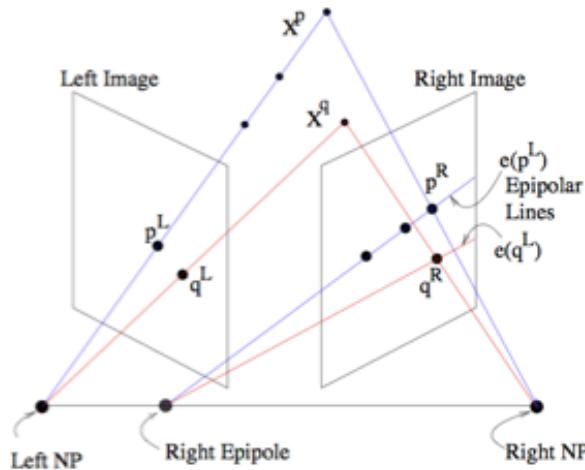


## Projection, stereo and panoramic images

$X^p$  and  $X^q$  are the physical location of objects.



NP are the nadir points of the cameras.

The line connecting the left and right NP is the baseline.

The projection of the left nadir, NP is seen as a the right epipole with w.r.t. the right image. The epipoles may or may not be within the border of the images.

The projection of  $X^p$  to left nadir, NP is seen as an epipole line in the right image. If more than one epipole line is present in the right image, they converge at the right epipole (which might not be within the boundaries of the image).

Once the points in the left image,  $X_L$  are matched with points in the right image,  $X_R$ , if there are at least 7 points, one can determine the “bifocal tensor”,

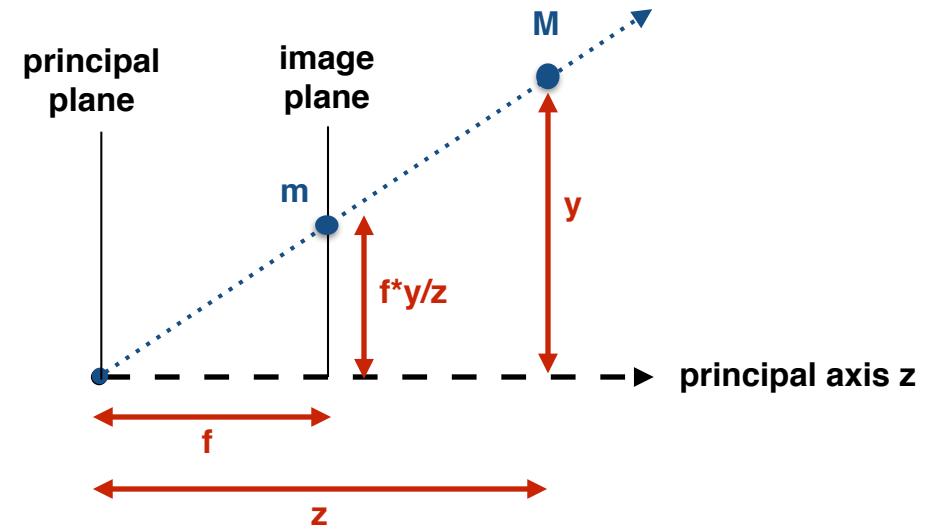
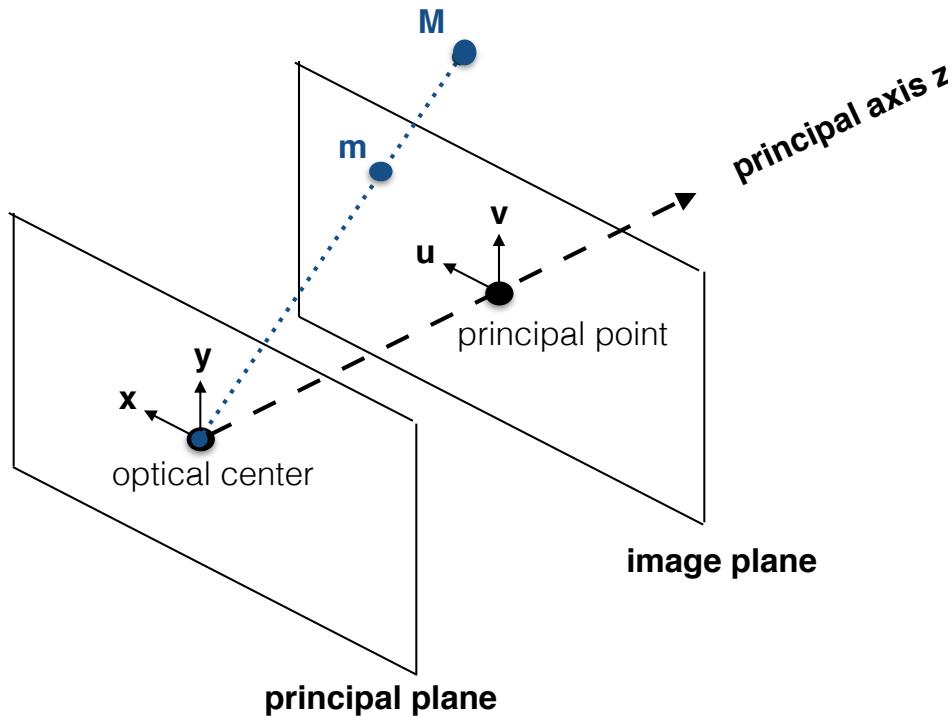
a.k.a. “fundamental matrix, relating the points in the 2 images using a 3x3 matrix of rank 2.

<http://www.cs.toronto.edu/~jepson/csc420/notes/epiPolarGeom.pdf>  
(note, the image is posted on an educational site and copied here without following up on permissions. Any further use of the image should follow up on the origins and permissions.)

$(X_L)^T * F * X_R = 0$  for any pair of points in the images.

Note: the “Essential matrix” is a matrix used if the camera details are known. The “bifocal tensor”, a.k.a. “fundamental matrix” does not need camera details.

## Pinhole camera geometry



camera projection matrix  $P$  is defined in  $z^* \mathbf{m} = P^* \mathbf{M}$   
 where  $\mathbf{M} = (x, y, z, 1)^T$  and  $\mathbf{m} = (f^*x/z, f^*y/z, 1)^T$

The right side of the projection eqn can be modified by rotation matrix  $\mathbf{R}$  and translation vector  $\mathbf{t}$  to put the coordinates in the (external) world coordinate system. The matrix is  $\begin{vmatrix} \mathbf{R} & \mathbf{t} \\ 0 & 1 \end{vmatrix}$   
 The six parameters are called *external parameters*.  
 rows of  $\mathbf{R}$  and the *optical center* describe the *camera reference frame* in world coordinates.

The left side of the projection eqn can be modified to change coordinates in the image frame.

$$K = \begin{vmatrix} f/s_x & f/s_x * \cot\theta & o_x \\ 0 & f/s_y & o_y \\ 0 & 0 & 1 \end{vmatrix} \quad K \text{ is the } \textit{camera calibration matrix},$$

result is pixel coords in image plane.

## Pinhole camera geometry (cont.)

*intrinsic parameters:*

$f$  is the focal distance in mm

$o_x, o_y$  is the principal point in pixel coordinates

$s_x$  is the width of the pixel in mm

$s_y$  is the height of the pixel in mm

$\phi$  is the angle between the axes, usually 90 degrees

The aspect ratio  $s_x/s_y$  is usually 1

$P$  can be rewritten as  $P = \mathbf{K}^*[ I | 0 ]^* \mathbf{G} = \mathbf{K}^*[ \mathbf{R} | \mathbf{t} ]$

A scale factor  $\lambda$  applied is  $P = \lambda * \mathbf{K}^*[ \mathbf{R} | \mathbf{t} ]$

This is a 3x4 full rank matrix and can be factorized by QR factorization.

$P$  can be rewritten as  $P = [ \mathbf{P}_{3x3} | \mathbf{p}_4 ]$  where  $\mathbf{P}_{3x3}$  is the first 3 rows of  $P$  and  $\mathbf{p}_4$  is the 4th column.

If  $\lambda=1$ , the matrix is normalized and the distance of  $\mathbf{M}$  from the focal plane of the camera is *depth*.

For the image plane at infinity, the image of points doesn't depend on camera position.

The angle between 2 rays is  $\cos \theta = \mathbf{m1}^T * \omega * \mathbf{m2} / (\sqrt{\mathbf{m1}^T * \omega * \mathbf{m1}} * \sqrt{\mathbf{m2}^T * \omega * \mathbf{m2}})$

(Notes on pinhole camera followed by a few notes on multi-view geometry are from  
[http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL\\_COPIES/FUSIELLO4/tutorial.html](http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/FUSIELLO4/tutorial.html))

## multi-view geometry

With point correspondences  $\mathbf{m}_i \longleftrightarrow \mathbf{M}_i$ , can solve for camera projection matrix P. Using the Kroenecker delta product and vec operator, coefficients are rewritten to their linearly independent forms of a matrix of size  $2n \times 12$  called the coefficient matrix A

From a set of n point correspondences, we obtain a  $2n \times 12$  coefficient matrix A, where n must be  $> 6$ .

In general A will have rank 11 (provided that the points are not all coplanar) and the solution is the *1-dimensional right null-space of A*.

The linear system of equations for inexact data is solved using least squares.

The least-squares solution for  $\text{vec } \mathbf{P}^T$  is the singular vector corresponding to the smallest *singular value* decomposition of A and the direct linear transform.

The equation is usually written in form  $A * h = 0$  where h is  $\text{vec } \mathbf{P}^T$  (its the one dimensional reshaping of the homograph matrix).

## multi-view geometry (cont.)

In general:

- dot product of a point and a line is zero if the point lies on the line
- if the *intrinsic parameters* are known, the relationship between corresponding points is given by the *essential matrix*:  $\mathbf{m}_r^T * \mathbf{E} * \mathbf{m}_l^T$
- when no camera details are available, one can use the *fundamental matrix*.

It's a 3x3 rank 2 homogeneous matrix. It has 7 degrees of freedom.

For any point  $\mathbf{m}_l$  in the left image, the corresponding epipolar line  $\mathbf{l}_r$  in the right image can be expressed as  $\mathbf{l}_r = \mathbf{F} * \mathbf{m}_l$  and vice versa  $\mathbf{l}_r = \mathbf{F}^T * \mathbf{m}_r$

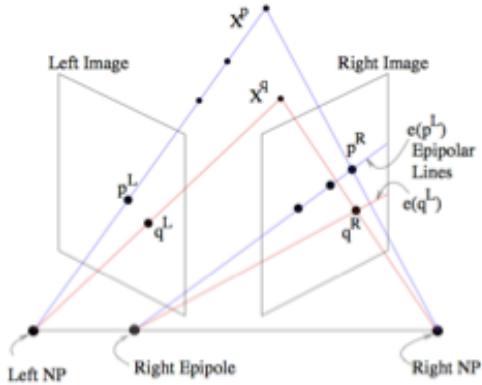
- the *essential and fundamental matrices* are related through  $\mathbf{F} = \mathbf{K}_r^{-T} * \mathbf{E} * \mathbf{K}_l^{-T}$  where  $\mathbf{K}_r$  and  $\mathbf{K}_l$  are the left and right camera matrices

Regarding 3D reconstruction:

- when both *intrinsic* and *extrinsic* camera parameters are known, the reconstruction is solved unambiguously by triangulation.
- when only *intrinsic* parameters are known, extrinsic parameters can be estimated and the reconstruction can be solved up to an unknown scale factor, that is  $\mathbf{R}$  can be estimated, but  $\mathbf{t}$  can be only up to a scale factor. the epipolar geometry is the essential matrix and the solvable projection is euclidean (rigid+ uniform scale)
- when neither *intrinsic* nor *extrinsic* parameters are known, i.e., the only information available are pixel correspondences, the reconstruction can be solved up to an unknown, global projective transformation of the world.

In Trifocal geometry, a trifocal tensor is used.

## Projection, stereo and panoramic images



<http://www.cs.toronto.edu/~jepson/csc420/notes/epiPolarGeom.pdf>  
(note, the image is posted on an educational site and copied here without following up on permissions. Any further use of the image should follow up on the origins and permissions.)

**panoramic** images here are from  
Brown & Lowe 2003

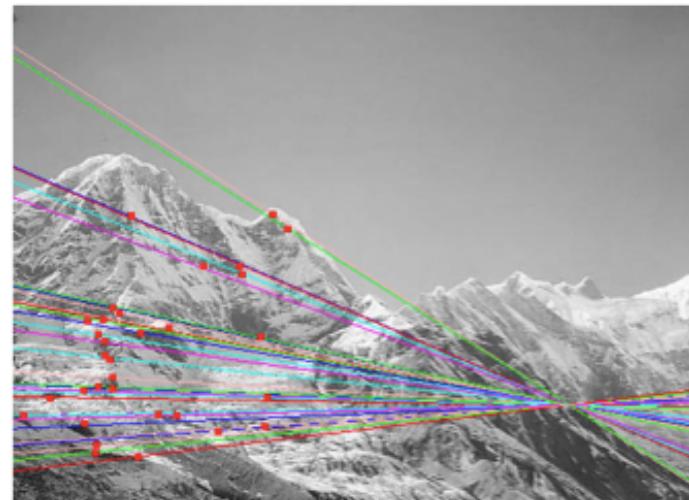
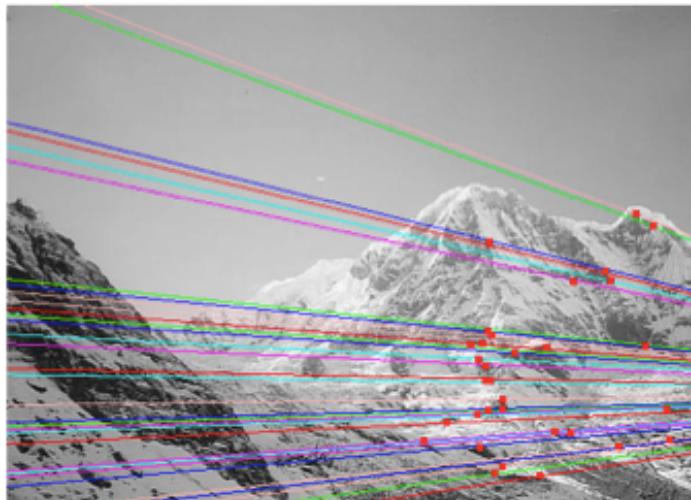


same camera objectives (nadir), but  
different orientation for the 2 images  
(that is, rotated around the same nadir)

## Point Correspondence

Need to create list of matched points between the images in order to solve for the epipolar projection

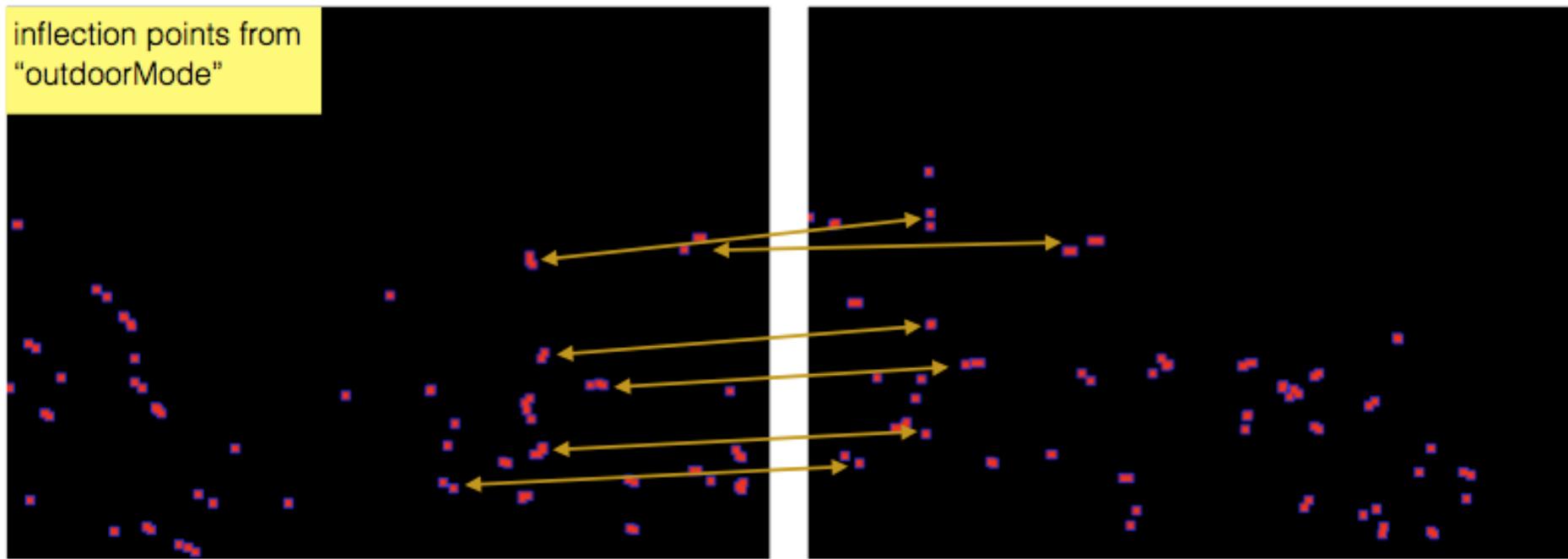
manually making a point list from the corners from the edge extractor used with "outdoor mode":



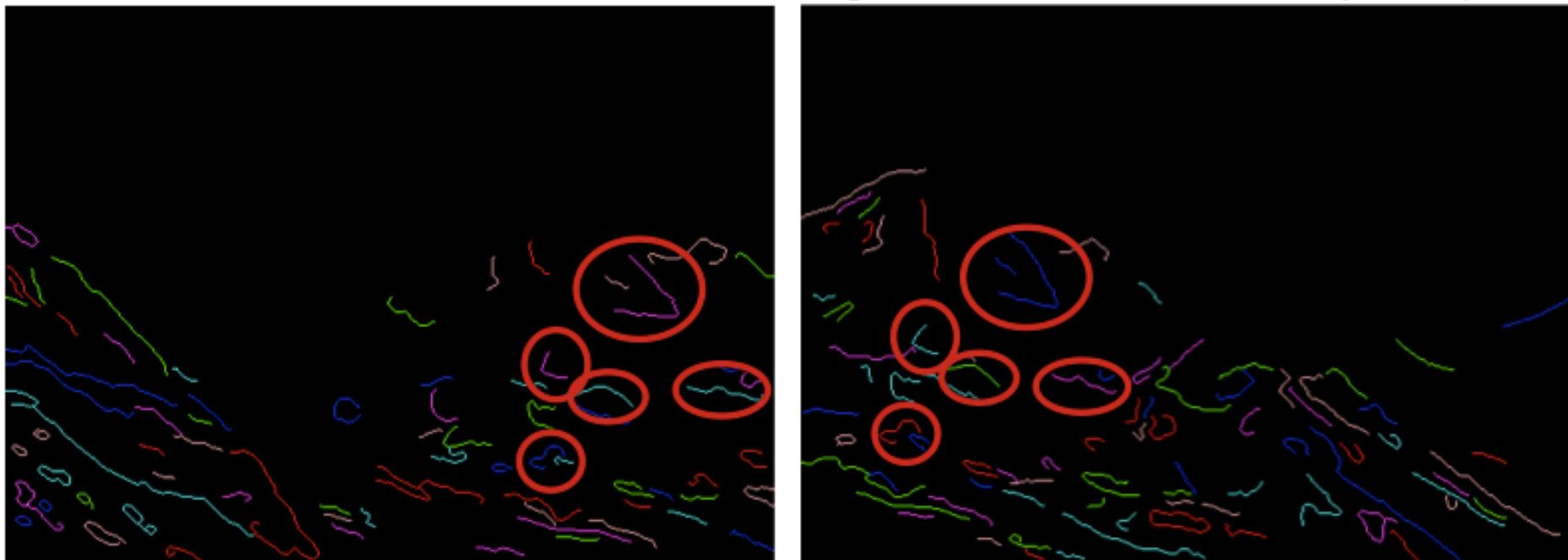
stereo projection fit to 32 points already known to match shows what the epipolar projections should be when the corner find + corner match + stereo projection solve are correctly automated.

**nMatched=32  
avgDist=0.281  
stDev=0.508**

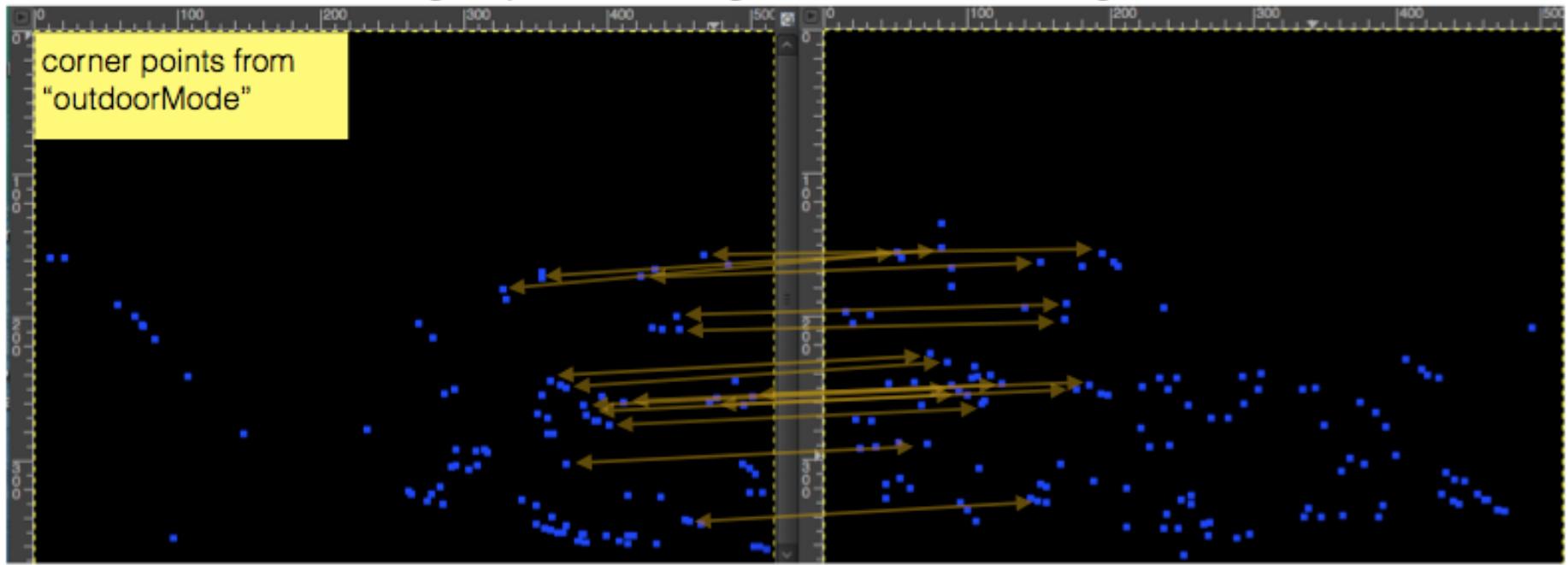
The Brown & Lowe 2003 images: point matching difficult because image intersection << difference



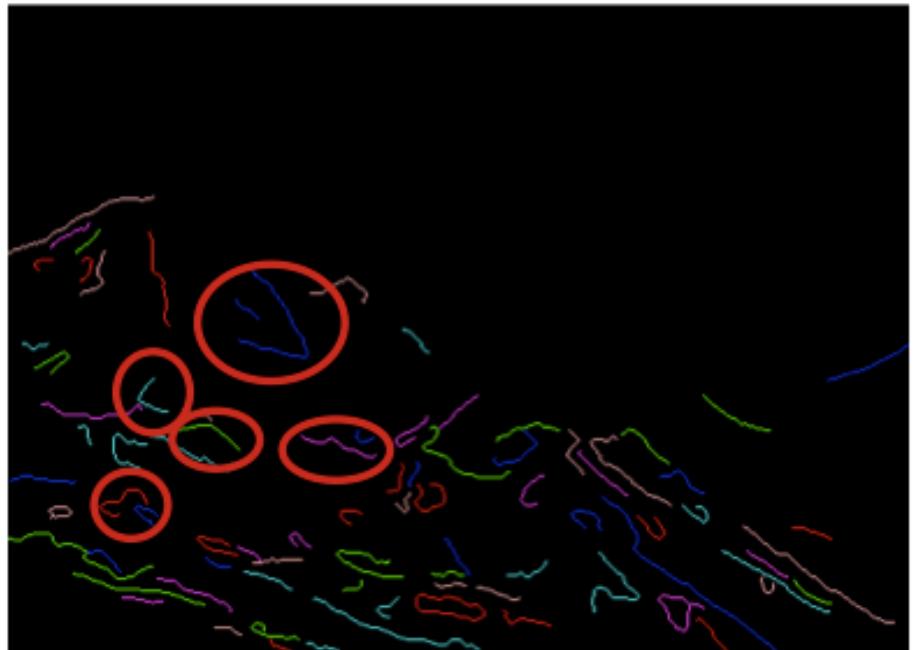
could consider distinct curves and their immediate neighbors, but that would be many more points:



The Brown & Lowe 2003 images: point matching difficult because image intersection < difference



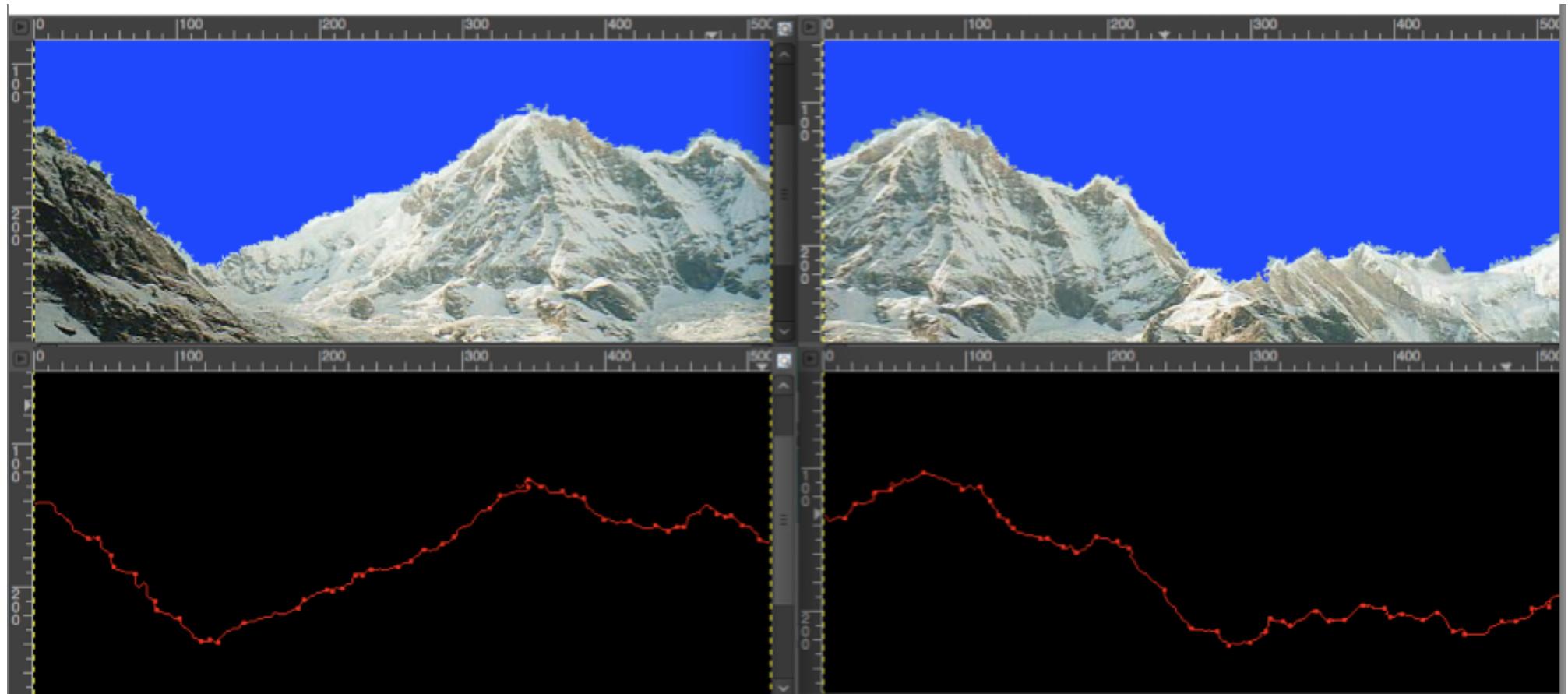
could consider distinct curves and their immediate neighbors, but that would be many more points:



For the outdoor images, can find the sky and create a sky mask and also create corners from just the skyline.  
(see skyline\_extraction.pdf)

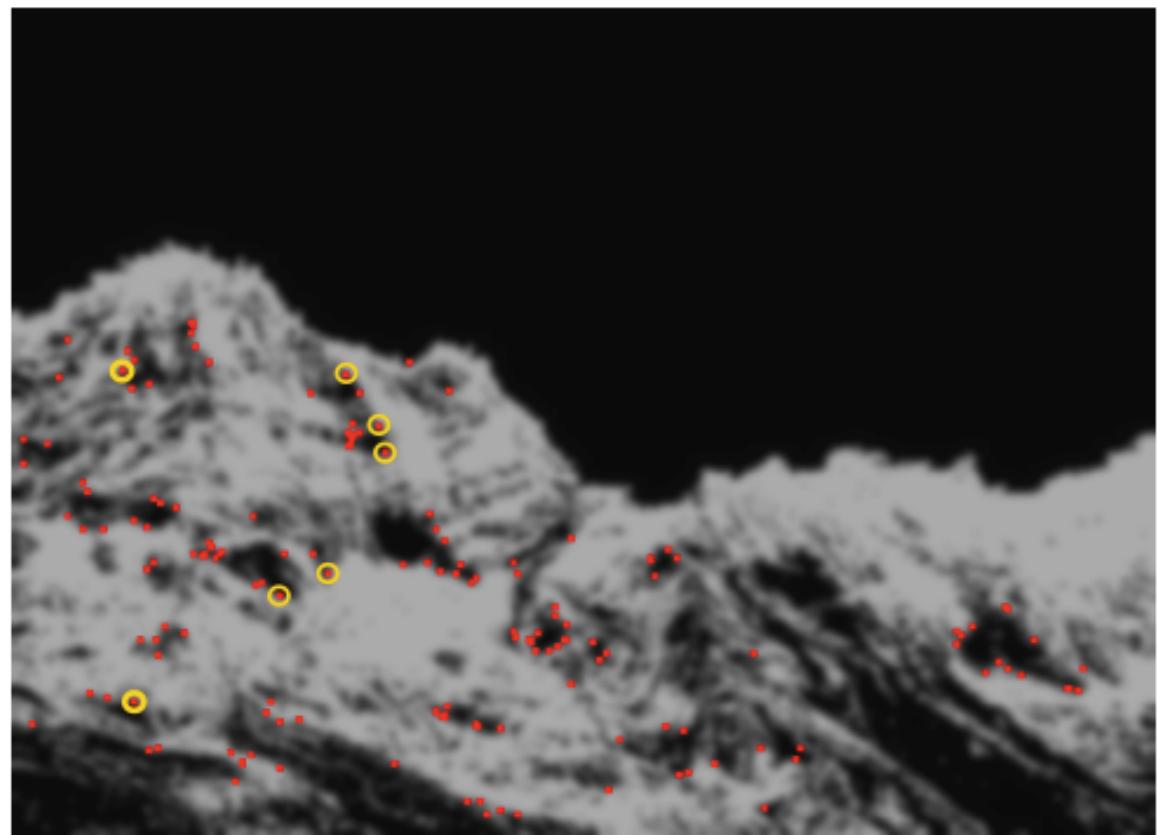
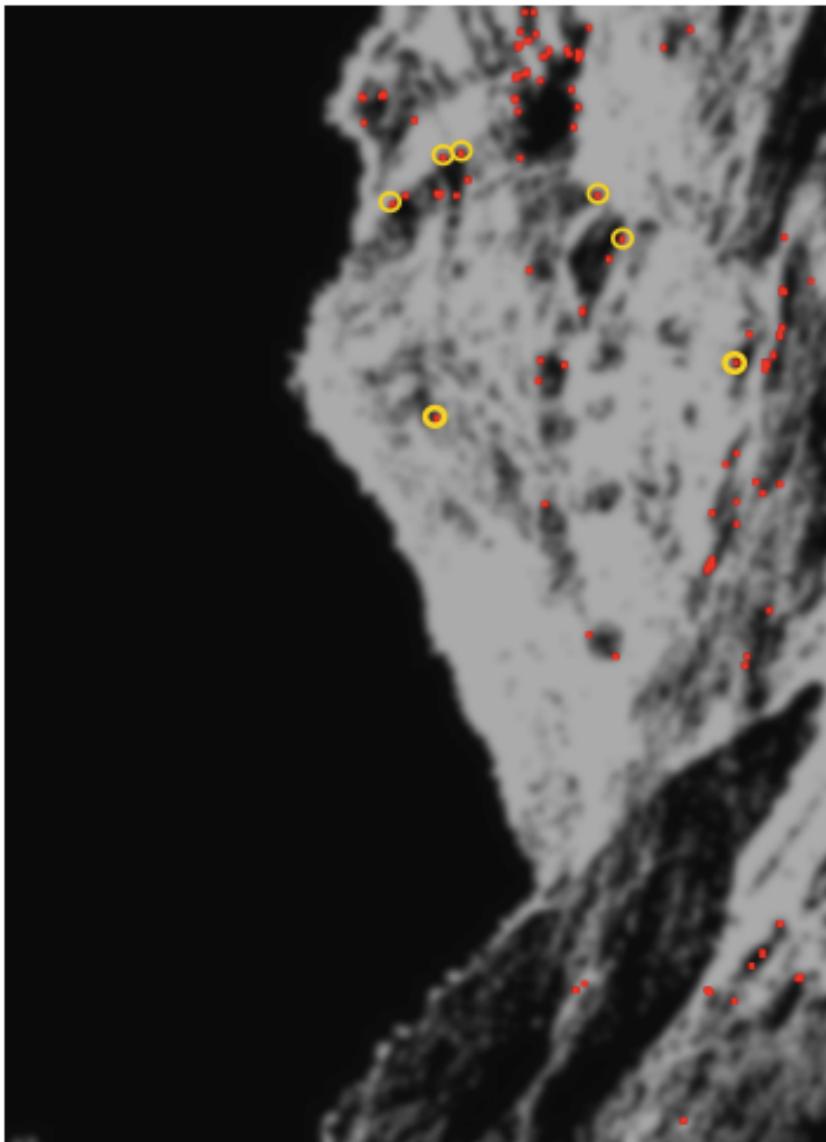
This sky mask can help to pre-process the image before feature finding and correspondence.

