BU Ratio Method Analysis – 60H

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60H Unblinding Workshop

2/25/19

Disclaimer

- Almost all plots are from fits to the 5033A dataset (or 5033D)
- Only the single ratio fit plot is from the gold data, which I'll identify later
- This is condensed version of my report (with parts still to be written or updated) – DocDB 16270

High Level Summary

- Lead Analysts: Nick Kinnaird, James Mott
- Positron reconstruction: Recon West
- Software Release: v9_17_00
- Dataset: gm2pro_daq_full_run1_60h_5036A_goldList
- Gain correction method: Default
- Pileup correction method: Asymmetric shadow window, doublets only
- Lost muon extraction: Triple coincidence, excluded from fit

High Level Summary

- CBO model: Exponential envelope, frequency from tracking analysis
- VW model: Exponential envelope, constant frequency, excluded from fit
- 149.15 ns bins
- Fit Range: 30 – 650 μs
- Energy threshold: 1700 MeV
- R = -20.35 ppm (blinded) ± ~1.327 ppm (stat.) ± ~0.095 ppm (syst.)
- X^2/NDF = 4204 / 4150
- P value = 0.2768

Fit Function

$$R(t) = \frac{2f(t) - f_+(t) - f_-(t)}{2f(t) + f_+(t) + f_-(t)}$$

$$f_\pm(t) = f(t \pm T_a/2)$$

$$f(t) = N_{cbo}(t) \cdot N_{2cbo}(t) \cdot (1 + A \cdot A_{cbo}(t) \cdot \cos(\omega_a t + \phi))$$

$$N_{cbo}(t) = 1 + A_{cbo-N} \cdot e^{-t/\tau_{cbo}} \cdot \cos(\omega_{cbo}(t) \cdot t + \phi_{cbo-N})$$

$$\cancel{N_{2cbo}(t) = 1 + A_{2cbo-N} \cdot e^{-t/\tau_{cbo}} \cdot \cos(2 \cdot \omega_{cbo}(t) \cdot t + \phi_{2cbo-N})}$$

$$\cancel{A_{cbo}(t) = 1 + A_{cbo-A} \cdot e^{-t/\tau_{cbo}} \cdot \cos(\omega_{cbo}(t) \cdot t + \phi_{cbo-A})}$$

$$\omega_a = 2\pi \cdot 0.2291\,\mathrm{MHz} \cdot (1 + R \times 10^{-6})$$

These gave stable fit parameters for 5033A but not the gold data set – still need to play around some.

Analysis Procedures

Histogramming

- Apply artificial dead time (ADT) of 6 ns
- Randomize times with ±149.15 ns
- Apply energy threshold of 1700 MeV
- Split data into 4 histograms with weighting as:

$$u_+(t) : u_-(t) : v_1(t) : v_2(t) = e^{T/2\tau} : e^{-T/2\tau} : 1 : 1$$

- With times in the first two histograms shifted down and up half a g-2 period respecitively
- Form the ratio histogram and fit

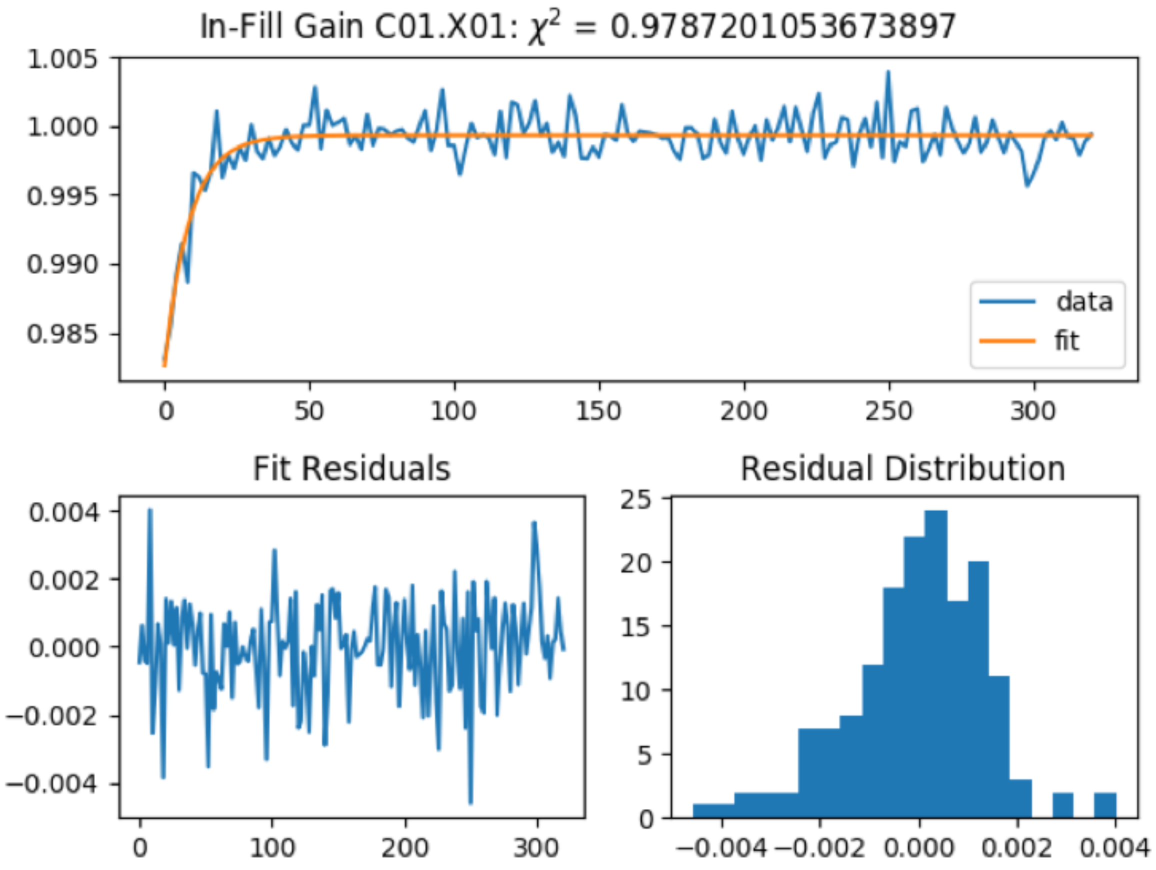
$$U(t) = u_+(t) + u_-(t)$$

$$V(t) = v_1(t) + v_2(t)$$

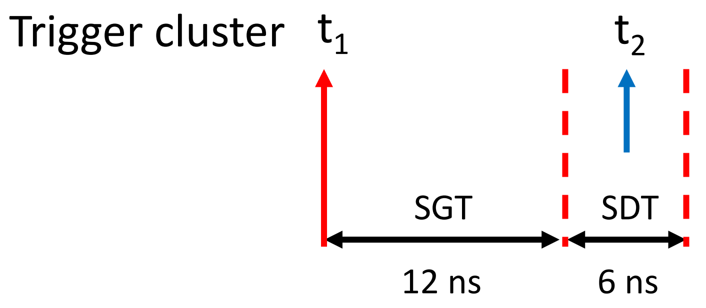
$$R(t) = \frac{V(t) - U(t)}{V(t) + U(t)}$$

Gain Correction

- Default from calibration team



Pileup Correction



- For each fill and each calorimeter create a vector of time – energy pairs from the clusters
- For each cluster look in a time window (SDT) a gap time away (SGT) for a shadow hit
- Construct pileup doublets if a shadow hit is found:

$$E_{doublet} = C \cdot (E_1 + E_2)$$

C set to 1 by default

$$t_{doublet} = \frac{t_1 \cdot E_1 + (t_2 - SGT) \cdot E_2}{E_1 + E_2}$$

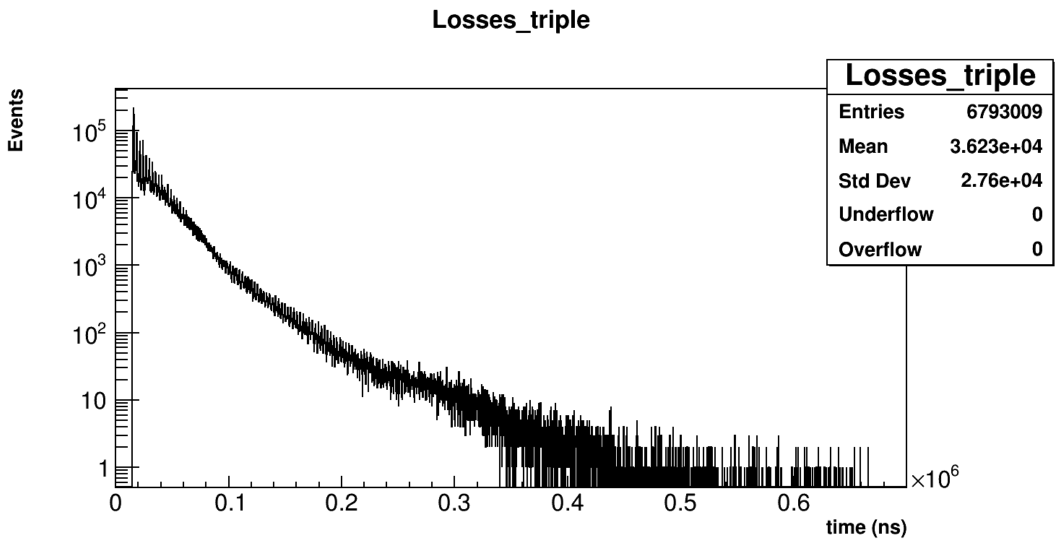
- Construct pileup spectra: P = D – S (doublets minus singlets)
- Subtract pileup spectra off calorimeter spectra

Lost Muon Extraction

- Triple coincidence of clusters in 3 consecutive calorimeters are made with an energy cut of 10 MeV < E < 250 MeV and 5 ns < dt < 8.5 ns

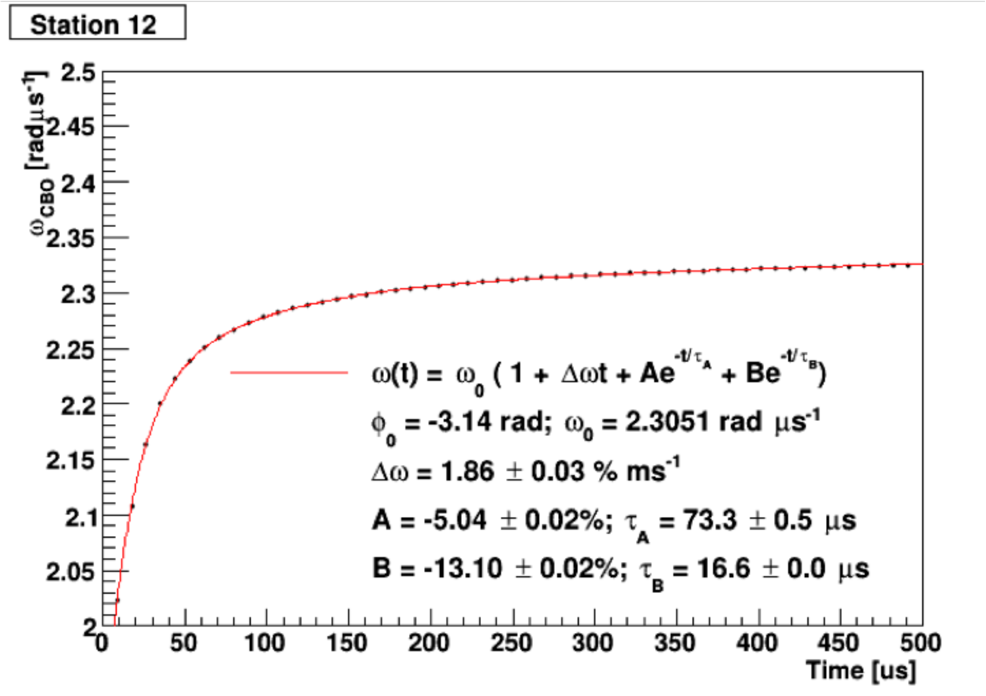
$$\Lambda(t) = 1 - \kappa_{loss} \int_0^t L(t')e^{(t'/\gamma\tau_\mu)}dt'$$

- Not included in ratio fit



CBO

- Frequency from tracking analysis:
- Function pieces as:
- Not all cbo terms included in calorimeter sum fit, all included in individual calo fits



$$N_{cbo}(t) = 1 + A_{cbo-N} \cdot e^{-t/\tau_{cbo}} \cdot \cos(\omega_{cbo}(t) \cdot t + \phi_{cbo-N}),$$

$$N_{2cbo}(t) = 1 + A_{2cbo-N} \cdot e^{-t/\tau_{cbo}} \cdot \cos(2 \cdot \omega_{cbo}(t) \cdot t + \phi_{2cbo-N}),$$

$$A_{cbo}(t) = 1 + A_{cbo-A} \cdot e^{-t/\tau_{cbo}} \cdot \cos(\omega_{cbo}(t) \cdot t + \phi_{cbo-A}),$$

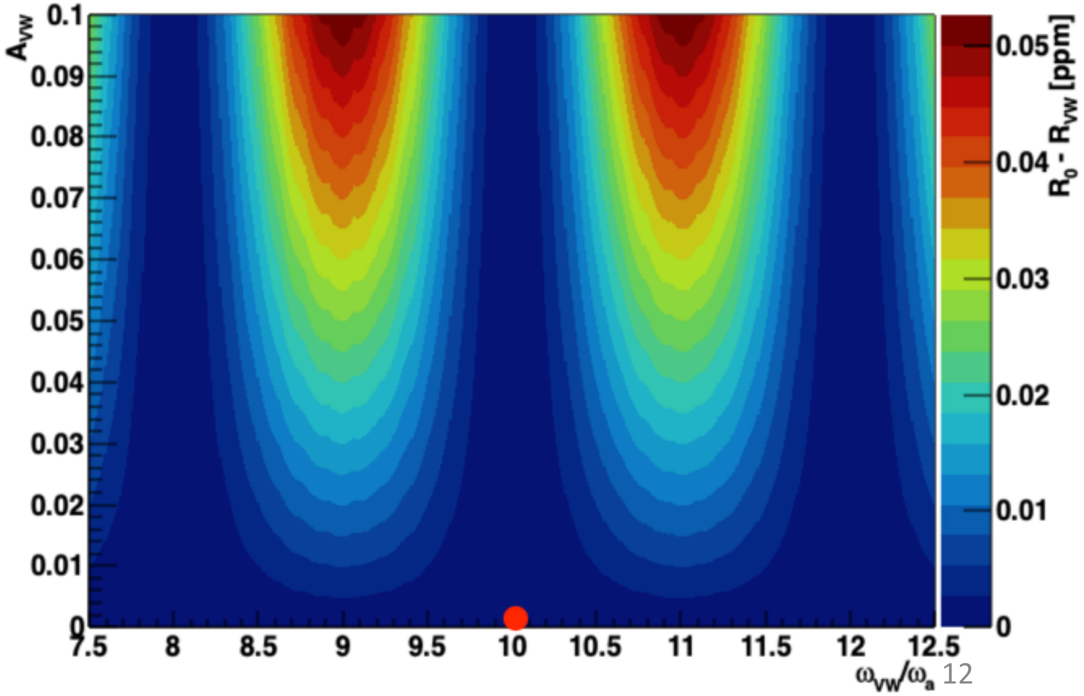
$$\phi_{cbo}(t) = 1 + A_{cbo-\phi} \cdot e^{-t/\tau_{cbo}} \cdot \cos(\omega_{cbo}(t) \cdot t + \phi_{cbo-\phi})$$

VW

- Form as:

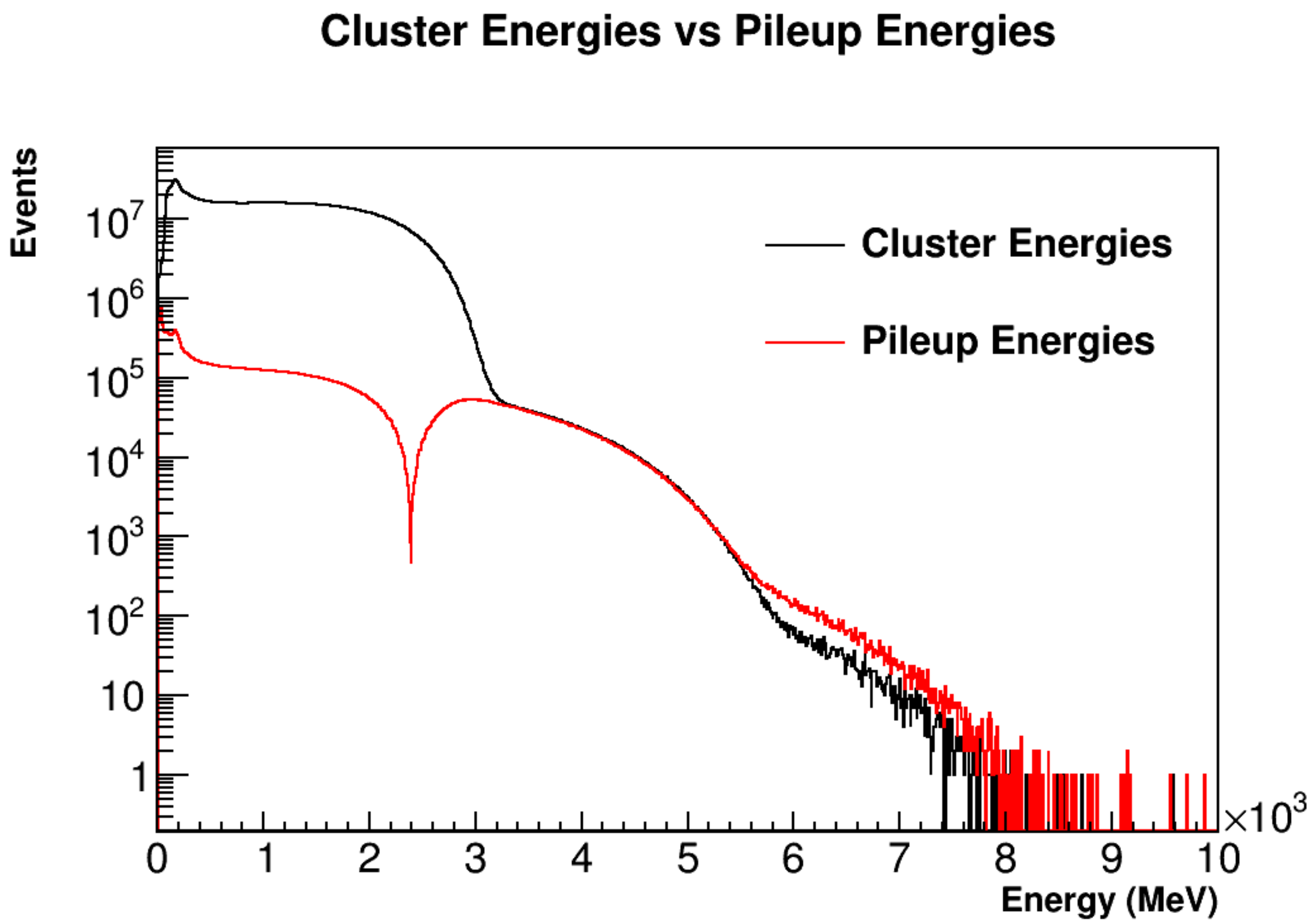
$$V(t) = 1 + A_{VW} e^{-t/\tau_{VW}} \cos(\omega_{VW} t + \phi_{VW})$$

