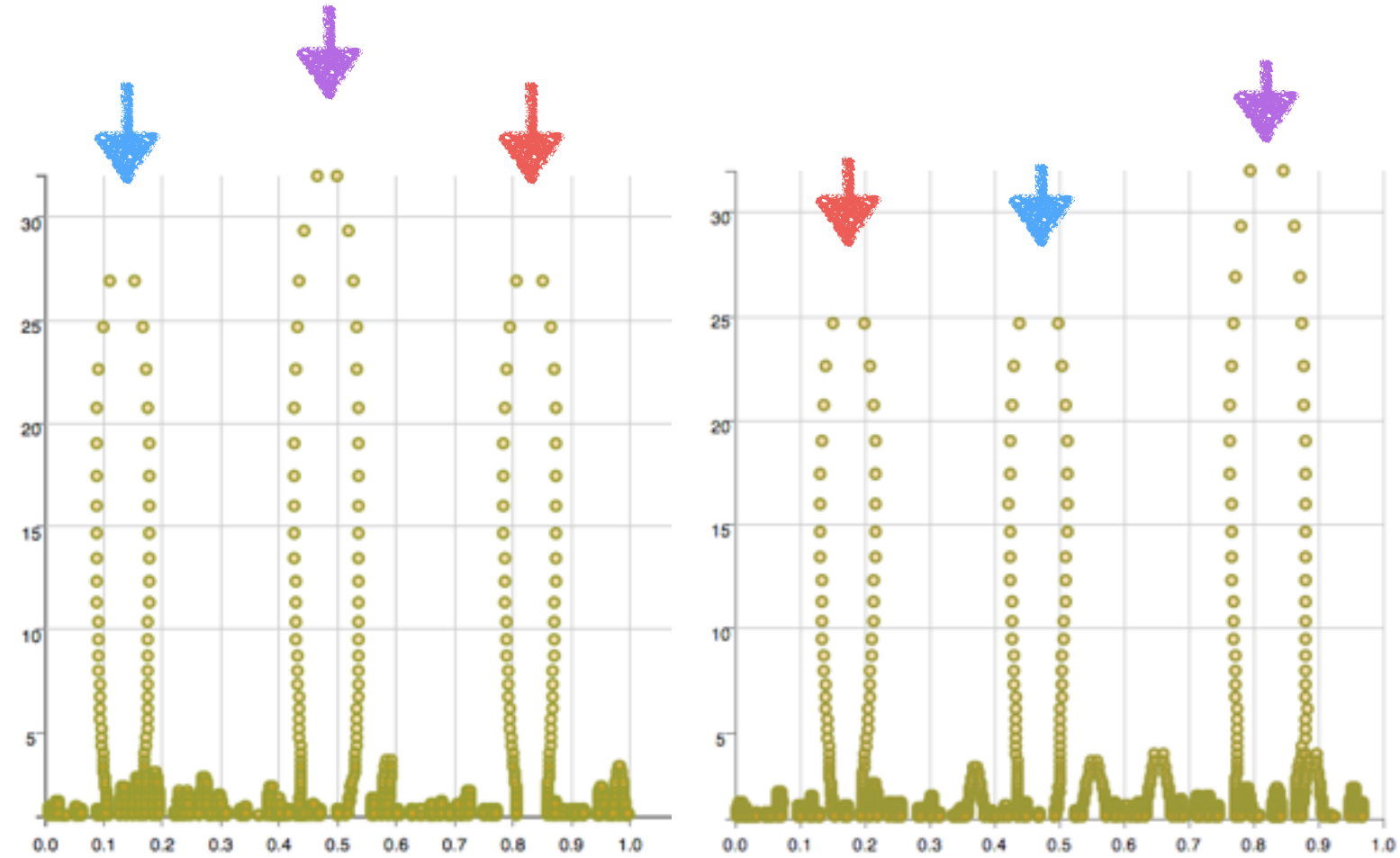
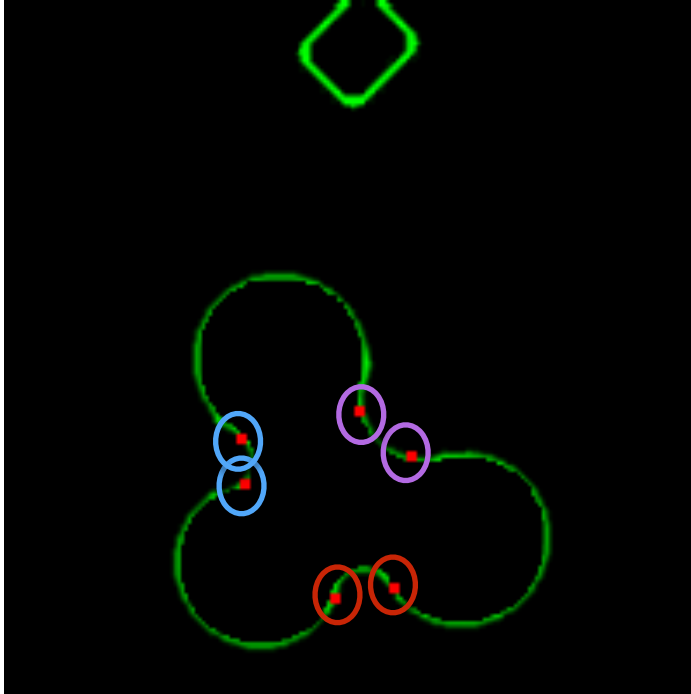
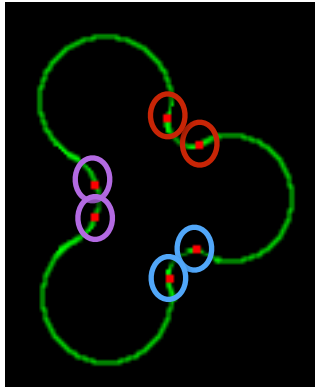
scale should be 1  
rotation should be 360 - 110 (250)



## After Refinement

```
rotationInRadians=4.354237
rotationInDegrees=249.4794076616157
scale=1.2752376
translationX=71.0
translationY=95.0
```

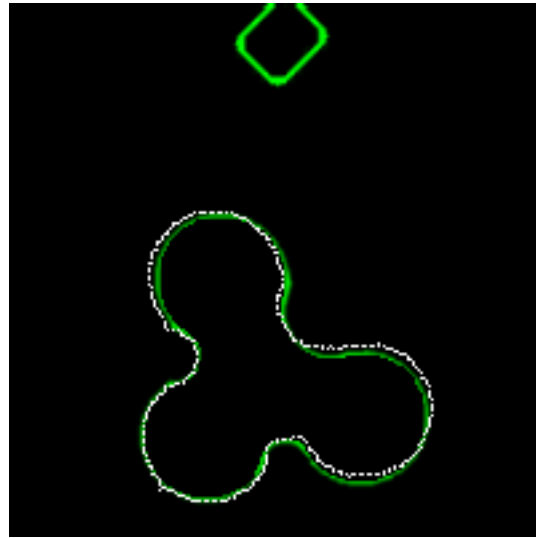
# coordinate transformation, after matching contours



scale should be 1.0  
rotation should be 360 - 135

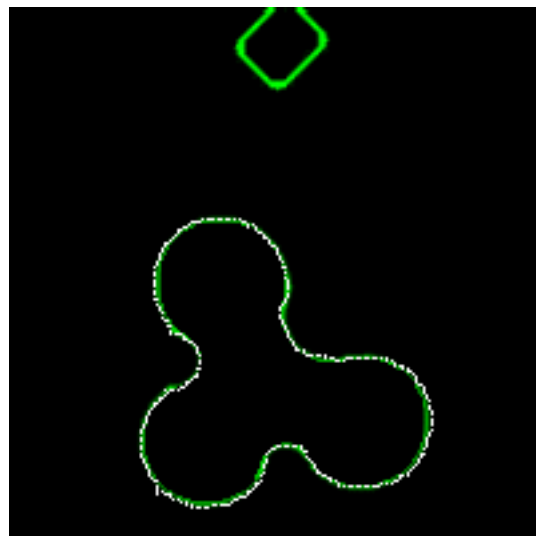
Contour matcher solution scale=1.0218971967697144  
Contour matcher solution shift=0.3104316294193268  
CONTOUR PEAK1: (32.000237, 0.497126) (34, 78) (35, 72) CONTOUR PEAK2: (32.700951, 0.818444) (103, 120) (115, 132)  
CONTOUR PEAK1: (26.908875, 0.146552) (70, 93) (61, 99) CONTOUR PEAK2: (25.768024, 0.468300) (69, 136) (69, 127)  
CONTOUR PEAK1: (26.908875, 0.846264) (60, 45) (69, 54) CONTOUR PEAK2: (25.768024, 0.175793) (105, 164) (97, 166)  
offsetImgX1=10 offsetImgY1=10  
offsetImgX2=46 offsetImgY2=0  
rotationInRadians=3.9818497  
rotationInDegrees=228.14318077005984  
scale=1.0218972  
translationX=32.712353  
translationY=70.68979

## apply coordinate transformation



```
offsetImgX1=10 offsetImgY1=10  
offsetImgX2=46 offsetImgY2=0  
rotationInRadians=3.9818497  
rotationInDegrees=228.14318077005984  
scale=1.0218972  
translationX=32.712353  
translationY=70.68979
```

scale should be 1  
rotation should be 360 - 135

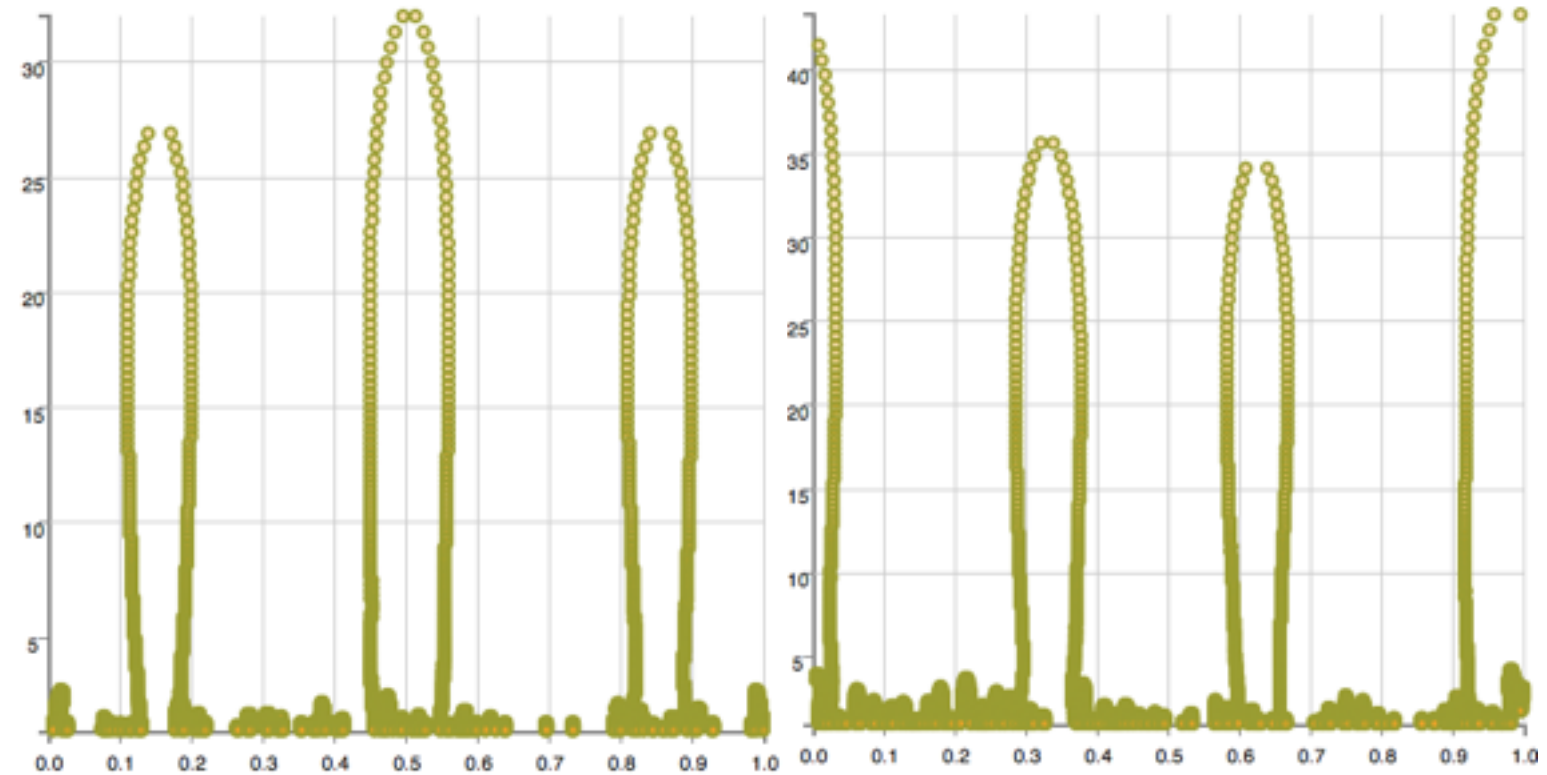
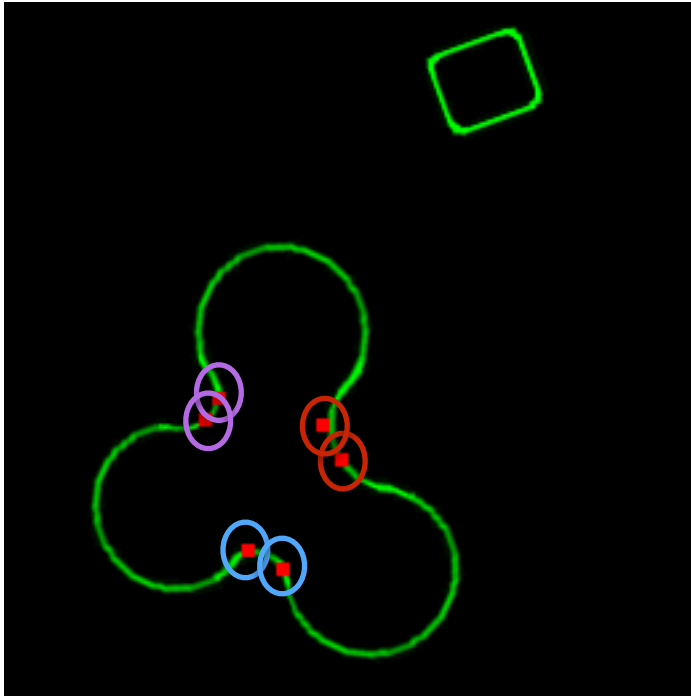
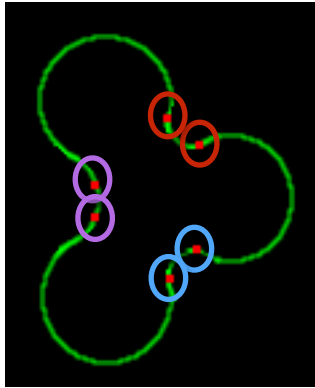


### After Refinement

```
rotationInRadians=3.9406004  
rotationInDegrees=225.77977139269535  
scale=1.0218972  
translationX=33.0  
translationY=72.0
```



