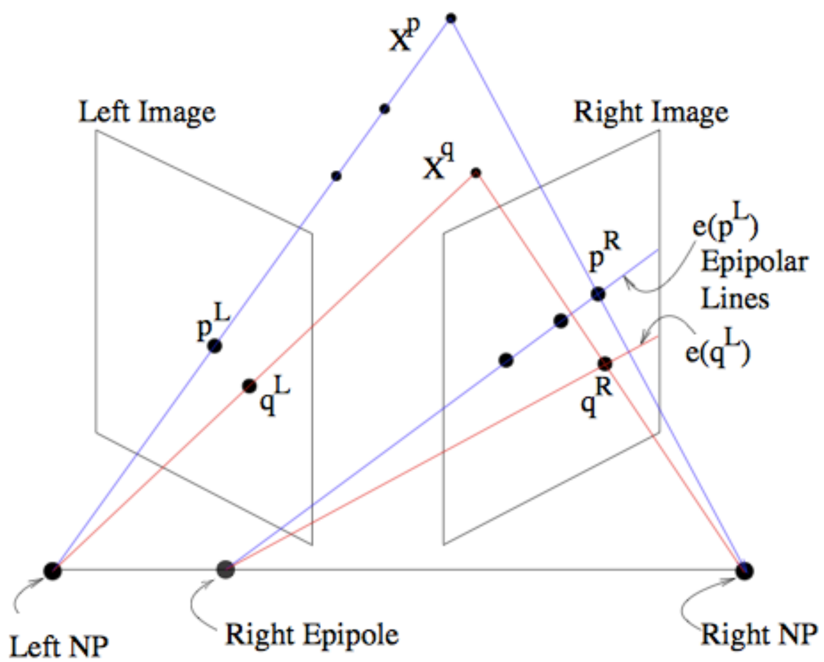
Projection, stereo and panoramic images

X^p and X^q are the physical location of objects.

NP are the nadir points of the cameras.



http://www.cs.toronto.edu/~jepson/csc420/notes/epiPolarGeom.pdf
(note, the image is posted on an educational site and copied here without following up on permissions. Any further use of the image should follow up on the origins and permissions.)

The projection of the left nadir, NP is seen as a the right epipole with w.r.t. the right image. The epipoles may or may not be within the border of the images.

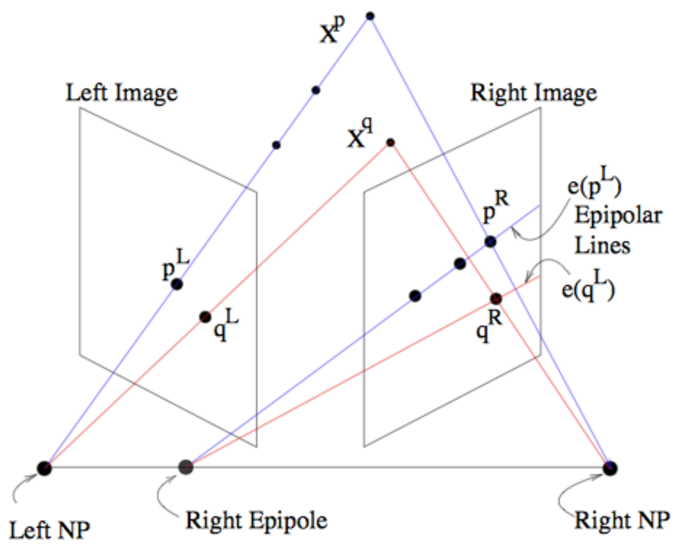
The projection of X^p to left nadir, NP is seen as an epipole line in the right image. If more than one epipole line is present in the right image, they converge at the right epipole (which might not be within the boundaries of the image).

Once the points in the left image, X_L are matched with points in the right image, X_R, if there are at least 7 points, one can determine the "bifocal tensor",
a.k.a. "fundamental matrix, relating the points in the 2 images using a 3x3 matrix of rank 2.

$(X_L)^T * F * X_R = 0$ for any pair or points in the images.

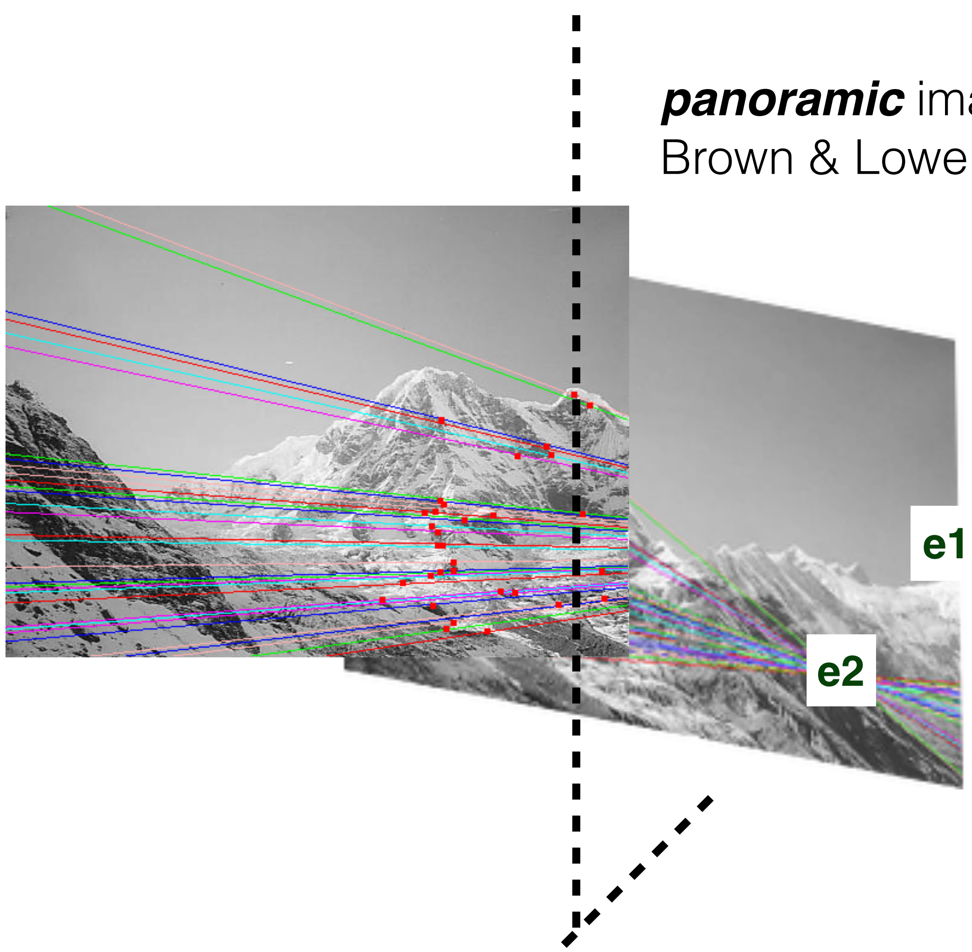
Note: the "Essential matrix" is a matrix used if the camera details are known. The "bifocal tensor", a.k.a. "fundamental matrix" does not need camera details.

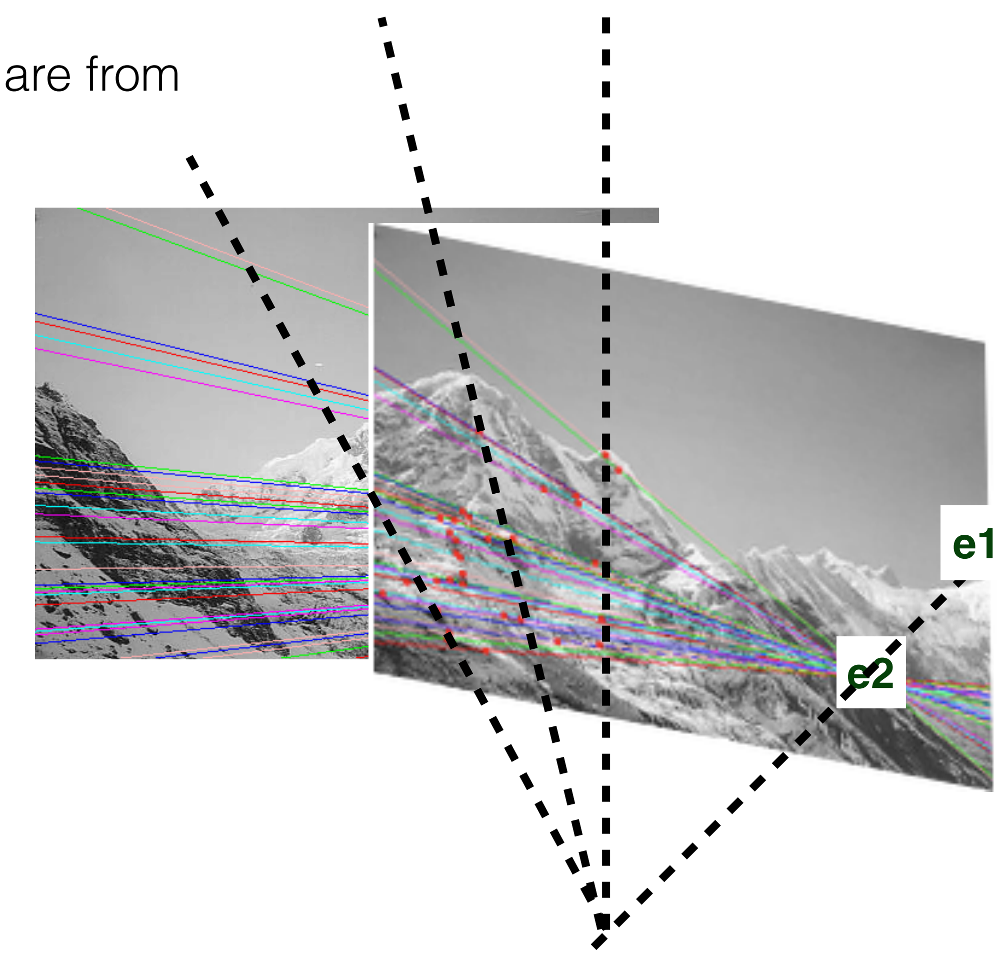
Projection, stereo and panoramic images



http://www.cs.toronto.edu/~jepson/csc420/notes/epiPolarGeom.pdf
(note, the image is posted on an educational site and copied here without following up on permissions. Any further use of the image should follow up on the origins and permissions.)

panoramic images here are from Brown & Lowe 2003



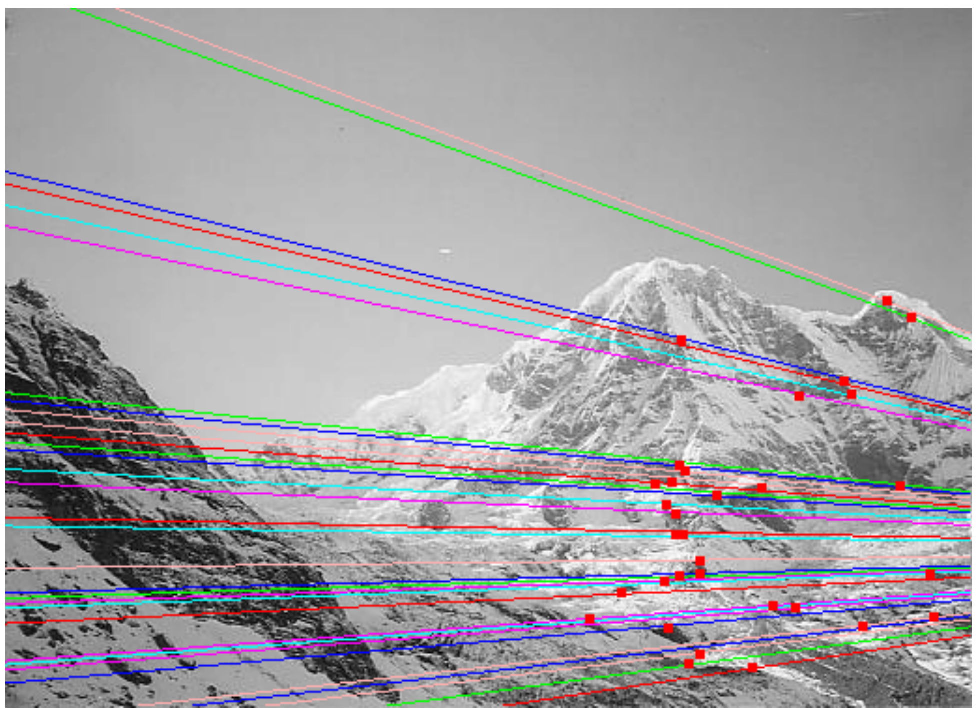


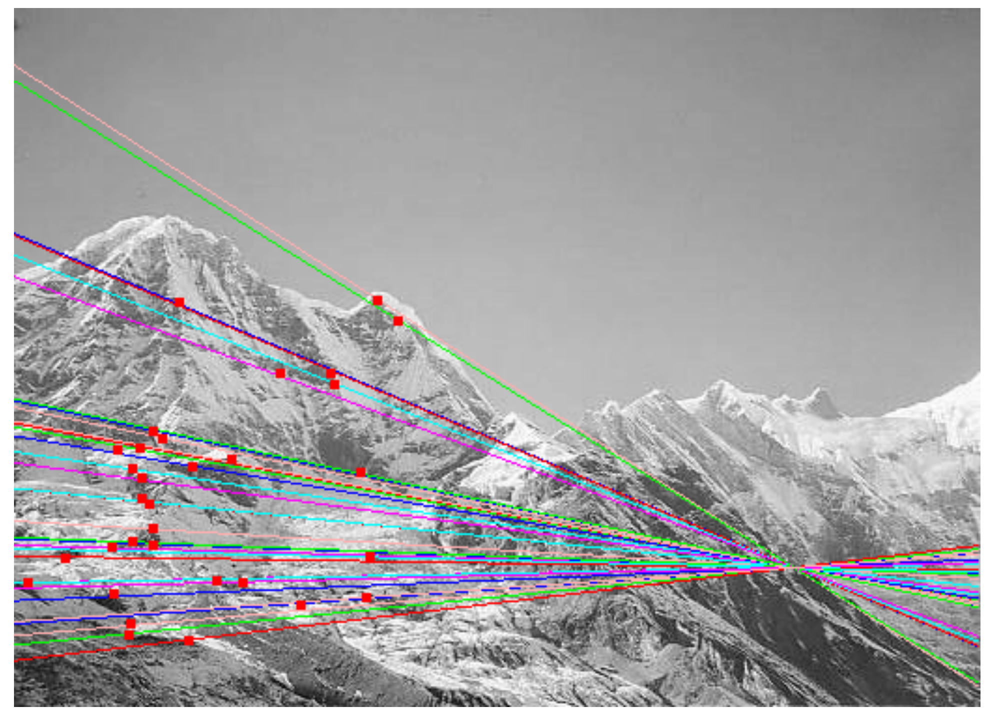
same camera objectives (nadirs), but different orientation for the 2 images (that is, rotated around the same nadir)

Point Correspondence

Need to create list of matched points between the images in order to solve for the epipolar projection

<u>manually making a point list from the corners from the edge extractor used with "outdoor mode":</u>

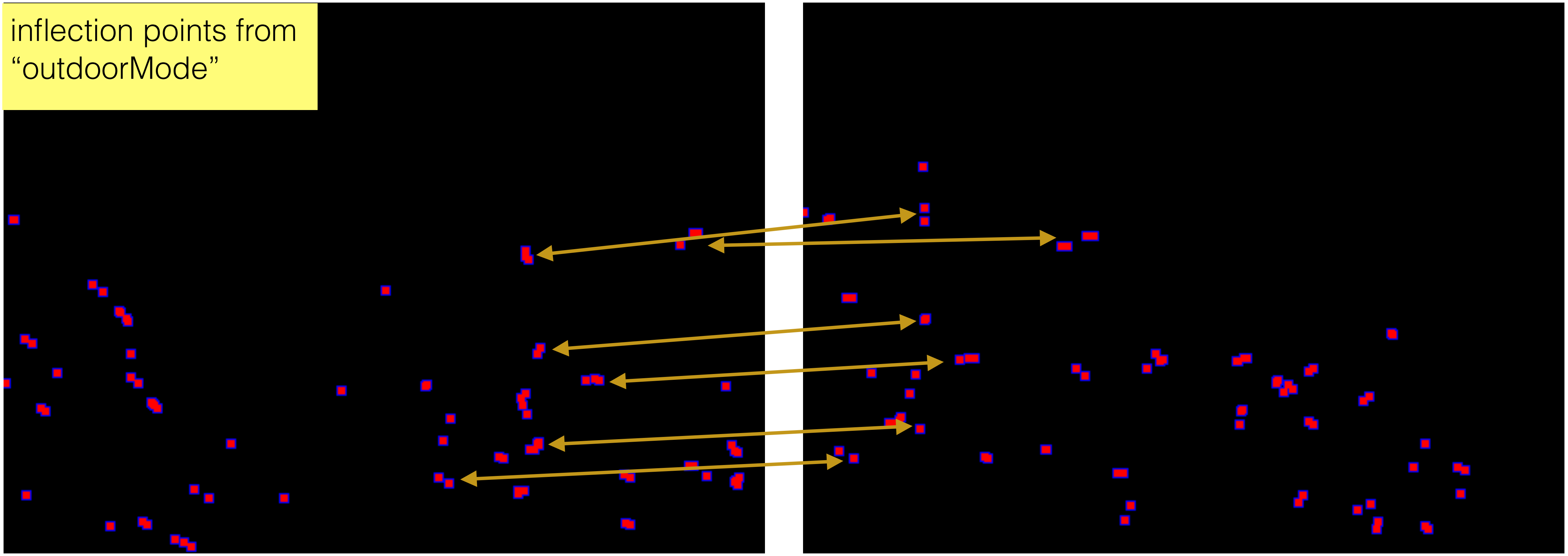




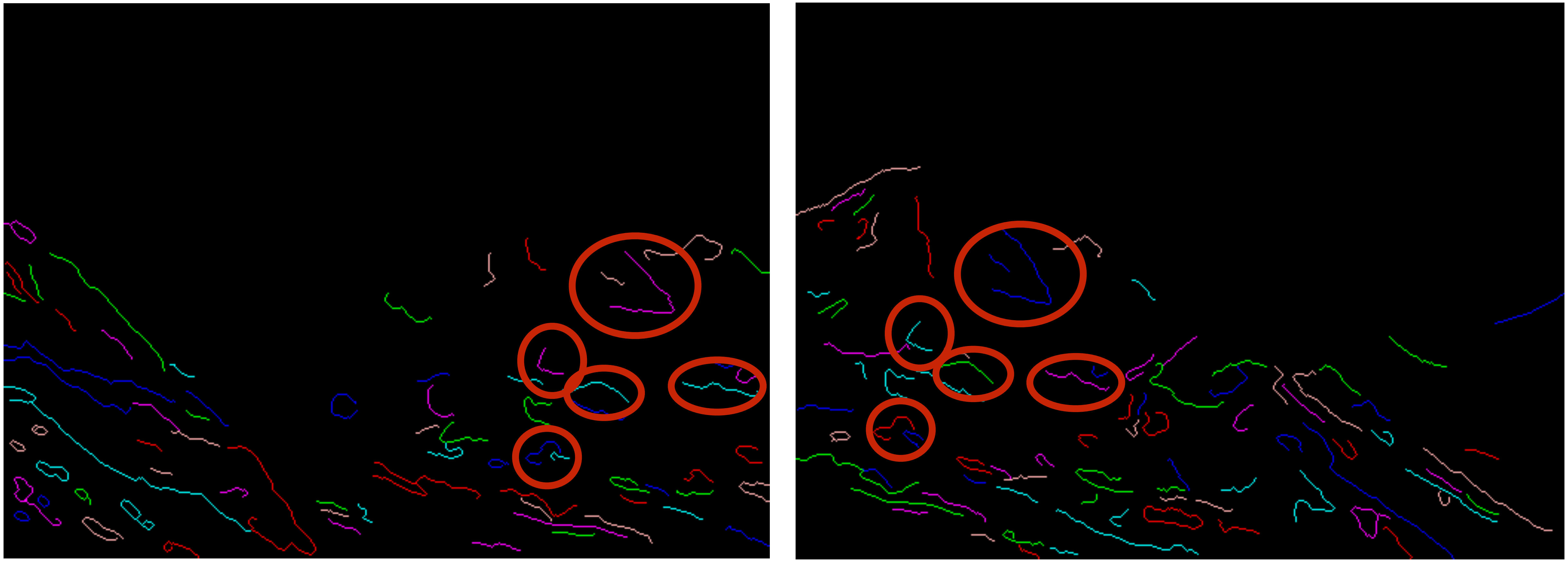
stereo projection fit to 32 points already known to match shows what the epipolar projections should be when the corner find + corner match + stereo projection solve are correctly automated.

```
nMatched=32
avgDist=0.281
stDev=0.508
```