Note that the skyline corners will be improved before real use, this is a quick look.

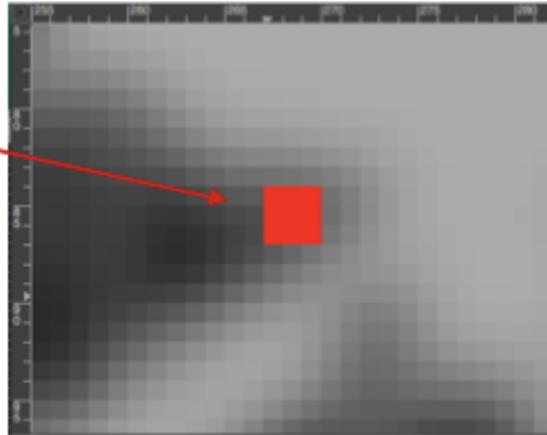
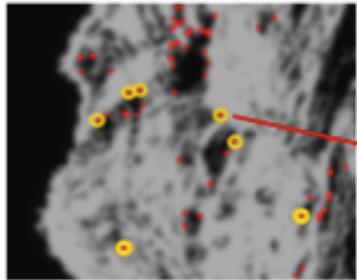


Need to match the corners using **features** because corner locations alone are often not enough to match correctly, especially when the number of truly matchable is smaller than the number of corners or when objects such as ridges exist giving large number of false matches to a different ridge of corners.

For features, need to determine an orientation of the corner region which is consistent w.r.t. the corner in any image. The highlighted corners are some examined in detail. The left is rotated to make sure the methods correctly handle rotation.



orientation of the feature: the edge already exists from making the corner, so can determine the tangent at the corner.



A vector perpendicular to  $k_{\max}$  on this edge can be used to calculate the orientation of this region (where the region is defined by being a CSS corner).

deriving the perpendicular angle from the points directly to the left and right of the maximum of curvature for points which have curvature

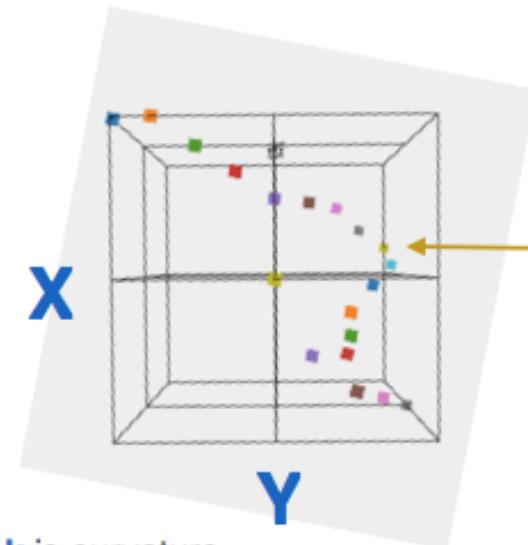
```

k      x      y
0.24, 267, 84      dx,dy=(1,1), perp=45
0.26, 268, 85 <-max k
0.21, 268, 86      dx,dy=(0,1), perp==0

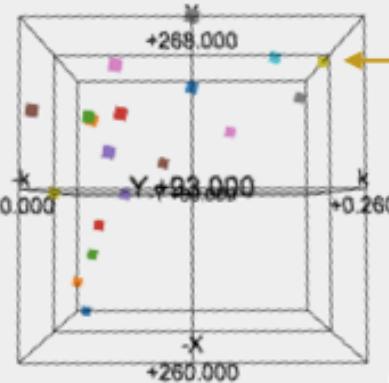
```

orientation =  $(45 + 0)/2. = 22.5$

rotated



**k** is curvature



**X**

### maximum in curvature

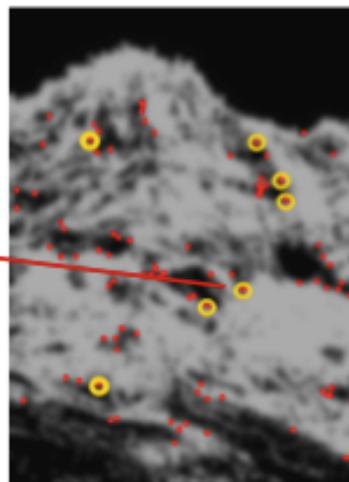
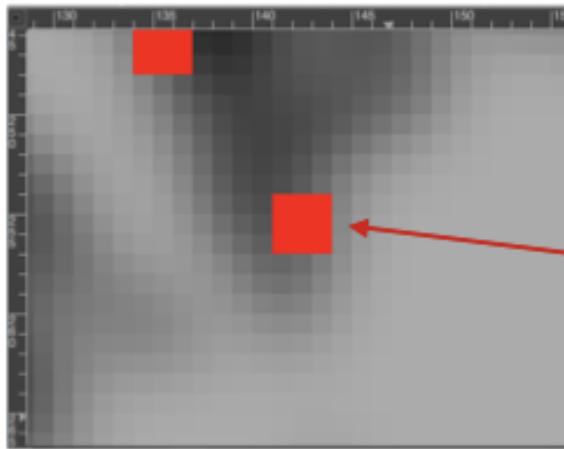
extract curvature from the highest resolution scale space curves:

```

idx=164 (268.0,85.0)
k      x      y
0.08  for (258, 79)
0.04  for (259, 79)
0.01  for (260, 80)
0.00  for (261, 80)
0.02  for (262, 81)
0.03  for (263, 82)
0.06  for (264, 83)
0.10  for (265, 83)
0.17  for (266, 83)
0.24  for (267, 84)
0.26  for (268, 85)
0.21  for (268, 86)
0.13  for (267, 87)
0.04  for (266, 88)
0.04  for (266, 89)
0.07  for (266, 90)
0.06  for (265, 90)
0.00  for (266, 91)
0.07  for (267, 92)
0.13  for (268, 93)
0.15  for (268, 94)

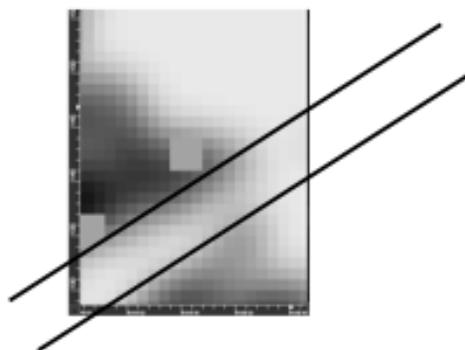
```

orientation.



extract the final curvature from the highest resolution  
scale space curves:  
idx=112 (143.0,255.0)

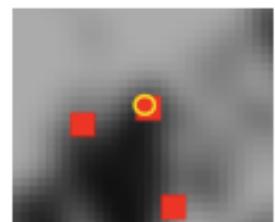
```
k[110]=0.01  for (143, 253)  
k[111]=0.13  for (143, 254)  
k[112]=0.34  for (143, 255)  dx,dy=(-1,0)  
k[113]=0.43  for (142, 255) <-- k_max  
k[114]=0.35  for (141, 255)  dx,dy=(-1,0)  
k[115]=0.28  for (140, 255)  
k[116]=0.24  for (139, 255)
```



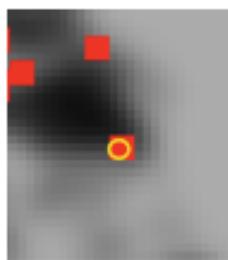
$$\text{orientation} = ((-90)+(-90))/2. = -90$$

rotated

**orientation**, corners from Brown & Lowe 2003 left and right panorama images

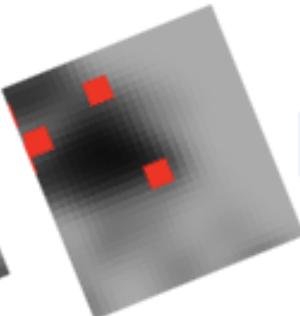
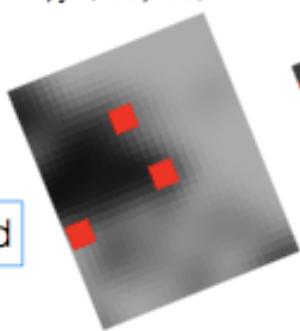


```
k[0]=0.25 x,y=(204, 66)
k[1]=0.32 x,y=(205, 66)
k[2]=0.32 x,y=(206, 66)
k[3]=0.30 x,y=(207, 67)
k[4]=0.26 x,y=(208, 68)
```



```
k[0]=0.22 x,y=(169, 197)
k[1]=0.32 x,y=(169, 198)
k[2]=0.41 x,y=(169, 199)
k[3]=0.38 x,y=(168, 200)
k[4]=0.27 x,y=(167, 200)
```

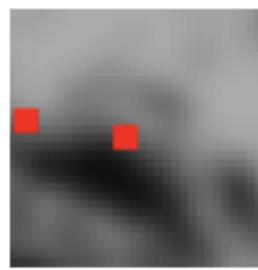
67.5  
rotated



337.5  
rotated



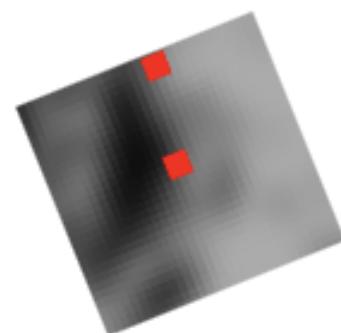
```
k[0]=0.11 x,y=(332, 161)
k[1]=0.37 x,y=(331, 161)
k[2]=0.58 x,y=(330, 161)
k[3]=0.47 x,y=(330, 162)
k[4]=0.30 x,y=(330, 163)
```



```
k[0]=0.07 x,y=(56, 315)
k[1]=0.09 x,y=(55, 314)
k[2]=0.31 x,y=(54, 313)
k[3]=0.23 x,y=(53, 313)
k[4]=0.03 x,y=(52, 313)
```

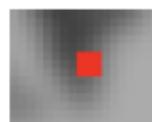
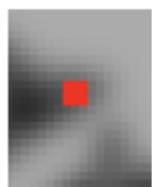


135.0  
rotated

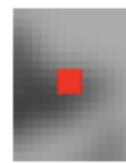
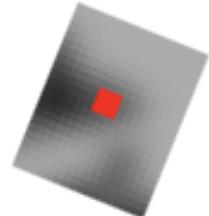


67.5  
rotated

```
k[0]=0.17 x,y=(266, 83)
k[1]=0.24 x,y=(267, 84)
k[2]=0.26 x,y=(268, 85)
k[3]=0.21 x,y=(268, 86)
k[4]=0.13 x,y=(267, 87)
```

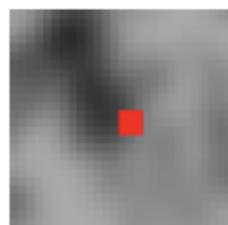


22.5  
rotated

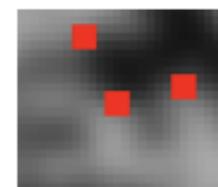


270.0  
rotated

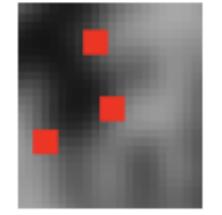
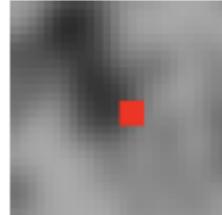
```
k[0]=0.13 x,y=(143, 254)
k[1]=0.34 x,y=(143, 255)
k[2]=0.43 x,y=(142, 255)
k[3]=0.35 x,y=(141, 255)
k[4]=0.28 x,y=(140, 255)
```



```
k[0]=0.20 x,y=(52, 170)
k[1]=0.23 x,y=(53, 171)
k[2]=0.25 x,y=(54, 171)
k[3]=0.21 x,y=(55, 171)
k[4]=0.13 x,y=(56, 171)
```



0.0  
rotated



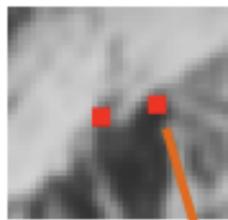
270.0  
rotated

orientation and junctions. corners  
from Brown & Lowe 2003 left and right panorama images

k[0]=0.09 x,y=(204, 65)  
k[1]=0.19 x,y=(205, 65)  
k[2]=0.21 x,y=(206, 65)  
k[3]=0.09 x,y=(206, 64)  
k[4]=0.03 x,y=(206, 63)

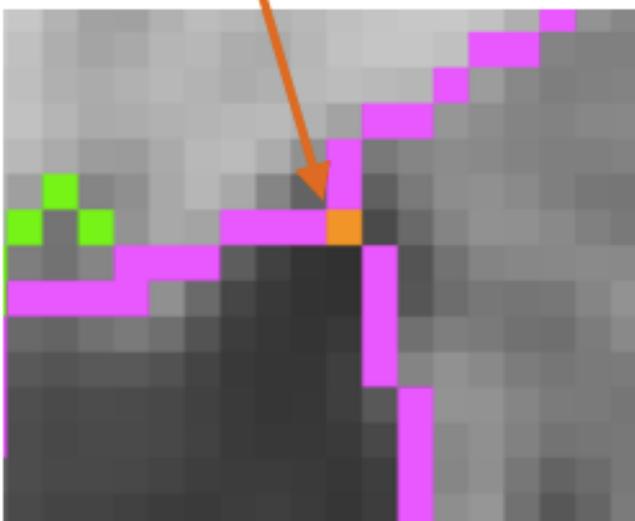
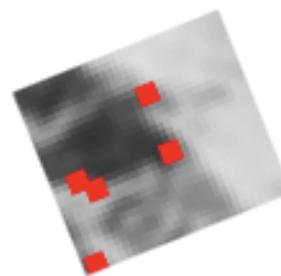
k[0]=0.21 x,y=(170, 198)  
k[1]=0.32 x,y=(170, 199)  
k[2]=0.41 x,y=(170, 200)  
k[3]=0.38 x,y=(169, 201)  
k[4]=0.27 x,y=(168, 201)

315.0



337.5

rotated

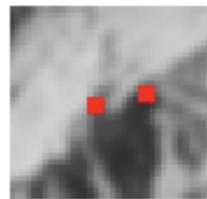


The left image corner is in a *junction* and that requires more complex analysis to determine here that the better edge for it is the lower right edge instead of the upper edge (the later gives an inconsistent orientation w.r.t. the right image).

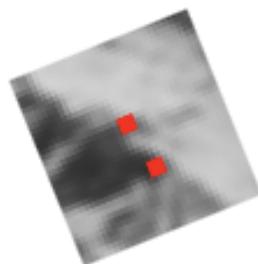
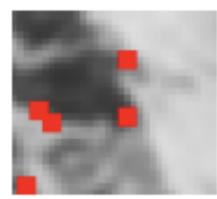
Without the more complex analysis for now, will just create a CornerRegion for each combination and one of the 3 will match the right.

## orientation and junctions (continued)

k[0]=0.02 x,y=(205, 65)  
k[1]=0.21 x,y=(206, 65)  
k[2]=0.02 x,y=(207, 66)



k[0]=0.21 x,y=(170, 198)  
k[1]=0.32 x,y=(170, 199)  
k[2]=0.41 x,y=(170, 200)  
k[3]=0.38 x,y=(169, 201)  
k[4]=0.27 x,y=(168, 201)

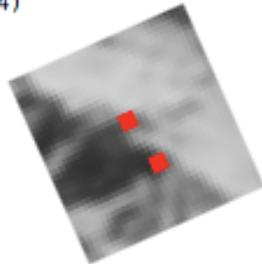


67.5

rotated

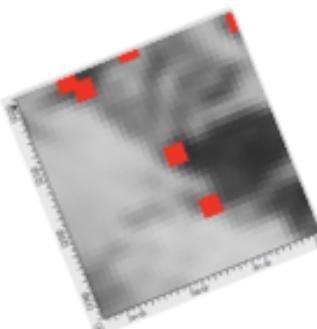
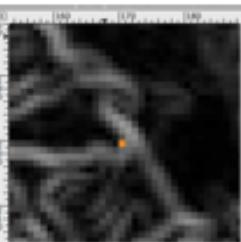
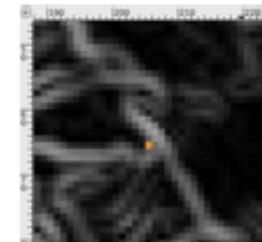
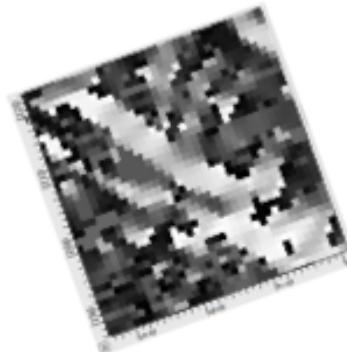
337.5

rotated

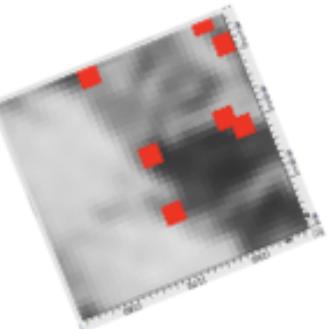


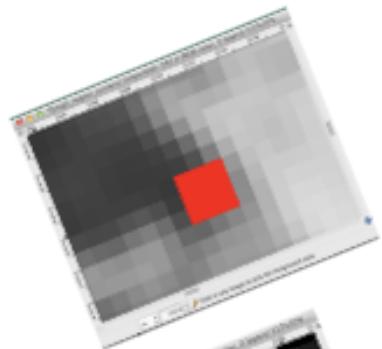
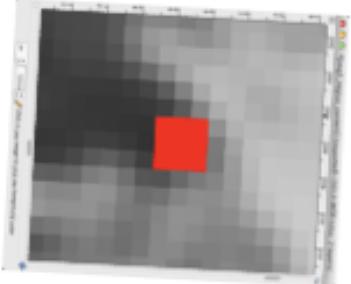
66

rotated

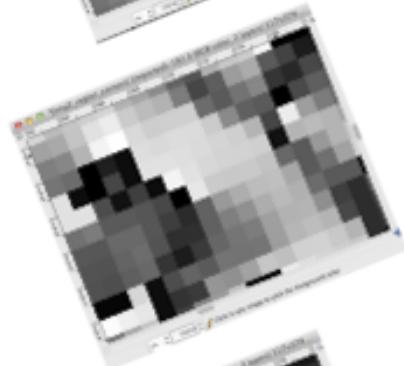
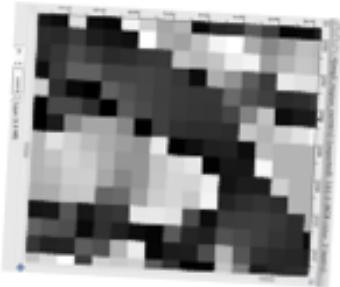


250 158

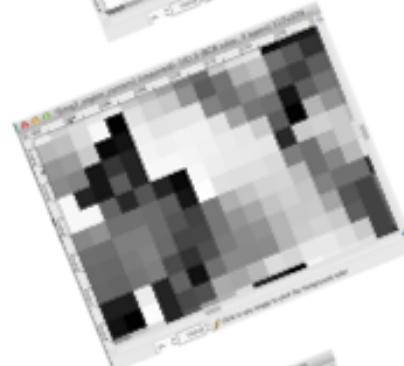
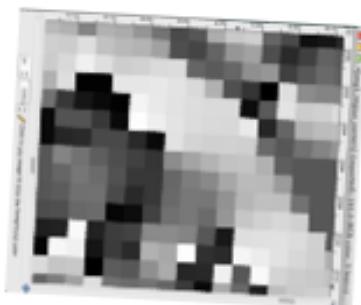




greyscale rotated to “dominant orientation”

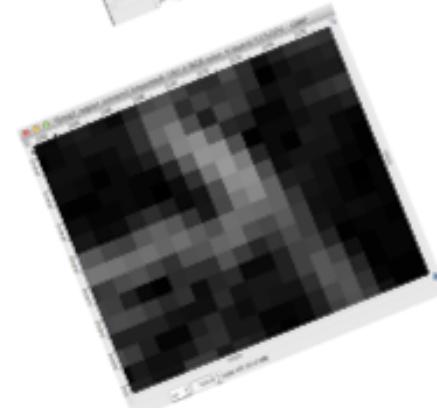
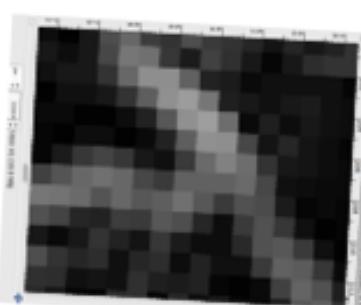


gradient theta rotated to “dominant orientation”



gradient theta rotated to “dominant orientation” with “dominant orientation” subtracted from theta

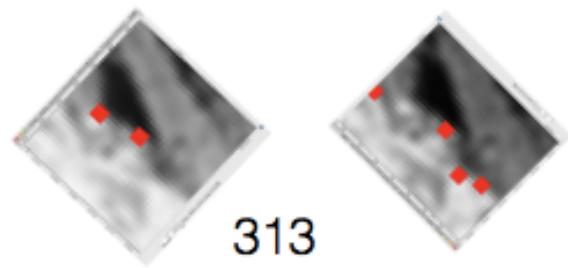
can see that image 2 should have preferred match w/ orientation 67.5 instead of 93



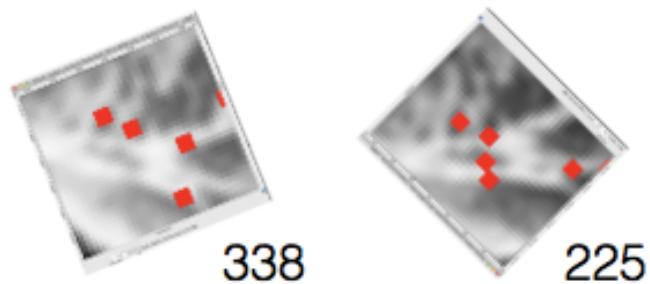
can see that SSD on gradient theta could be a good descriptor with the orientation corrections and quadrant consideration (e.g. 0 diff 350 is 10, not 350)

can see that SSD on gradient could be a good descriptor but may not be as tolerant of skew as subcells of histograms of orientation?

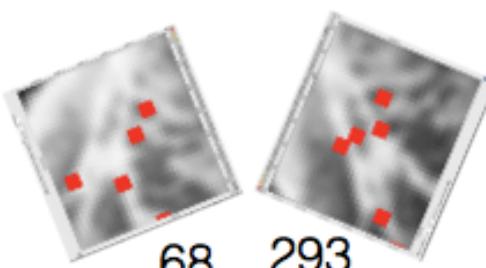
will use gradient, but binned into 2x2 cells, and 16 of them surrounding the corner.



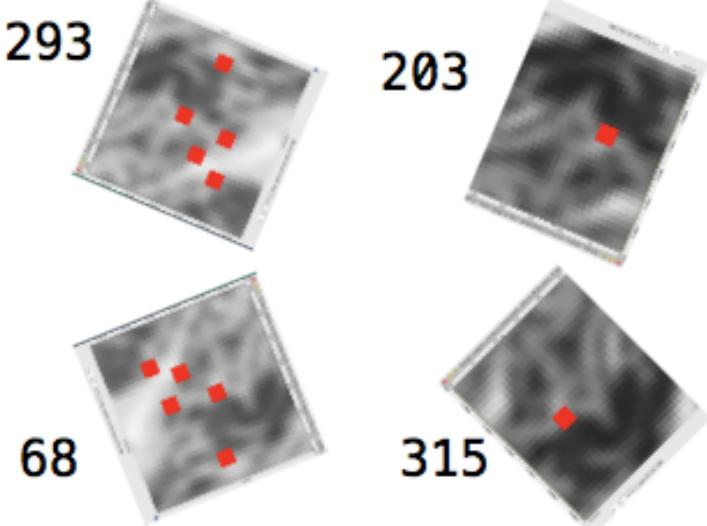
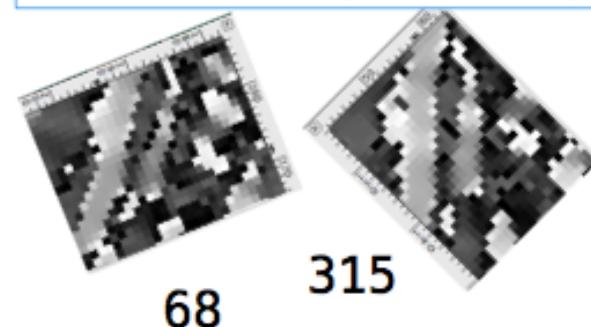
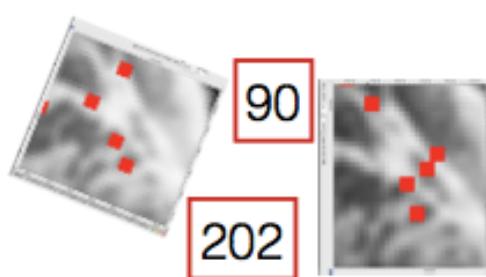
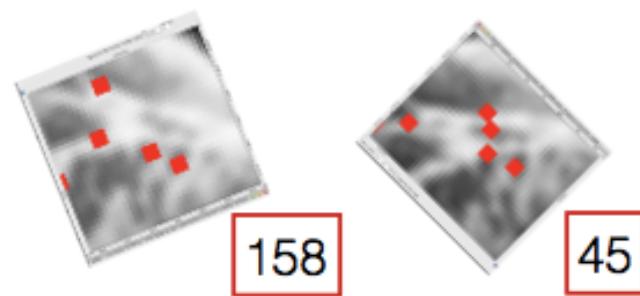
[1]



[2]

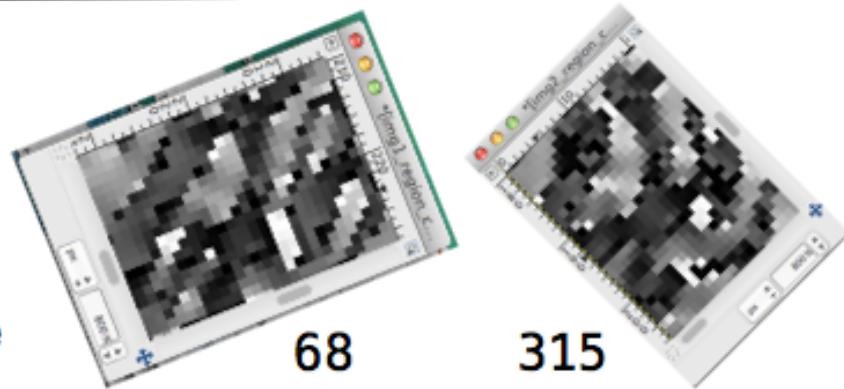


gradient theta rotated to "dominant orientation" with "dominant orientation" subtracted from theta (ref for method?)

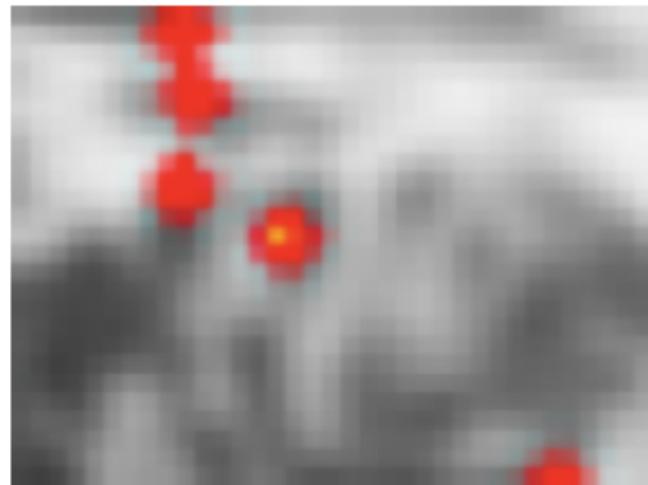
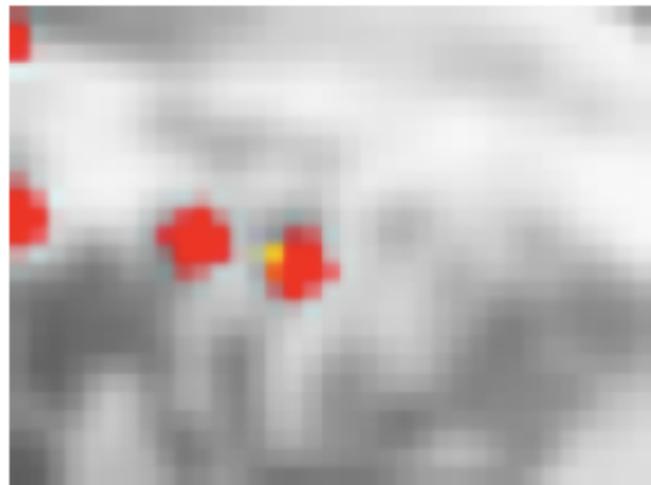


[3]

good for  
normalization  
test or preference  
for gradient theta  
(220, 220)(9, 194)



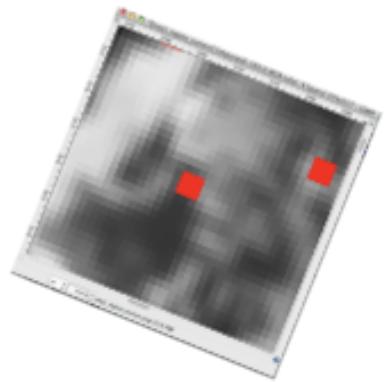
gradient theta rotated to "dominant orientation" with "dominant orientation" subtracted from theta (ref for method?)



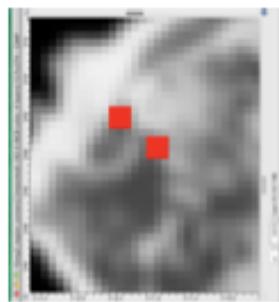
left and right clips from the Brown & Lowe showing projection and illumination differences.

Binning the intensity descriptor pixels into  $2 \times 2$  cells and subtracting the mean of the descriptor intensity values from itself helps to remove the illumination differences and projection differences for those within that small range.

