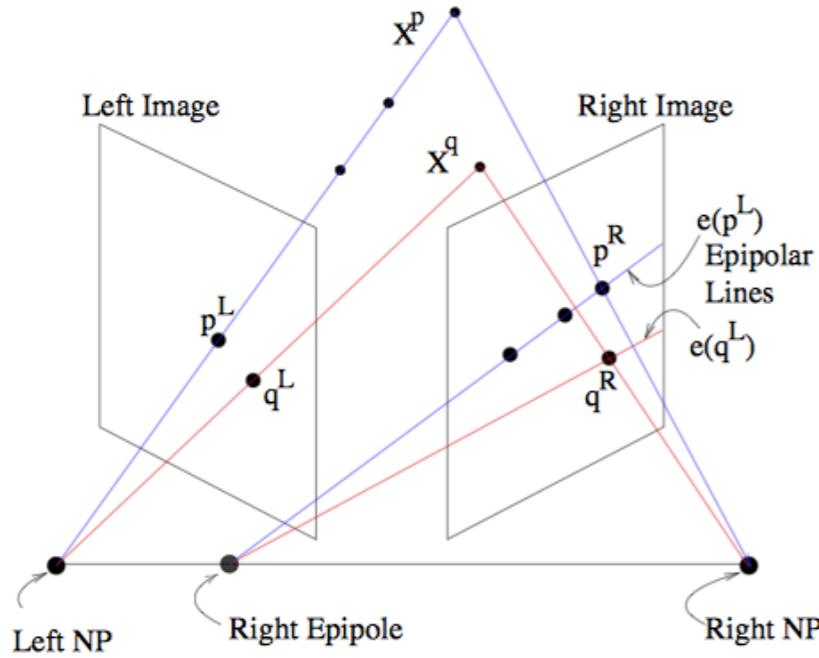


## Projection, stereo and panoramic images

$X^p$  and  $X^q$  are the physical location of objects.

NP are the nadir points of the cameras.



The projection of the left nadir, NP is seen as a the right epipole with w.r.t. the right image. The epipoles may or may not be within the border of the images.

The projection of  $X^p$  to left nadir, NP is seen as an epipole line in the right image. If more than one epipole line is present in the right image, they converge at the right epipole (which might not be within the boundaries of the image).

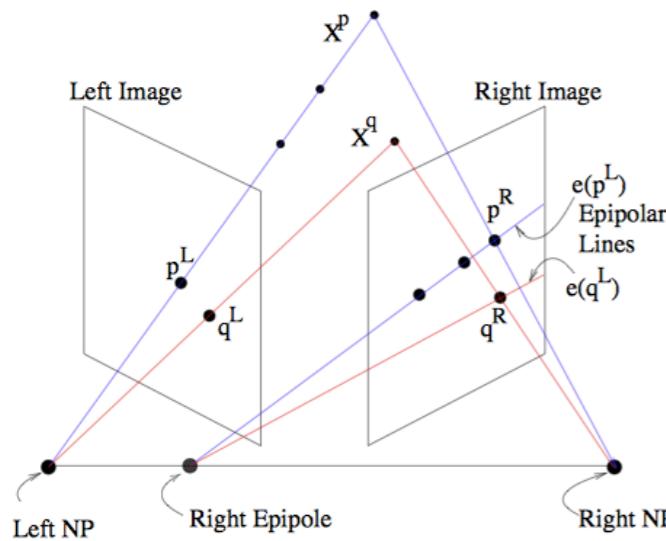
Once the points in the left image,  $X_L$  are matched with points in the right image,  $X_R$ , if there are at least 7 points, one can determine the “bifocal tensor”,  
a.k.a. “fundamental matrix, relating the points in the 2 images using a  
3x3 matrix of rank 2.

<http://www.cs.toronto.edu/~jepson/csc420/notes/epiPolarGeom.pdf>  
(note, the image is posted on an educational site and copied here without following up on permissions. Any further use of the image should follow up on the origins and permissions.)

$(X_L)^T * F * X_R = 0$  for any pair of points in the images.

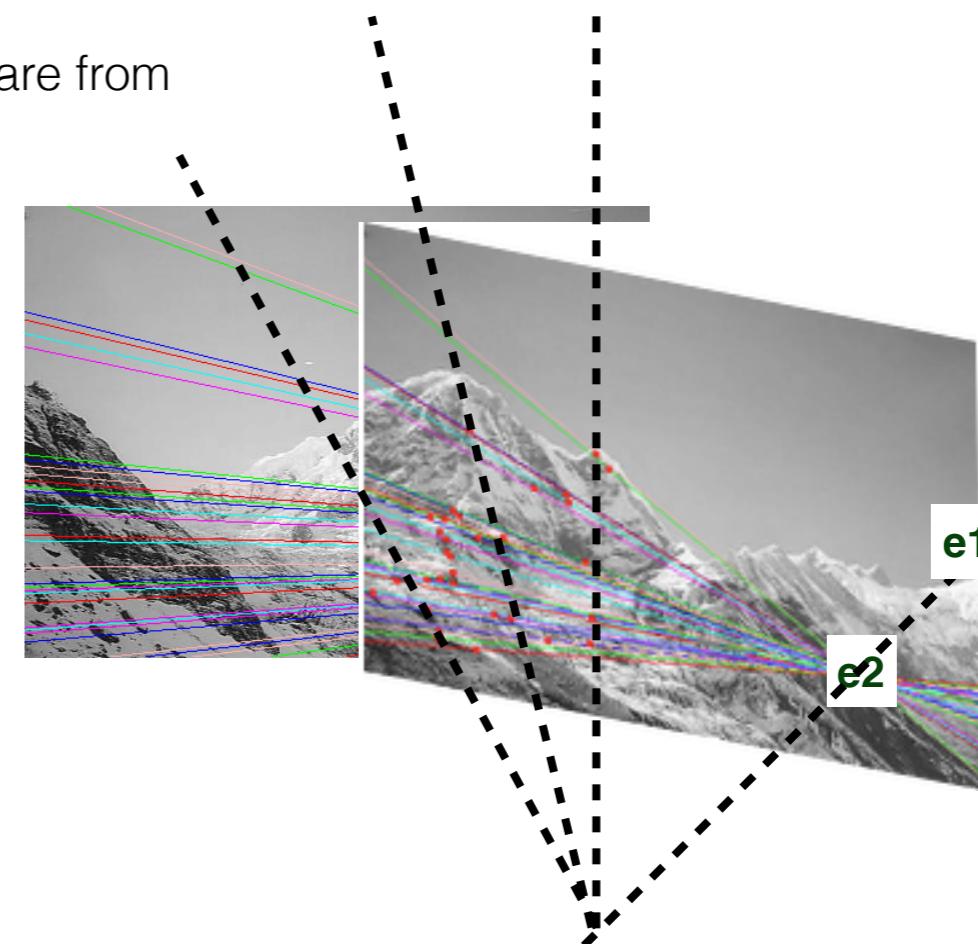
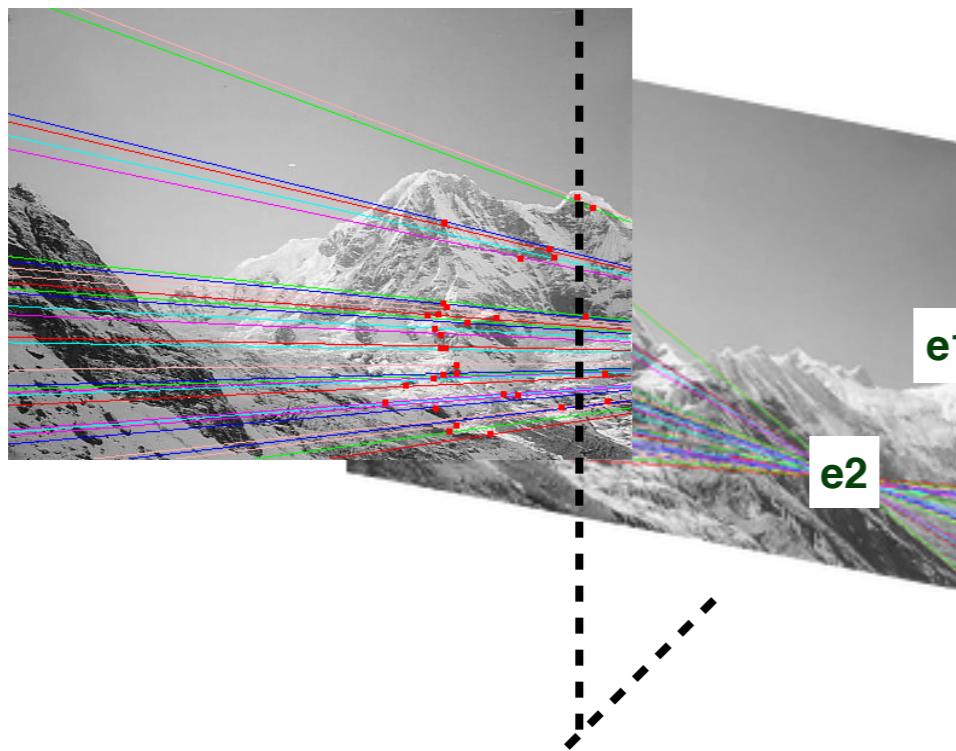
Note: the “Essential matrix” is a matrix used if the camera details are known. The “bifocal tensor”, a.k.a. “fundamental matrix” does not need camera details.

## Projection, stereo and panoramic images



<http://www.cs.toronto.edu/~jepson/csc420/notes/epiPolarGeom.pdf>  
(note, the image is posted on an educational site and copied here without following up on permissions. Any further use of the image should follow up on the origins and permissions.)

**panoramic** images here are from  
Brown & Lowe 2003

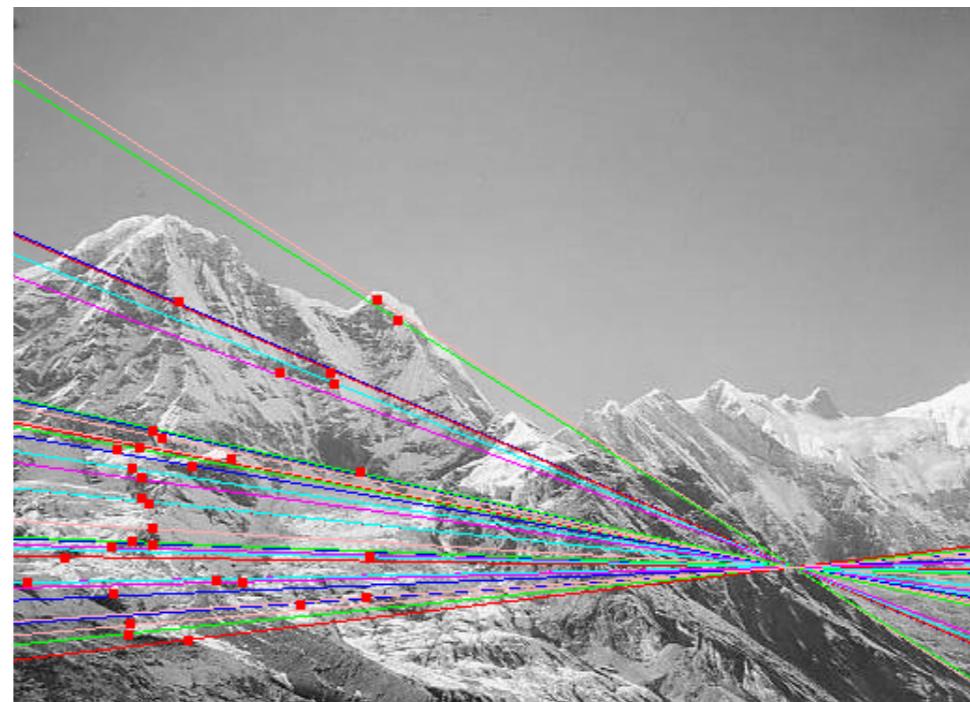
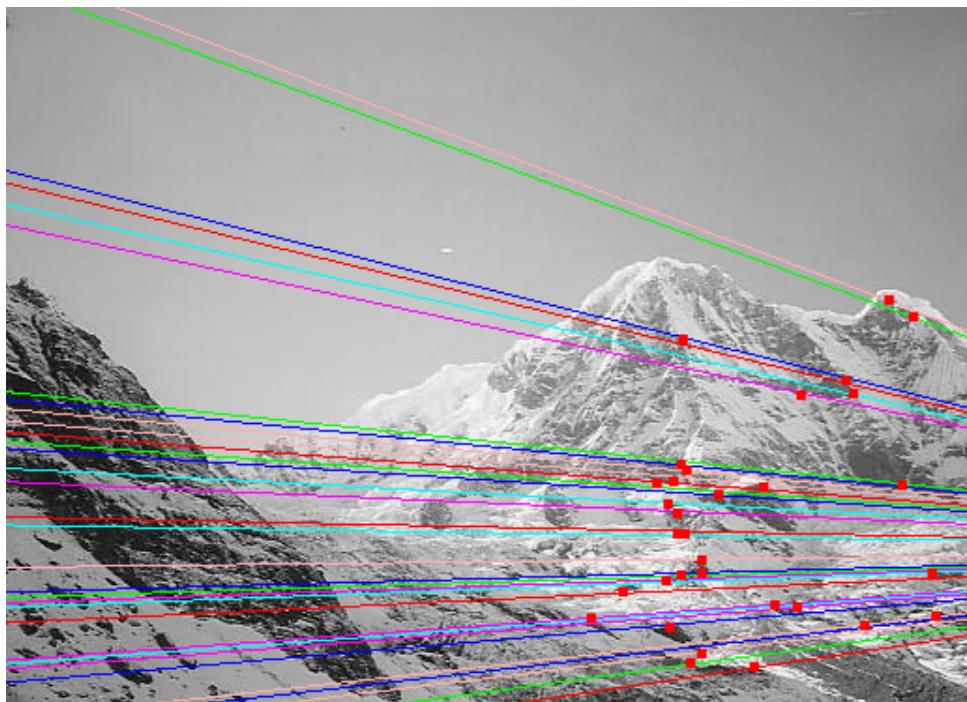


same camera objectives (nadir), but  
different orientation for the 2 images  
(that is, rotated around the same nadir)

## Point Correspondence

Need to create list of matched points between the images in order to solve for the epipolar projection

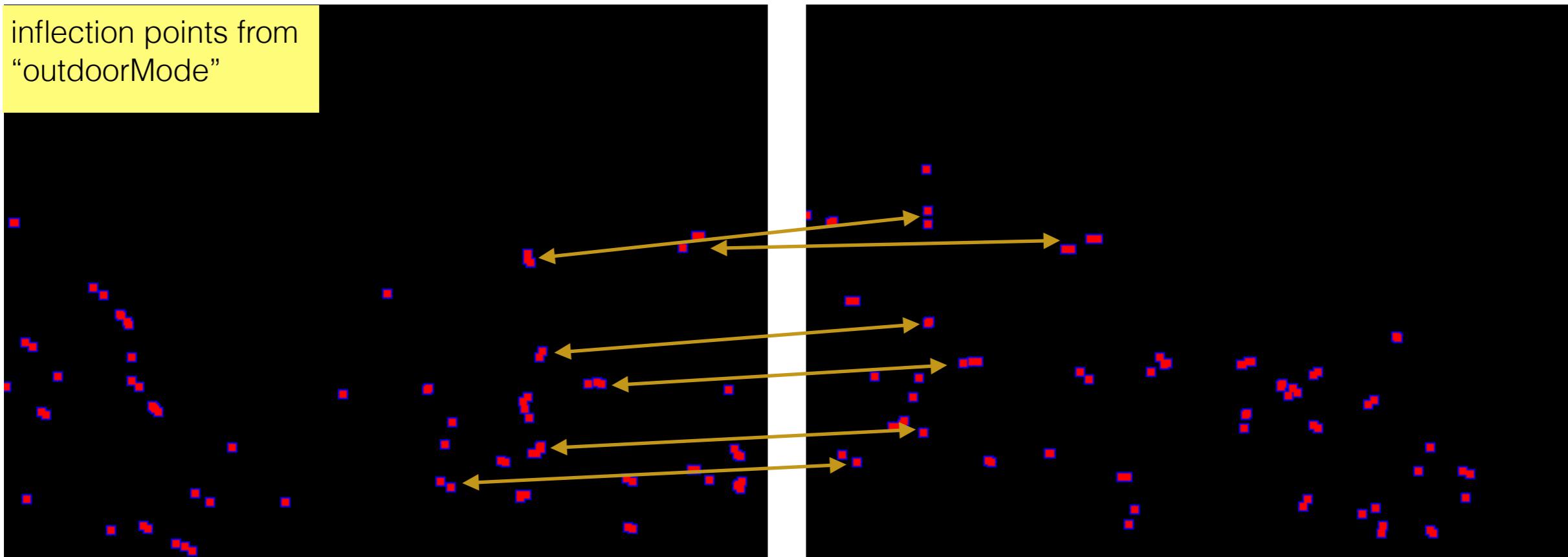
manually making a point list from the corners from the edge extractor used with “outdoor mode”:



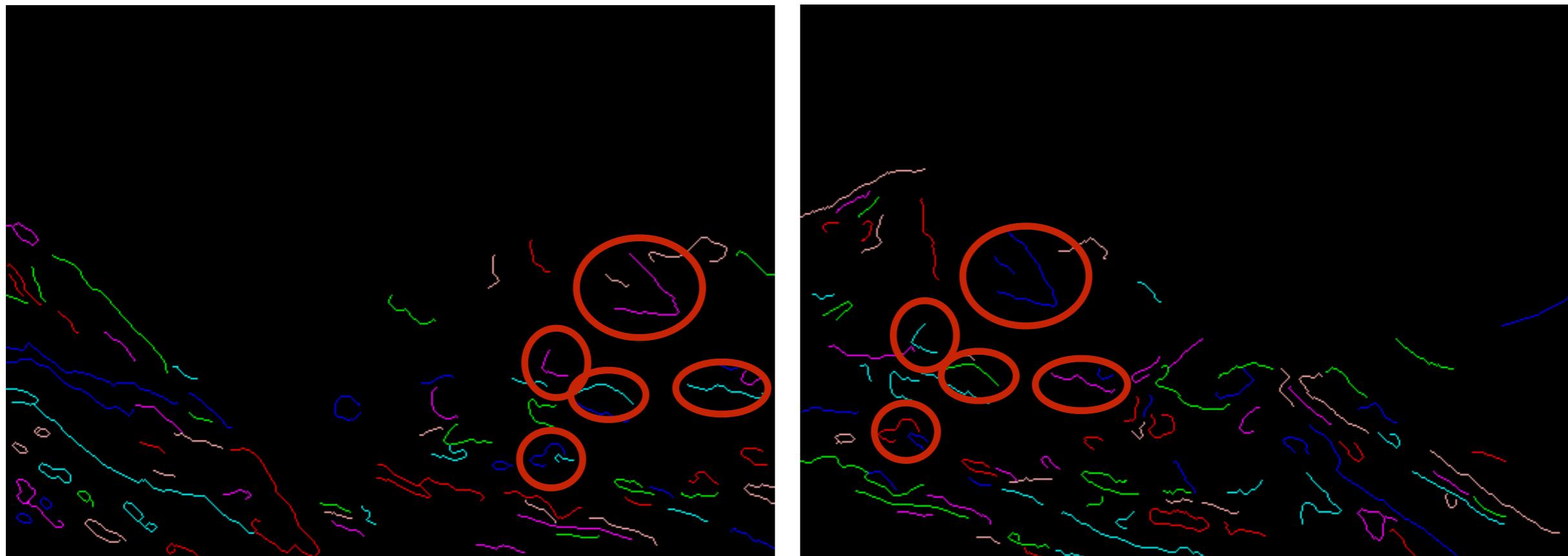
stereo projection fit to 32 points already known to match shows what the epipolar projections should be when the corner find + corner match + stereo projection solve are correctly automated.