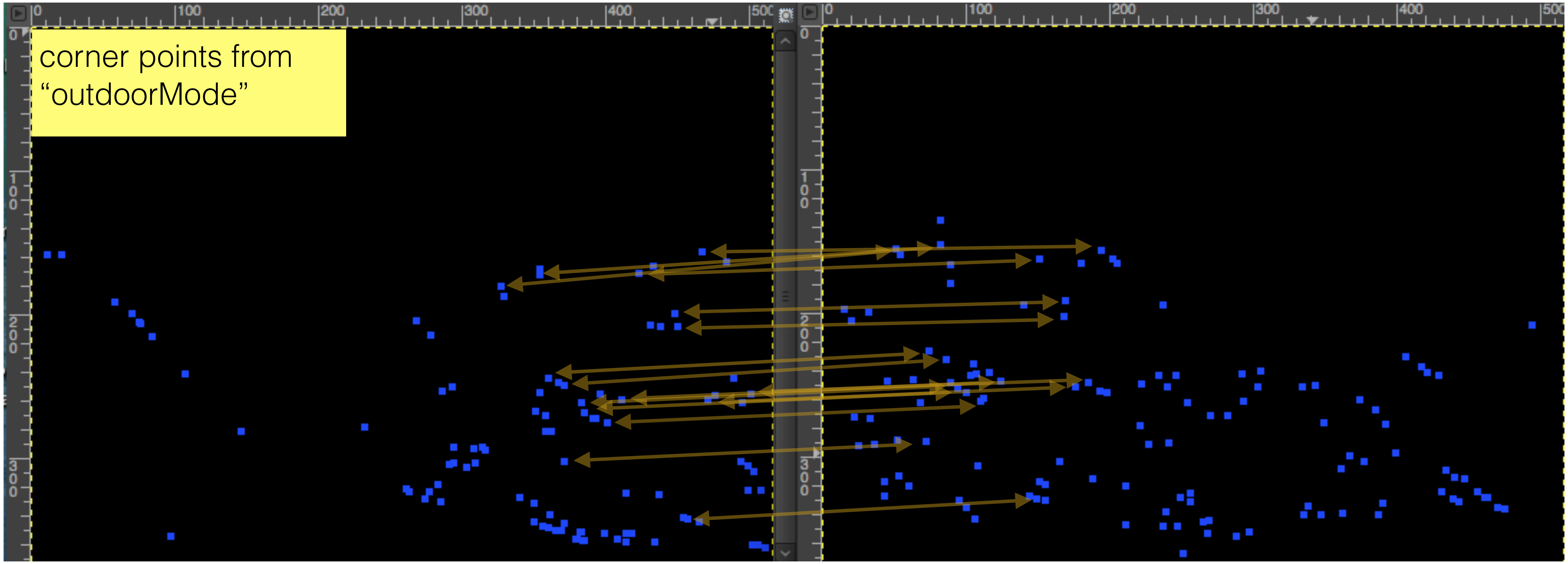
The Brown & Lowe 2003 images: point matching difficult because image intersection << difference



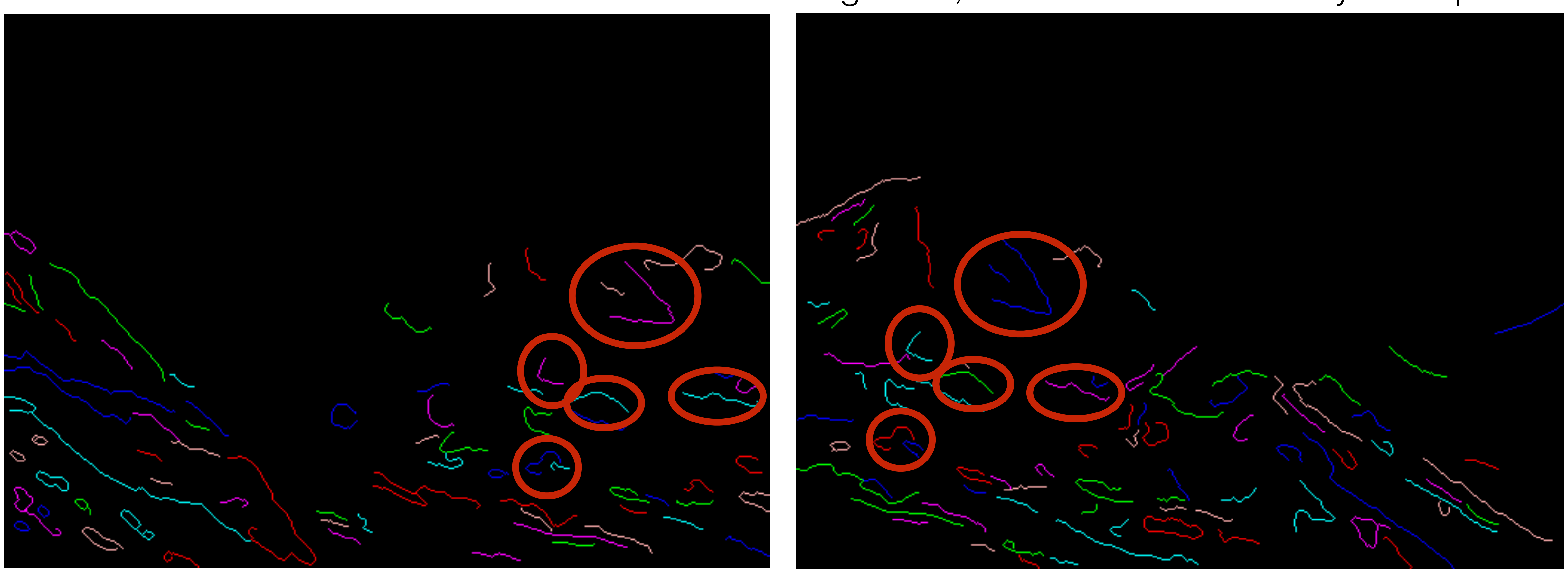
could consider distinct curves and their immediate neighbors, but that would be many more points:



The Brown & Lowe 2003 images: point matching difficult because image intersection < difference

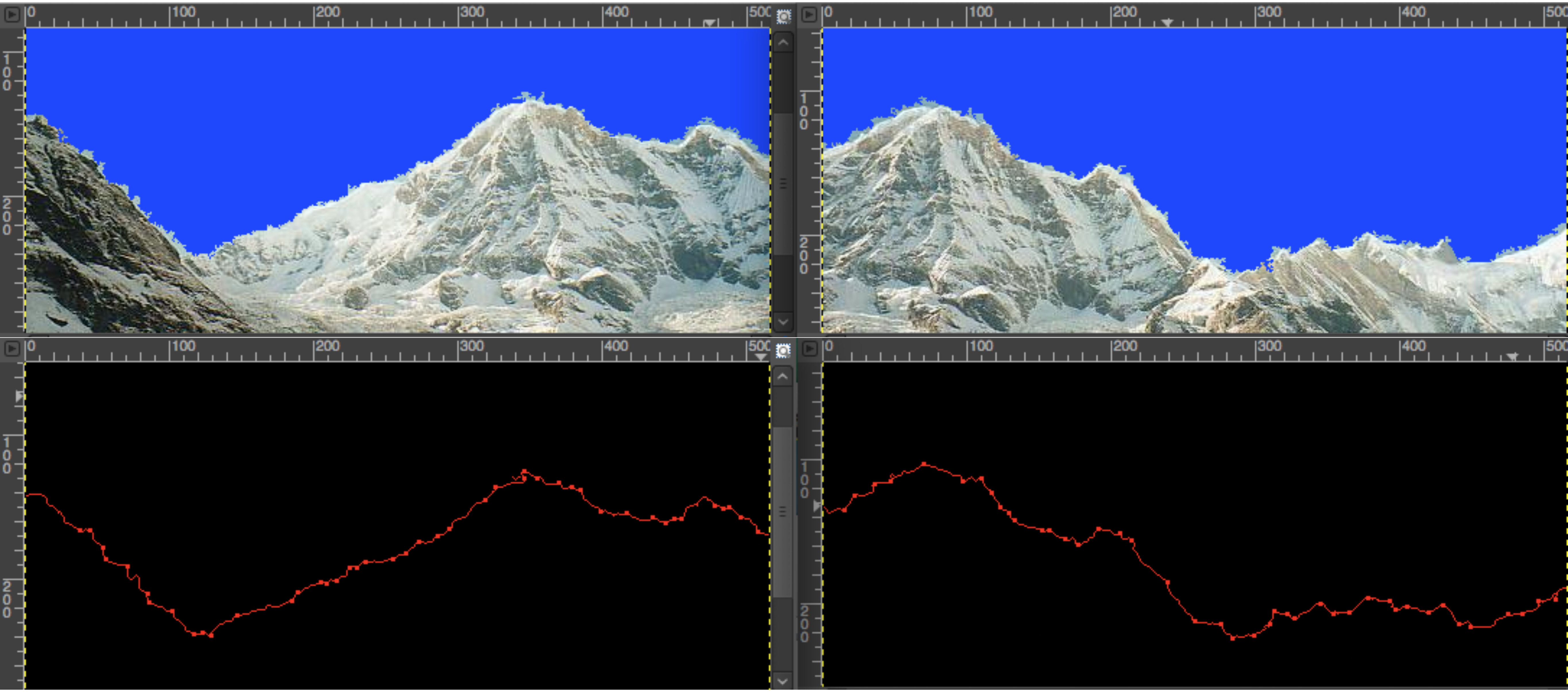


could consider distinct curves and their immediate neighbors, but that would be many more points:



For the outdoor images, can find the sky and create a sky mask and also create corners from just the skyline.
(see skyline_extraction.pdf)

This sky mask helps to pre-process the image before feature finding and correspondence.



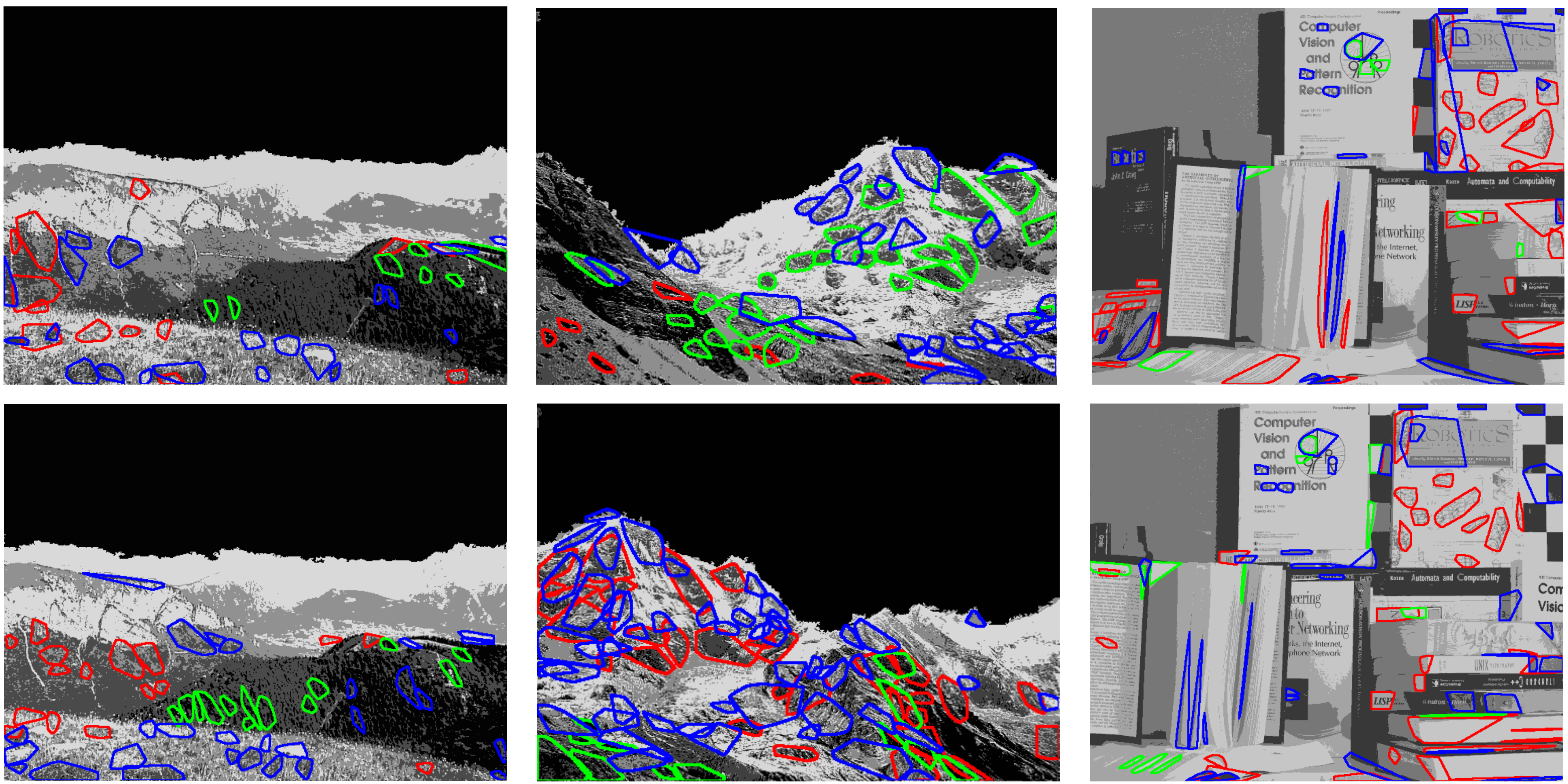
Trying blob feature creation and matching methods in next slides.

— perform histogram equalization if mean is too far from median or the two images are too different for those params.

— color segmentation of ***k=4*** to reduce image to 4 bands of intensities.

— contiguous pixel group finder for each of the 4 bands using point limits of: smallestGroupLimit = ***100***; largestGroupLimit = ***1000***;

Would prefer fewer features because the pairwise calculations of transformation are (N1*(N1-1)/2) * (N2*(N2-1)/2) which is roughly N^4.
Note that trying combinations of rotation and translation (and scale) increase with the image sizes but are very large numbers too for reasonable accuracy.



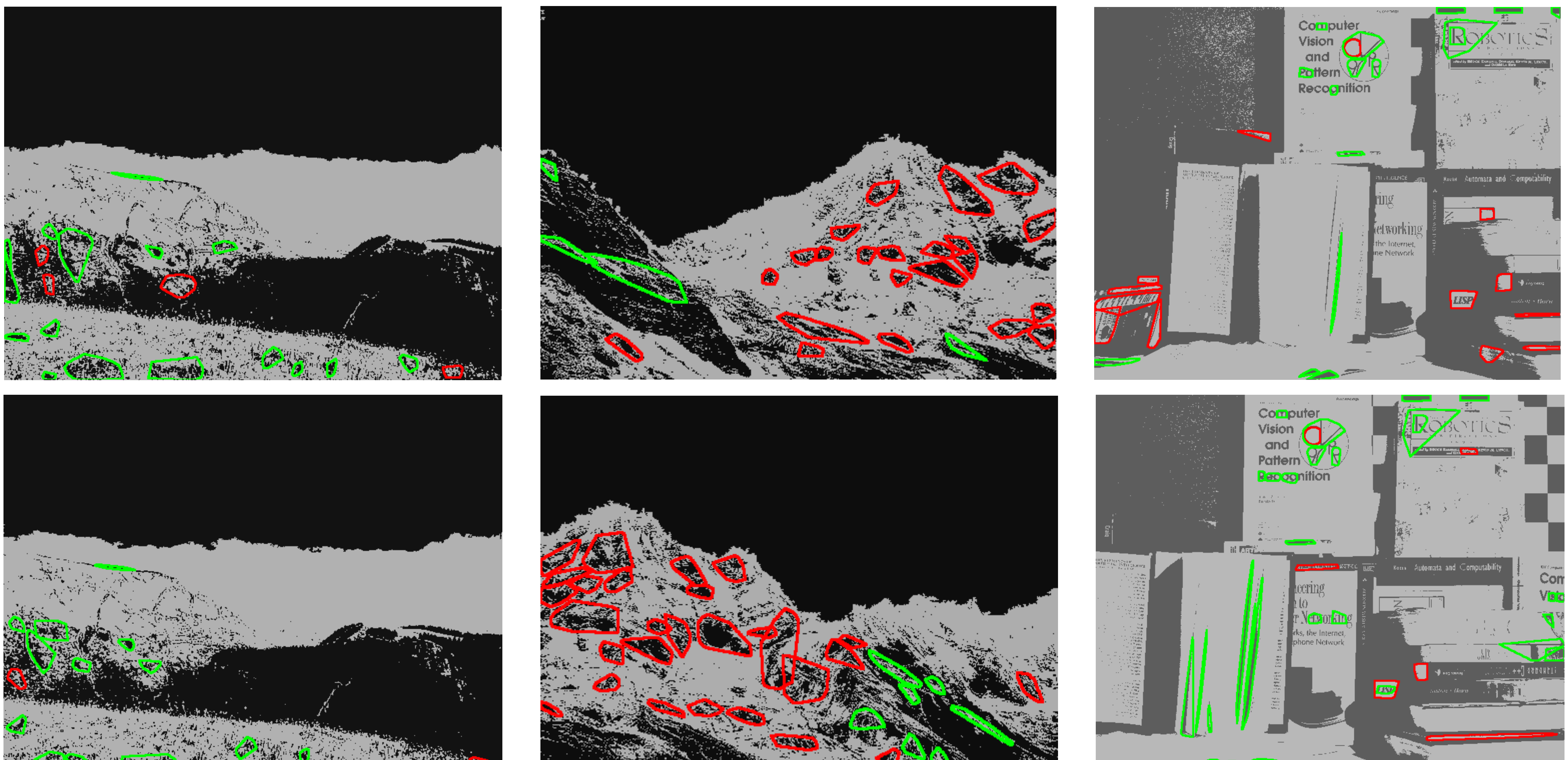
Even better:

— perform histogram equalization if mean is too far from median or the two images are too different for those params.

— color segmentation of ***k=2*** to reduce image to 2 bands of intensities.

— contiguous pixel group finder for each of the 2 bands using point limits of: smallestGroupLimit = ***100***; largestGroupLimit = ***1000***;

This is a smaller number of features that almost has all of the major blobs one would want to use for matching.



Even better:

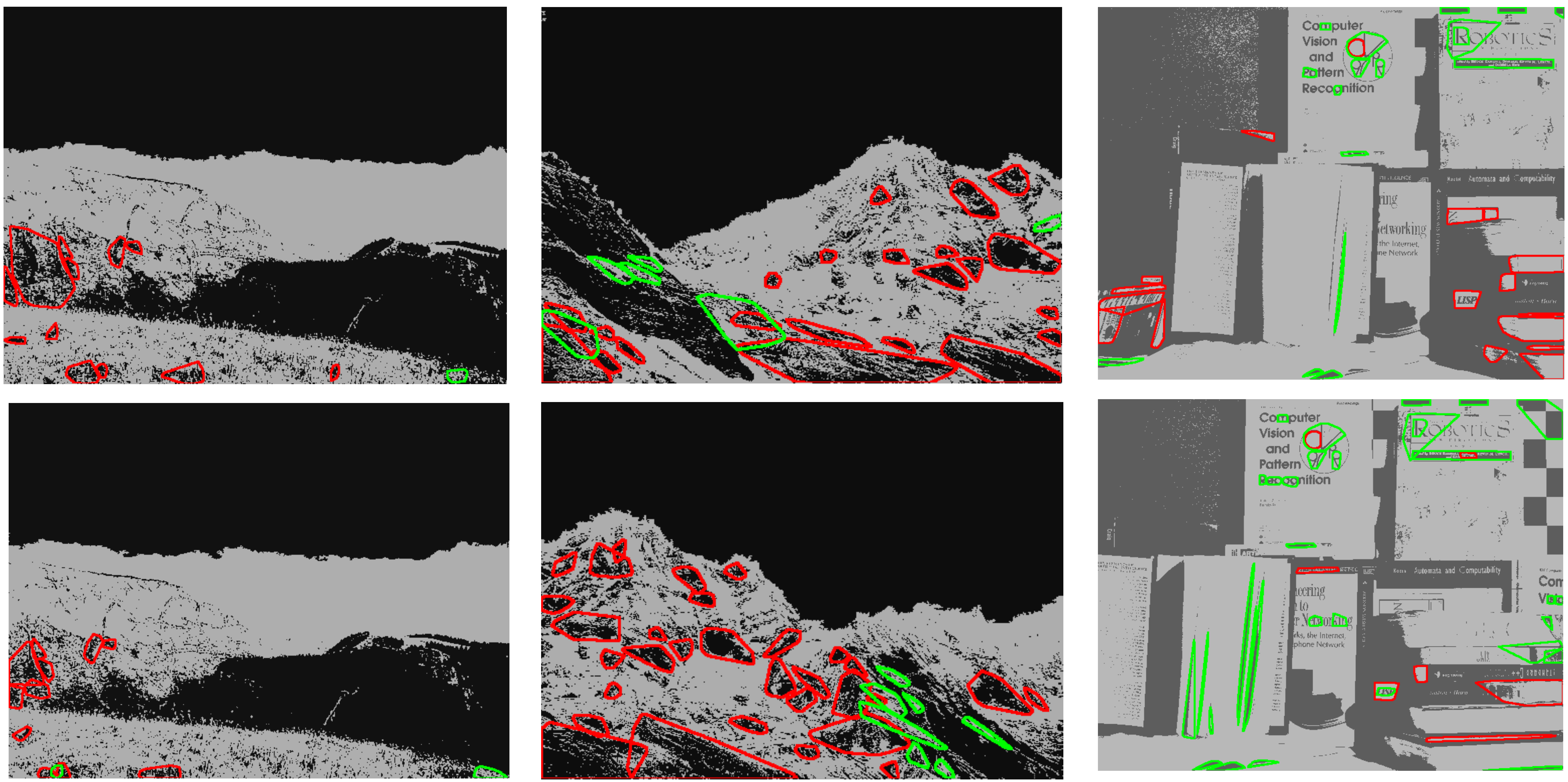
— perform histogram equalization if mean is too far from median or the two images are too different for those stats.

— color segmentation of ***k=2*** to reduce image to 2 bands of intensities.

— contiguous pixel group finder for each of the 2 bands using point limits of: smallestGroupLimit = ***100***; largestGroupLimit = ***5000***;

correspondence looks like enough for an initial Euclidean solution

NOTE that blobs as first transformation solution works better for images like the middle where in contrast if done with corners, the ridge line in the differences would match more strongly than the true corner matches.