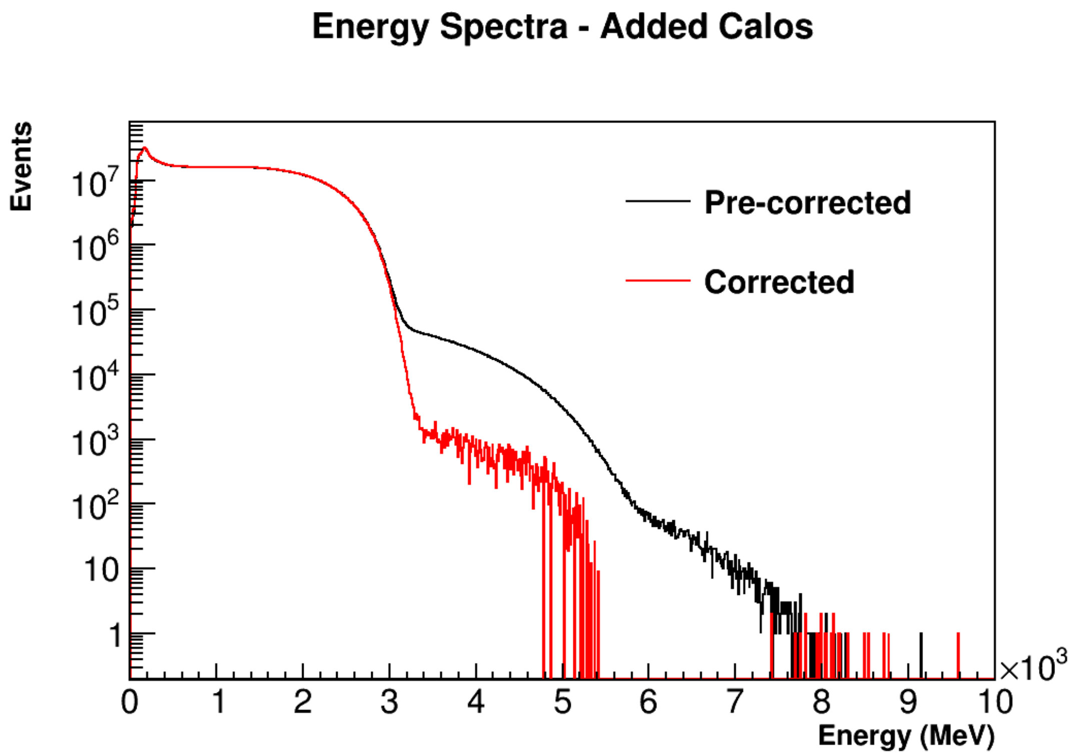
- Not included in ratio fit, since it divides out due to the VW frequency being very near an even multiple of the g-2 frequency



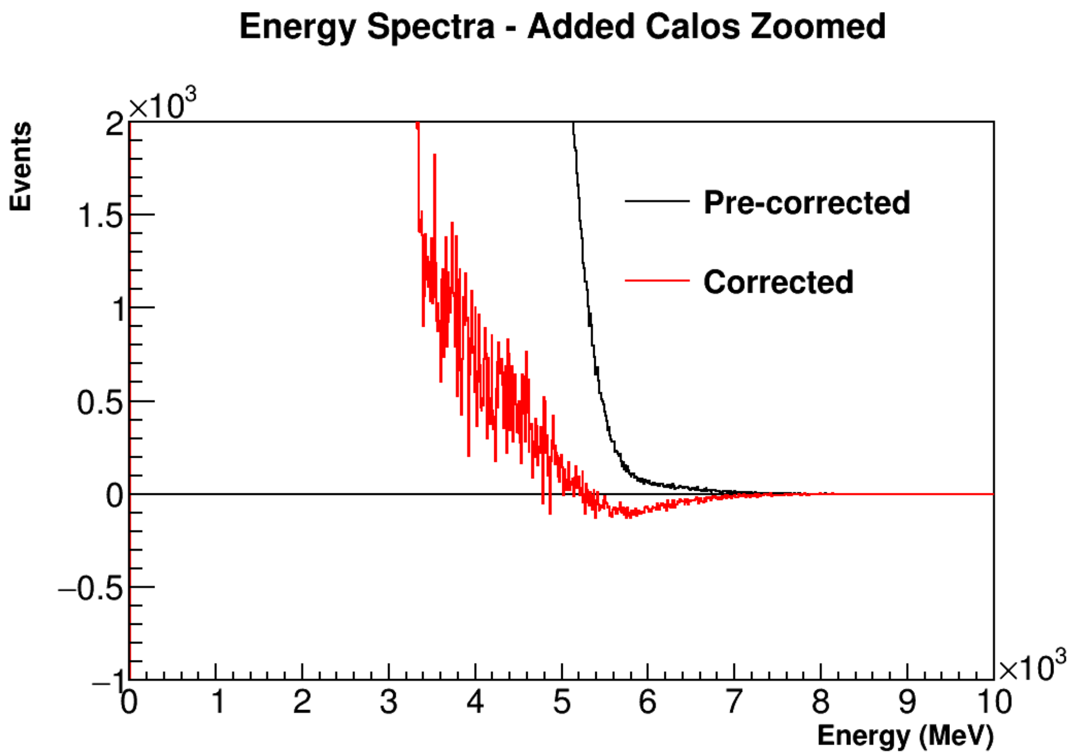
Analysis Results

Pileup Correction

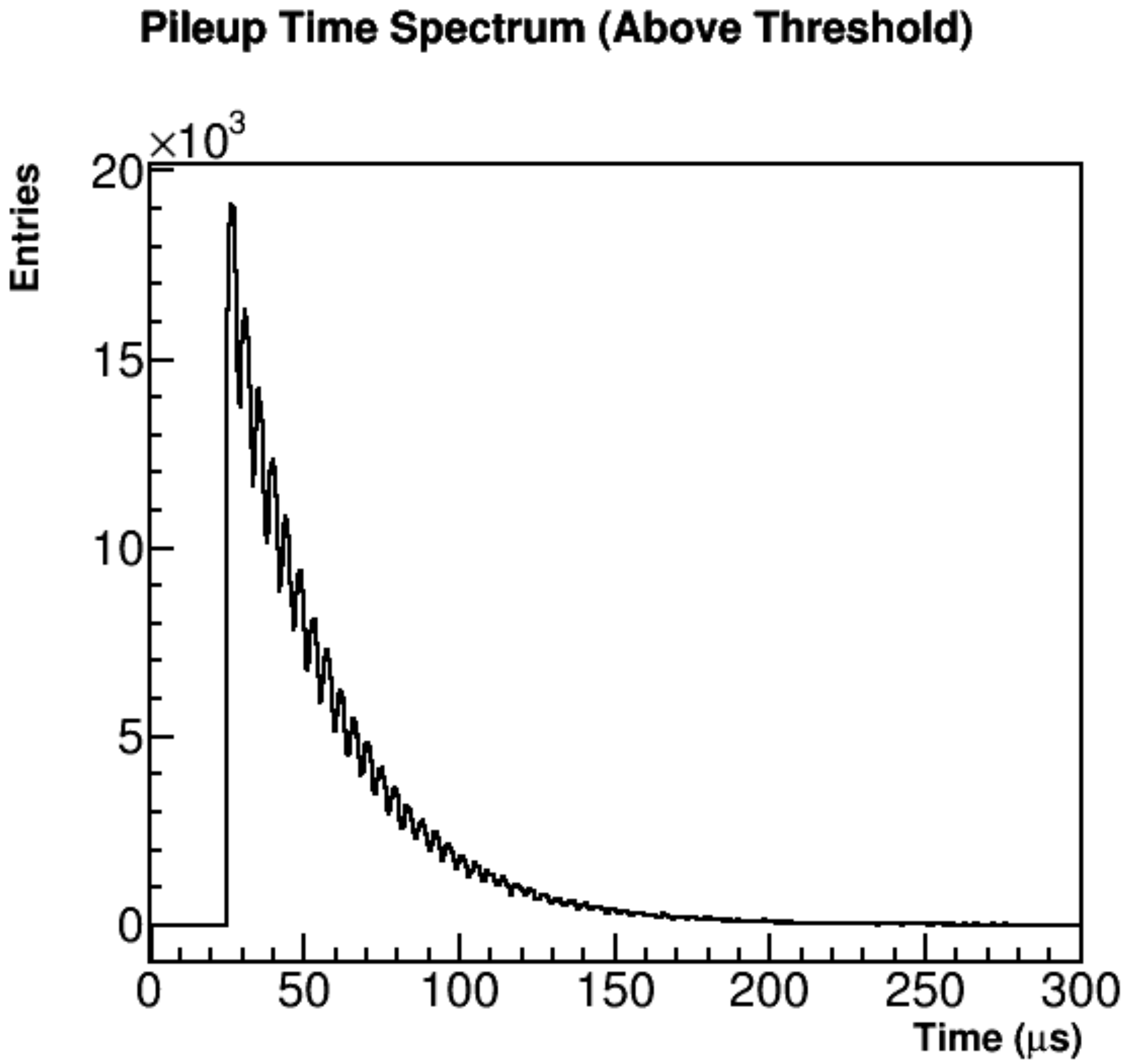


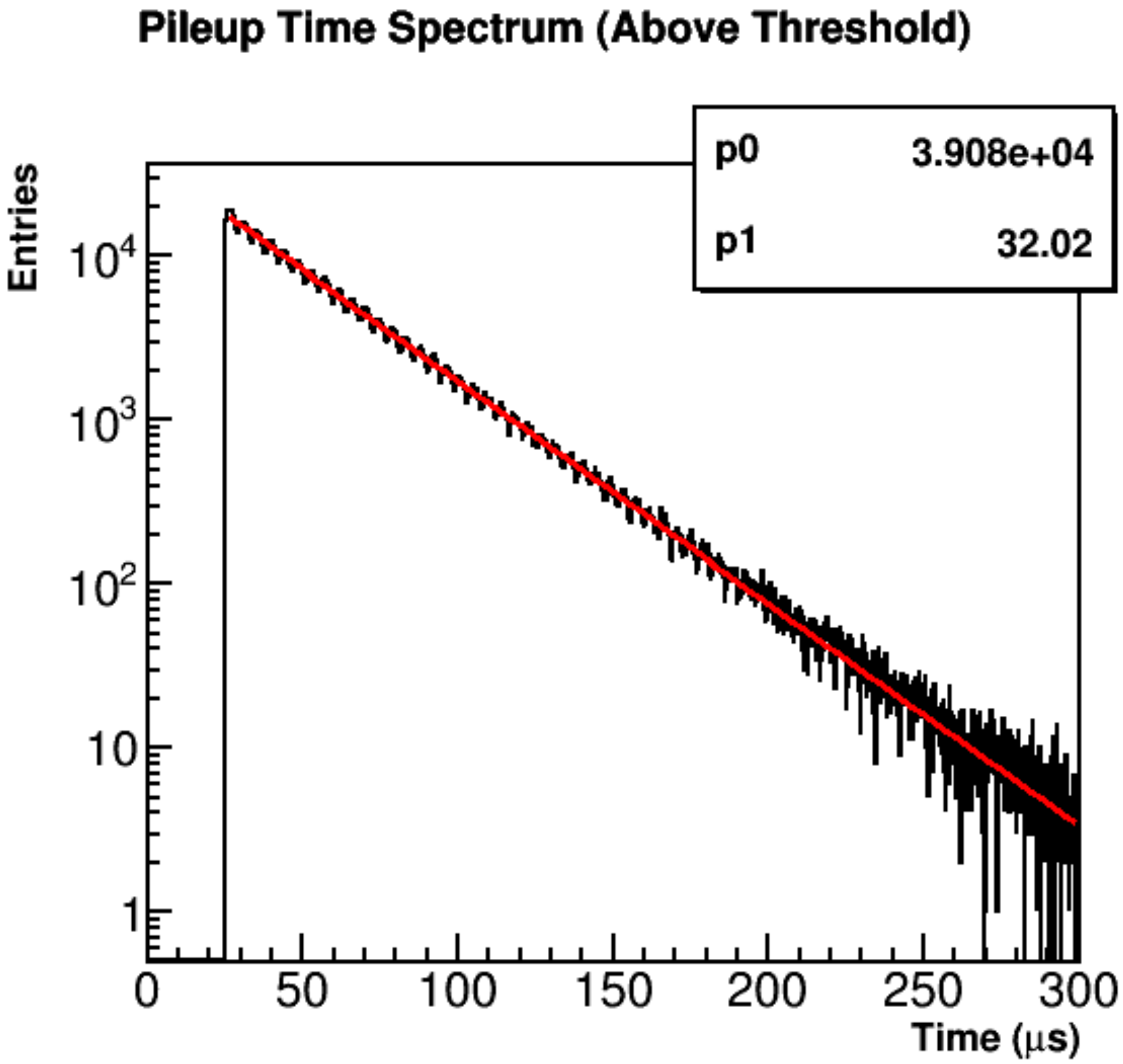


Energy Spectra - Added Calos
Events
Pre-corrected
Corrected
Energy (MeV)

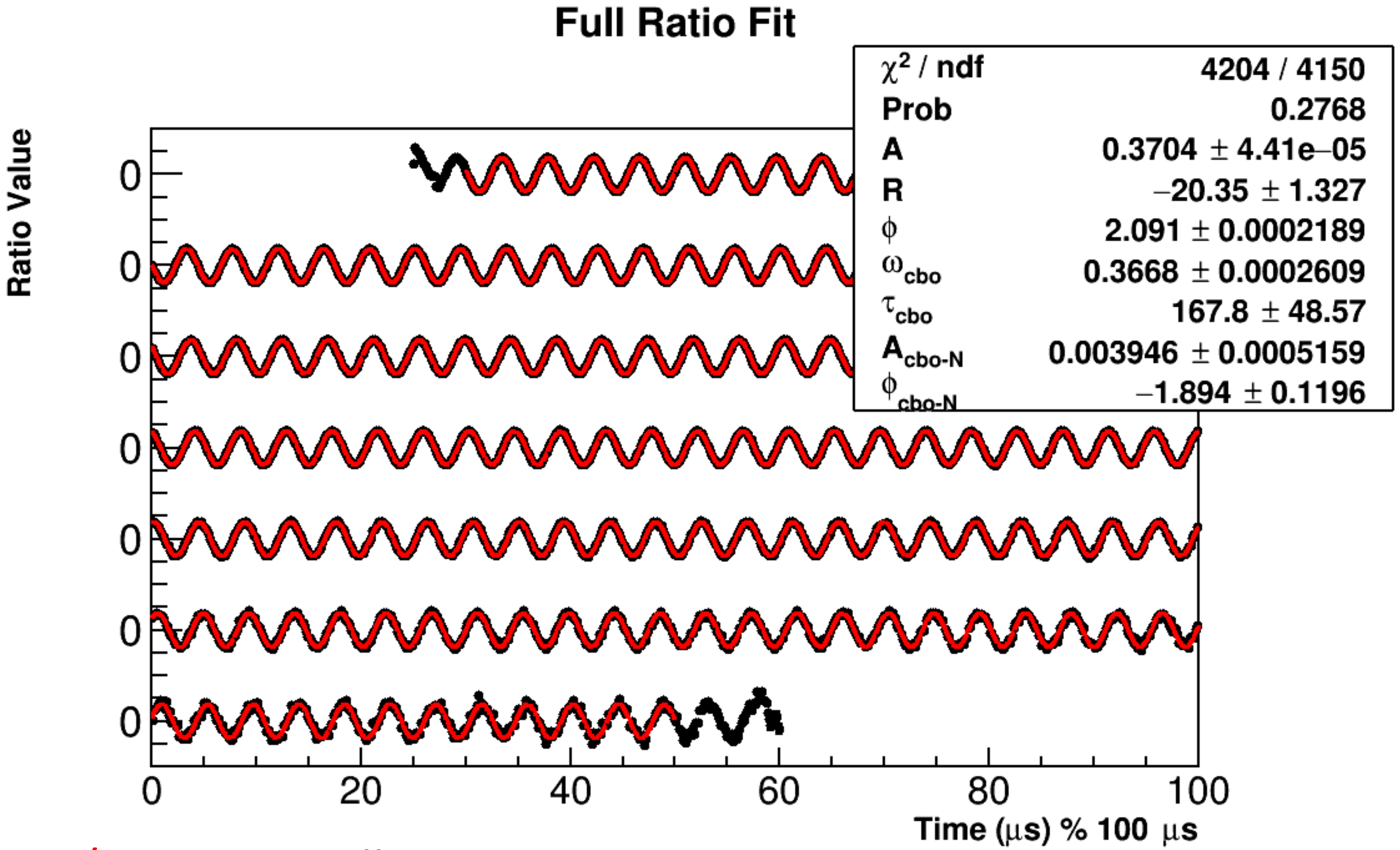


Energy Spectra - Added Calos Zoomed
Events
Pre-corrected
Corrected
Energy (MeV)


Pileup Time Spectrum (Above Threshold)
Entries
×10^3
Time (μs)