**nMatched=32  
avgDist=0.281  
stDev=0.508**

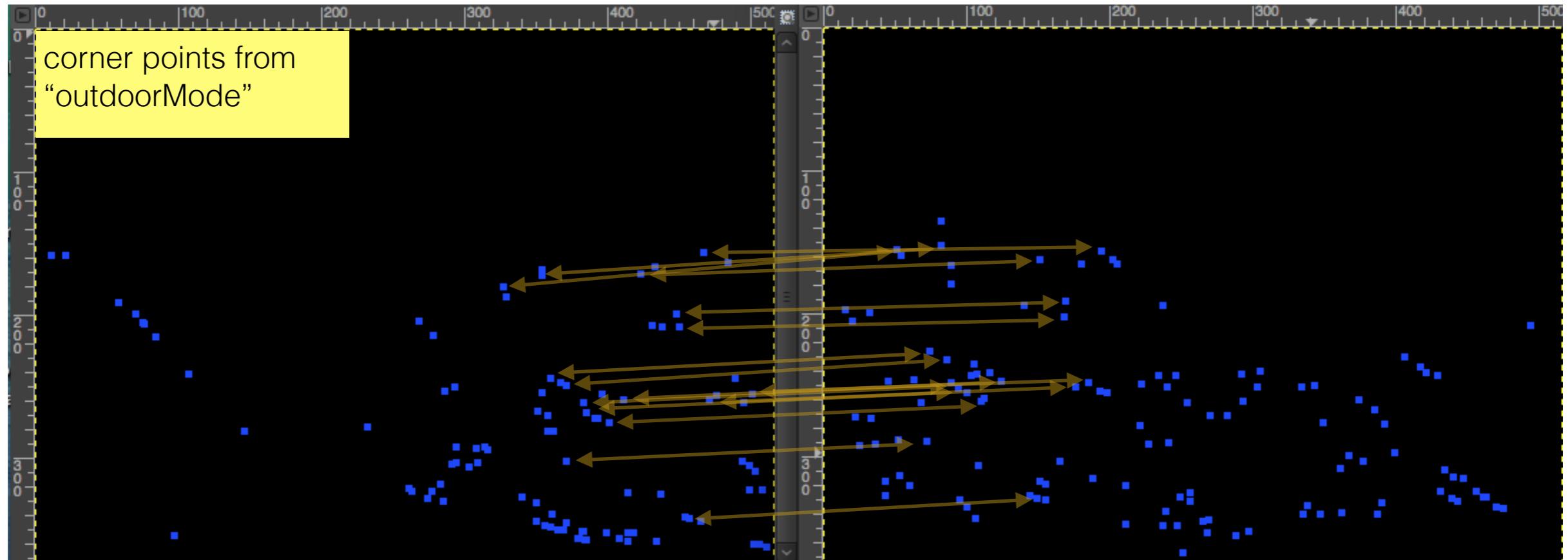
The Brown & Lowe 2003 images: point matching difficult because image intersection << difference



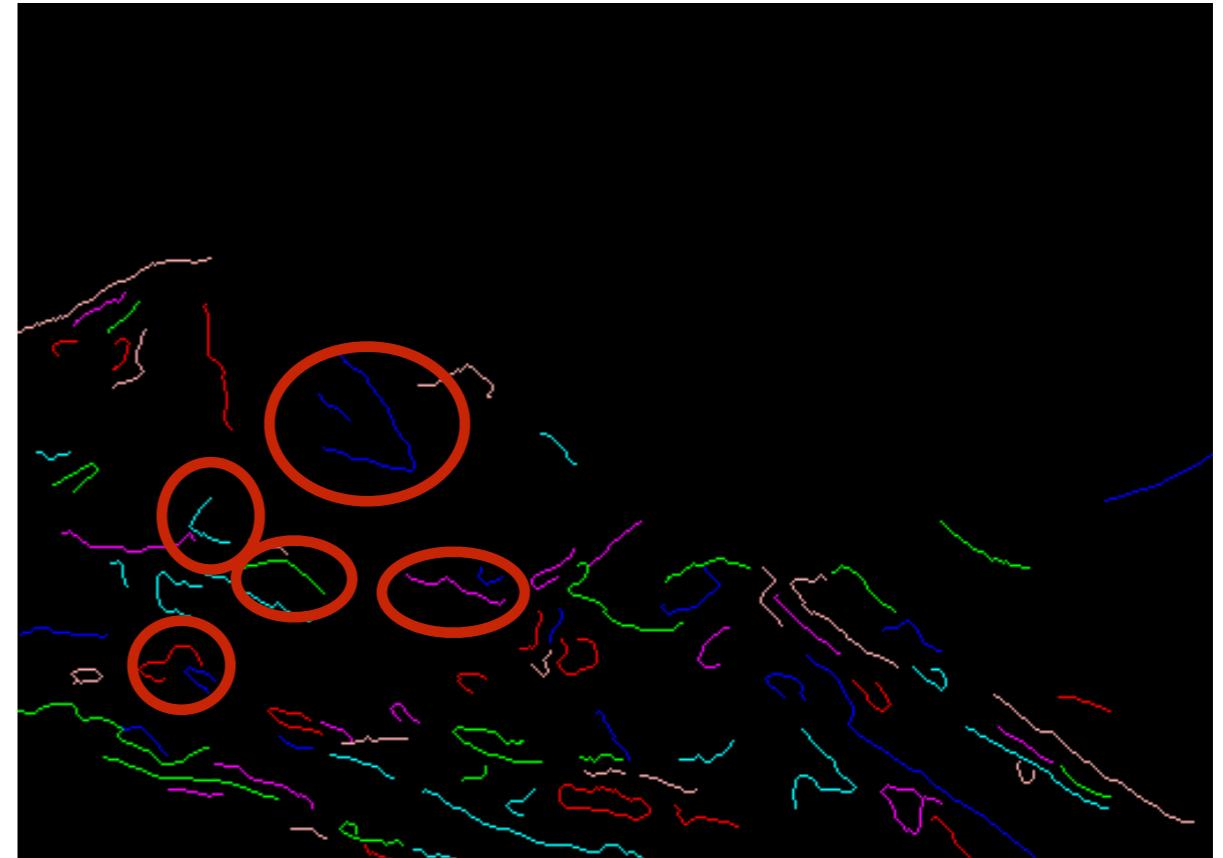
could consider distinct curves and their immediate neighbors, but that would be many more points:



The Brown & Lowe 2003 images: point matching difficult because image intersection < difference

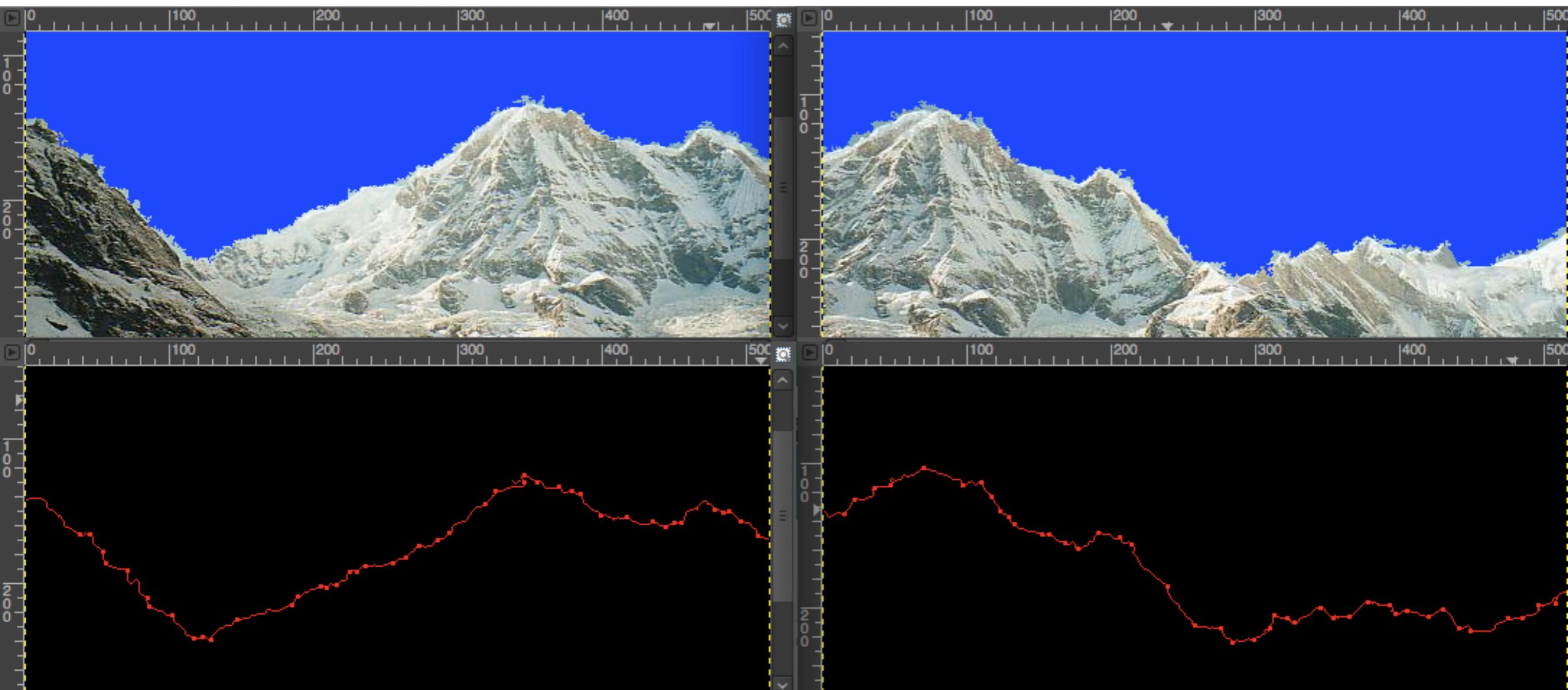


could consider distinct curves and their immediate neighbors, but that would be many more points:



For the outdoor images, can find the sky and create a sky mask and also create corners from just the skyline.  
(see skyline\_extraction.pdf)

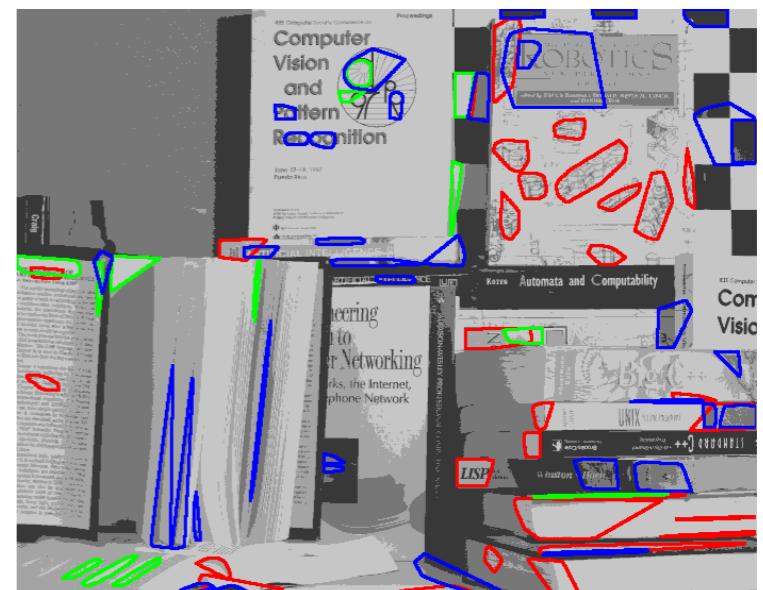
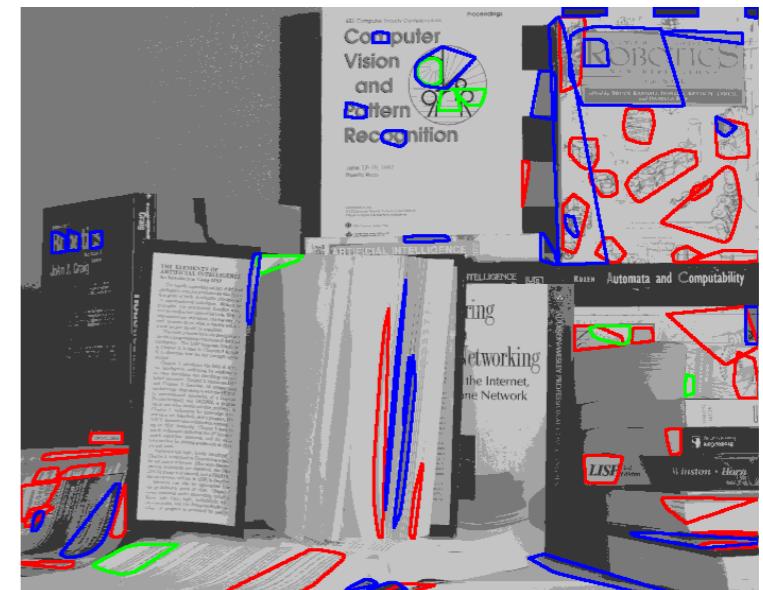
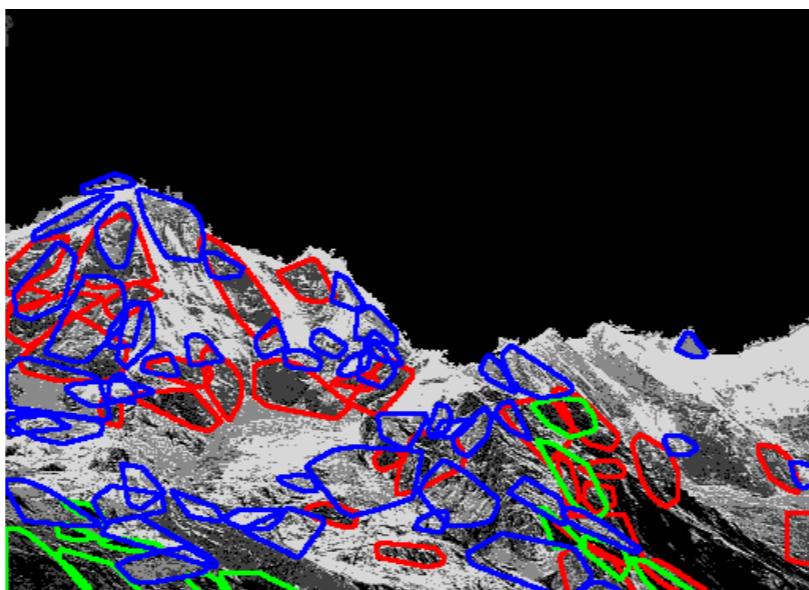
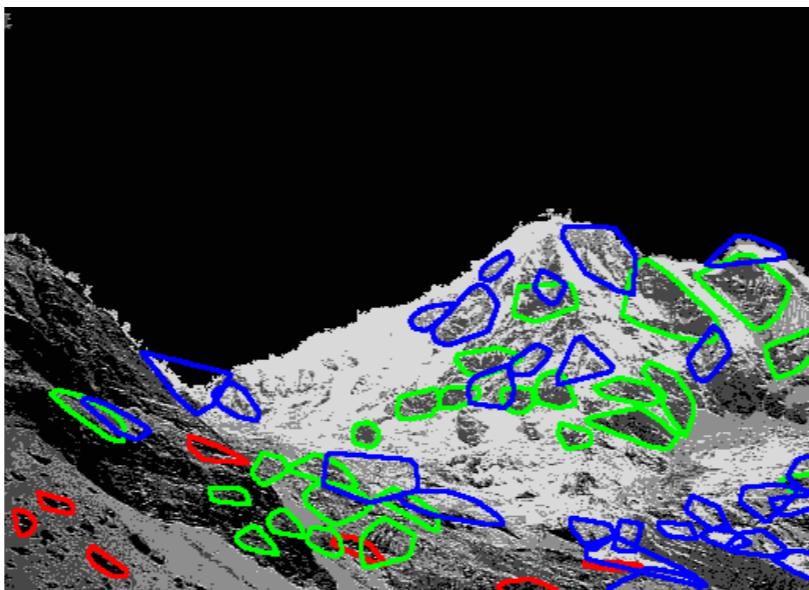
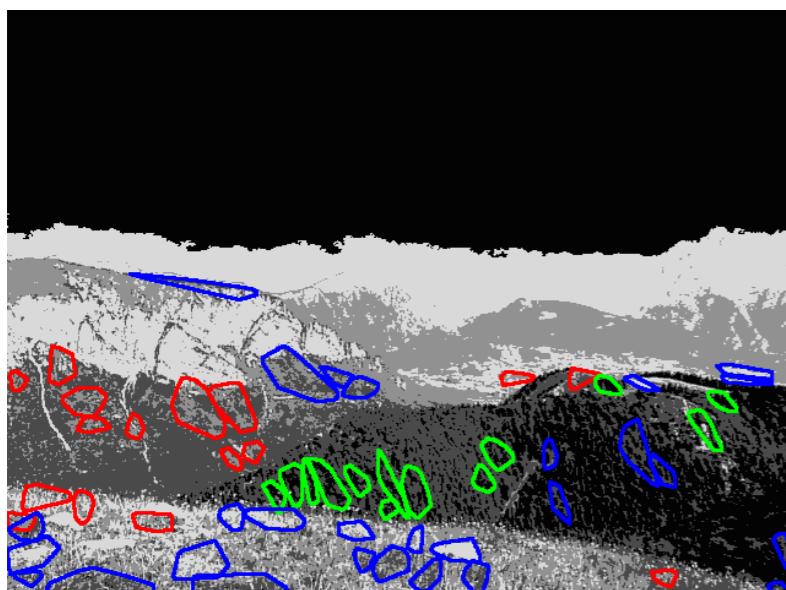
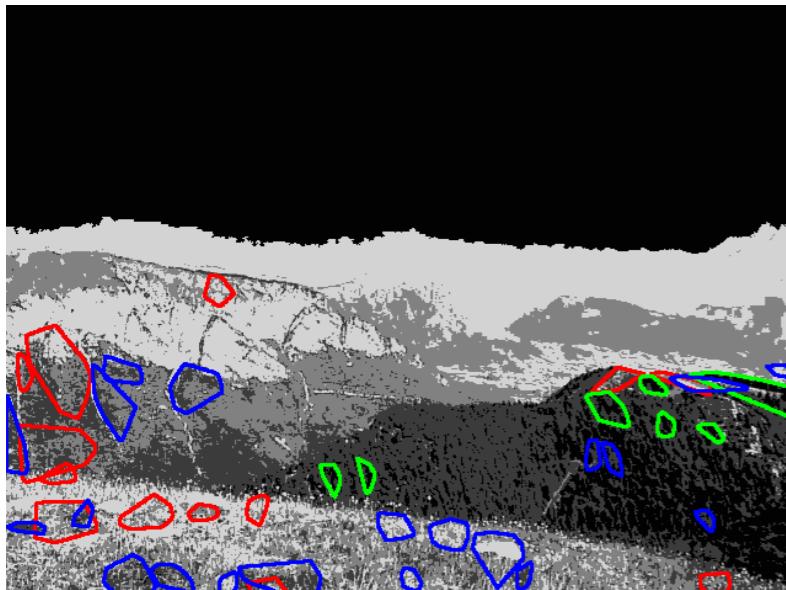
This sky mask helps to pre-process the image before feature finding and correspondence.