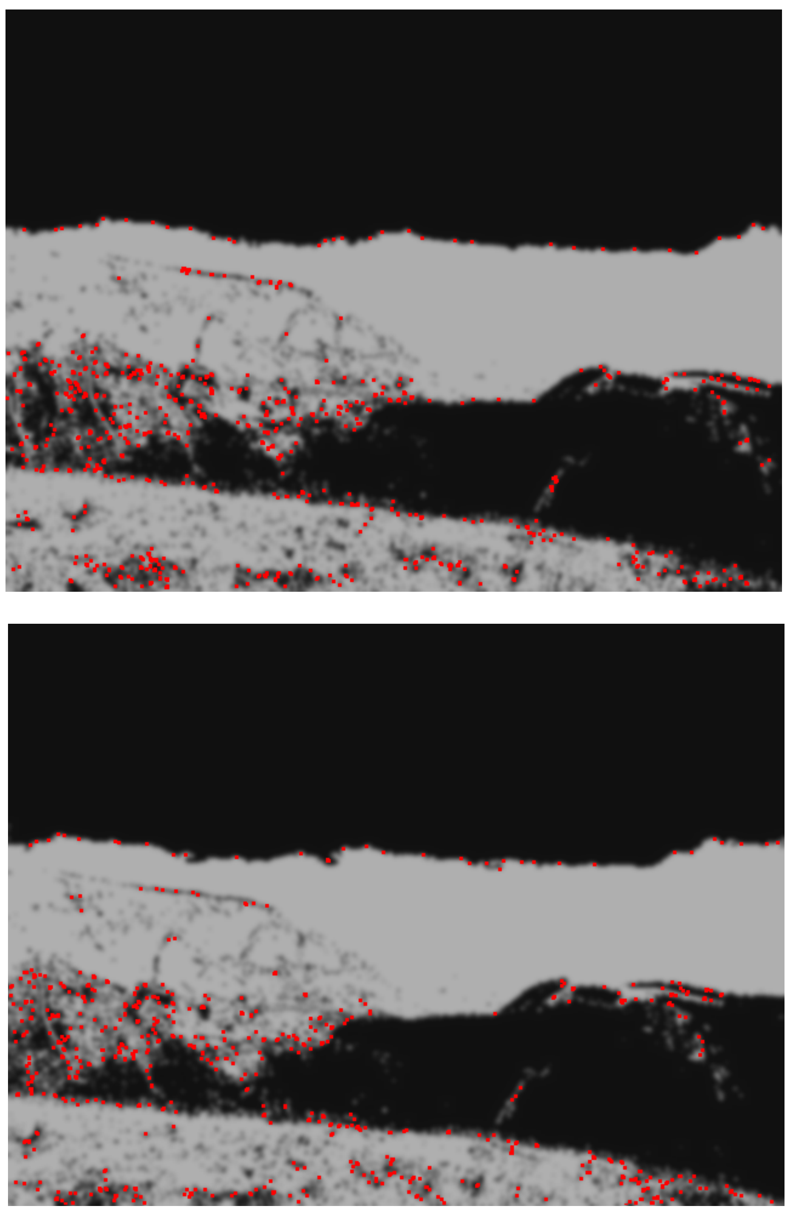
<u>After Euclidean transformation calculation, need to make lists of matching points for input to epipolar projection solver.</u>

— gaussian blur with sigma=2
— scale space curvature corners

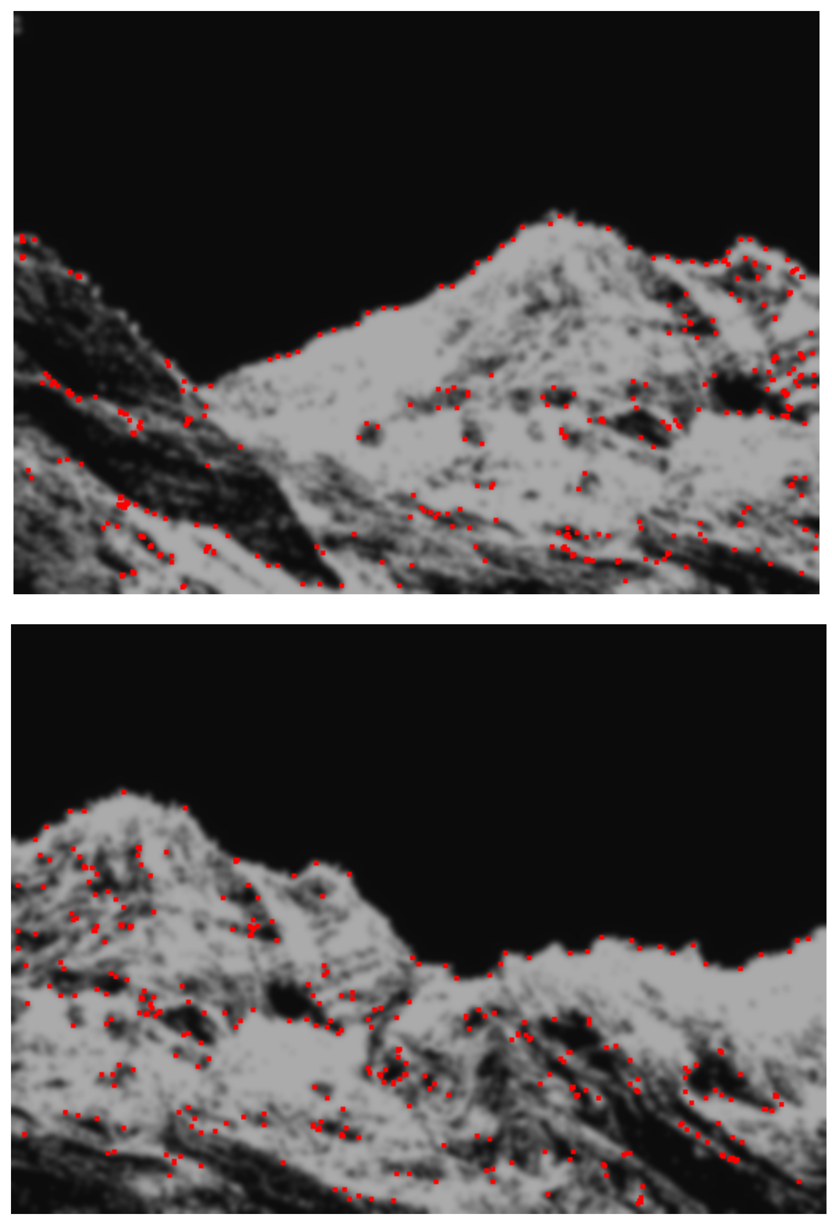
While finding the matches, may need to increase the tolerance or change the solution slightly across the image due to projection.

With the transformation already roughly solved, making point lists even for a large number of corners is fast.

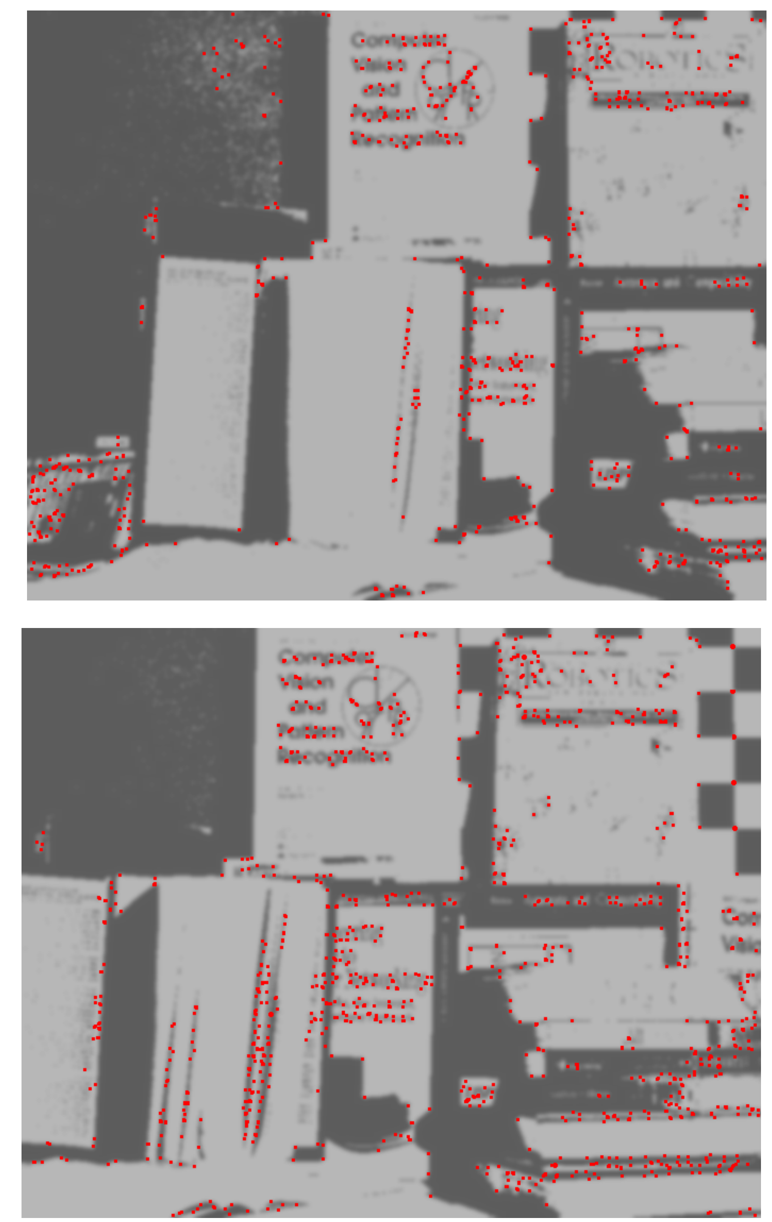
panoramic image pair



panoramic image pair



rectified image pair



<u>Alternatively for the blob (feature) finding, can reduce the image size:</u>

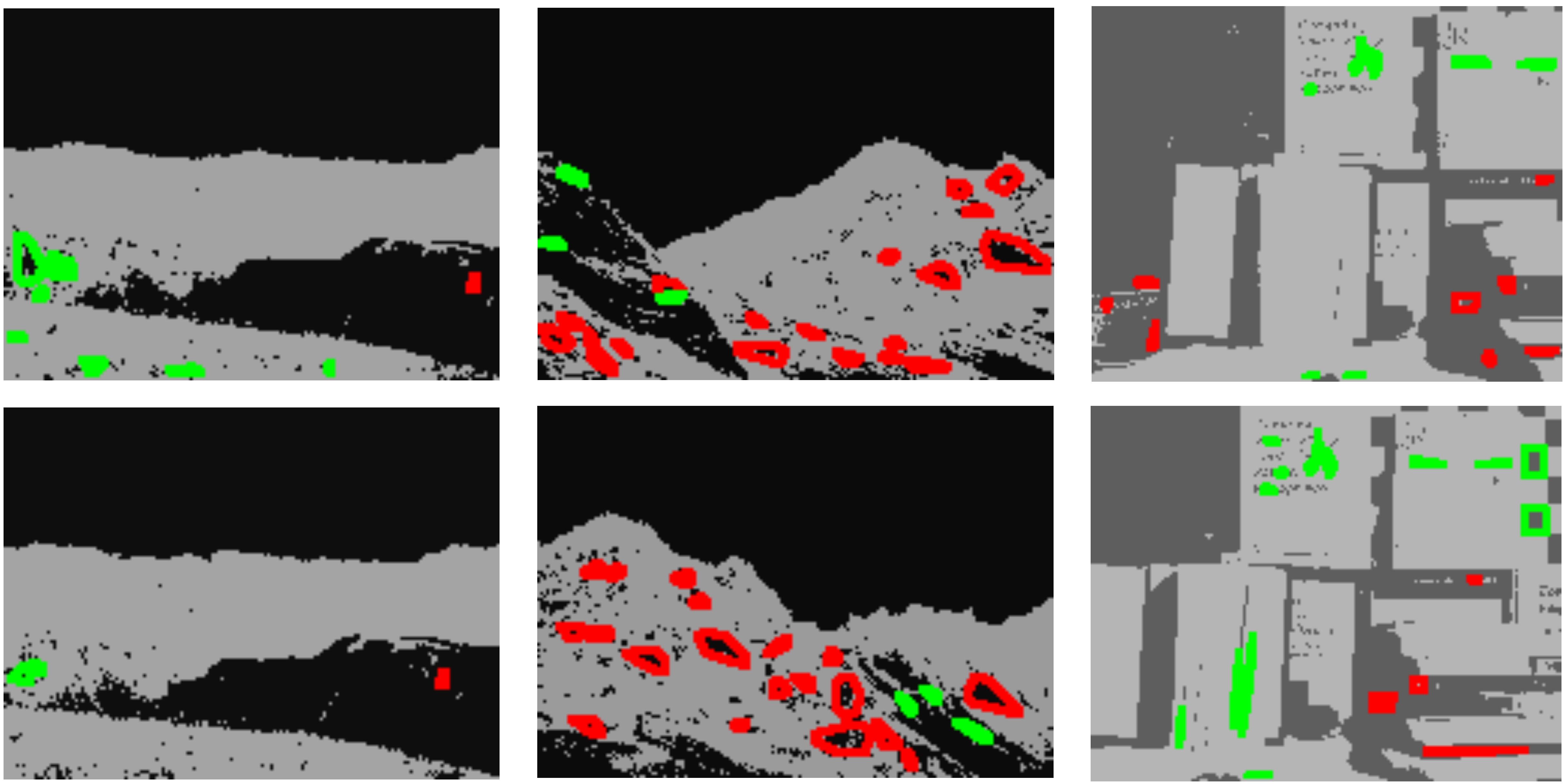
— perform histogram equalization if mean is too far from median or the two images are too different for those params.

— ***bin the image to < 200 X 200***

— color segmentation of ***k=2*** to reduce image to 2 bands of intensities.

— contiguous pixel group finder for each of the 2 bands using point limits of:
smallestGroupLimit = ***100/(binFactor^2)***; largestGroupLimit = ***5000/(binFactor^2)***;

This solution doesn't have a better matchable to non-matchable blob fraction than the full image, so will not use the binned processing.

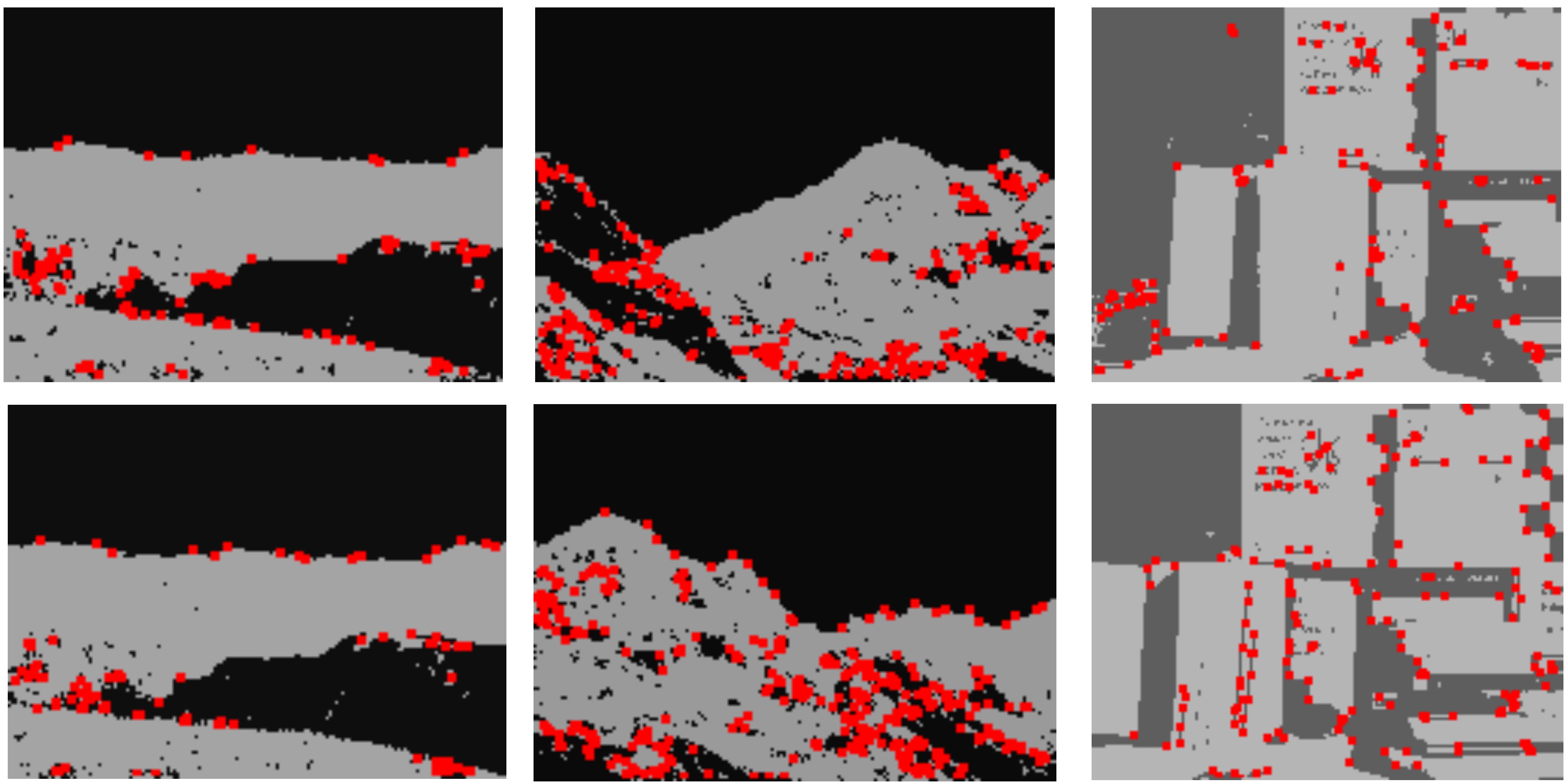


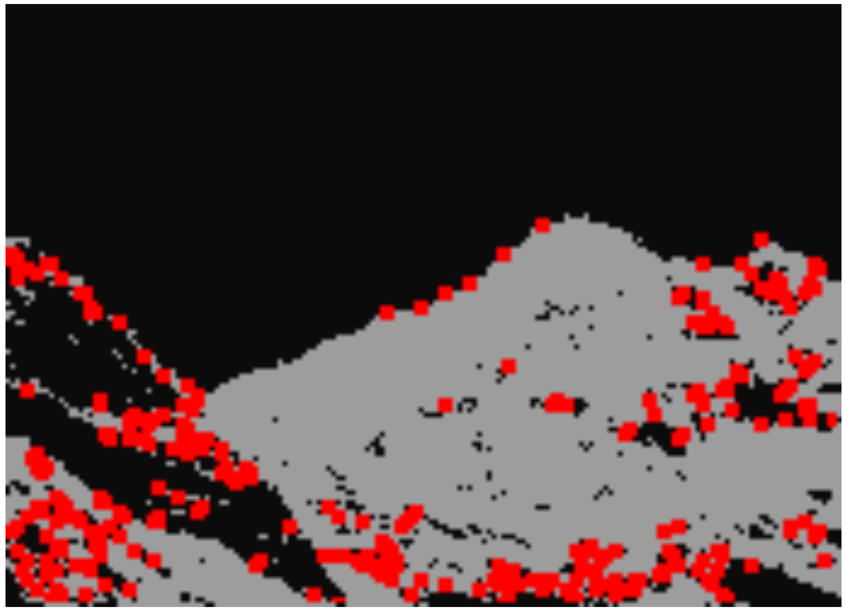
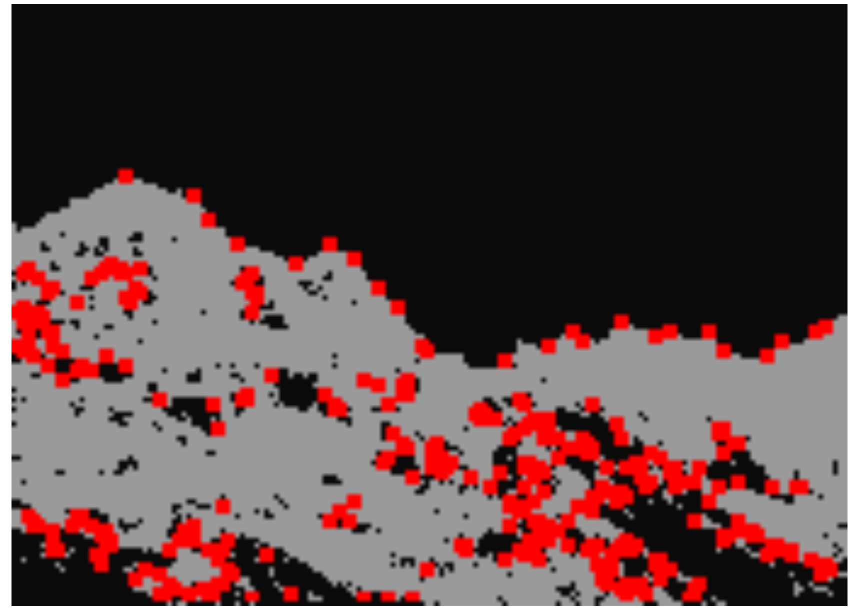
<u>After Euclidean transformation from blobs, need to make lists of matching points for input to epipolar projection solver.</u>

— No gaussian blur
— scale space curvature corners

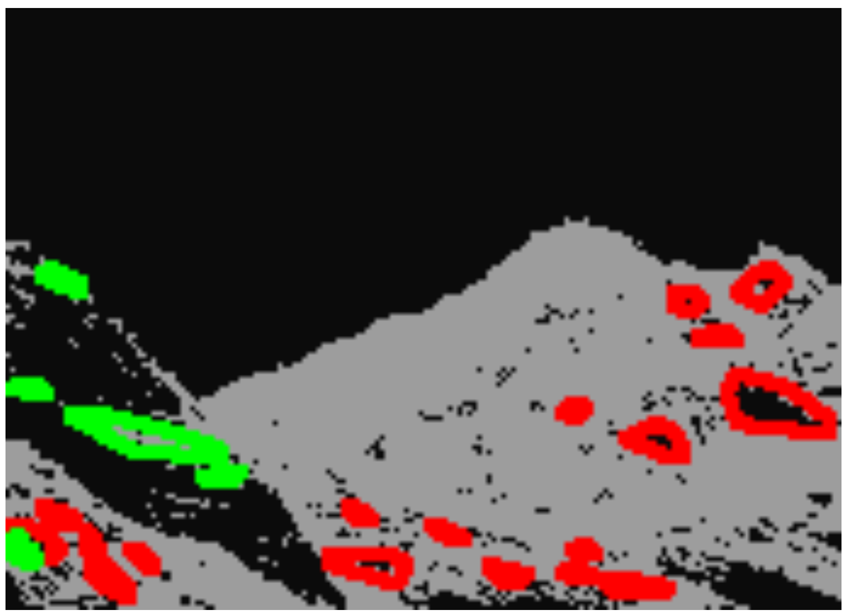
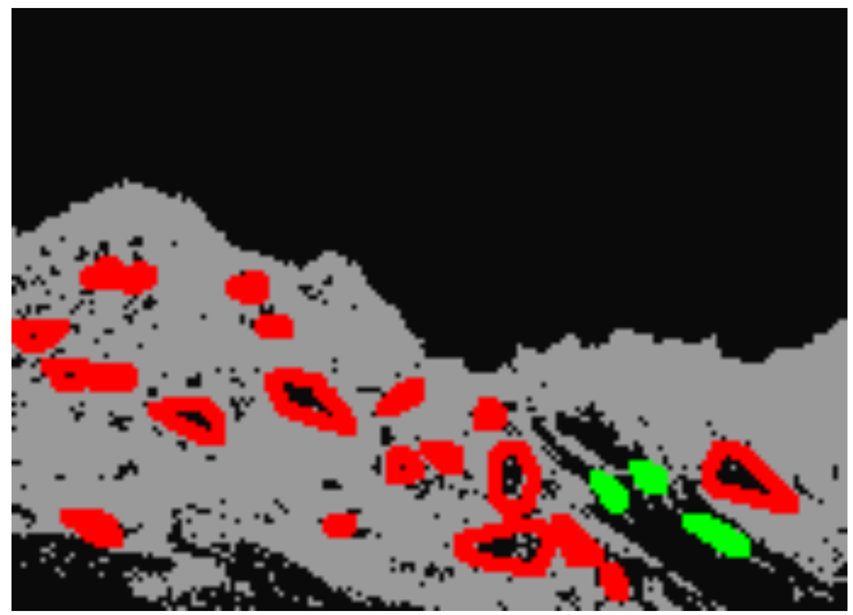
Note that this stage looks like it it best to perform using the full image, even though solving the Euclidean with the binned features is still a good first stage.

