Analysis Note for 60H Dataset Relative Unblinding

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High Level Summary

- Lead Analyst: Nick Kinnaird
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- Positron Reconstruction Method: Recon West
- Software Release: V9_11_00
- Dataset: gm2pro_daq_full_run1_60h_5033A_withfullDQC
- Histogramming Method: Weighted Ratio
- Gain Correction Method: Default in reconstruction
- Pileup Correction Method: Asymmetic shadow window
- Lost Muon Spectrum Extraction: put something here?
- Models for CBO and VW: Exponential envelopes, frequency from tracking analysis
- $R = -19.something \pm 1.somethingppm$ (blinding with common string)
- $\chi^2/NDF = 4211/4200$something

Final fit function:

$$R(t) = \frac{2f(t) - f_+(t) - f_-(t)}{2f(t) + f_+(t) + f_-(t)}$$
$$f_\pm(t) = f(t \pm T_a/2)$$
$$f(t) = C(t)(1 + A\cos(\omega_a t + \phi))$$
$$C(t) = 1 + A_{cbo} e^{-t/\tau_{cbo}} \cos(\omega_{cbo} t + \phi_{cbo})$$

Chapter 1

Analysis Procedures

1.1 Key parameters in reconstruction method

Find out procedures used in 60 hr production dataset

1.2 Analysis Data Preparation Procedure

- git branch: gm2analyses branch feature/KinnairdAnalyses
- Majority of code located in gm2analyses/macros/RatioMacro folder.

1. Submit jobs to OSG to run the rootTreesAndLostMuons.fcl file which produces root trees of positron hits using the ClusterTree analyzer module and coincident MIP hits using the TestCoincidenceFinder analyzer module.
2. Submit jobs to Fermigrid to produce histograms from root trees using the ClusterTreeToHistsPileup.C macro in RatioMacro/HistMaking. Beyond standard threshold histograms this macro produces pileup and lost muon histograms all within the same root file.

1.3 Histogramming Procedure

Method: Weighted Ratio (threshold)

1. Loop through all clusters and apply an artificial deadtime to combine hits within 6 ns into a single pulse using the same procedure and code that the pileup method uses (see below). Drop clusters with time $< 25\mu s$ or time $> 600\mu s$.
2. Histograms are constructed with ROOT's TH1F class with 149.15 ns bins from $0 - 699.96095\mu s$ corresponding to 4693 bins.
3. Randomize times by $\pm 149.15/2ns$ and fill histograms for energies > 1.7 GeV. Randomization uses ROOT's default TRandom3 class.
4. Generate a random number per cluster to determine which

Should this be a section at the front of the report? Or should it be in the appendix as I've place it here?

Appendix A

Ratio Method Derivation

Consider the 5 parameter function:

$$N_5(t) = N_0 e^{-t/\tau}(1 + A\cos(\omega_a t + \phi)), \tag{A.1}$$

which describes some ideal dataset in histogram format. Here ϕ will be set to zero for simplicity. Now define the variables $u_+(t)$, $u_-(t)$, $v_1(t)$, and $v_2(t)$ as

$$\begin{aligned} u_+(t) &= \frac{1}{4}N_5(t + T/2) \\ u_-(t) &= \frac{1}{4}N_5(t - T/2) \\ v_1(t) &= \frac{1}{4}N_5(t) \\ v_2(t) &= \frac{1}{4}N_5(t), \end{aligned} \tag{A.2}$$

where the 1/4 out front stands for randomly splitting the whole dataset into 4 equally weighted sub-datasets, and T is the g-2 period known to high precision. This corresponds to a weighting of 1:1:1:1 between the datasets. To be explicit here regarding the signs, the counts that are filled into the histogram described by u_+ have their times shifted as $t \to t - T/2$, which is what the function $N_5(t + T/2)$ describes, and vice versa for u_-. To form the ratio define the variables:

$$\begin{aligned} U(t) &= u_+(t) + u_-(t) \\ V(t) &= v_1(t) + v_2(t) \\ R(t) &= \frac{V(t) - U(t)}{V(t) + U(t)}. \end{aligned} \tag{A.3}$$

Plugging in and dividing the common terms $(N_0 e^{-t/\tau}/4)$,

$$R(t) = \frac{2(1 + A\cos(\omega_a t)) - e^{-T/2\tau}(1 + A\cos(\omega_a t + \omega_a T/2)) - e^{T/2\tau}(1 + A\cos(\omega_a t - \omega_a T/2))}{2(1 + A\cos(\omega_a t)) + e^{-T/2\tau}(1 + A\cos(\omega_a t + \omega_a T/2)) + e^{T/2\tau}(1 + A\cos(\omega_a t + \omega_a T/2))}. \tag{A.4}$$

Now set $\omega_a T/2 = \delta$, and note that T is really

$$\begin{aligned} T = T_{guess} &= \frac{2\pi}{\omega_a} + \Delta T, \\ \Delta T &= T_{guess} - T_{true}. \end{aligned} \tag{A.5}$$

Now

$$\delta = \frac{\omega_a}{2} T_{guess} = \frac{\omega_a}{2}(\frac{2\pi}{\omega_a} + \Delta T) = \pi + \pi \frac{\Delta T}{T_{true}} = \pi + \pi(\delta T). \tag{A.6}$$

δ can be redefined as

$$\delta = \pi(\delta T), \tag{A.7}$$

by flipping the sign of any cosine terms that contain δ (coming from the π that has been dropped). If one makes this subsitution for $R(t)$ and then makes approximations in the smallness of δT, then one arrives at the conventional 3 parameter function

$$R(t) = A\cos(\omega_a t) - C_1, \tag{A.8}$$

$$C_1 = \frac{1}{16}\Big(\frac{T}{\tau}\Big)^2. \tag{A.9}$$

(Go back and show this derivation here at some point - taking inspiration but not copying from Paley's thesis.)