## Trying blob feature creation and matching methods in next slides.

- perform histogram equalization if mean is too far from median or the two images are too different for those params.
- color segmentation of **k=4** to reduce image to 4 bands of intensities.
- contiguous pixel group finder for each of the 4 bands using point limits of: smallestGroupLimit = **100**; largestGroupLimit = **1000**;

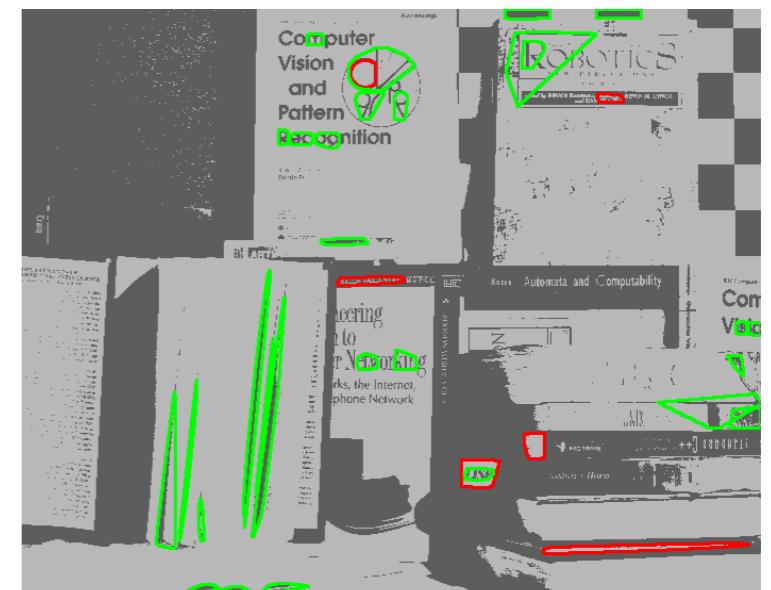
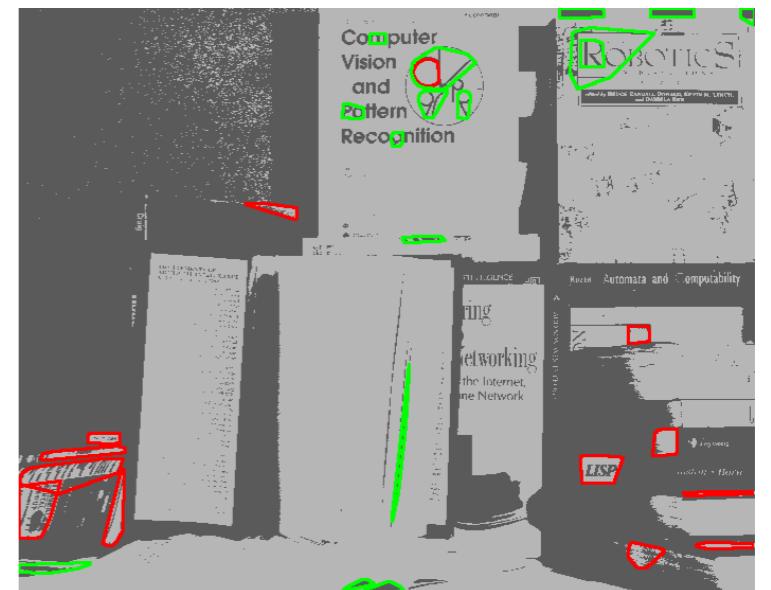
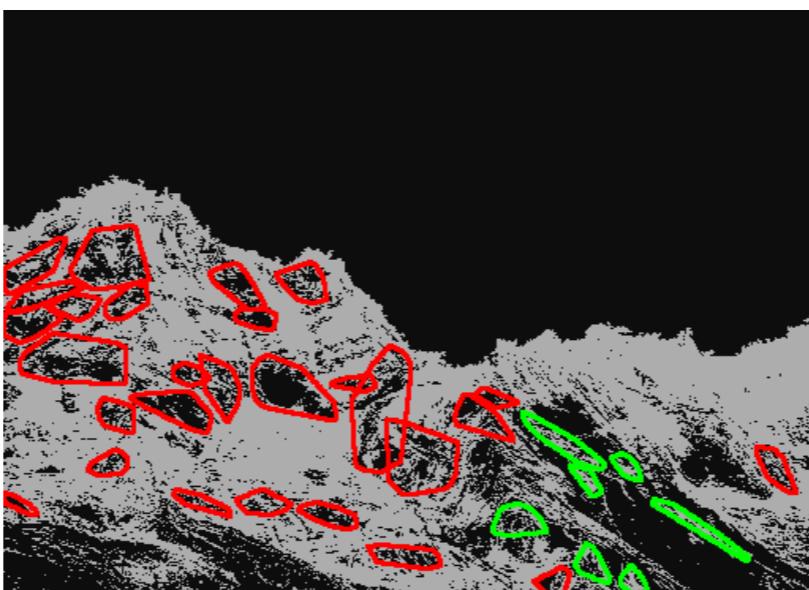
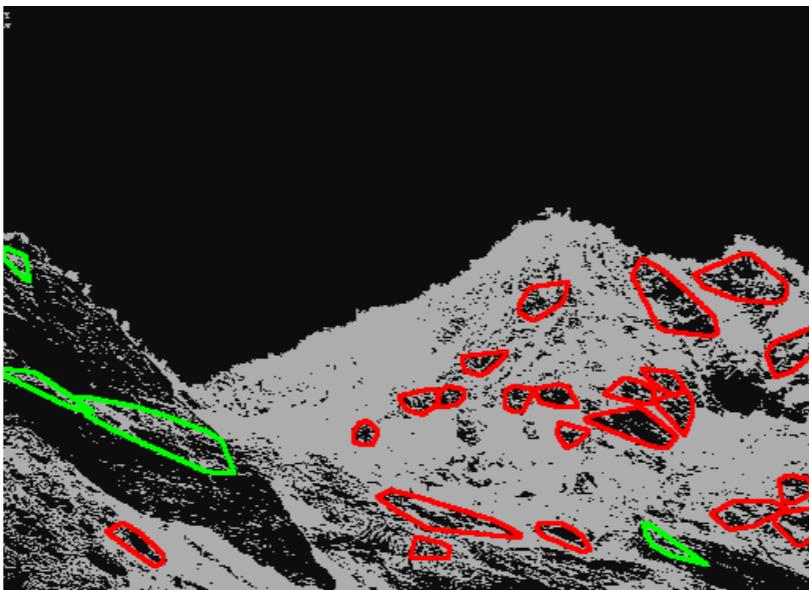
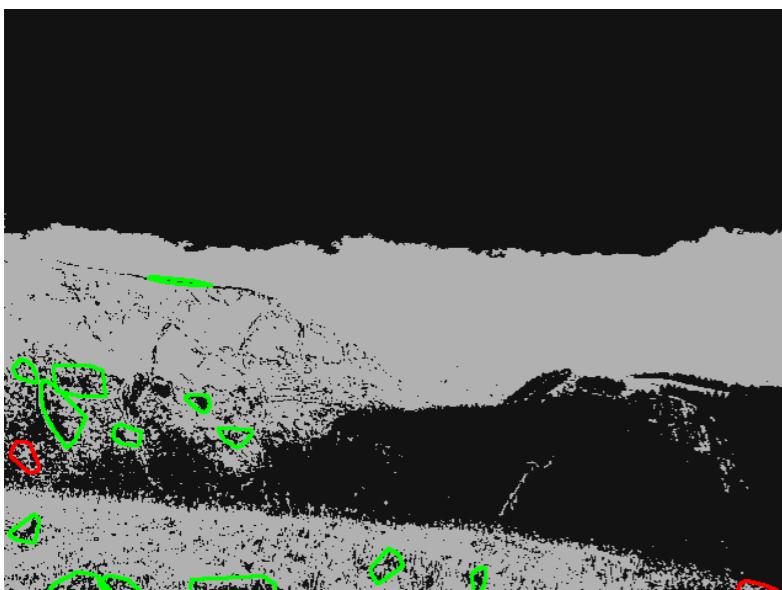
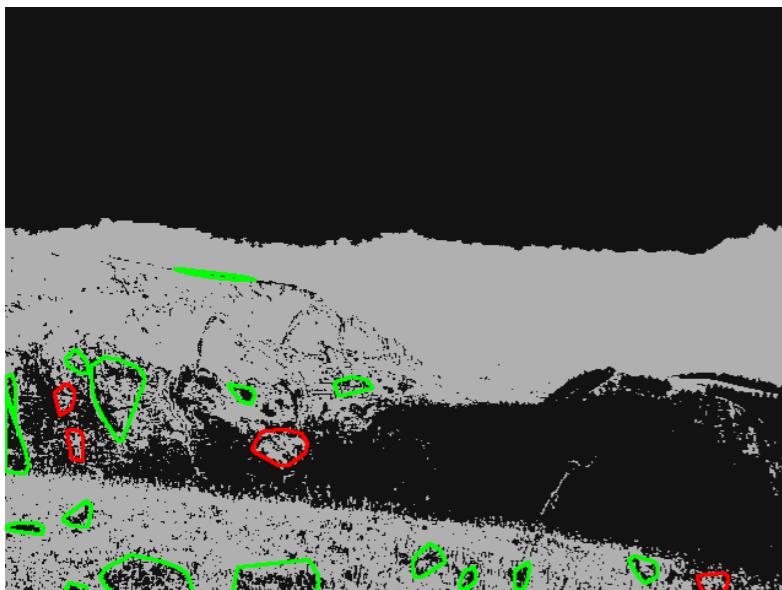
Would prefer fewer features because the pairwise calculations of transformation are  $(N_1*(N_1-1)/2) * (N_2*(N_2-1)/2)$  which is roughly  $N^4$ . Note that trying combinations of rotation and translation (and scale) increase with the image sizes but are very large numbers too for reasonable accuracy.



## Even better:

- perform histogram equalization if mean is too far from median or the two images are too different for those params.
- color segmentation of **k=2** to reduce image to 2 bands of intensities.
- contiguous pixel group finder for each of the 2 bands using point limits of:  
smallestGroupLimit = **100**; largestGroupLimit = **1000**;

This is a smaller number of features that almost has all of the major blobs one would want to use for matching.

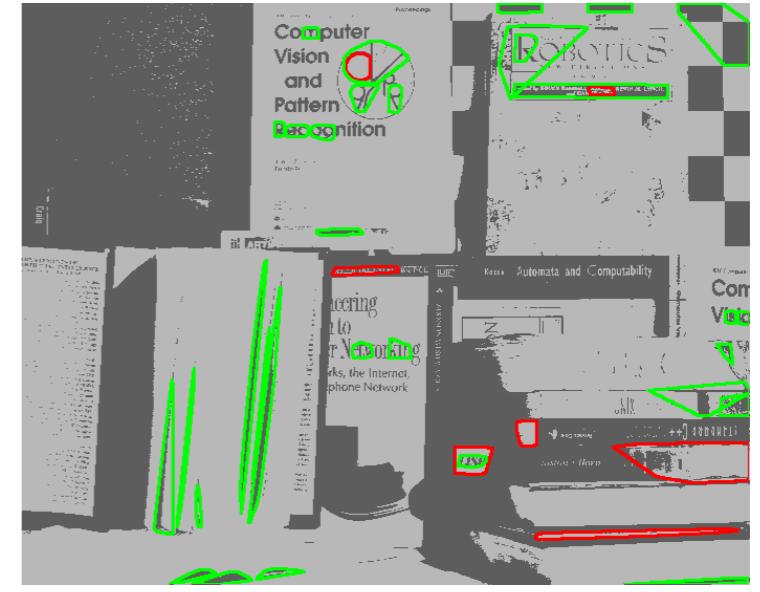
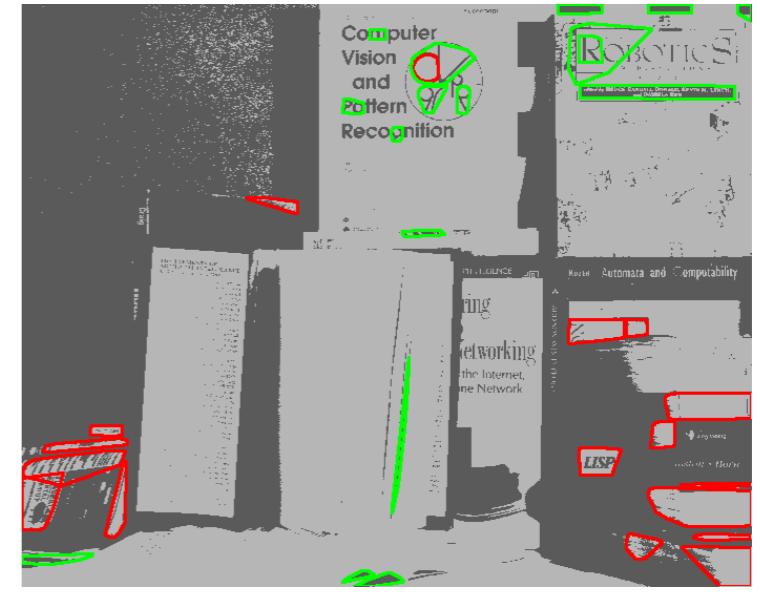
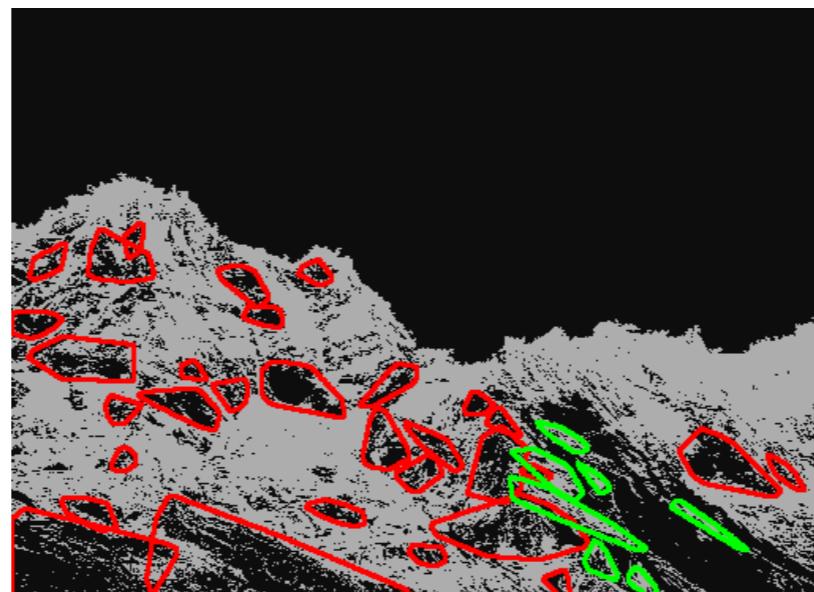
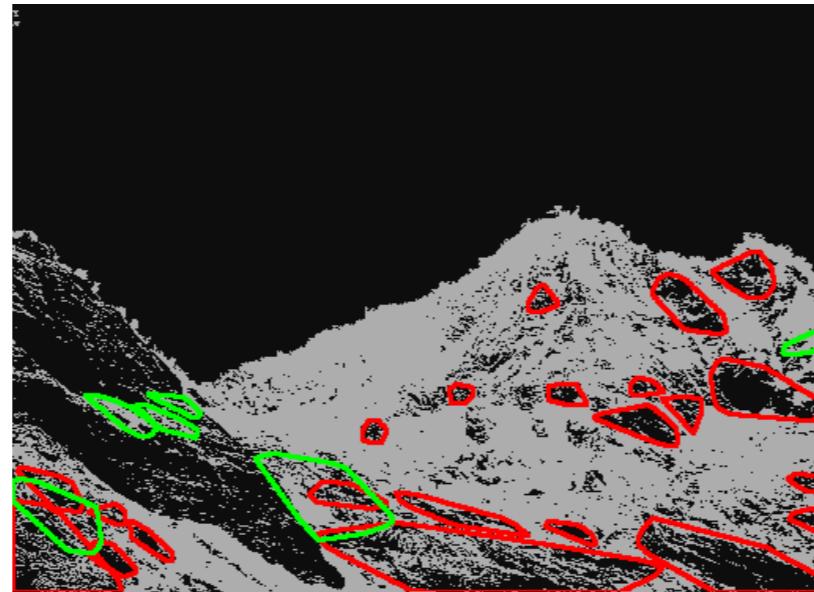
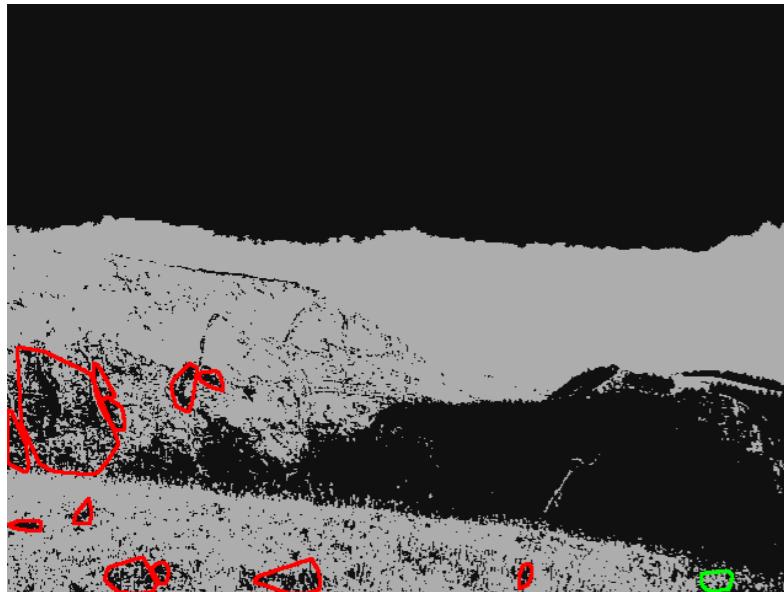




## Even better:

- perform histogram equalization if mean is too far from median or the two images are too different for those stats.
- color segmentation of **k=2** to reduce image to 2 bands of intensities.
- contiguous pixel group finder for each of the 2 bands using point limits of: smallestGroupLimit = **100**; largestGroupLimit = **5000**;

**correspondence looks like enough for an initial Euclidean solution**



NOTE that blobs as first transformation solution works better for images like the middle where in contrast if done with corners, the ridge line in the differences would match more strongly than the true corner matches.



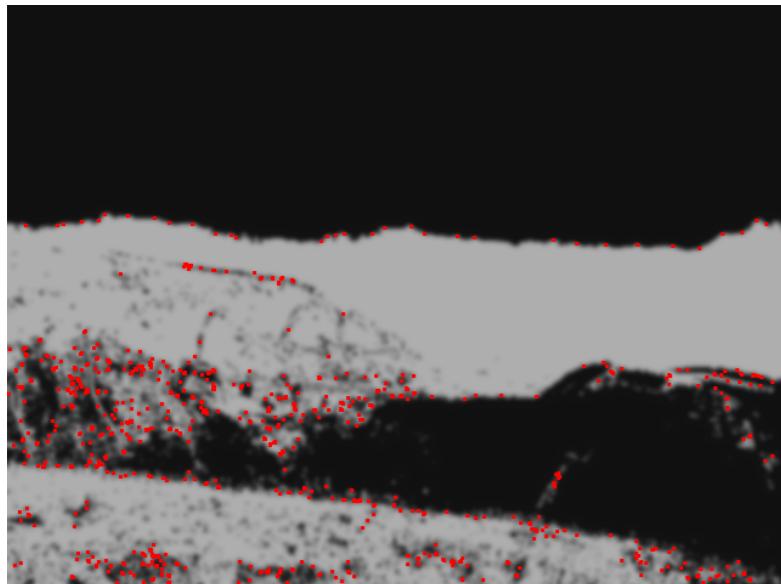
## **After Euclidean transformation calculation, need to make lists of matching points for input to epipolar projection solver.**

- gaussian blur with sigma=2
- scale space curvature corners

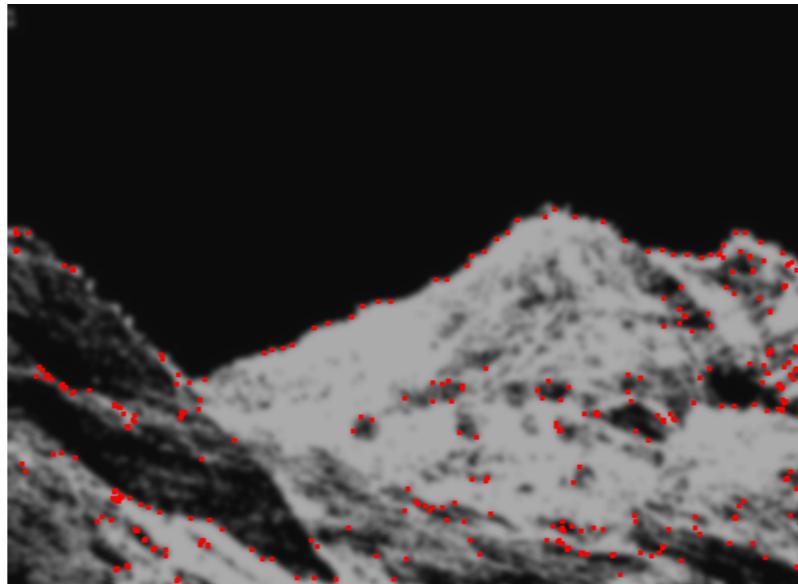
While finding the matches, may need to increase the tolerance or change the solution slightly across the image due to projection.