# coordinate transformation, after matching contours

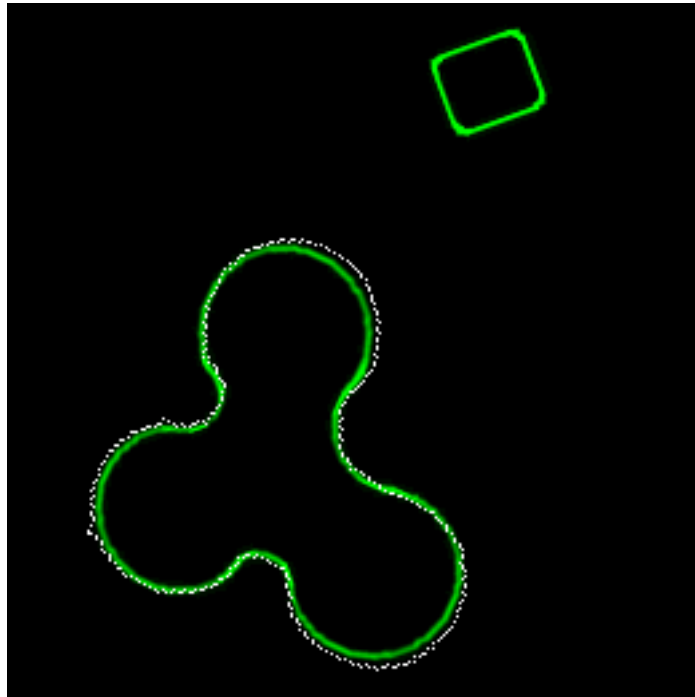


reversed to have CCW ordering

scale should be 1.3  
rotation should be 360 - 160

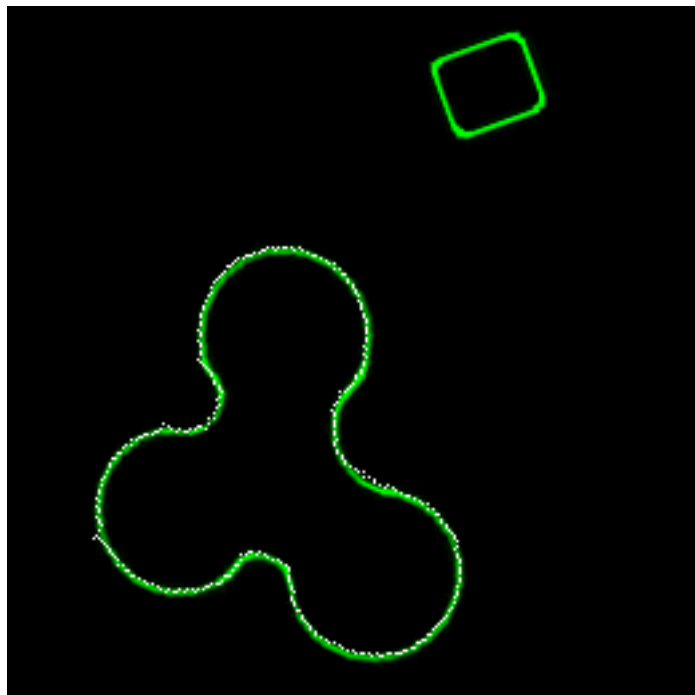
```
Contour matcher solution shift=-0.648127555847168
Contour matcher solution cost=3.0
CONTOUR PEAK1: (32.000237, 0.497126) (34, 78) (35, 72) CONTOUR PEAK2: (43.336529, 0.025109) (119, 158) (126, 171)
CONTOUR PEAK1: (26.908875, 0.146552) (70, 93) (61, 99) CONTOUR PEAK2: (35.660648, 0.672489) (75, 156) (80, 148)
CONTOUR PEAK1: (26.908875, 0.846264) (60, 45) (69, 54) CONTOUR PEAK2: (34.148750, 0.377729) (104, 212) (91, 205)
offsetImgX1=10 offsetImgY1=10
offsetImgX2=29 offsetImgY2=6
rotationInRadians=3.4556763
rotationInDegrees=197.995668339729
scale=1.3542565
translationX=17.566353
translationY=81.42969
```

apply coordinate transformation



rotationInRadians=3.4556763  
rotationInDegrees=197.995668339729  
scale=1.3542565  
translationX=17.566353  
translationY=81.42969

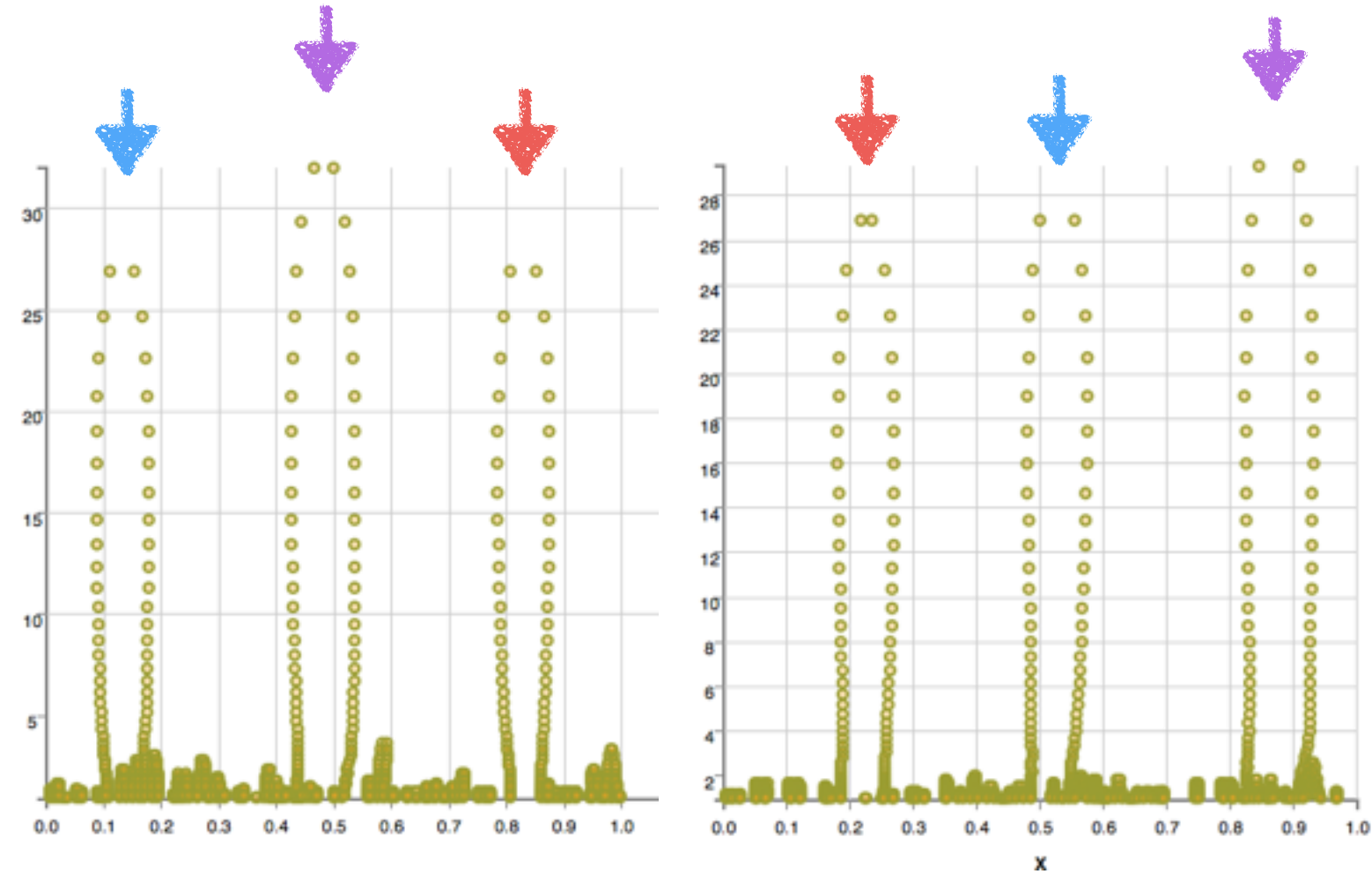
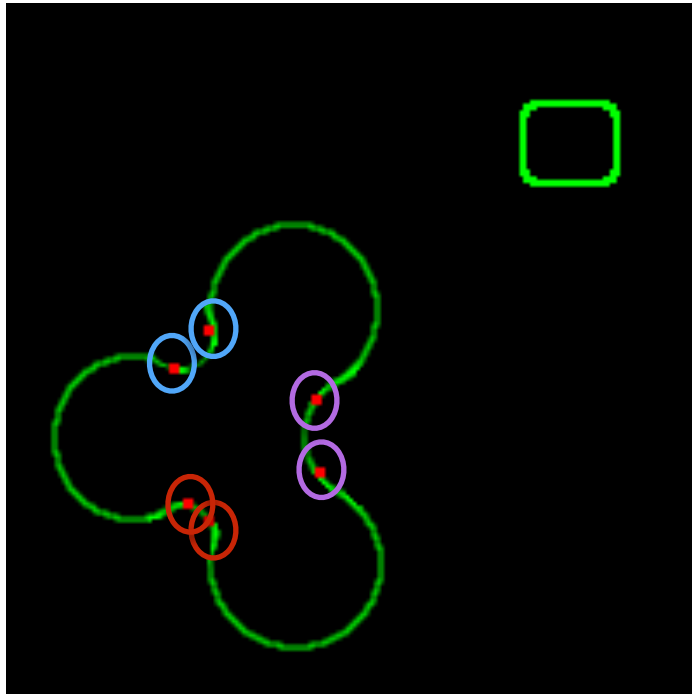
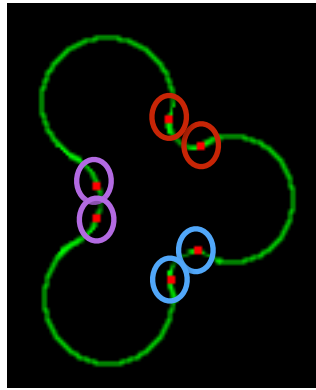
scale should be 1  
rotation should be 360 - 160



**After Refinement**

rotationInRadians=3.4993095  
rotationInDegrees=200.49566784011543  
scale=1.3042566  
translationX=20.0  
translationY=84.0

# coordinate transformation, after matching contours

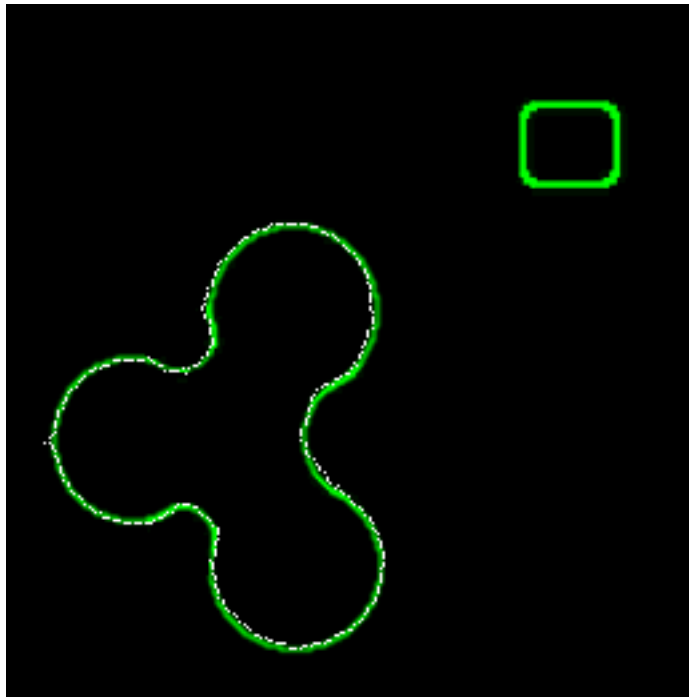


reversed to have CCW ordering

scale should be 1.3  
rotation should be 180

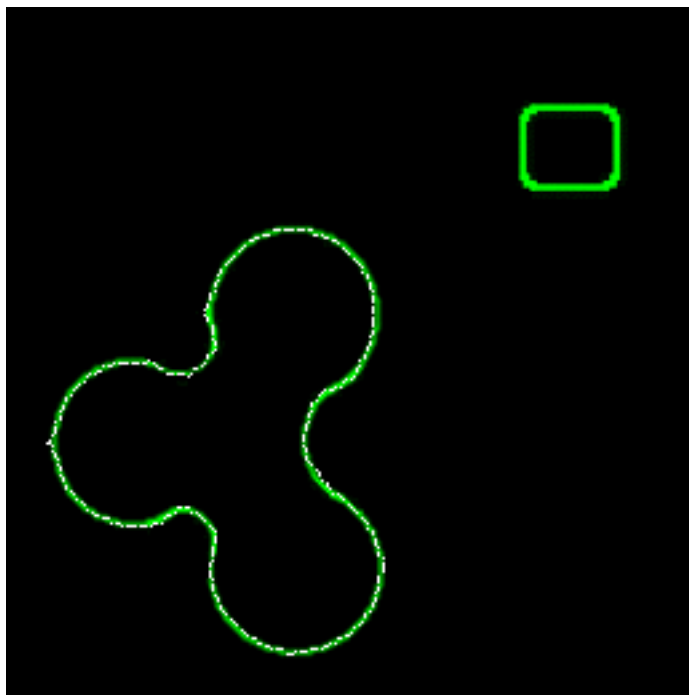
Contour matcher solution scale=1.2968404293060303  
Contour matcher solution shift=0.22837476432323456  
CONTOUR PEAK1: (32.000237, 0.497126) (34, 78) (35, 72) CONTOUR PEAK2: (41.499199, 0.873068) (111, 158) (112, 167)  
CONTOUR PEAK1: (26.908875, 0.146552) (70, 93) (61, 99) CONTOUR PEAK2: (35.660648, 0.522075) (67, 137) (77, 127)  
CONTOUR PEAK1: (26.908875, 0.846264) (60, 45) (69, 54) CONTOUR PEAK2: (34.896511, 0.222958) (75, 194) (69, 188)  
offsetImgX1=10 offsetImgY1=10  
offsetImgX2=14 offsetImgY2=33  
rotationInRadians=3.1657186  
rotationInDegrees=181.38231235356184  
scale=1.2968404  
translationX=5.891382  
translationY=70.504585

## apply coordinate transformation



offsetImgX1=10 offsetImgY1=10  
offsetImgX2=14 offsetImgY2=33  
rotationInRadians=3.1657186  
rotationInDegrees=181.38231235356184  
scale=1.2968404  
translationX=5.891382  
translationY=70.504585

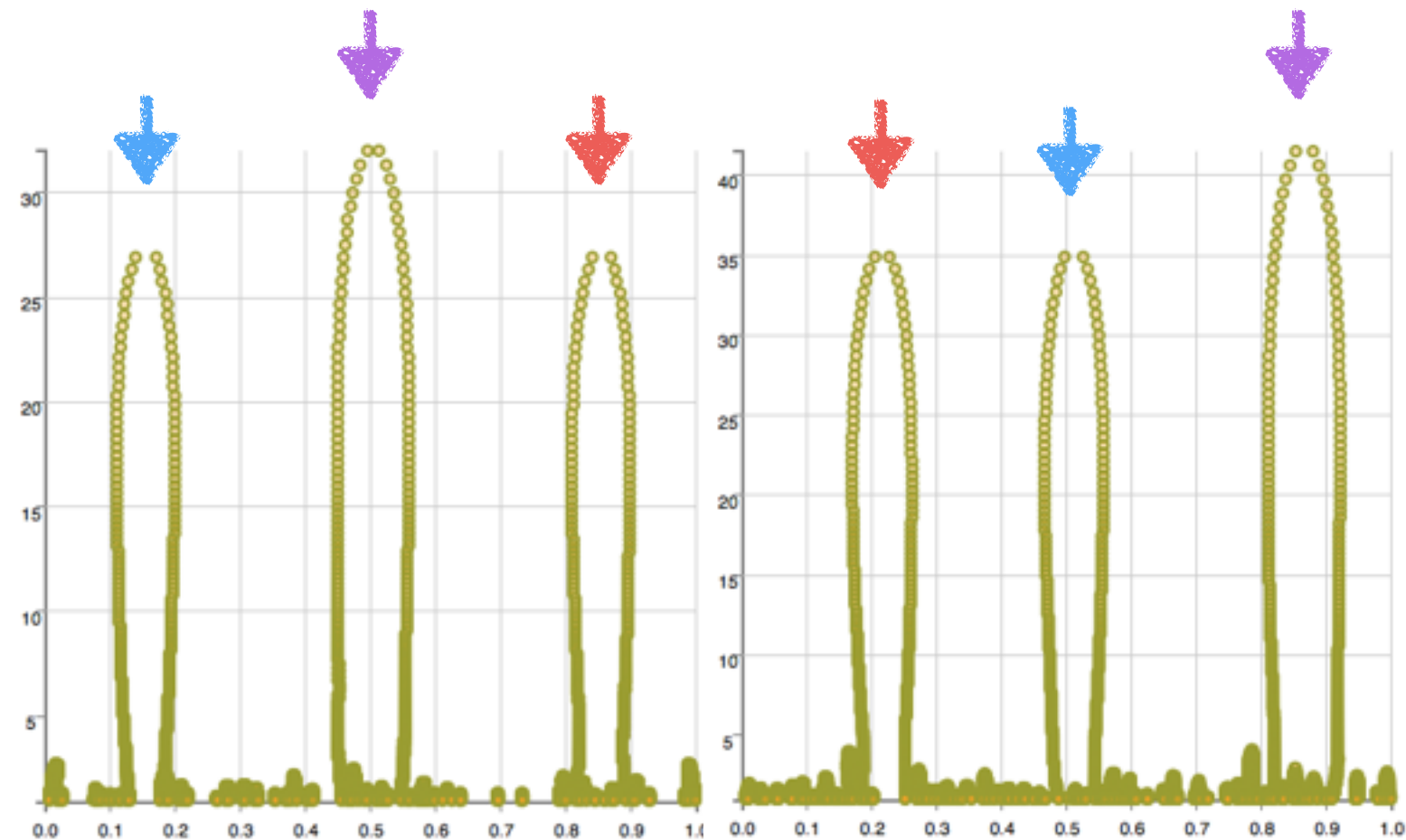
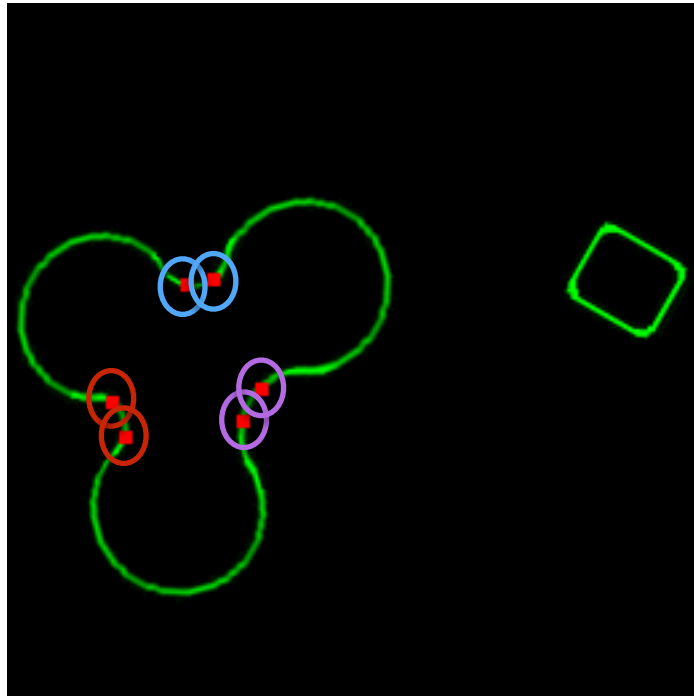
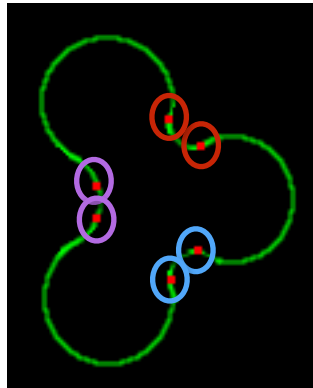
scale should be 1.3  
rotation should be 180