Note that the rectangular radius around the central pixel is somewhat large, 12 pixels (note too that the snapshot above is larger than a descriptor's region).



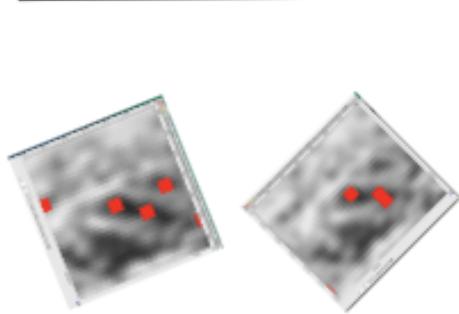
0 or 22.5



[4]

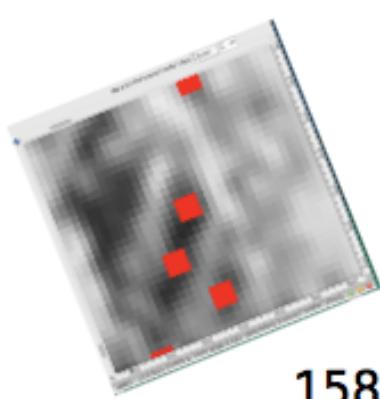
270

---

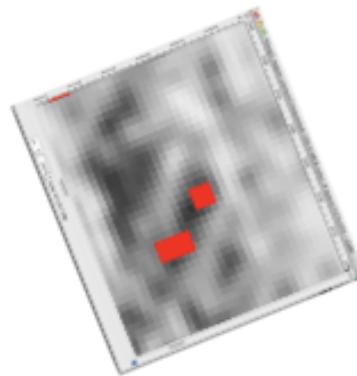


68

315



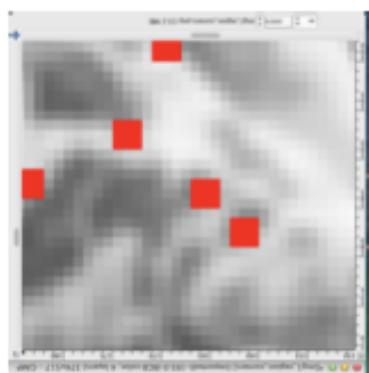
158



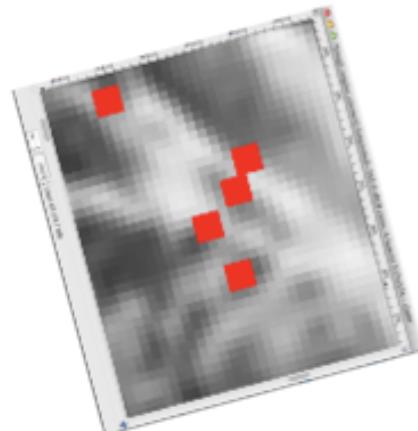
[5]

68

---



180



[6]

75?

### orientation. corners from Venturi dataset

$k[0]=0.15$ x,y=(461, 449)	$k[0]=0.13$ x,y=(157, 466)
$k[1]=0.23$ x,y=(460, 448)	$k[1]=0.19$ x,y=(156, 466)
$k[2]=0.29$ x,y=(459, 447)	$k[2]=0.22$ x,y=(155, 466)
$k[3]=0.26$ x,y=(459, 446)	$k[3]=0.20$ x,y=(154, 466)
$k[4]=0.16$ x,y=(459, 445)	$k[4]=0.16$ x,y=(153, 467)



202.5

90.0

rotated

$k[0]=0.18$ x,y=(293, 496)
$k[1]=0.24$ x,y=(292, 495)
$k[2]=0.29$ x,y=(291, 494)
$k[3]=0.25$ x,y=(291, 493)
$k[4]=0.13$ x,y=(291, 492)



112.5

rotated

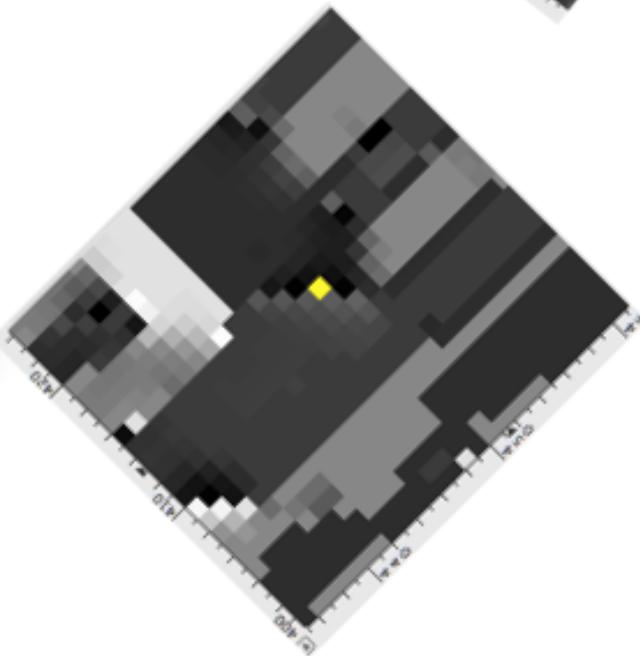
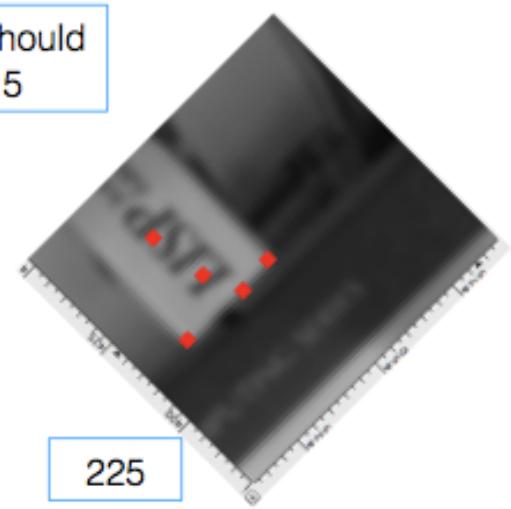
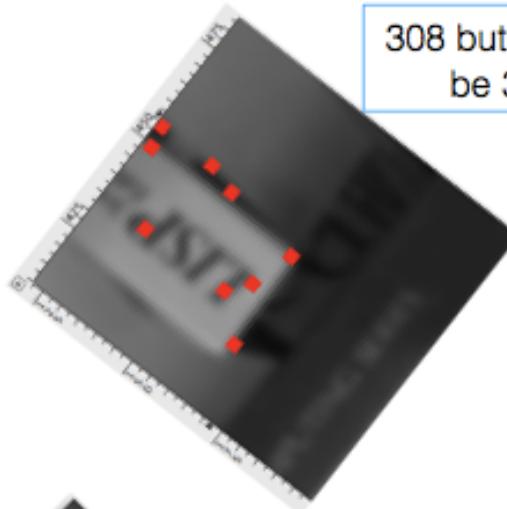
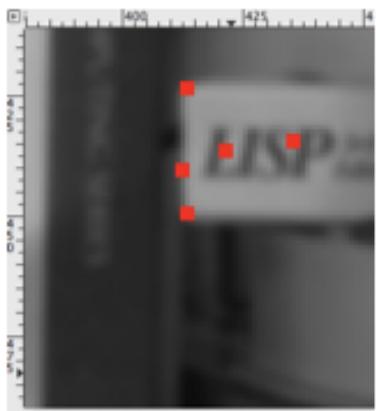
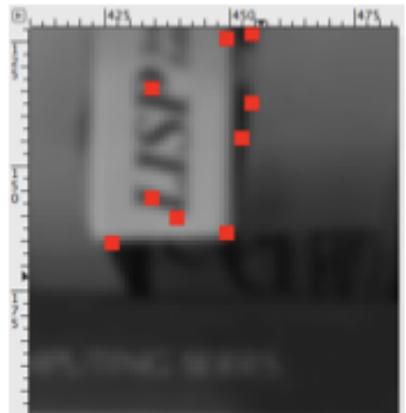
$k[0]=0.27$ x,y=(449, 222)	$k[0]=0.26$ x,y=(382, 448)
$k[1]=0.33$ x,y=(448, 221)	$k[1]=0.29$ x,y=(383, 447)
$k[2]=0.37$ x,y=(447, 220)	$k[2]=0.31$ x,y=(384, 446)
$k[3]=0.35$ x,y=(446, 220)	$k[3]=0.29$ x,y=(384, 445)
$k[4]=0.26$ x,y=(445, 220)	$k[4]=0.23$ x,y=(384, 444)



67.5

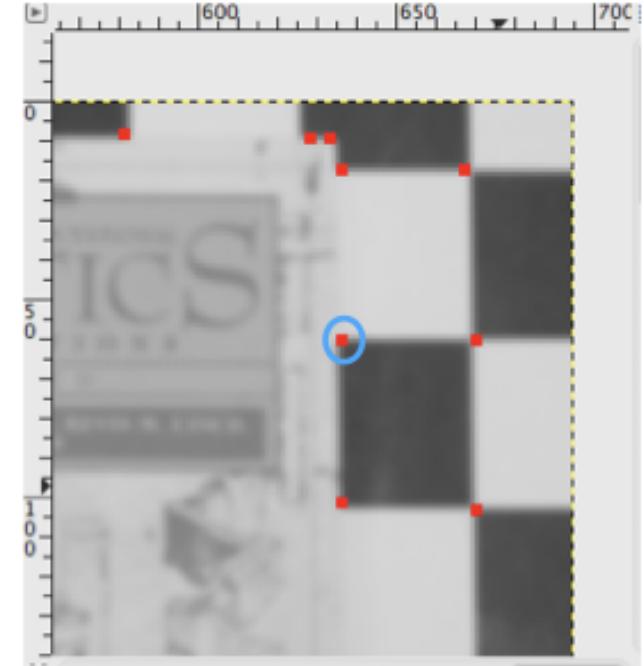
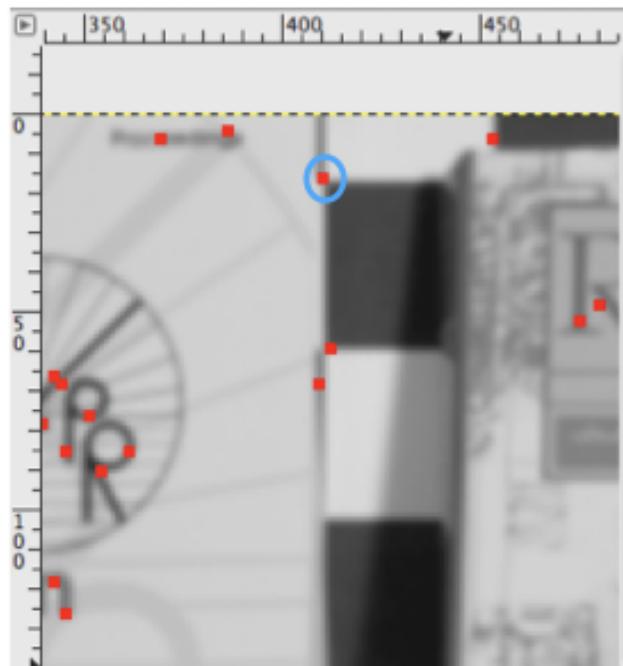
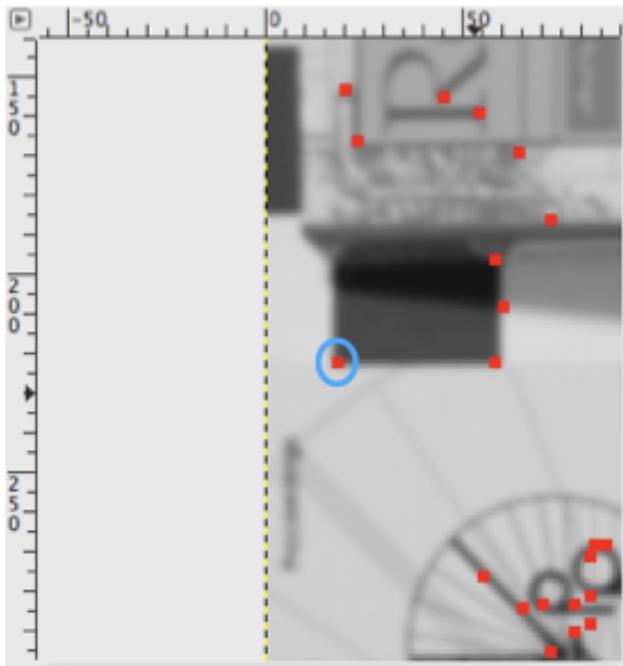
337.5

rotated



the theta descriptor SSD is larger than the error from auto-correlation. discard in favor of clearer matches and note for a future algorithm which recognizes contours and matches within...

projection shows a different neighboring region for the book. so theta is very good for identifying the book consistently, but object needs to be on an unvarying background/foreground or one should only compare theta within the contour (bounds) of the book.



true match (not found)

false match (found)

this is an example where a repeated pattern is better matched elsewhere in the image due to 2 things: (1) it's a pattern (very big texture) and (2) the background/foreground changes near the corner change the appearance enough that the true match isn't found.

where “found” is referring to the matching algorithm to make rough correspondence lists before use of the epipolar projection solver.

might need to consider the top k matches, especially if the image is known to contain repeated patterns/textures.

Euclidean transformation from pairwise calc of known points:

rotationInRadians=4.5554905

rotationInDegrees=261.01037890524134 scale=1.004318

translationX=266.80643

translationY=5.852234 originX=0.0 originY=0.0

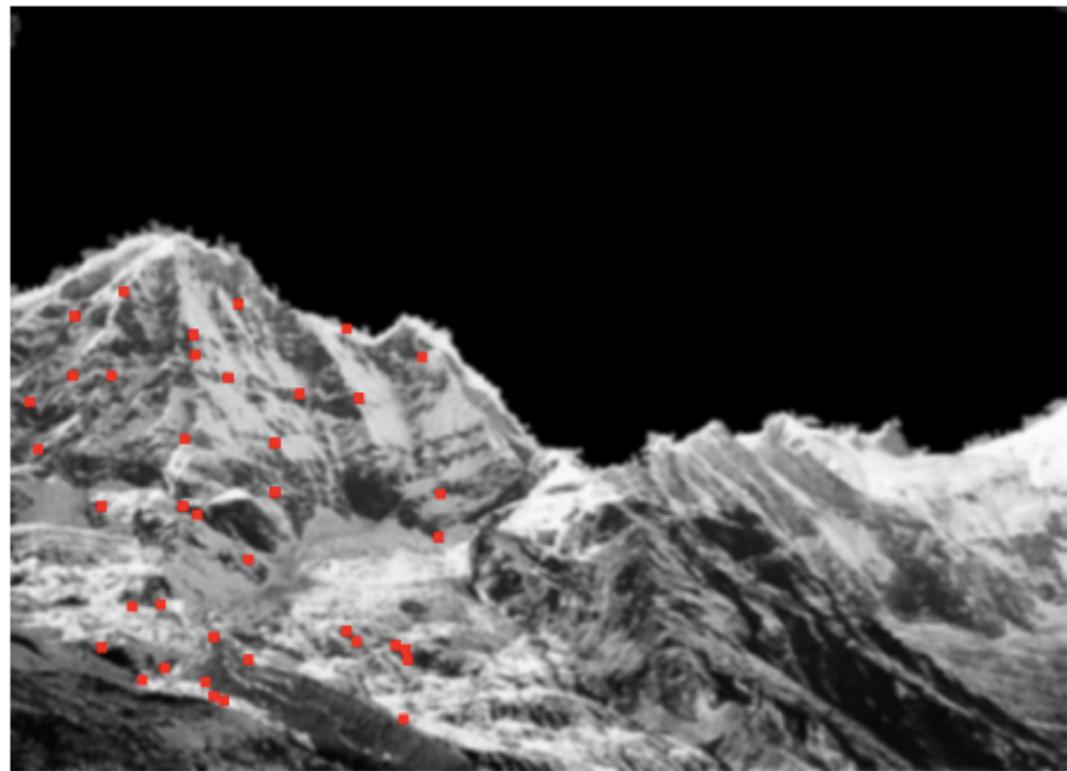
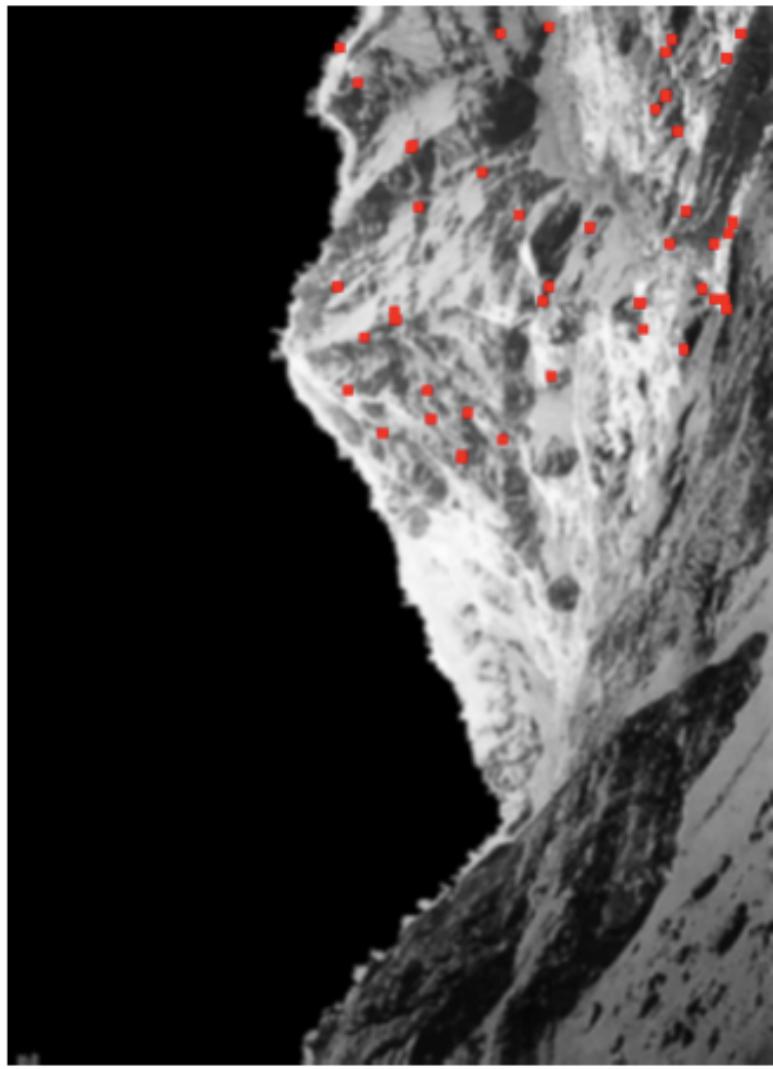
Feature matches of corner regions followed by a look at the implied rotations by frequency then filtering with that to see the most frequent translation in X then filtering by that to find the most frequent translation in Y:

solution

rotation=270.0+-20

translationX=215.0+-55.847068786621094

translationY=-25.0+=29.851972579956055



result is a list of correspondence, usable as input to the epipolar solver which uses RANSAC to discard outliers while solving the fundamental matrix.

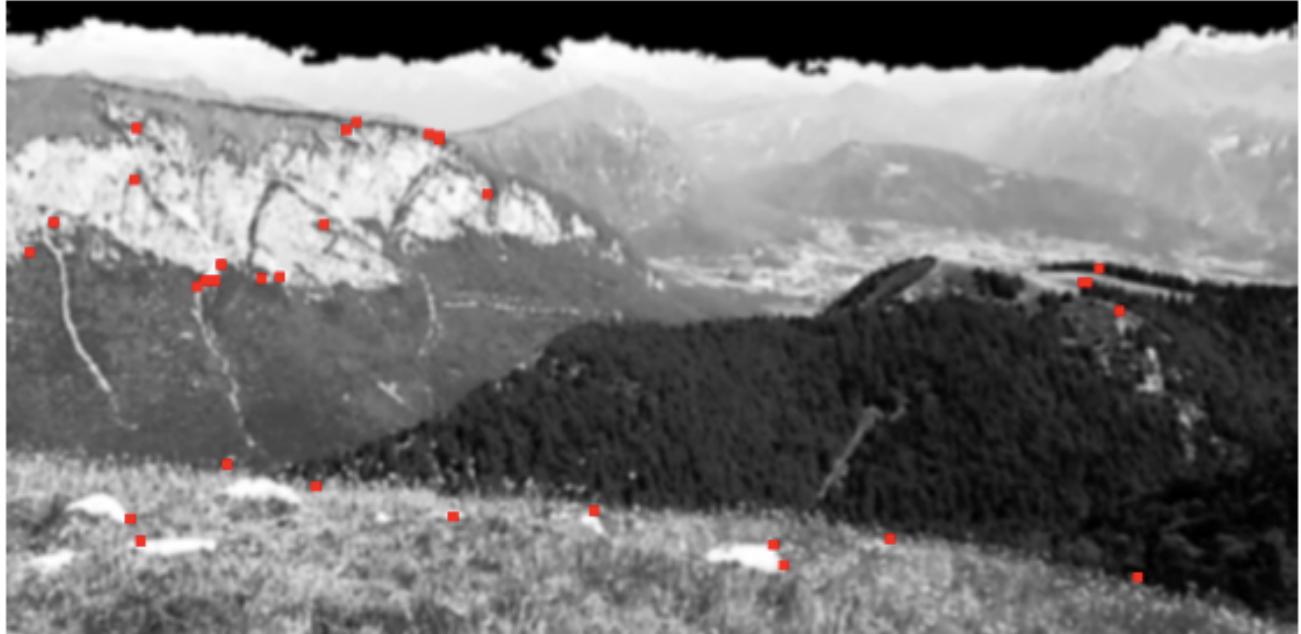
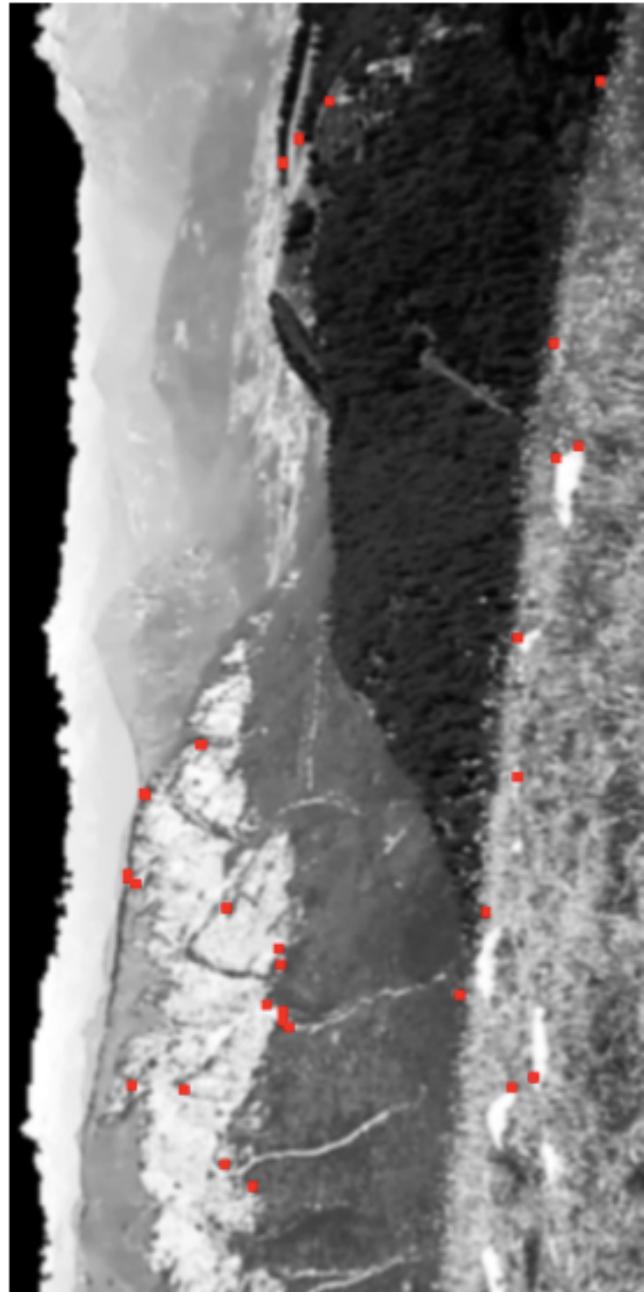
Euclidean transformation from pairwise calc of known points:

```
params=rotationInRadians=4.712166  
rotationInDegrees=269.9872145691191 scale=1.0279127  
translationX=614.1122 translationY=-4.460491  
originX=0.0 originY=0.0
```

Feature matches of corner regions followed by a look at the implied rotations by frequency then filtering with that to see the most frequent translation in X then filtering by that to find the most frequent translation in Y:

solution

```
rotation=280.0+-20  
translationX=625.0+-31.55487060546875  
translationY=5.0+=0.0
```



result is a list of correspondence, usable as input to the epipolar solver which uses RANSAC to discard outliers while solving the fundamental matrix.

Euclidean transformation from pairwise calc of known points:

```
params=rotationInRadians=4.512667  
rotationInDegrees=258.556783710076  
scale=1.0265783  
translationX=639.77747 translationY=12.565971  
originX=0.0 originY=0.0
```

Feature matches of corner regions followed by a look at the implied rotations by frequency then filtering with that to see the most frequent translation in X then filtering by that to find the most frequent translation in Y:

```
solution  
rotation=270.0+-20  
translationX=625.0+-55.98248291015625  
translationY=25.0+=44.471153259277344
```



On already rectified images, the rough euclidean solution that is used to remove more distant matched pairs probably needs a 2nd order term, at least for translations.

Have algorithm to match features, but still need to know scale between the images. For many sets this is already known to be '1', and in those cases, the correspondence list can be very fast to make by evaluating translations combined w/ features, but for images of different cameras, etc, the scale isn't known much less the general euclidean transformation.

A scale between both images can be found if one can identify common objects present in both. That requires segmentation tailored for unique patterns in the image.

Several segmentation algorithms were tried and the best for producing several matchable blobs tended to be KMPP on greyscale intensities with k=2, followed by a DFS traversal for contiguous pixels of set size within a range.

## Blob finding for scale

- perform histogram equalization if mean is too far from median or the two images are too different for those stats.
- color segmentation of **k=2** to reduce image to 2 bands of intensities.
- contiguous pixel group finder for each of the 2 bands using point limits of: smallestGroupLimit = **100**; largestGroupLimit = **5000**
- implemented a blob ordered perimeter extractor to create closed curves (concave hulls) usable as contours for matching.

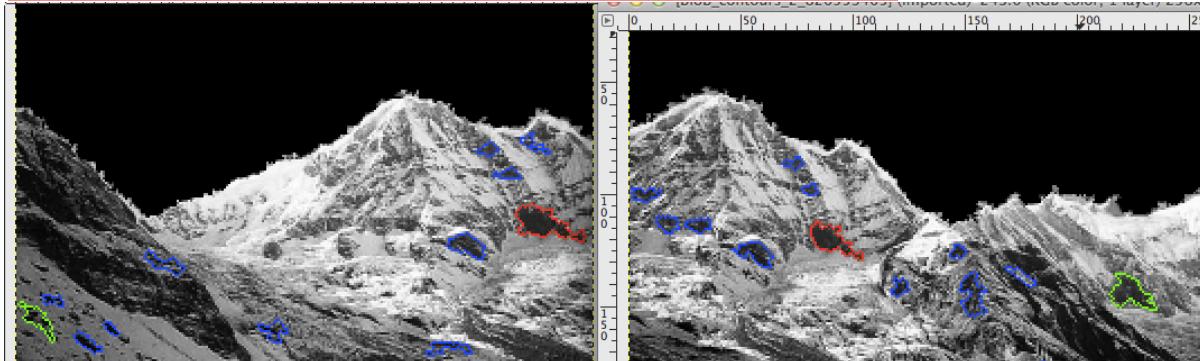
The blob finding performed on images binned to < 300 x 300 and on the full images seems to produce feasible matching features even though there are few.

The blob contours are used to solve for scale and to roughly solve for rotation and translation.

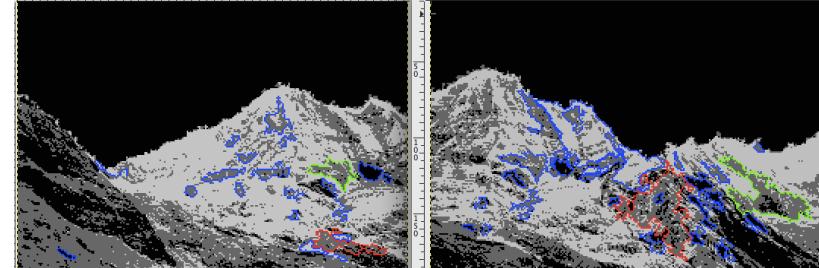
Once scale is calculated from blobs, **features** are used to create correspondence lists for input to epipolar projection solver (see previous pages).

## A look at blob contours (to use for scale solutions)

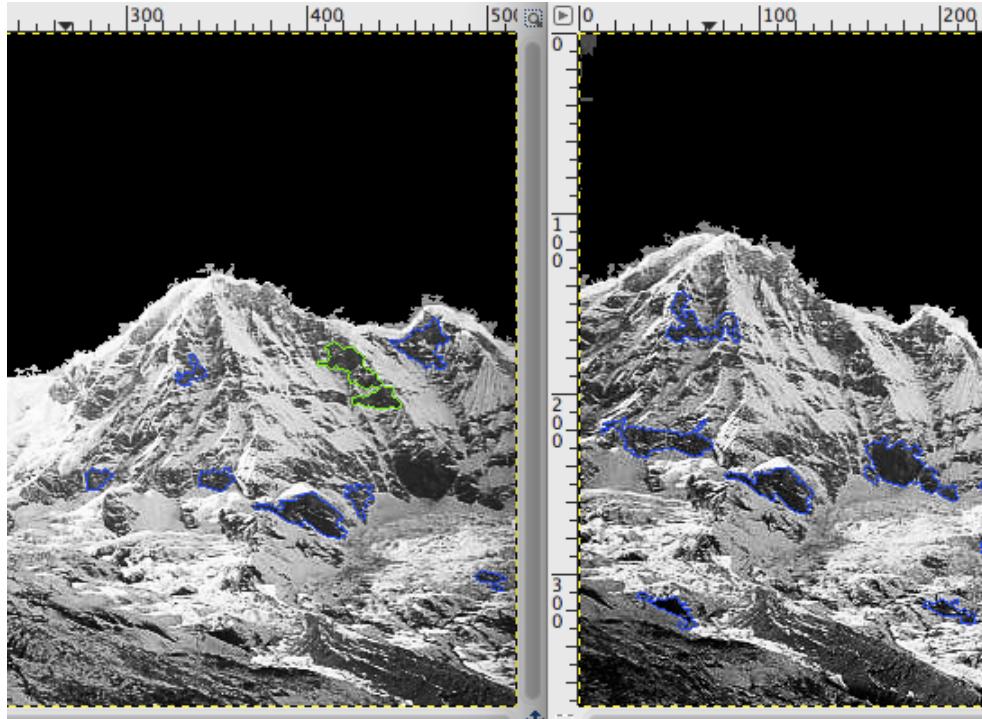
k=2 segmentation blobs, binned to < 300x300



k=3 segmentation blobs, < 300x300



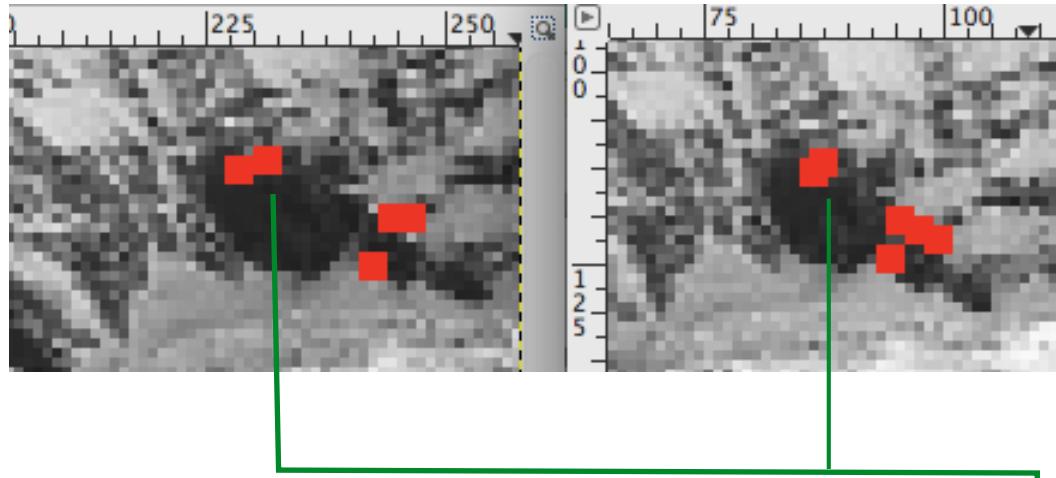
k=2 segmentation blobs, image not binned



In summary with other images, a feasible approach to using blob contours to solve for scale may be to use binary segmentation on binned images and images not binned and combine the results w/ expectation that there may only be one or two blobs in the solutions.

k=2 segmentation blobs from images binned to < 300x300, blob perimeters extracted and ordered, perimeter inflection points matched

note, details of the matching contours in a blob are displayed on the binned image without segmentation:



```
[0](236,123) [0](90,120) cost=1.0 scale=0.86 nMatched=4  
(245,123) theta1=90    (95,120) theta2=203  intensity=4618.6(4808.2),  
(246,123) theta1=45    (97,121) theta2=90  
(242,128) theta1=23    (94,124) theta2=225  
(228,118) theta1=203   (86,115) theta2=270  intensity=2531.4(2671.8),  
(231,117) theta1=135   (87,114) theta2=113  intensity=523.5(1835.1),  
(244,123) theta1=248   (98,121) theta2=90
```

A false match to another feature has better overall matches for contour points so the method has to be improved. might try best orientation and increasing dither value.