With the transformation already roughly solved, making point lists even for a large number of corners is fast.

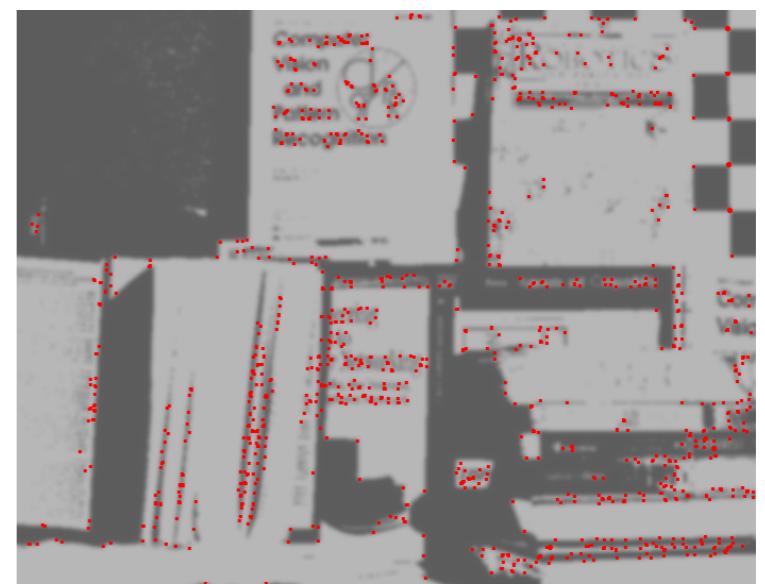
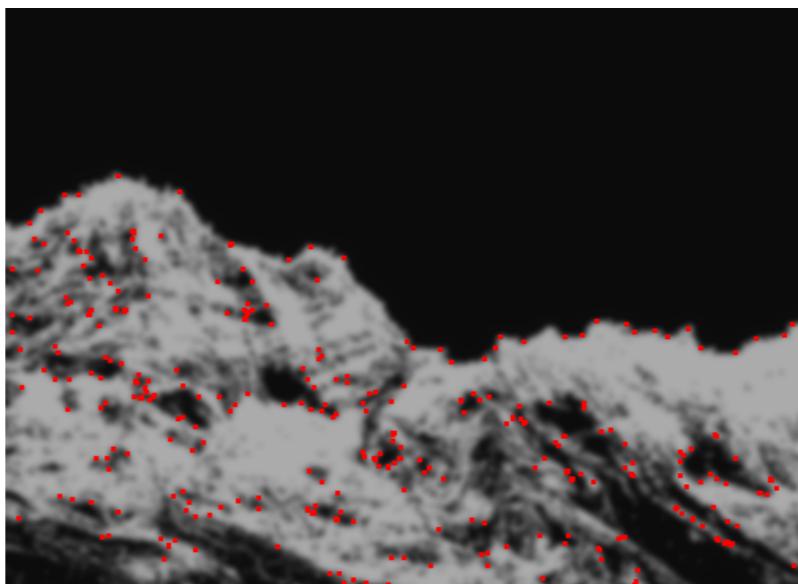
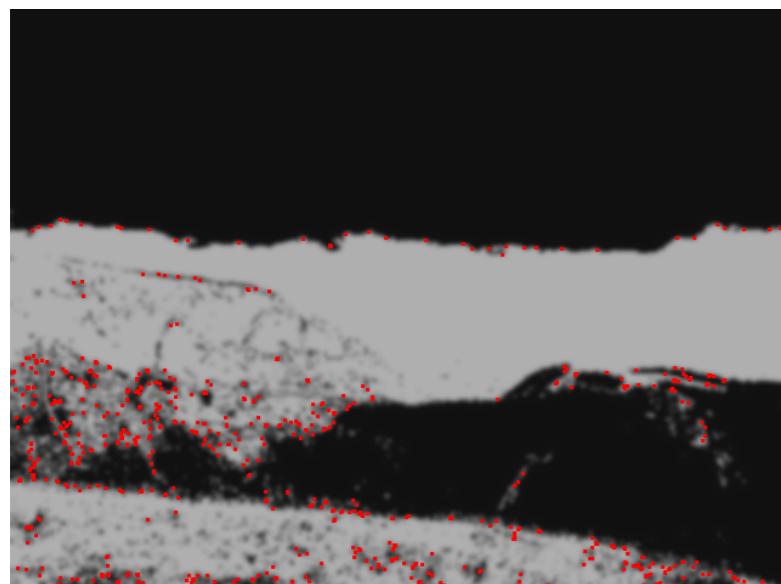
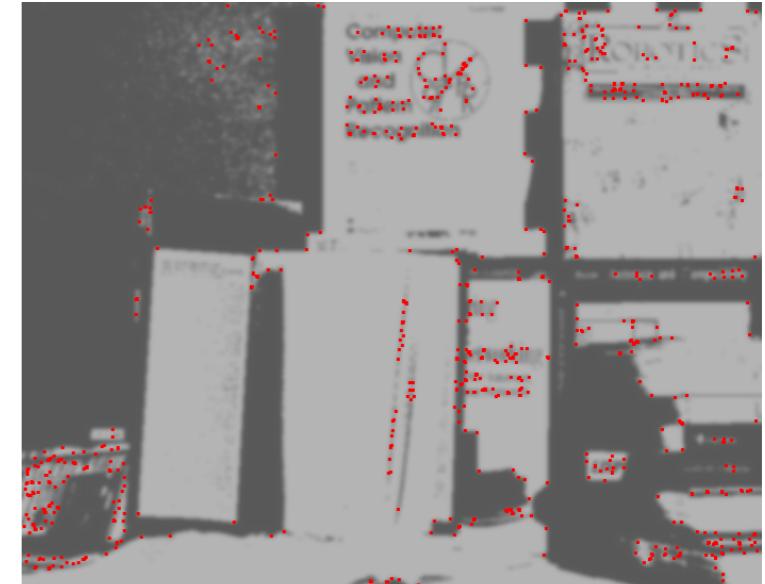
panoramic image pair



panoramic image pair



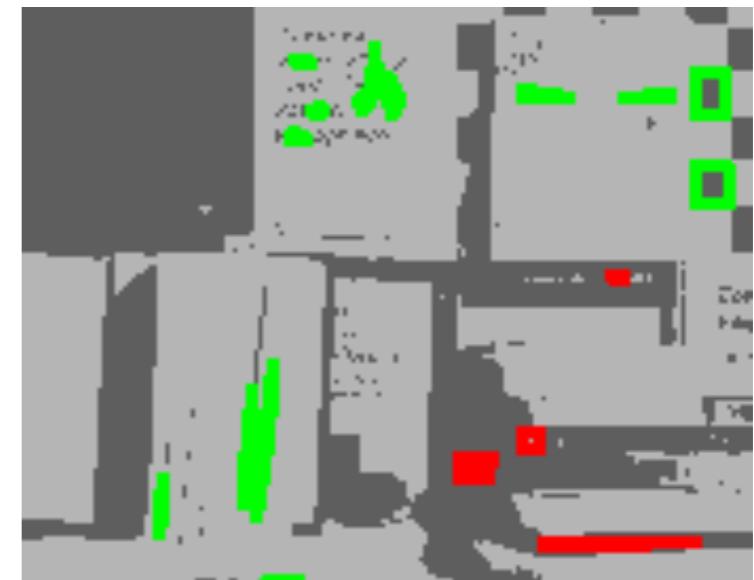
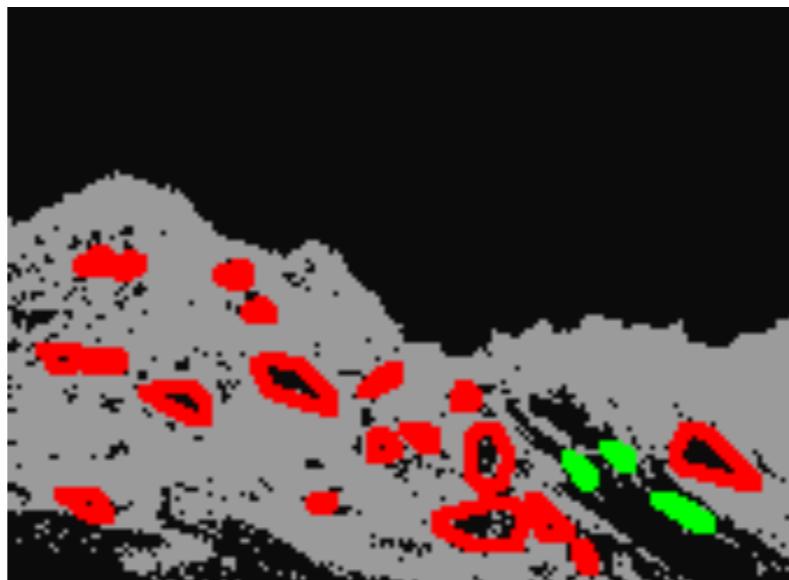
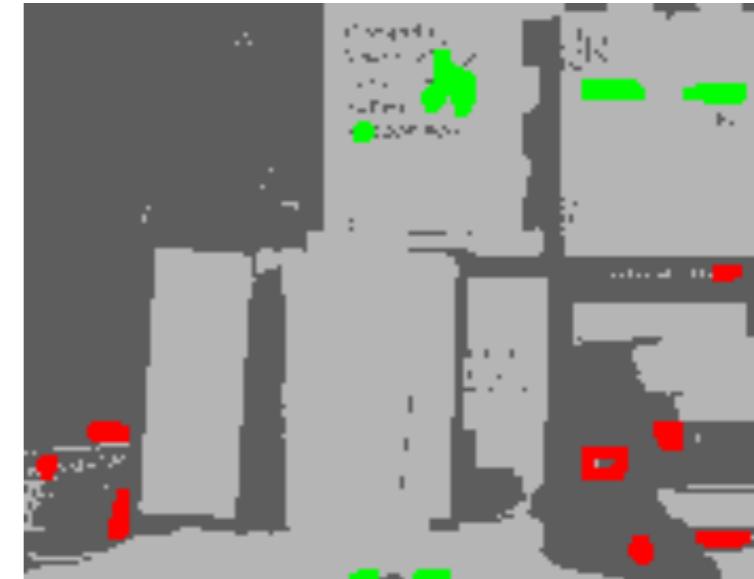
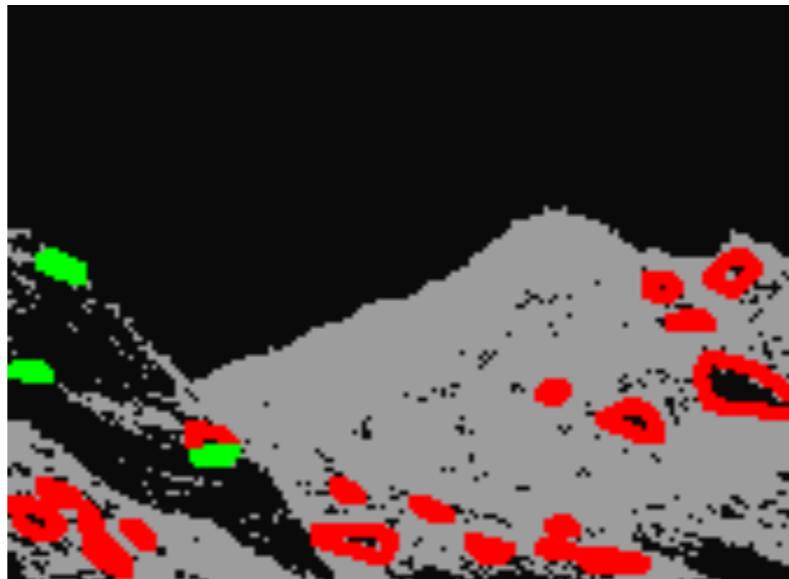
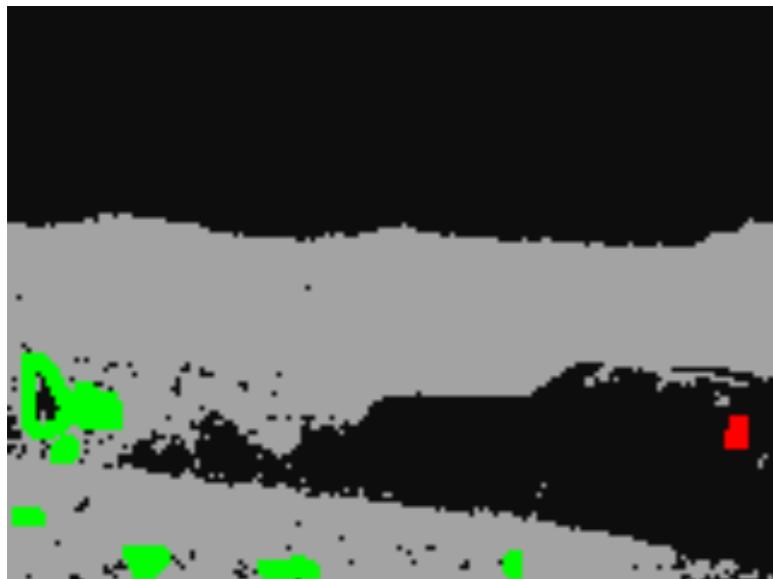
rectified image pair



## Alternatively for the blob (feature) finding, can reduce the image size:

- perform histogram equalization if mean is too far from median or the two images are too different for those params.
- ***bin the image to < 200 X 200***
- color segmentation of ***k=2*** to reduce image to 2 bands of intensities.
- contiguous pixel group finder for each of the 2 bands using point limits of:  
smallestGroupLimit = ***100/(binFactor^2)***; largestGroupLimit = ***5000/(binFactor^2)***;

This solution doesn't have a better matchable to non-matchable blob fraction than the full image, so will not use the binned processing.

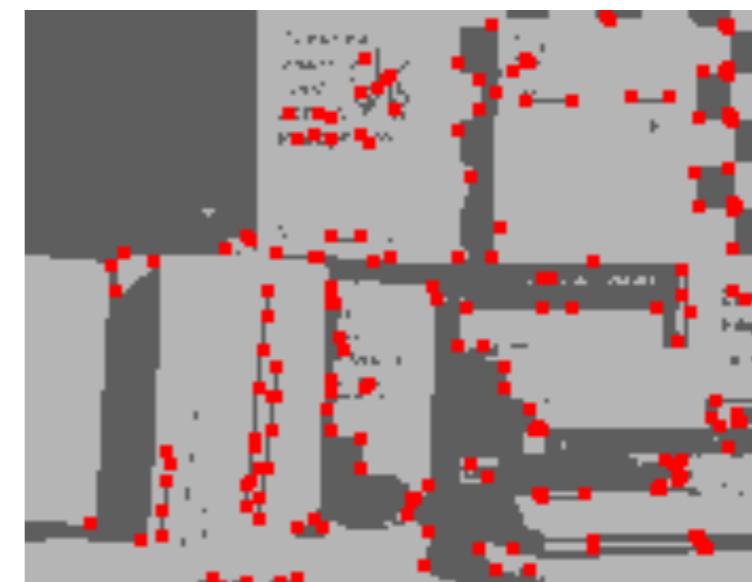
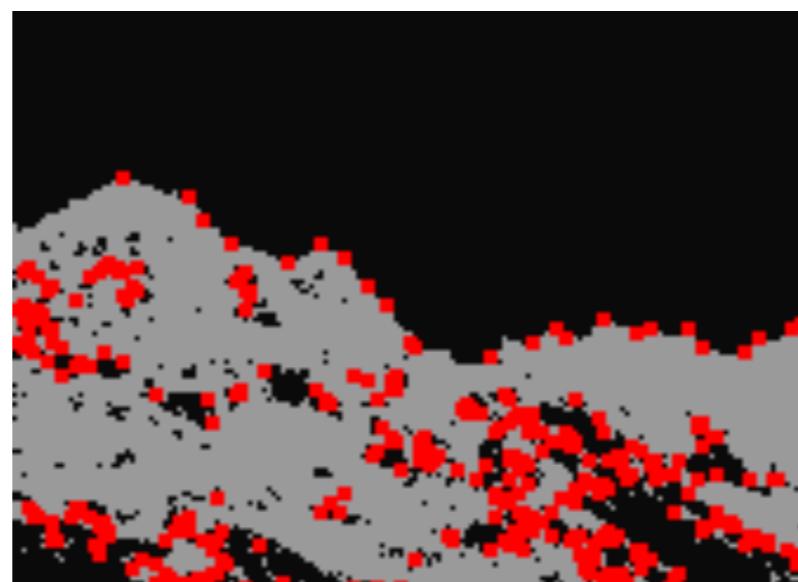
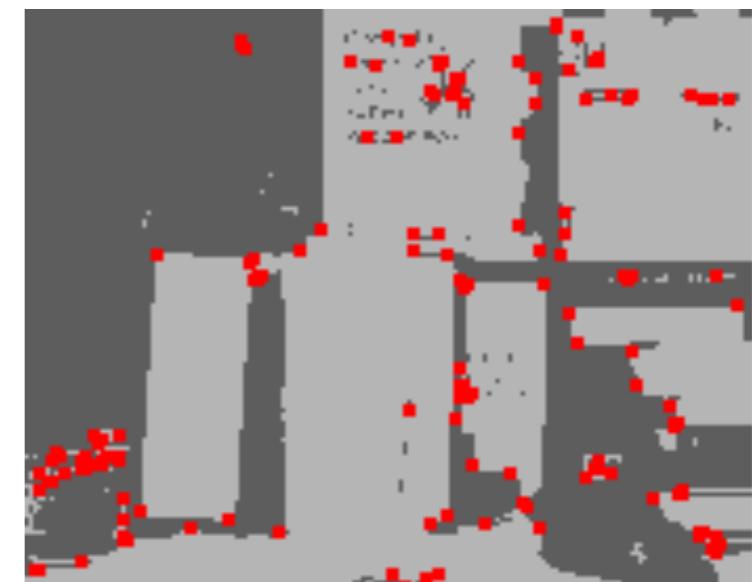
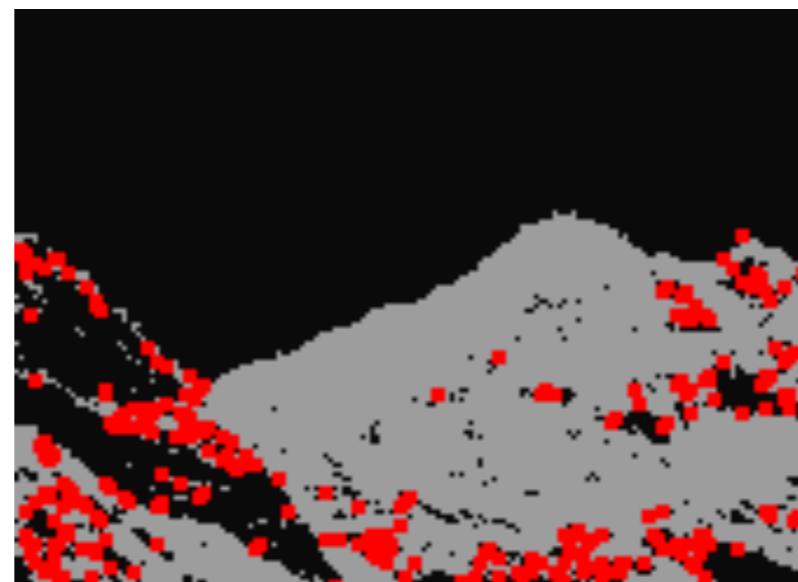
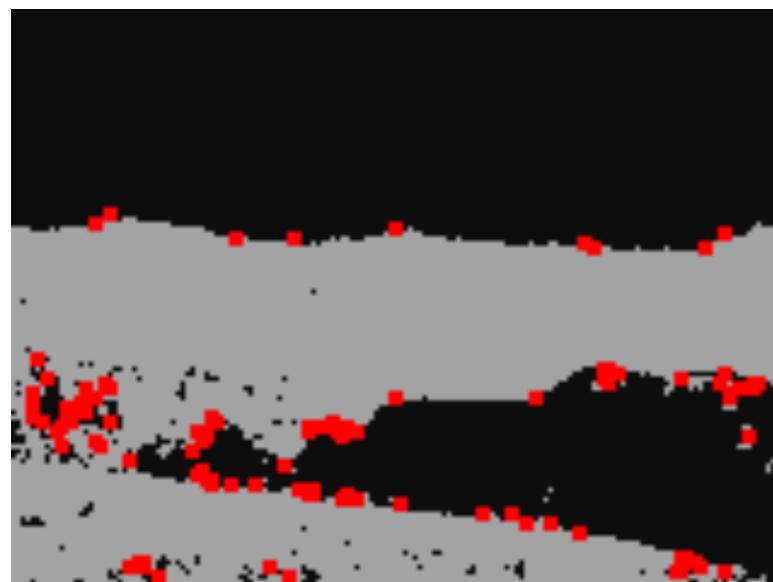