The binned blob features do not look better than the full image size blobs, but here are the corners from them just to have followed it through.

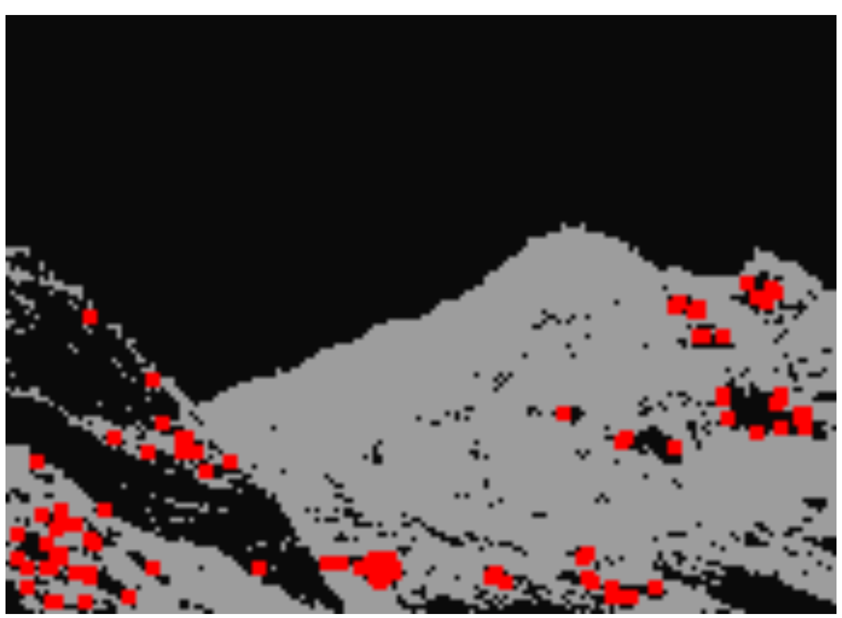
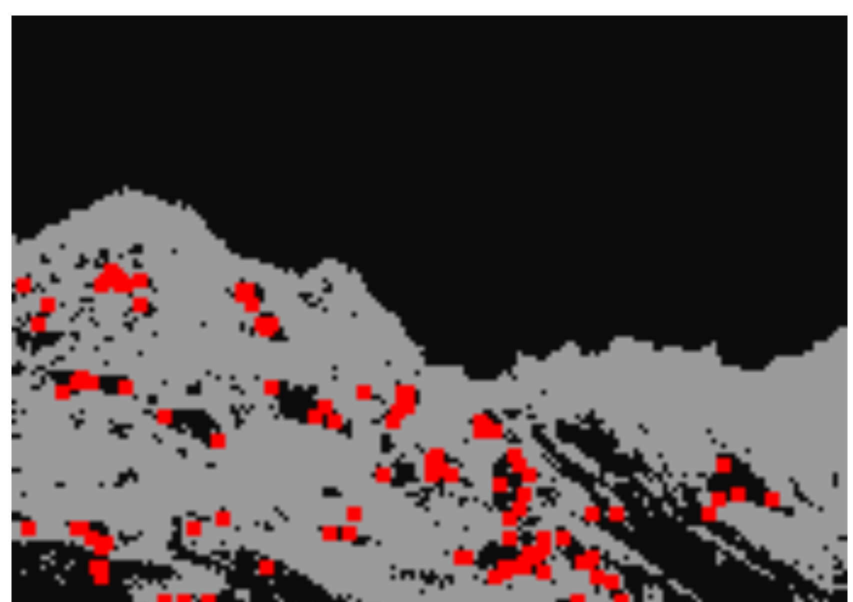


corners from images pre-processed by segmentation and binning (not using a Gaussian pyramid yet, but will later).

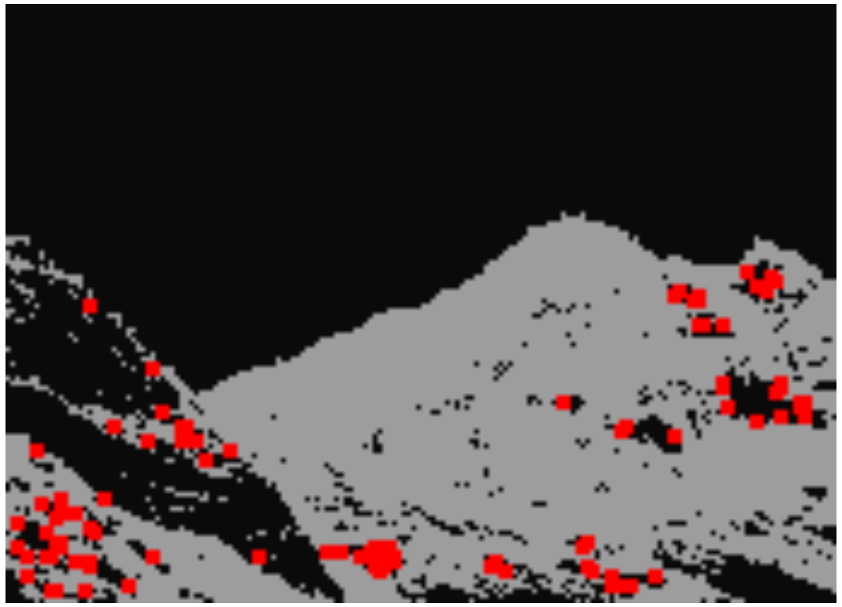
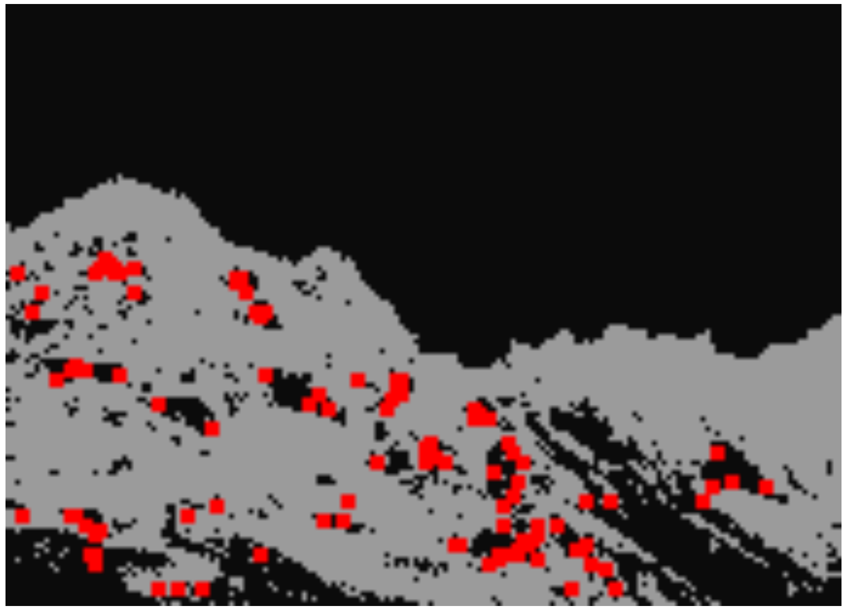



contiguous regions within size ranges were found then convex hulls constructed around them.

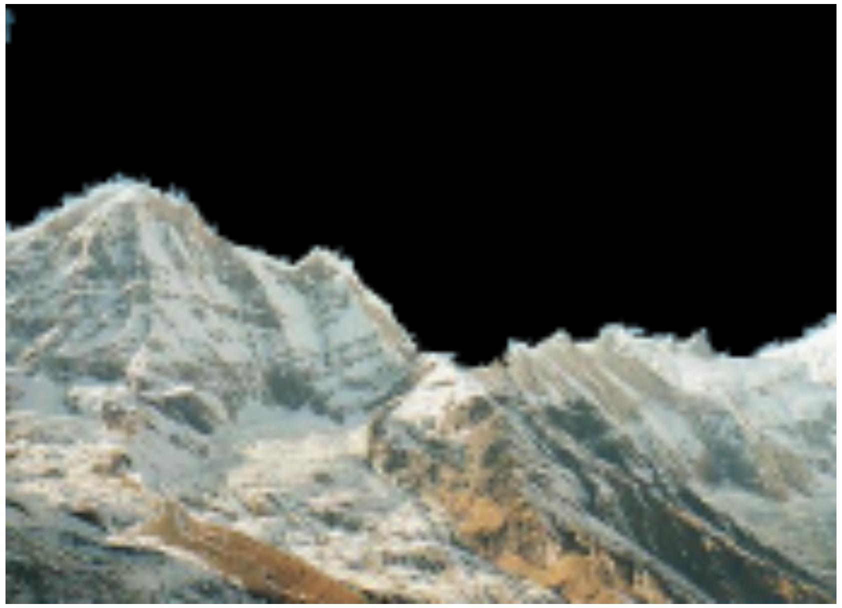



corners filtered to those hulls to reduce the corner list to regions of interest.

the number of possible true matches is less than half of the total number of corners, but feature matching with color image descriptors should help match them.

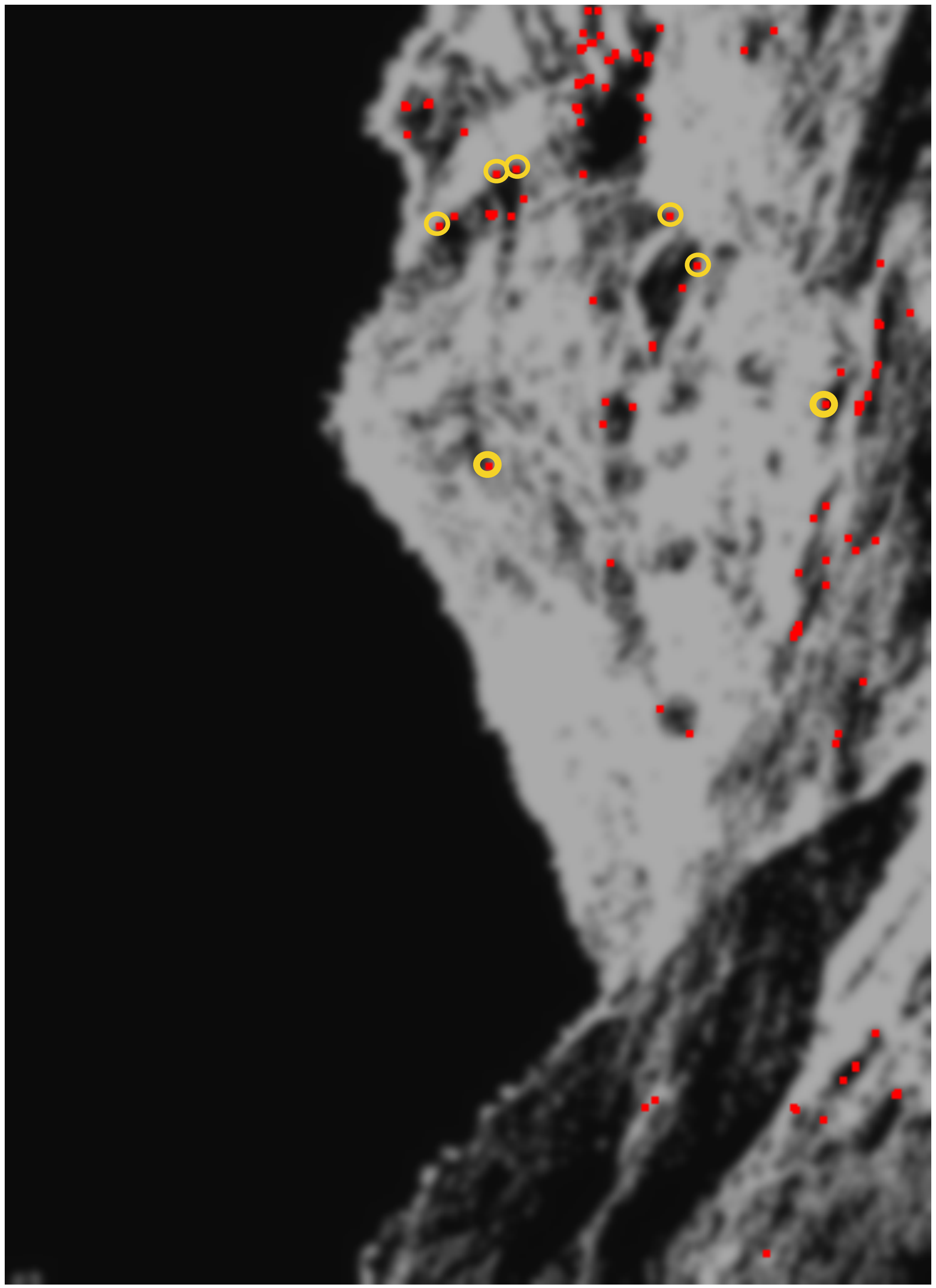



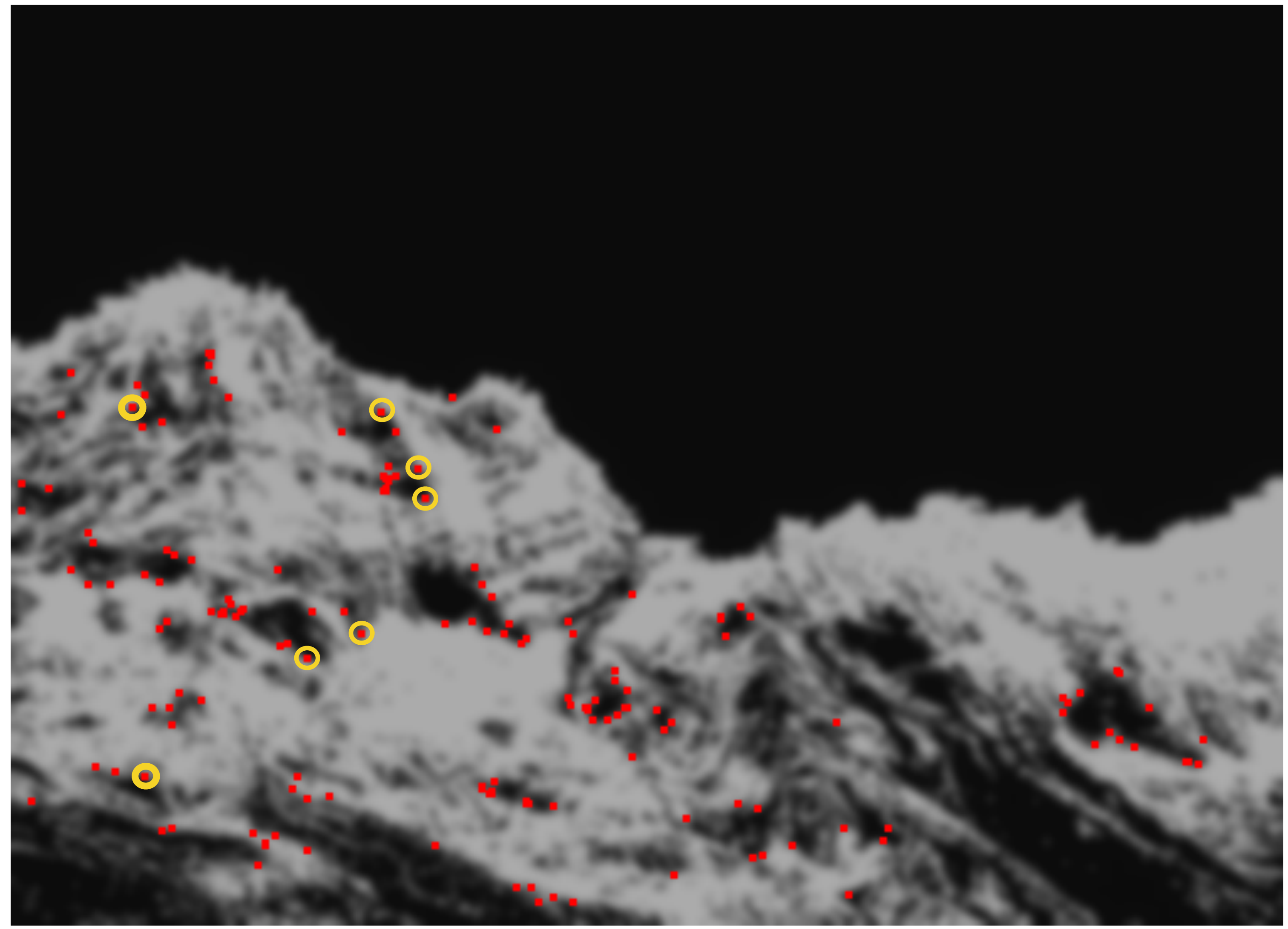
corners filtered

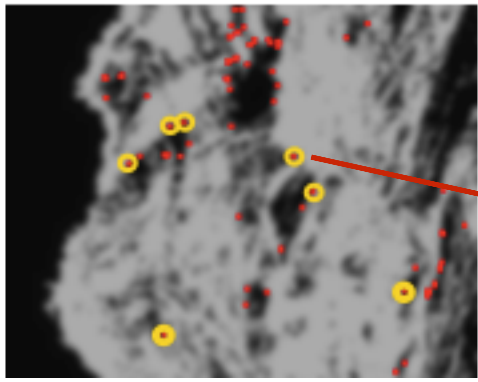
need to determine an orientation of the corner region which is consistent w.r.t. the corner in any image.

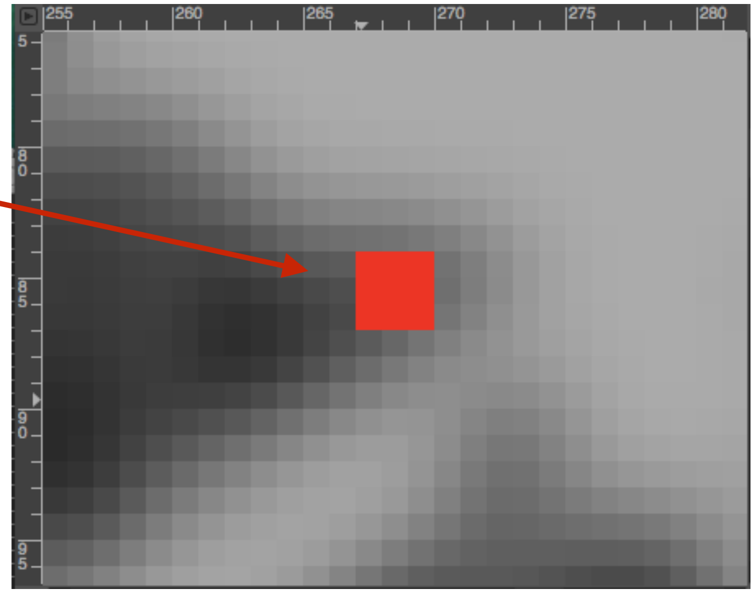
Trying with the full images first to make sure the method and math are correct, then will see if can derive the same answer using the binned images.

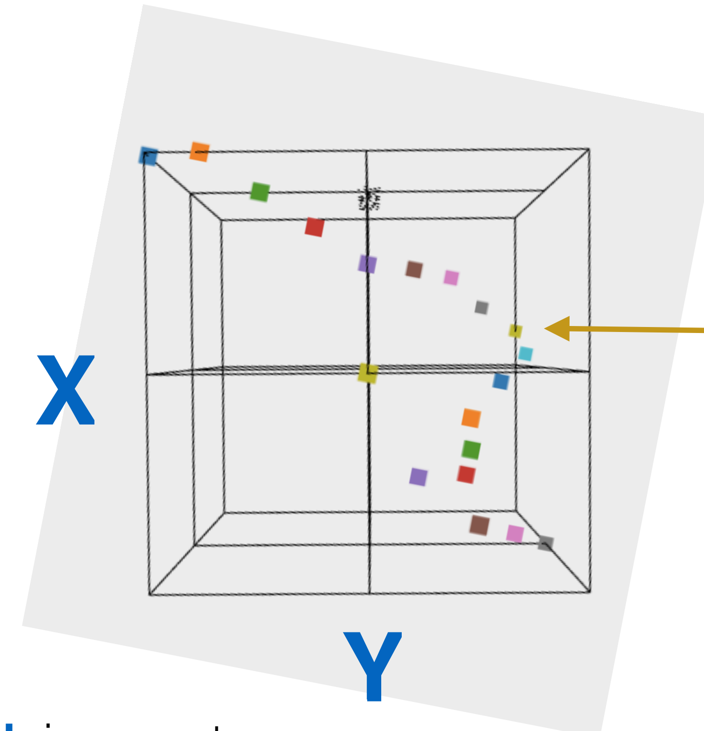




orientation: since the corners are made with CSS the edge information already exists and can be used here.