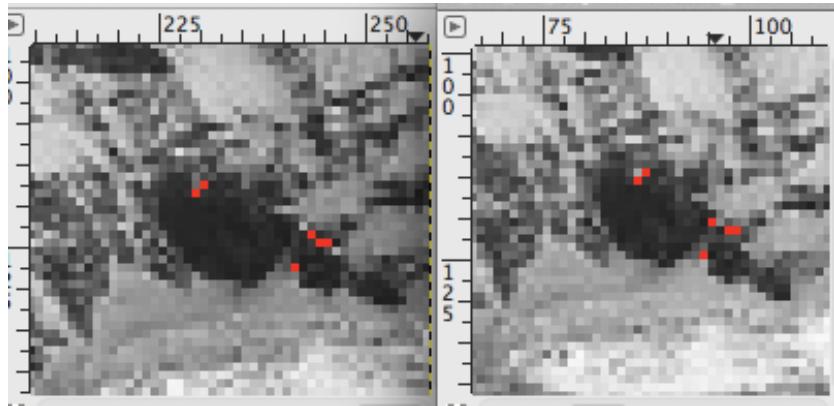
```
[0](236,123) [1](225,144) cost=11.0 scale=0.98 nMatched=5  
(227,125) theta1=0    (228,142) theta2=203  1048.2(1482.4),  
(226,124) theta1=203   (230,144) theta2=248  
(245,123) theta1=90    (225,148) theta2=203  
(246,123) theta1=45    (225,147) theta2=23  
(242,128) theta1=23    (218,140) theta2=293  1223.6(1637.5),  
(241,127) theta1=68    (219,139) theta2=338  1414.0(1802.3),  
(244,123) theta1=248   (229,143) theta2=225  582.4(2856.6),  <== best match (but false)  
(228,118) theta1=203   (232,148) theta2=248  2557.6(3910.4),  
(231,117) theta1=135   (233,148) theta2=270  2662.2(3554.0)
```

to improve, might bootstrap from the best matching point's orientation to correct for errors in blob perimeters due to segmentation, etc..

best orientation applied to other points and a dither of 1 or 2.

one caveat is that the orientation found locally is pointing outwards from the blob, giving a better distinguishing region to compare, but using one orientation for all features instead may sometimes be comparing regions within the blob (the blob was chosen as a relatively uniform patch in images).

Can skip an improved comparison step if the differences between img1 and img2 feature orientations are approximately the same constant number.



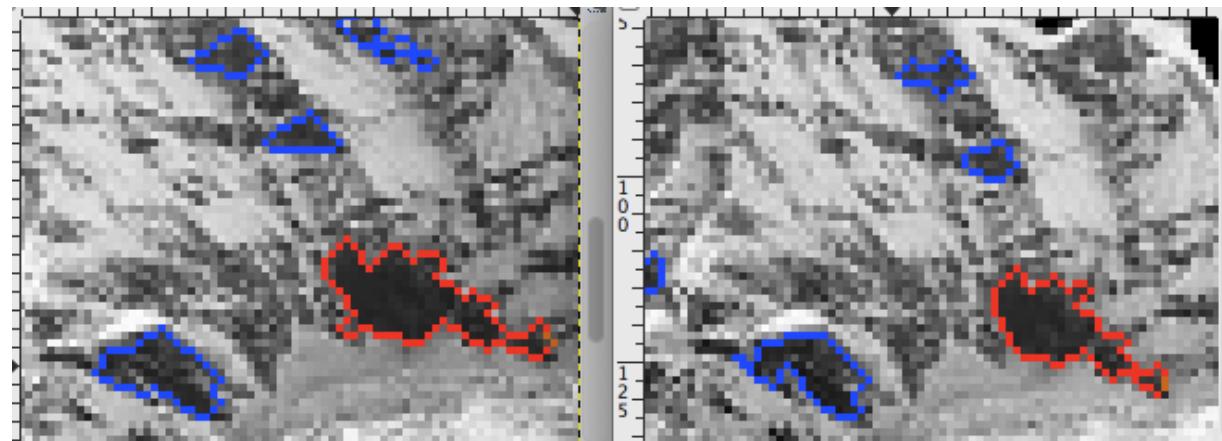
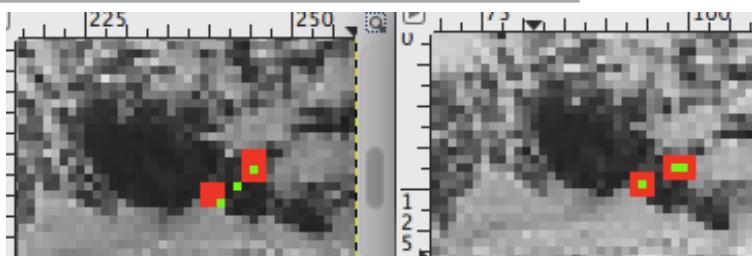
```
[0] [0] dither=2, using previous best rotations for all peaks
(243,123) (95,120) intSqDiff=598.3(2155.0)
(244,124) (97,121) intSqDiff=452.4(2820.2)
(241,127) (94,124) intSqDiff=273.9(4022.0)
(229,118) (86,115) intSqDiff=179.3(1226.4)
(230,117) (87,114) intSqDiff=175.5(1835.1)
(245,124) (98,121) intSqDiff=316.7(2304.6) nMaxStats=6
[0] [1]
(227,123) (228,142) intSqDiff=390.9(1482.4)
(226,123) (230,144) intSqDiff=588.6(1617.9)
(246,124) (225,148) intSqDiff=945.1(2646.4)
(244,124) (225,147) intSqDiff=990.8(2360.6)
(239,129) (219,139) intSqDiff=1345.3(1370.8)
(228,119) (232,148) intSqDiff=1062.1(2164.9)
(230,119) (233,148) intSqDiff=1096.7(1414.1) nMaxStats=9
```

The improvement works pretty well.  
 The correct match has smaller sum square of differences now making the true match of [0][0] easier to distinguish from the false match of [0][1].

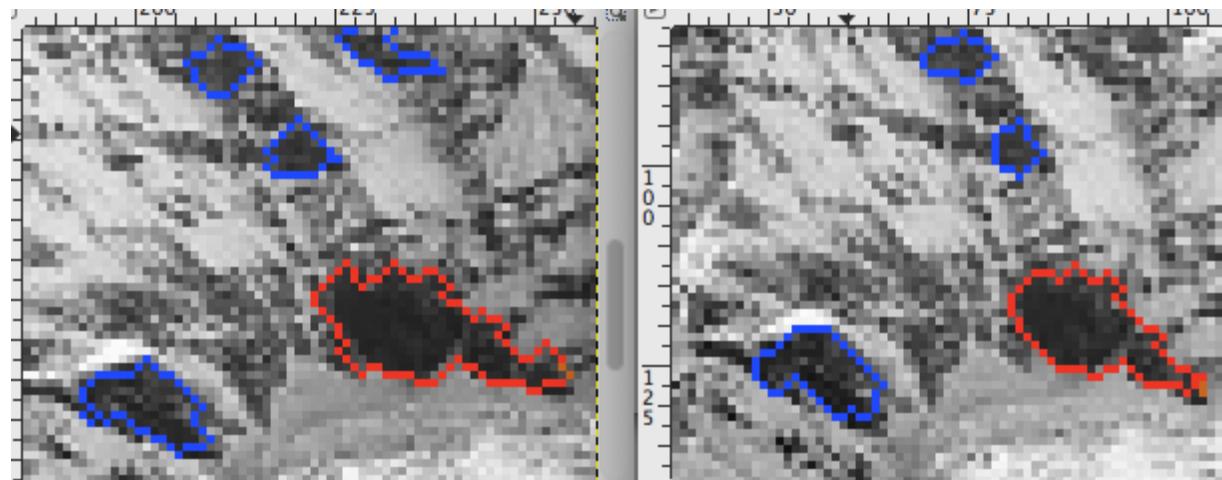
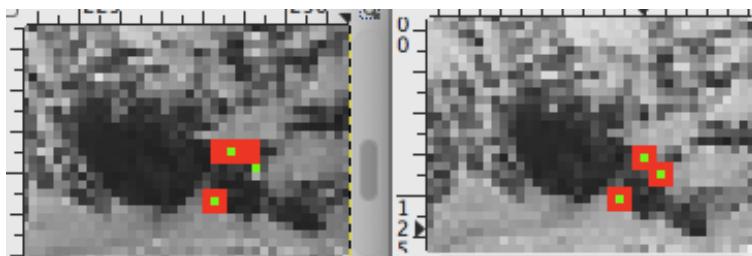
For the **binned images** especially, an edge corrector using the greyscale image as a guide is helpful to remove some of the defects present in the perimeter curve due to blobs extracted as contiguous pixels (in contrast to edges which are extracted from subtracted gaussian blurred images).

**Red** are the contour matched points. **Light green** are the feature improved locations.

before image guided corrections:

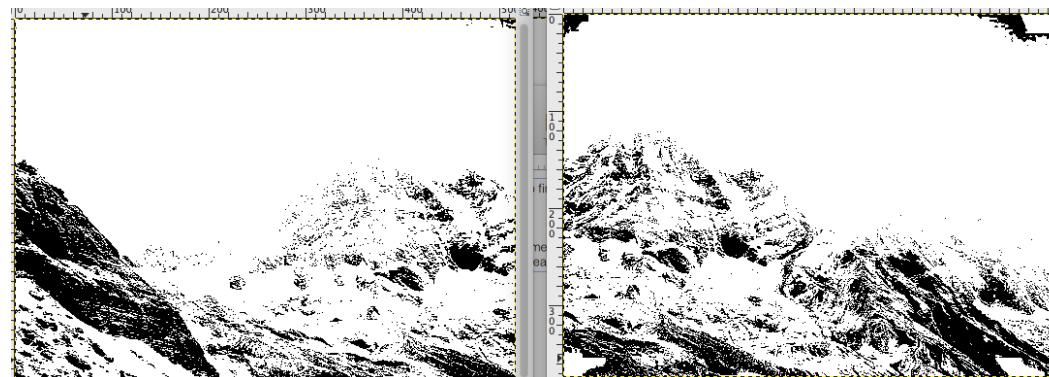


after image guided corrections:

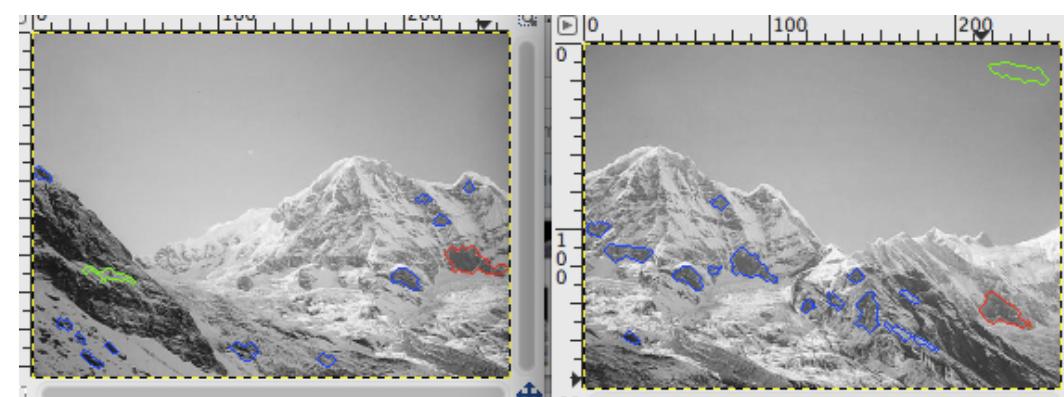
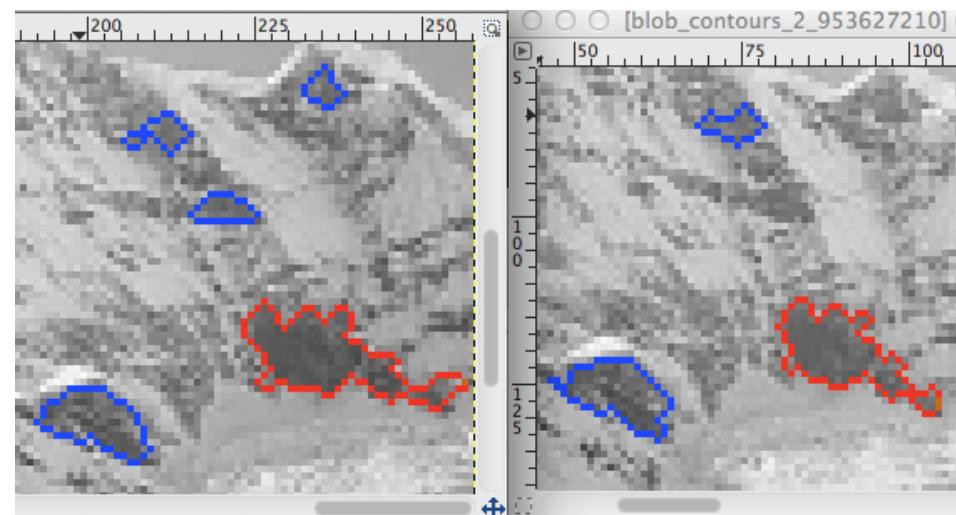
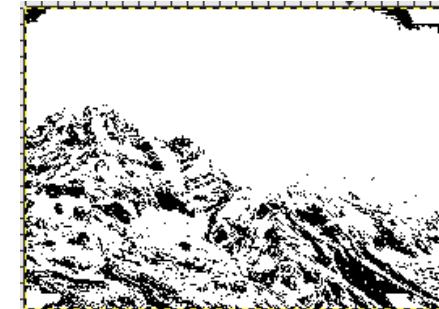
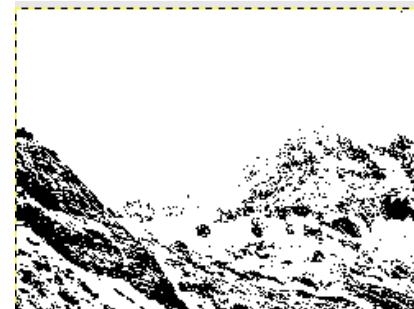


alternate (faster) way to process image to find same blobs.  
no sky subtraction needed for pre-processing.

```
img0 = img.copyToGreyscale();  
imageProcessor.applyImageSegmentation(img0, 2);  
imageProcessor.applyAdaptiveMeanThresholding(img0, 20);
```

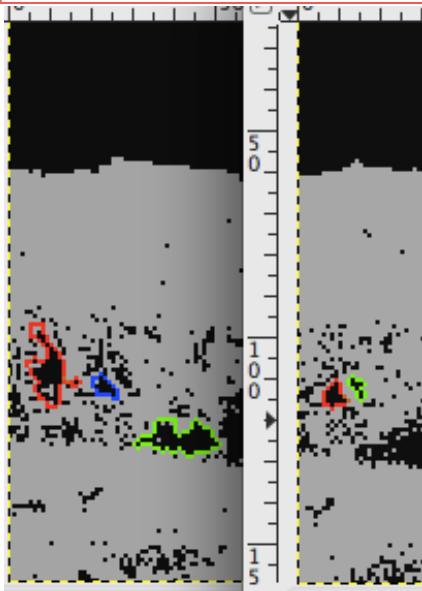


same, but binned, and  
adapMeanThr=20/binFactor=20/3

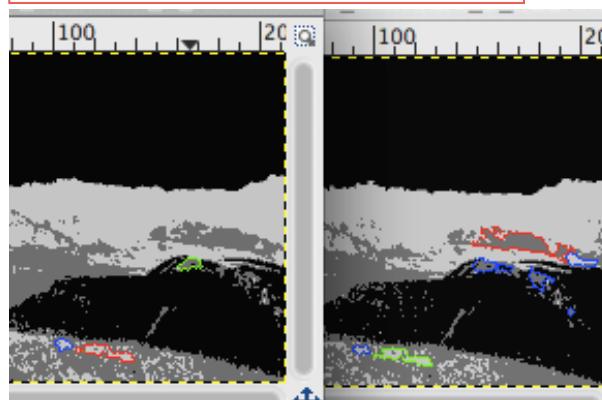


## A look at blob contours (to use for scale solutions)

k=2 segmentation blobs,  
< 300x300, no histeq

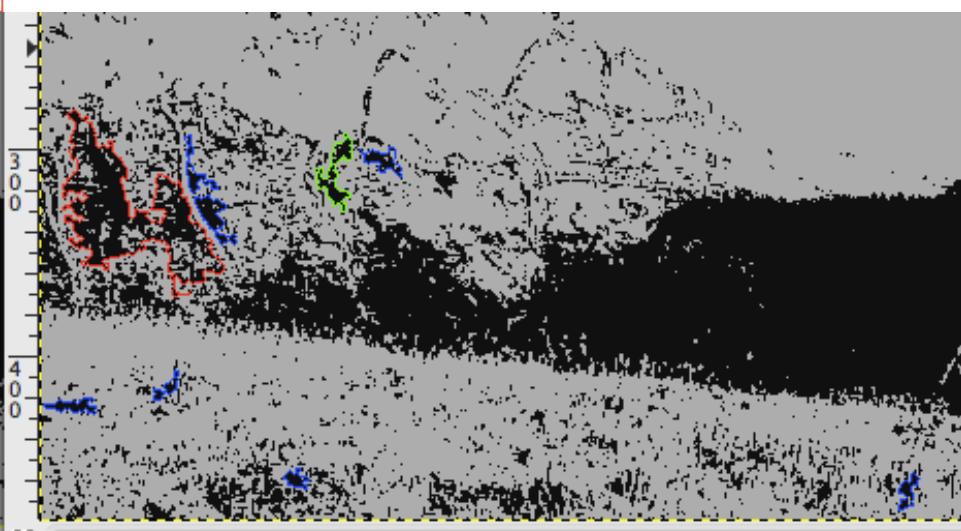
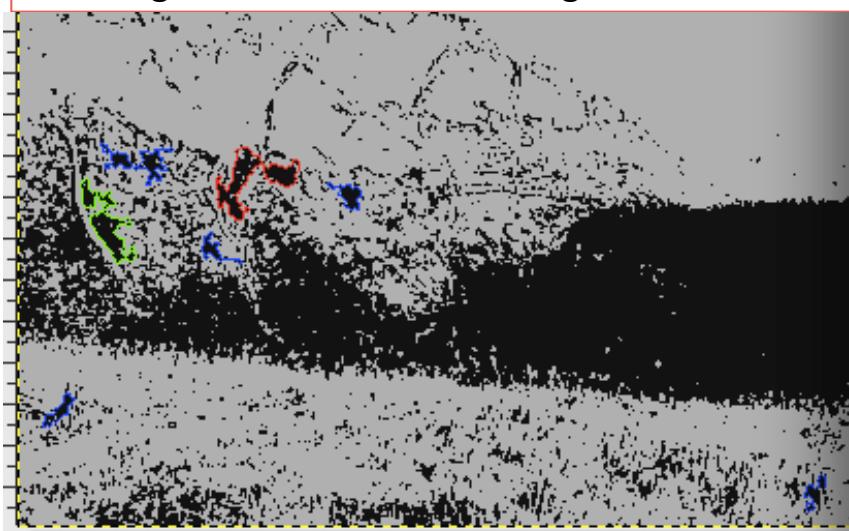


k=3 segmentation blobs,  
< 300x300, no histeq

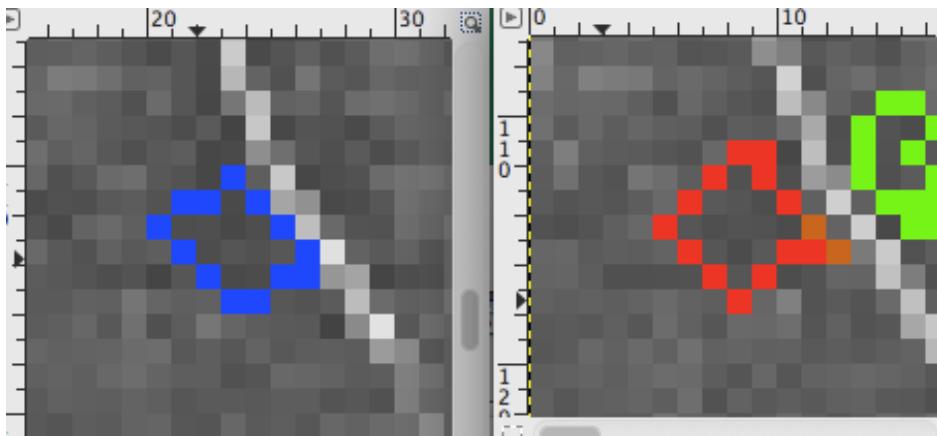


binary segmentation on binned images and images not binned is not the best solution for these images. still exploring a better means.

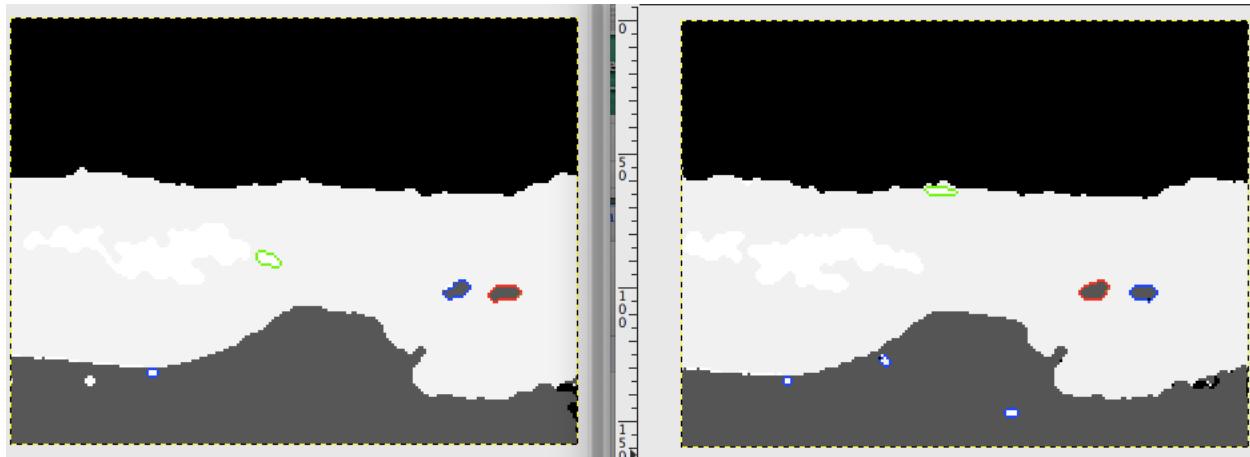
k=2 segmentation blobs, image not binned



k=3 segmentation blobs, image not binned  
has about 4 strong blobs in common



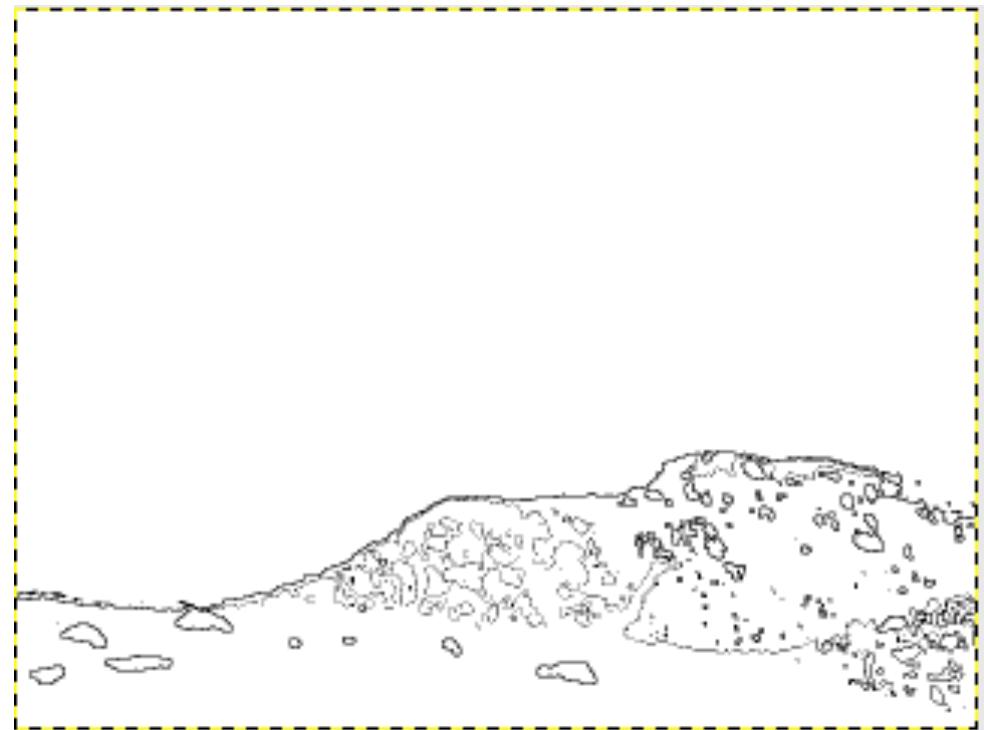
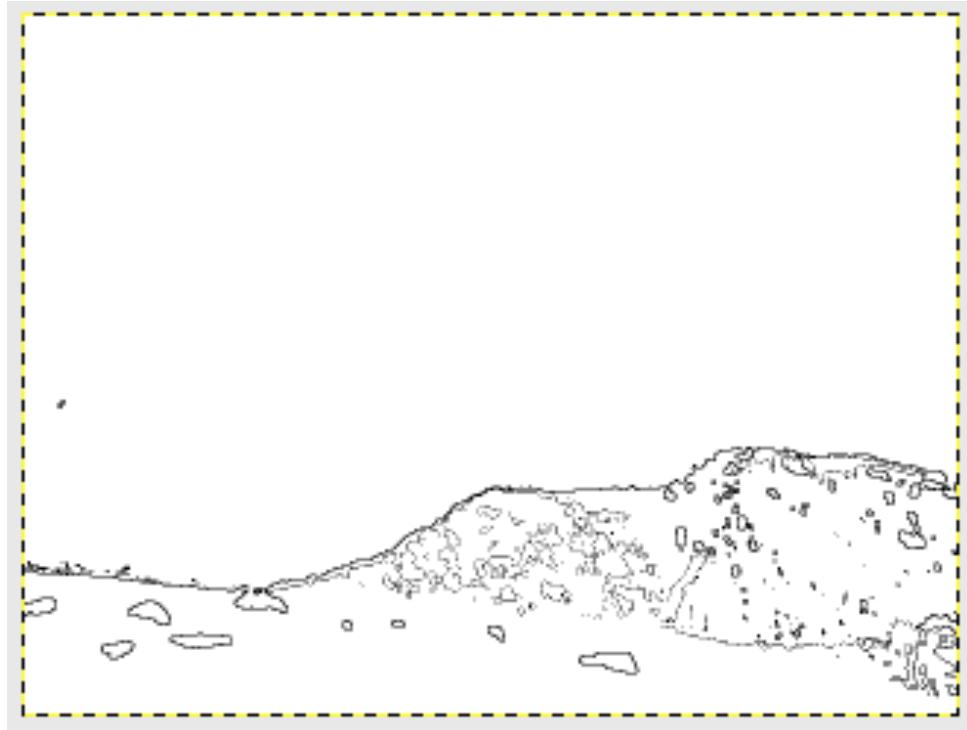
inflection points not always best choice for points of interest. For example, these small rectangles have small scale space contours that are revisited and included if larger contours (from closed curves) are not found.

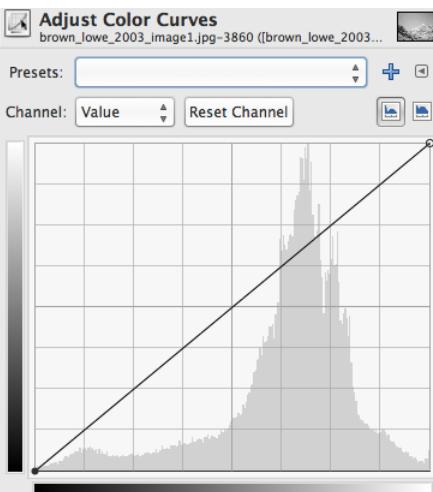


There are also other characteristics that can be seen with segmentation that would make good matching features. This would make a good image to make corners from. The unsegmented unbinned image is a very busy high resolution image with details such as grass texture that make many edges and corners.

The scale solution still needs closed contours though so need to consider the small circles that have small scale space contours if larger contours (from closed curves) are not found.

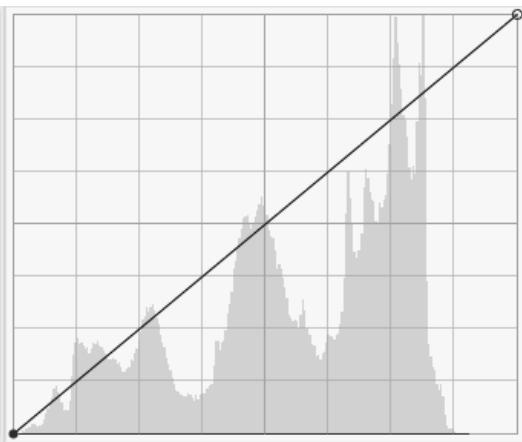
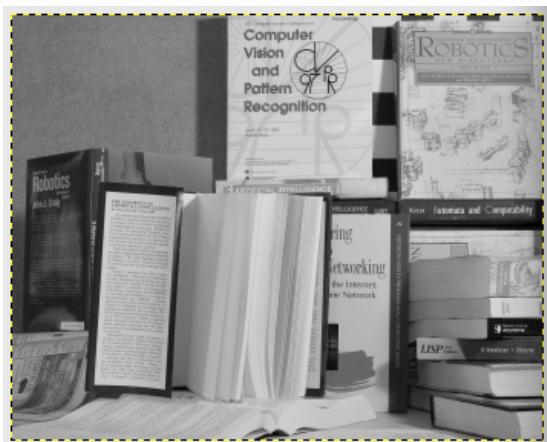
Another approach: Color segmentation followed by adaptive mean thresholding removes the grass as noise and sky variation. The foreground rocks are matchable, but there are many small blobs too. Those will not pass the number of points low filter.



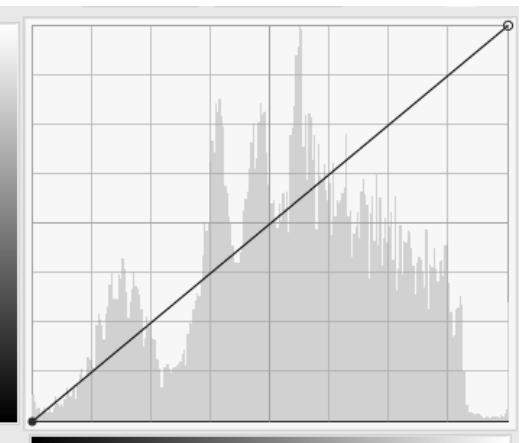


When most of pixels are bright, prefer to use binary segmentation and binning to find dark blobs.