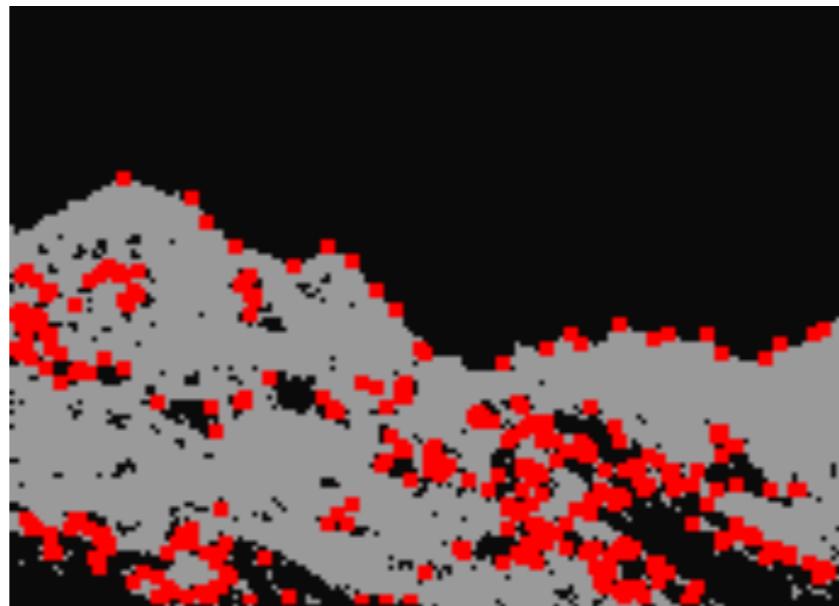
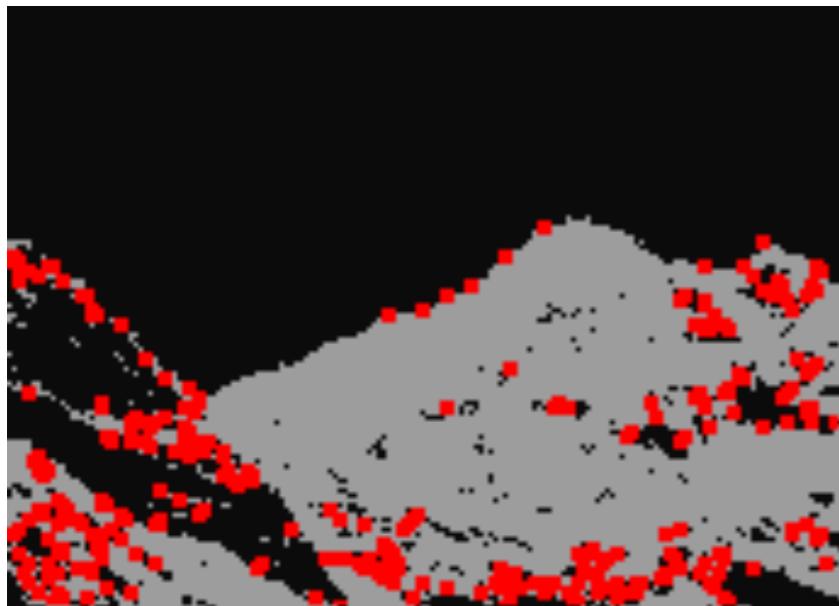
## After Euclidean transformation from blobs, need to make lists of matching points for input to epipolar projection solver.

- No gaussian blur
- scale space curvature corners

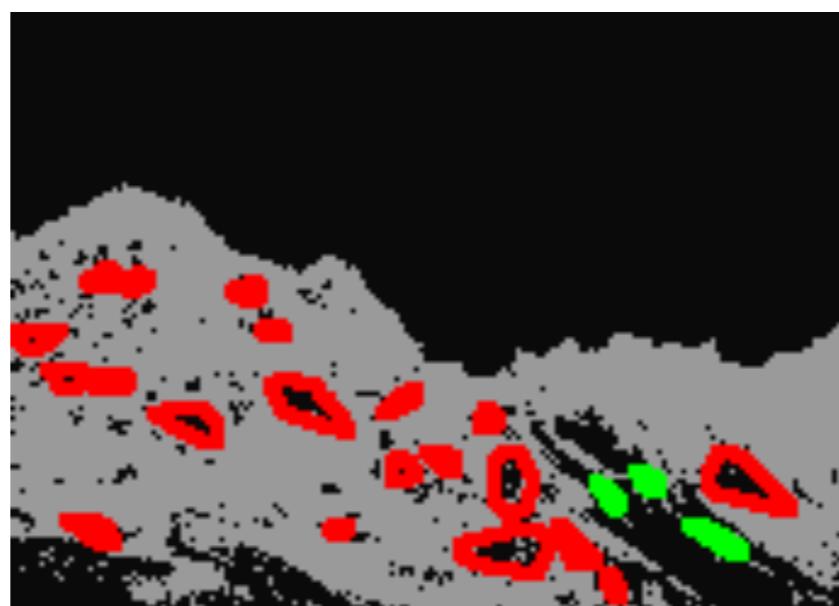
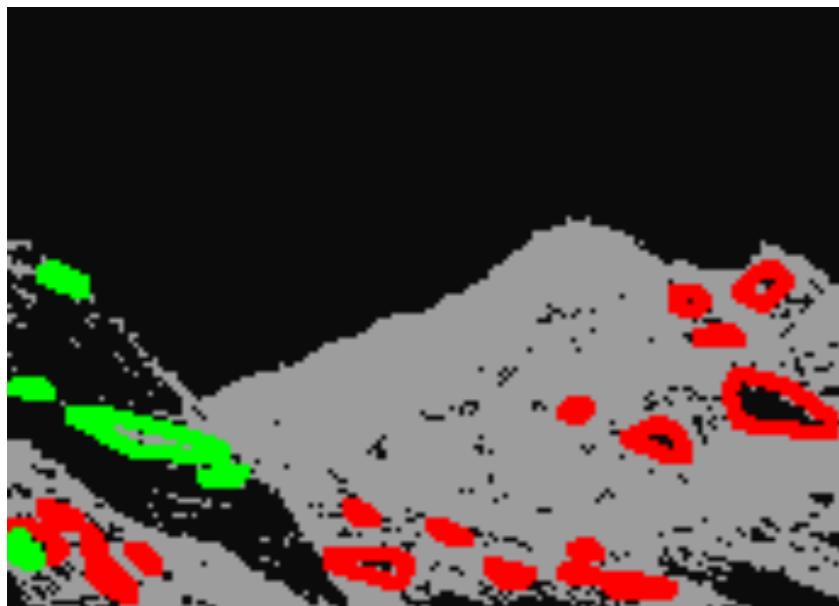
**Note that this stage looks like it is best to perform using the full image, even though solving the Euclidean with the binned features is still a good first stage.**

The binned blob features do not look better than the full image size blobs, but here are the corners from them just to have followed it through.

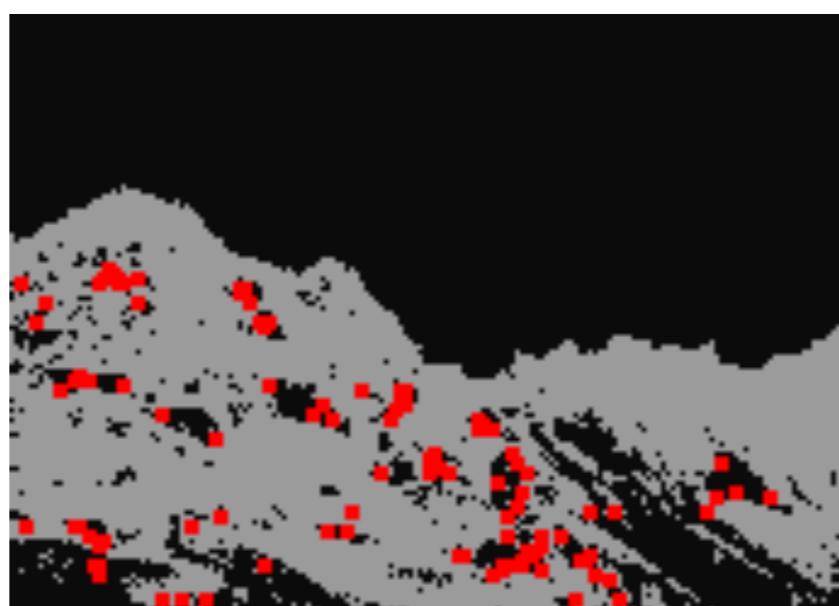
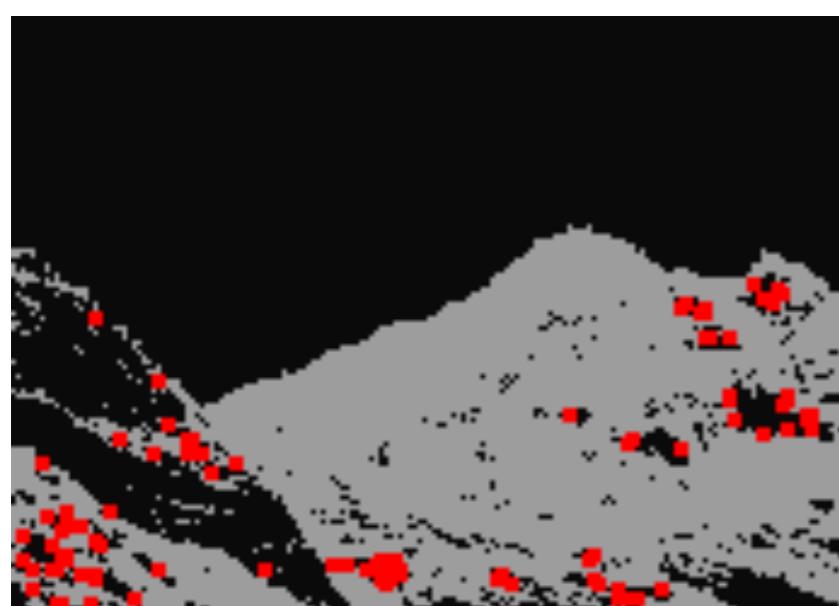




corners from images pre-processed by segmentation and binning (not using a Gaussian pyramid yet, but will later).

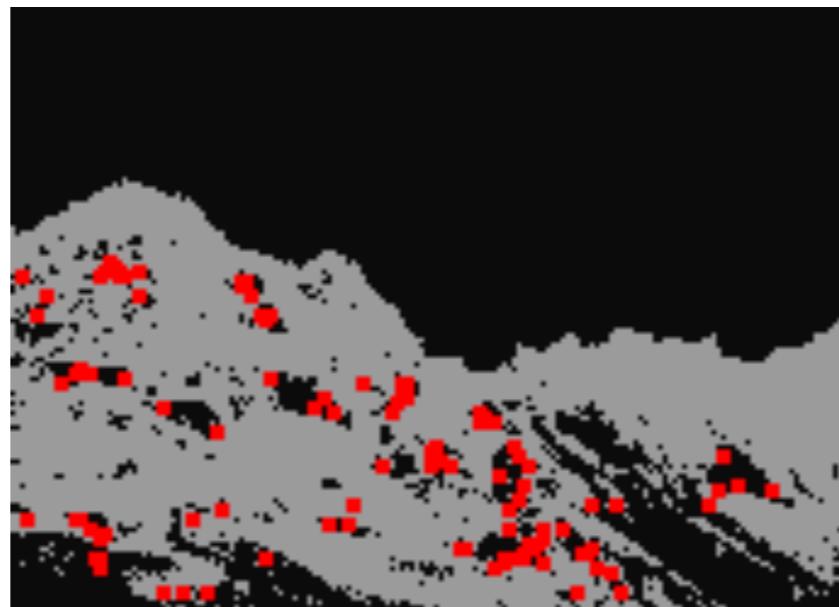
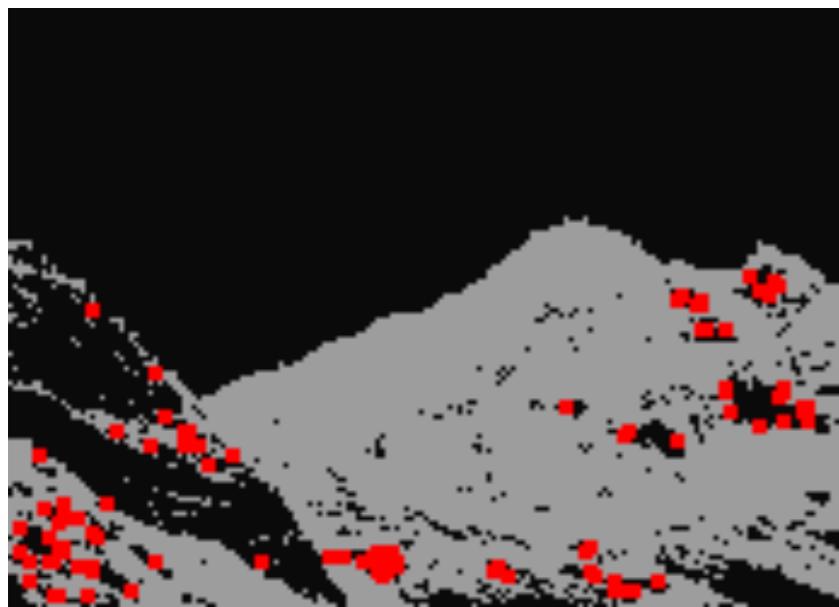


contiguous regions within size ranges were found then convex hulls constructed around them.

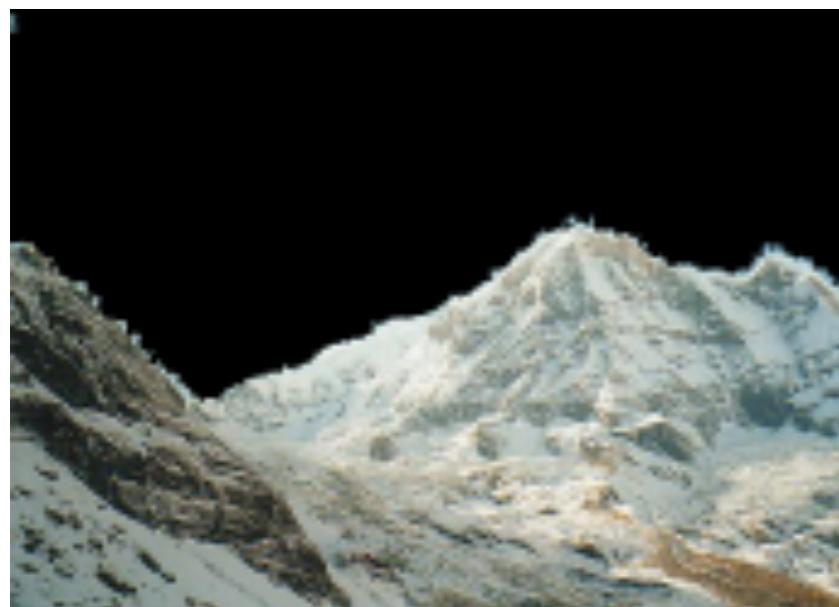


corners filtered to those hulls to reduce the corner list to regions of interest.

the number of possible true matches is less than half of the total number of corners, but feature matching with color image descriptors should help match them.