## After Refinement

rotationInRadians=3.1482654  
rotationInDegrees=180.38231801755862  
scale=1.2968404  
translationX=6.0  
translationY=71.0

# coordinate transformation, after matching contours



reversed to have CCW ordering

reversed to have CCW ordering

scale should be 1.3  
rotation should be 360 - 210

Contour matcher solution scale=1.2968404293060303

Contour matcher solution shift=-0.5097379088401794

CONTOUR PEAK1: (32.000237, 0.497126) (34, 78) (35, 72) CONTOUR PEAK2: (41.499199, 0.134956) (95, 144) (88, 156)

CONTOUR PEAK1: (26.908875, 0.146552) (70, 93) (61, 99) CONTOUR PEAK2: (34.896511, 0.785398) (67, 105) (77, 103)

CONTOUR PEAK1: (26.908875, 0.846264) (60, 45) (69, 54) CONTOUR PEAK2: (34.896511, 0.490044) (44, 162) (39, 149)

offsetImgX1=10 offsetImgY1=10

offsetImgX2=1 offsetImgY2=70

rotationInRadians=2.6591716

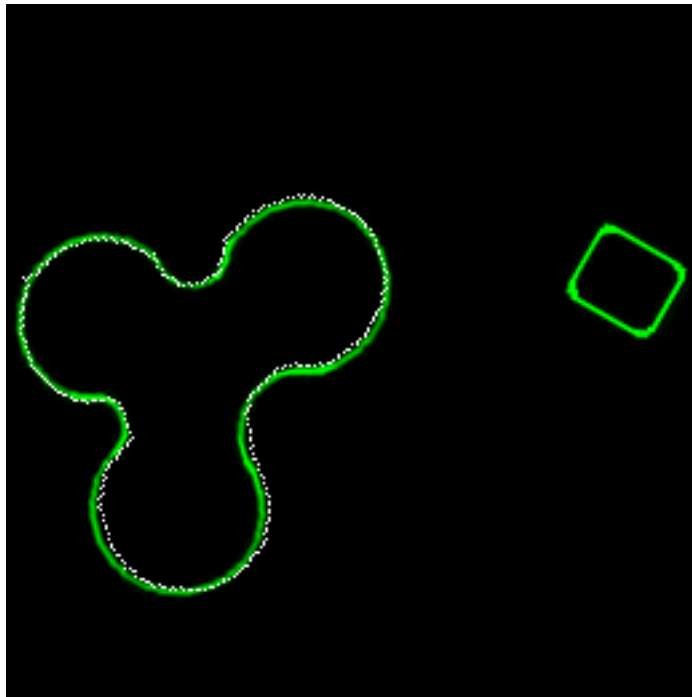
rotationInDegrees=152.3593086078036

scale=1.2968404

translationX=-10.876109

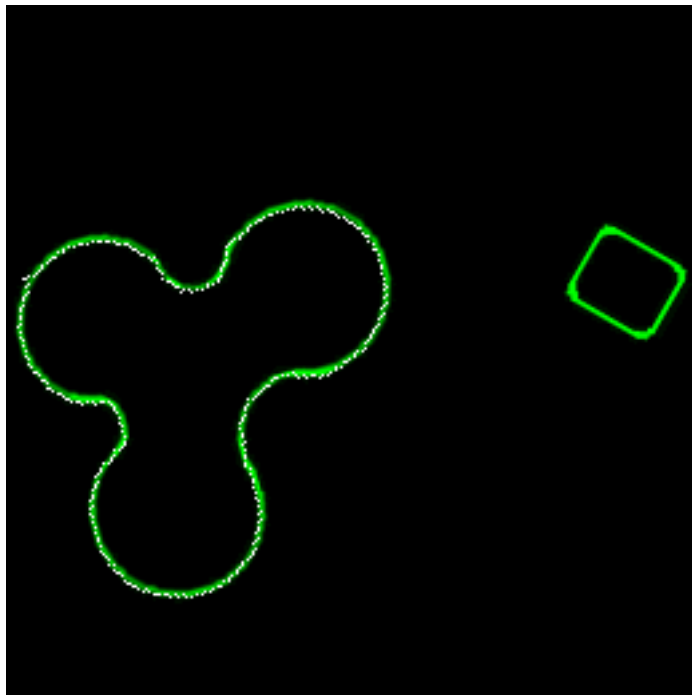
translationY=41.450577

## apply coordinate transformation



```
offsetImgX1=10 offsetImgY1=10  
offsetImgX2=1 offsetImgY2=70  
rotationInRadians=2.6591716  
rotationInDegrees=152.3593086078036  
scale=1.2968404  
translationX=-10.876109  
translationY=41.450577
```

scale should be 1.3  
rotation should be 360 - 210

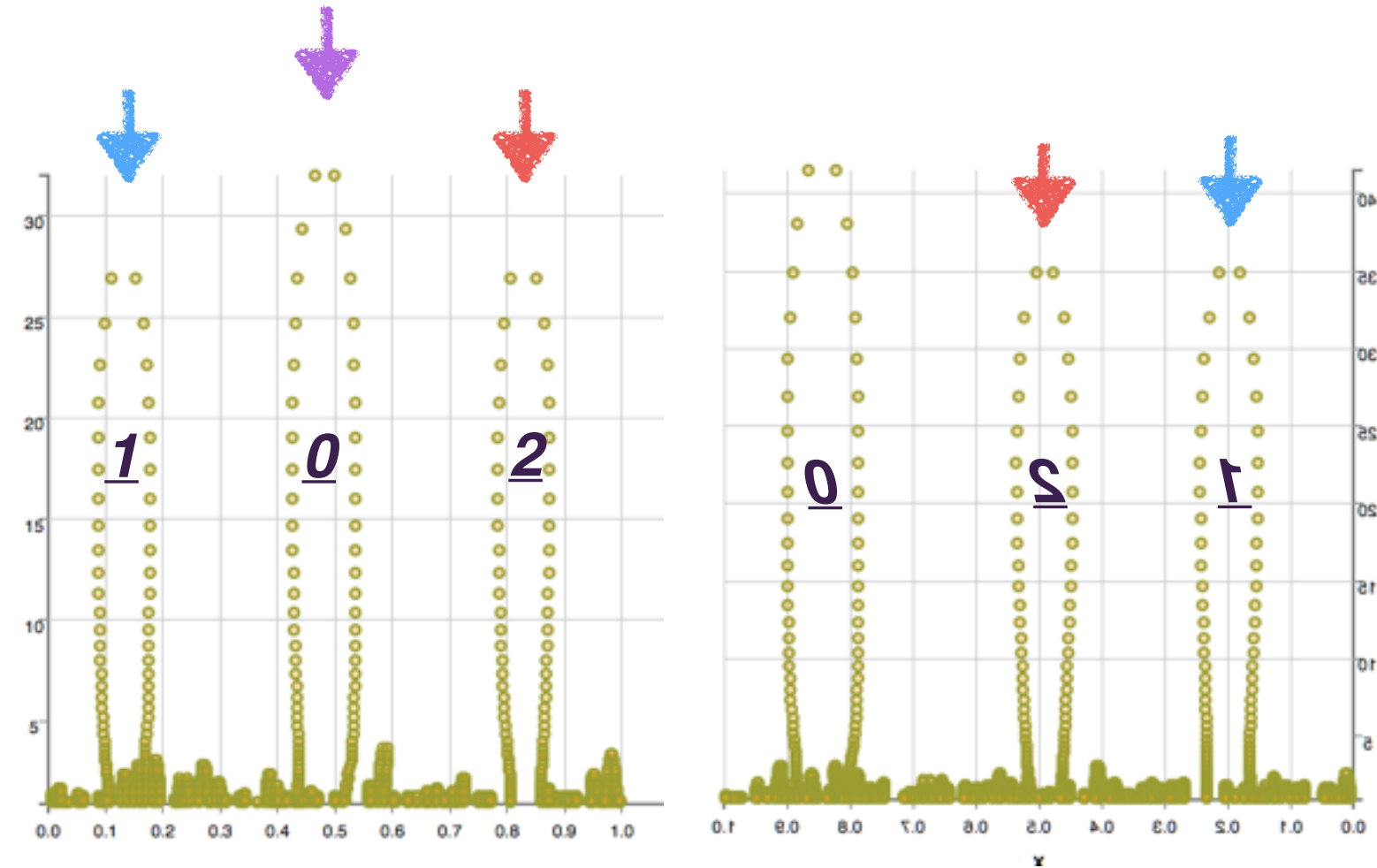
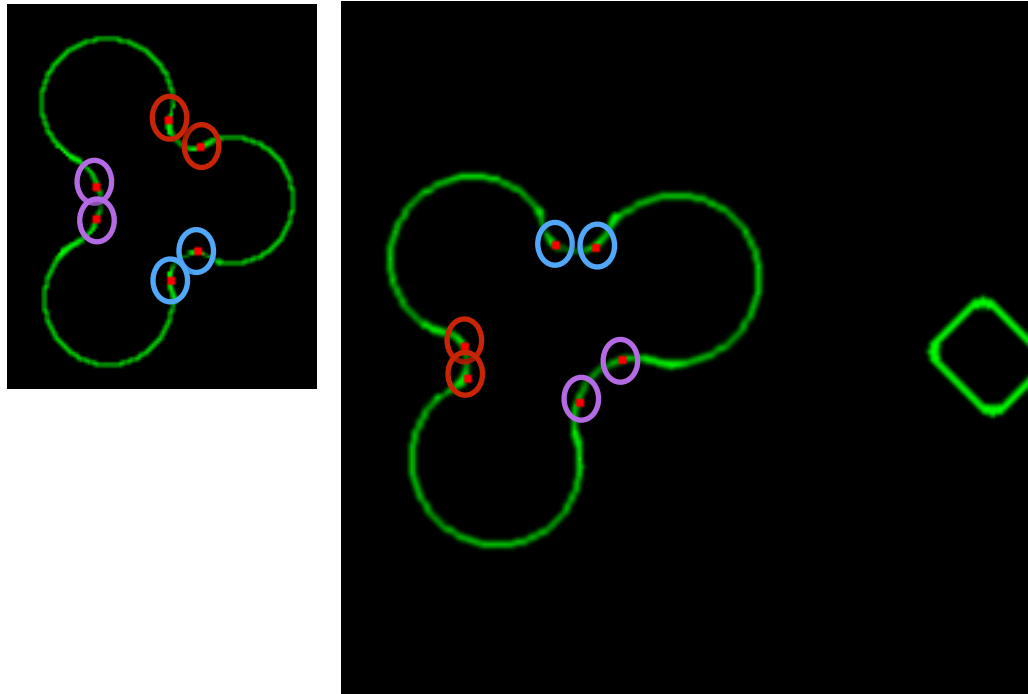


## After Refinement

```
rotationInRadians=2.6155384  
rotationInDegrees=149.85930910741718  
scale=1.2968404  
translationX=-12.0  
translationY=43.0
```



# coordinate transformation, after matching contours

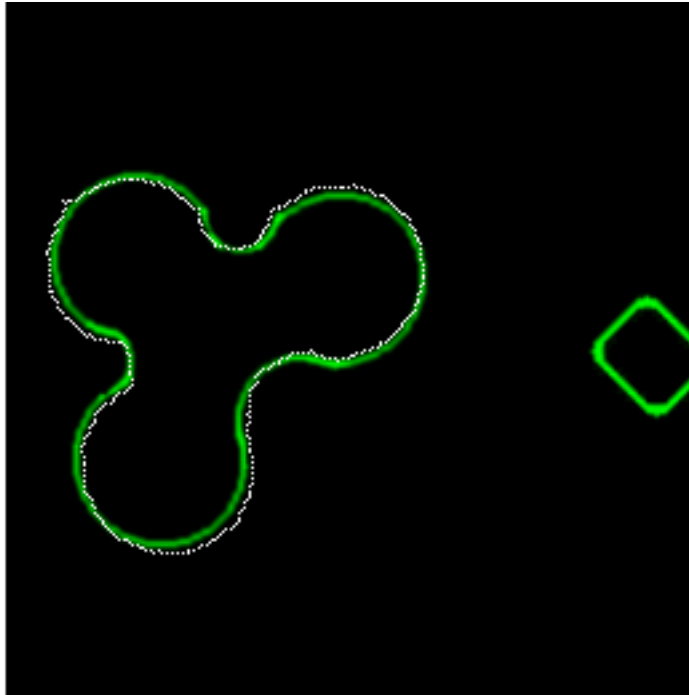


reversed to have CCW ordering

scale should be 1.3  
rotation should be 360 - 225

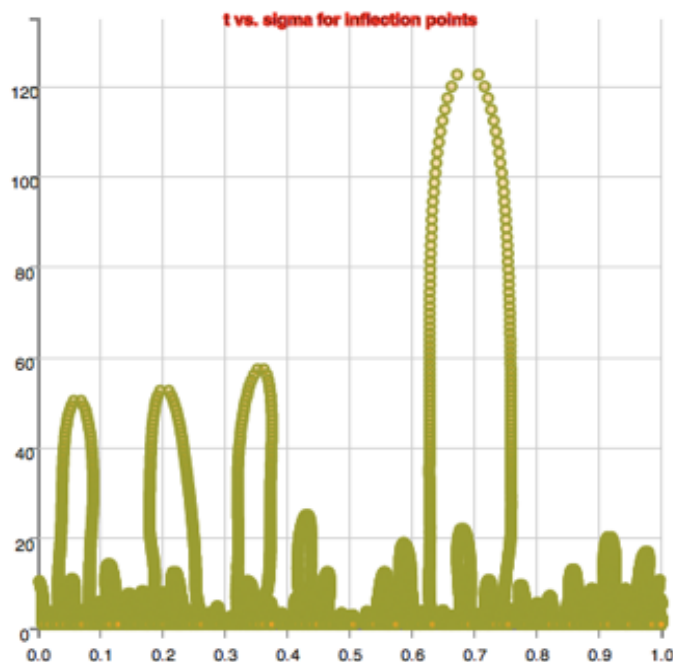
Contour matcher solution scale=1.325237512588501  
Contour matcher solution shift=-0.5053889751434326  
CONTOUR PEAK1: (32.000237, 0.497126) (34, 78) (35, 72) CONTOUR PEAK2: (42.407913, 0.153422) (98, 136) (91, 143)  
CONTOUR PEAK1: (26.908875, 0.146552) (70, 93) (61, 99) CONTOUR PEAK2: (34.896511, 0.802428) (81, 91) (92, 92)  
CONTOUR PEAK1: (26.908875, 0.846264) (60, 45) (69, 54) CONTOUR PEAK2: (34.896511, 0.508830) (46, 140) (45, 129)  
offsetImgX1=10 offsetImgY1=10  
offsetImgX2=14 offsetImgY2=61  
rotationInRadians=2.4194849  
rotationInDegrees=138.6262707153875  
scale=1.3252375  
translationX=-5.8268623  
translationY=25.13414

## apply coordinate transformation



offsetImgX1=10 offsetImgY1=10  
offsetImgX2=14 offsetImgY2=61  
rotationInRadians=2.4194849  
rotationInDegrees=138.6262707153875  
scale=1.3252375  
translationX=-5.8268623  
translationY=25.13414