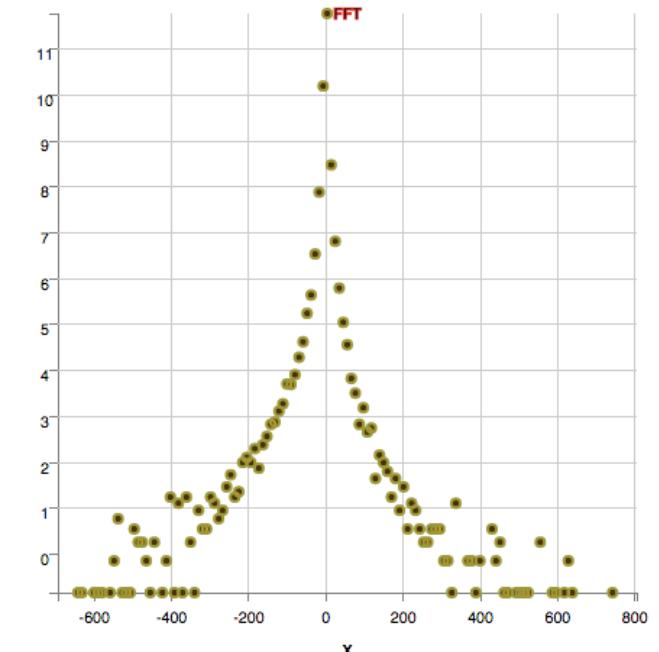
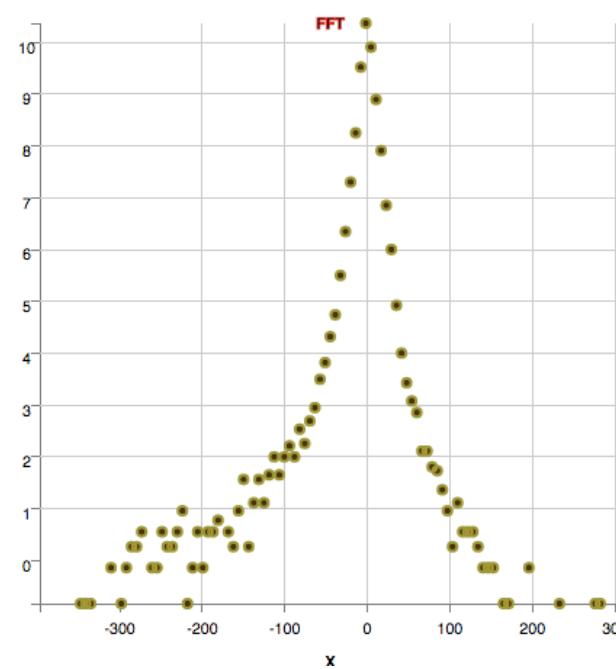
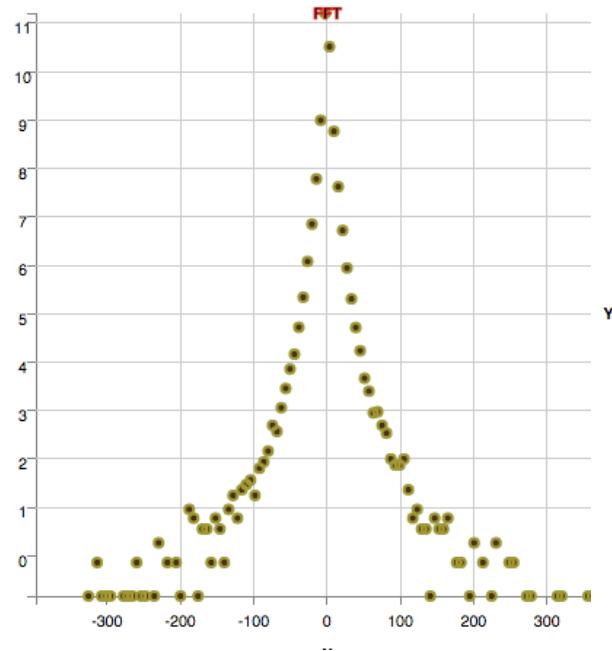
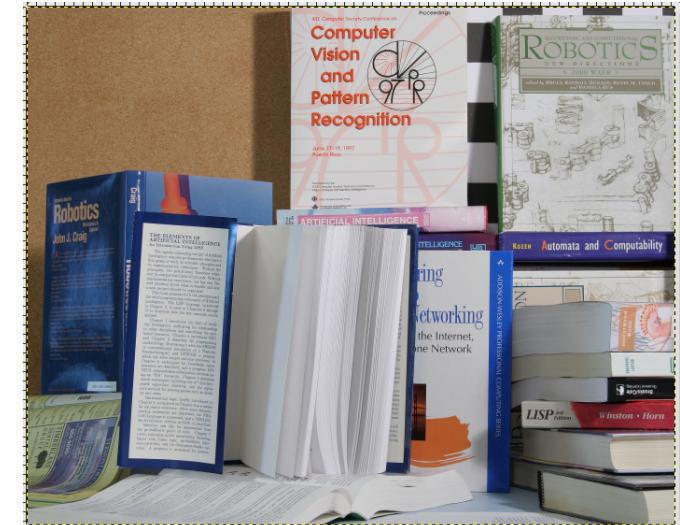
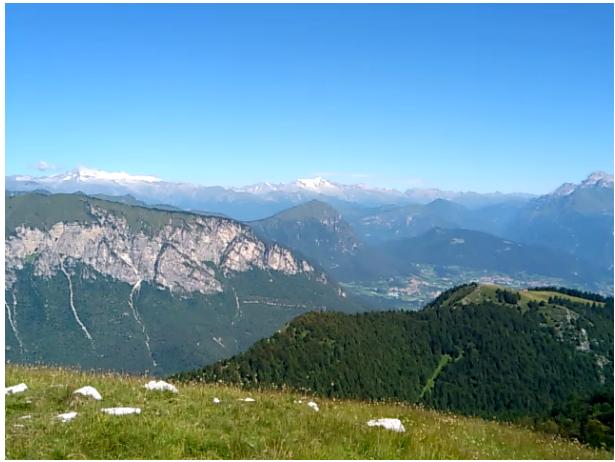
```
stats=min=0.0
[junit] max=255.0
[junit] mean=159.15577697753906
[junit] median=163.0
[junit] mode=165.19515991210938
[junit] quartiles=[145.0, 163.0, 180.0, 255.0]
[junit] histogram=histogram=[(9.717362, 116) (29.152088, 780)
(48.58681, 2144) (68.02154, 5951) (87.45626, 6139) (106.89098, 8183)
(126.325714, 13823) (145.76044, 36746) (165.19516, 57768) (184.62988,
37465) (204.0646, 11988) (223.49933, 7597) (242.93407, 4009) ]
[junit] histogram errors=[(9.717363, 0.77178496) (9.717363,
0.20207894) (9.717363, 0.15404044) (9.717363, 0.10978928) (9.717363,
0.12205033) (9.717363, 0.116789326) (9.717363, 0.099361) (9.717363,
0.06792254) (9.717363, 0.058328137) (9.717363, 0.07409418) (9.717363,
0.1321732) (9.717363, 0.17136808) (9.717363, 0.24313246) ]
histAreas=[0.23737653, 0.7626235]
```



```
stats=min=0.0
[junit] max=230.0
[junit] mean=142.4168701171875
[junit] median=150.0
[junit] mode=193.89910888671875
[junit] quartiles=[113.0, 150.0, 190.0, 230.0]
[junit] histogram=histogram=[(9.233291, 1934) (27.699871,
11960) (46.166454, 21237) (64.63303, 25864) (83.09962, 16128)
(101.5662, 15522) (120.032776, 58025) (138.49936, 38071)
(156.96594, 25586) (175.43253, 55022) (193.89911, 82024)
(212.36569, 34202) (230.83226, 150) ]
[junit] histogram errors=[(9.23329, 0.129593) (9.23329,
0.053420994) (9.23329, 0.050305214) (9.23329, 0.054534614)
(9.23329, 0.07460825) (9.23329, 0.08468471) (9.23329,
0.049914666) (9.23329, 0.063751675) (9.23329, 0.08130212)
(9.23329, 0.06033505) (9.23329, 0.052829705) (9.23329,
0.08098979) (9.23329, 1.1787614) ]
histAreas=[0.31374013, 0.686259871]
```



```
stats=min=0.0
[junit] max=255.0
[junit] mean=130.99832153320312
[junit] median=134.0
[junit] mode=140.81146240234375
[junit] quartiles=[92.0, 134.0, 173.0, 255.0]
[junit] histogram=histogram=[(9.38743, 2005) (28.16229, 10477)
(46.93715, 25751) (65.71201, 10222) (84.48687, 33586) (103.26173,
32719) (122.03659, 32886) (140.81146, 42862) (159.58632, 33073)
(178.36118, 31383) (197.13603, 25580) (215.9109, 21222)
(234.68576, 3989) ]
[junit] histogram errors=[(9.38743, 0.1964549) (9.38743,
0.054399375) (9.38743, 0.04402962) (9.38743, 0.08140163) (9.38743,
0.052351285) (9.38743, 0.057749763) (9.38743, 0.061792303)
(9.38743, 0.05860566) (9.38743, 0.07022972) (9.38743, 0.07583307)
(9.38743, 0.08779022) (9.38743, 0.10032542) (9.38743, 0.2337324) ]
histAreas=[0.5551447, 0.44485524]
```



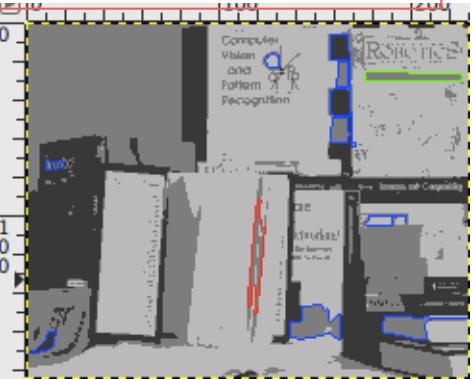
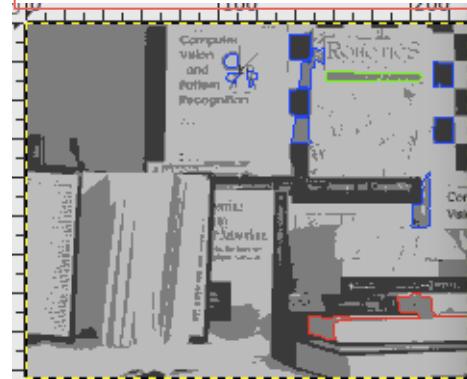
FFT of brightness, logarithm of histogram to show spatial frequency (lots of added components...)

# A look at blob contours (to use for scale solutions)

k=2 segmentation blobs, binned to < 300x300



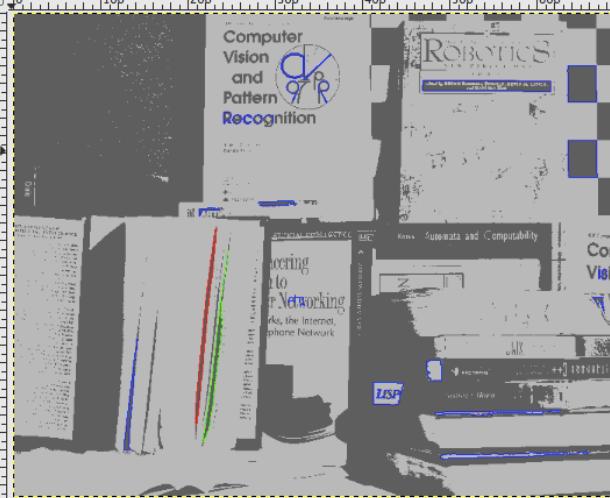
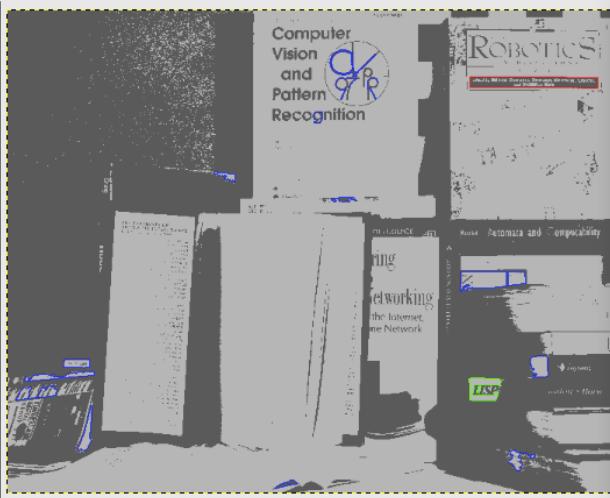
k=3 segmentation blobs, binned to < 300x300



In summary with other images, a feasible approach to using blob contours to solve for scale may be to use binary segmentation ( $k=2$ ) on binned images and images not binned, and combine the results w/ expectation that there may only be one or two blobs in the solutions.

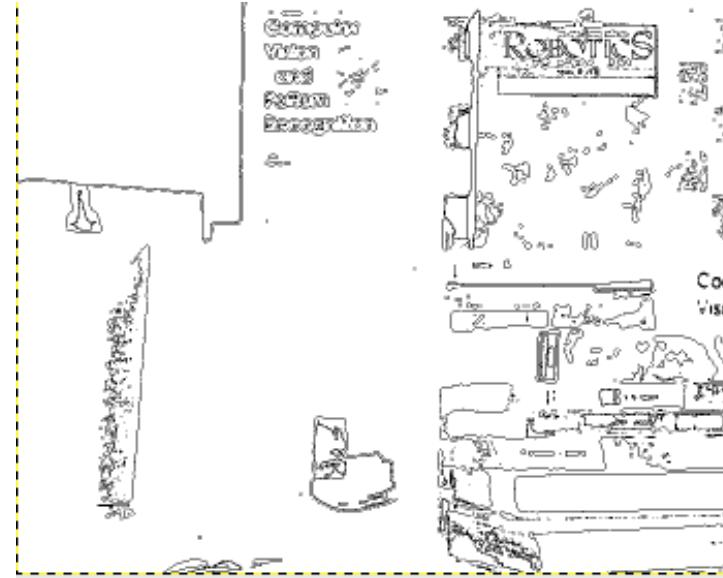
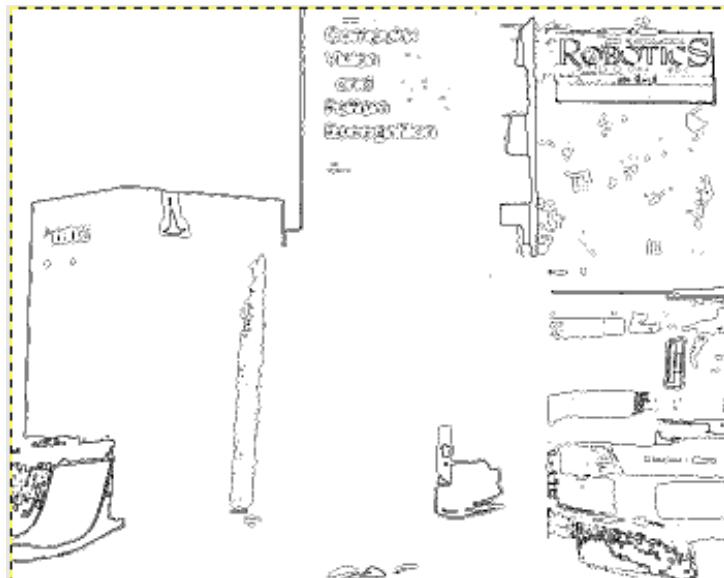
In this case, the number of possible matches for  $k=2$  is very low, but still significant. The  $k=3$  binned to < 300x300, however, has many more features in common.

k=2 segmentation blobs, image not binned

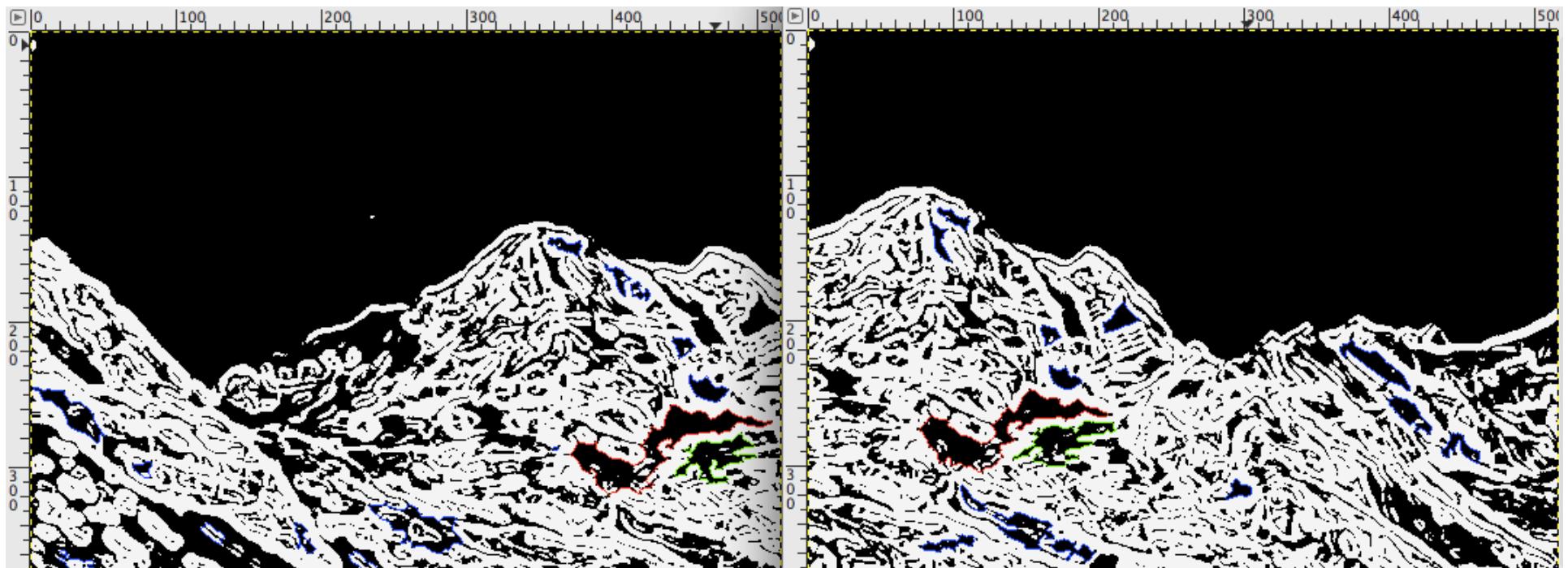


Adaptive Thresholding works well for these images with different lighting and details such as text.

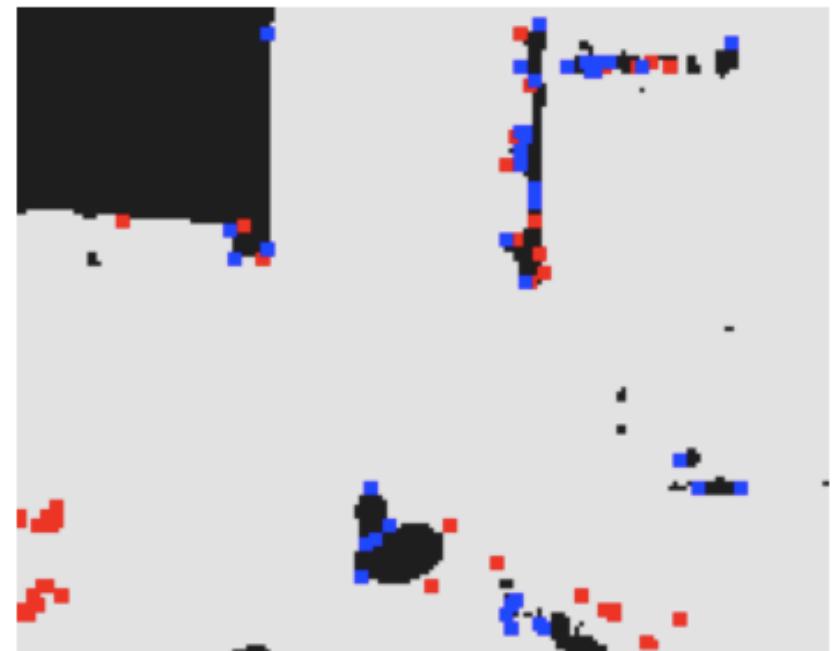
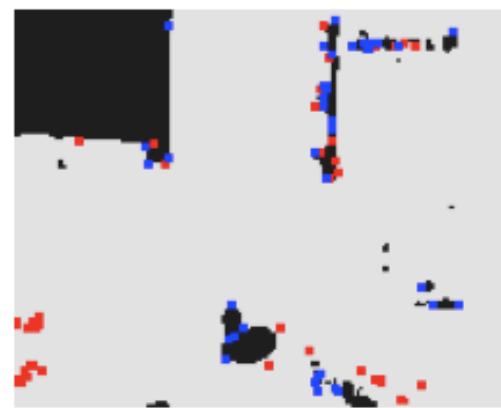
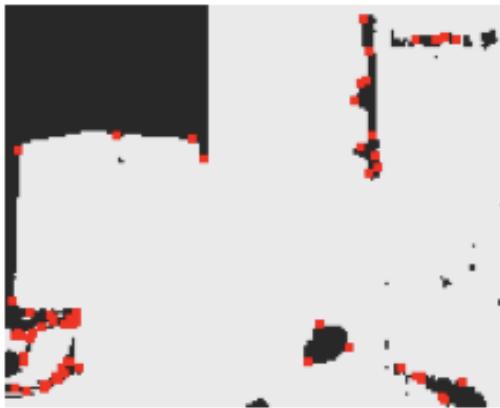
If color segmentation is used first, can reduce the noise to these:



```
segImg = imgGrey.copyImage();
CannyEdgeFilter filter = new CannyEdgeFilter();
CannyEdgeFilterSettings settings = new CannyEdgeFilterSettings();
settings.setUseOutdoorMode();
filter.setSetters(settings);
filter.applyFilter(segImg);
EdgeFilterProducts edgeFilterProducts = filter.getEdgeFilterProducts();
segImg = edgeFilterProducts.getGradientXY();
imageProcessor.applyImageSegmentation(segImg, 2);
for (int i = 0; i < segImg.getWidth(); ++i) {
    for (int j = 0; j < segImg.getHeight(); ++j) {
        int v = 100*segImg.getValue(i, j);
        segImg.setValue(i, j, v);
    }
}
```

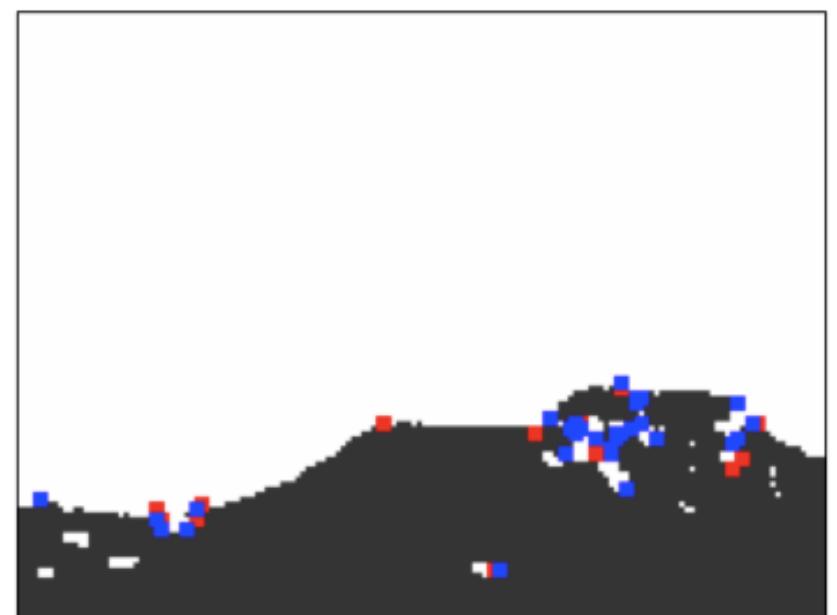
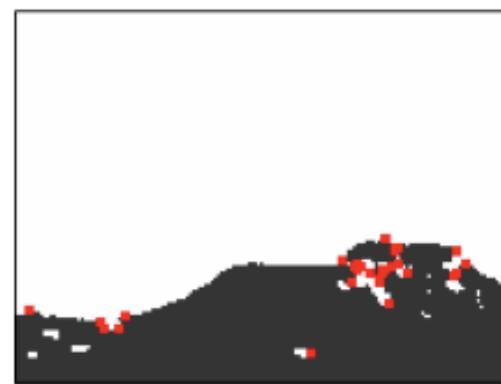
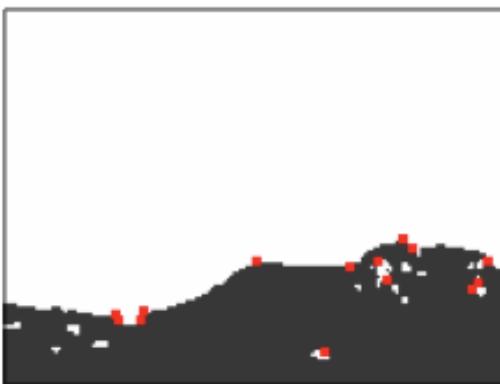


A quick look at color segmentation with k=3 followed by binary segmentation, binned to size < 200 x 200, followed by corners worked fine for 2 image sets, but not the third so the other methods in previous slides are preferred (corners filtered to blobs, matching, calculating transformation, then creating correspondence with all corners).

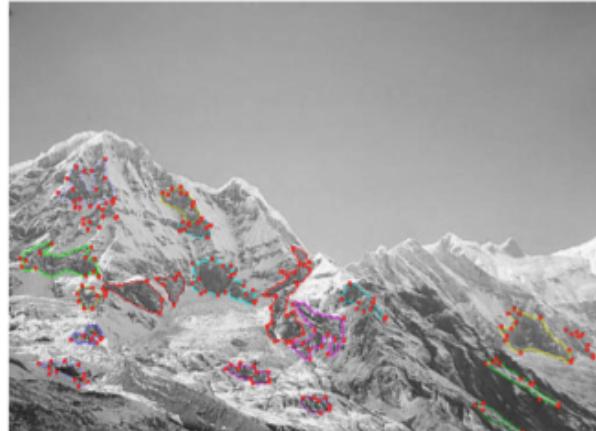
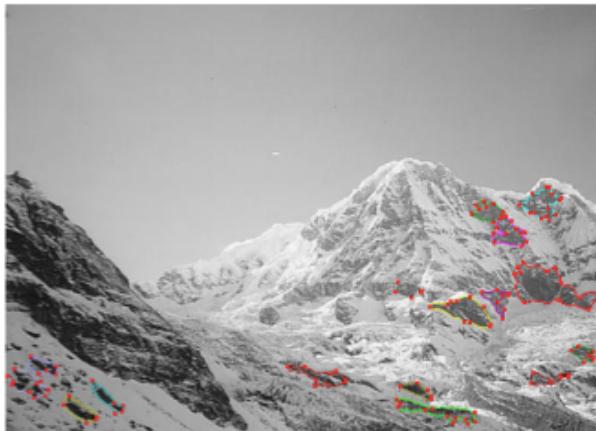


For the stereoscopic above, there's more than translation  
so this can only be the start of the solution

## previous notes from a look at binning and segmentation



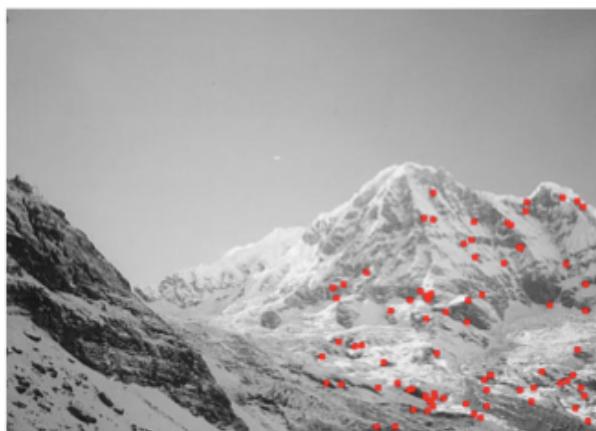
current summary of calculating euclidean transformation with blobs, then creating a correspondence list for the epipolar solver.



corners of blobs in kmpp segmentation.