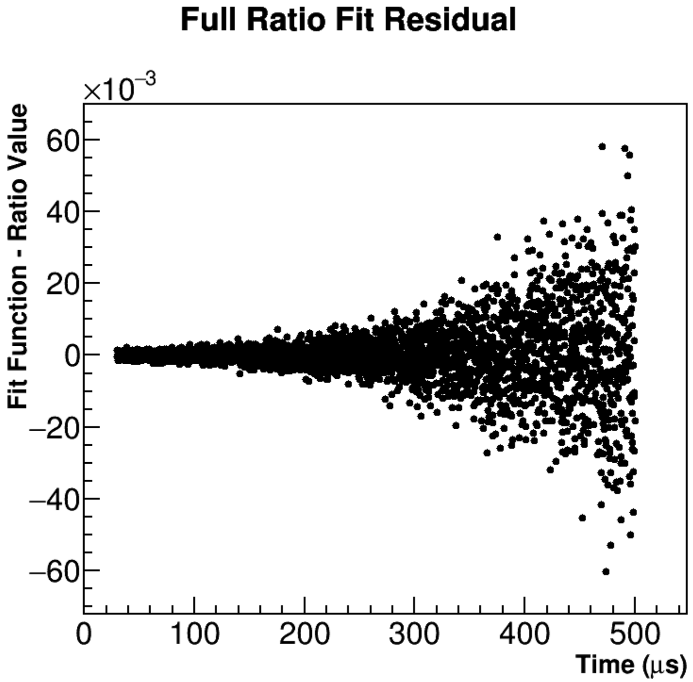

Pileup Time Spectrum (Above Threshold)
Entries
p0 3.908e+04
p1 32.02
Time (μs)

Fit – Gold Data

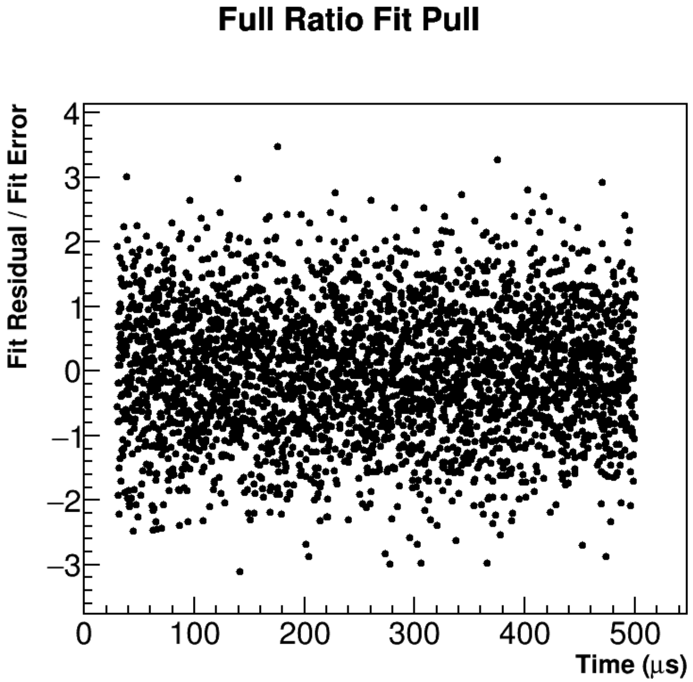


Missing 190/1,498,587 Fills

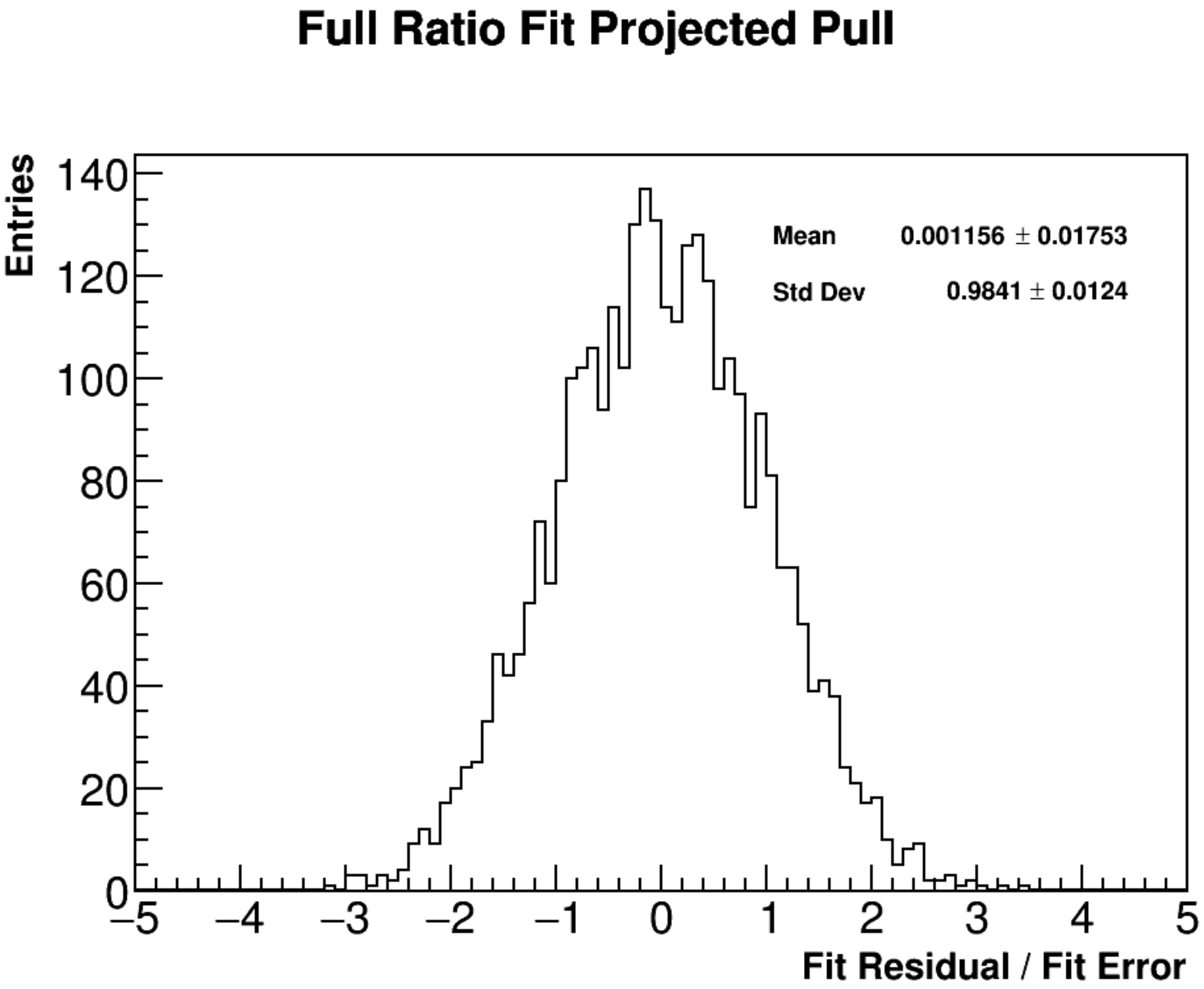
Fit



Full Ratio Fit Residual
×10⁻³
Fit Function - Ratio Value
Time (μs)

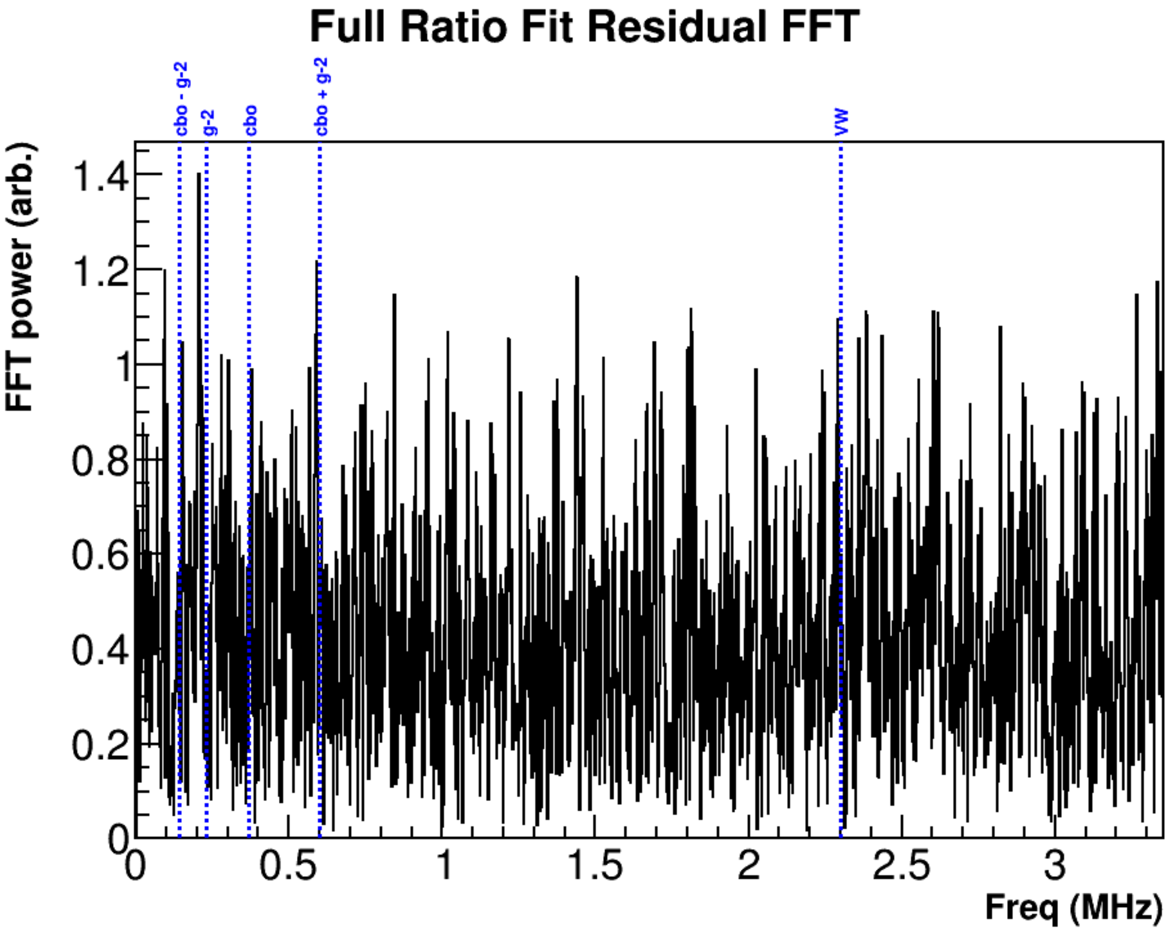


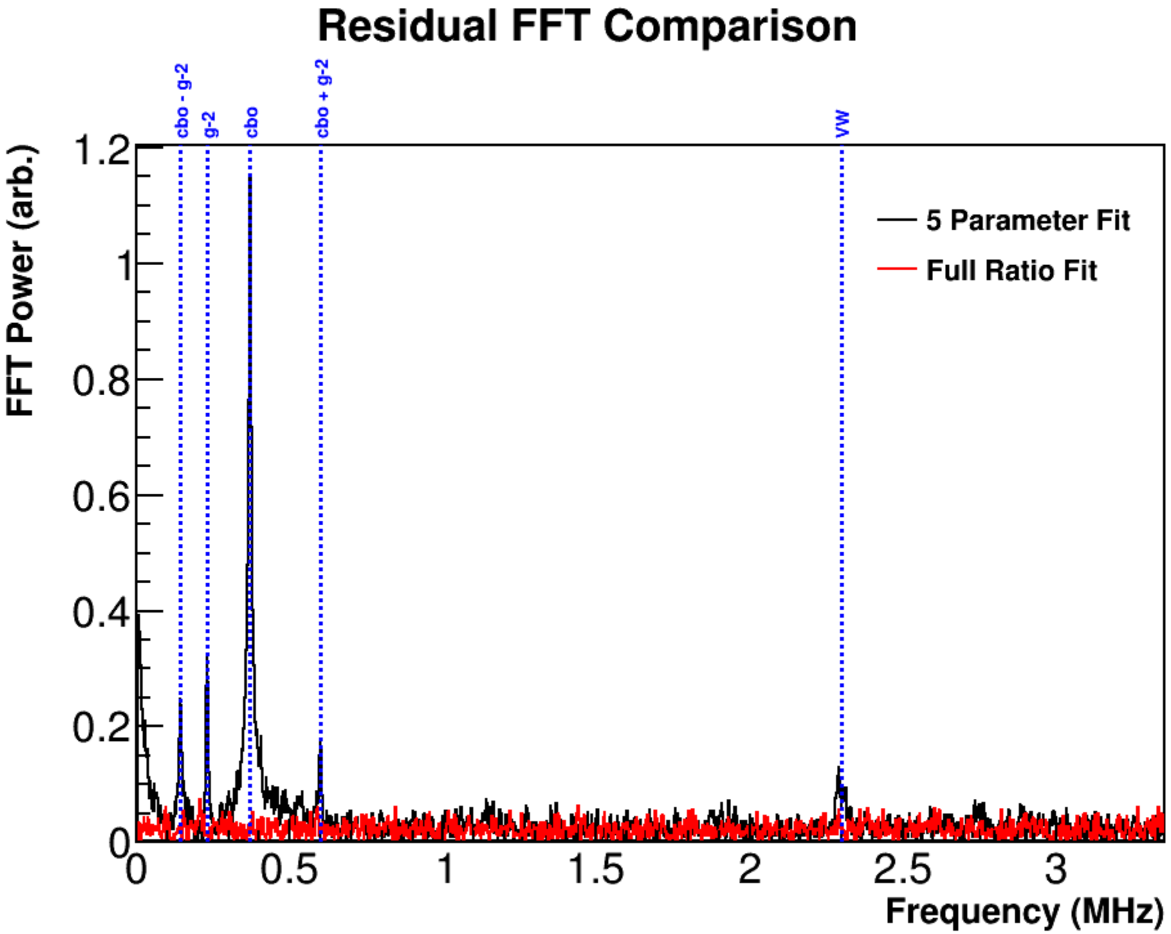
Full Ratio Fit Pull
Fit Residual / Fit Error
Time (μs)



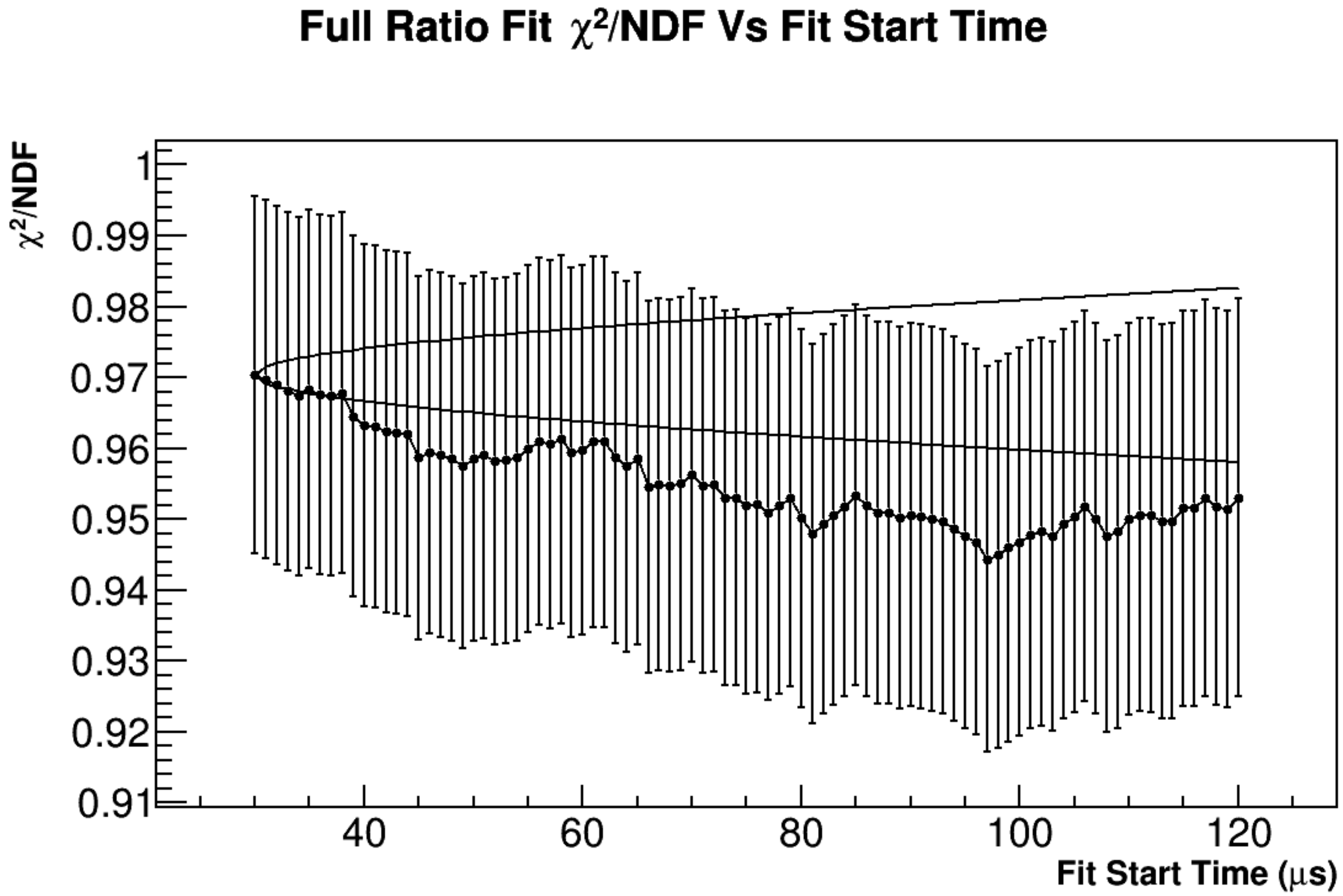
Full Ratio Fit Projected Pull
Entries
Mean 0.001156 ± 0.01753
Std Dev 0.9841 ± 0.0124
Fit Residual / Fit Error