If one instead weights the counts in the histograms as

$$u_+(t) : u_-(t) : v_1(t) : v_2(t) = e^{T/2\tau} : e^{-T/2\tau} : 1 : 1, \tag{A.10}$$

so that

$$u_+(t) = \frac{e^{T/2\tau}}{2 + e^{T/2\tau} + e^{-T/2\tau}} N_5(t + T/2) \tag{A.11}$$

$$u_-(t) = \frac{e^{-T/2\tau}}{2 + e^{T/2\tau} + e^{-T/2\tau}} N_5(t - T/2) \tag{A.12}$$

$$v_1(t) = \frac{1}{2 + e^{T/2\tau} + e^{-T/2\tau}} N_5(t) \tag{A.13}$$

$$v_2(t) = \frac{1}{2 + e^{T/2\tau} + e^{-T/2\tau}} N_5(t), \tag{A.14}$$

then instead $R(t)$ becomes

$$R(t) = \frac{2(1 + A\cos(\omega_a t)) - (1 - A\cos(\omega_a t + \delta)) - (1 - A\cos(\omega_a t - \delta))}{2(1 + A\cos(\omega_a t)) + (1 - A\cos(\omega_a t + \delta)) + (1 - A\cos(\omega_a t + \delta))}, \tag{A.15}$$

where the $e^{\pm T/2\tau}$ terms now cancel out and δ has replaced the $\omega_a T/2$ terms here and the appropriate signs have been switched. (These factors out front aren't so far off from 1/4 since $e^{\pm T/2\tau} \approx e^{\pm 4.35/2*64.4} \approx 1.034, .967$.) In the code, this amounts to generating a random double between 0 and 1 per pulse and then filling the associated histogram based according to the relative probabilities of the different weights. As a reminder, the pulses that had their times shifted by $t \to t - T/2$ are the pulses that should be weighted by $e^{T/2\tau}$ since they will be filled into the u_+ histogram, and vice versa for the u_- histogram.

Then, using the trig identity

$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b) \tag{A.16}$$

so that

$$\cos(\omega_a t \pm \delta) = \cos(\omega_a t)\cos\delta \mp \sin(\omega_a t)\sin\delta, \tag{A.17}$$

replacing those terms in $R(t)$, cancelling the sin terms and simplifying:

$$R(t) = \frac{2A\cos(\omega_a t)(1+\cos\delta)}{4+2A\cos(\omega_a t)(1-\cos\delta)}. \tag{A.18}$$

In the limit that

$$\delta = \pi(\delta T) \rightarrow 0 \tag{A.19}$$

since δT is small,

$$R(t) = A\cos(\omega_a t) \tag{A.20}$$

with no approximations having been made.

(Go back and expand out these $\cos\delta$ terms to show the level of precision needed for ω_a. I can sort of see what to do for R(t) but not sure how to get from that to ω_a exactly.)

While the 3 parameter function suffices for initial fits and data containing slow modulations, it does not suffice for faster oscillation features such as the coherent betatron oscillation (CBO). In that case it is more useful to fit with a higher parameter function (historically 9 terms):

$$R_9(t) = \frac{V(t)-U(t)}{V(t)+U(t)} = \frac{v_1(t)+v_2(t)-u_+(t)-u_-(t)}{v_1(t)+v_2(t)+u_+(t)+u_-(t)} \tag{A.21}$$

where $u_+(t)$, $u_-(t)$, $v_1(t)$, and $v_2(t)$ now derive themselves from the more general formula

$$N(t) = N_{cbo}(t)e^{-t/\tau}(1+A_{cbo}(t)\cos(\omega_a t+\phi_{cbo}(t))), \tag{A.22}$$

where the number, asymmetry, and phase of the incoming positrons are modulated by the CBO. In the E821 experiment these were all well described by the normal constant term plus an additional exponentially decaying and oscillatory term:

$$N_{cbo}(t) = N_0(1 + e^{-t/\tau_{cbo}} A_{N_{cbo}} \cos(\omega_{cbo} t + \phi_{N_{cbo}})), \tag{A.23}$$

$$A_{cbo}(t) = A(1 + e^{-t/\tau_{cbo}} A_{A_{cbo}} \cos(\omega_{cbo} t + \phi_{A_{cbo}})), \tag{A.24}$$

$$\phi_{cbo}(t) = \phi_0(1 + e^{-t/\tau_{cbo}} A_{\phi_{cbo}} \cos(\omega_{cbo} t + \phi_{\phi_{cbo}})), \tag{A.25}$$

where τ_{cbo} and ω_{cbo} are the lifetime and frequency of the CBO oscillations respectively. (It is important to remember that these forms were effective in getting the E821 fits to converge, but are not necessarily the exact form of the CBO effects.) This results in 11 unknown parameters. When forming the $R_9(t)$ variable, the N_0 and $e^{-t/\tau}$ cancel out in the same way as before. In the past τ_{cbo} and ω_{cbo} were extracted from the data in separate analyses and taken as given in the final ratio fit, with 9 free parameters. It is unecessary to simplify this general form for $R_9(t)$ when fitting the data as was done for the 3 parameter ratio function, and indeed improves the accuracy of the fit by avoiding approximations.