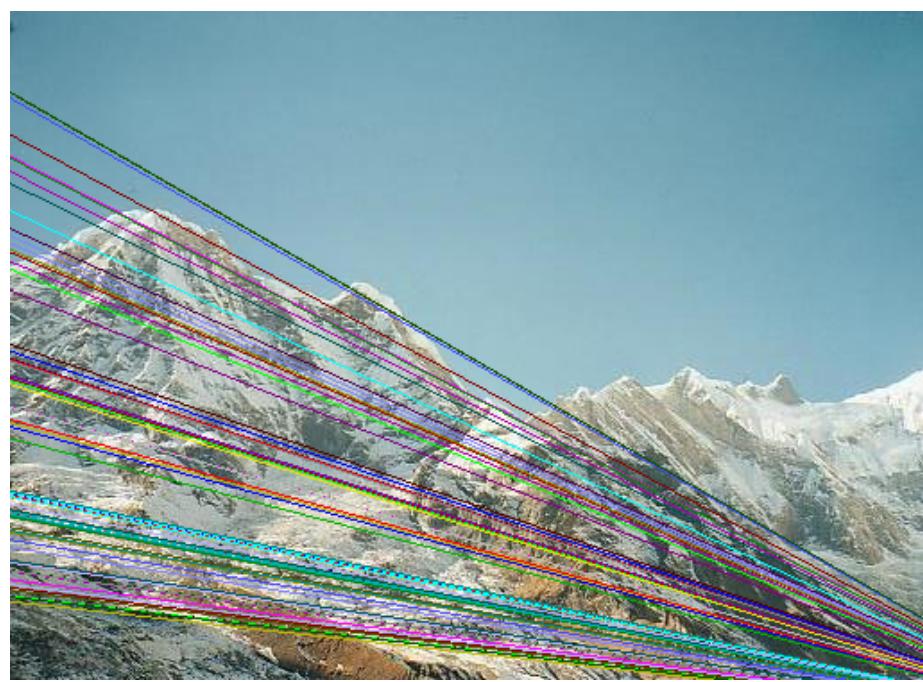
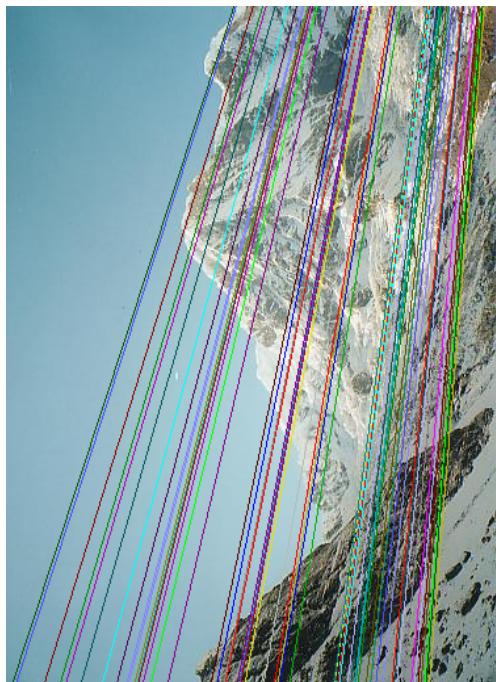
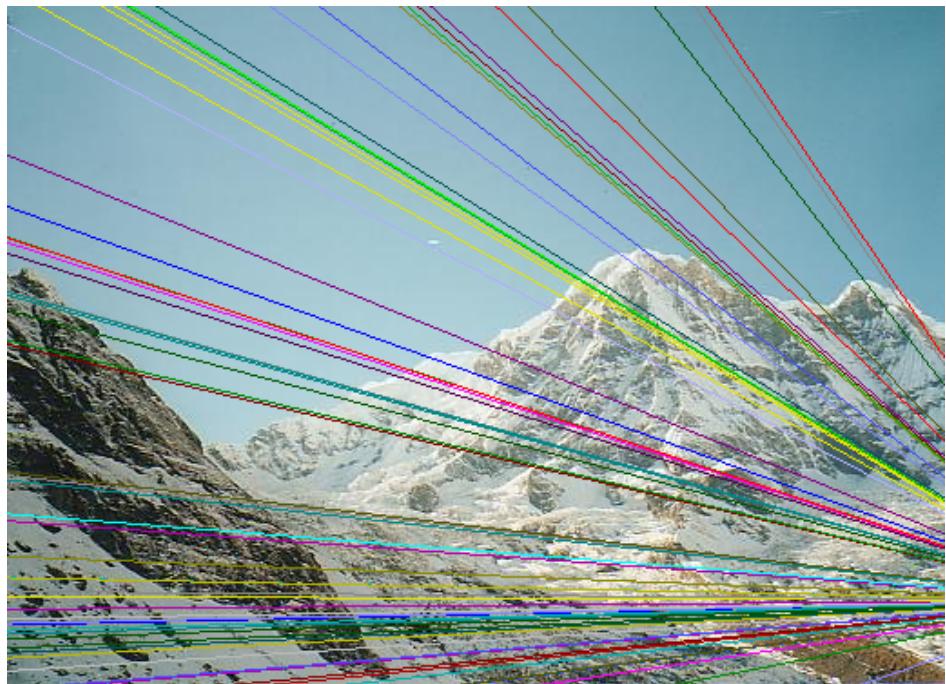


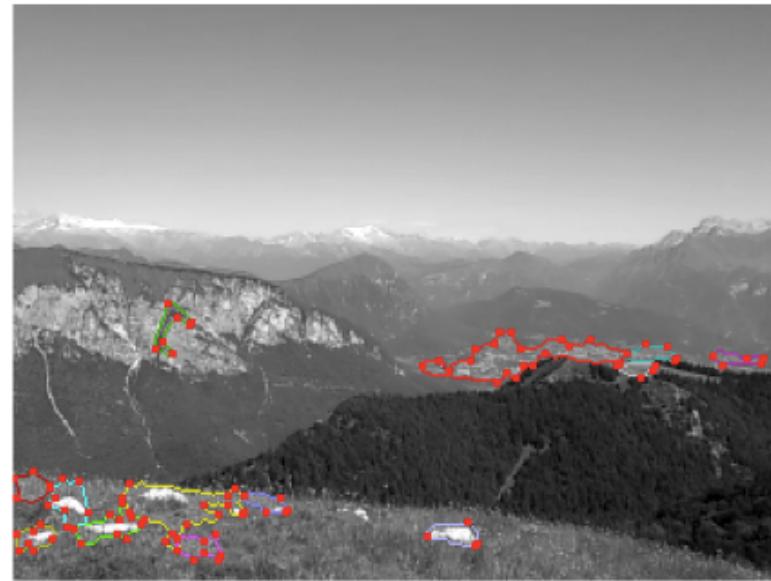
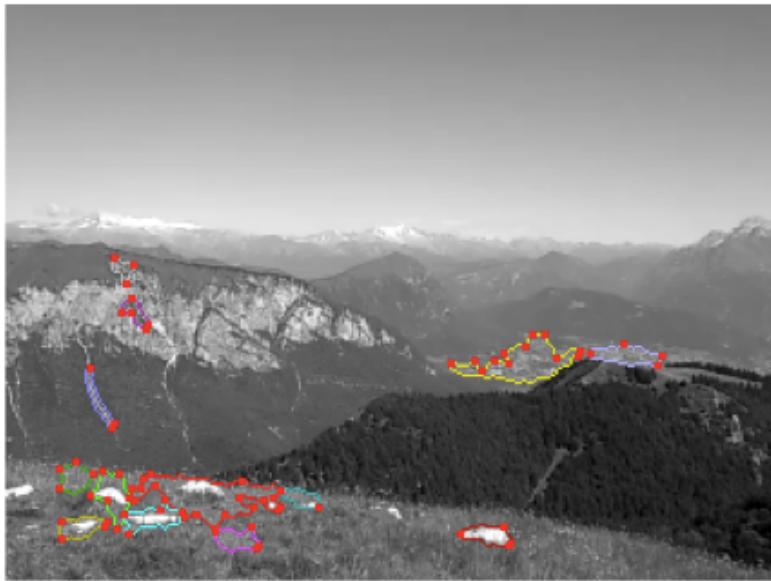
matched corners from euclidean solution



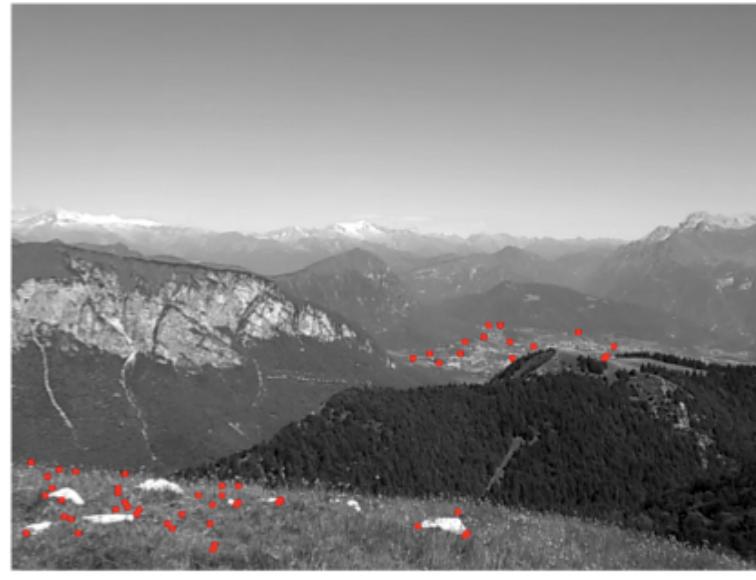
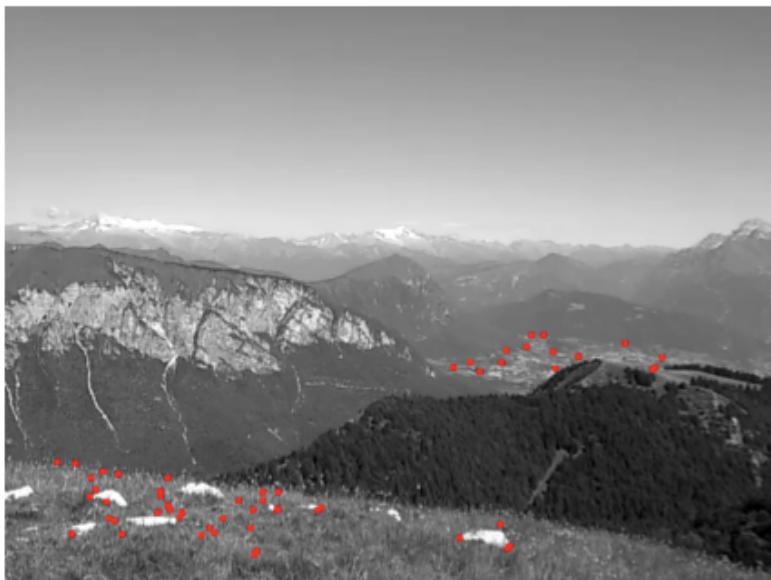
matched corners from canny edge corners found with above blob calculated euclidean solution

(before RANSAC and epipolar solver)

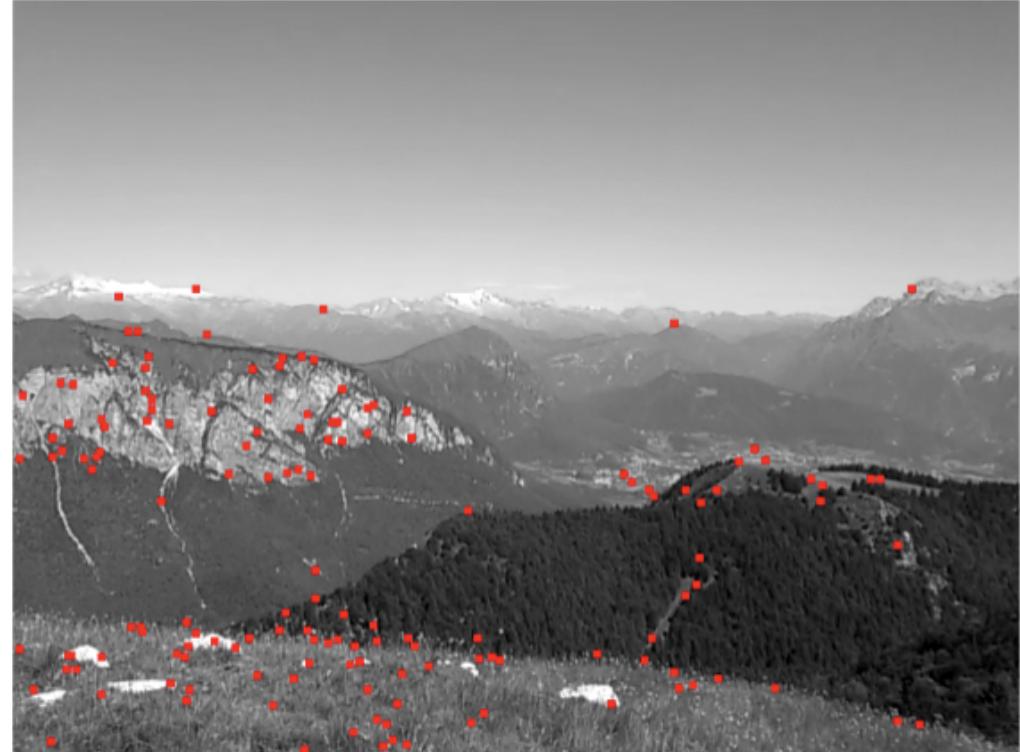
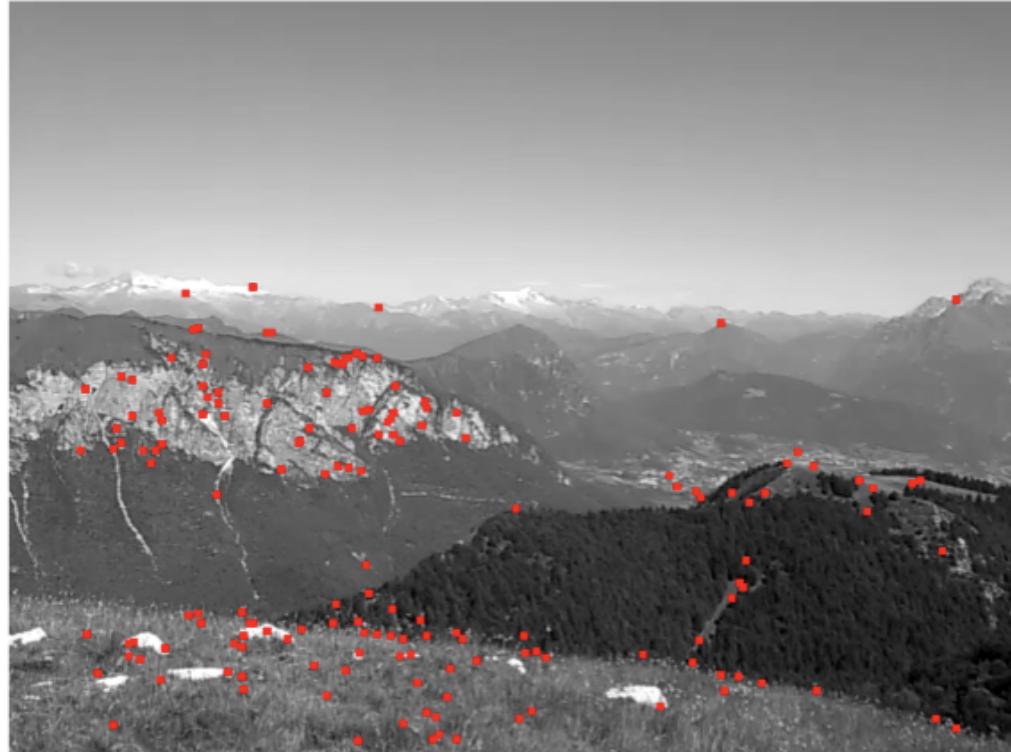




corners of blobs  
in kmpp  
segmentation.



matched corners  
from euclidean  
solution



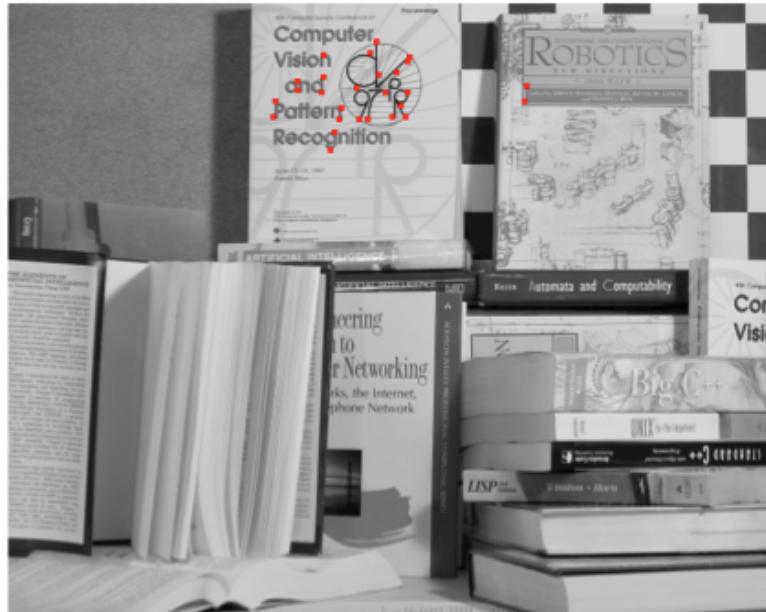
matched corners from canny edge corners found with  
blob calculated euclidean solution

(before RANSAC and epipolar solver)





corners of  
blobs in kmpp  
segmentation.

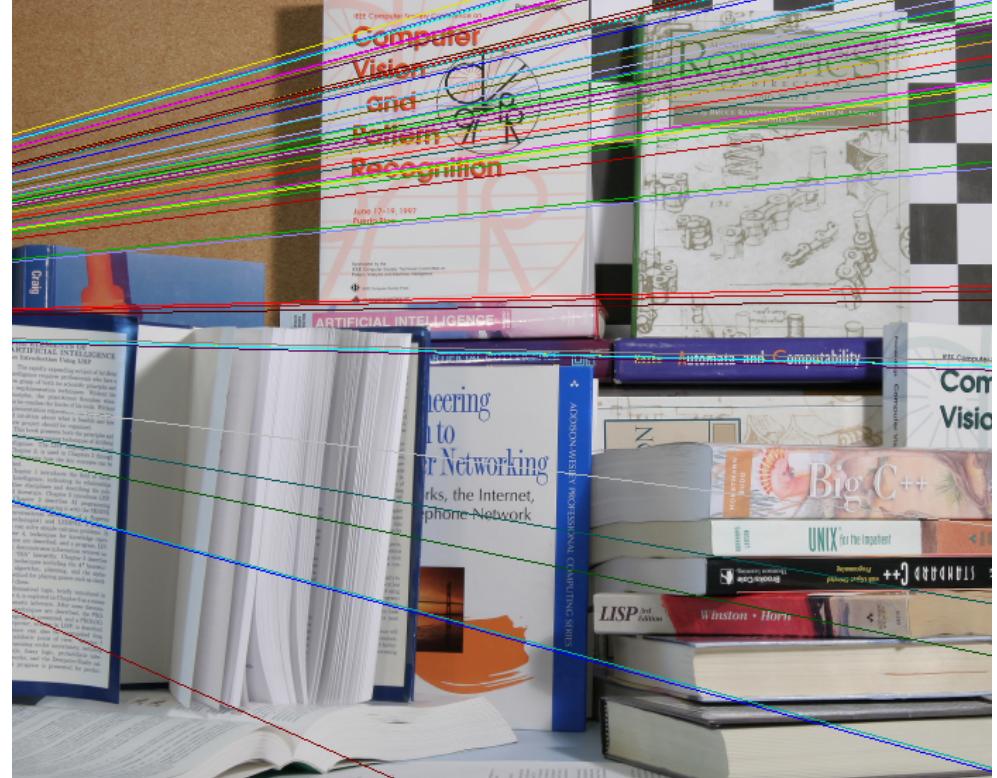


matched corners  
from euclidean  
solution

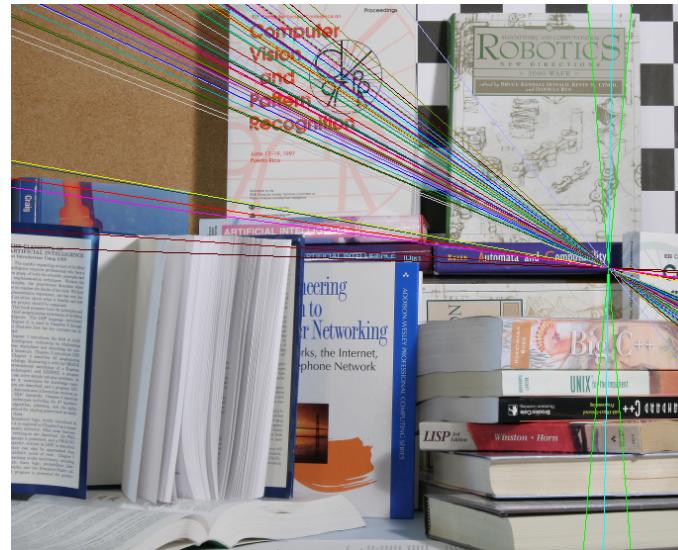


matched corners from canny edge corners found  
with blob calculated euclidean solution

(before RANSAC and epipolar solver)



needs improved corners (blobs, segmentation)





corners of blobs in  
kmpp segmentation.

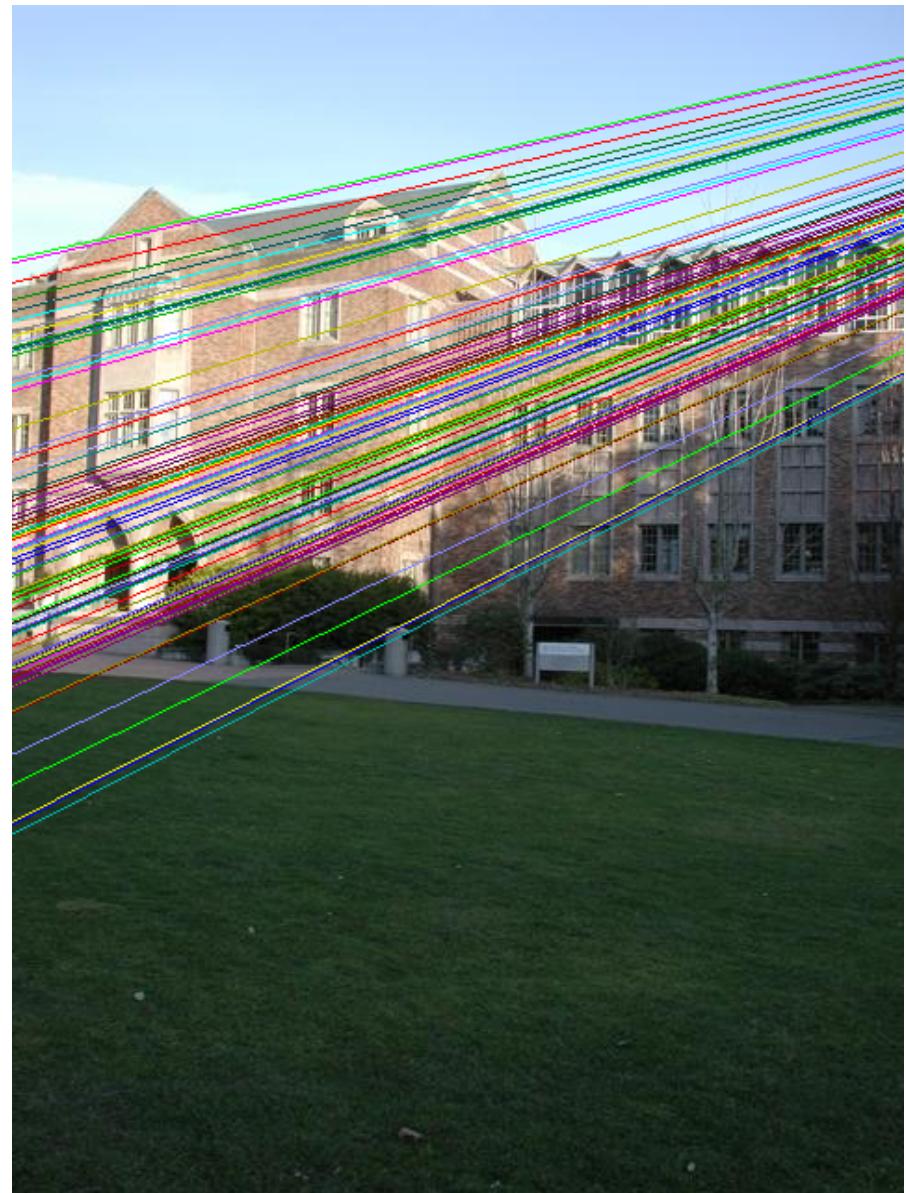
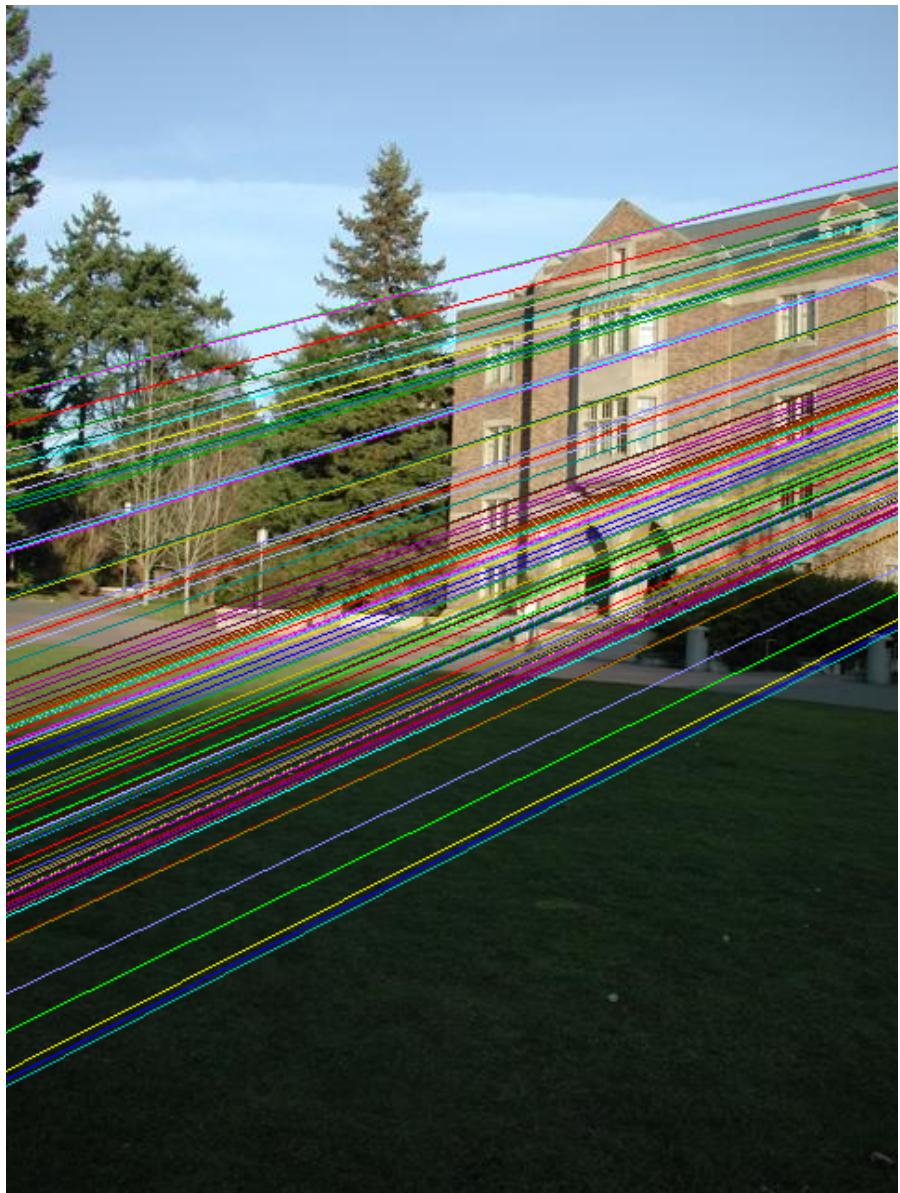


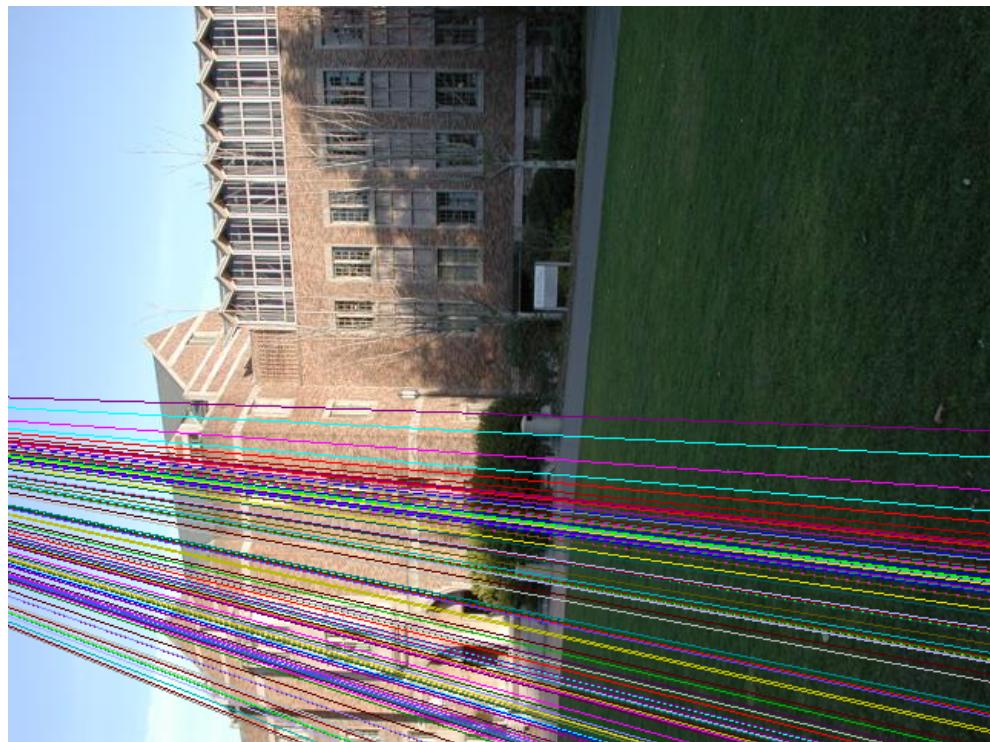
matched corners from  
euclidean solution

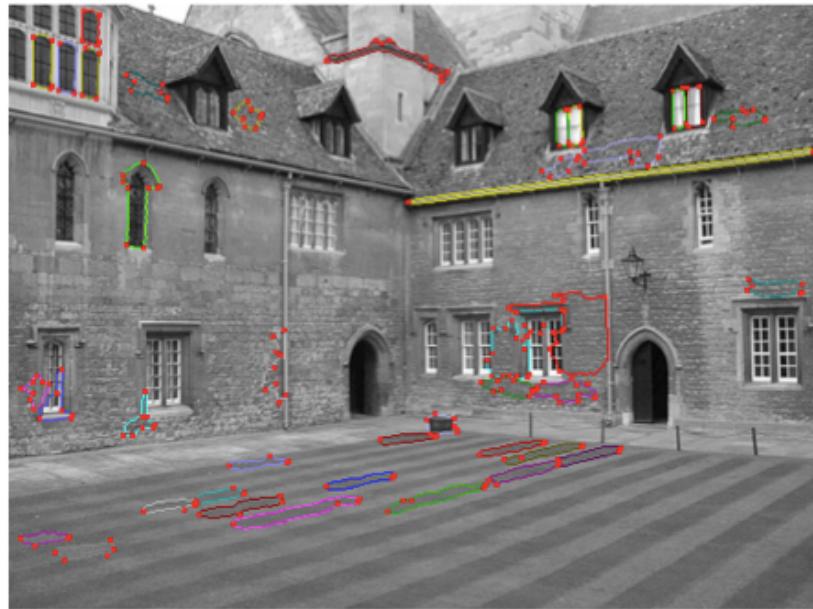


matched corners from canny edge corners found  
with blob calculated euclidean solution

(before RANSAC and epipolar solver)







corners of  
blobs in kmpp  
segmentation.