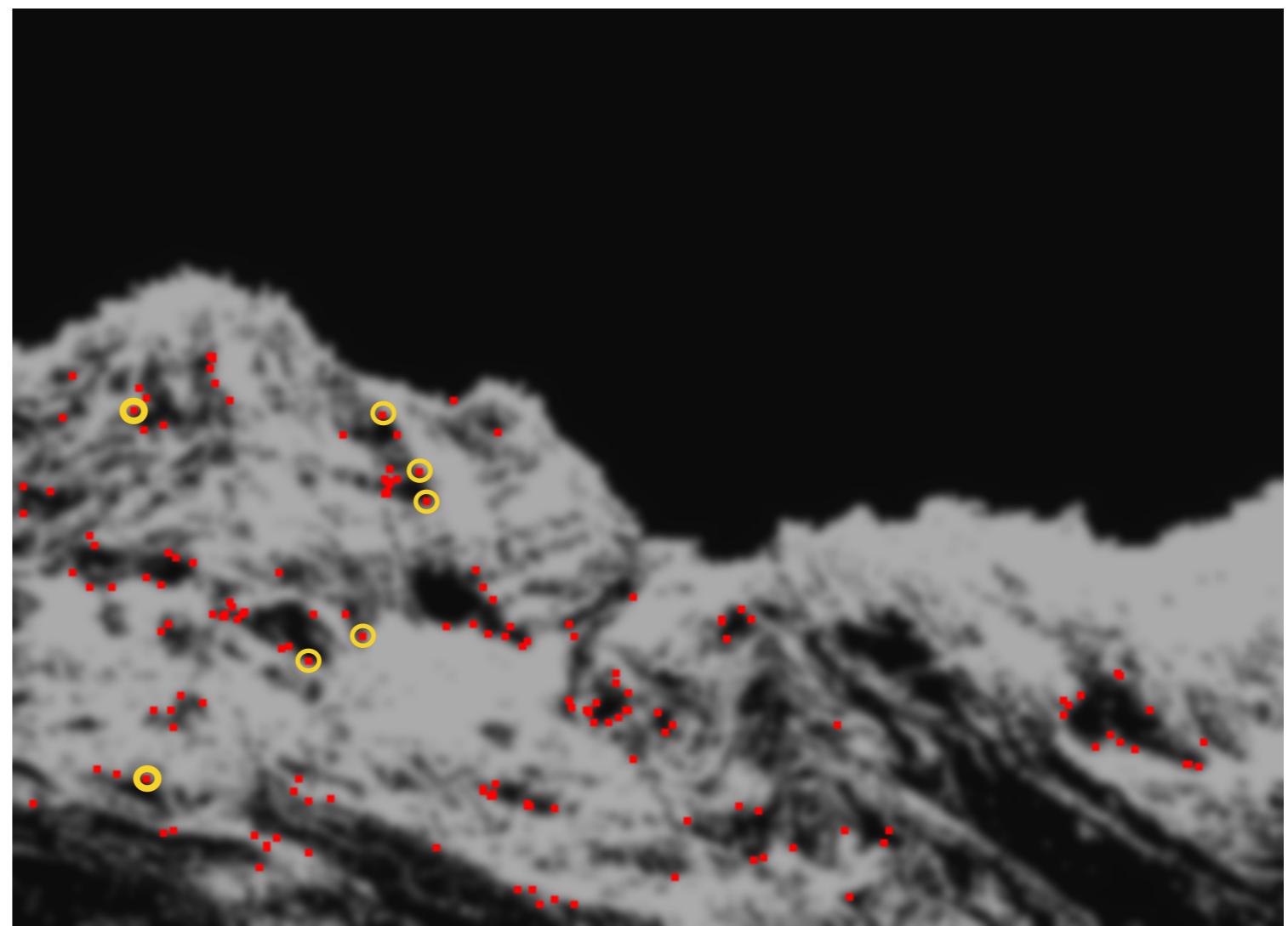
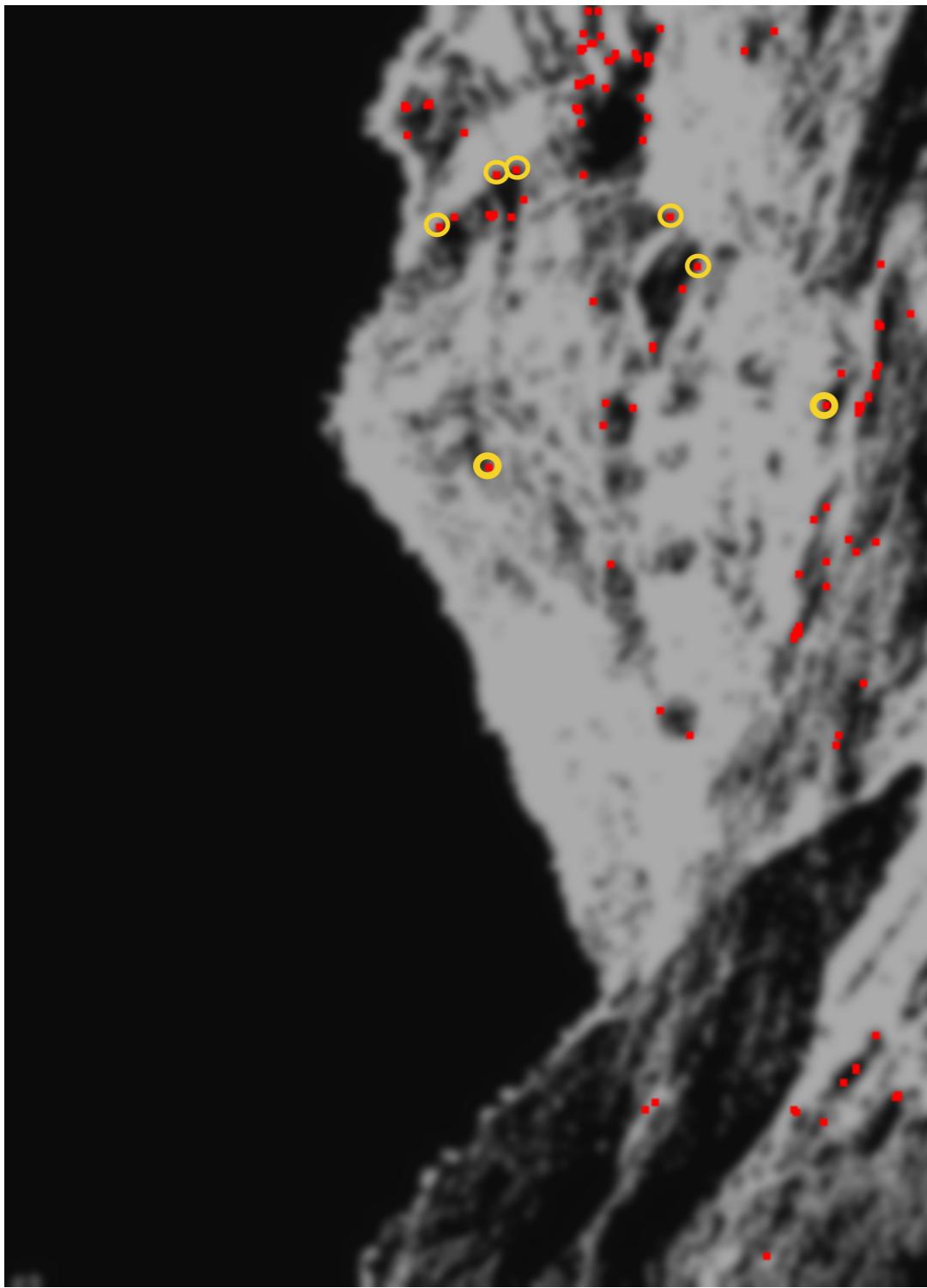


corners filtered

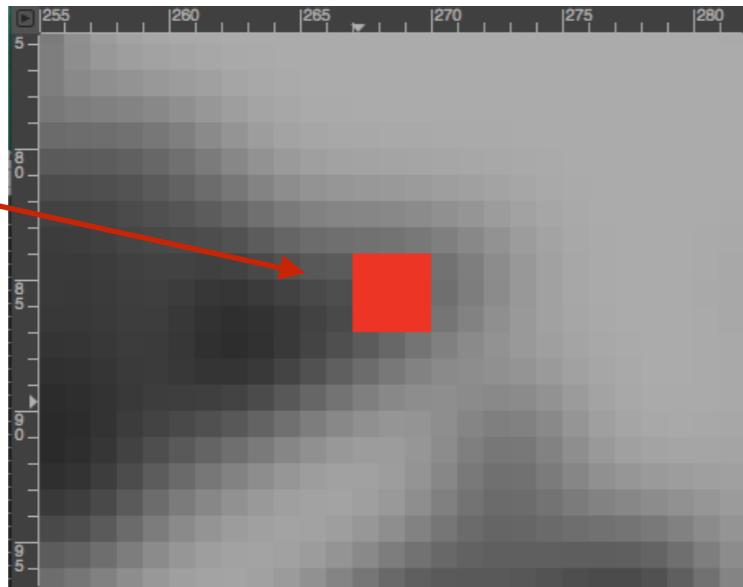
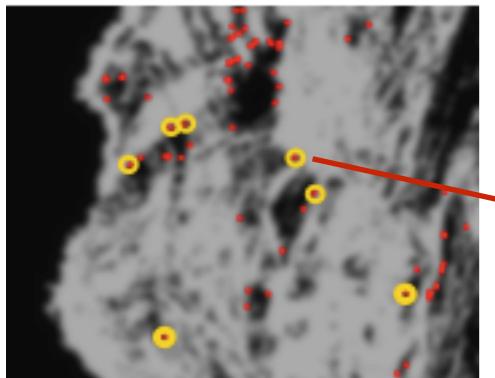


need to determine an orientation of the corner region which is consistent w.r.t. the corner in any image.

Trying with the full images first to make sure the method and math are correct, then will see if can derive the same answer using the binned images.



orientation: since the corners are made with CSS the edge information already exists and can be used here.



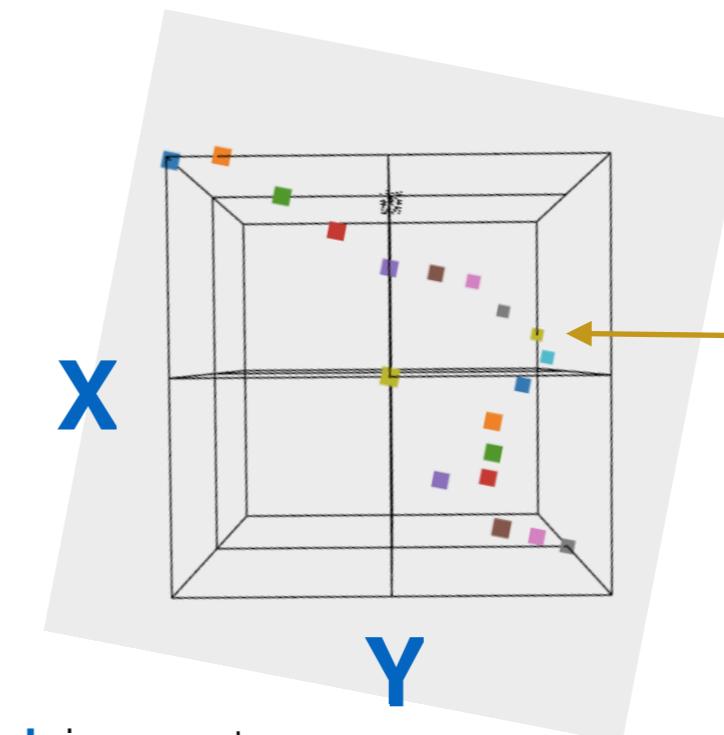
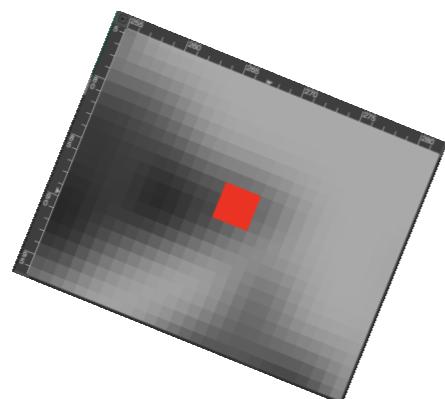
A vector perpendicular to  $k_{\max}$  on this edge can be used to calculate the orientation of this region (where the region is defined by being a CSS corner).

deriving the perpendicular angle from the points directly to the left and right of the maximum of curvature for points which have curvature

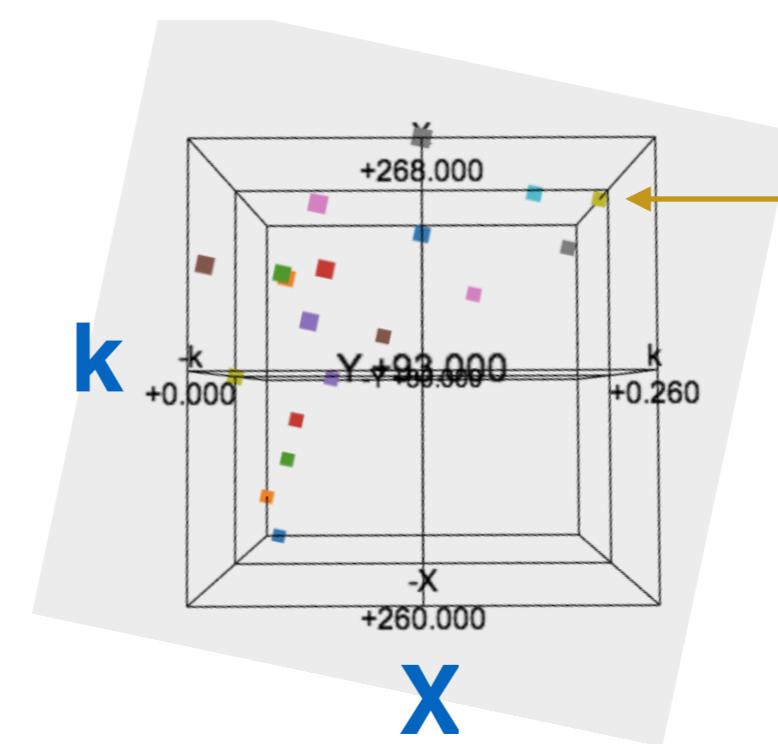
$k$	$x$	$y$
0.24, 267, 84	$dx, dy=(1,1)$ ,	$perp=45$
0.26, 268, 85	$<-max k$	
0.21, 268, 86	$dx, dy=(0,1)$ ,	$perp==0$

orientation =  $(45 + 0)/2. = 22.5$

rotated



$\mathbf{k}$  is curvature



maximum in curvature

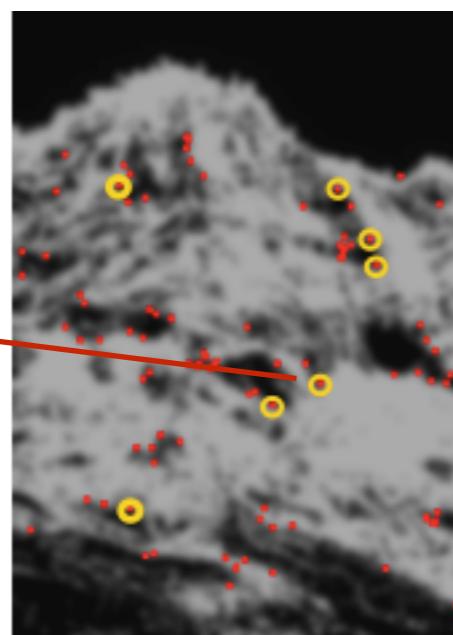
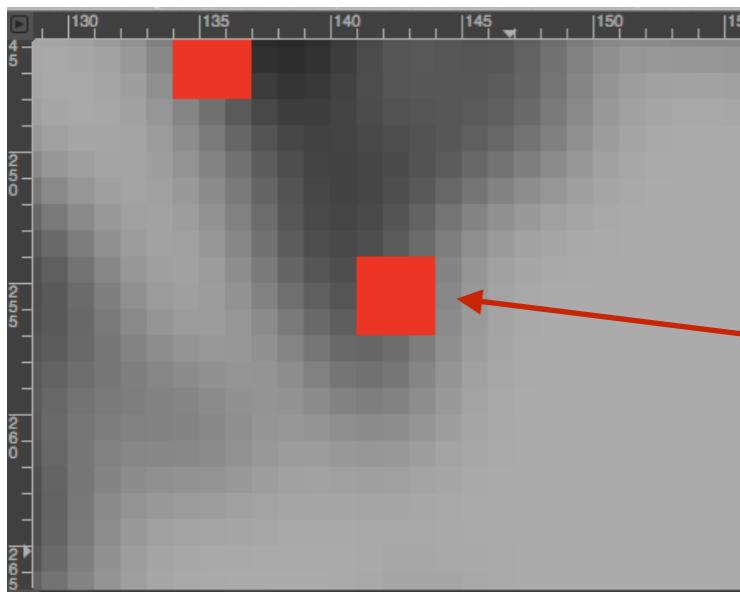
extract curvature from the highest resolution scale space curves:

```

idx=164 (268.0,85.0)
k      x      y
0.08   for (258, 79)
0.04   for (259, 79)
0.01   for (260, 80)
0.00   for (261, 80)
0.02   for (262, 81)
0.03   for (263, 82)
0.06   for (264, 83)
0.10   for (265, 83)
0.17   for (266, 83)
0.24   for (267, 84)
0.26   for (268, 85)
0.21   for (268, 86)
0.13   for (267, 87)
0.04   for (266, 88)
0.04   for (266, 89)
0.07   for (266, 90)
0.06   for (265, 90)
0.00   for (266, 91)
0.07   for (267, 92)
0.13   for (268, 93)
0.15   for (268, 94)

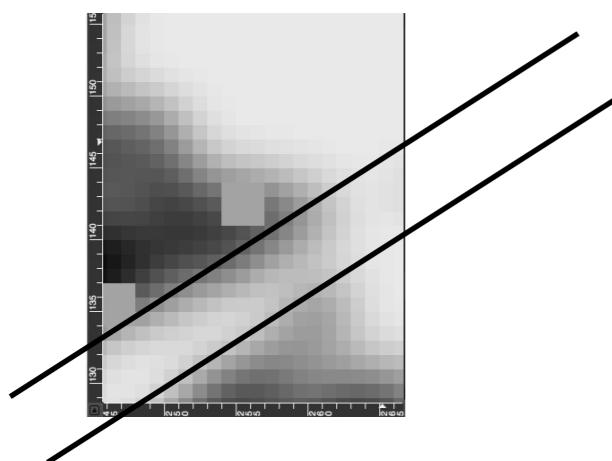
```

orientation.



extract the final curvature from the highest resolution  
scale space curves:  
idx=112 (143.0, 255.0)

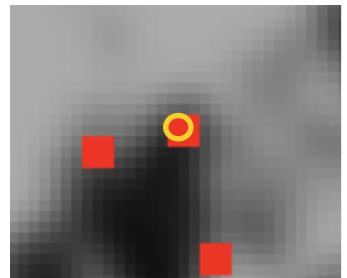
```
k[110]=0.01  for (143, 253)  
k[111]=0.13  for (143, 254)  
k[112]=0.34  for (143, 255)  dx,dy=(-1,0)  
k[113]=0.43  for (142, 255) <-- k_max  
k[114]=0.35  for (141, 255)  dx,dy=(-1,0)  
k[115]=0.28  for (140, 255)  
k[116]=0.24  for (139, 255)
```



$$\text{orientation} = ((-90)+(-90))/2. = -90$$

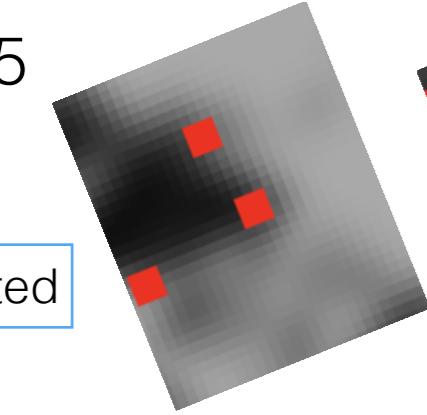
rotated

orientation. corners from Brown & Lowe 2003 left and right panorama images



$k[0]=0.25 \quad x,y=(204, 66)$   
 $k[1]=0.32 \quad x,y=(205, 66)$   
 $k[2]=0.32 \quad x,y=(206, 66)$   
 $k[3]=0.30 \quad x,y=(207, 67)$   
 $k[4]=0.26 \quad x,y=(208, 68)$

67.5



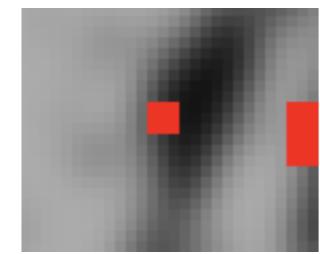
rotated



$k[0]=0.22 \quad x,y=(169, 197)$   
 $k[1]=0.32 \quad x,y=(169, 198)$   
 $k[2]=0.41 \quad x,y=(169, 199)$   
 $k[3]=0.38 \quad x,y=(168, 200)$   
 $k[4]=0.27 \quad x,y=(167, 200)$

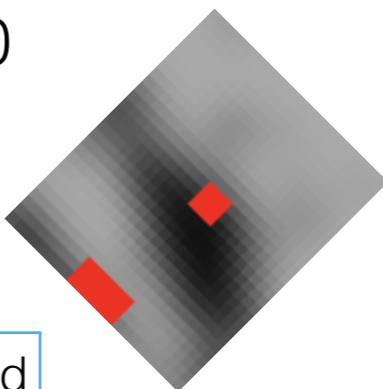
337.5

rotated

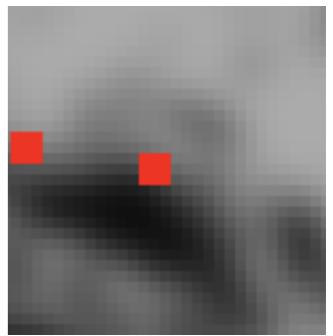


$k[0]=0.11 \quad x,y=(332, 161)$   
 $k[1]=0.37 \quad x,y=(331, 161)$   
 $k[2]=0.58 \quad x,y=(330, 161)$   
 $k[3]=0.47 \quad x,y=(330, 162)$   
 $k[4]=0.30 \quad x,y=(330, 163)$