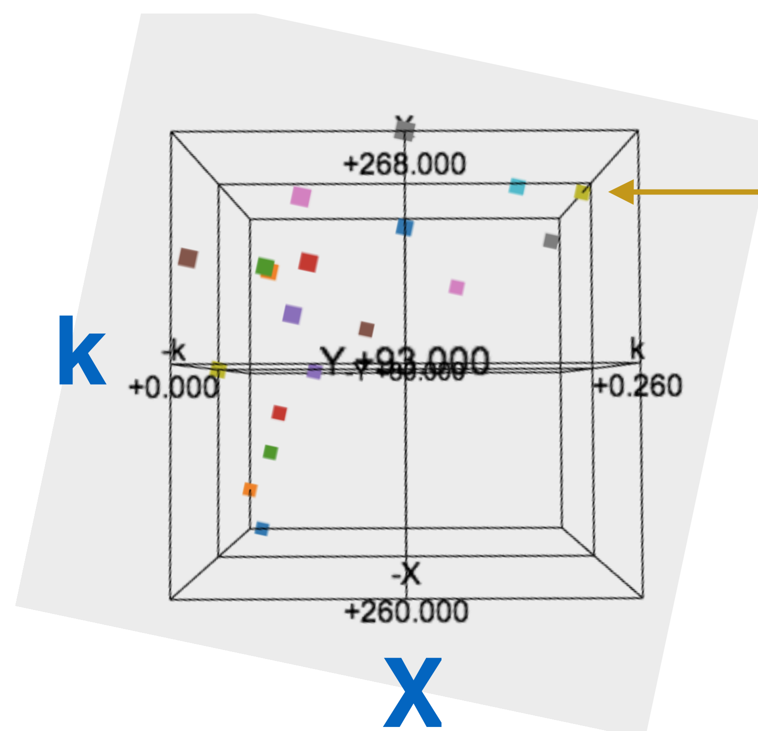






k is curvature

maximum in curvature

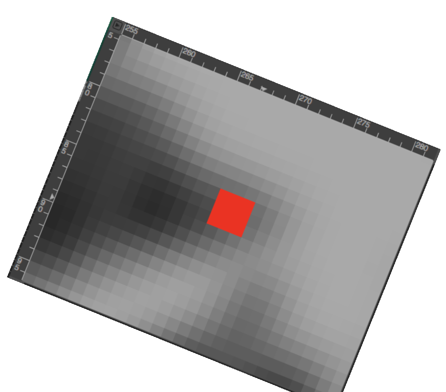


A vector perpendicular to k_max on this edge can be used to calculate the orientation of this region (where the region is defined by being a CSS corner).

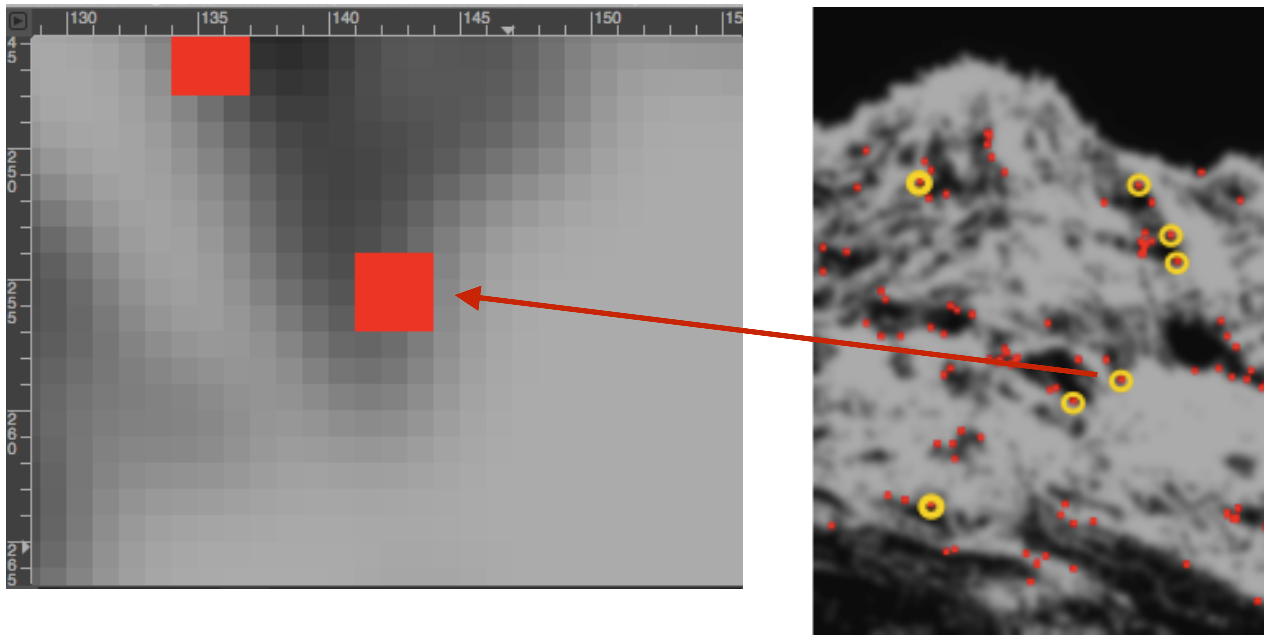
```
deriving the perpendicular angle from the points directly to
the left and right of the maximum of curvature for points which
have curvature
k       x     y
0.24, 267, 84          dx,dy=(1,1), perp=45
0.26, 268, 85 <—max k
0.21, 268, 86          dx,dy=(0,1), perp==0

orientation = (45 + 0)/2. = 22.5
```

```
extract curvature
from the highest
resolution
scale space curves:

idx=164 (268.0,85.0)
k            x    y
0.08  for (258, 79)
0.04  for (259, 79)
0.01  for (260, 80)
0.00  for (261, 80)
0.02  for (262, 81)
0.03  for (263, 82)
0.06  for (264, 83)
0.10  for (265, 83)
0.17  for (266, 83)
0.24  for (267, 84)
0.26  for (268, 85)
0.21  for (268, 86)
0.13  for (267, 87)
0.04  for (266, 88)
0.04  for (266, 89)
0.07  for (266, 90)
0.06  for (265, 90)
0.00  for (266, 91)
0.07  for (267, 92)
0.13  for (268, 93)
0.15  for (268, 94)
```

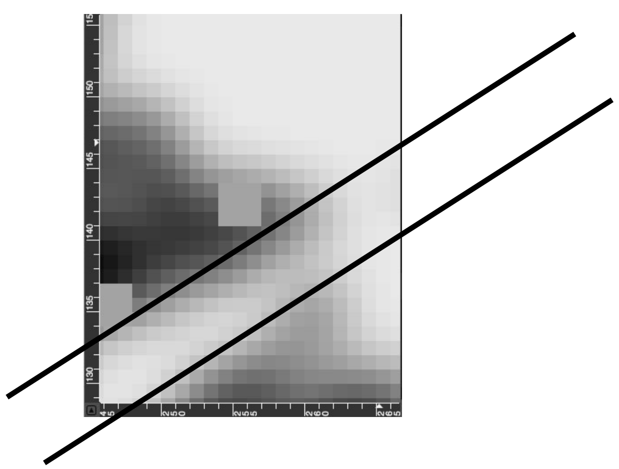
rotated



orientation.



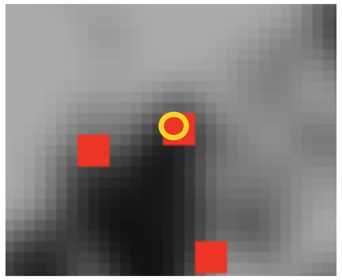
```
extract the final curvature from the highest resolution
scale space curves:
idx=112 (143.0,255.0)

k[110]=0.01  for (143, 253)
k[111]=0.13  for (143, 254)
k[112]=0.34  for (143, 255)  dx,dy=(-1,0)
k[113]=0.43  for (142, 255) <-- k_max
k[114]=0.35  for (141, 255)  dx,dy=(-1,0)
k[115]=0.28  for (140, 255)
k[116]=0.24  for (139, 255)
```



orientation =((-90)+(-90))/2. = -90

rotated

orientation. corners from Brown & Lowe 2003 left and right panorama images

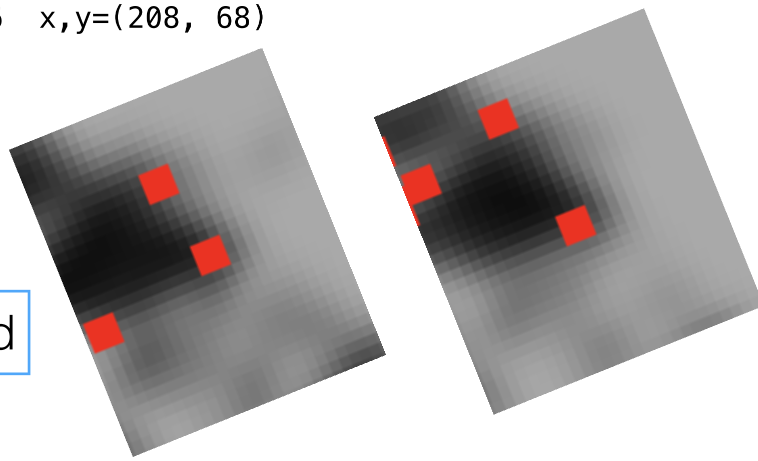


```
k[0]=0.25  x,y=(204, 66)
k[1]=0.32  x,y=(205, 66)
k[2]=0.32  x,y=(206, 66)
k[3]=0.30  x,y=(207, 67)
k[4]=0.26  x,y=(208, 68)
```

```
k[0]=0.22  x,y=(169, 197)
k[1]=0.32  x,y=(169, 198)
k[2]=0.41  x,y=(169, 199)
k[3]=0.38  x,y=(168, 200)
k[4]=0.27  x,y=(167, 200)
```

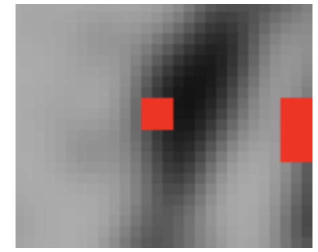
67.5

rotated

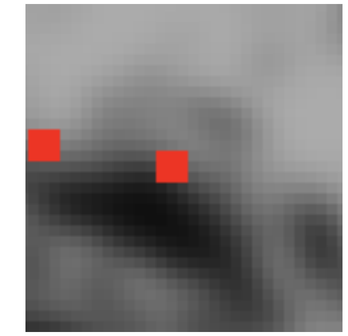


337.5

rotated



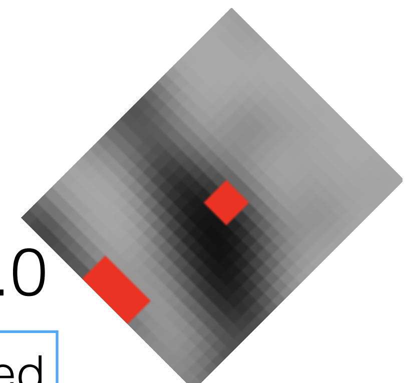
```
k[0]=0.11  x,y=(332, 161)
k[1]=0.37  x,y=(331, 161)
k[2]=0.58  x,y=(330, 161)
k[3]=0.47  x,y=(330, 162)
k[4]=0.30  x,y=(330, 163)
```

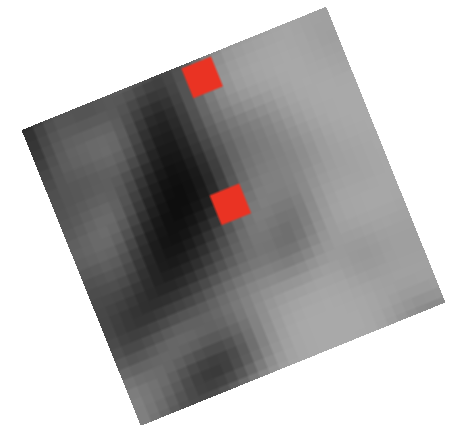


```
k[0]=0.07  x,y=(56, 315)
k[1]=0.09  x,y=(55, 314)
k[2]=0.31  x,y=(54, 313)
k[3]=0.23  x,y=(53, 313)
k[4]=0.03  x,y=(52, 313)
```

135.0

rotated



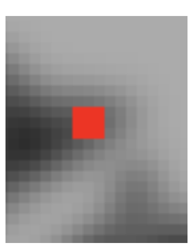


67.5

rotated

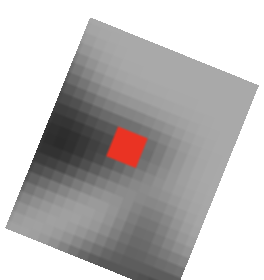
```
k[0]=0.17  x,y=(266, 83)
k[1]=0.24  x,y=(267, 84)
k[2]=0.26  x,y=(268, 85)
k[3]=0.21  x,y=(268, 86)
k[4]=0.13  x,y=(267, 87)
```

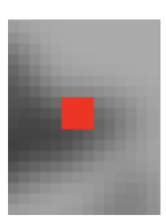
```
k[0]=0.13  x,y=(143, 254)
k[1]=0.34  x,y=(143, 255)
k[2]=0.43  x,y=(142, 255)
k[3]=0.35  x,y=(141, 255)
k[4]=0.28  x,y=(140, 255)
```



22.5

rotated



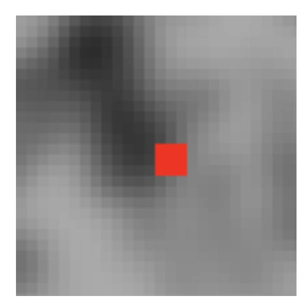


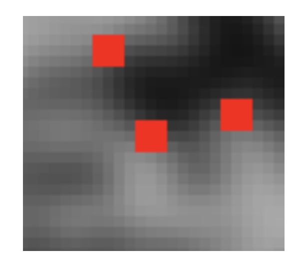
270.0

rotated

```
k[0]=0.16  x,y=(195, 183)
k[1]=0.26  x,y=(195, 184)
k[2]=0.32  x,y=(195, 185)
k[3]=0.29  x,y=(195, 186)
k[4]=0.25  x,y=(194, 187)
```

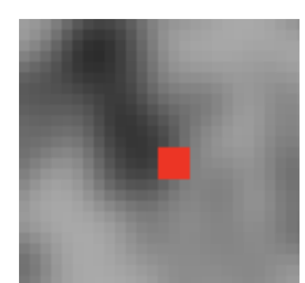
```
k[0]=0.20  x,y=(52, 170)
k[1]=0.23  x,y=(53, 171)
k[2]=0.25  x,y=(54, 171)
k[3]=0.21  x,y=(55, 171)
k[4]=0.13  x,y=(56, 171)
```

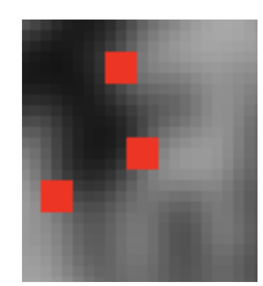




0.0

rotated





270.0

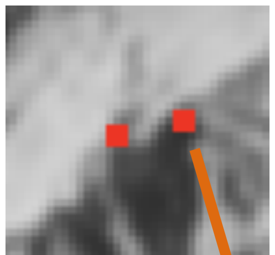
rotated

orientation and junctions. corners

from Brown & Lowe 2003 left and right panorama images

```
k[0]=0.09  x,y=(204, 65)
k[1]=0.19  x,y=(205, 65)
k[2]=0.21  x,y=(206, 65)
k[3]=0.09  x,y=(206, 64)
k[4]=0.03  x,y=(206, 63)
```

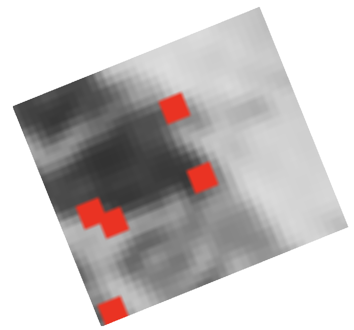
315.0

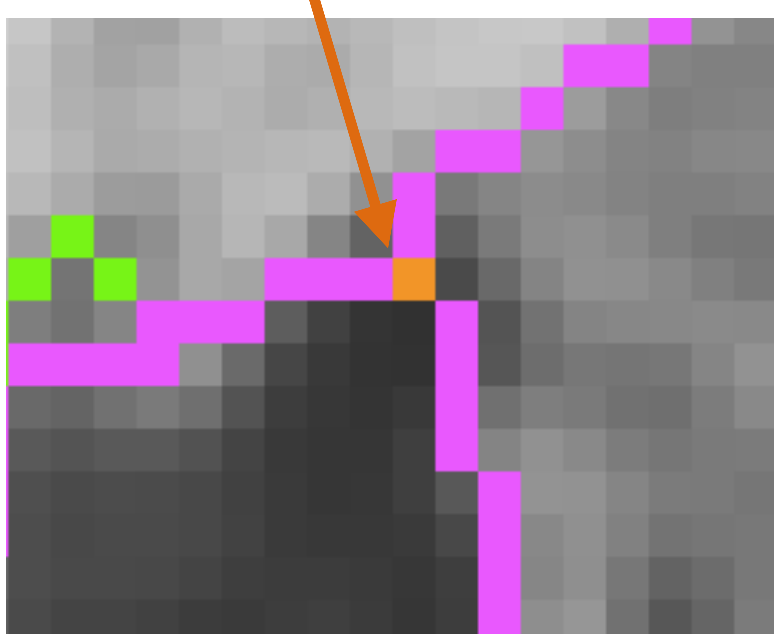


```
k[0]=0.21  x,y=(170, 198)
k[1]=0.32  x,y=(170, 199)
k[2]=0.41  x,y=(170, 200)
k[3]=0.38  x,y=(169, 201)
k[4]=0.27  x,y=(168, 201)
```

337.5

rotated



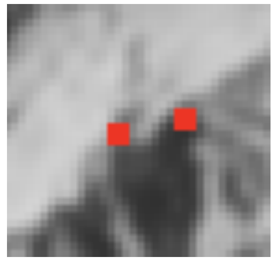


The left image corner is in a *junction* and that requires more complex analysis to determine here that the better edge for it is the lower right edge instead of the upper edge (the later gives an inconsistent orientation w.r.t. the right image).

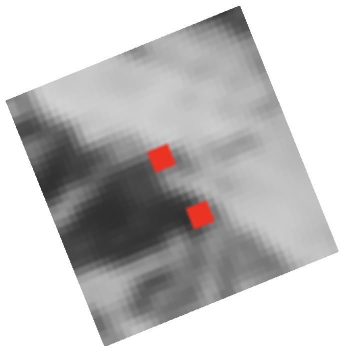
Without the more complex analysis for now, will just create a CornerRegion for each combination and one of the 3 will match the right.

orientation and junctions (continued)

```
k[0]=0.02  x,y=(205, 65)
k[1]=0.21  x,y=(206, 65)
k[2]=0.02  x,y=(207, 66)
```



```
k[0]=0.21  x,y=(170, 198)
k[1]=0.32  x,y=(170, 199)
k[2]=0.41  x,y=(170, 200)
k[3]=0.38  x,y=(169, 201)
k[4]=0.27  x,y=(168, 201)
```

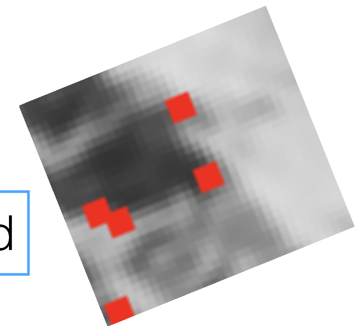


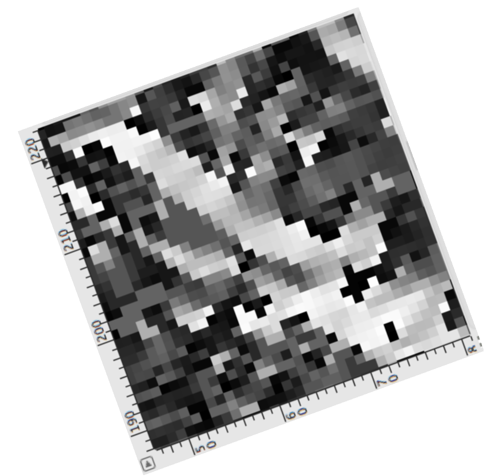
67.5

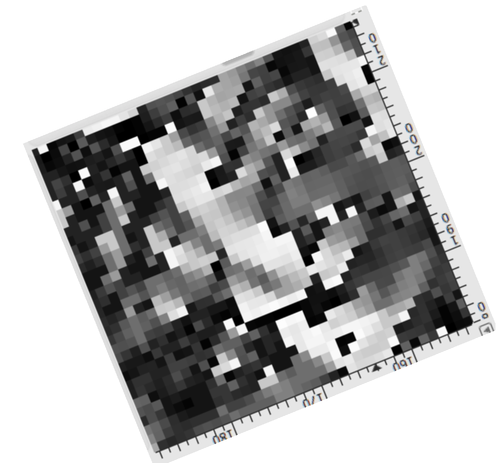
rotated

337.5

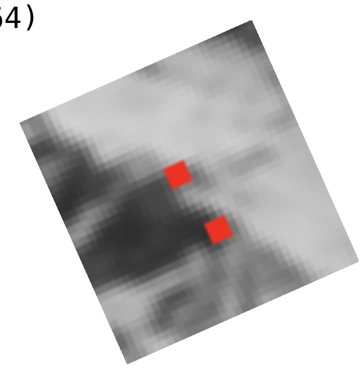
rotated





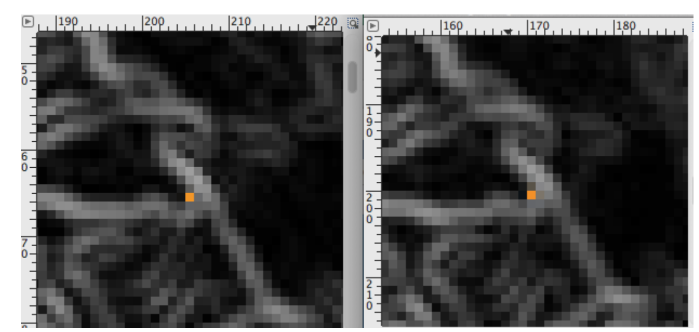


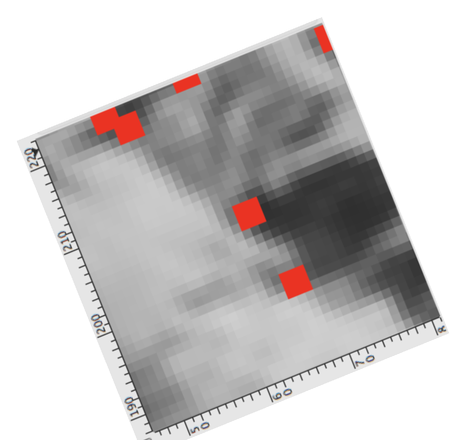
```
k[0]=0.02  x,y=(205, 65)
k[1]=0.21  x,y=(206, 65)
k[2]=0.02  x,y=(206, 64)
```