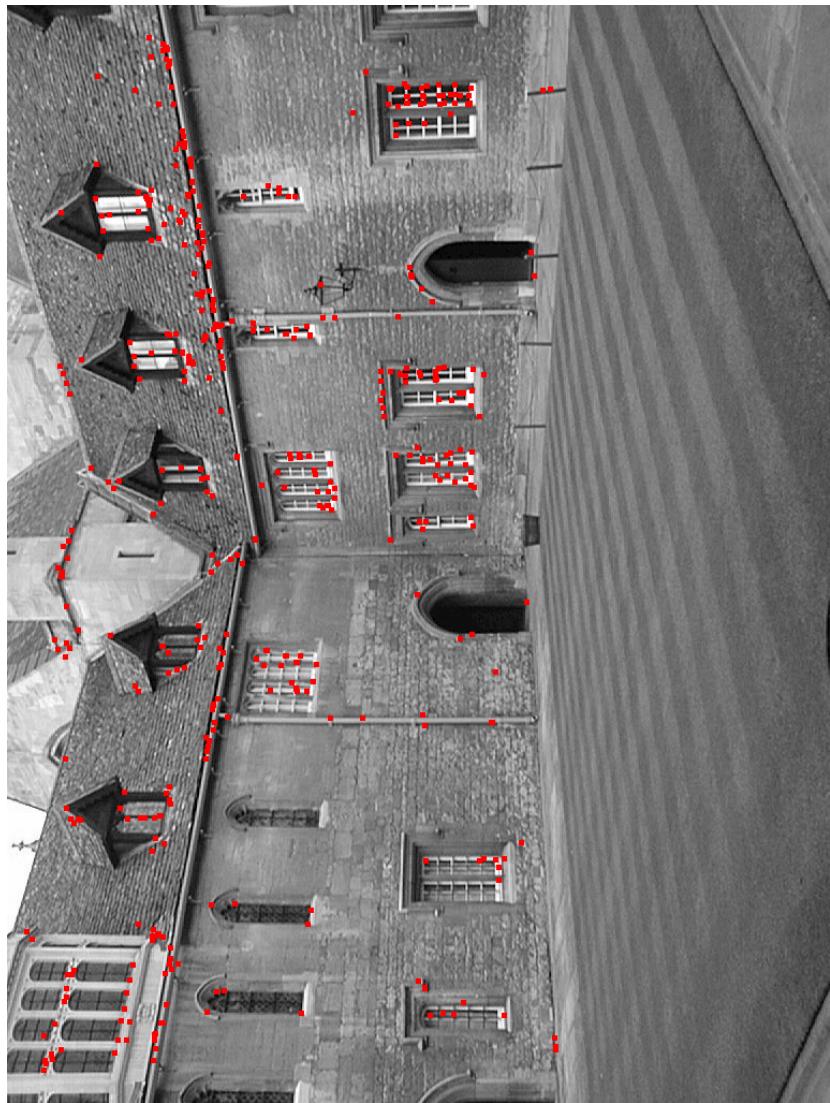


matched  
corners from  
euclidean  
solution

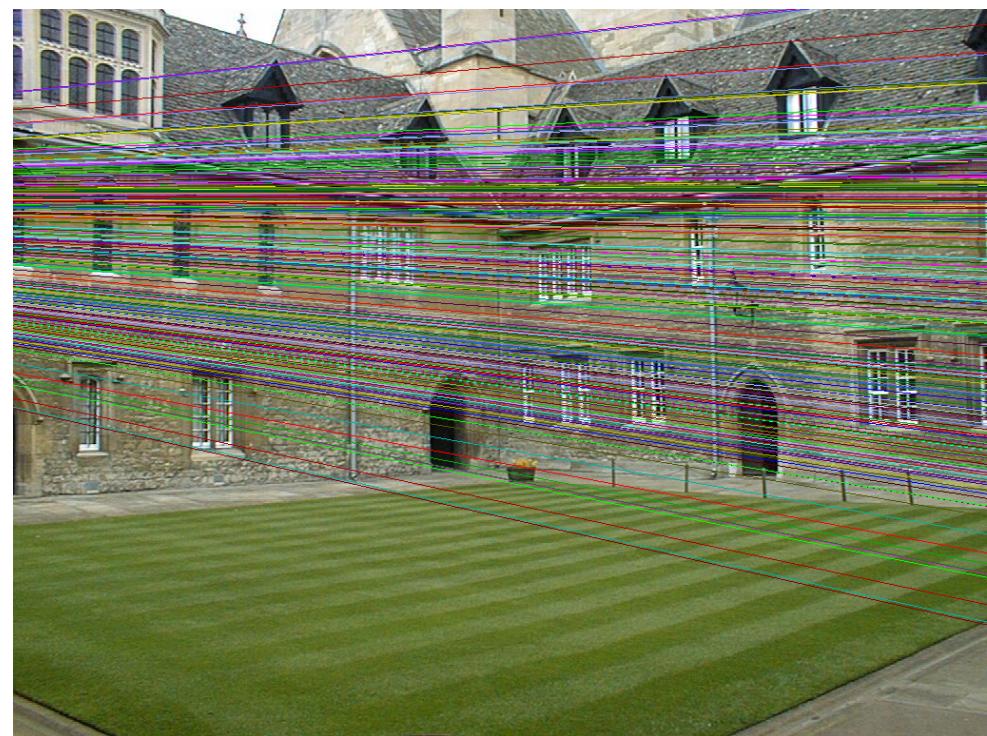
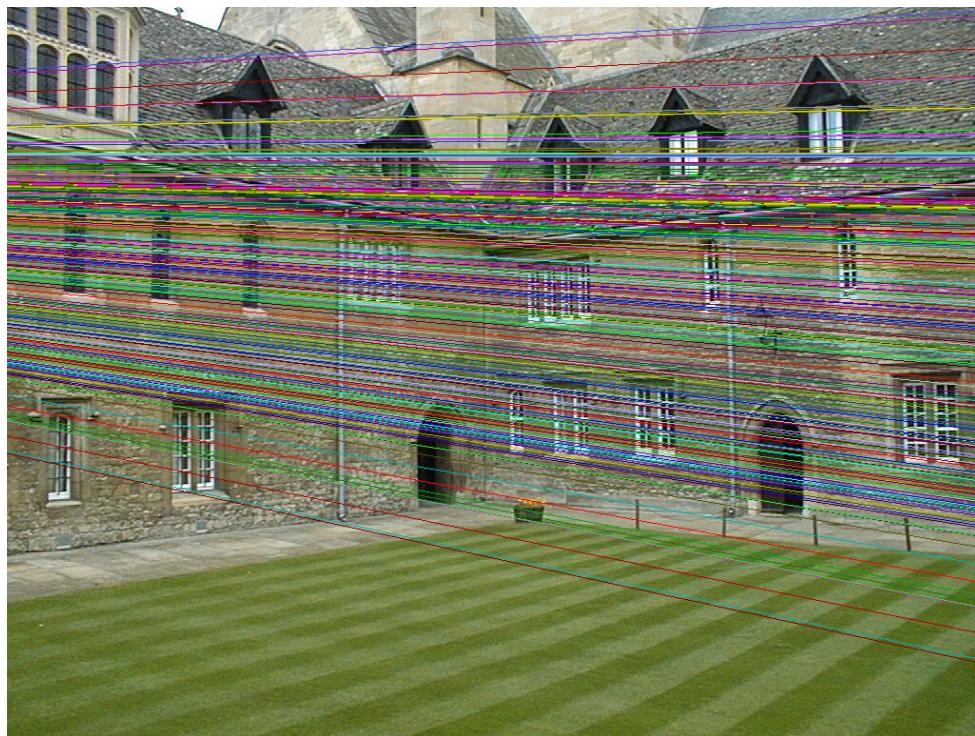


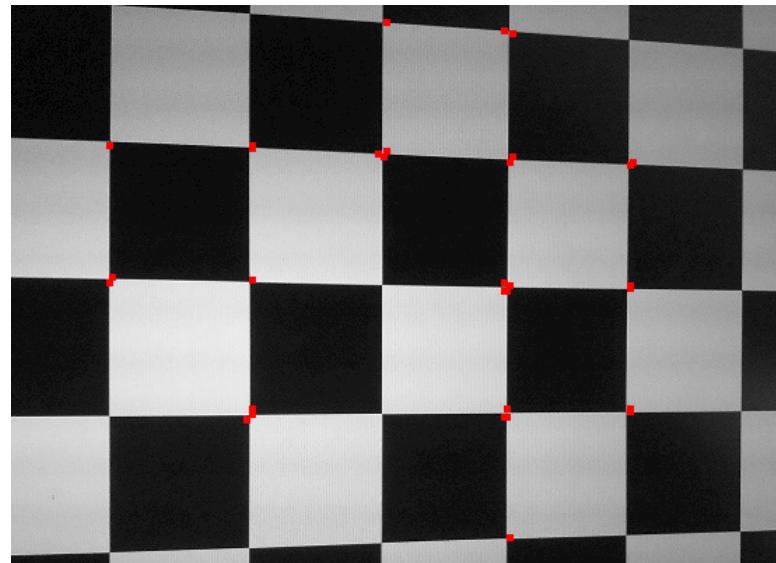
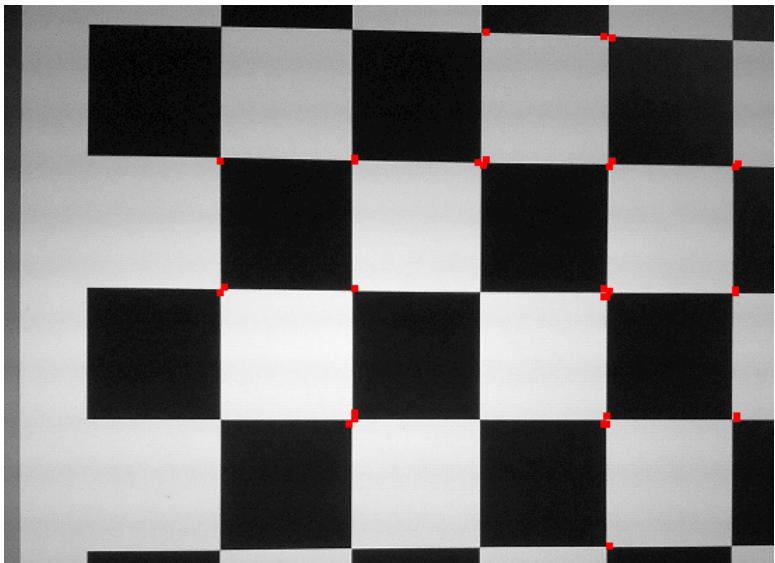
matched corners from canny edge corners found with  
blob calculated euclidean solution.

(before RANSAC and epipolar solver)

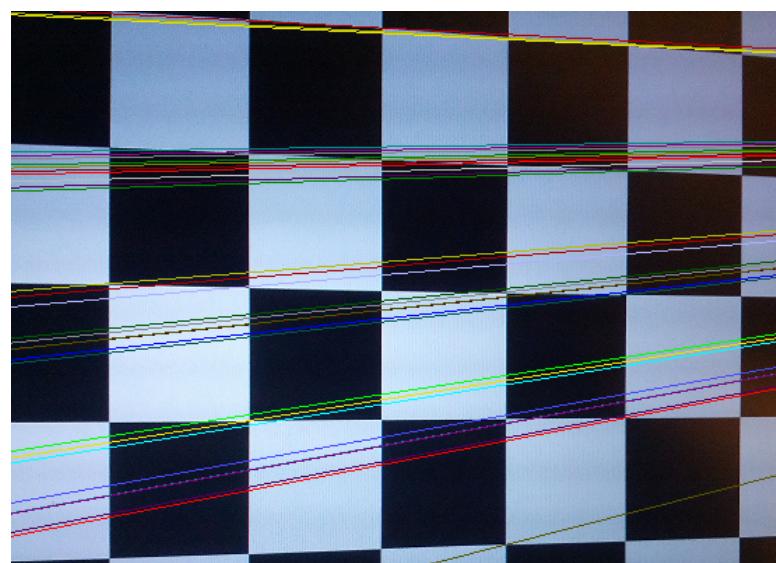


same steps, but image1 is landscape

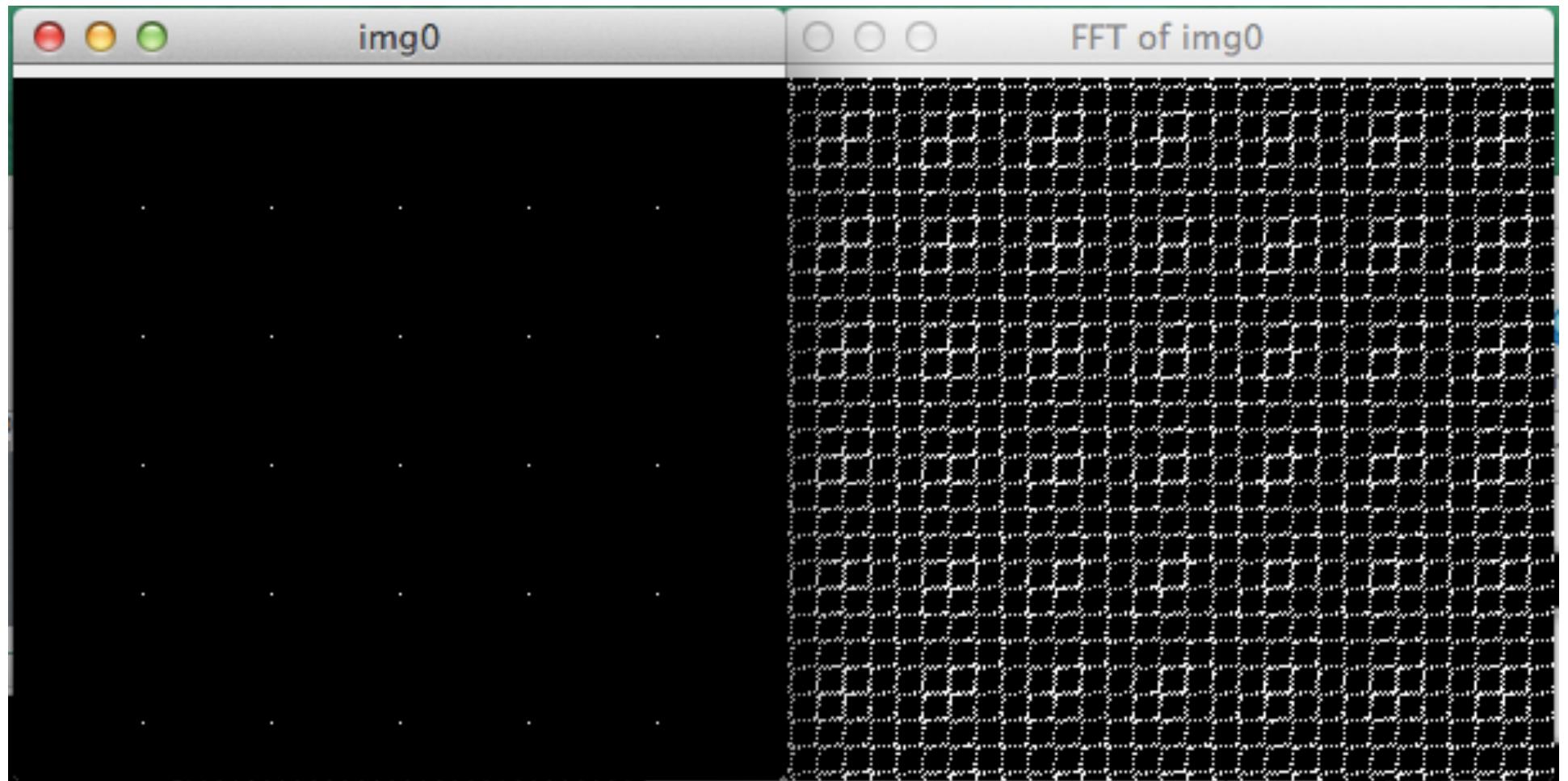




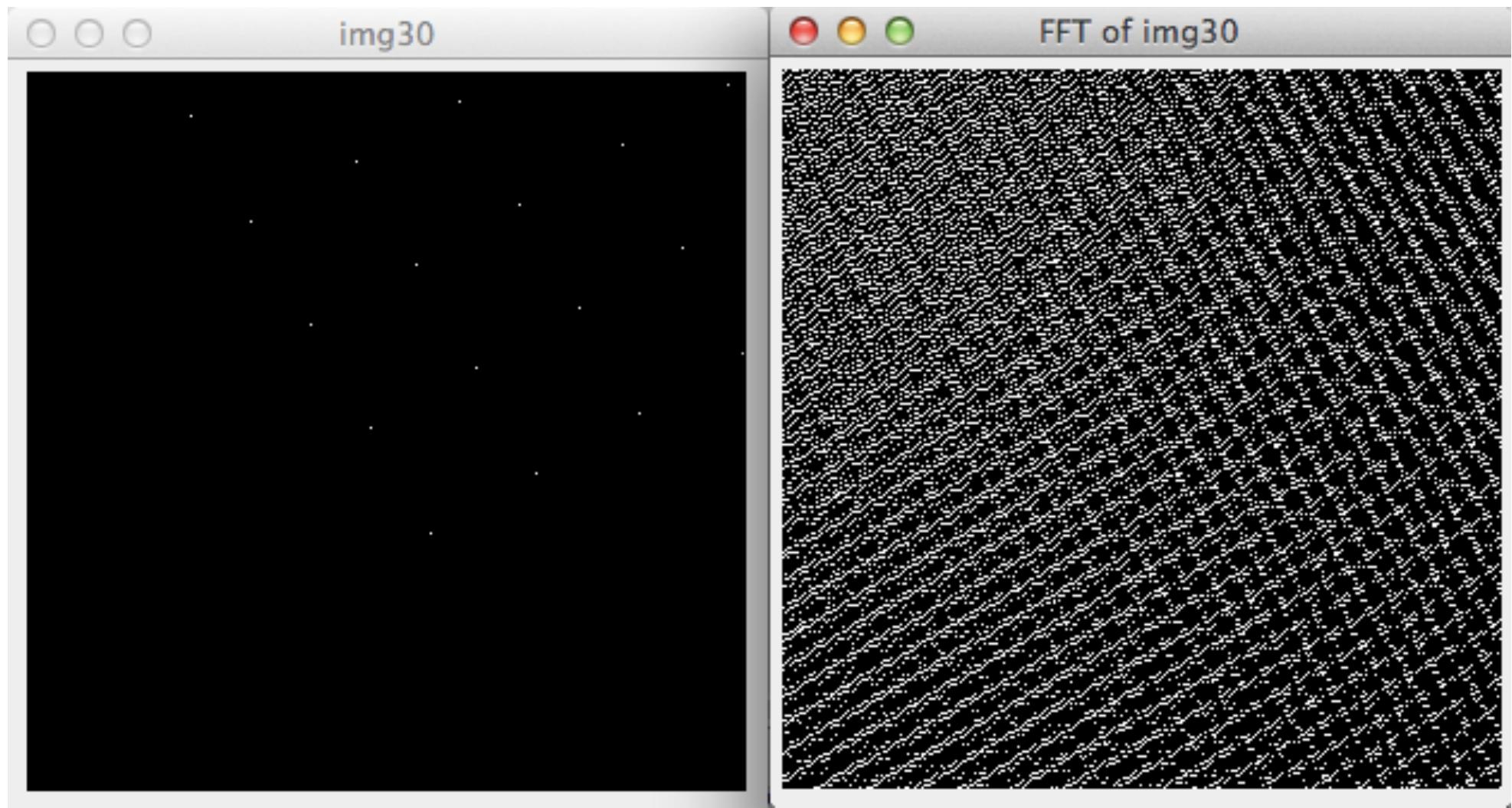
matched  
corners



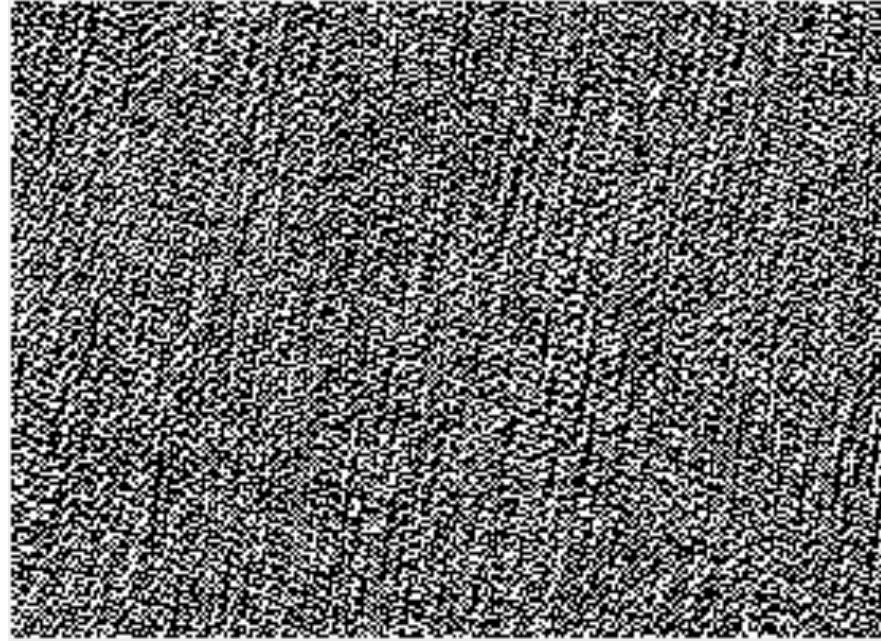
epipolar  
projection  
lines



FFT of a grid of points

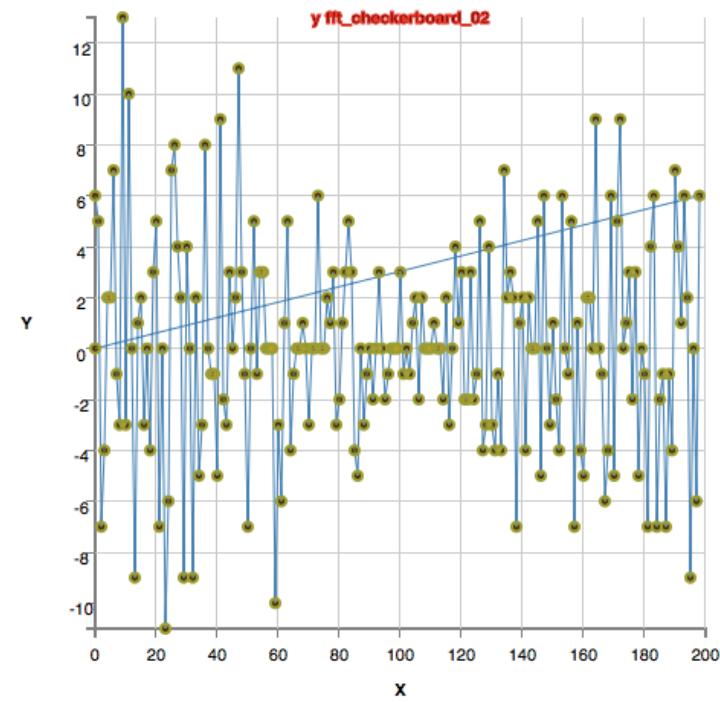
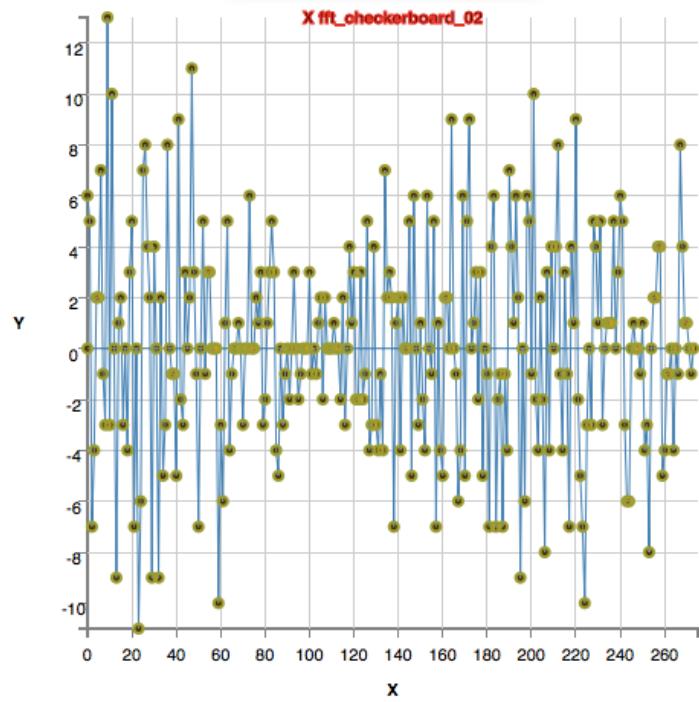
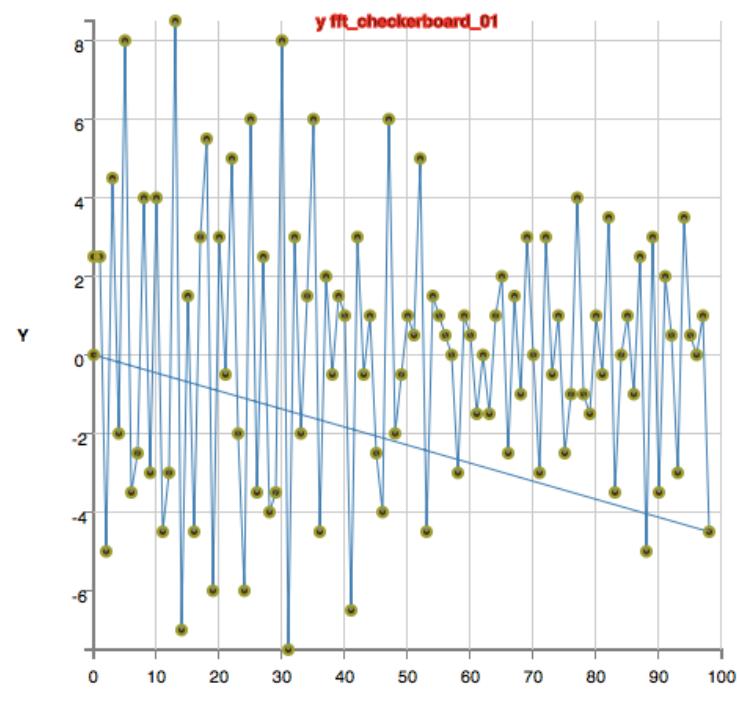
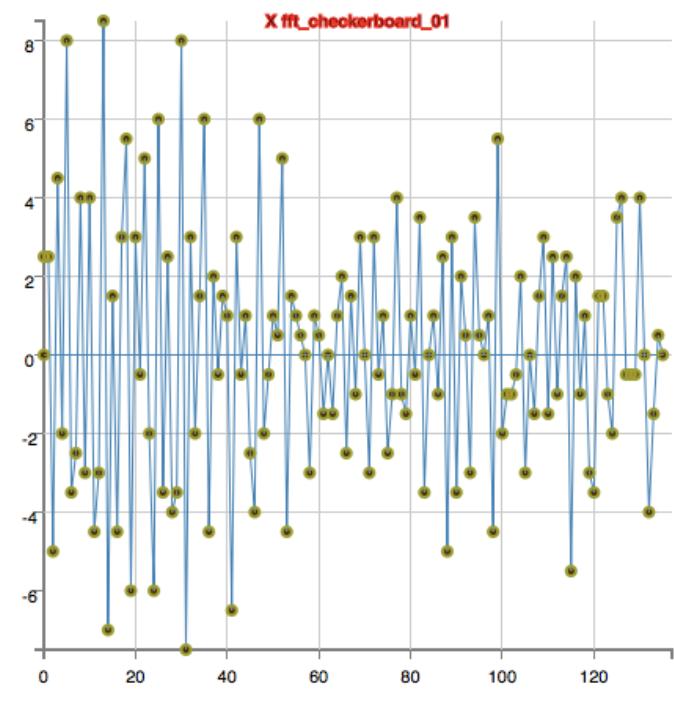


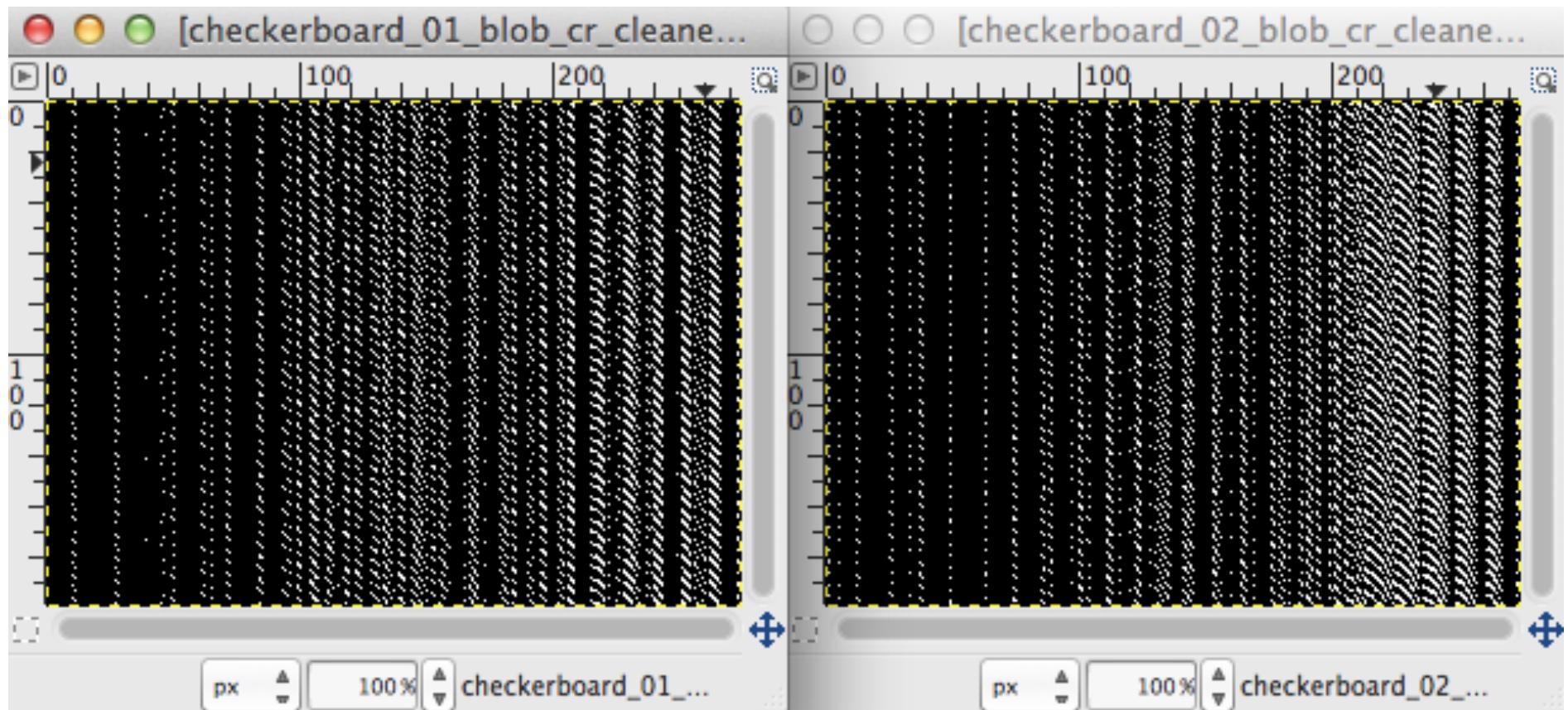
FFT of a same grid of points rotated by 30 degrees



FFT of the corner regions

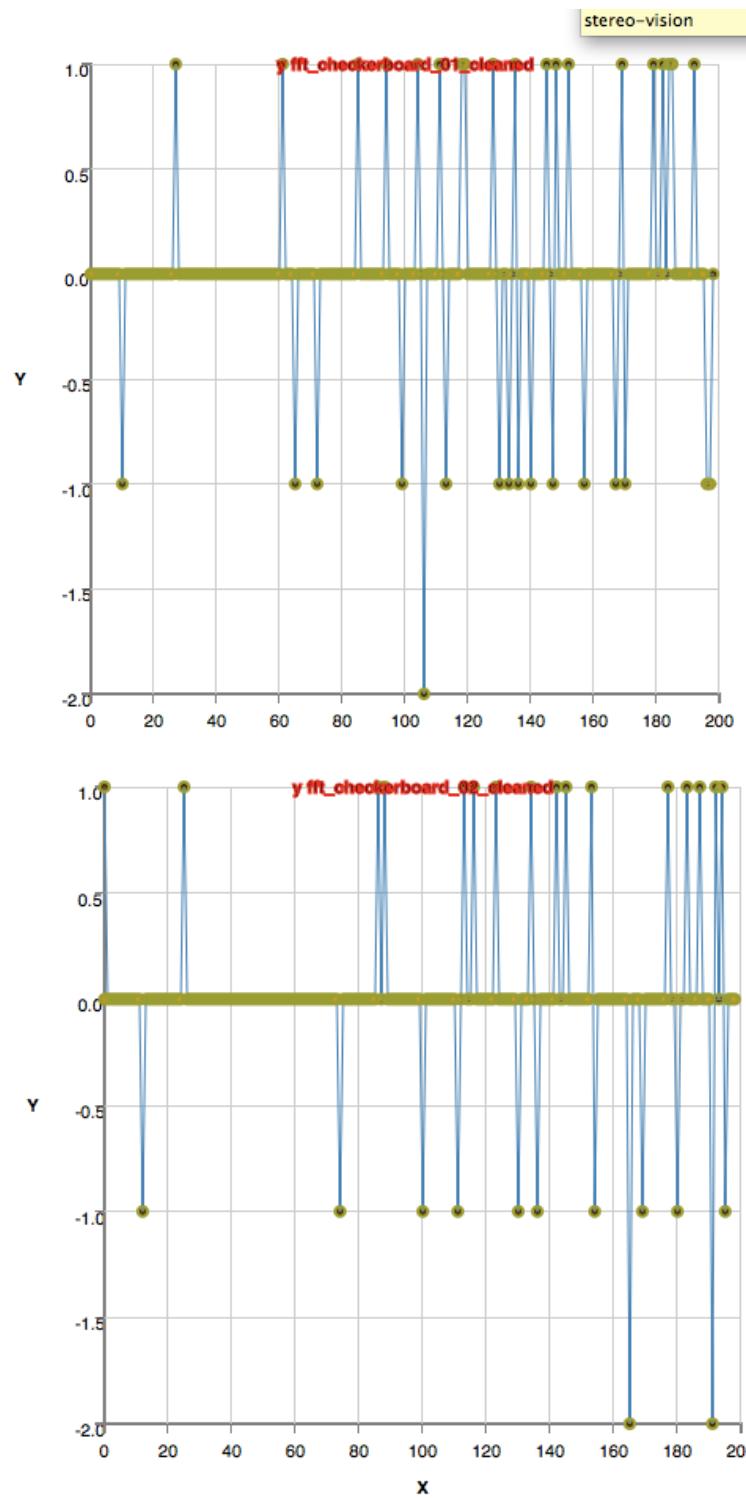
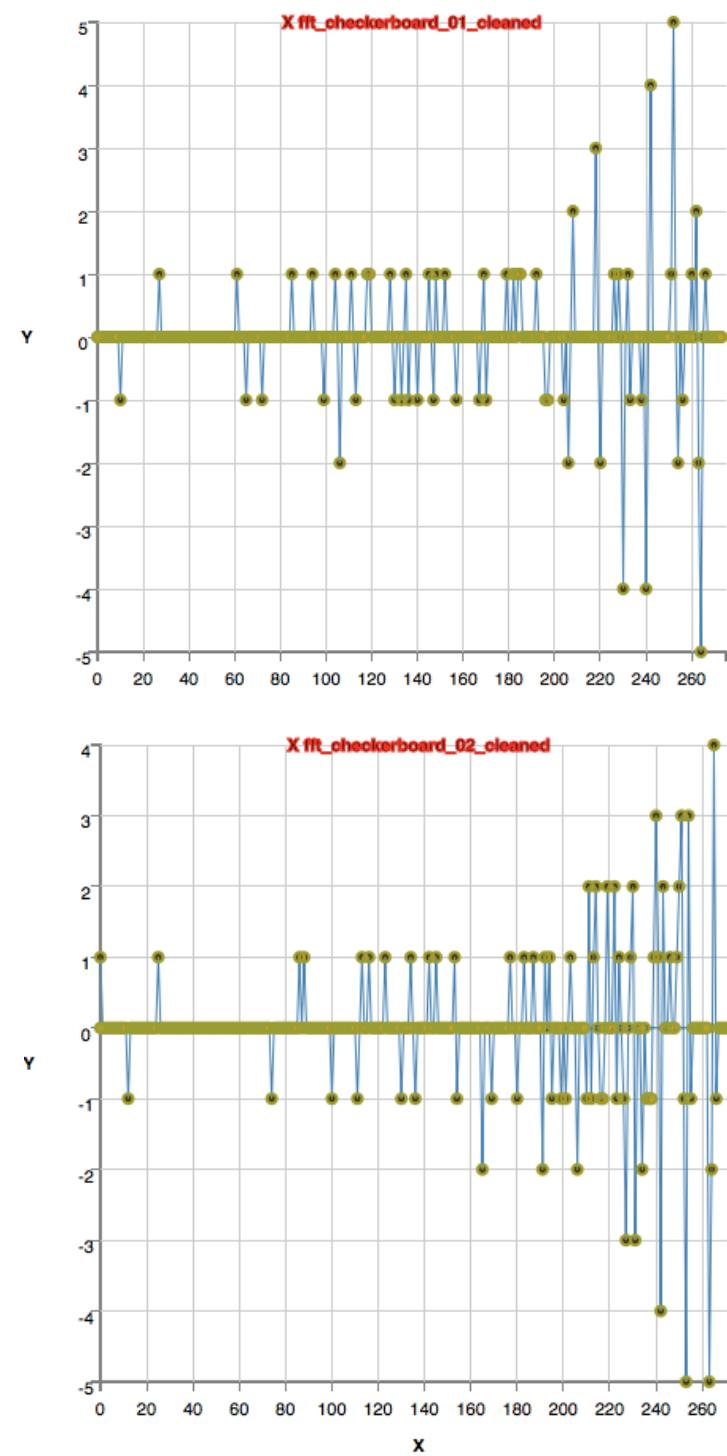
# FFT of the corner regions