135.0

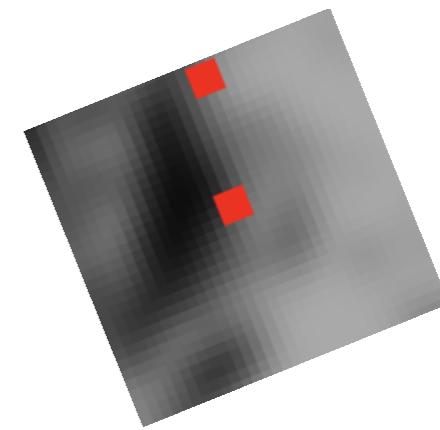


rotated



$k[0]=0.07 \quad x,y=(56, 315)$   
 $k[1]=0.09 \quad x,y=(55, 314)$   
 $k[2]=0.31 \quad x,y=(54, 313)$   
 $k[3]=0.23 \quad x,y=(53, 313)$   
 $k[4]=0.03 \quad x,y=(52, 313)$

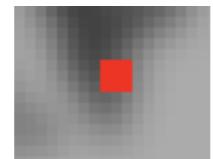
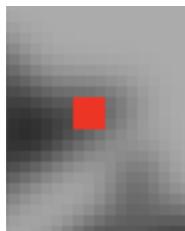
67.5



rotated

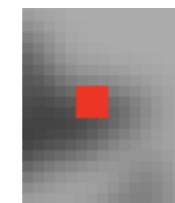
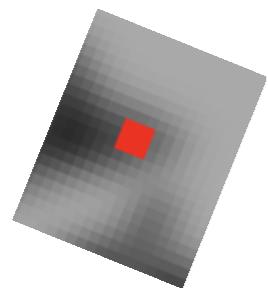
$k[0]=0.17 \quad x,y=(266, 83)$   
 $k[1]=0.24 \quad x,y=(267, 84)$   
 $k[2]=0.26 \quad x,y=(268, 85)$   
 $k[3]=0.21 \quad x,y=(268, 86)$   
 $k[4]=0.13 \quad x,y=(267, 87)$

22.5



270.0

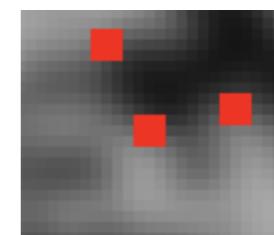
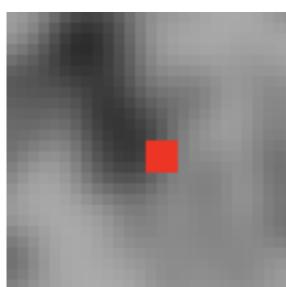
rotated



rotated

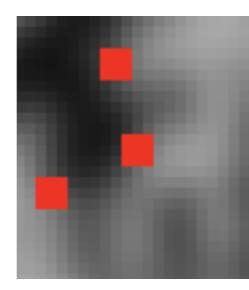
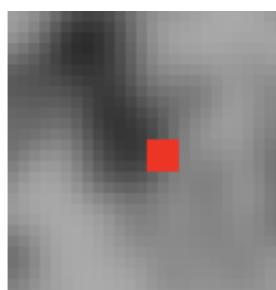
$k[0]=0.16 \quad x,y=(195, 183)$   
 $k[1]=0.26 \quad x,y=(195, 184)$   
 $k[2]=0.32 \quad x,y=(195, 185)$   
 $k[3]=0.29 \quad x,y=(195, 186)$   
 $k[4]=0.25 \quad x,y=(194, 187)$

0.0



270.0

rotated



rotated

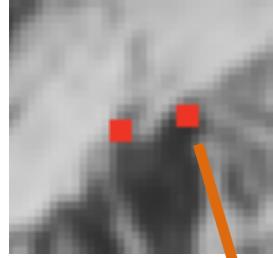
## orientation and junctions.

corners  
from Brown & Lowe 2003 left and right panorama images

k[0]=0.09 x,y=(204, 65)  
k[1]=0.19 x,y=(205, 65)  
k[2]=0.21 x,y=(206, 65)  
k[3]=0.09 x,y=(206, 64)  
k[4]=0.03 x,y=(206, 63)

k[0]=0.21 x,y=(170, 198)  
k[1]=0.32 x,y=(170, 199)  
k[2]=0.41 x,y=(170, 200)  
k[3]=0.38 x,y=(169, 201)  
k[4]=0.27 x,y=(168, 201)

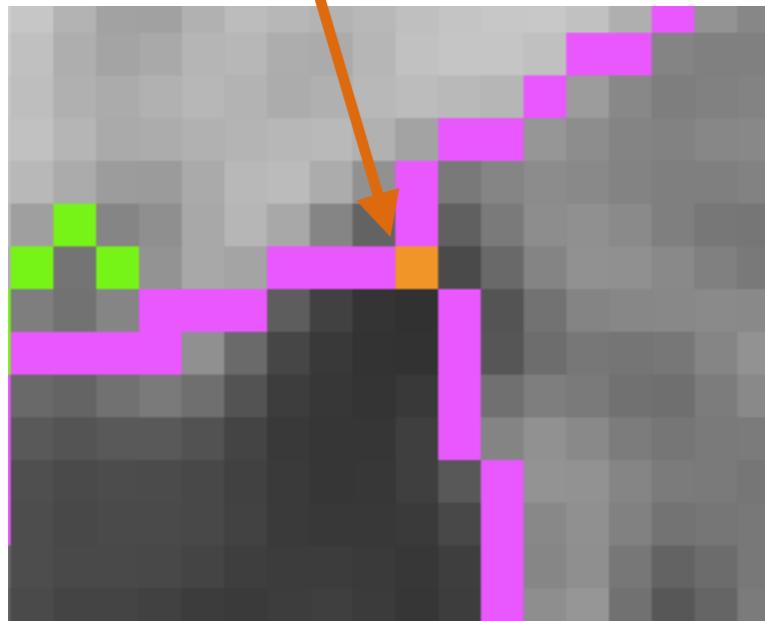
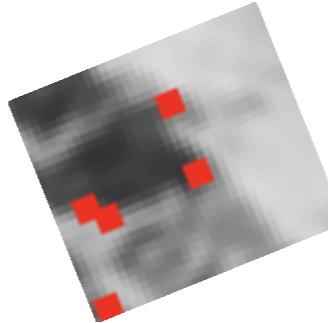
315.0



337.5



rotated

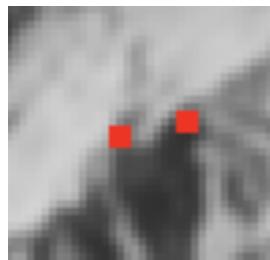


The left image corner is in a *junction* and that requires more complex analysis to determine here that the better edge for it is the lower right edge instead of the upper edge (the later gives an inconsistent orientation w.r.t. the right image).

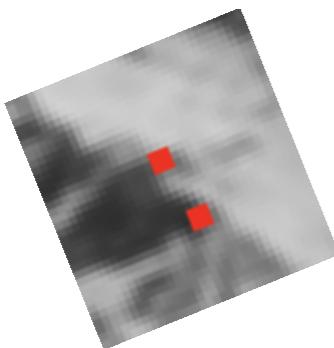
Without the more complex analysis for now, will just create a CornerRegion for each combination and one of the 3 will match the right.

## orientation and junctions (continued)

k[0]=0.02 x,y=(205, 65)  
k[1]=0.21 x,y=(206, 65)  
k[2]=0.02 x,y=(207, 66)



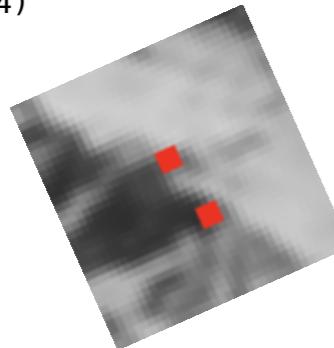
67.5



rotated

k[0]=0.02 x,y=(205, 65)  
k[1]=0.21 x,y=(206, 65)  
k[2]=0.02 x,y=(206, 64)

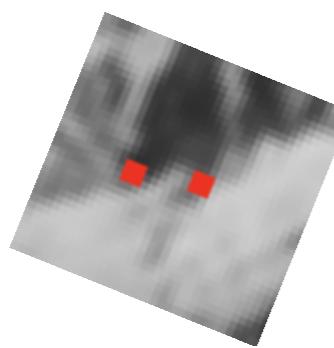
66



rotated

k[0]=0.02 x,y=(207, 66)  
k[1]=0.21 x,y=(206, 65)  
k[2]=0.02 x,y=(206, 64)

202.5



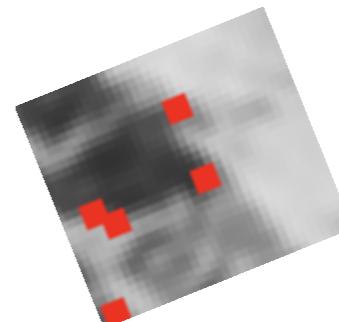
rotated

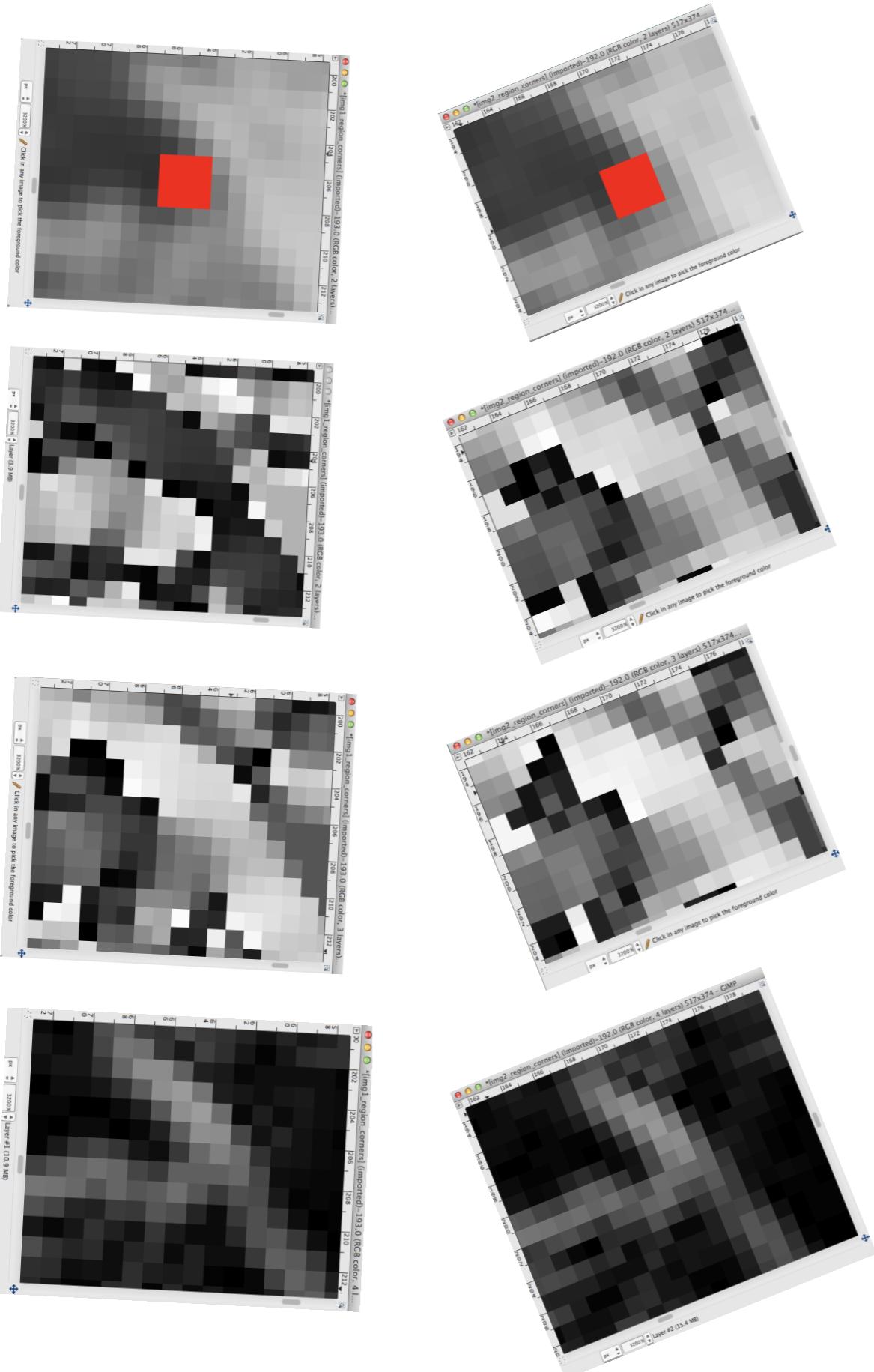
k[0]=0.21 x,y=(170, 198)  
k[1]=0.32 x,y=(170, 199)  
k[2]=0.41 x,y=(170, 200)  
k[3]=0.38 x,y=(169, 201)  
k[4]=0.27 x,y=(168, 201)

337.5



rotated





greyscale rotated to “dominant orientation”

gradient theta rotated to  
“dominant orientation”

gradient theta rotated to  
“dominant orientation” with  
“dominant orientation” subtracted  
from theta

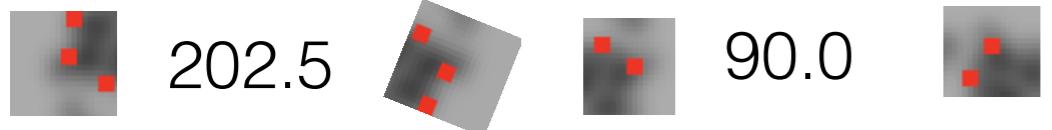
can see that image  
1 should have  
preferred match w/  
orientation 67.5  
instead of 93

can see that SSD on gradient theta could be a good  
descriptor with the orientation corrections and quadrant  
consideration (e.g. 0 diff 350 is 10, not 350)

can see that SSD on gradient could be a good descriptor  
but may not be as tolerant of skew as subcells of  
histograms of orientation?

## orientation. corners from Venturi dataset

$k[0]=0.15$ x,y=(461, 449)	$k[0]=0.13$ x,y=(157, 466)
$k[1]=0.23$ x,y=(460, 448)	$k[1]=0.19$ x,y=(156, 466)
$k[2]=0.29$ x,y=(459, 447)	$k[2]=0.22$ x,y=(155, 466)
$k[3]=0.26$ x,y=(459, 446)	$k[3]=0.20$ x,y=(154, 466)
$k[4]=0.16$ x,y=(459, 445)	$k[4]=0.16$ x,y=(153, 467)

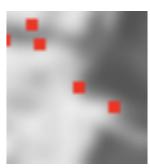


202.5

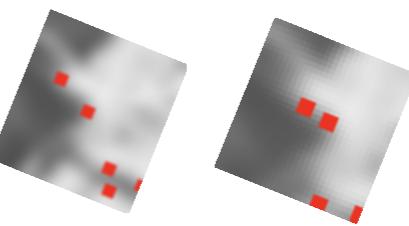
90.0

rotated

$k[0]=0.18$ x,y=(293, 496)
$k[1]=0.24$ x,y=(292, 495)
$k[2]=0.29$ x,y=(291, 494)
$k[3]=0.25$ x,y=(291, 493)
$k[4]=0.13$ x,y=(291, 492)



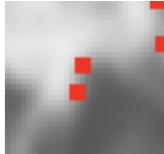
202.5



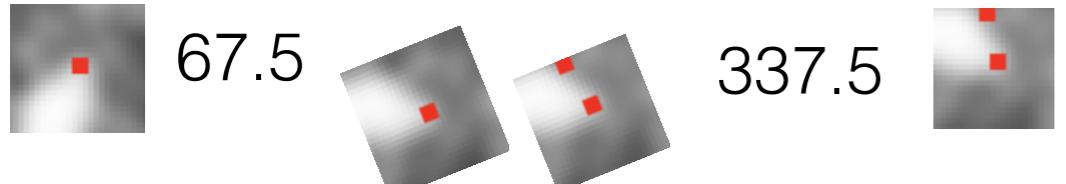
112.5

rotated

$k[0]=0.18$ x,y=(105, 298)
$k[1]=0.33$ x,y=(105, 297)
$k[2]=0.36$ x,y=(106, 296)
$k[3]=0.26$ x,y=(107, 296)
$k[4]=0.14$ x,y=(108, 296)



$k[0]=0.27$ x,y=(449, 222)	$k[0]=0.26$ x,y=(382, 448)
$k[1]=0.33$ x,y=(448, 221)	$k[1]=0.29$ x,y=(383, 447)
$k[2]=0.37$ x,y=(447, 220)	$k[2]=0.31$ x,y=(384, 446)
$k[3]=0.35$ x,y=(446, 220)	$k[3]=0.29$ x,y=(384, 445)
$k[4]=0.26$ x,y=(445, 220)	$k[4]=0.23$ x,y=(384, 444)