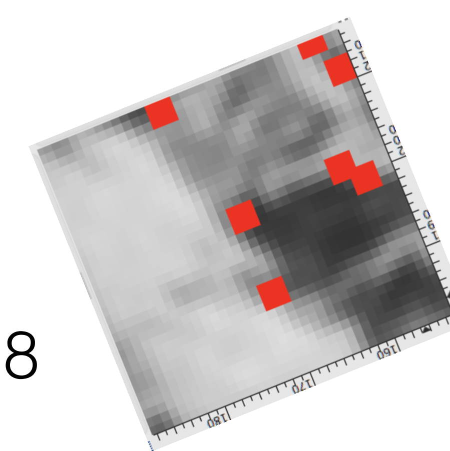
66

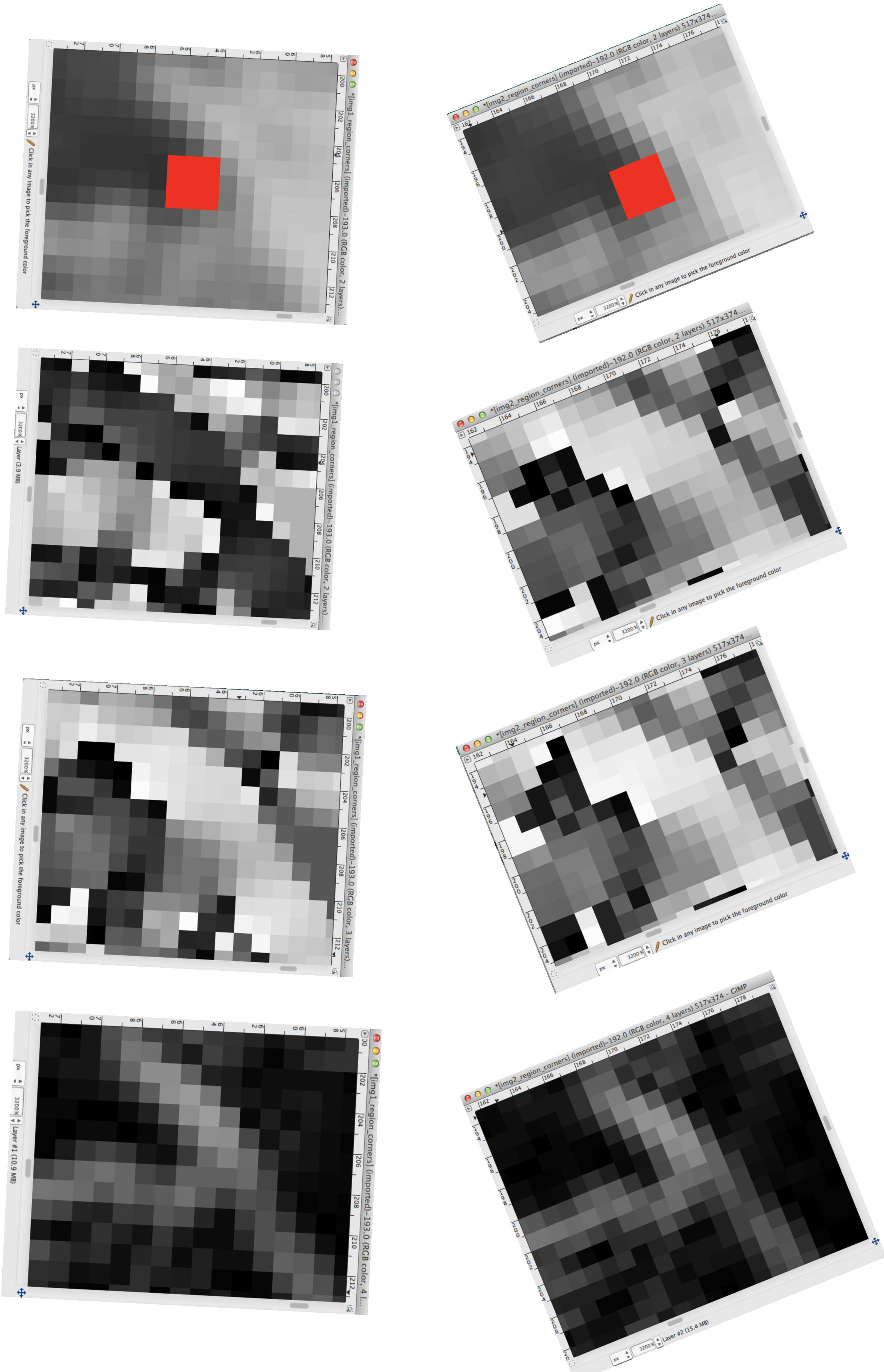
rotated





250 158





93

338

greyscale rotated to “dominant orientation”

gradient theta rotated to “dominant orientation”

gradient theta rotated to “dominant orientation” with “dominant orientation” subtracted from theta

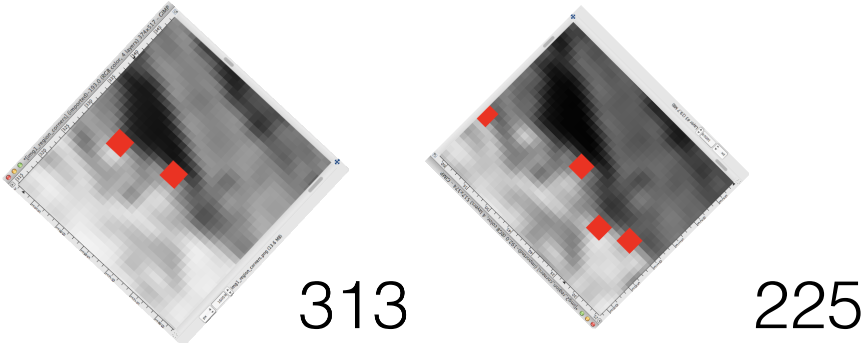
can see that image 2 should have preferred match w/ orientation 67.5 instead of 93

can see that SSD on gradient theta could be a good descriptor with the orientation corrections and quadrant consideration (e.g. 0 diff 350 is 10, not 350)

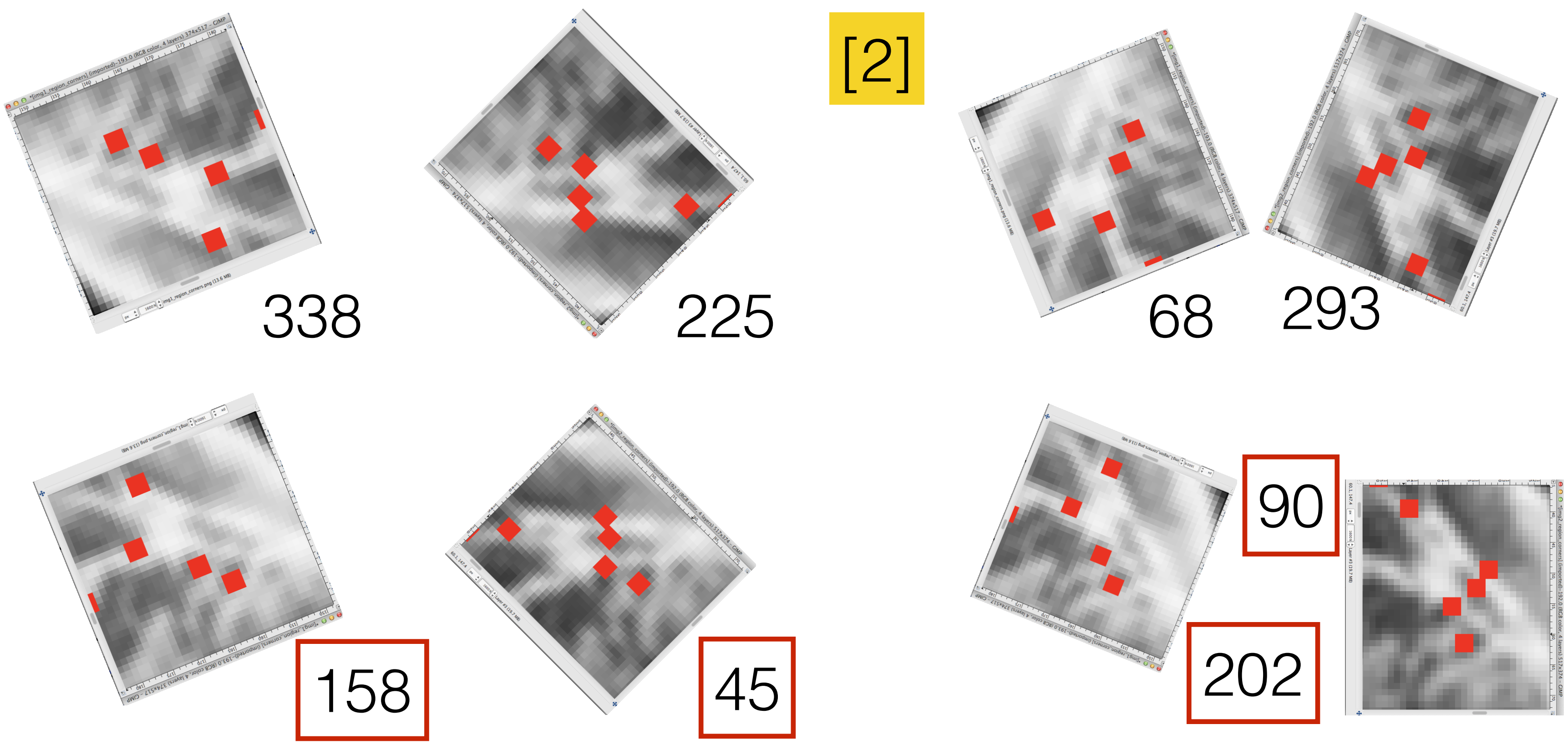
can see that SSD on gradient could be a good descriptor but may not be as tolerant of skew as subcells of histograms of orientation?

will use gradient, but binned into 2x2 cells, and 16 of them surrounding the corner.

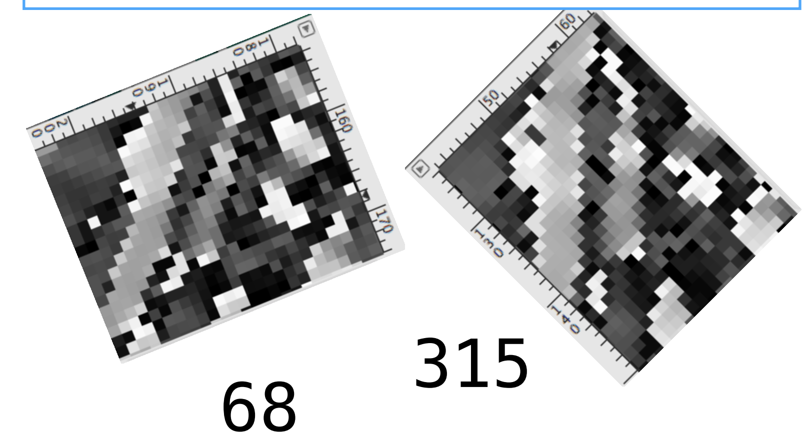
[1]



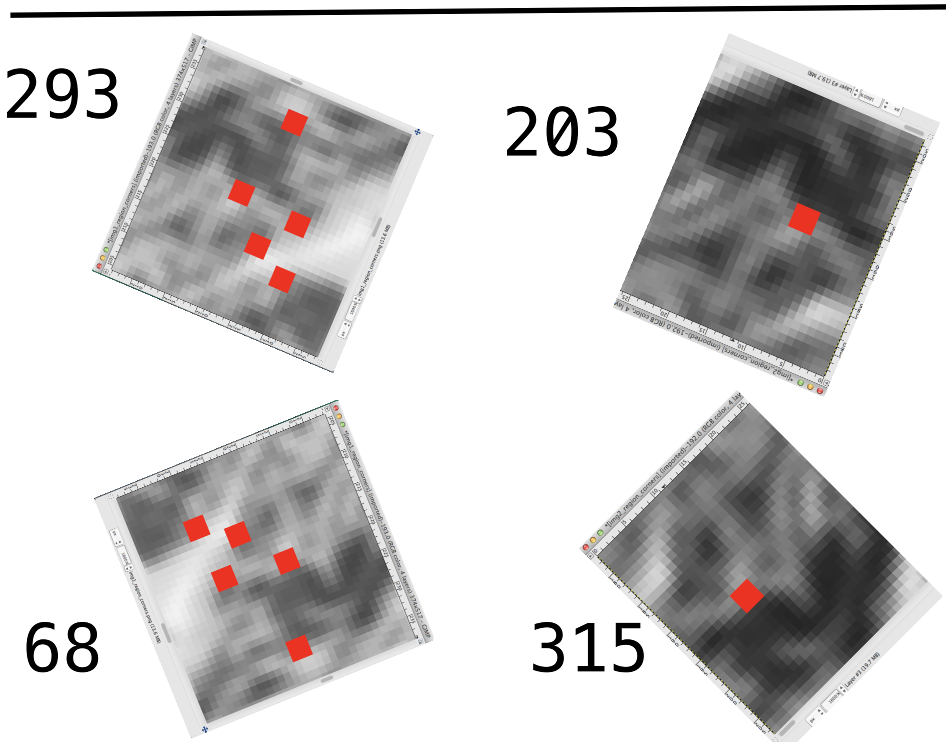
[2]



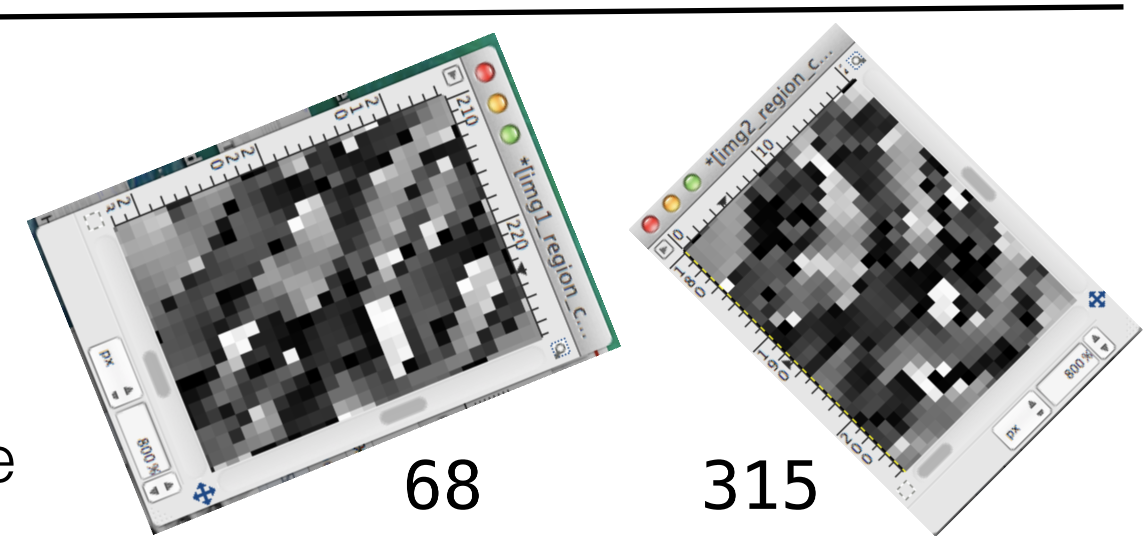
gradient theta rotated to “dominant orientation” with “dominant orientation” subtracted from theta (ref for method?)



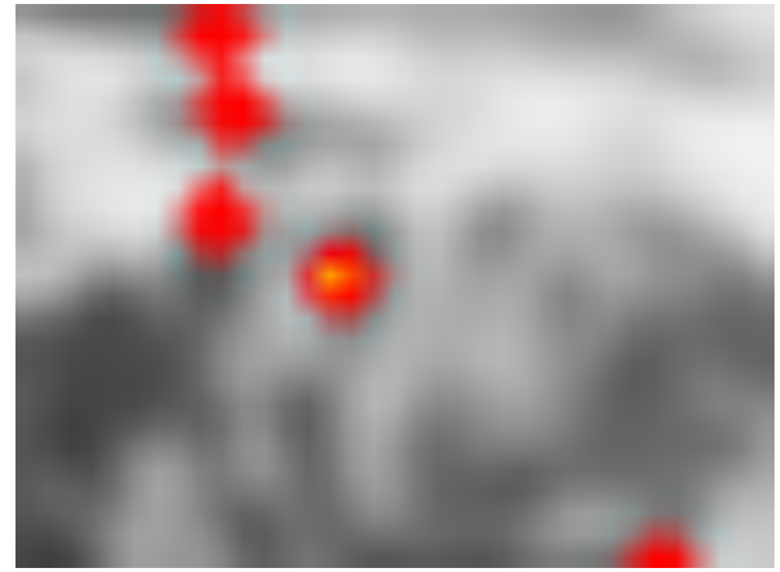
[3]



good for normalization test or preference for gradient theta (220, 220)(9, 194)



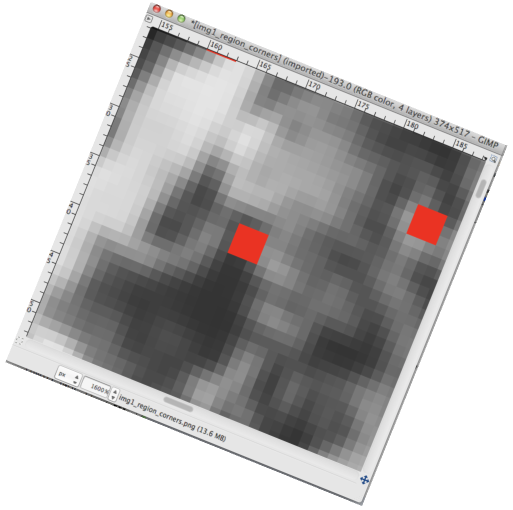
gradient theta rotated to “dominant orientation” with “dominant orientation” subtracted from theta (ref for method?)



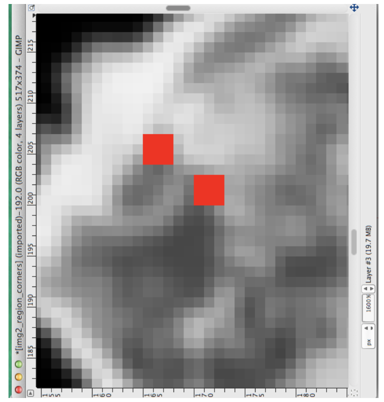
left and right clips from the Brown & Lowe showing projection and illumination differences.

Binning the intensity descriptor pixels into 2x2 cells and subtracting the mean of the descriptor intensity values from itself helps to remove the illumination differences and projection differences for those within that small range.

Note that the rectangular radius around the central pixel is somewhat large, 12 pixels (note too that the snapshot above is larger than a descriptor's region).

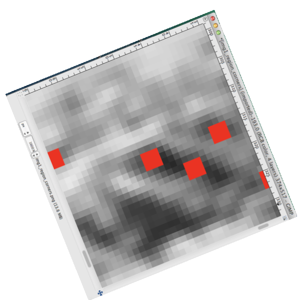


0 or 22.5

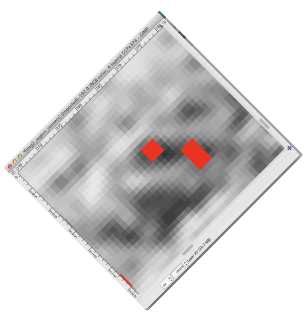


270

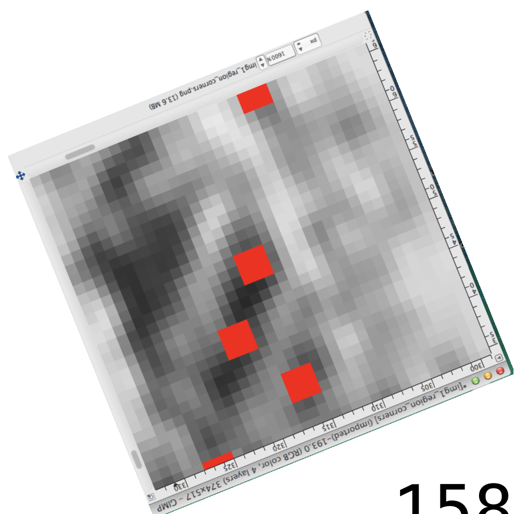
[4]



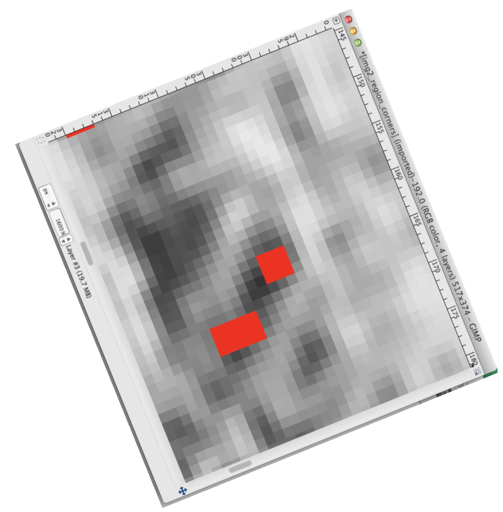
68



315

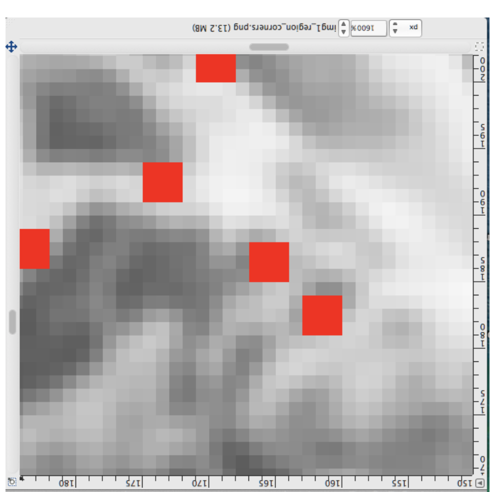


158

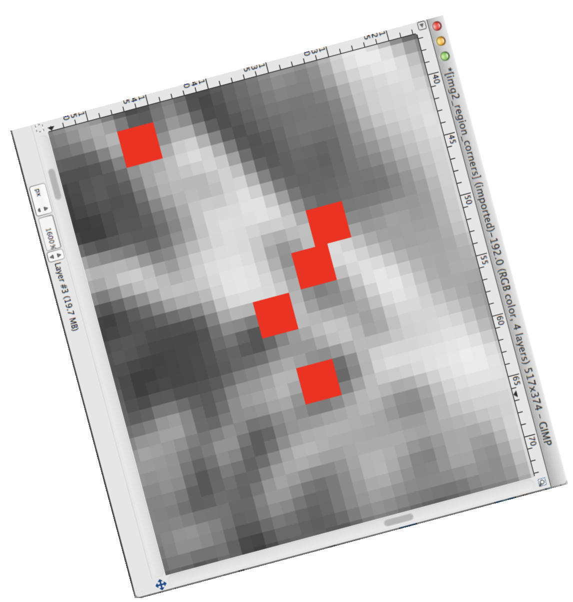


68

[5]



180

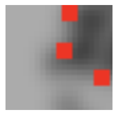


75?