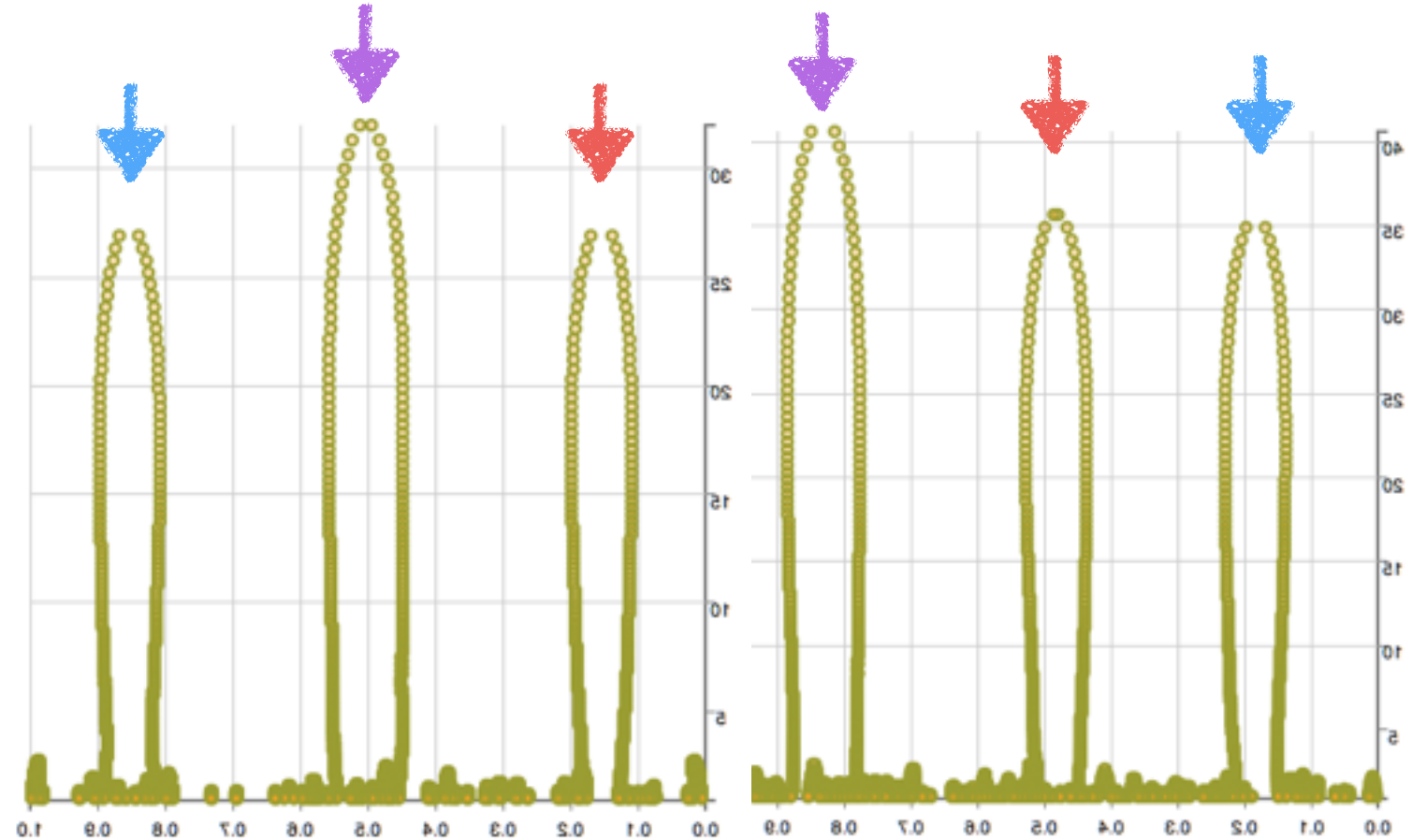
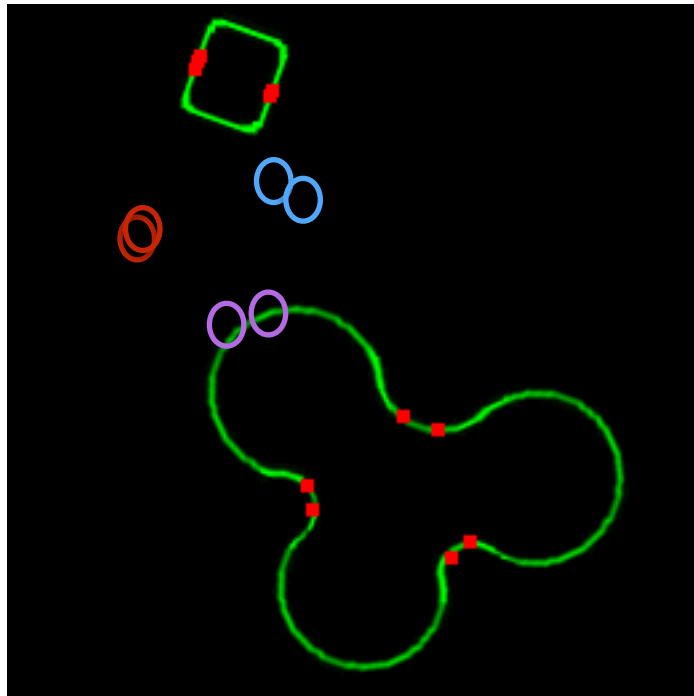
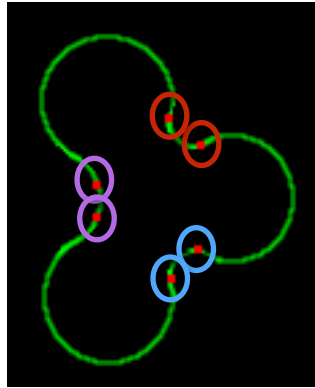
scale should be 1.3  
rotation should be 360 - 225



## After Refinement

rotationInRadians=2.3322184  
rotationInDegrees=133.6262717146147  
scale=1.2752376  
translationX=-3.0  
translationY=27.0

# coordinate transformation, after matching contours



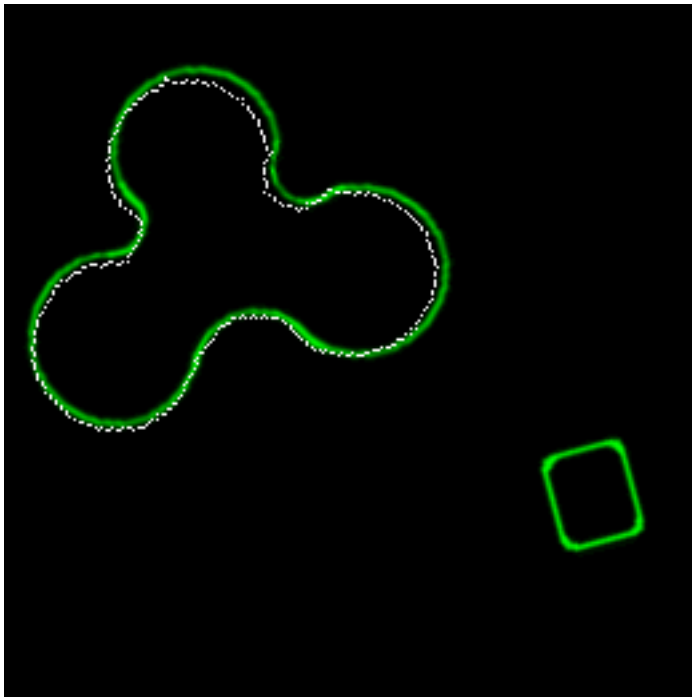
reversed to have CCW ordering

reversed to have CCW ordering

scale should be 1.3  
rotation should be 360 - 255

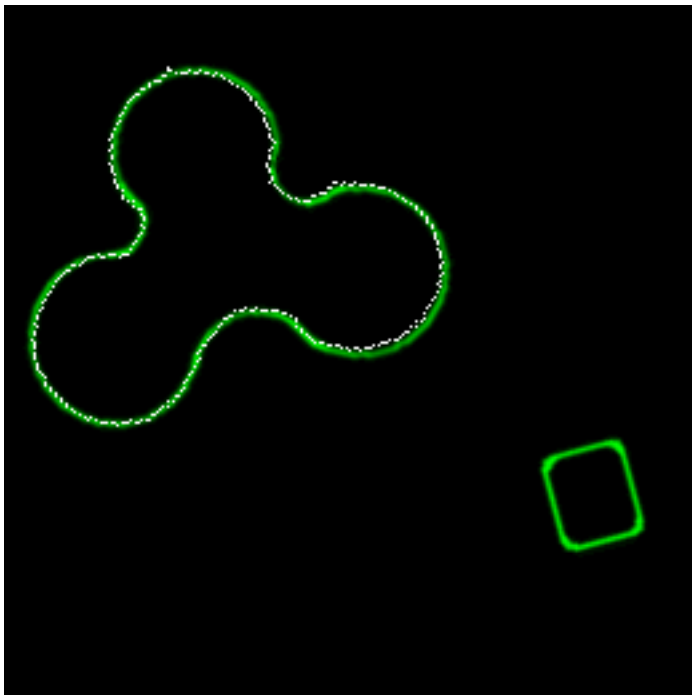
CONTOUR PEAK1: (32.000237, 0.497126) (34, 78) (35, 72) CONTOUR PEAK2: (40.609955, 0.168514) (98, 115) (82, 118)  
CONTOUR PEAK1: (26.908875, 0.146552) (70, 93) (61, 99) CONTOUR PEAK2: (34.896511, 0.817073) (101, 65) (113, 73)  
CONTOUR PEAK1: (26.908875, 0.846264) (60, 45) (69, 54) CONTOUR PEAK2: (35.660648, 0.517738) (49, 89) (51, 86)  
offsetImgX1=10 offsetImgY1=10  
offsetImgX2=5 offsetImgY2=20  
rotationInRadians=1.8595577  
rotationInDegrees=106.54481071213507  
scale=1.2690517  
translationX=3.6613295  
translationY=-1.6037707

## apply coordinate transformation



rotationInRadians=1.8595577  
rotationInDegrees=106.54481071213507  
scale=1.2690517  
translationX=3.6613295  
translationY=-1.6037707

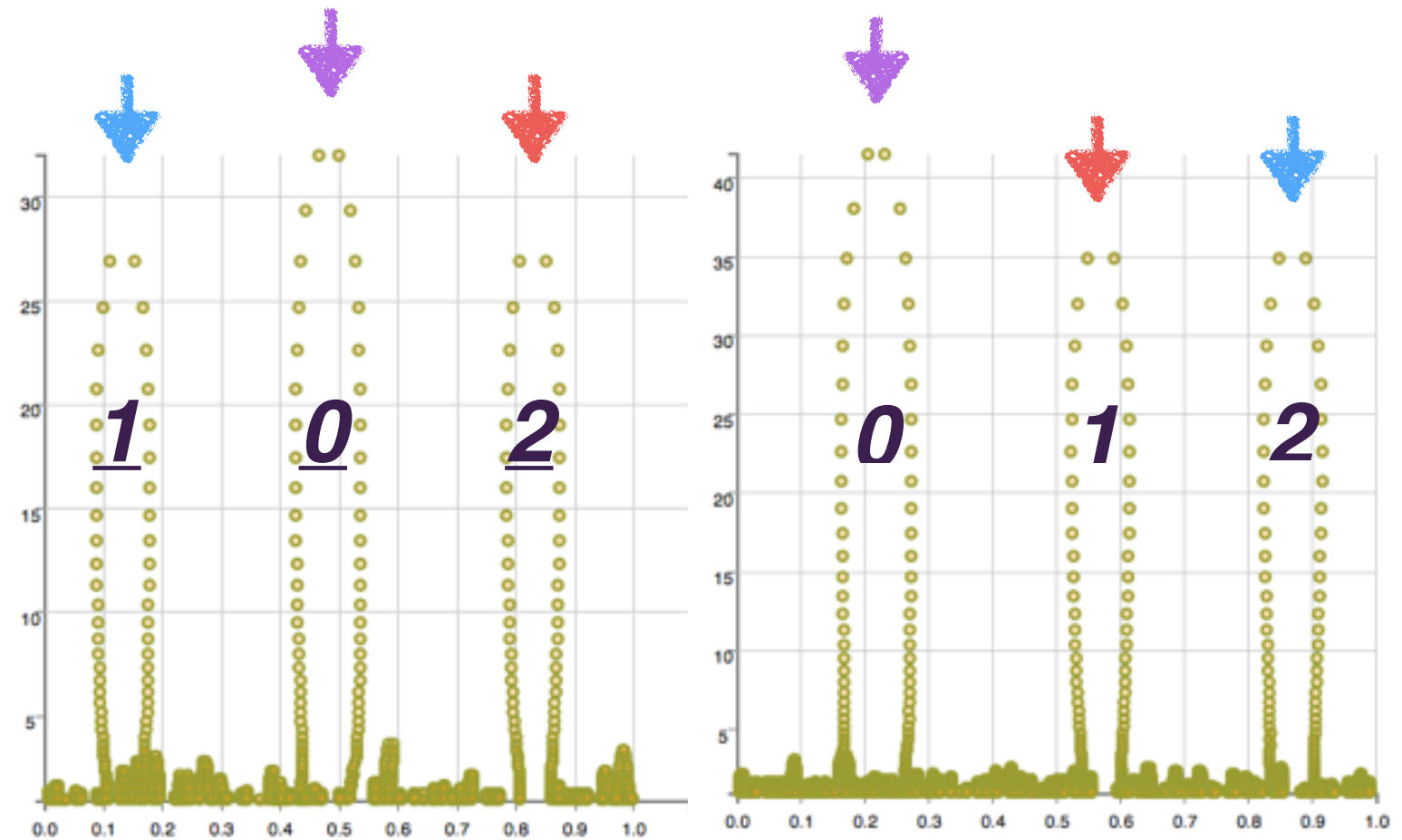
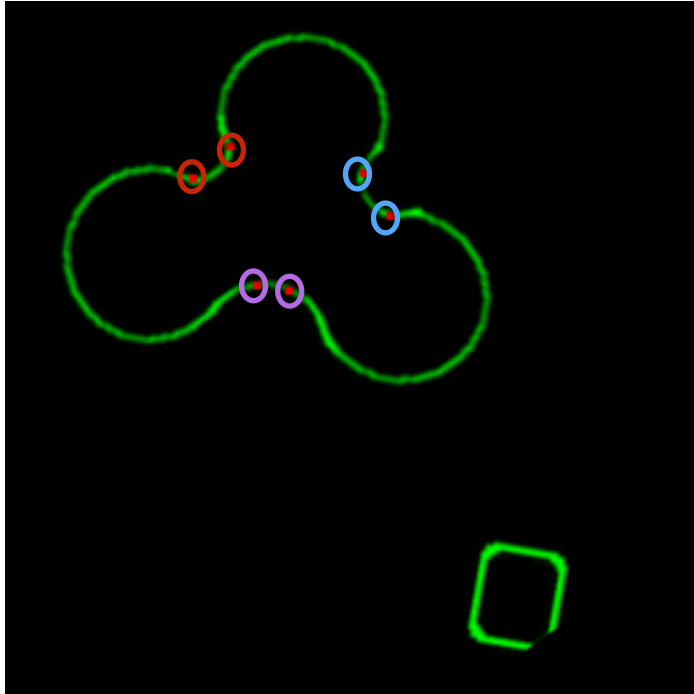
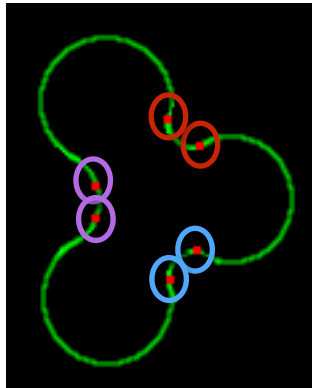
scale should be 1.3  
rotation should be 360 - 255



### After Refinement

rotationInRadians=1.8558925  
rotationInDegrees=106.33480971591386  
scale=1.2845517  
translationX=4.0  
translationY=-6.0