Fit Residual FFT

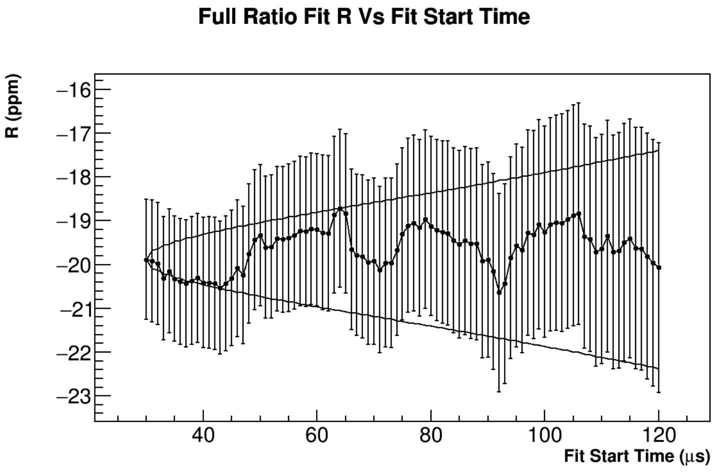


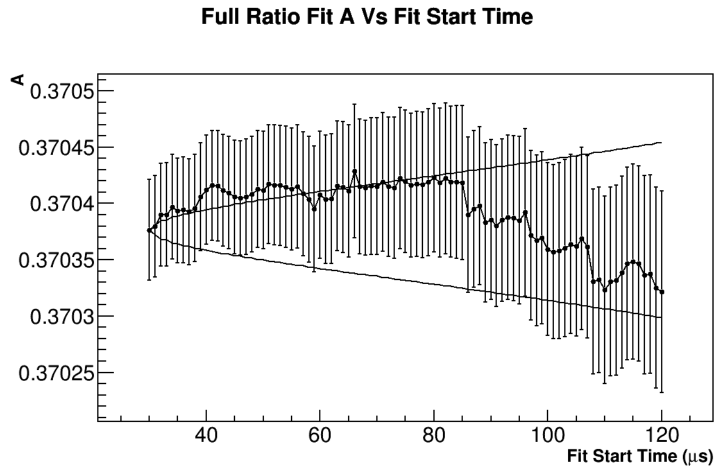


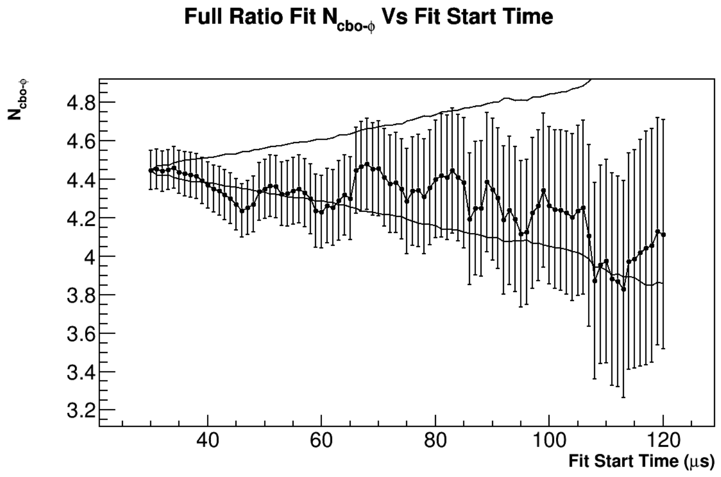
Fit Start Time Scan

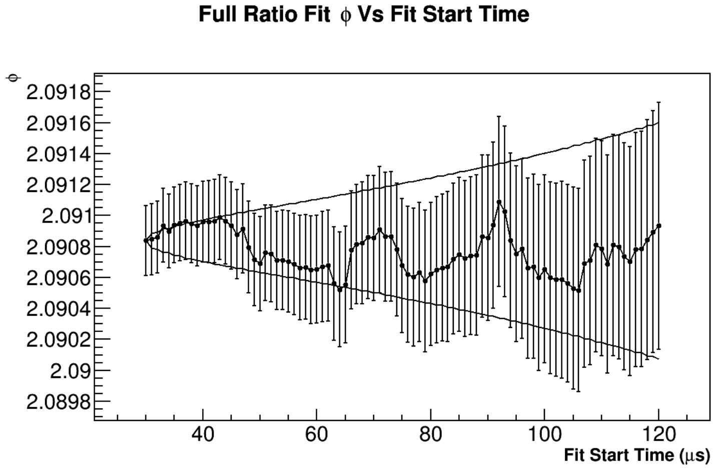


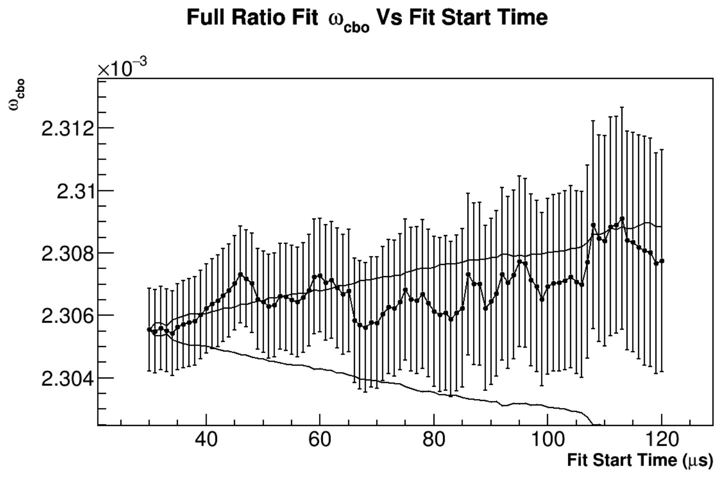
Fit Start Time Scan

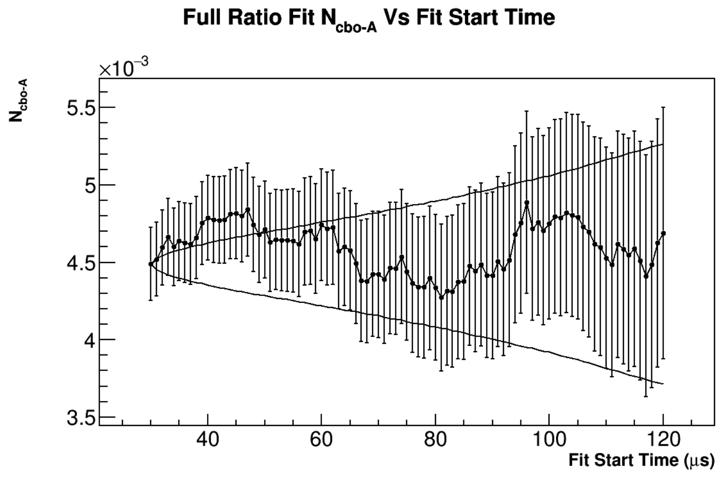




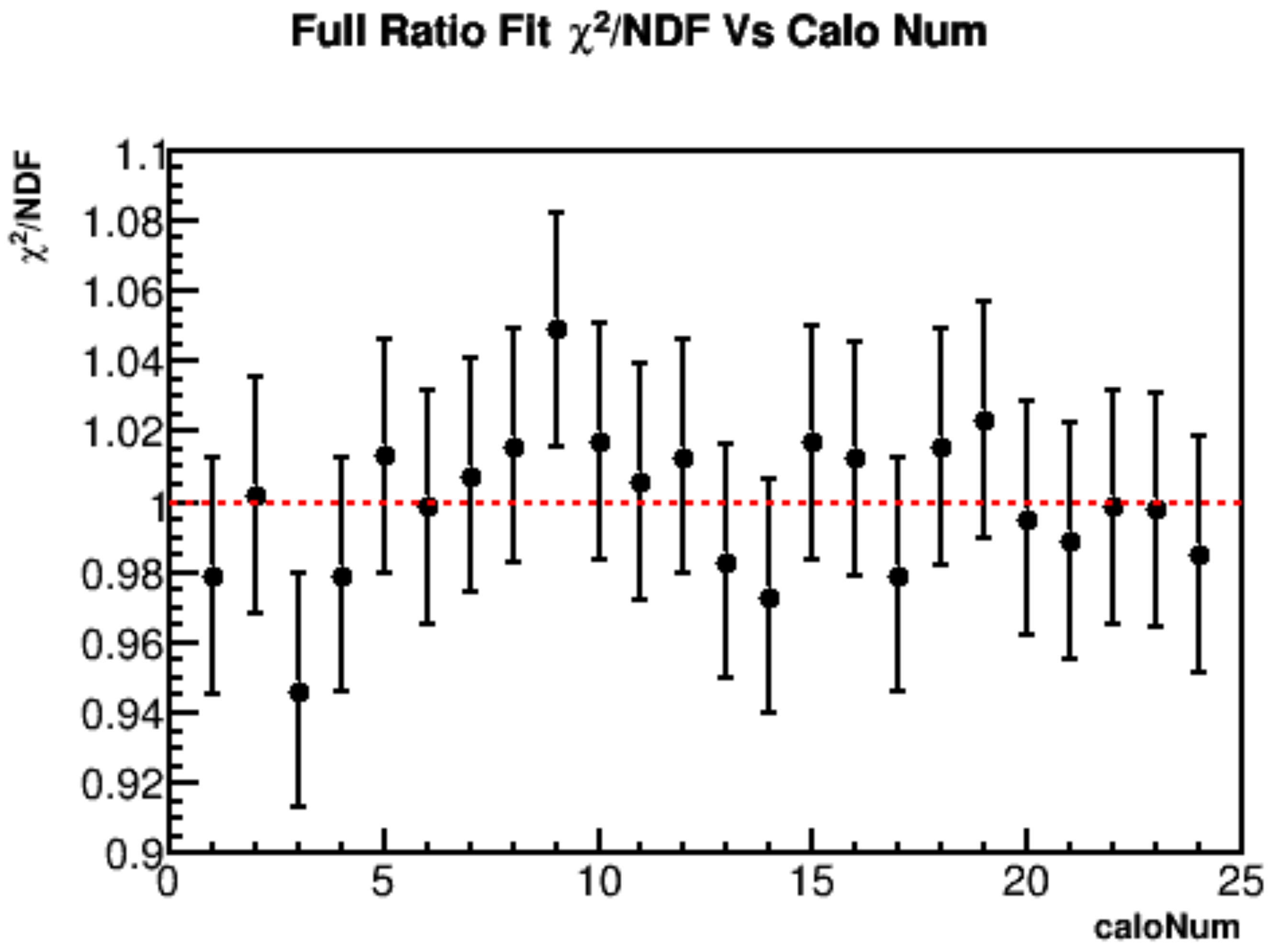




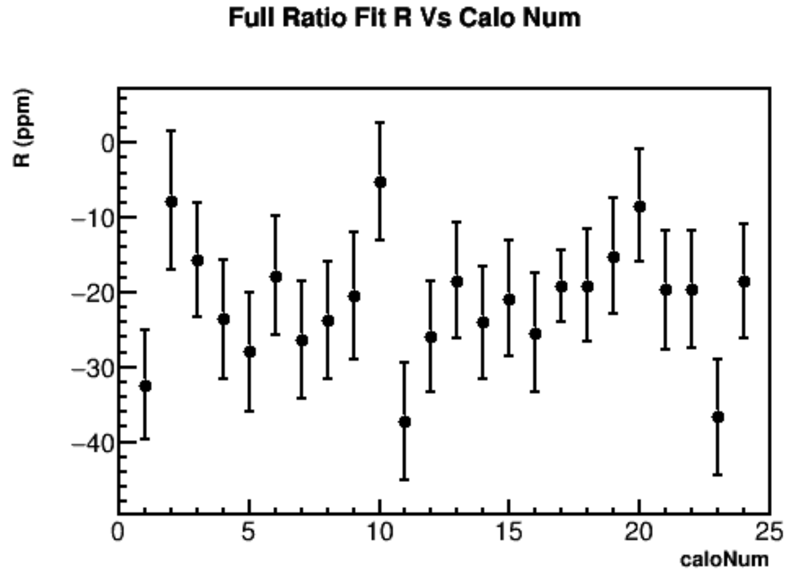


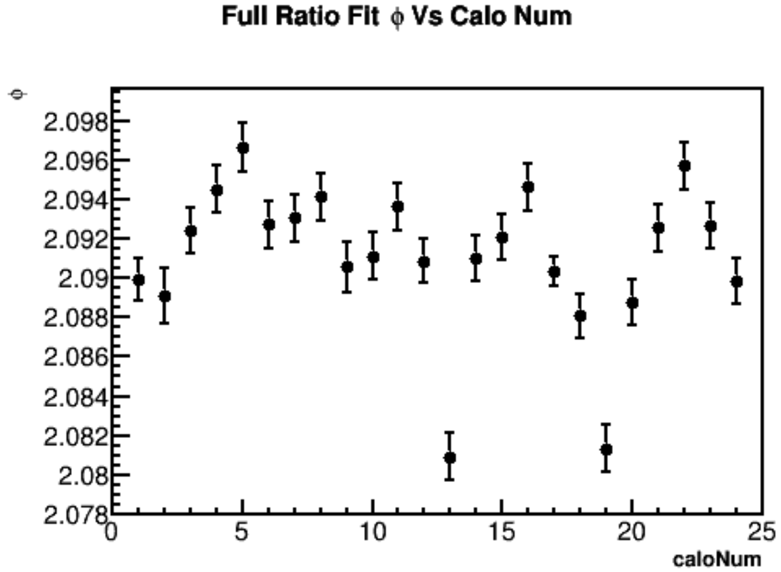


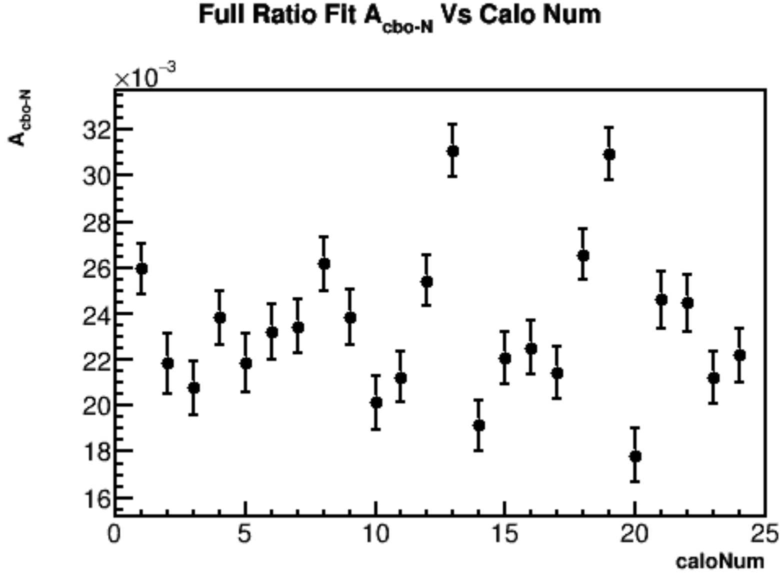
Per Calo Fits

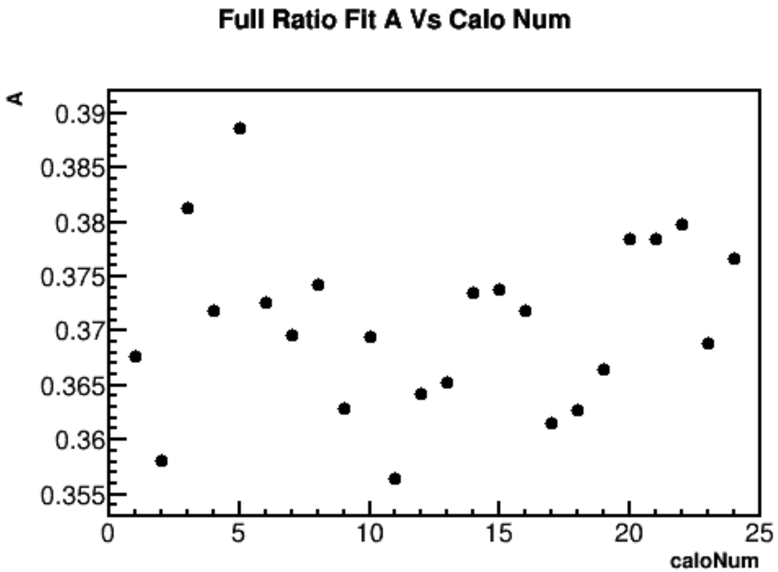


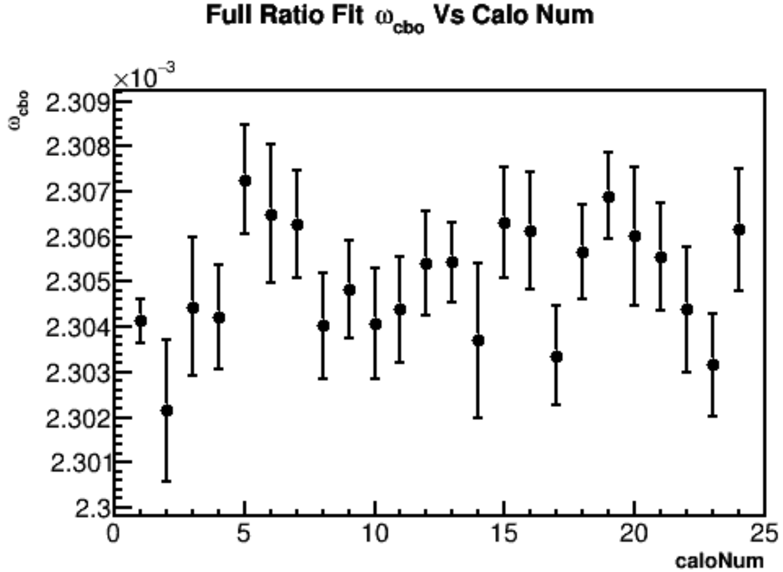
Per Calo Fits

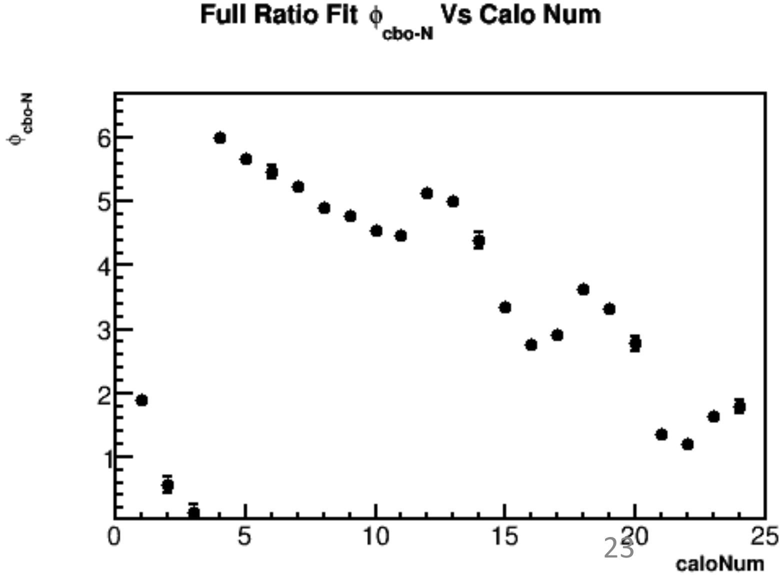




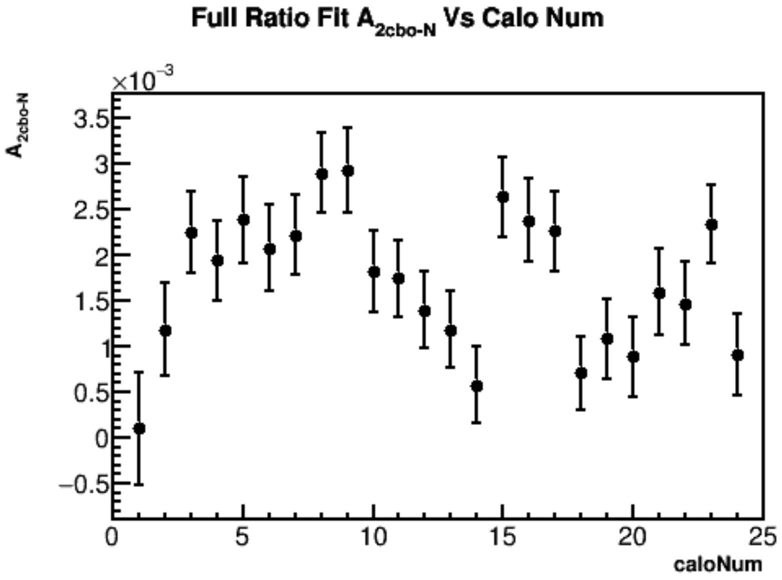


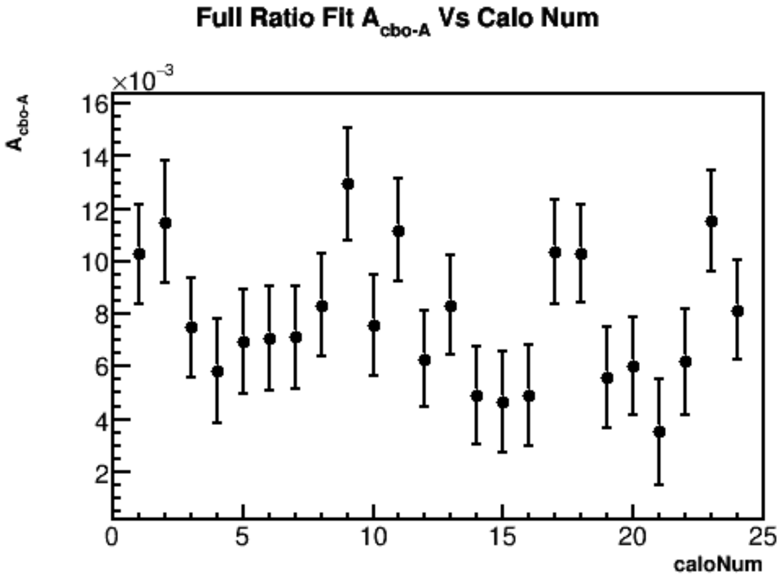


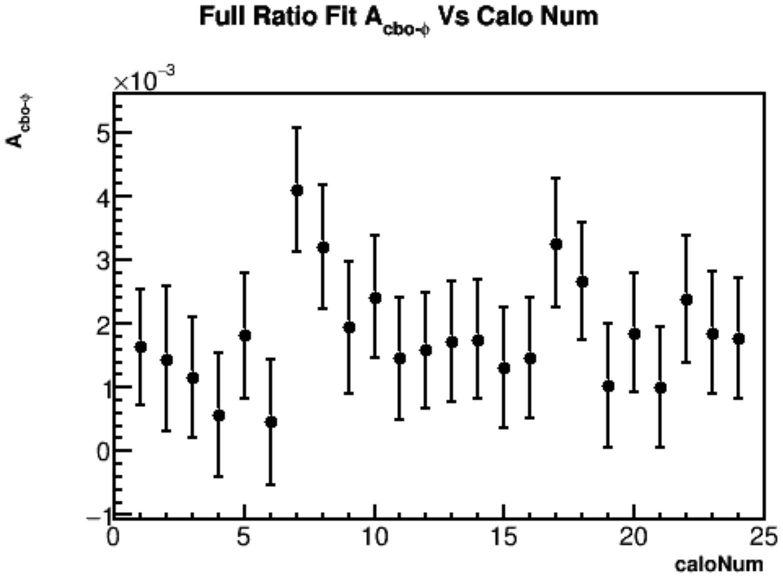


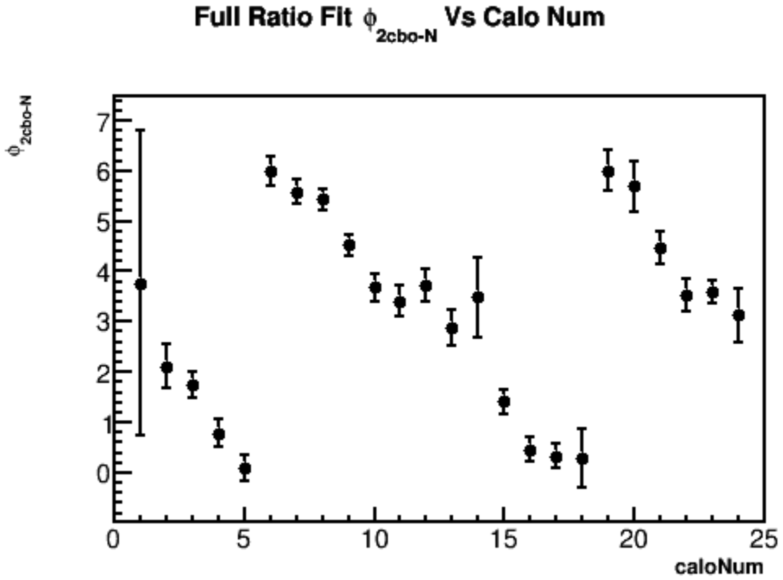


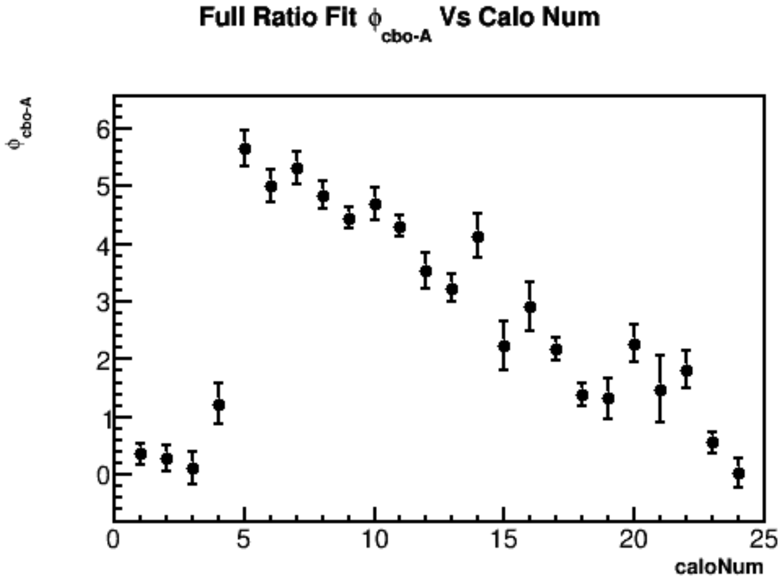
Per Calo Fits

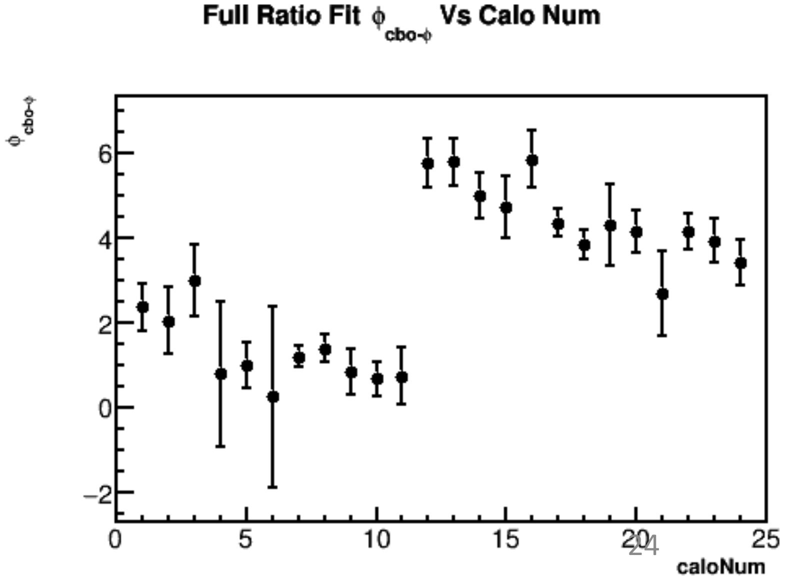












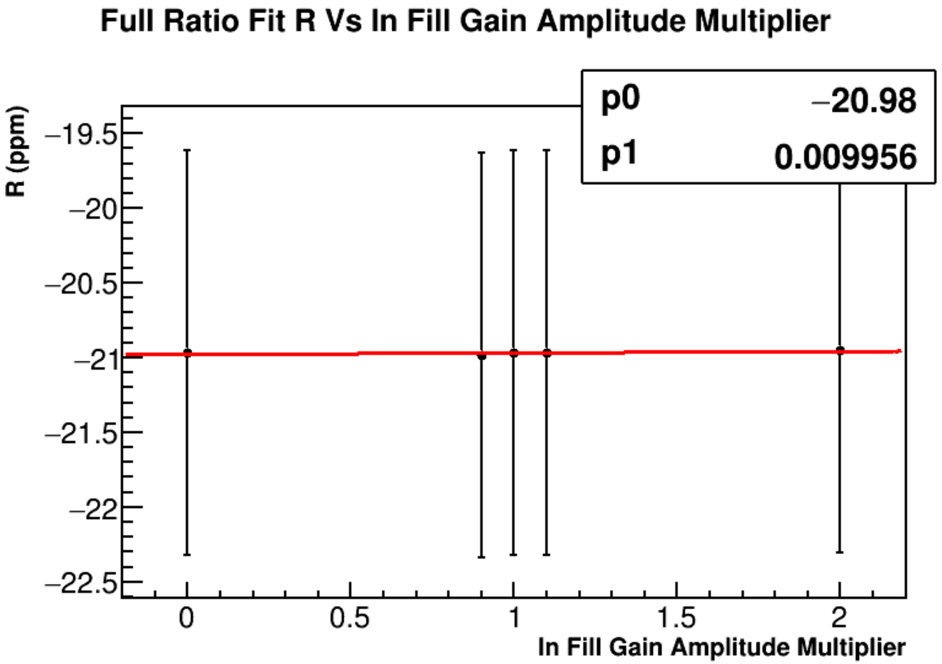