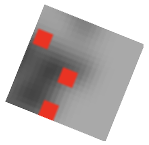
[6]

orientation. corners from Venturi dataset

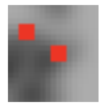
```
k[0]=0.15  x,y=(461, 449)
k[1]=0.23  x,y=(460, 448)
k[2]=0.29  x,y=(459, 447)
k[3]=0.26  x,y=(459, 446)
k[4]=0.16  x,y=(459, 445)
```



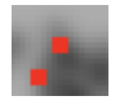
202.5



```
k[0]=0.13  x,y=(157, 466)
k[1]=0.19  x,y=(156, 466)
k[2]=0.22  x,y=(155, 466)
k[3]=0.20  x,y=(154, 466)
k[4]=0.16  x,y=(153, 467)
```



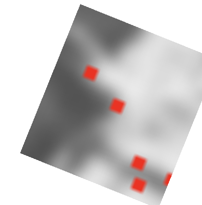
90.0

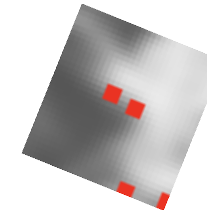


rotated

```
k[0]=0.18  x,y=(293, 496)
k[1]=0.24  x,y=(292, 495)
k[2]=0.29  x,y=(291, 494)
k[3]=0.25  x,y=(291, 493)
k[4]=0.13  x,y=(291, 492)
```

202.5



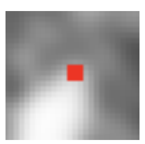


```
k[0]=0.18  x,y=(105, 298)
k[1]=0.33  x,y=(105, 297)
k[2]=0.36  x,y=(106, 296)
k[3]=0.26  x,y=(107, 296)
k[4]=0.14  x,y=(108, 296)
```

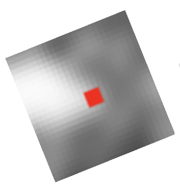
112.5

rotated

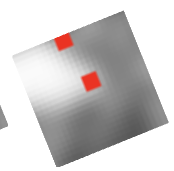
```
k[0]=0.27  x,y=(449, 222)
k[1]=0.33  x,y=(448, 221)
k[2]=0.37  x,y=(447, 220)
k[3]=0.35  x,y=(446, 220)
k[4]=0.26  x,y=(445, 220)
```



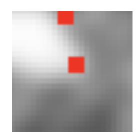
67.5



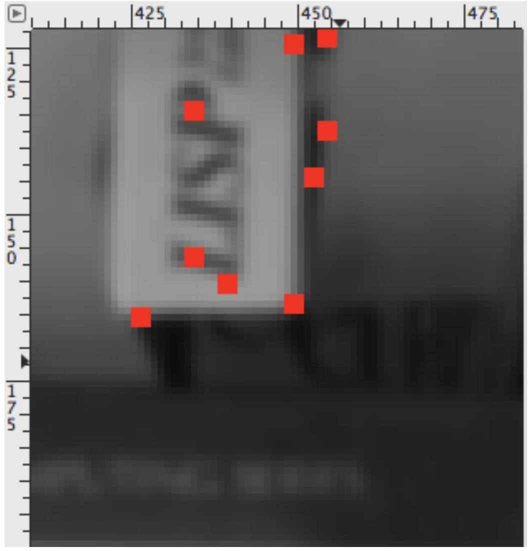
```
k[0]=0.26  x,y=(382, 448)
k[1]=0.29  x,y=(383, 447)
k[2]=0.31  x,y=(384, 446)
k[3]=0.29  x,y=(384, 445)
k[4]=0.23  x,y=(384, 444)
```

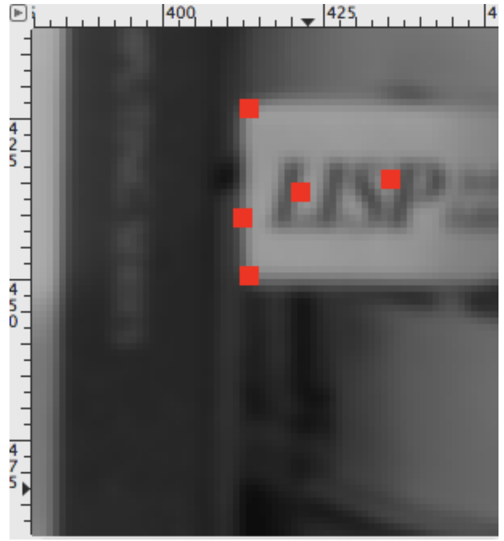


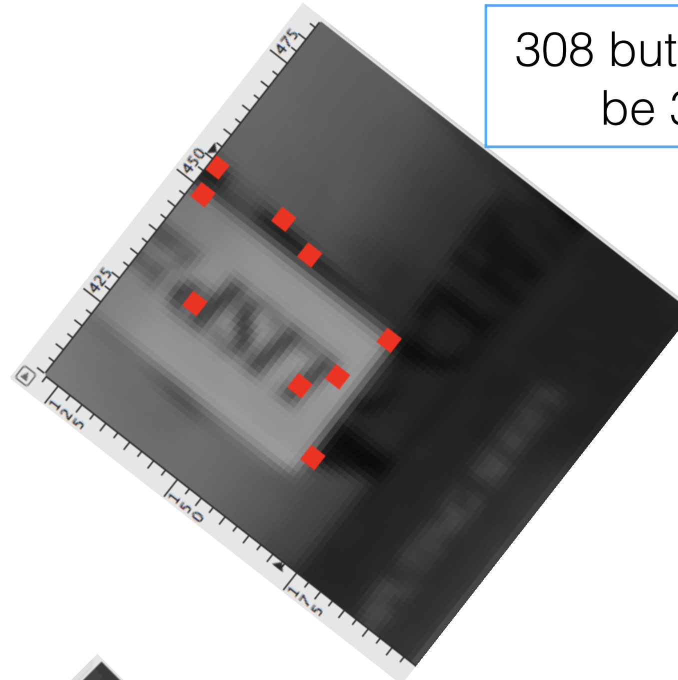
337.5



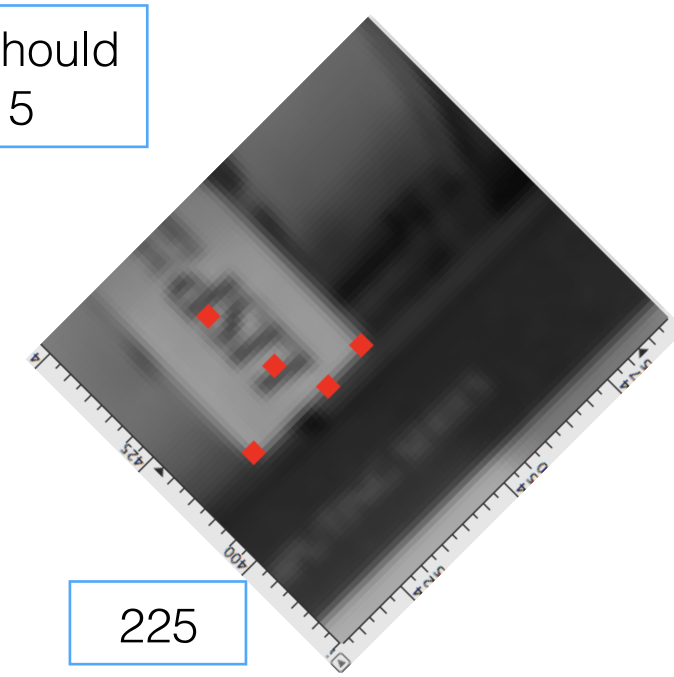
rotated



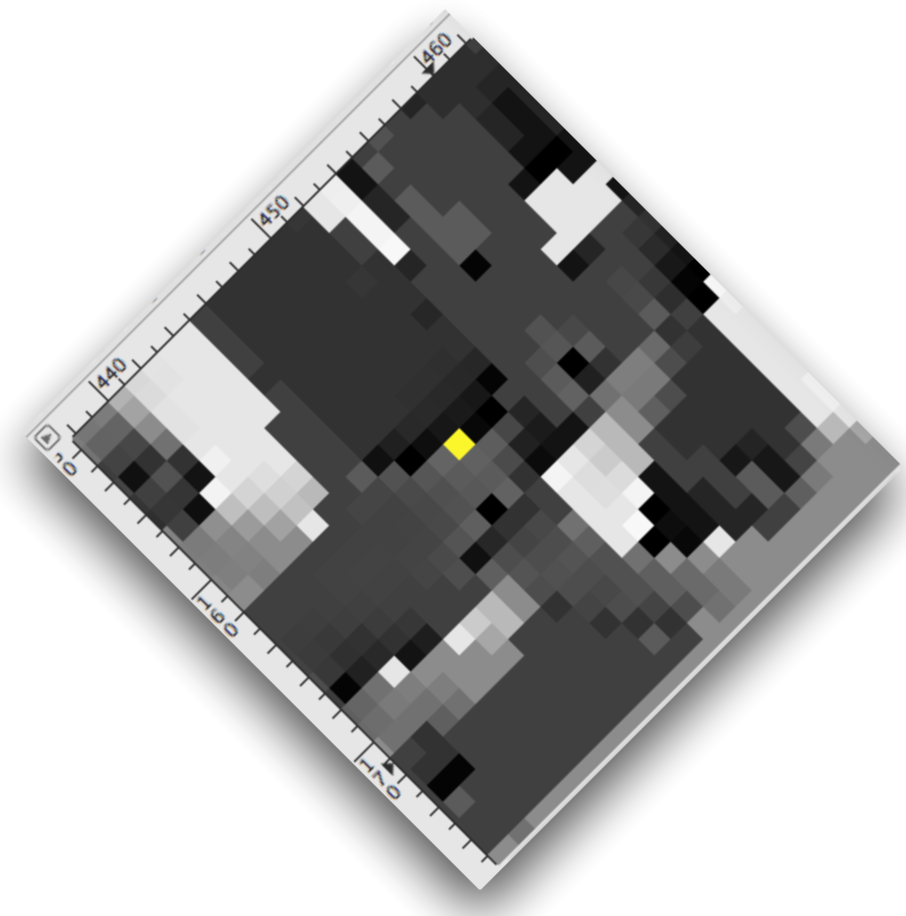


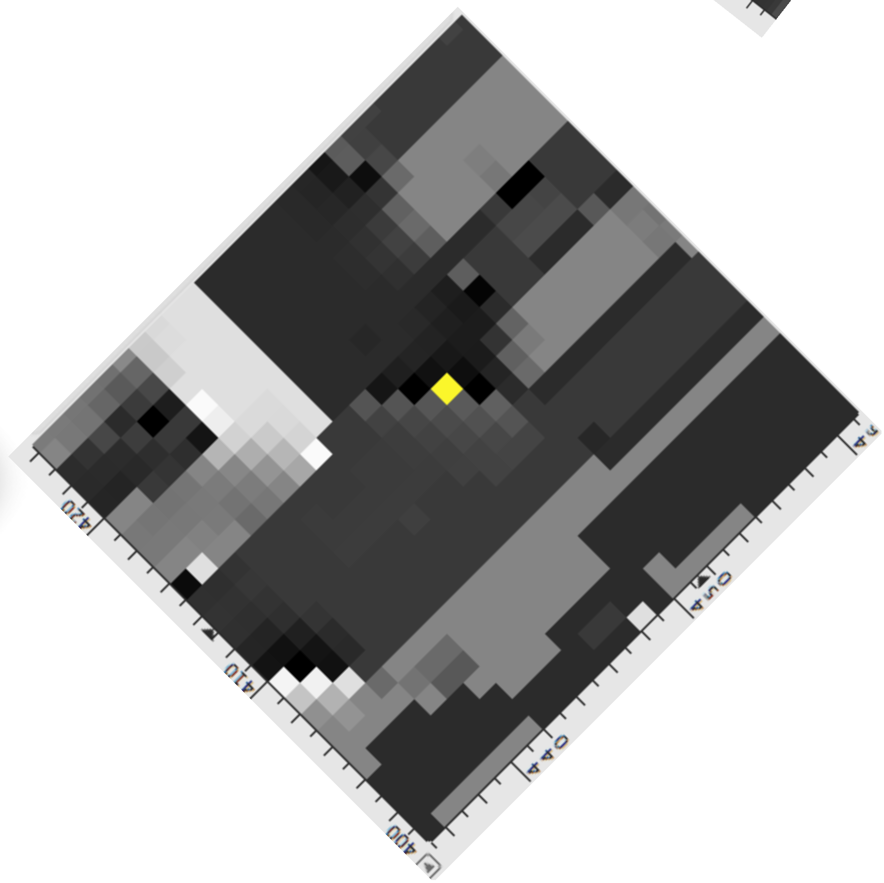


308 but should be 315



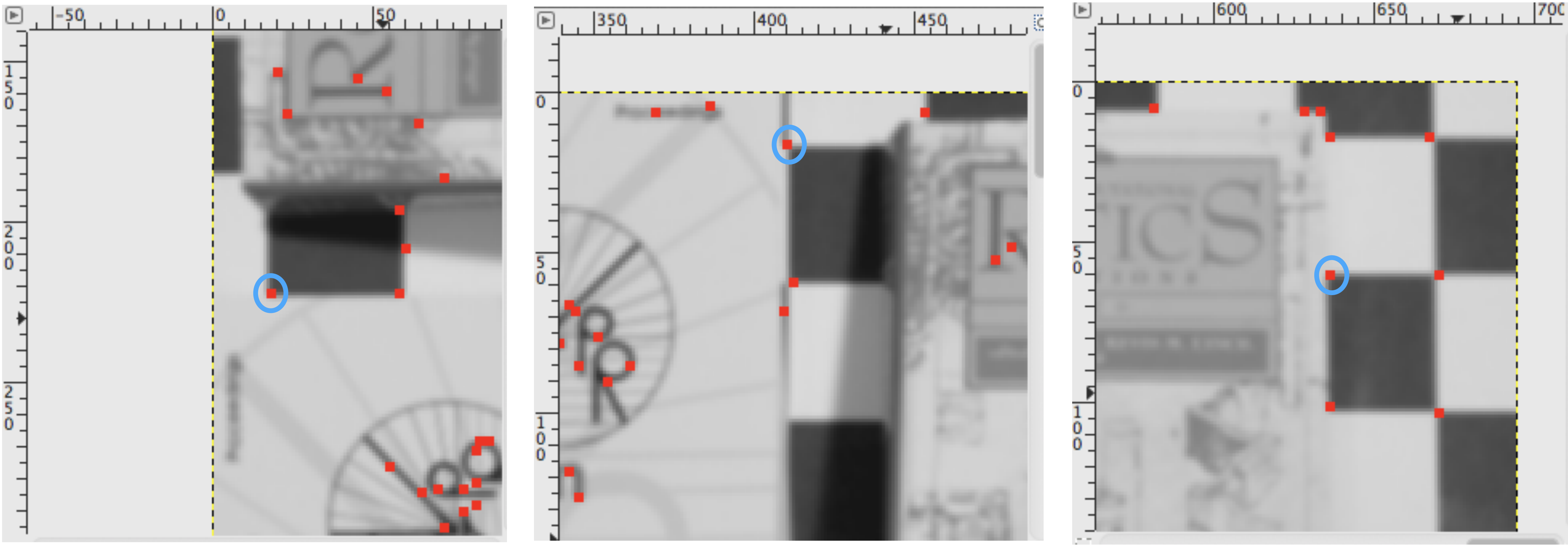
225





projection shows a different neighboring region for the book. so theta is very good for identifying the book consistently, but object needs to be on an unvarying background/ foreground or one should only compare theta within the contour (bounds) of the book.

the theta descriptor SSD is larger than the error from auto-correlation. discard in favor of clearer matches and note for a future algorithm which recognizes contours and matches within…



true match (not found)

false match (found)

this is an example where a repeated pattern is better matched elsewhere in the image due to 2 things: (1) it's a pattern (very big texture) and (2) the background/foreground changes near the corner change the appearance enough that the true match isn't found.

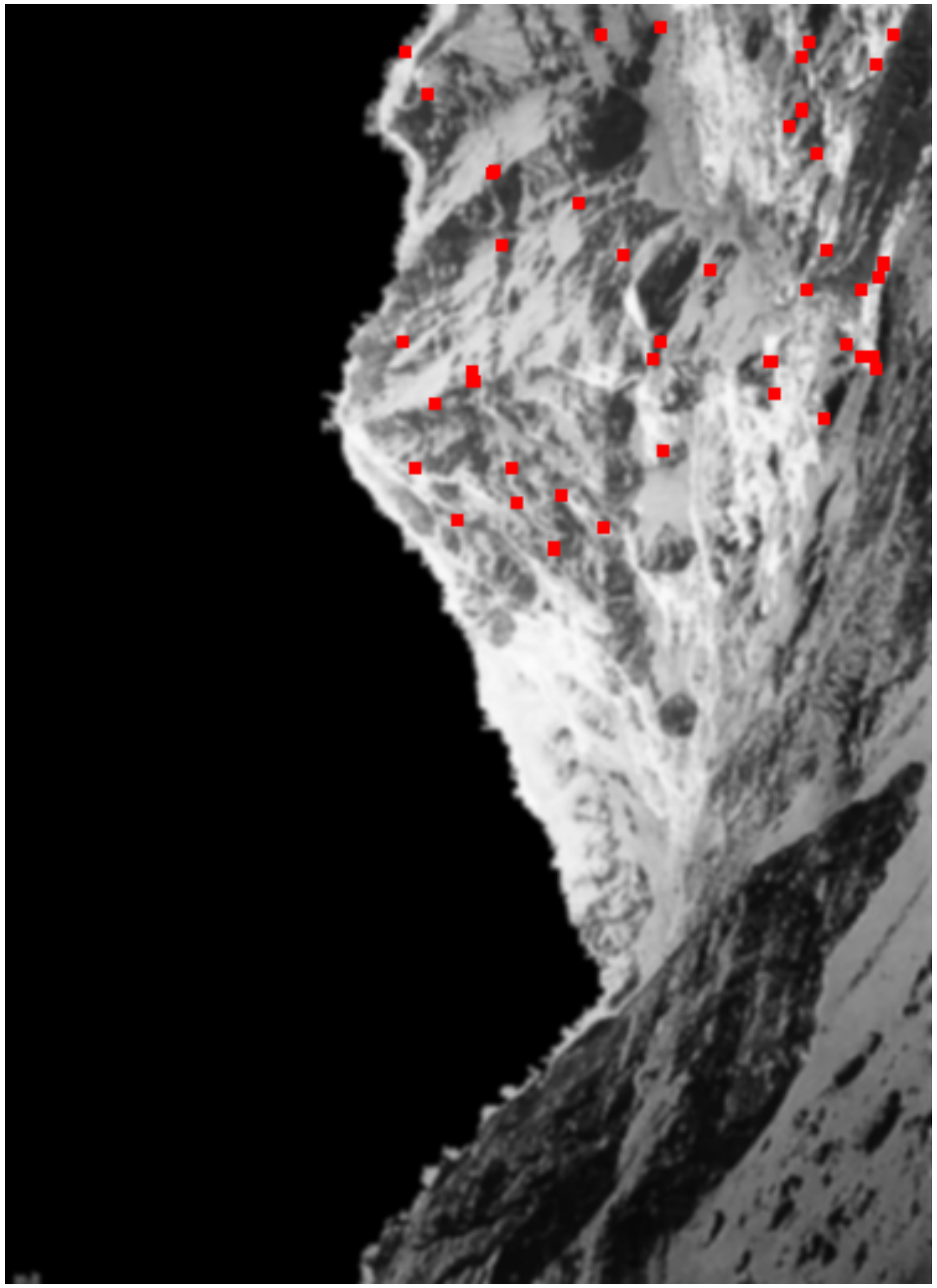
where "found" is referring to the matching algorithm to make rough correspondence lists before use of the epipolar projection solver.