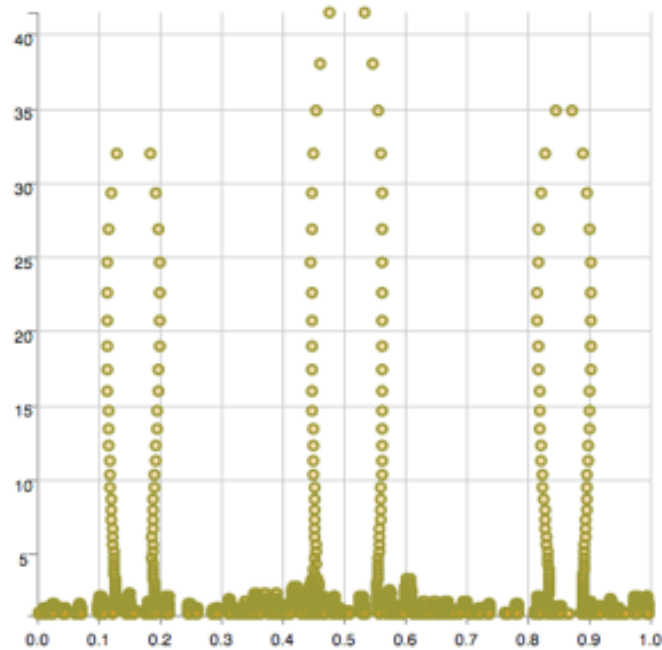
# coordinate transformation, after matching contours



scale should be 1.3  
rotation should be 360 - 280

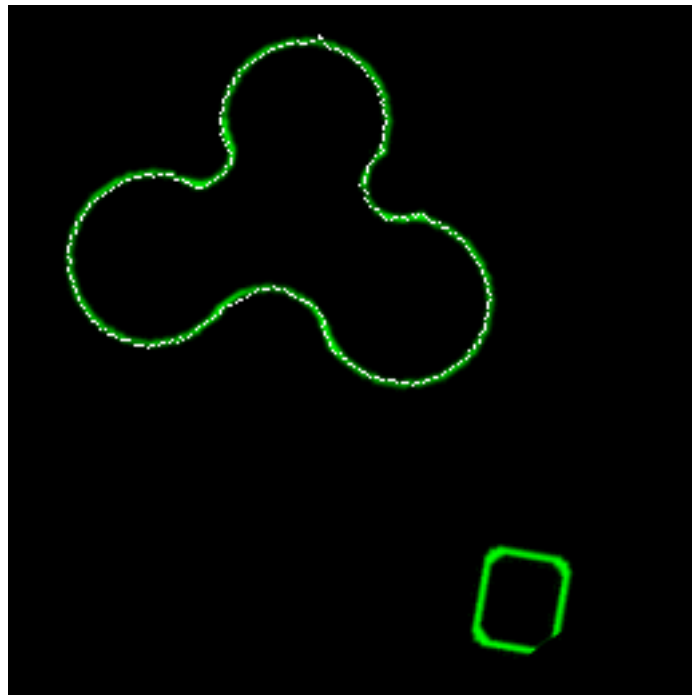
Contour matcher solution scale=1.2968404293060303  
Contour matcher solution shift=-0.4310373365879059  
CONTOUR PEAK1: (32.000237, 0.497126) (34, 78) (35, 72) CONTOUR PEAK2: (41.499199, 0.213656) (106, 108) (96, 106)  
CONTOUR PEAK1: (26.908875, 0.146552) (70, 93) (61, 99) CONTOUR PEAK2: (35.660648, 0.865639) (134, 70) (137, 76)  
CONTOUR PEAK1: (26.908875, 0.846264) (60, 45) (69, 54) CONTOUR PEAK2: (35.660648, 0.566079) (74, 67) (82, 60)  
offsetImgX1=10 offsetImgY1=10  
offsetImgX2=19 offsetImgY2=9  
rotationInRadians=1.4486057  
rotationInDegrees=82.9989903033234  
scale=1.2968404  
translationX=28.18638  
translationY=-16.518988

apply coordinate transformation



offsetImgX1=10 offsetImgY1=10  
offsetImgX2=19 offsetImgY2=9  
rotationInRadians=1.4486057  
rotationInDegrees=82.9989903033234  
scale=1.2968404  
translationX=28.18638  
translationY=-16.518988

scale should be 1.3  
rotation should be 360 - 280

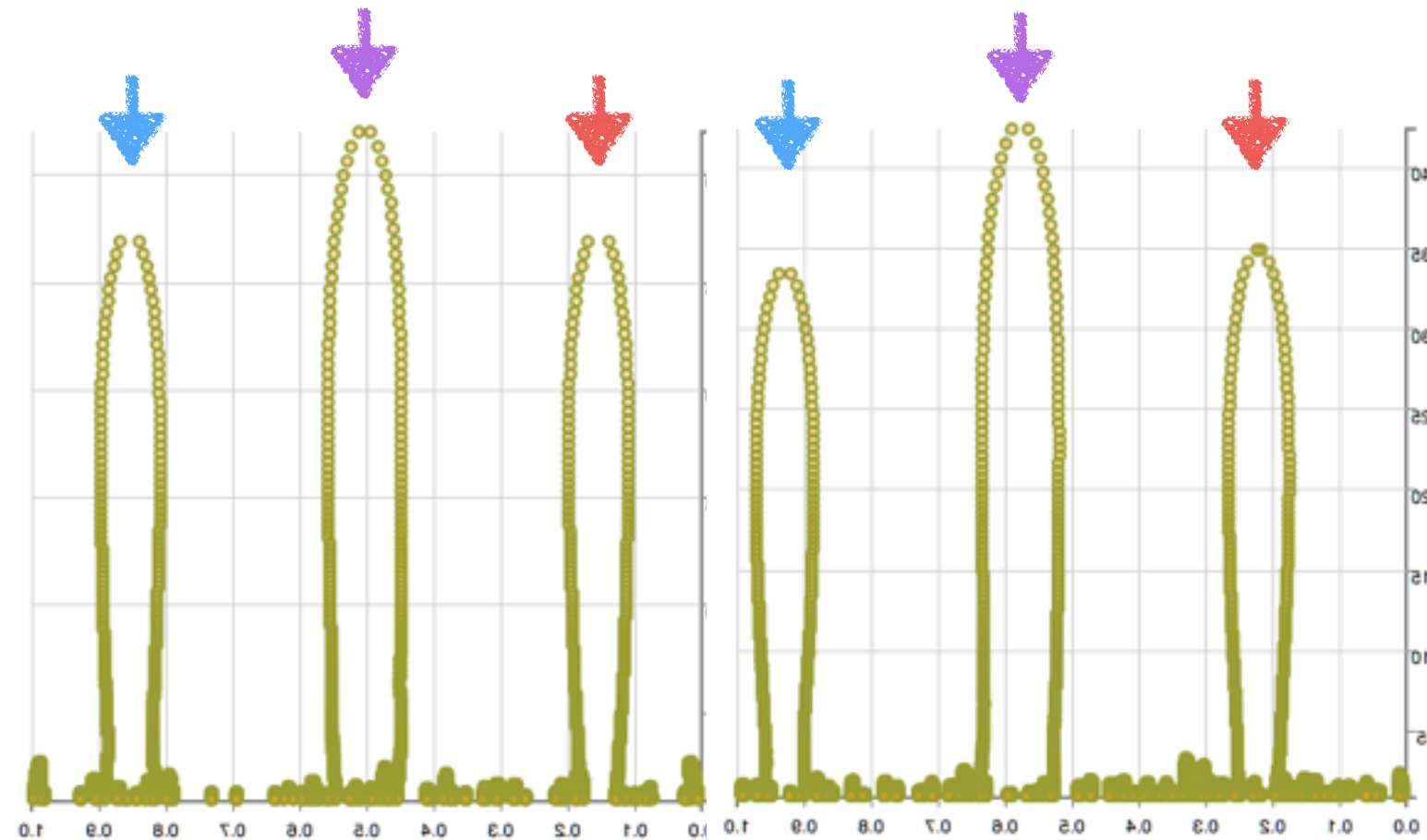
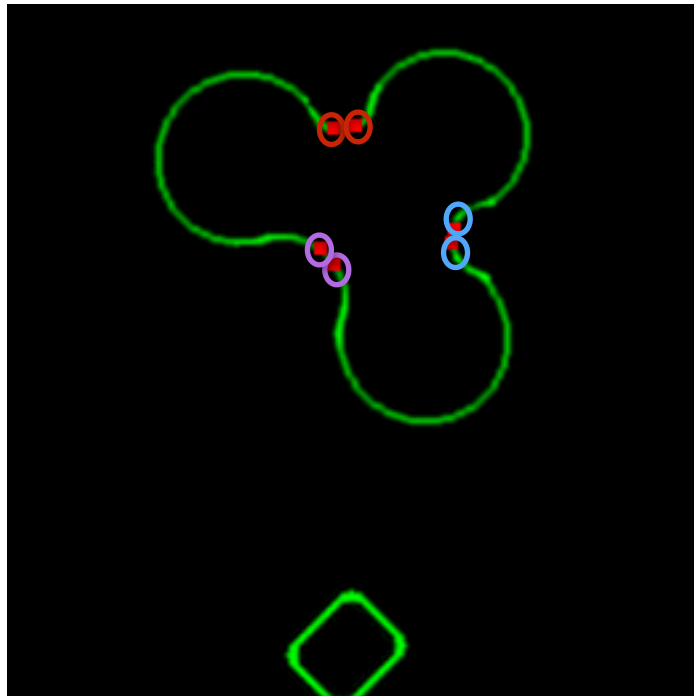
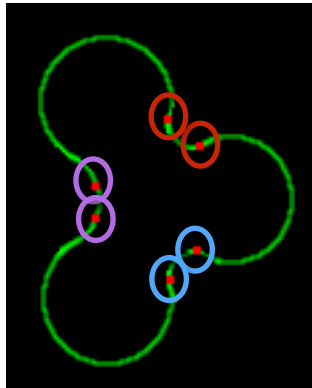


**After Refinement**

rotationInRadians=1.4049724  
rotationInDegrees=80.49899080293699  
scale=1.2968404  
translationX=29.0  
translationY=-17.0



# coordinate transformation, after matching contours



reversed to have CCW ordering

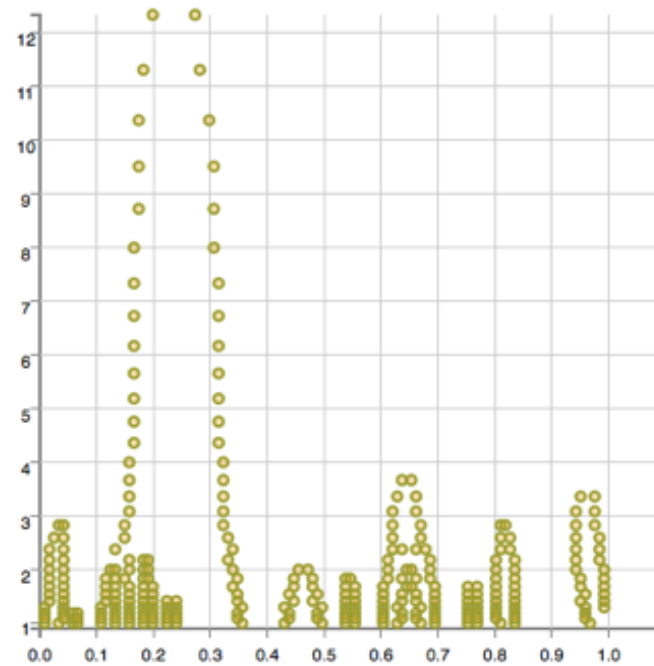
reversed to have CCW ordering

scale should be 1.3  
rotation should be 360 - 315

Contour matcher solution scale=1.325237512588501  
Contour matcher solution shift=-0.2311936765909195  
Contour matcher solution cost=89.0  
CONTOUR PEAK1: (32.000237, 0.497126) (34, 78) (35, 72)  
CONTOUR PEAK1: (26.908875, 0.146552) (70, 93) (61, 99)  
CONTOUR PEAK1: (26.908875, 0.846264) (60, 45) (69, 54)  
offsetImgX1=10 offsetImgY1=10  
offsetImgX2=52 offsetImgY2=14  
rotationInRadians=0.7365556  
rotationInDegrees=42.2015259002177  
scale=1.3252375  
translationX=60.744064  
translationY=-25.366236

CONTOUR PEAK2: (42.407913, 0.427617) (125, 102) (112, 89)  
CONTOUR PEAK2: (33.417011, 0.074610) (169, 80) (168, 93)  
CONTOUR PEAK2: (34.148750, 0.783964) (120, 45) (132, 45)

apply coordinate transformation



offsetImgX1=10 offsetImgY1=10  
offsetImgX2=52 offsetImgY2=14  
rotationInRadians=0.7365556  
rotationInDegrees=42.2015259002177  
scale=1.3252375  
translationX=60.744064  
translationY=-25.366236

scale should be 1.3  
rotation should be 360 - 315

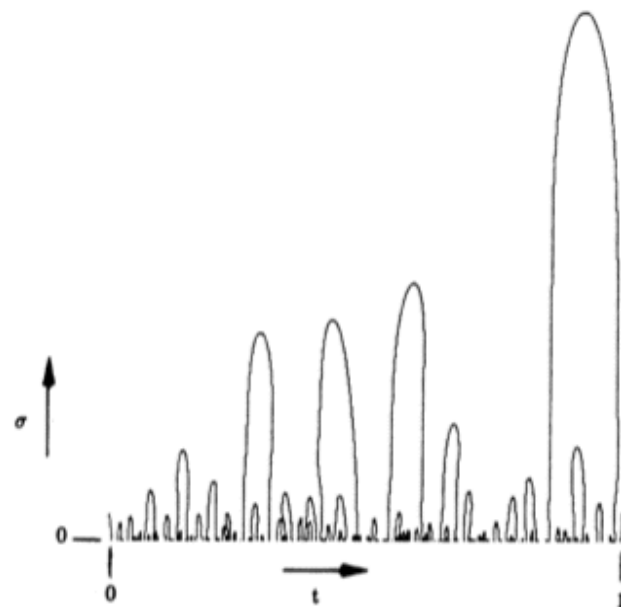
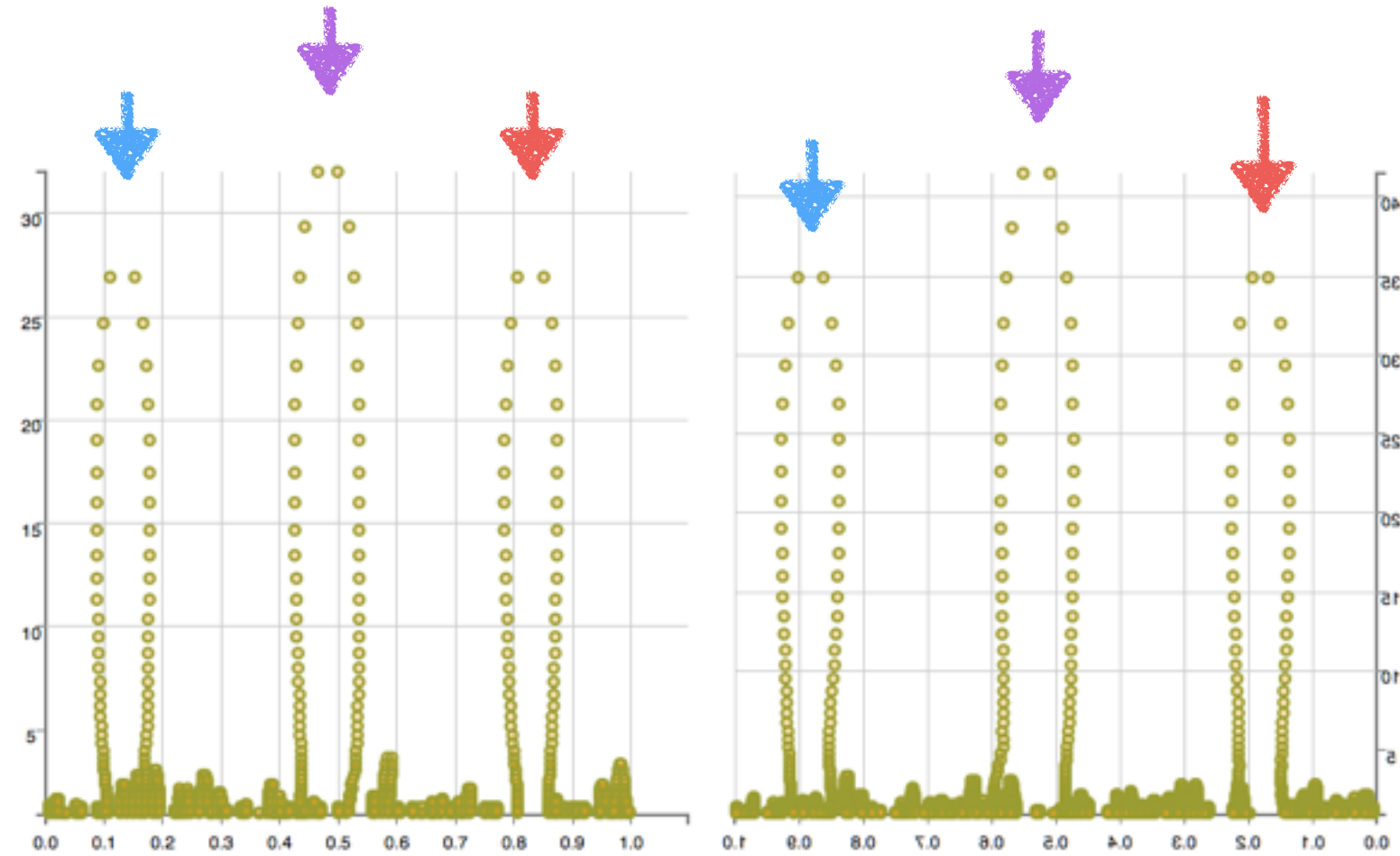
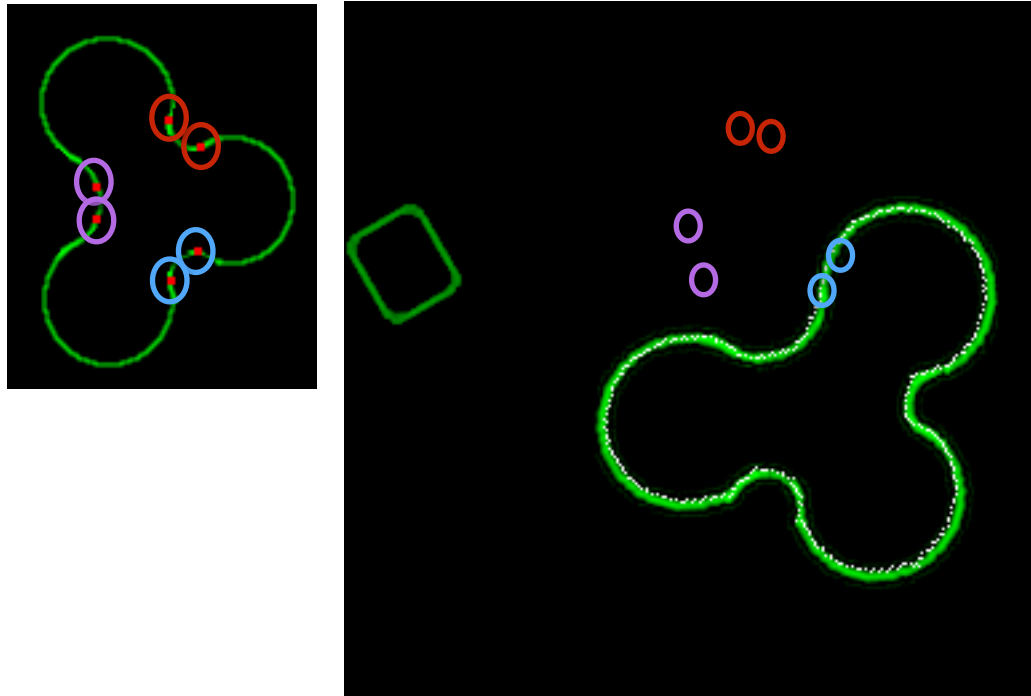


Fig. 3. Generalized scale space image of Africa.