Systematic Evaluations

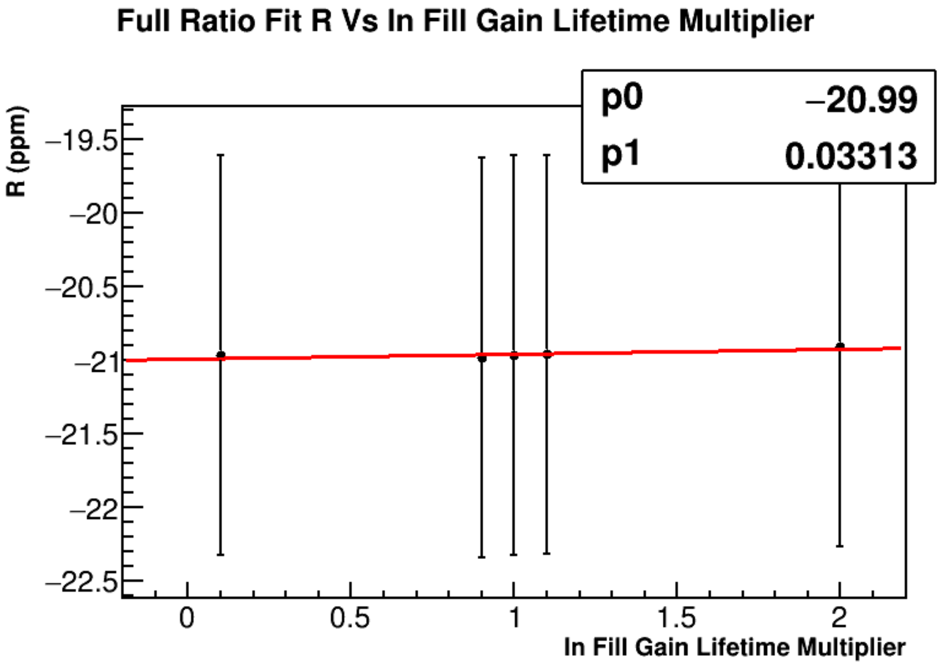
- Everything here evaluated on 5033A dataset

In-Fill Gain

$$E = E_0(1 - A \cdot e^{-t/\tau})$$

- Stored crystal in fill gain amplitude and lifetime parameters
- Undid the correction at the histogram phase and reapplied new correction factors (in multiples of the original factors)





In-Fill Gain

- Systematic error calculated in the usual way, slope times uncertainty

$$\delta R_A = \delta\alpha_A \times \frac{dR}{d\alpha_A}$$

$$\delta R_\tau = \delta\alpha_\tau \times \frac{dR}{d\alpha_\tau}$$

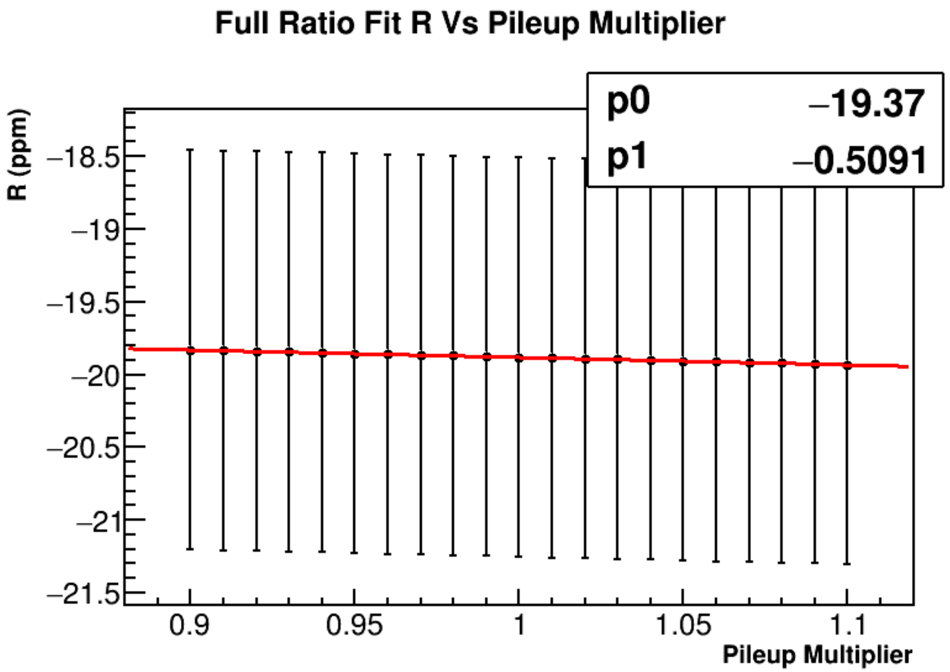
- Taking the uncertainty as 10%, I get a systematic error (added in quadrature) of 3.5 ppb
- Need to estimate the uncertainty in a better way, and want to take more points and recalculate this

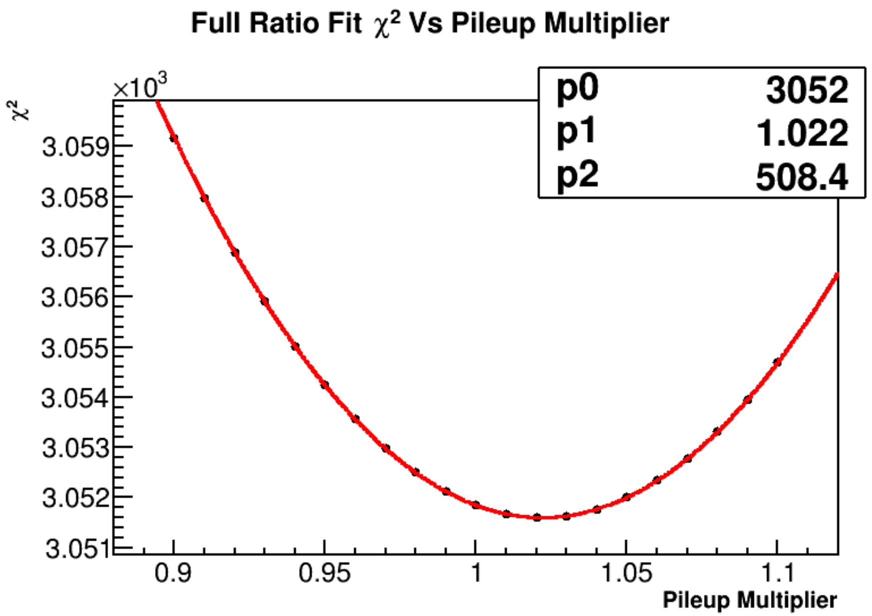
Other Gain

- For the SDTP, want to calculate R w/ and w/o it to calculate the systematic error
- Need to do this at the reconstruction level