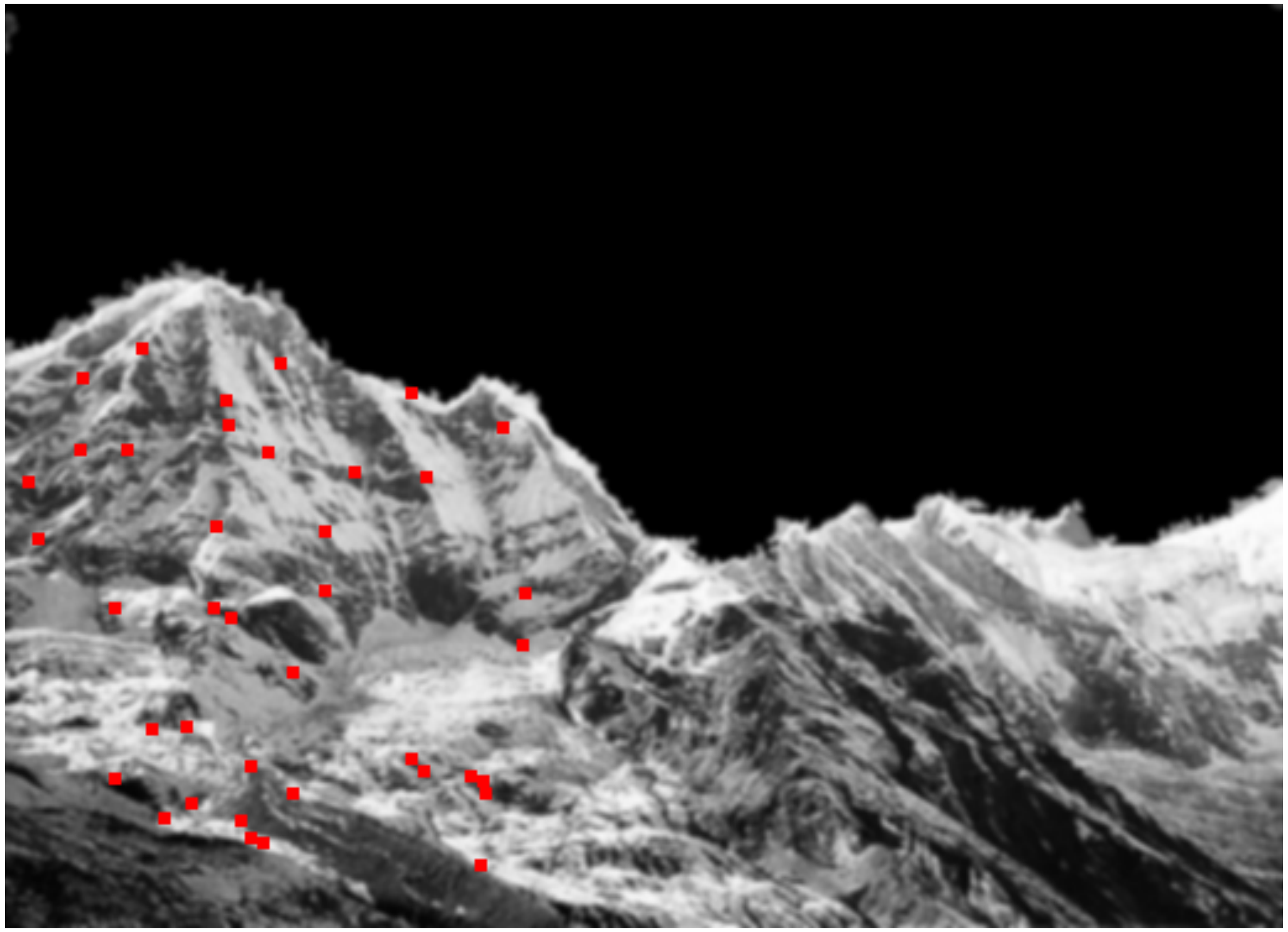
might need to consider the top k matches, especially if the image is known to contain repeated patterns/textures.

Euclidean transformation from pairwise
calc of known points:
rotationInRadians=4.5554905
rotationInDegrees=261.01037890524134 scale=1.004318
translationX=266.80643
translationY=5.852234 originX=0.0 originY=0.0

Feature matches of corner regions followed by a look at the implied rotations by frequency then filtering with that to see the most frequent translation in X then filtering by that to find the most frequent translation in Y:

solution
rotation=270.0+-20
translationX=215.0+-55.84706878621094
translationY=-25.0+=29.85197257956055



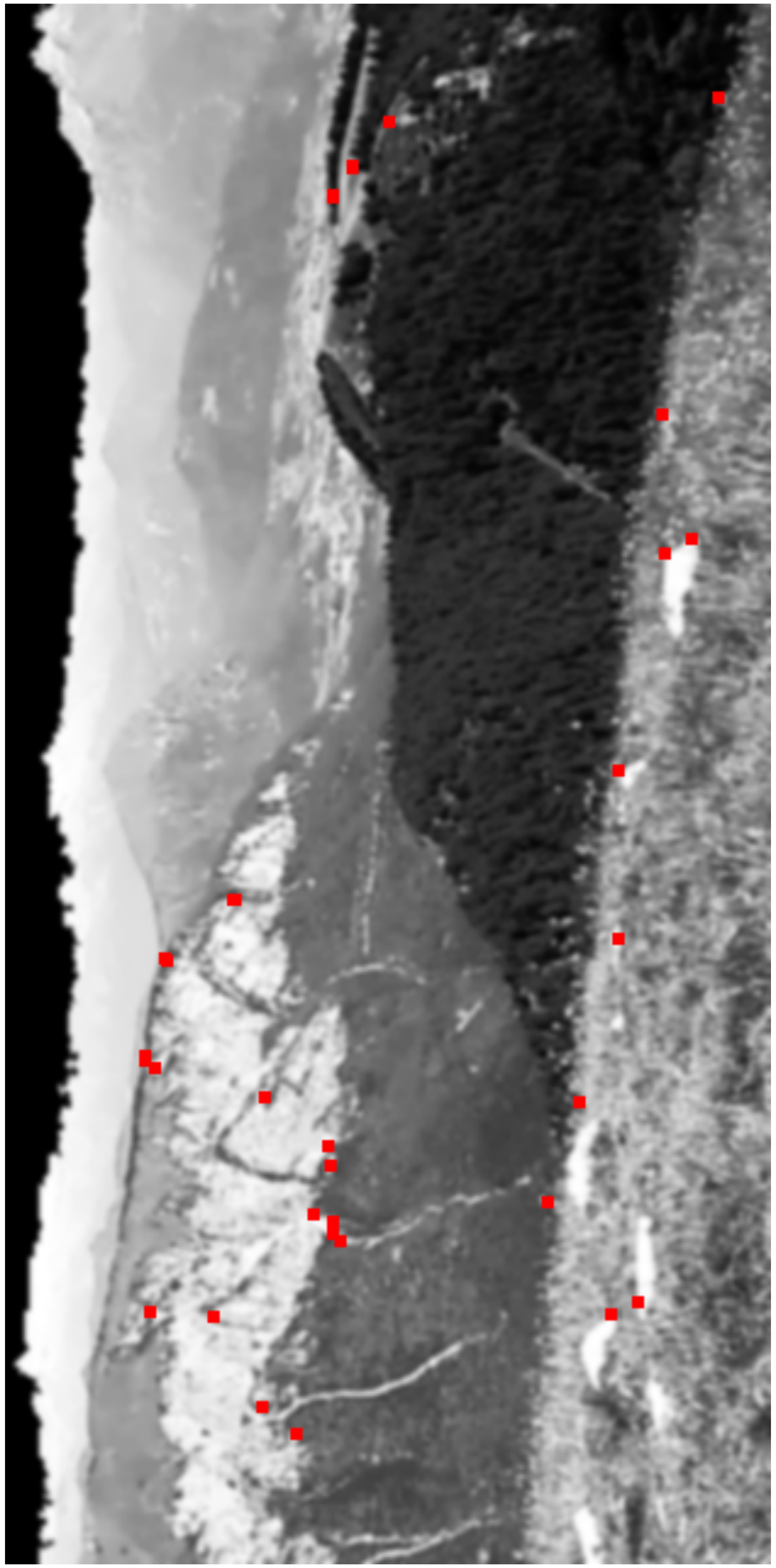


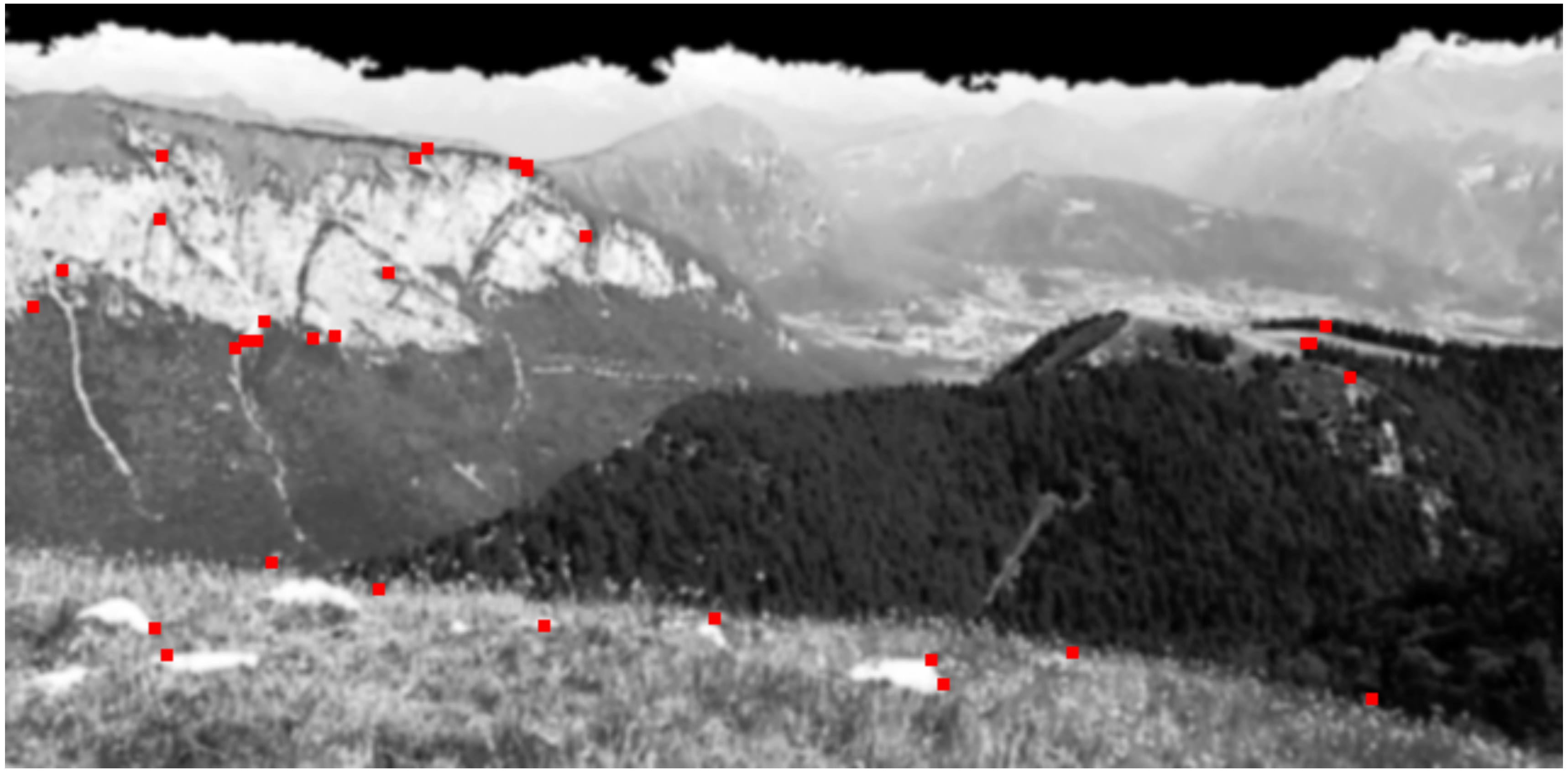
result is a list of correspondence, usable as input to the epipolar solver which uses RANSAC to discard outliers while solving the fundamental matrix.

Euclidean transformation from pairwise
calc of known points:
params=rotationInRadians=4.712166
rotationInDegrees=269.9872145691191 scale=1.0279127
translationX=614.1122 translationY=-4.460491
originX=0.0 originY=0.0

Feature matches of corner regions followed by a look at the implied rotations by frequency then filtering with that to see the most frequent translation in X then filtering by that to find the most frequent translation in Y:

solution
rotation=280.0+-20
translationX=625.0+-31.55487060546875
translationY=5.0+=0.0





result is a list of correspondence,
usable as input to the epipolar solver
which uses RANSAC to discard outliers
while solving the fundamental matrix.

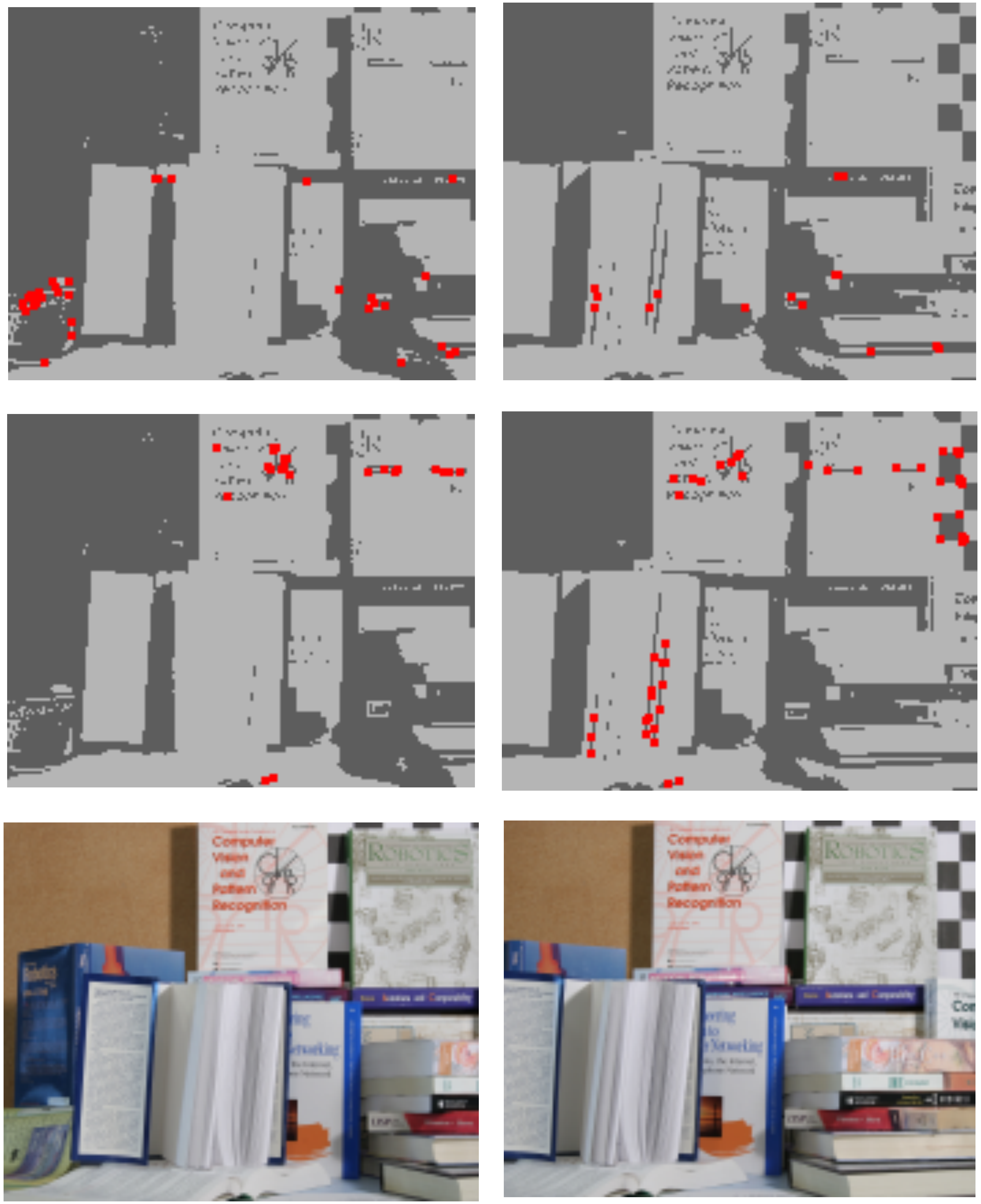
Euclidean transformation from pairwise
calc of known points:
params=rotationInRadians=4.512667
rotationInDegrees=258.55678371007 6
scale=1.0265783
translationX=639.77747 translationY=12.565971
originX=0.0 originY=0.0

Feature matches of corner regions followed by a look at the implied rotations by frequency then filtering with that to see the most frequent translation in X then filtering by that to find the most frequent translation in Y:

solution
rotation=270.0+-20
translationX=625.0+-55.98248291015625
translationY=25.0+=44.47115325927734 4

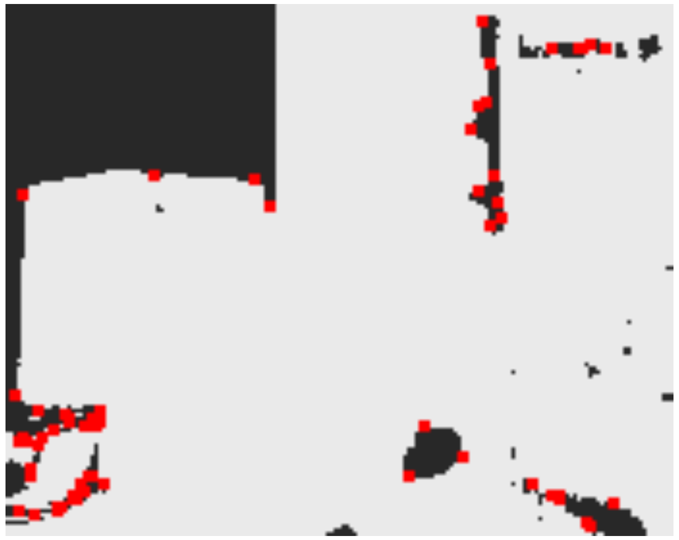
On already rectified images, the rough euclidean solution that is used to remove more distant matched pairs probably needs a 2nd order term, at least for translations.

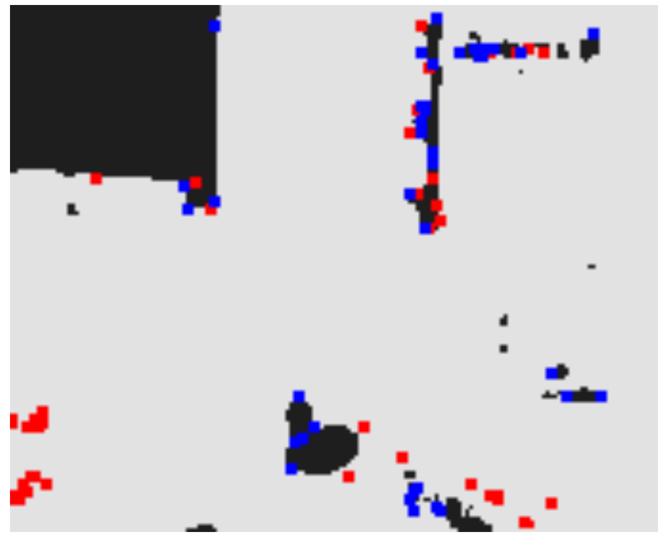
corners filtered (not finished
here, but keeping snapshots)

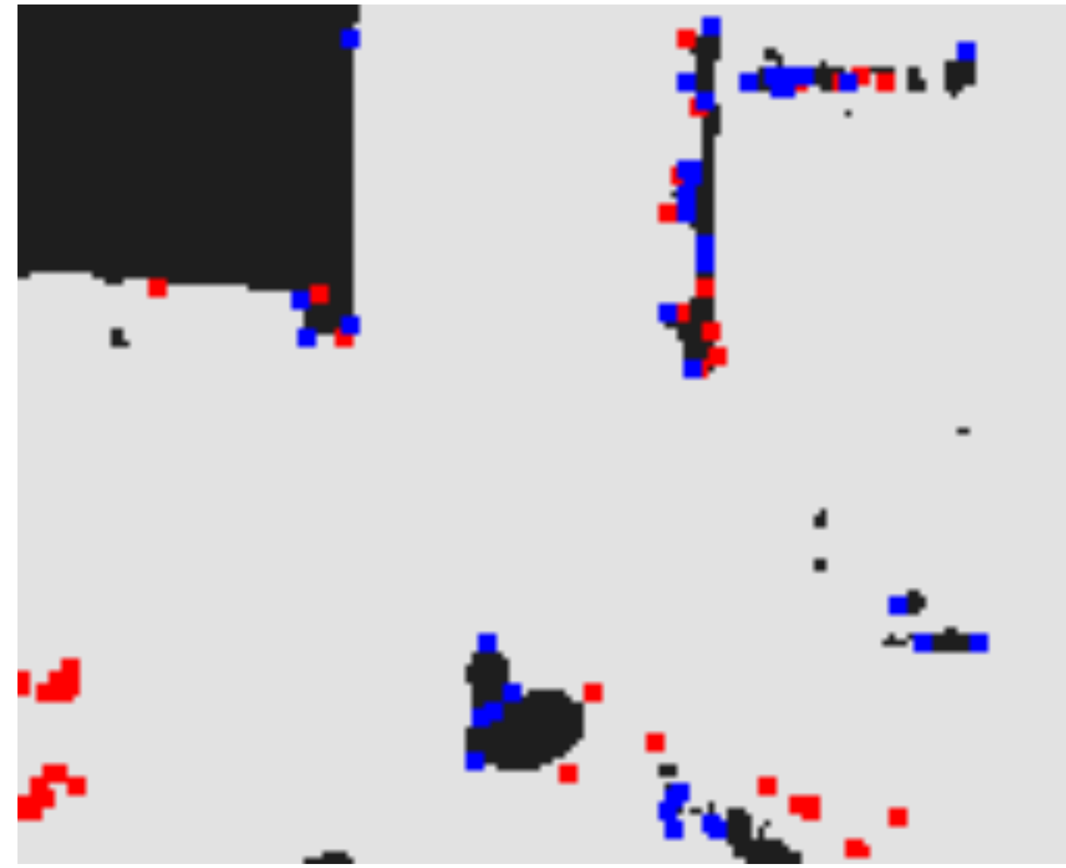


previous notes from a look at binning and segmentation

A quick look at color segmentation with k=3 followed by binary segmentation, binned to size < 200 x 200, followed by corners worked fine for 2 image sets, but not the third so the other methods in previous slides are preferred (corners filtered to blobs, matching, calculating transformation, then creating correspondence with all corners).







For the stereoscopic above, there's more than translation so this can only be the start of the solution

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previous notes from a
look at binning and
segmentation
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