**After Refinement**

rotationInRadians=0.7801888  
rotationInDegrees=44.70152540060411  
scale=1.2752376  
translationX=65.0  
translationY=-21.0

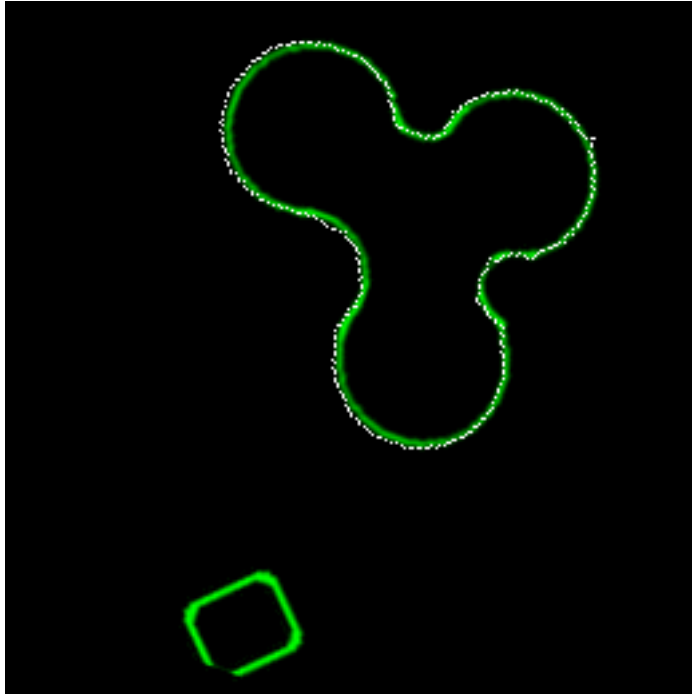
# coordinate transformation, after matching contours



scale should be 1.3  
rotation should be 360 - 335

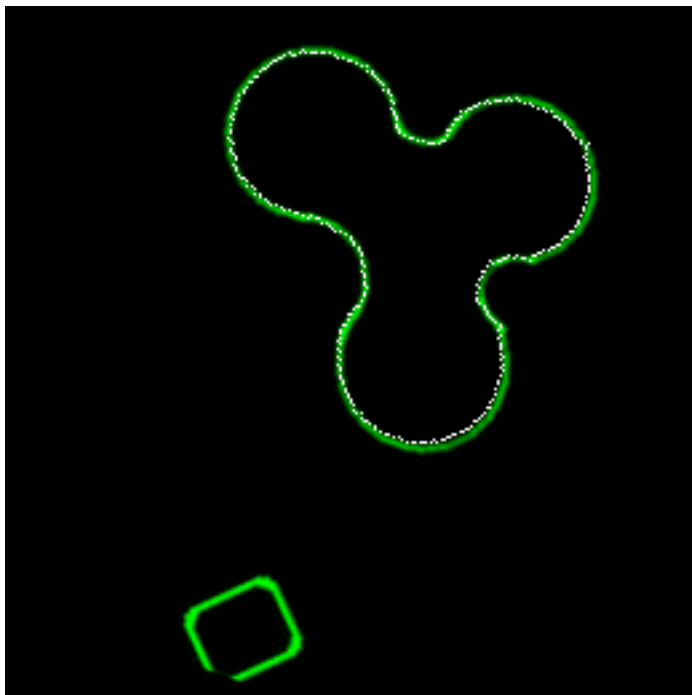
Contour matcher solution scale=1.325237512588501  
Contour matcher solution shift=-0.19089026749134064  
CONTOUR PEAK1: (32.000237, 0.497126) (34, 78) (35, 72) CONTOUR PEAK2: (42.407913, 0.467920) (134, 99) (131, 90)  
CONTOUR PEAK1: (26.908875, 0.146552) (70, 93) (61, 99) CONTOUR PEAK2: (34.896511, 0.115044) (182, 98) (178, 106)  
CONTOUR PEAK1: (26.908875, 0.846264) (60, 45) (69, 54) CONTOUR PEAK2: (34.896511, 0.818584) (150, 48) (158, 51)  
offsetImgX1=10 offsetImgY1=10  
offsetImgX2=63 offsetImgY2=12  
rotationInRadians=0.43239865  
rotationInDegrees=24.77461754427387  
scale=1.3252375  
translationX=81.613556  
translationY=-16.98058

## apply coordinate transformation



```
offsetImgX1=10 offsetImgY1=10  
offsetImgX2=63 offsetImgY2=12  
rotationInRadians=0.43239865  
rotationInDegrees=24.77461754427387  
scale=1.3252375  
translationX=81.613556  
translationY=-16.98058
```

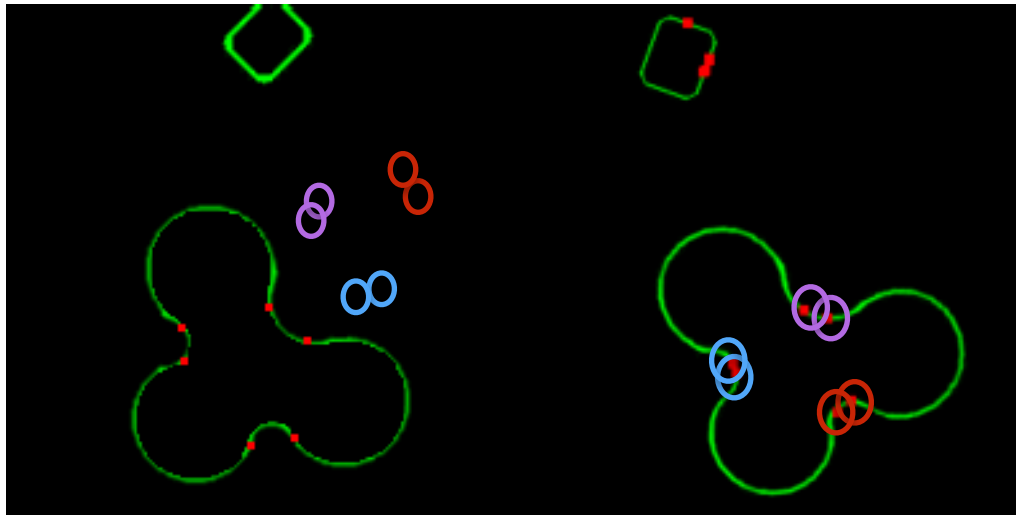
scale should be 1.3  
rotation should be 360 - 335



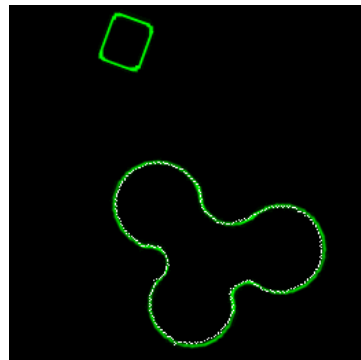
### After Refinement

```
rotationInRadians=0.43239865  
rotationInDegrees=24.77461754427387  
scale=1.2752376  
translationX=85.0  
translationY=-14.0
```

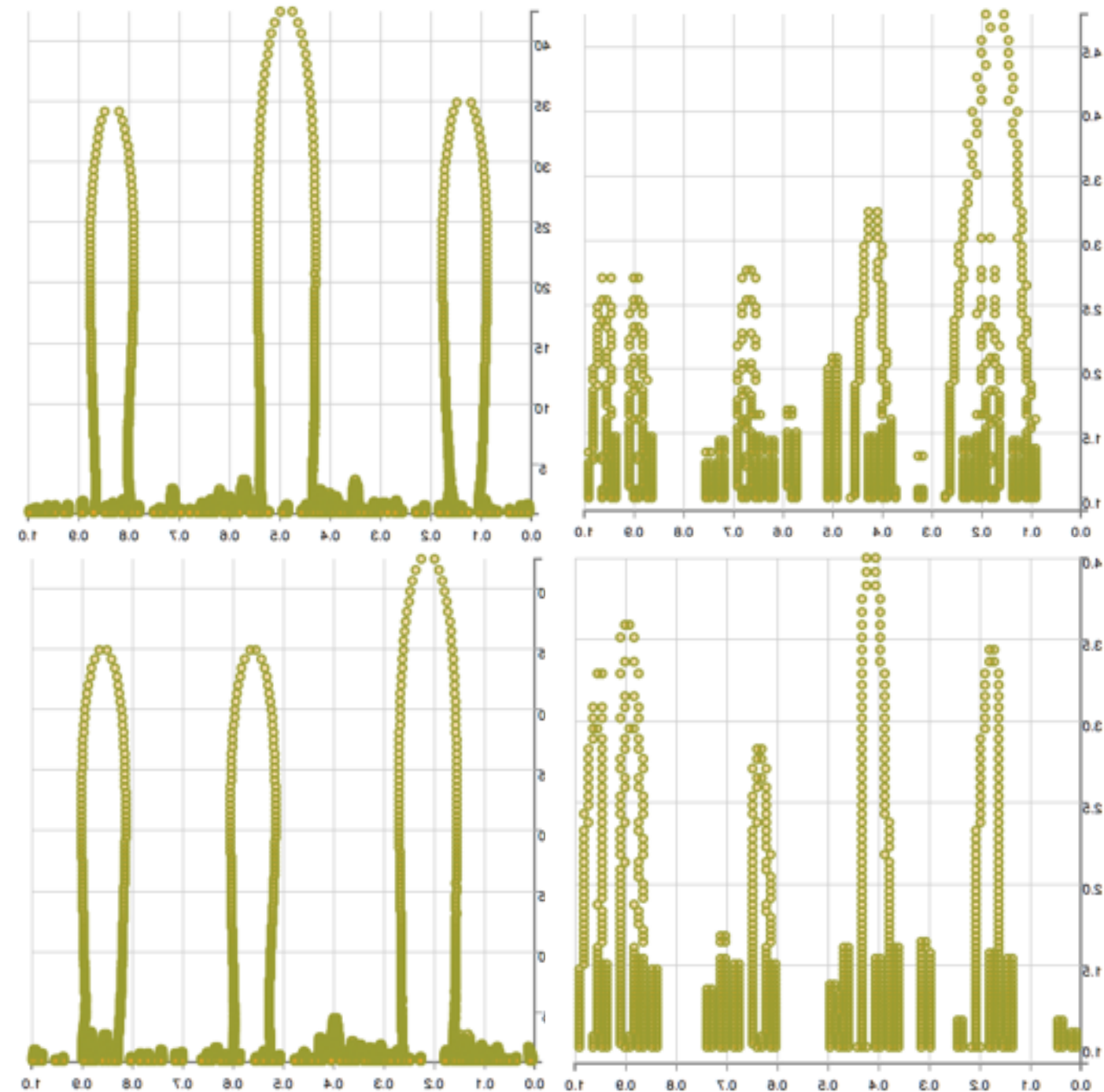
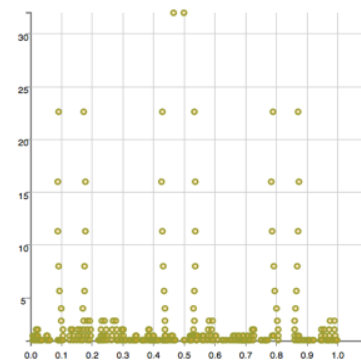
# coordinate transformation, after matching contours



rotationInRadians=4.7316465  
rotationInDegrees=271.1033767625259  
scale=1.0  
translationX=0.5612701  
translationY=0.34837633



rotationInRadians=4.7141933  
rotationInDegrees=270.10338242652267  
scale=1.0  
translationX=-0.19677997  
translationY=0.0