- Other gain systematics

Pileup Amplitude





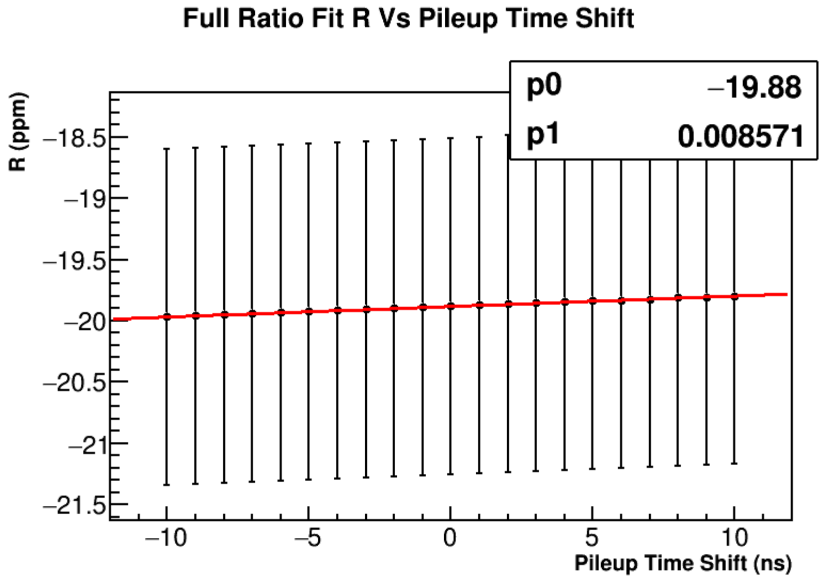
$$\delta\alpha_{pm} = \sqrt{1/508.4}$$

$$\delta R_{pm} = \delta\alpha_{pm} \times \frac{dR}{d\alpha_{pm}}$$

Systematic error: 22.6 ppb

Pileup Phase

- Time shift pulses before filling pileup histograms and refit

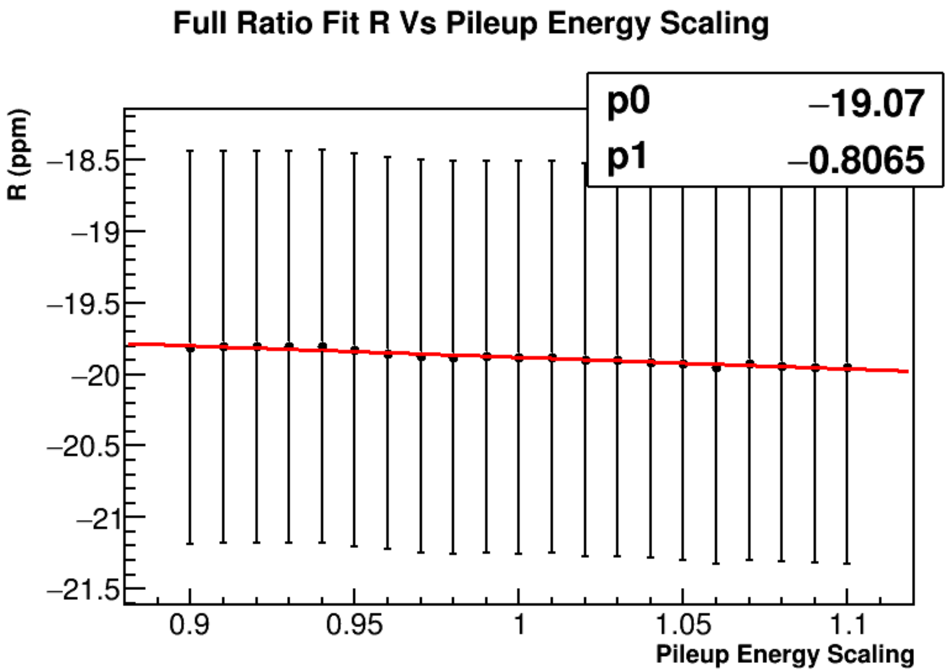


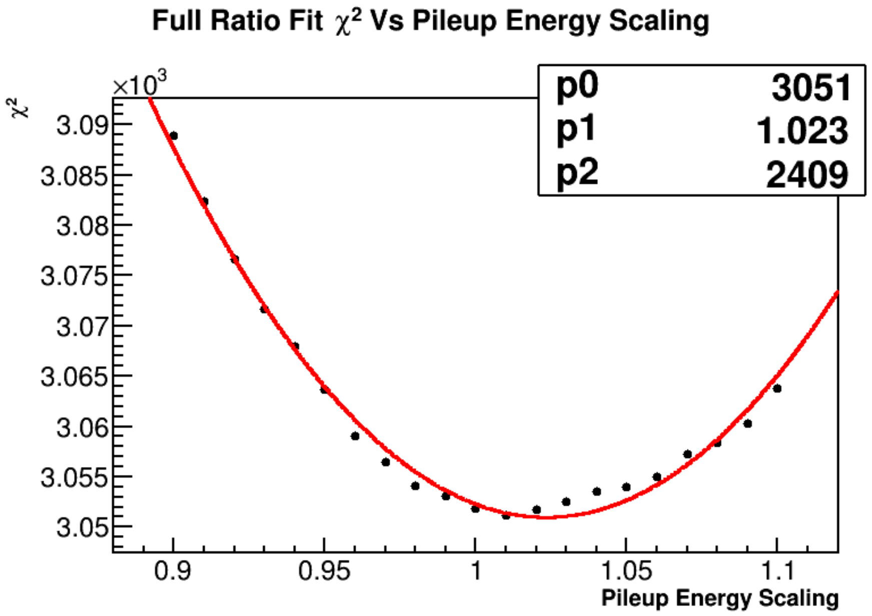
$$\delta R_{pp} = \delta\alpha_{pp} \times \frac{dR}{d\alpha_{pp}}$$

- Take the uncertainty on the pileup time conservatively at 3 ns
- Systematic error: 25.7 ppb

Pileup Phase

$$E_{doublet} = C \cdot (E_1 + E_2)$$





$$\delta\alpha_{pm} = \sqrt{1/2409}$$

$$\delta R_{pe} = \delta\alpha_{pe} \times \frac{dR}{d\alpha_{pe}}$$

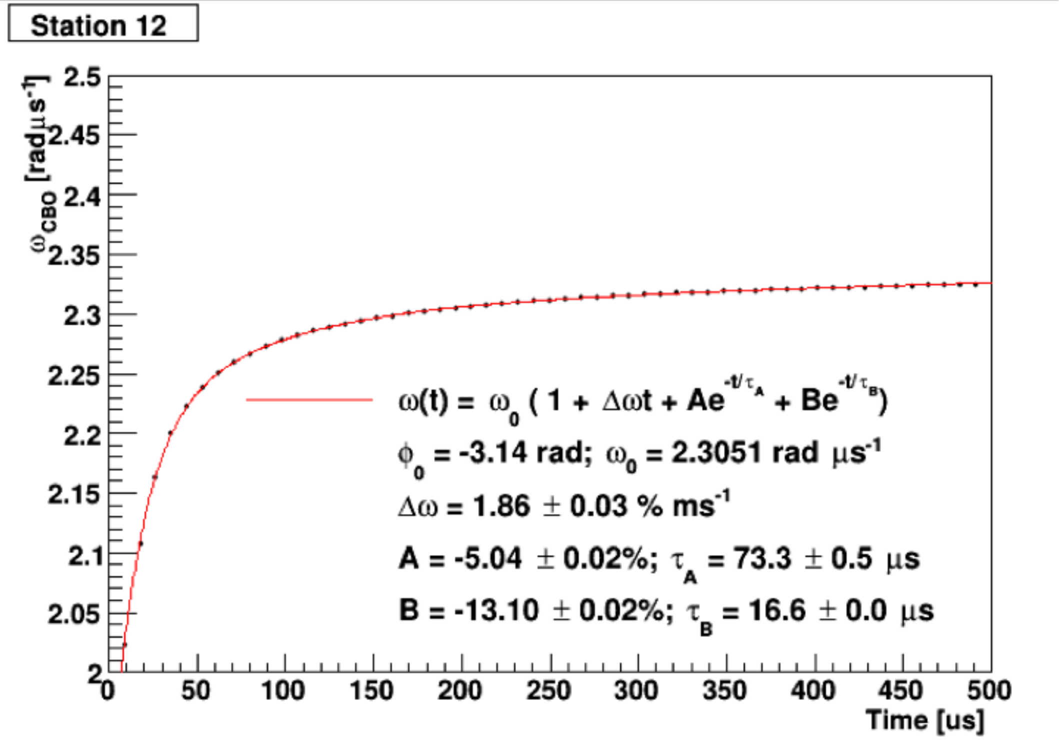
Systematic error: 16.1 ppb

Total Pileup Systematic Error

- 37.8 ppb added in quadrature

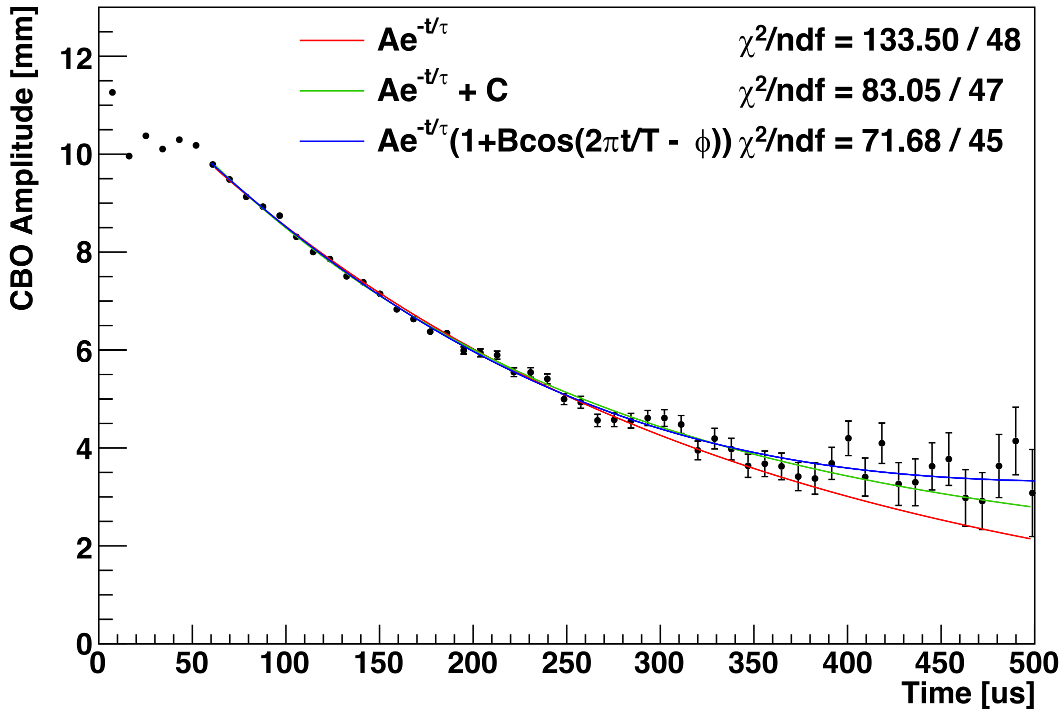
CBO Frequency

- Varied parameters in frequency function by ±2σ
- Systematic error conservatively estimated at 30 ppb



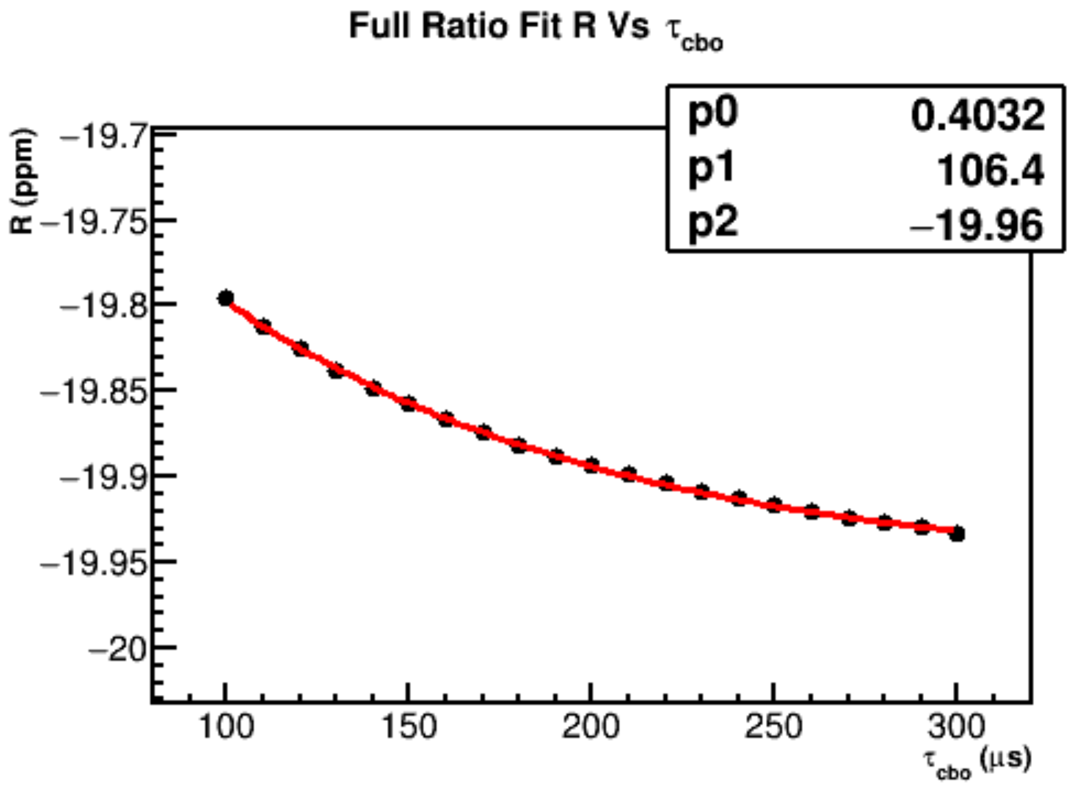
CBO Shape

- Fit with various shapes determined in tracking analysis
- Largest change on R taken as the systematic error at 21.1 ppb



CBO Lifetime

- The lifetime was originally fixed in the ratio fit, and is now going to be left free – but I calculated a systematic error for it which I'll include here
- Scanned over the lifetime parameter
- R vs lifetime fit to a parabola, and the change in R for the fixed lifetime ± the lifetime error from a T method fit taken as the systematic error at 12.1 ppb



Total CBO Systematic Error

- 38.6 ppb added in quadrature
- Still want to calculate a systematic error from excluding certain CBO pieces (like the phi cbo term)

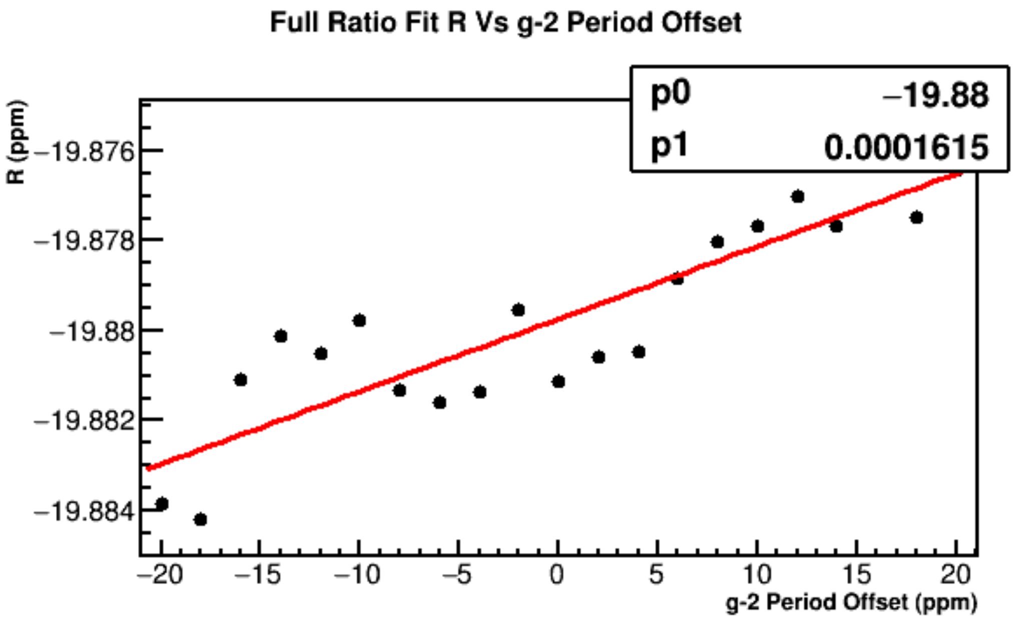
Lost muons and VW

- Lost muon function: Fit w/ and w/o lost muon function and parameter determined from T method fit: 36.1 ppb
- Need to calculate other lost muon systematic error parts

- VW function: Fit w/ and w/o VW function and parameters determined from T method fit: < 1 ppb

g-2 period

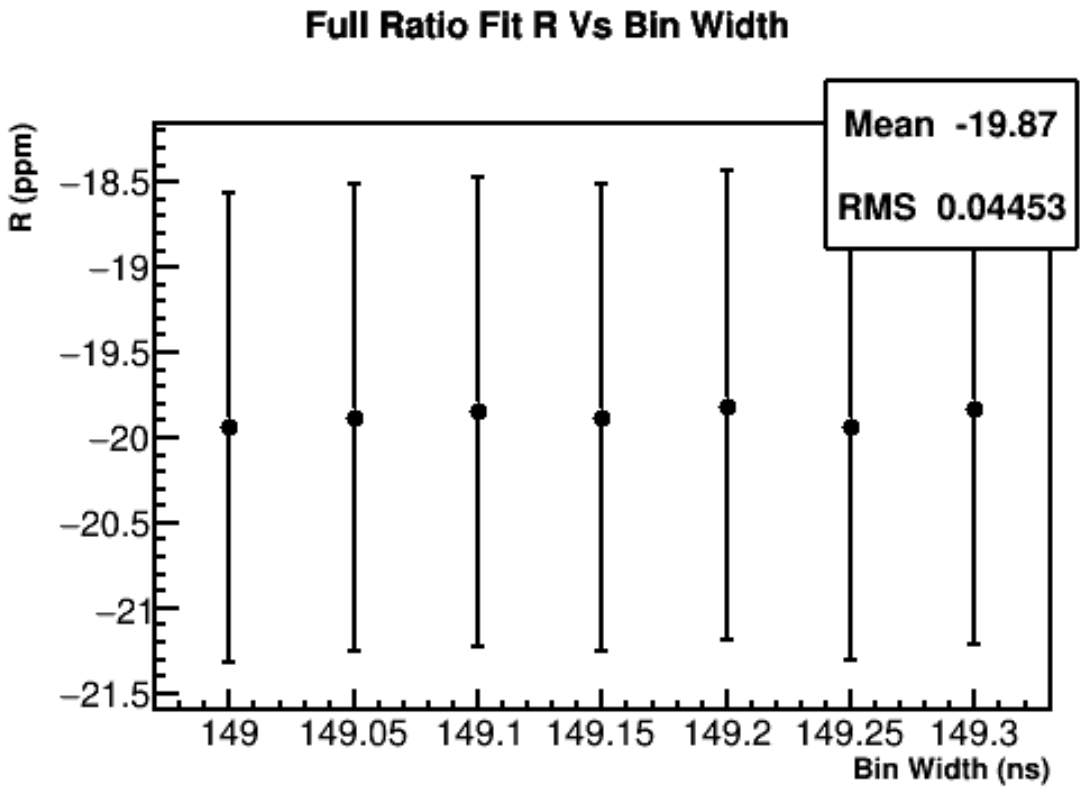
- g-2 period used in making the ratio histogram:
- Uncertainty taken at 10 ppm, systematic error calculated at 1.6 ppb



$$\delta R_{period} = \delta \alpha_{period} \times \frac{dR}{d\alpha_{period}}$$