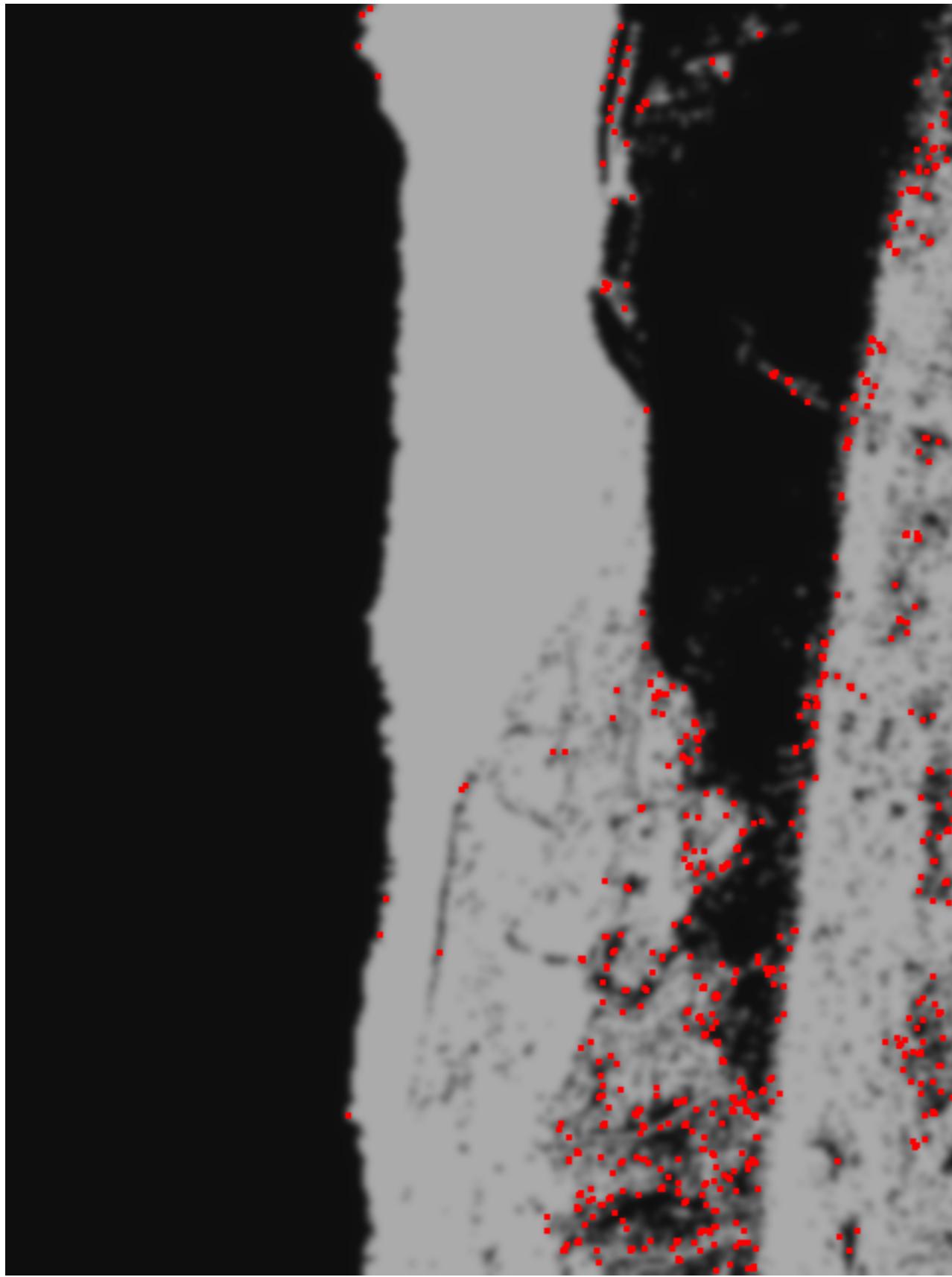


67.5

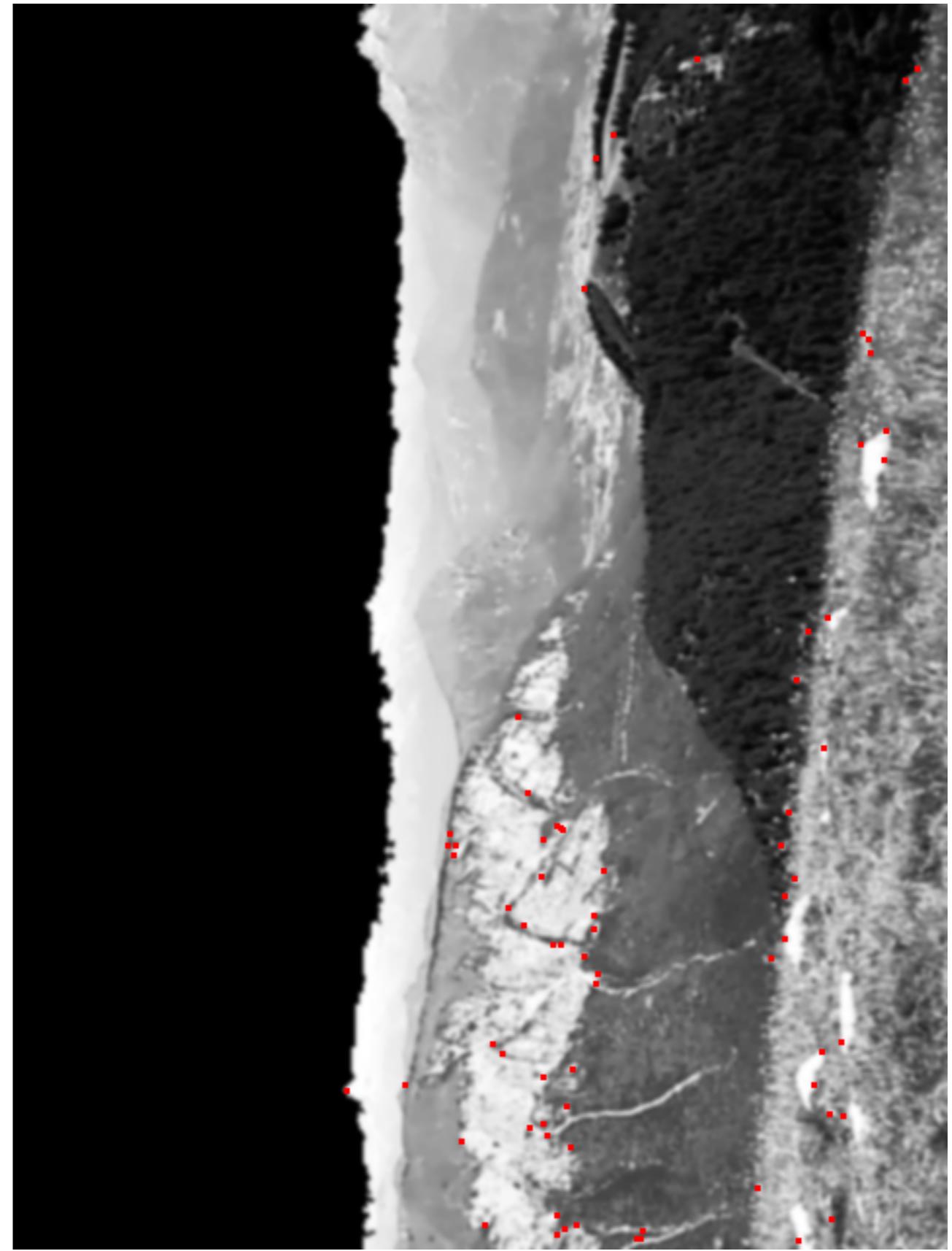
337.5

rotated

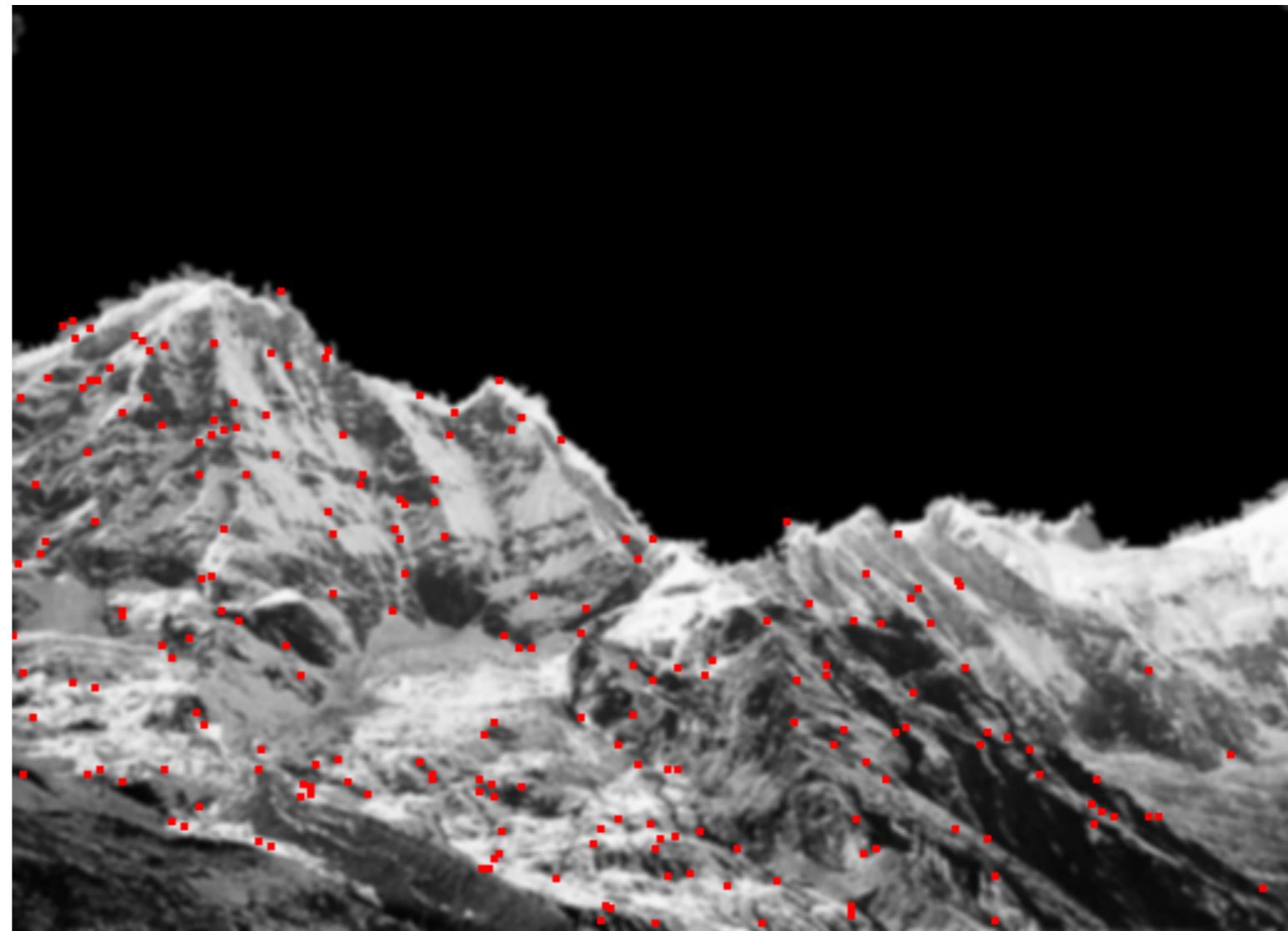
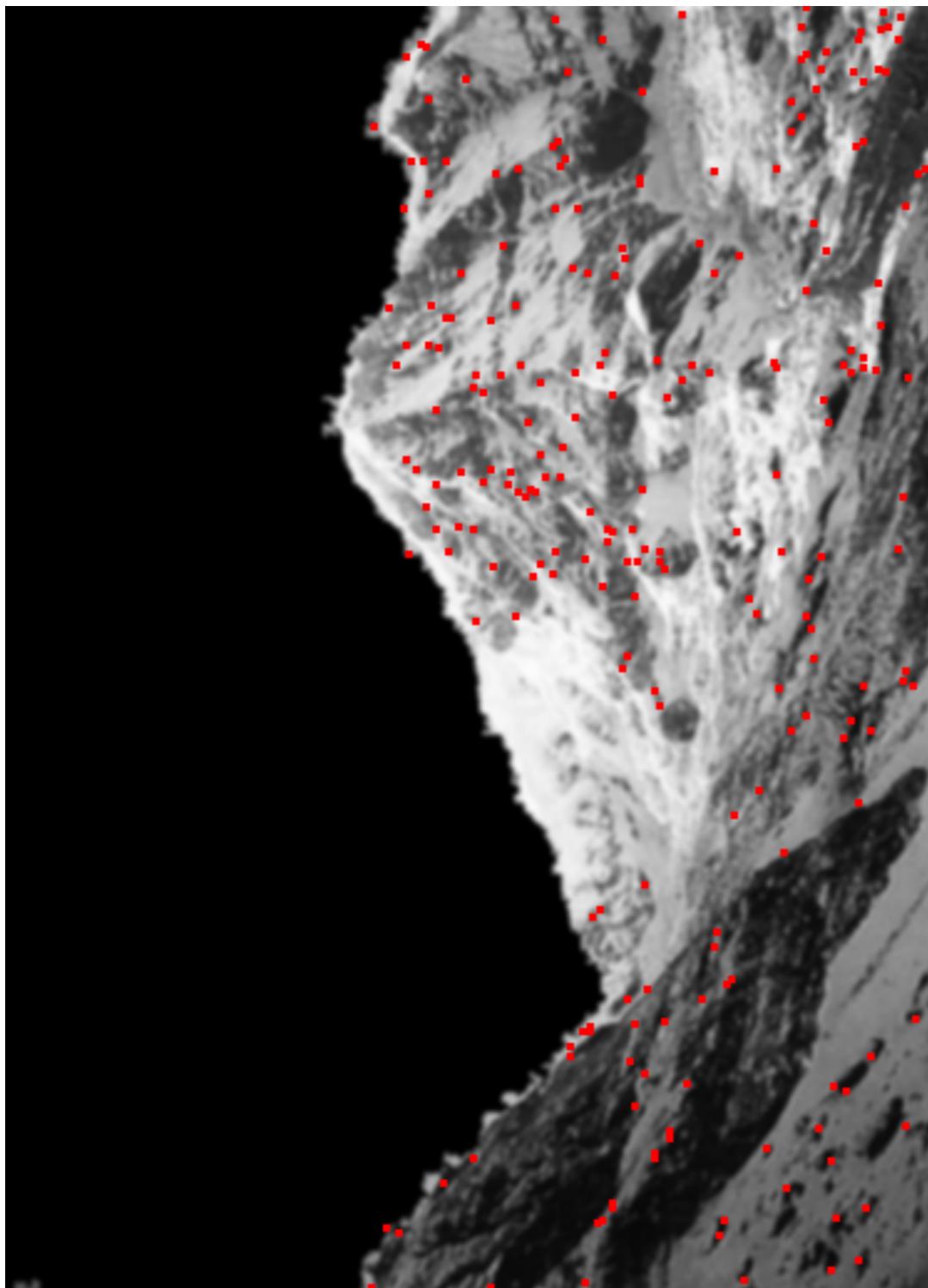
corners using color segmentation

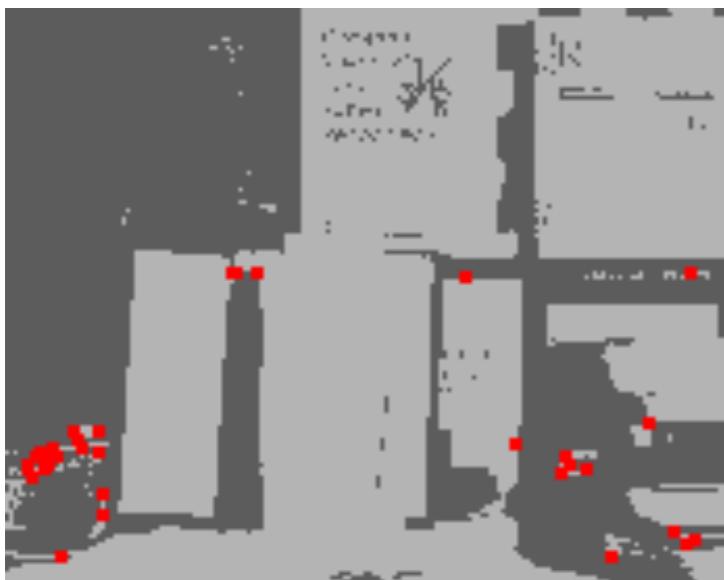


corners without color segmentation  
and filtered to keep sharp corners only

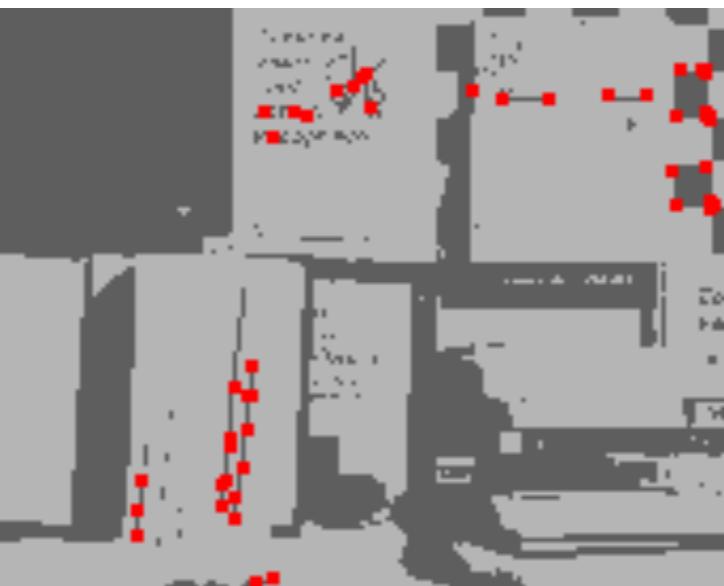


corners without color segmentation and filtered to keep sharp corners only

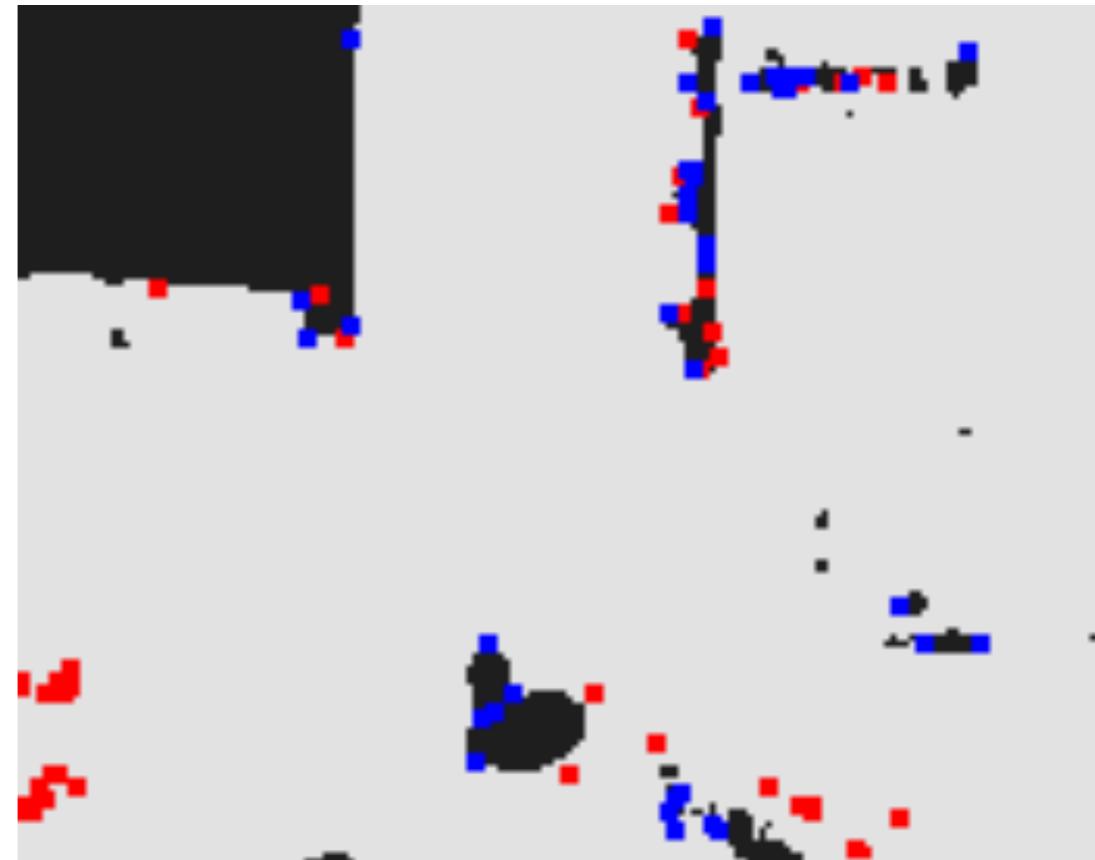
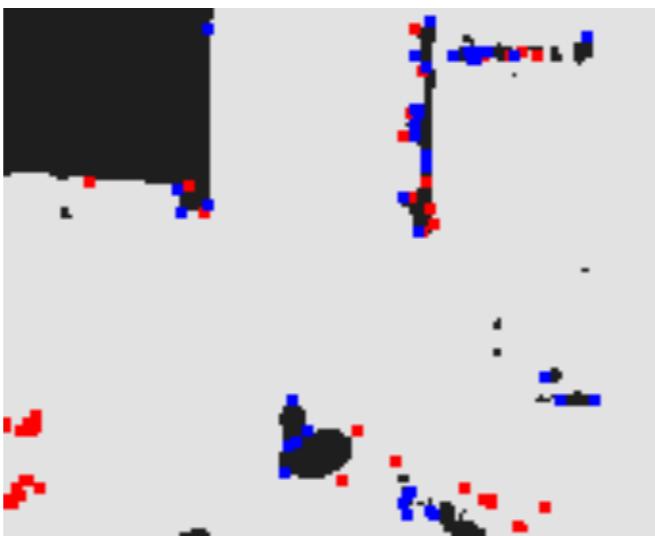




corners filtered (not finished here, but keeping snapshots)



A quick look at color segmentation with k=3 followed by binary segmentation, binned to size < 200 x 200, followed by corners worked fine for 2 image sets, but not the third so the other methods in previous slides are preferred (corners filtered to blobs, matching, calculating transformation, then creating correspondence with all corners).



For the stereoscopic above, there's more than translation  
so this can only be the start of the solution