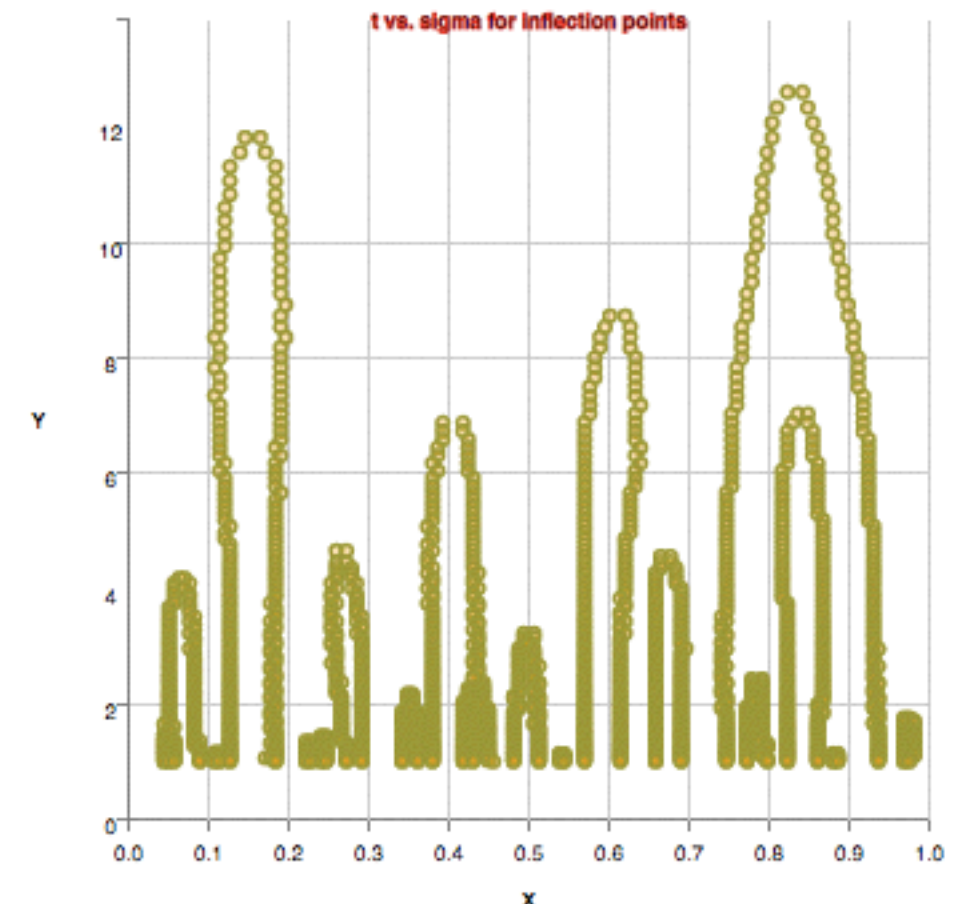
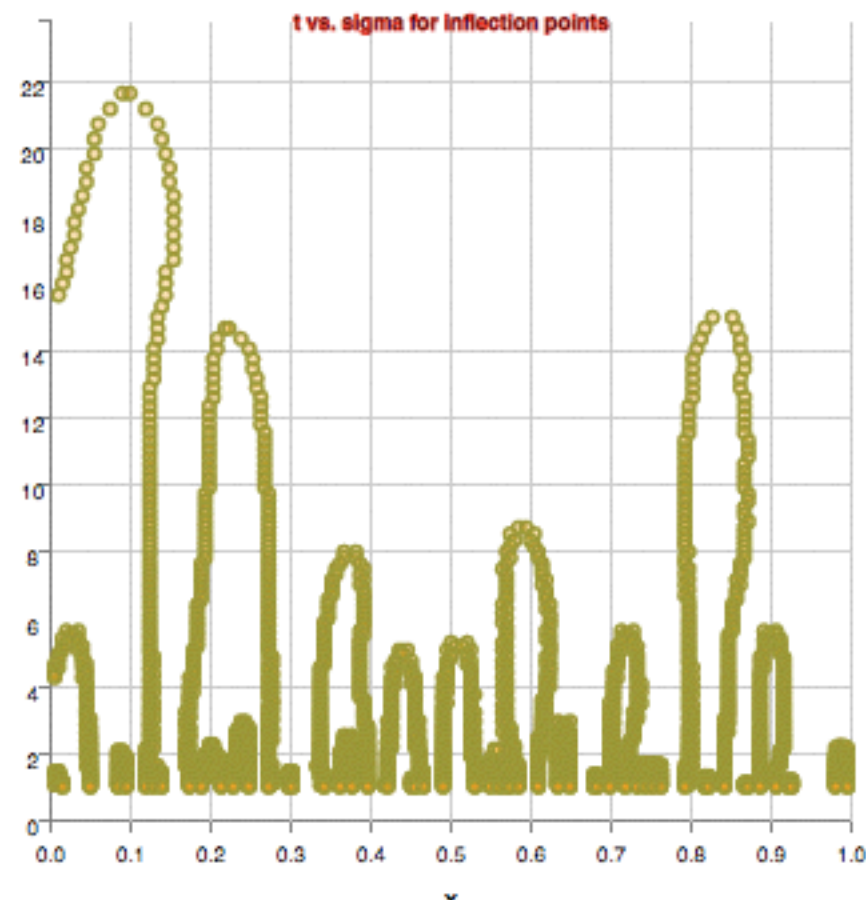
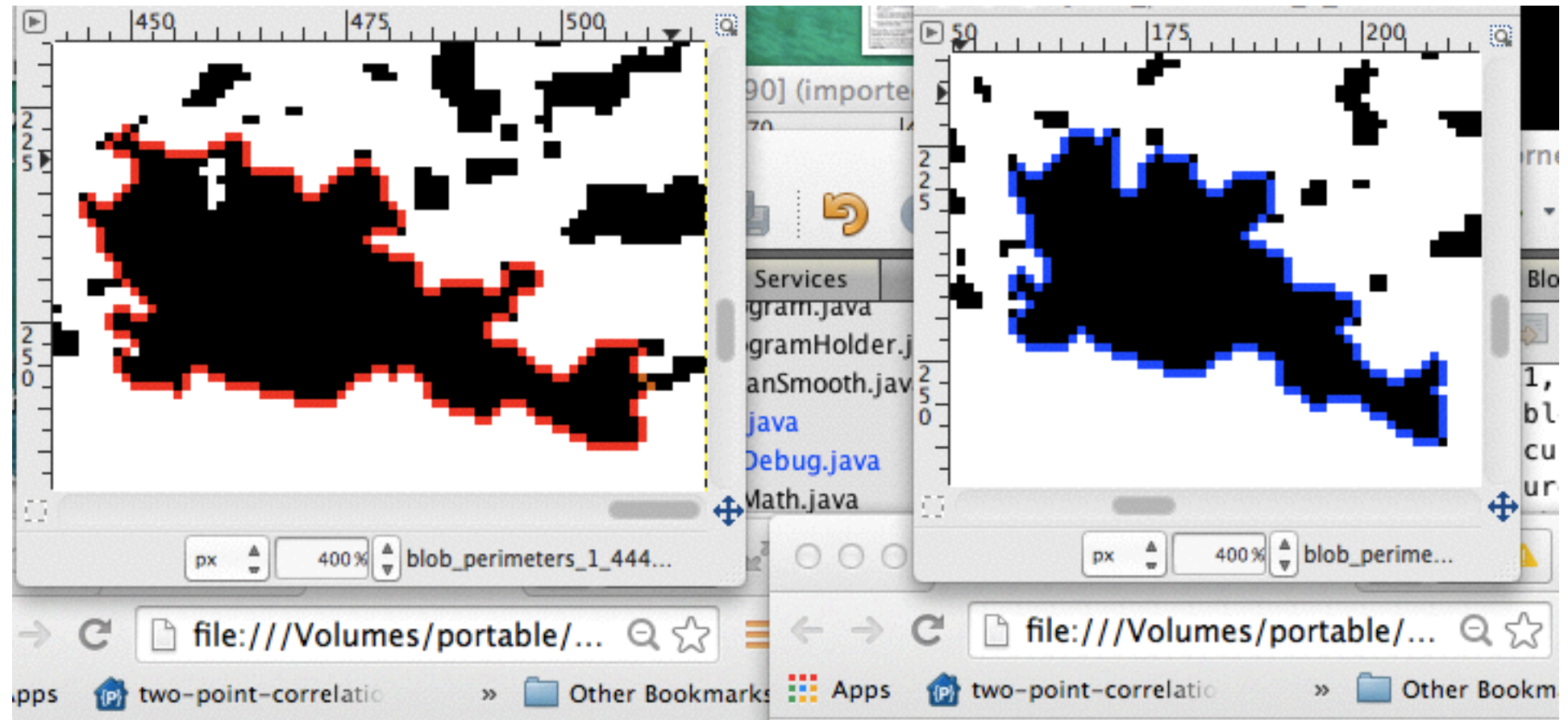


Contour matcher solution scale=1.0  
Contour matcher solution shift=0.27483445405960083  
Contour matcher solution cost=14.0  
CONTOUR PEAK1: (42.407913, 0.513245) (156, 110) (159, 99) CONTOUR PEAK2: (42.407913, 0.788080) (149, 155) (161, 159)  
CONTOUR PEAK1: (34.896511, 0.866446) (201, 83) (208, 96) CONTOUR PEAK2: (34.896511, 0.142384) (173, 201) (166, 207)  
CONTOUR PEAK1: (4.756843, 0.845455) (33, 160) (33, 160) CONTOUR PEAK2: (3.589427, 0.103604) (99, 33) (98, 34)  
CONTOUR PEAK1: (34.148750, 0.166667) (191, 144) (178, 149) CONTOUR PEAK2: (34.896511, 0.440397) (115, 187) (113, 182)  
CONTOUR PEAK1: (3.220988, 0.581818) (9, 170) (10, 168) CONTOUR PEAK2: (3.437247, 0.828829) (90, 9) (90, 9)  
CONTOUR PEAK1: (3.018335, 0.809091) (28, 158) (30, 159) CONTOUR PEAK2: (3.291518, 0.049550) (101, 27) (101, 28)



very important to have similar curves.

The same blob in different images with a couple of large bumps introduced from the image processing and then perimeter extraction instead of edges, shows a scale space map on left dominated by the difference. The contour matcher prefers the strongest peaks so does not give as good of a score for the true match (seen from 0.2 to 0.9)

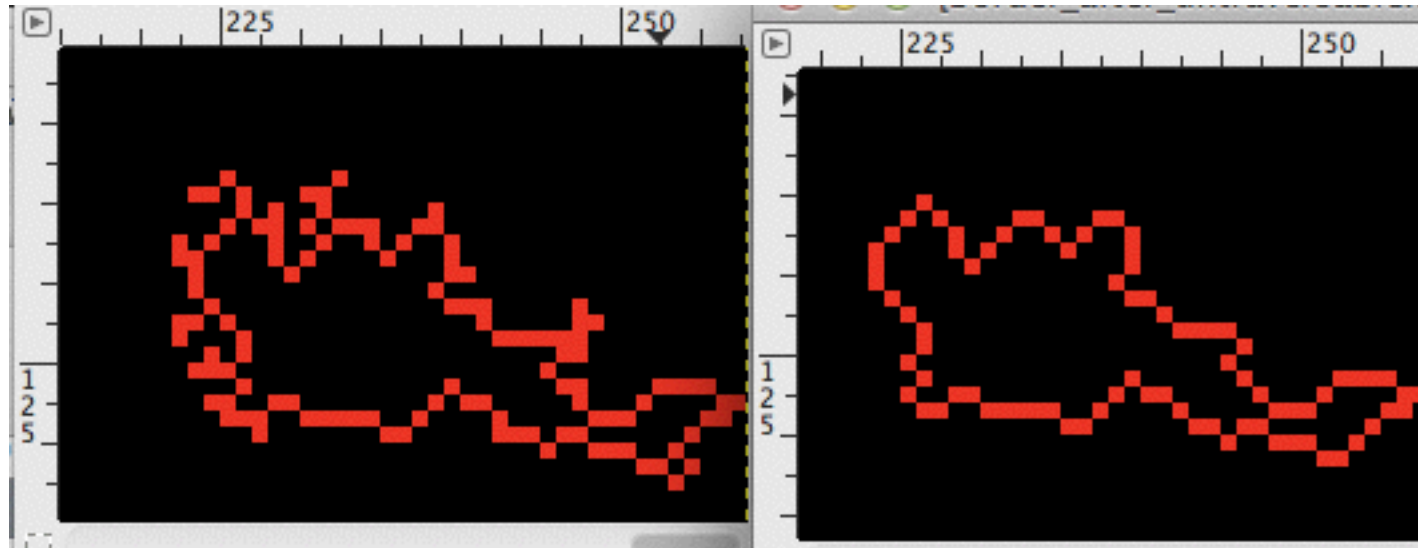


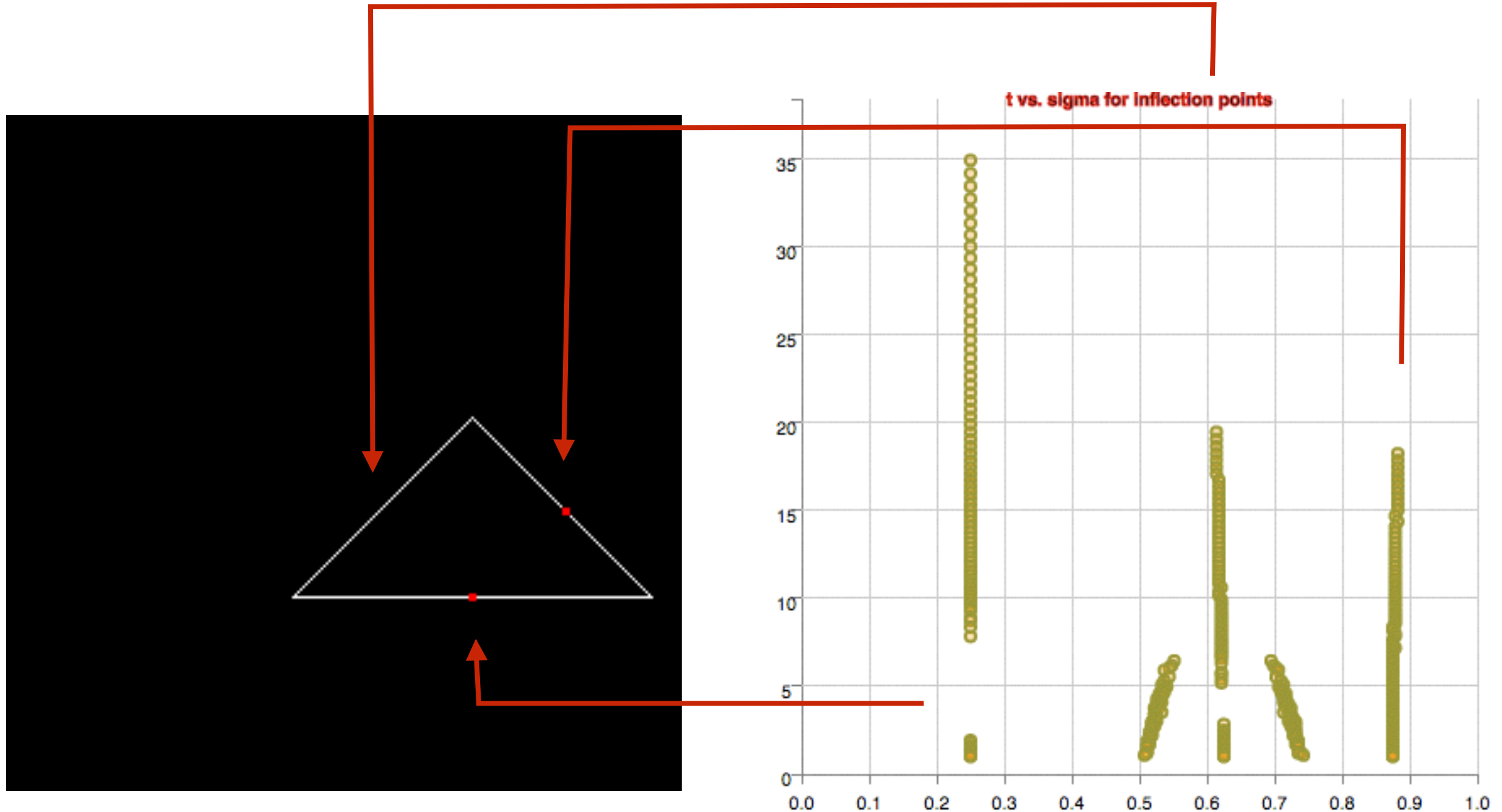


Note that the blob perimeters were not constructed from differences of gaussians, but instead from the borders of contiguous pixels in segmented images.

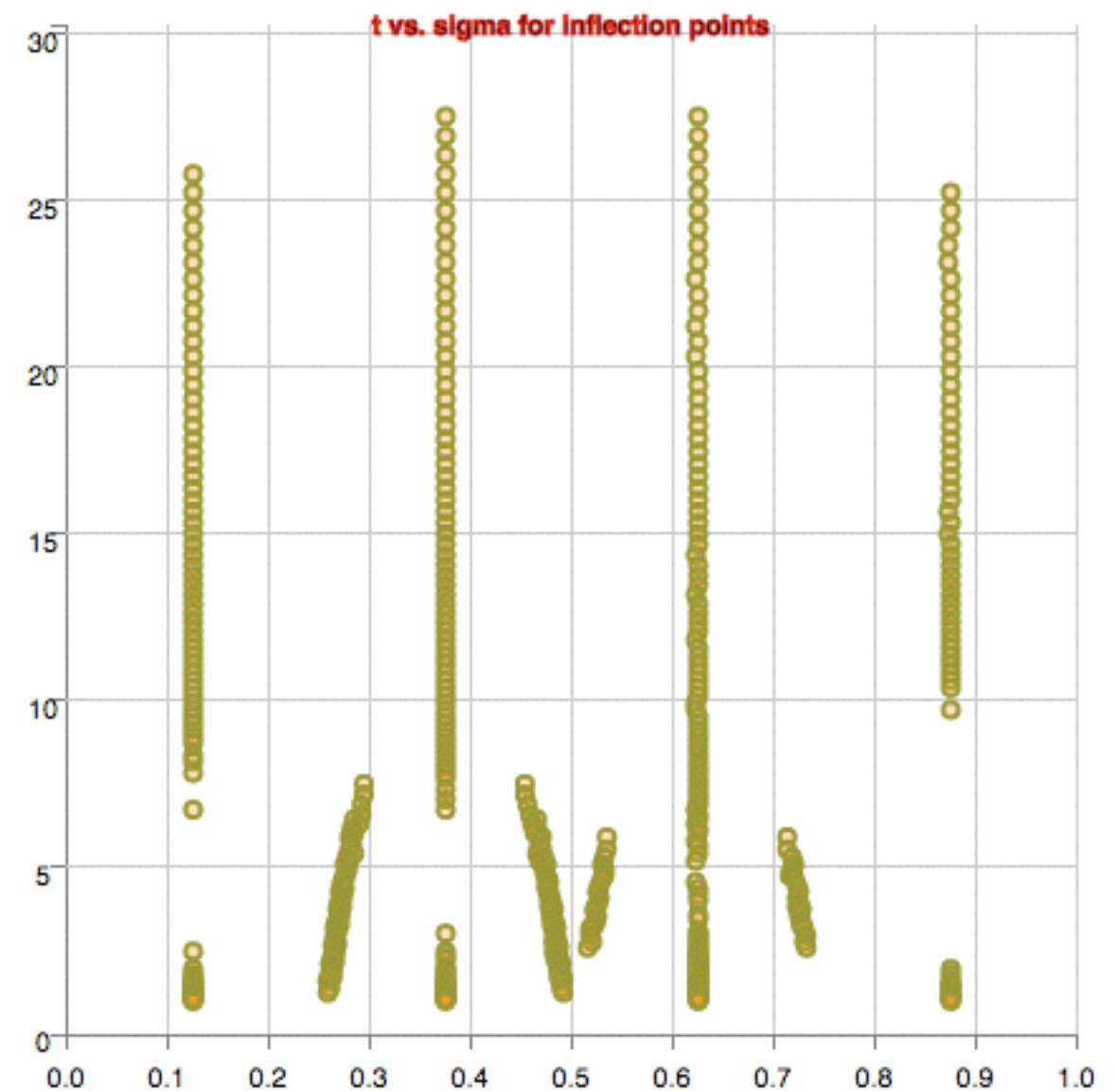
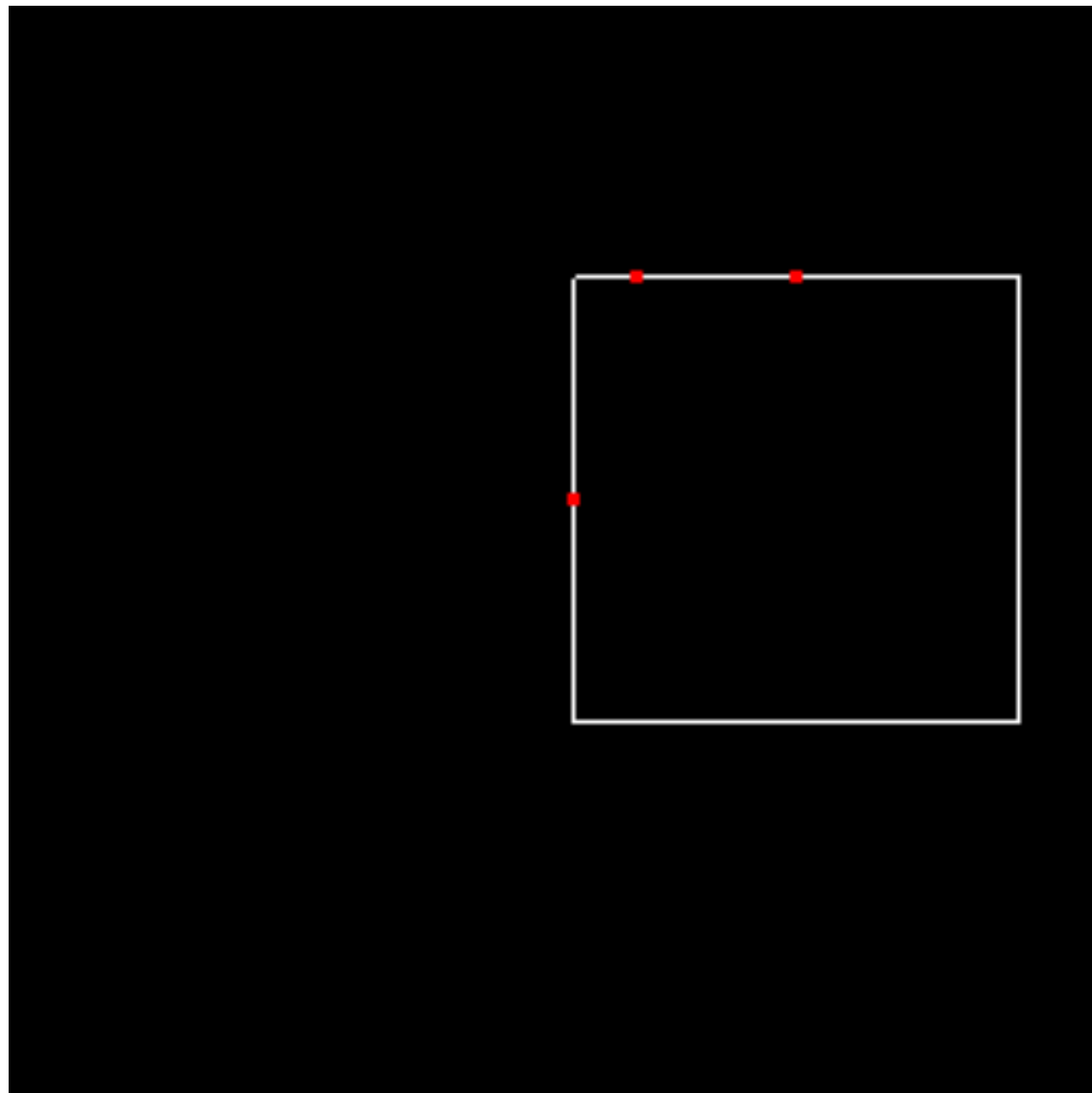
So a lot of processing of the perimeter is needed to make a usable contour.