Bin Width

- Performed histogramming and fitting stages with various bin widths
- Took the RMS spread in fitted R values as the systematic error at 44.5 ppb
- Want to redo this one with more points and a range defined more appropriately

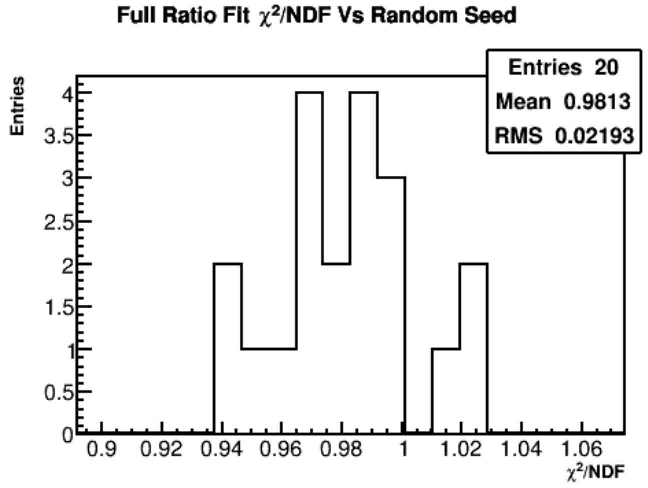


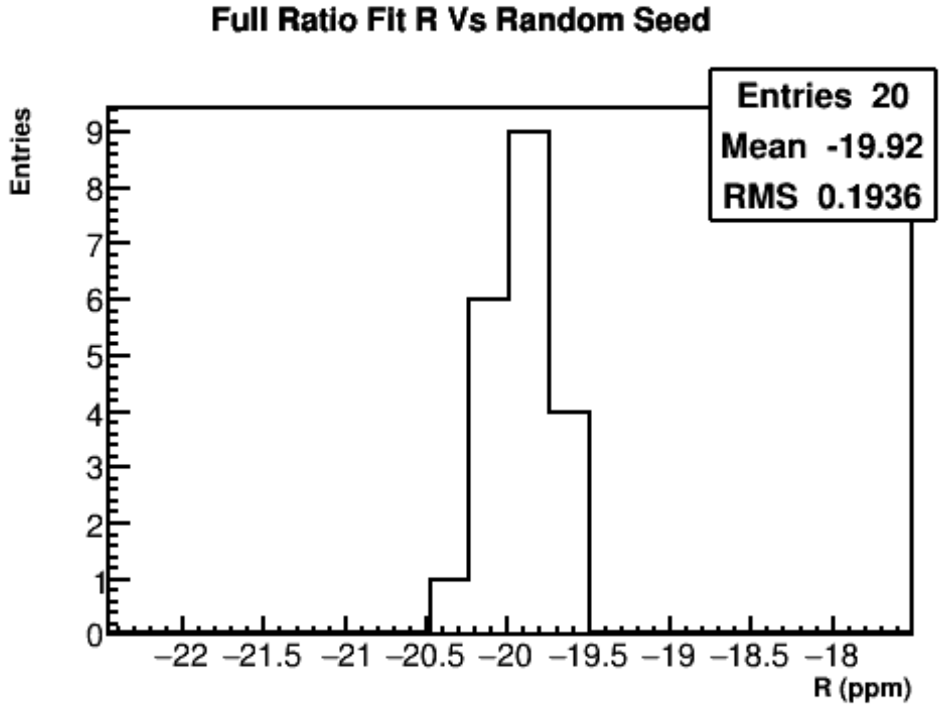
Randomization

- Performed fitting with 20 different random seeds
- The systematic error is calculated as (where N is the number of random seeds):

$$\delta R_{rand} = \sigma(R)/\sqrt{N-1}$$

- Systematic error calculated at 44.4 ppb
- Want to redo this with more seeds





(This done on 5033A, so mean doesn't correspond to the final R answer yet.)

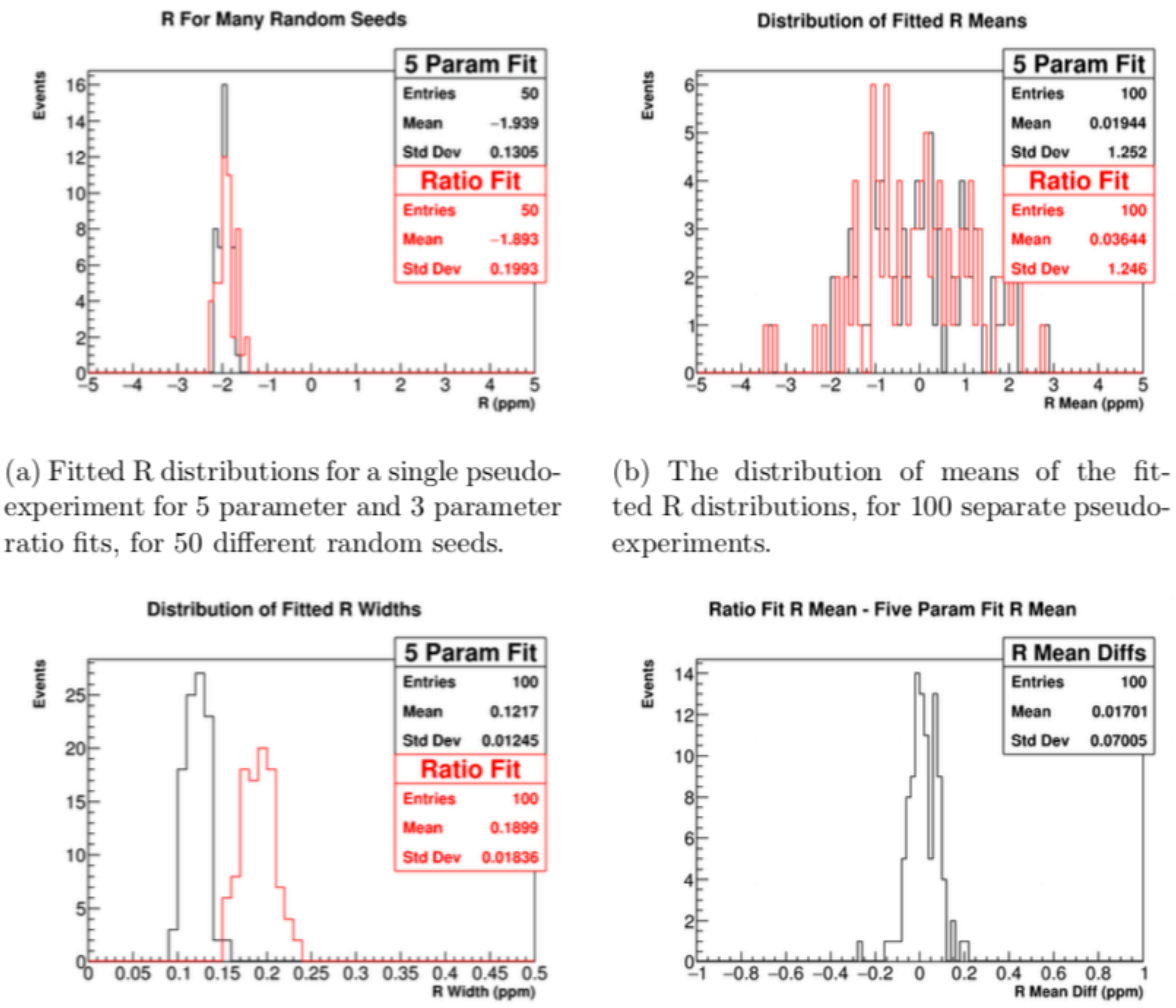
Systematic error table

Summary of Systematic Errors	
Systematic Error	60 H
Gain (incomplete)	3.5
Pileup	37.8
Lost Muons (incomplete)	36.1
CBO	38.6
VW	< 1
Bin Width	44.5
Randomization	44.4
Other (incomplete)	1.6
Total	90.5

Still some systematic errors to estimate/finish

T Method MC Comparison

- Performed 100 pseudo-experiments in MC
- Each pseudo-experiment consisted of a set of “positron” hits with stats comparable to the 60H dataset
- Each set of hits was time randomized with 50 random seeds
- 5 parameter fits and 3 parameter ratio fits were performed on all sets of hits



(a) Fitted R distributions for a single pseudo-experiment for 5 parameter and 3 parameter ratio fits, for 50 different random seeds.

(b) The distribution of means of the fitted R distributions, for 100 separate pseudo-experiments.

(c) The distribution of widths of the fitted R distributions, for 100 separate pseudo-experiments.

(d) The distribution of differences in fitted R means between 5 parameter fits and 3 parameter ratio fits per pseudo-experiment for 100 pseudo-experiments.

The width of the bottom right plot tells me the expected spread in average fitted R values for 50 different random seeds between a 5 parameter fit and a 3 parameter ratio fit– a difference of 70 ppb is consistent to 1σ

Summary

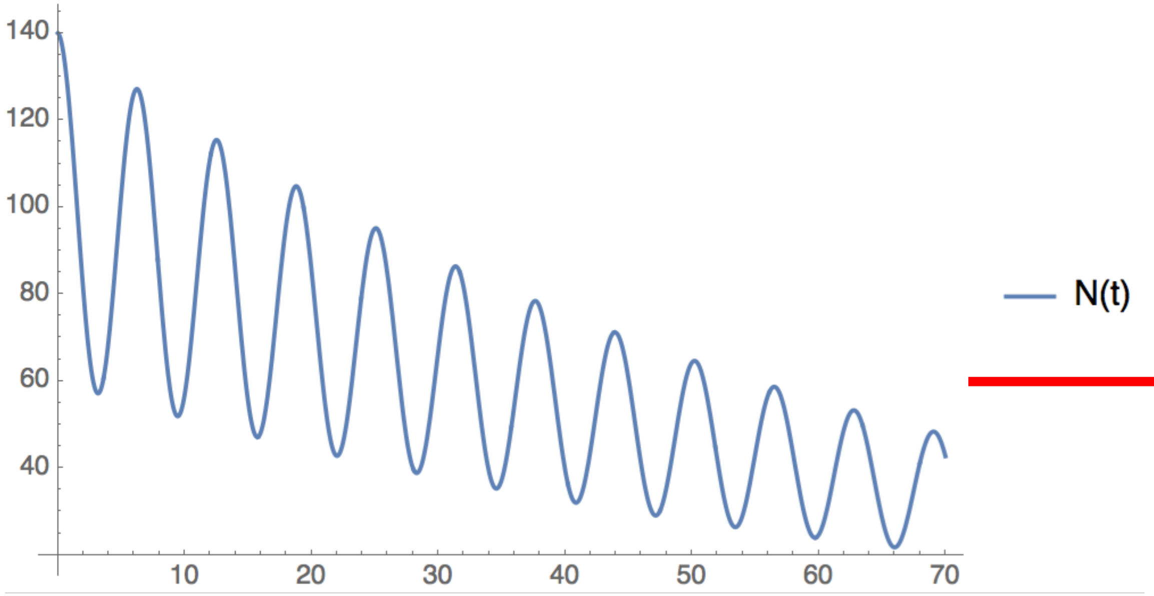
- R = -20.35 ppm (blinded) ± ~1.327 ppm (stat.) ± ~0.095 ppm (syst.)
- X^2/NDF = 4204 / 4150
- P value = 0.2768

- Need to make slight tuning adjustments on Gold dataset (and include missing file), then reproduce all fits, plots, and systematic studies for my report
- Continue with other systematic estimates

Backup

How the ratio method works

$$N_5(t) = N_0 e^{-t/\tau}(1 + A\cos(\omega_a t + \phi))$$



$$u_+(t) = N_5(t + T/2)/4$$
$$u_-(t) = N_5(t - T/2)/4$$
$$v_1(t) = v_2(t) = N_5(t)/4$$

Randomly split positron time spectra into 4 sets, two with time spectra shifted up and down by half a g-2 period, and two unchanged. (Equal weighting corresponding to ¼ factors.)

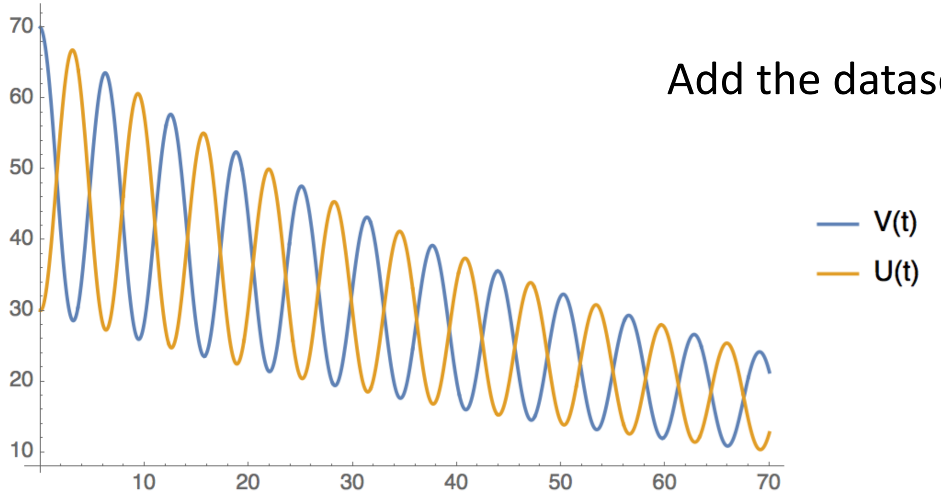
$$N_5(t \pm T/2) = N_0 e^{-t/\tau} e^{\mp T/2\tau}(1 + A\cos(\omega_a t \pm \omega_a \frac{T}{2} + \phi)) \qquad T \approx \frac{2\pi}{\omega_a}$$

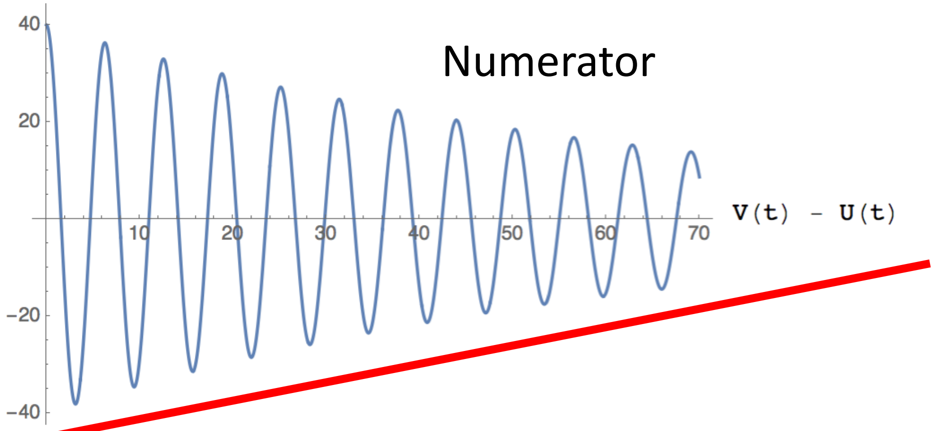
Add the datasets separately:

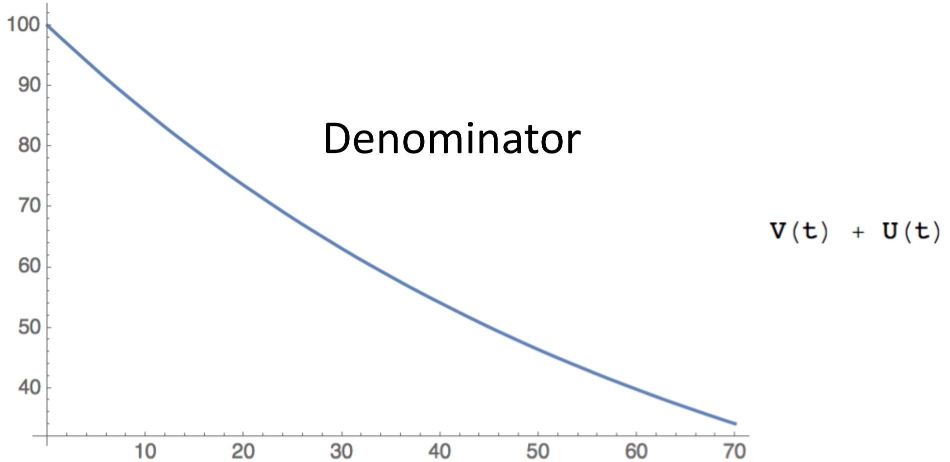
$$U(t) = u_+(t) + u_-(t)$$

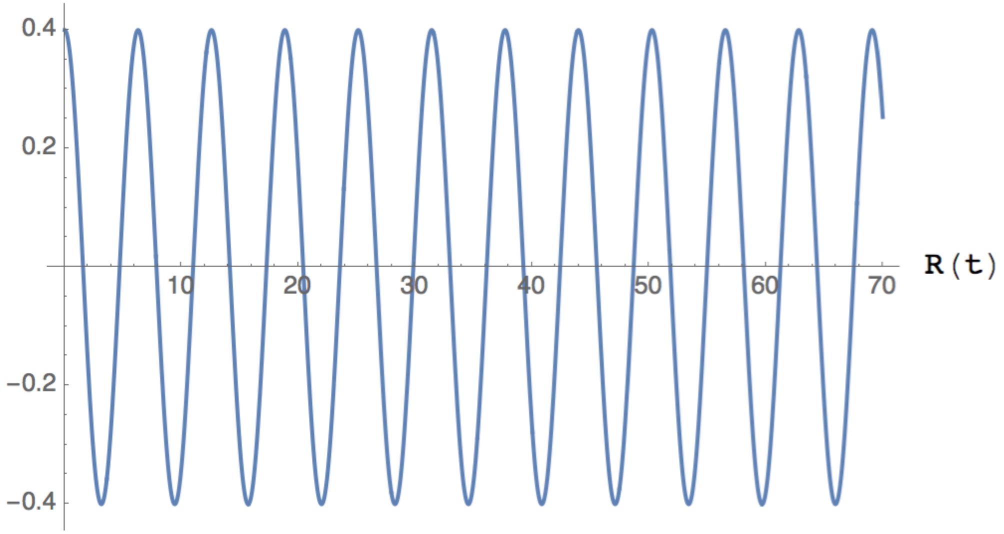
$$V(t) = v_1(t) + v_2(t)$$

$$R(t) = \frac{V(t) - U(t)}{V(t) + U(t)}$$









$$R(t) = A\cos(\omega_a t + \phi) - \frac{1}{16}\left(\frac{T}{\gamma\tau}\right)^2 + (h.o.)$$

Exponential gets divided out – fit is now down to 3 parameters.