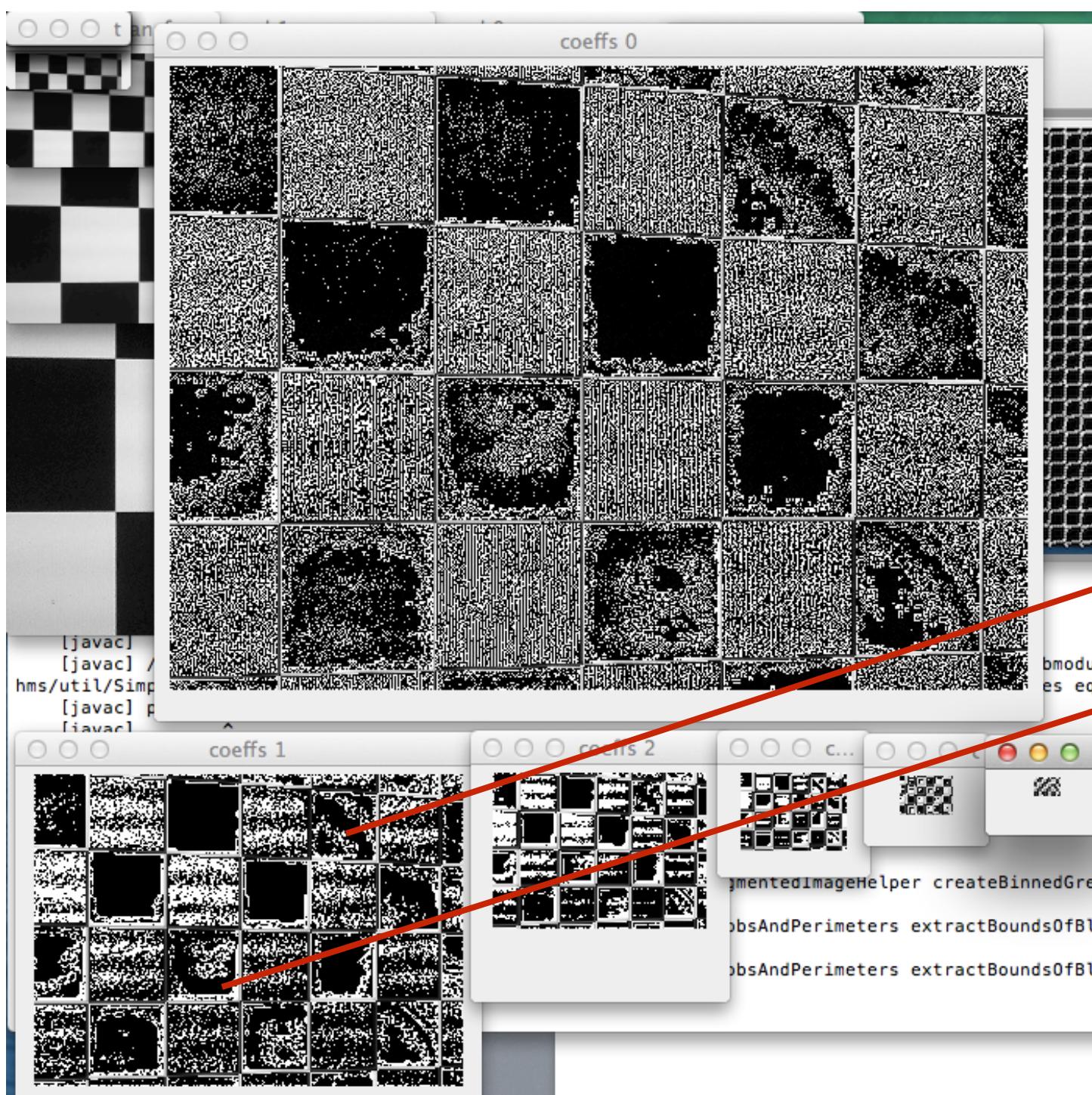


FFT of corner regions (artifact corners removed)

FFT of  
corner  
regions  
(artifacts  
removed)



a look at  
median transform  
coefficients  
for potential use  
in segmentation

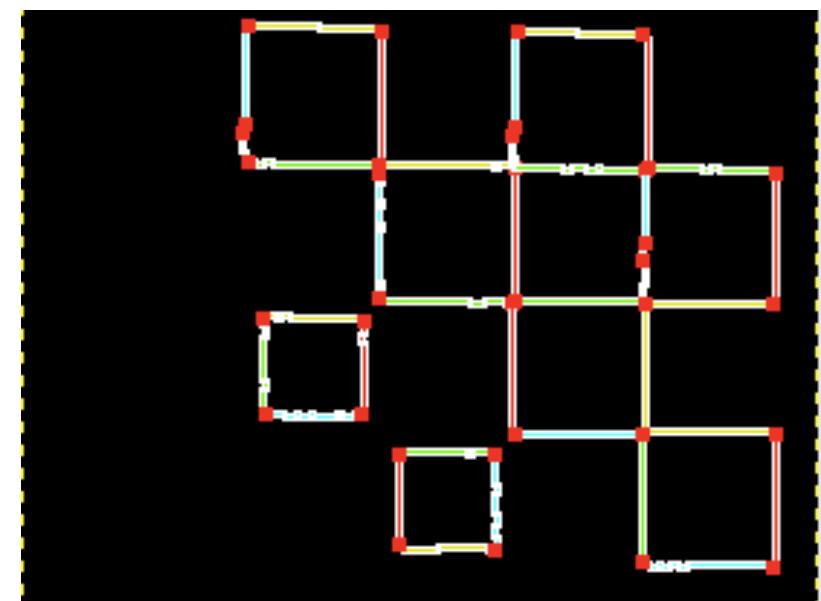
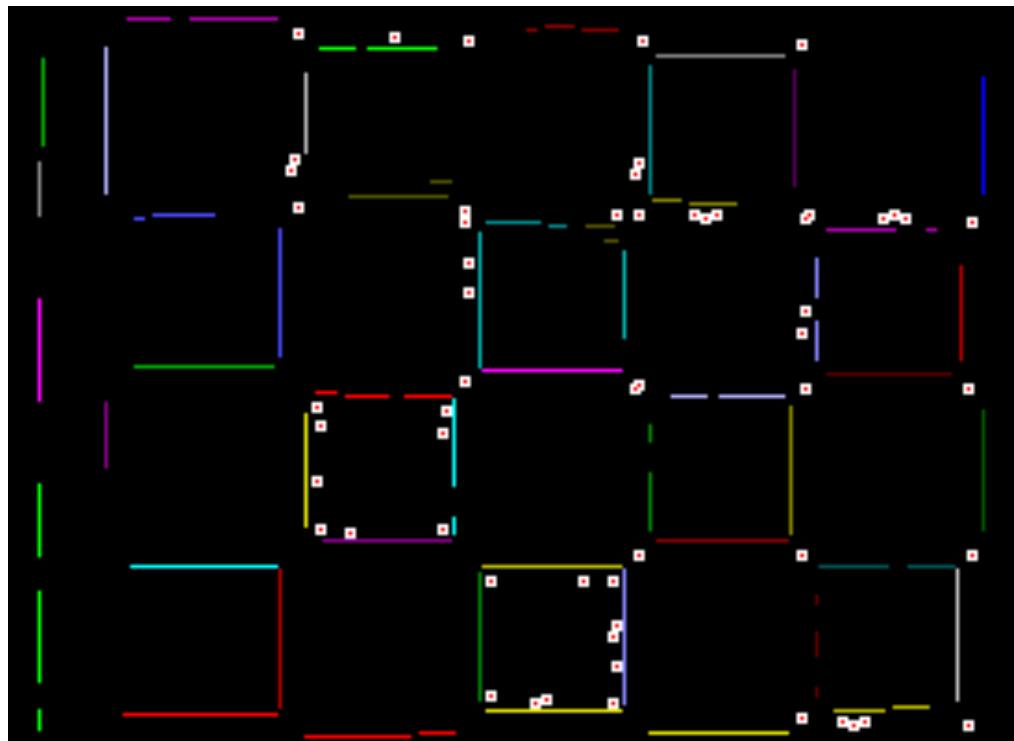


The a trous wavelet transform finest coefficient is now what the blob segmentation is based upon.

The checkerboard test image is an example showing corners due to horiz or vert line artifacts.

The Hough transform for lines looks useful in finding straight line segments of a minimum length and removing inner points that aren't junctions.

corners over plotted on the lines found from Hough Transform for lines w/ a small theta and small radius tolerance shown on the left (tolerances for considering adjacent points part of same line). On the right is the results of removing corners internal to line segments.

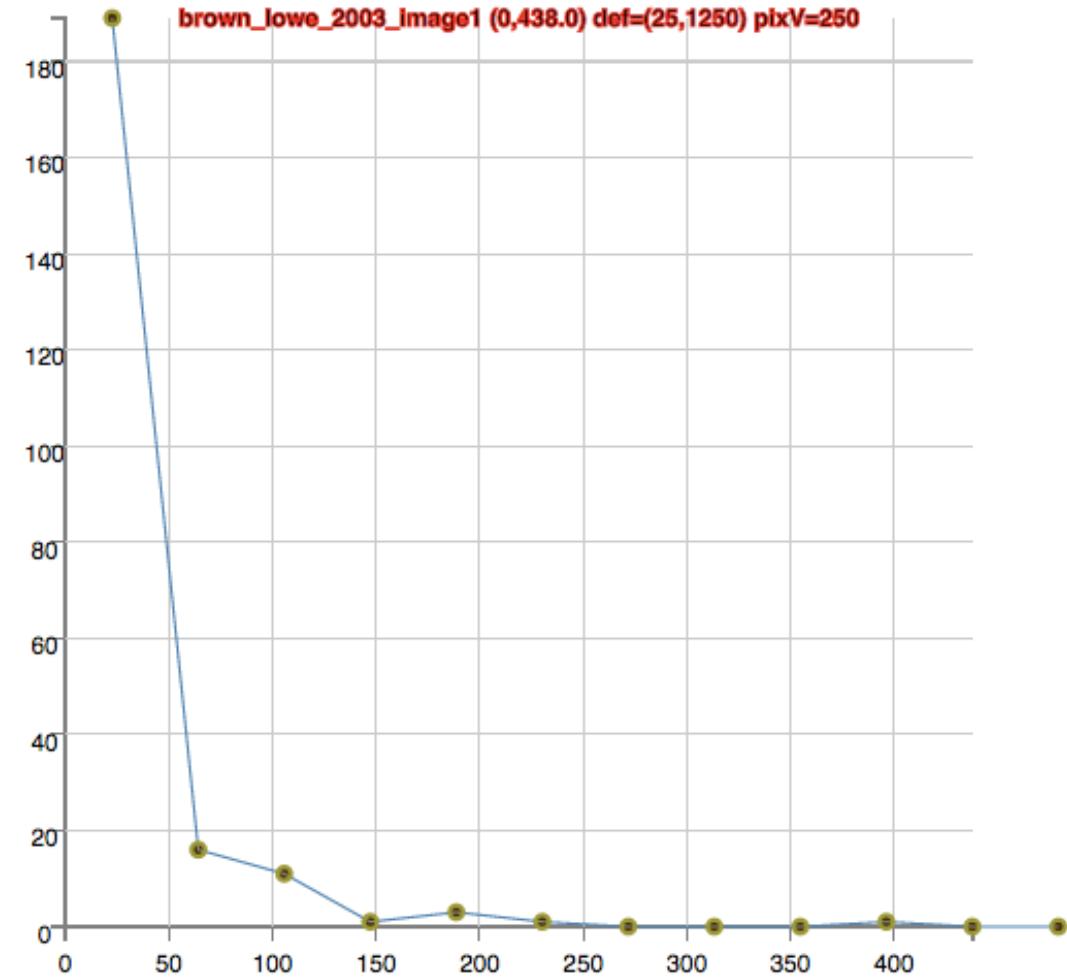
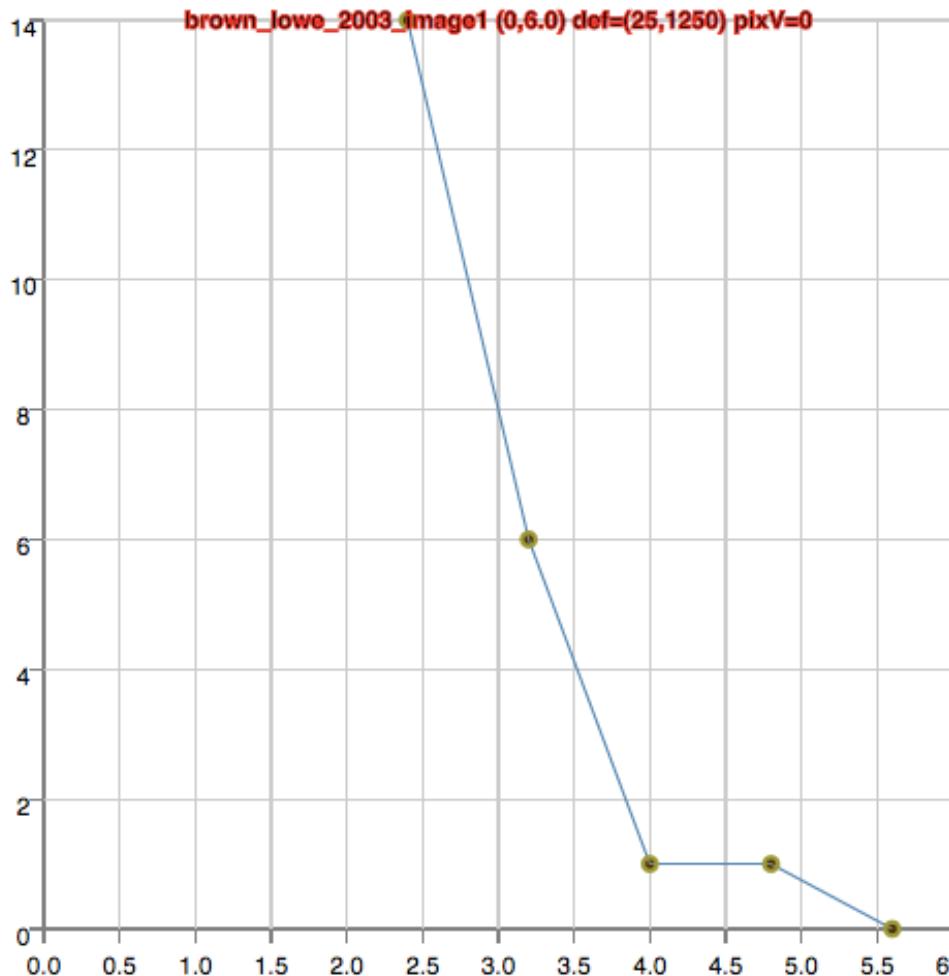


Note: improvements in blob extraction (and hence population of corners before this stage) is next.

Blobs are contiguous pixels of same intensity in the segmented images.

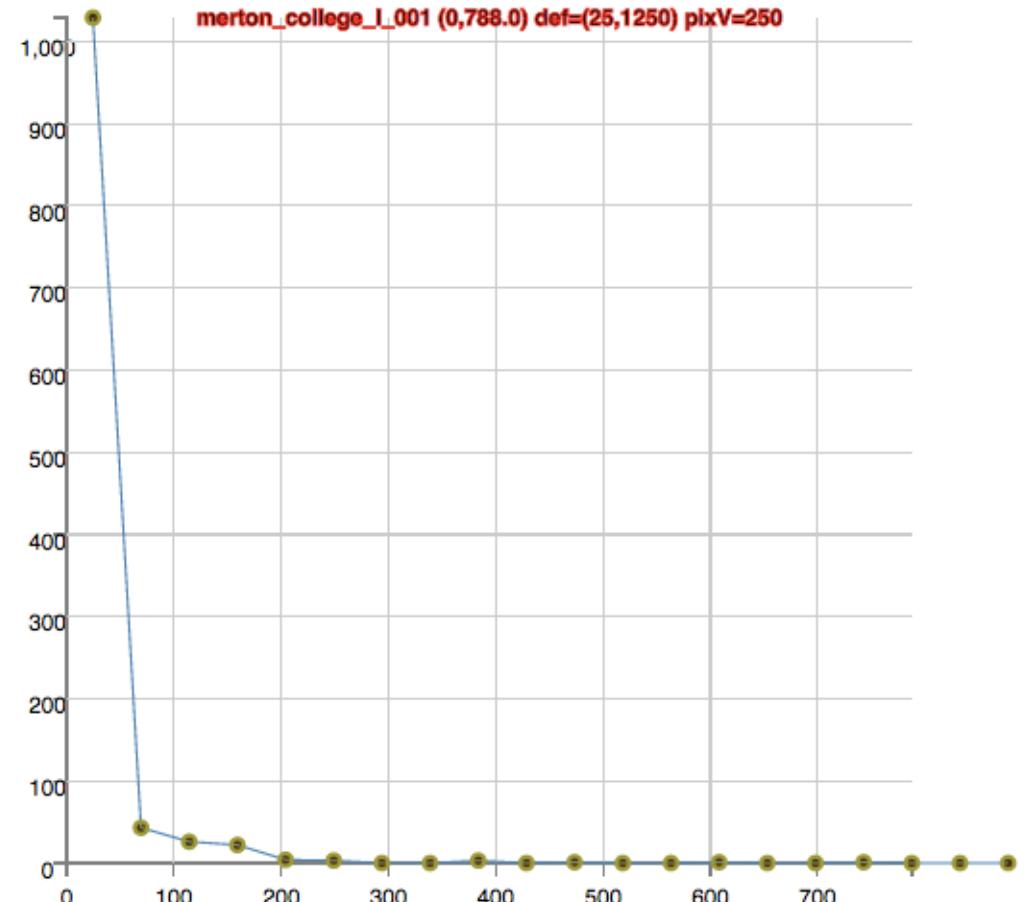
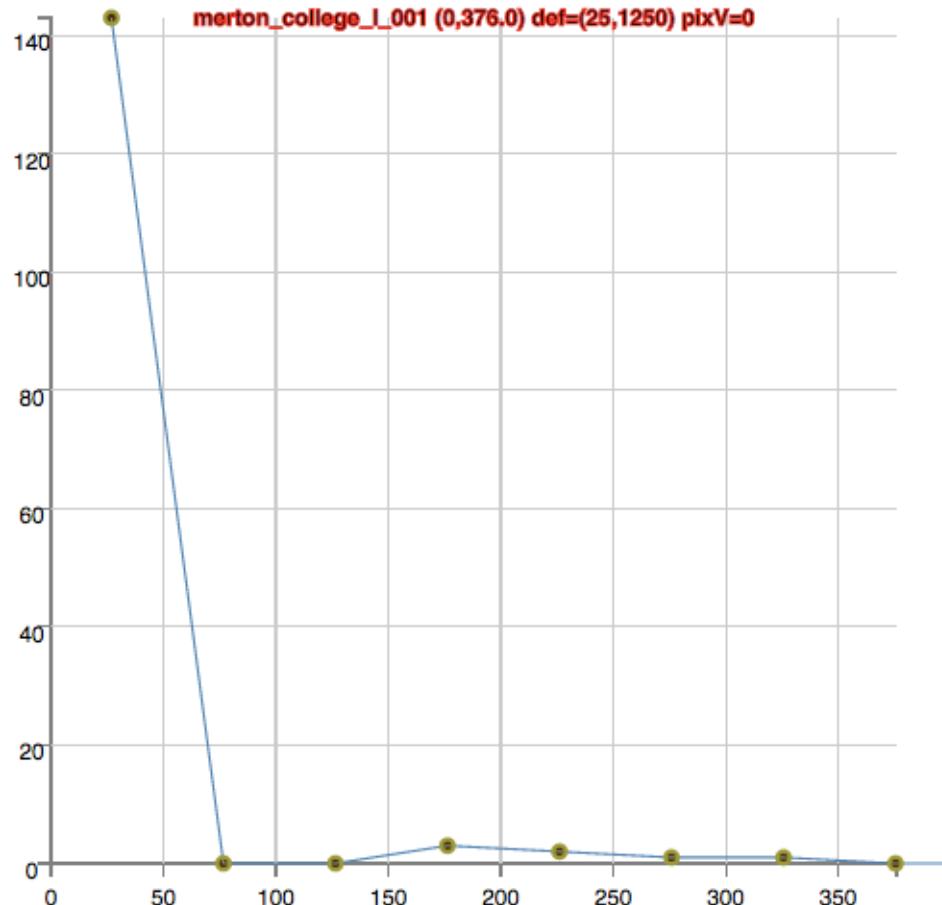
Currently, a fixed range of blob sizes is used and those ranges were determined from an empirical look at several test images.

Those ranges need to be adjusted by what is present in the images already, so looking at histograms here. Plotting histograms of blob sizes and a label of the default previously used.



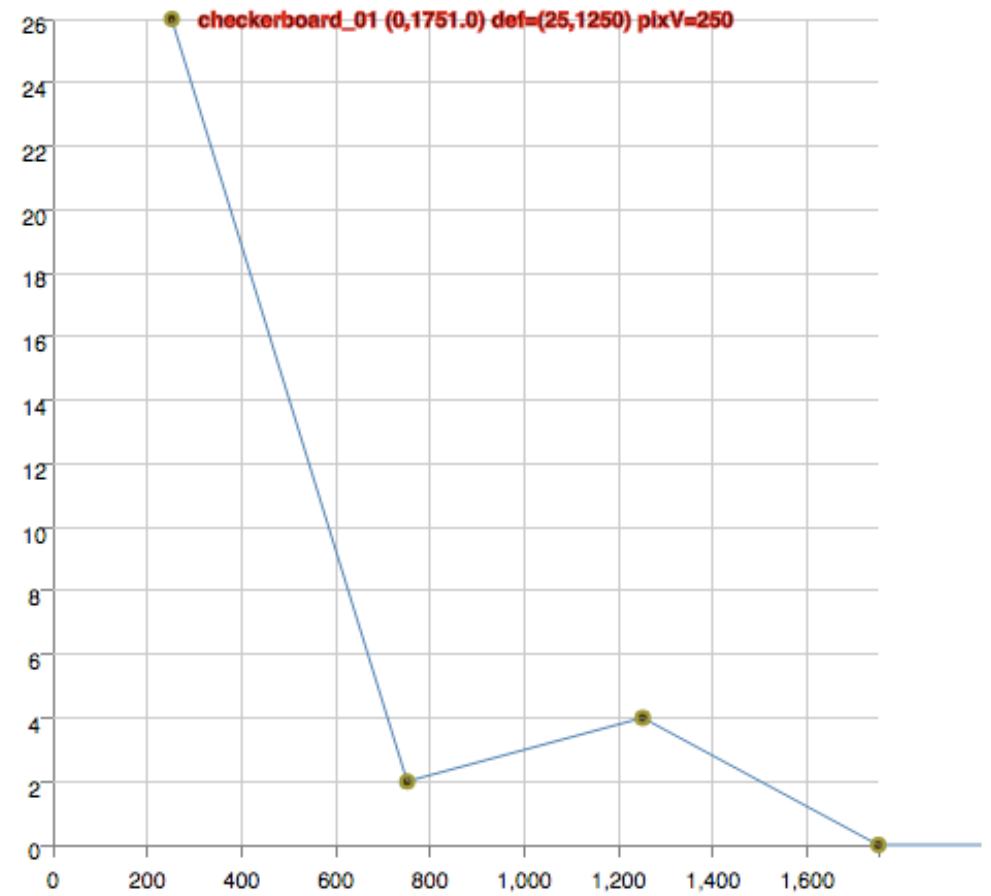
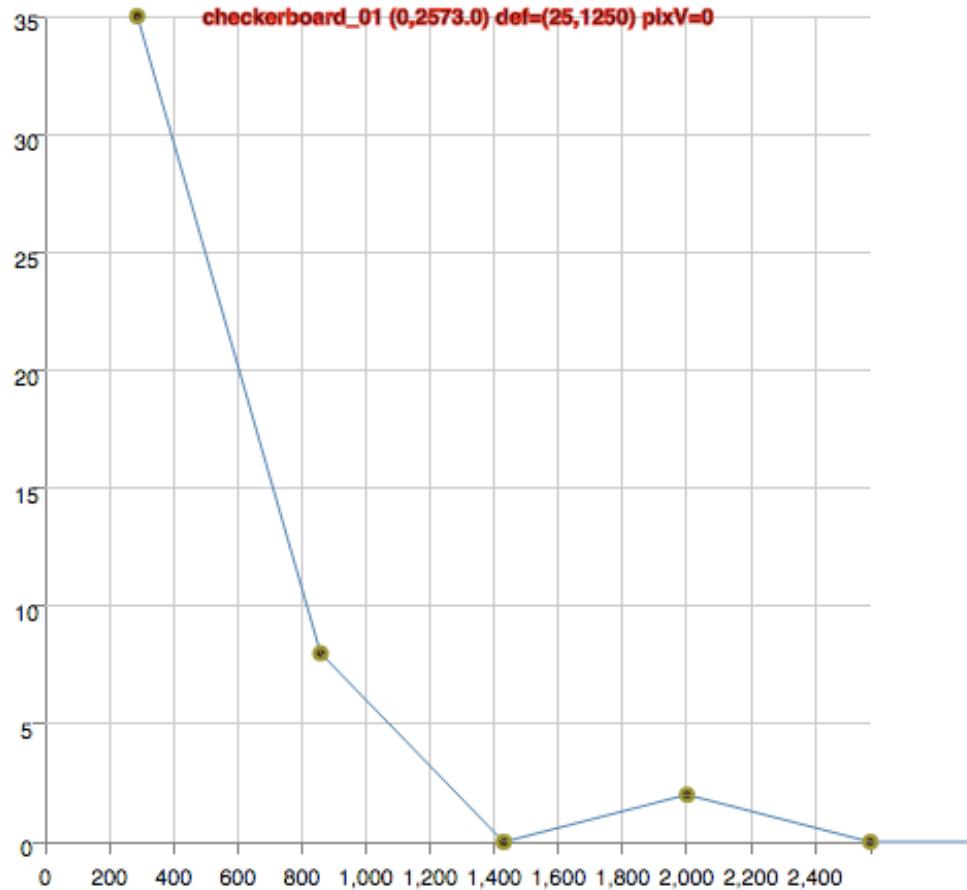
default range of 25:1250 is fine here

Plotting histograms of blob sizes and a label of the default previously used.



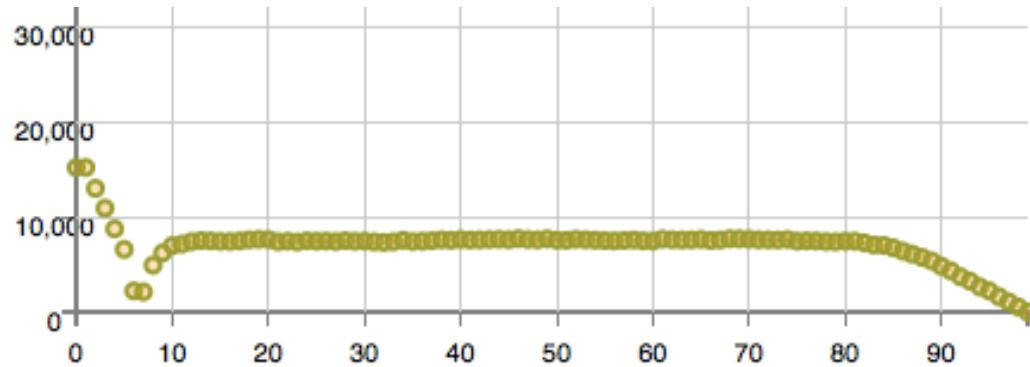
default range of 25:1250 is fine here

Plotting histograms of blob sizes and a label of the default previously used.

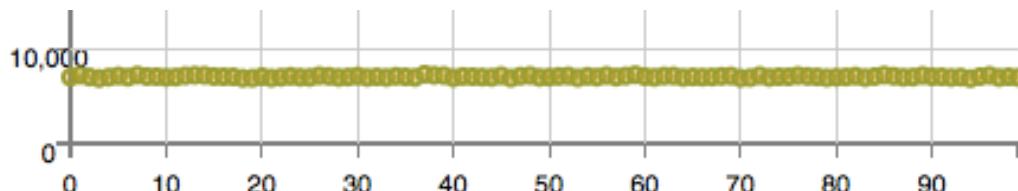
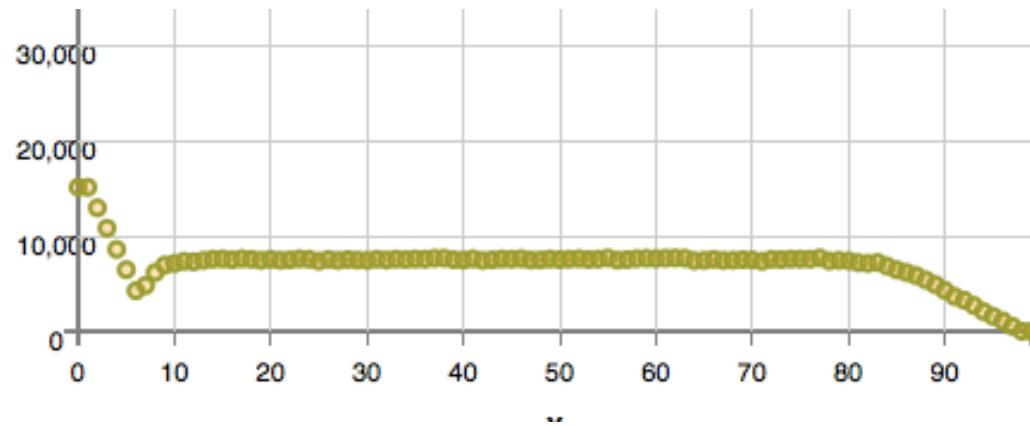


default range of 25:1250 does not include many blobs.  
The unbinned size range of 50:2500 would be much better.

On a different subject, looking at improving the random sampling for the RANSACSolver. Created a k bits random number selector for a distribution of indexes and created the same for a distribution of indexes altered to replicate the number by its value. Plotted is a look at the numbers drawn from 100,000 throws using these methods.



The new methods have a few biases.



randomly choosing k numbers, in contrast to the above, shows a very uniform distribution