Here's an example of the blob from a binned segmented image before and after additional line processing.

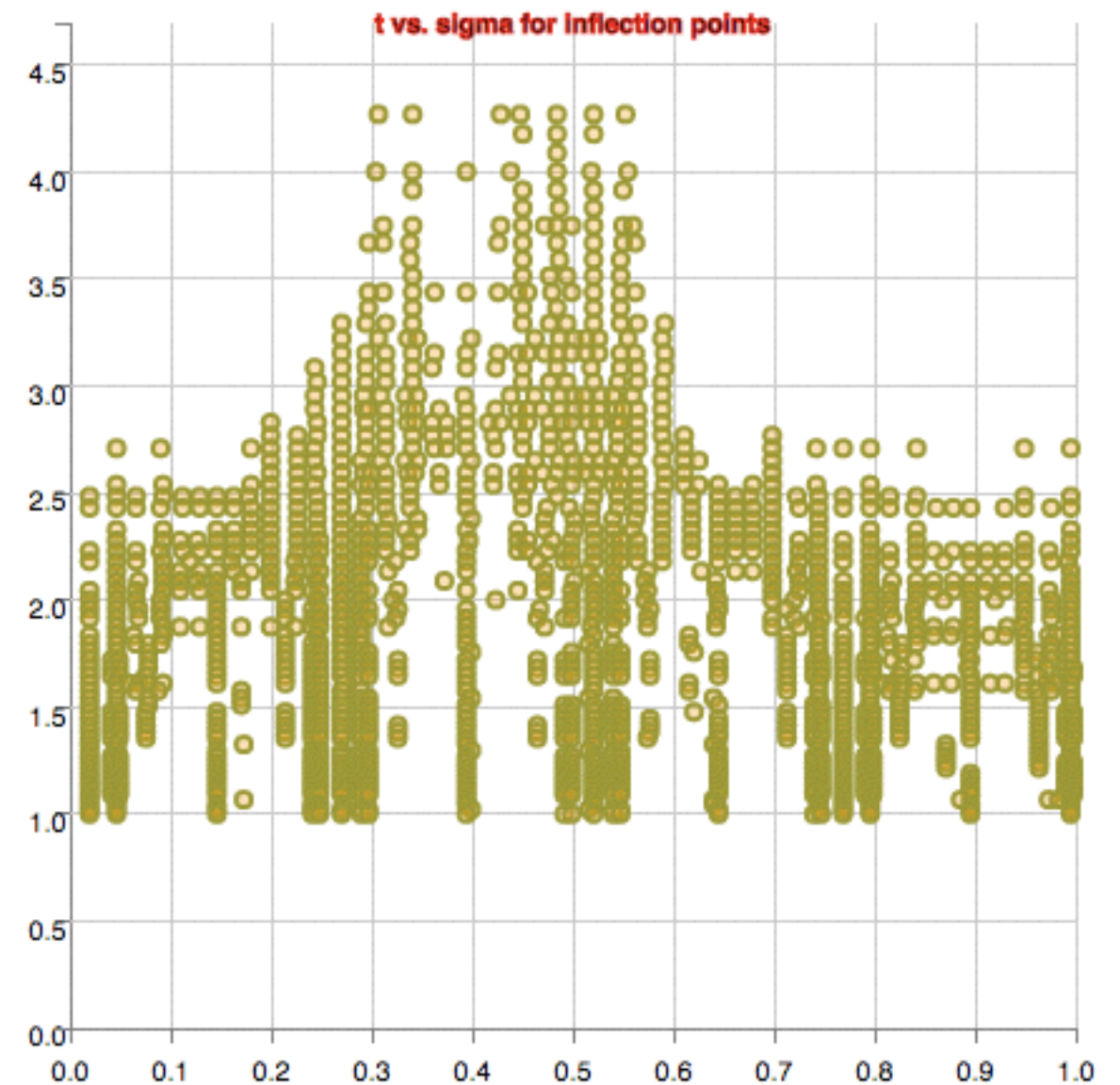
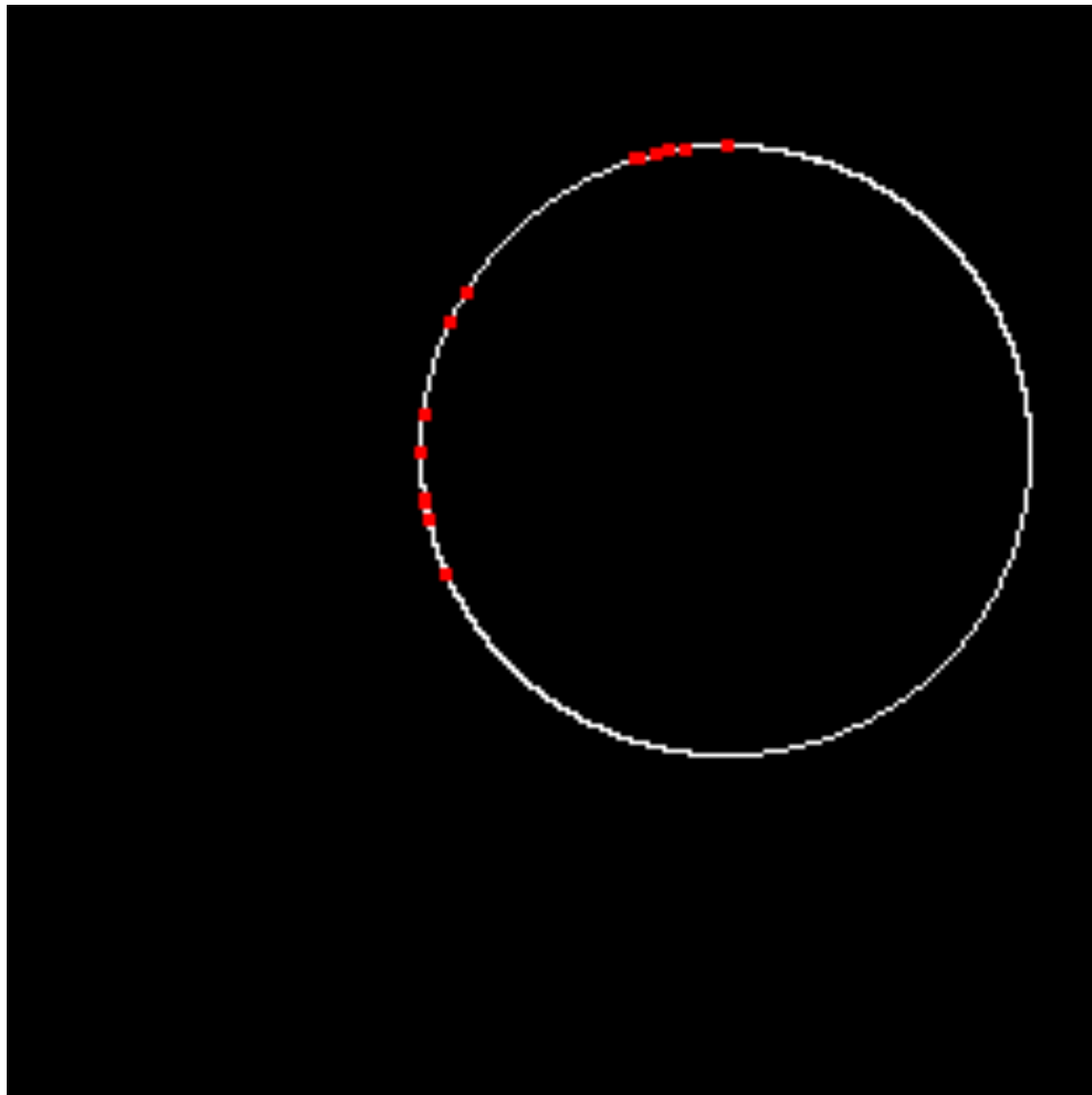




**The scale space images are dependent upon defining the zero crossings of curvature, and need to be improved. Finding wide minima combined with zero-crossings has improved the results.**



The scale space images are dependent upon defining the zero crossings of curvature, and need to be improved. Finding wide minima combined with zero-crossings has improved the results.



The scale space images are dependent upon defining the zero crossings of curvature, and need to be improved. Finding wide minima combined with zero-crossings has improved the results.

Considering a hybrid approach to use corners around an object, but the runtime might be larger than the contour approach (not considering corner extraction vs scale space image construction and moving feature comparison into the first search):

- (1) find the closed curve surrounding an object.

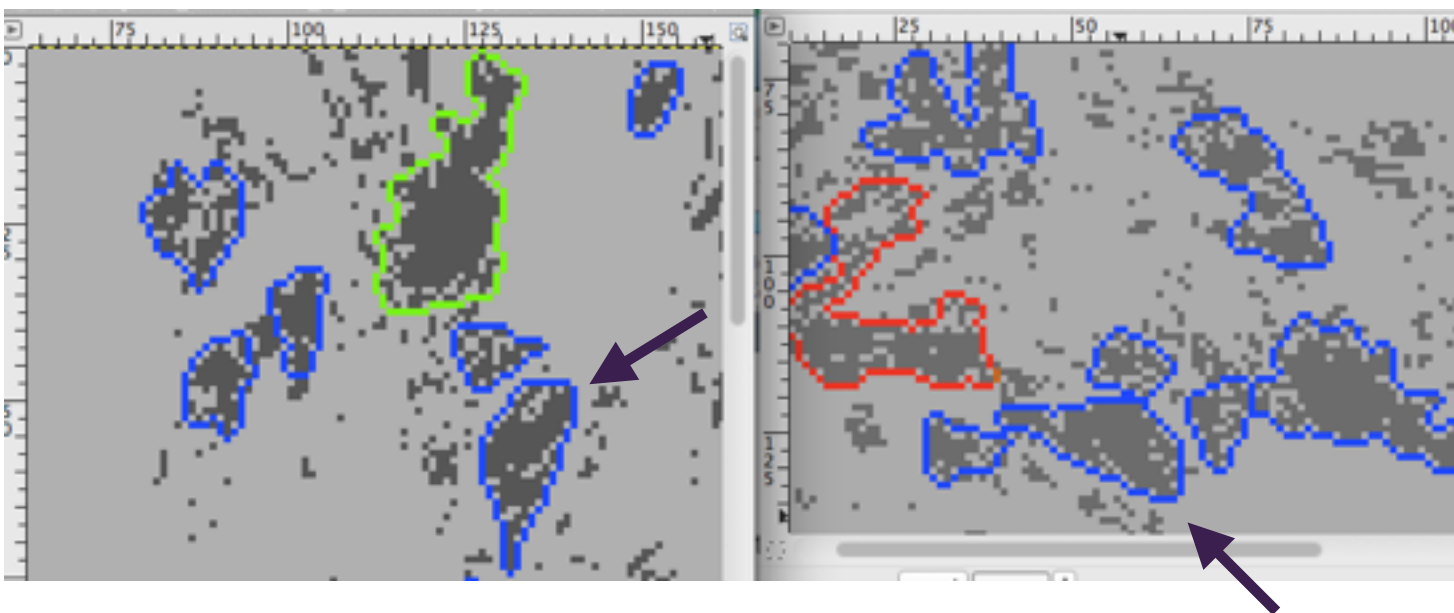
- (2) determine corners on the curve.

- (3) match pairs of points in one curve in an image with pairs of points in a curve in another image using location and feature descriptors. Like the contour matching, one could make an assumption of strongest characteristic to create a reference point used to determine a reference for the scores and penalties of matching. For a non-scale-space-image approach, the smallest error point descriptor along the first curve could be treated as the reference for all of the first curve's matches. Then one determines the best match for it along the second curve ( $O(N)$ ). That single point match needs a second correct match to make a euclidean transformation estimate. The second single point to match along the first curve could be found as the one with the next smallest feature error at a reasonable distance from the first point. Its match along the second curve would be found in  $O(N)$ . From those two matching points, one could estimate the euclidean transformation and then apply it one by one to the remaining points in the first curve and use a scoring and penalty system adapted from the contour matching (SSD instead of sigma, but additive not multiplicative scoring and penalty).

The first few steps of that match are stored in a min cost heap as the cost of the start of a solution. Then the next solution is started w/ the next smallest error point not already in a solution, etc until each point is a starter in a solution.

Then, just as in the contour matching algorithm, the best solution is applied to the remaining points and the best solution (min cost) is found.

Next is an example of blobs found in binary segmented images. The arrowed blob does not have strong inflection points (as currently implemented), but does have somewhat strong corners. (note the greyscale image is used for feature descriptor rather than segmented image shown).



note that for the contour matching, errors in scale due to curve length differences because of deviations in segmentation more strongly affect the scale estimate than a method based on point pair matches.

test images shown have scale = 1.

| <u>nCurvePoints1</u> | <u>nCurvePoints2</u> |
|----------------------|----------------------|
| 97                   | 78                   |
| 60                   | 60                   |

So, will implement the new hybrid method of point pair matching around blob contours to estimate scale.

Note that if the only goal is scale, the euclidean transformation for scale could be considered only in scale space, that is in number of points along the curve, but has the same vulnerability to the curve length. That method is safe for textures if the textures are all of the same size in an image, that is corner location predicted from starter solution is relative to location on the curve (does not predict location in image x,y space, only curve point index space). This approach would be better for an image with large projection differences across it too.

If one could be certain that the contours were extremely well formed and deviations such as a large bump present in only one were not possible, no occlusion, etc, the scale space image contour method would be preferred. The hybrid method seems necessary.