Less sensitivity to slow effects which divide out.

Better weighting procedure

(for the # of counts in the histograms, not the weighting of the counts themselves)

$$u_+(t) : u_-(t) : v_1(t) : v_2(t) = 1 : 1 : 1 : 1$$

$$\downarrow$$

$$u_+(t) : u_-(t) : v_1(t) : v_2(t) = e^{T/2\tau} : e^{-T/2\tau} : 1 : 1$$

Exponential factors now cancel out and the ratio equals the simple form exactly (in the absence of further fit parameters like CBO and the limit that T is exactly the g-2 period.)

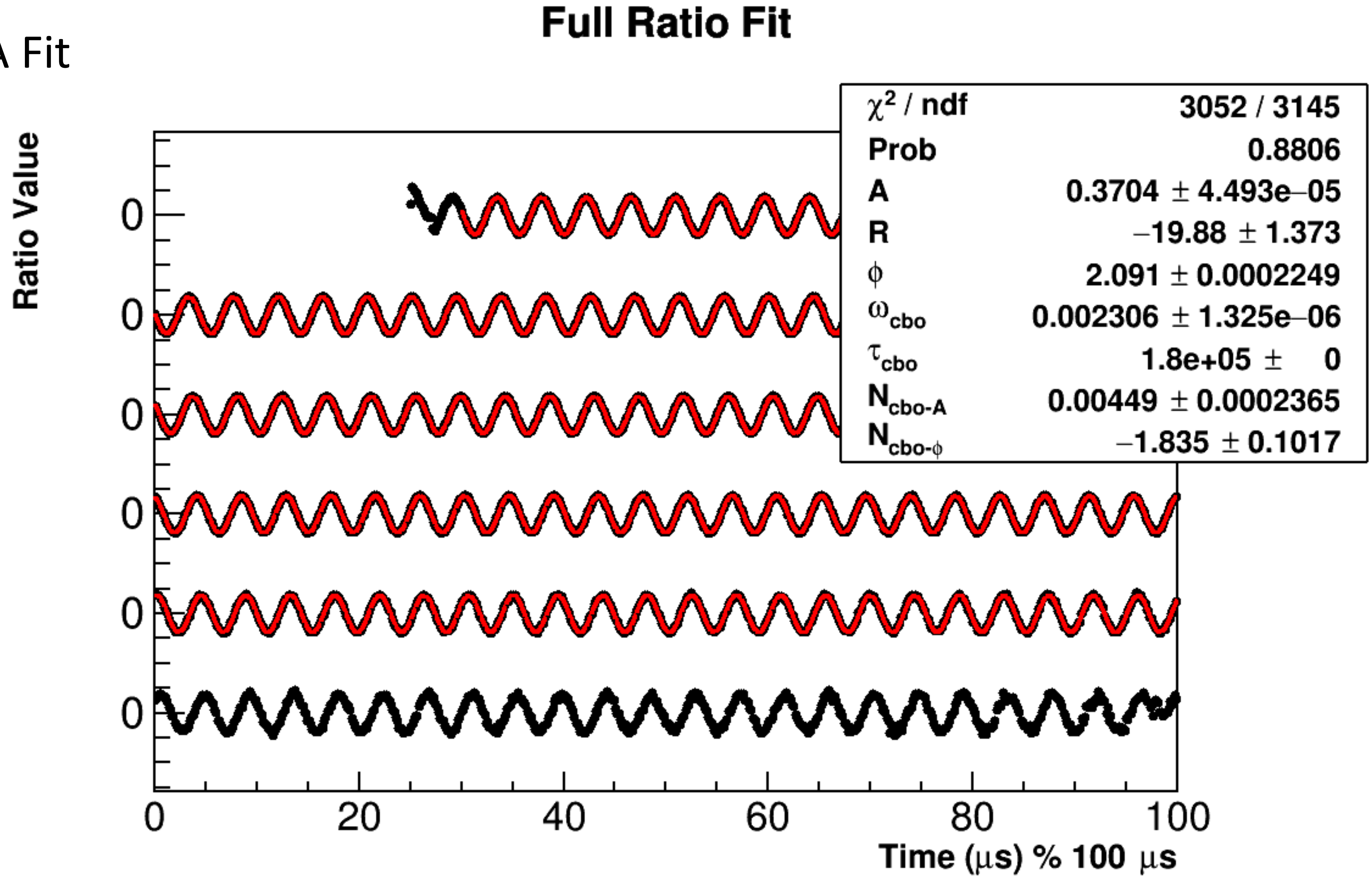
$$\approx \pi$$

$$N_5(t \pm T/2) = N_0 e^{-t/\tau} \cancel{e^{\mp T/2\tau}} (1 + A\cos(\omega_a t \pm \cancel{\omega_a \frac{T}{2}} + \phi))$$

$$R(t) = A\cos(\omega_a t + \phi) - \frac{1}{16}(\frac{T}{\gamma\tau})^2 + (h.o.) \longrightarrow R(t) = A\cos(\omega_a t + \phi)$$

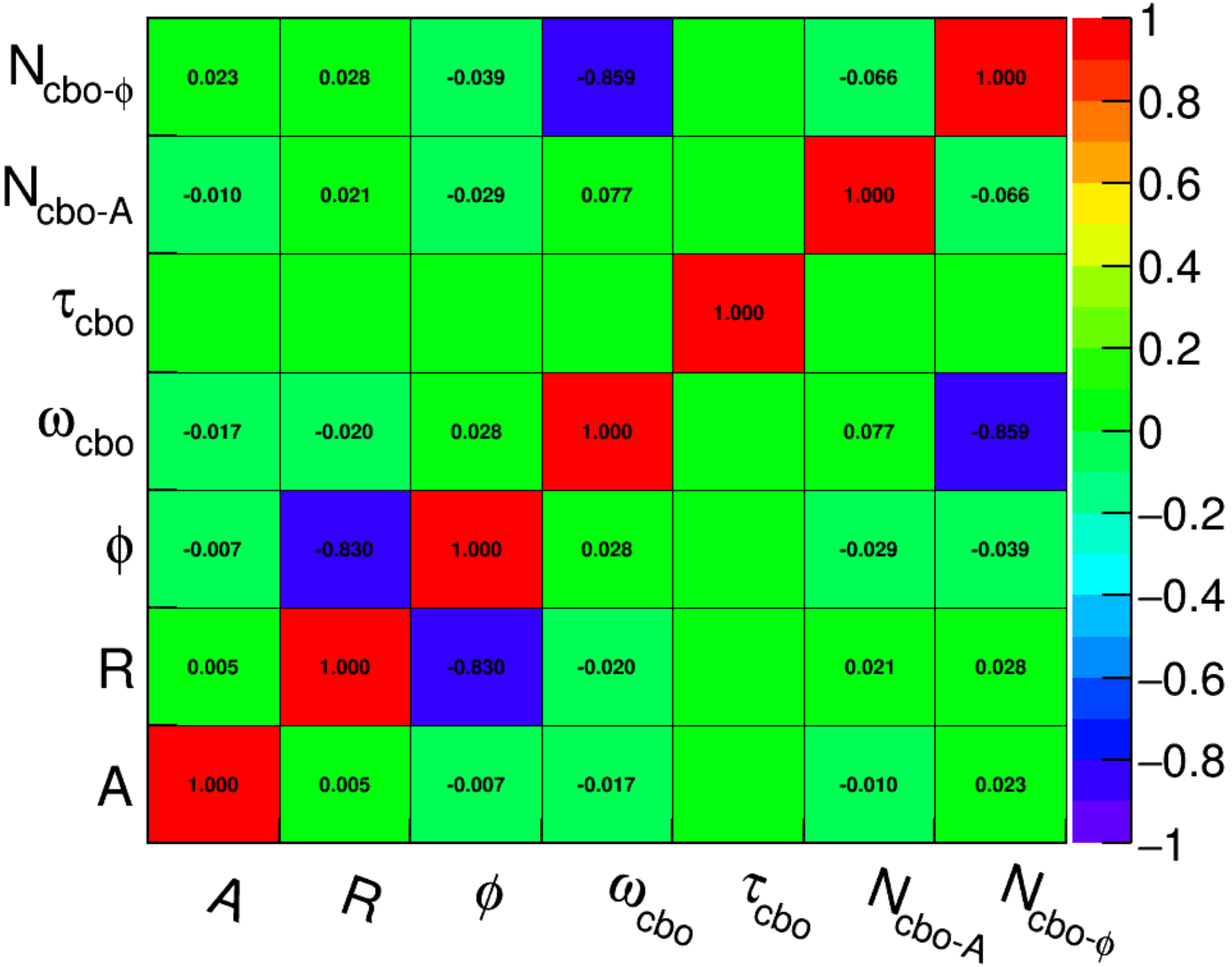
This weighting accounts for the muon decay during half a g-2 period.

5033A Fit

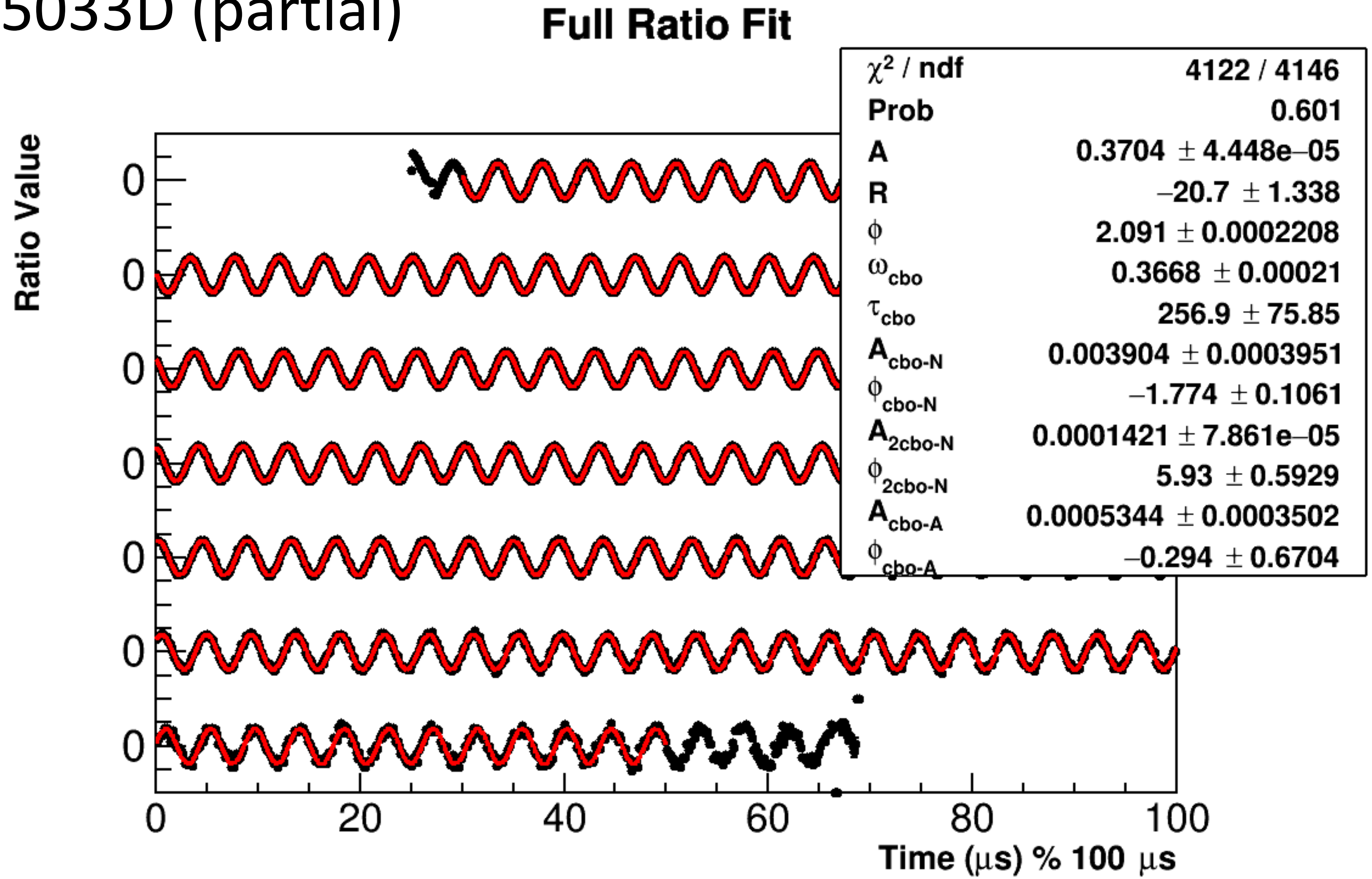

Full Ratio Fit
Ratio Value
0
0
0
0
0
0
0
20
40
60
80
100
Time (μs) % 100 μs
χ^2 / ndf 3052 / 3145
Prob 0.8806
A 0.3704 ± 4.493e−05
R −19.88 ± 1.373
ϕ 2.091 ± 0.0002249
ω_{cbo} 0.002306 ± 1.325e−06
τ_{cbo} 1.8e+05 ± 0
N_{cbo-A} 0.00449 ± 0.0002365
$N_{cbo-\phi}$ −1.835 ± 0.1017

Correlation matrix

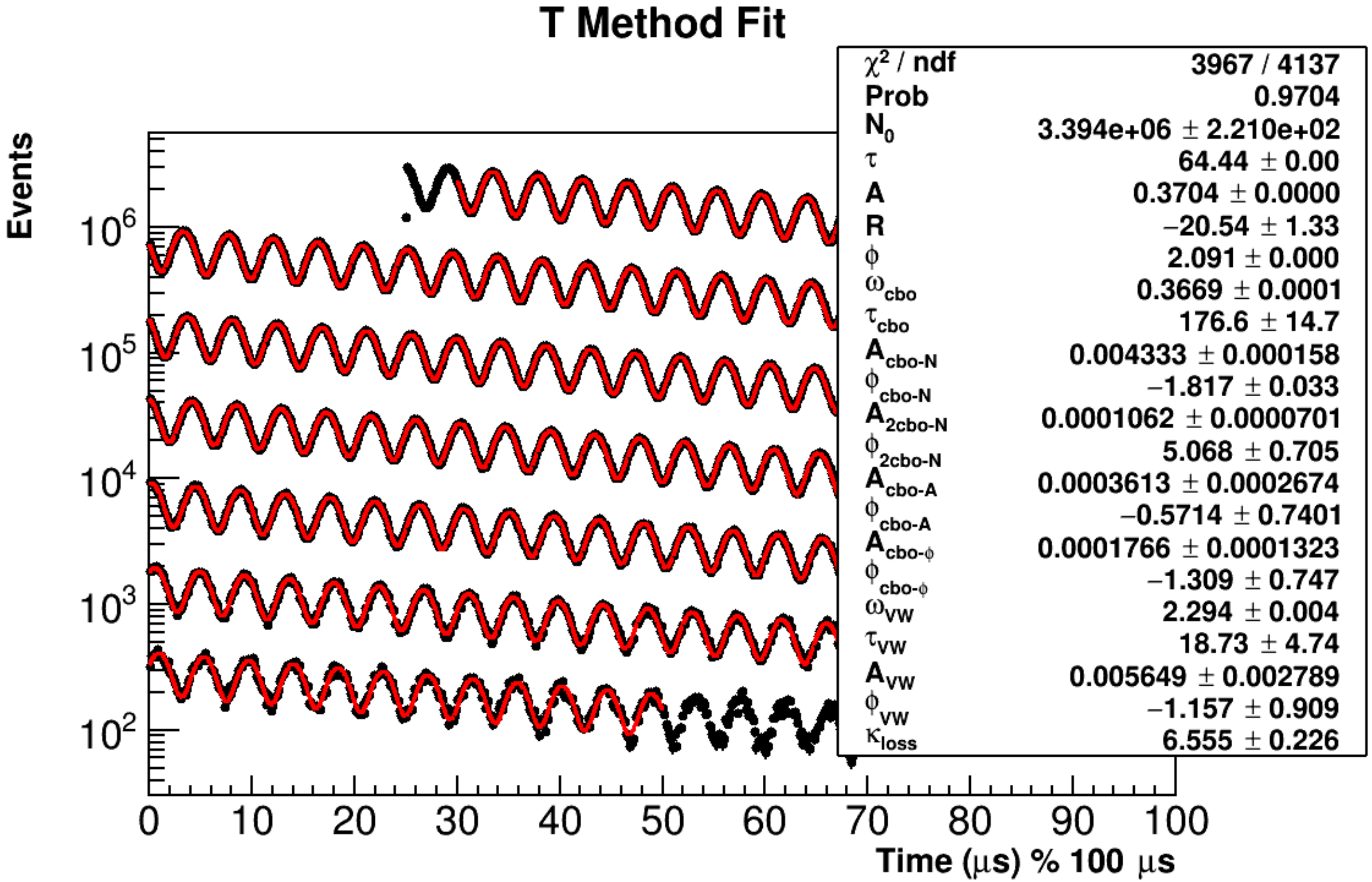
5033A



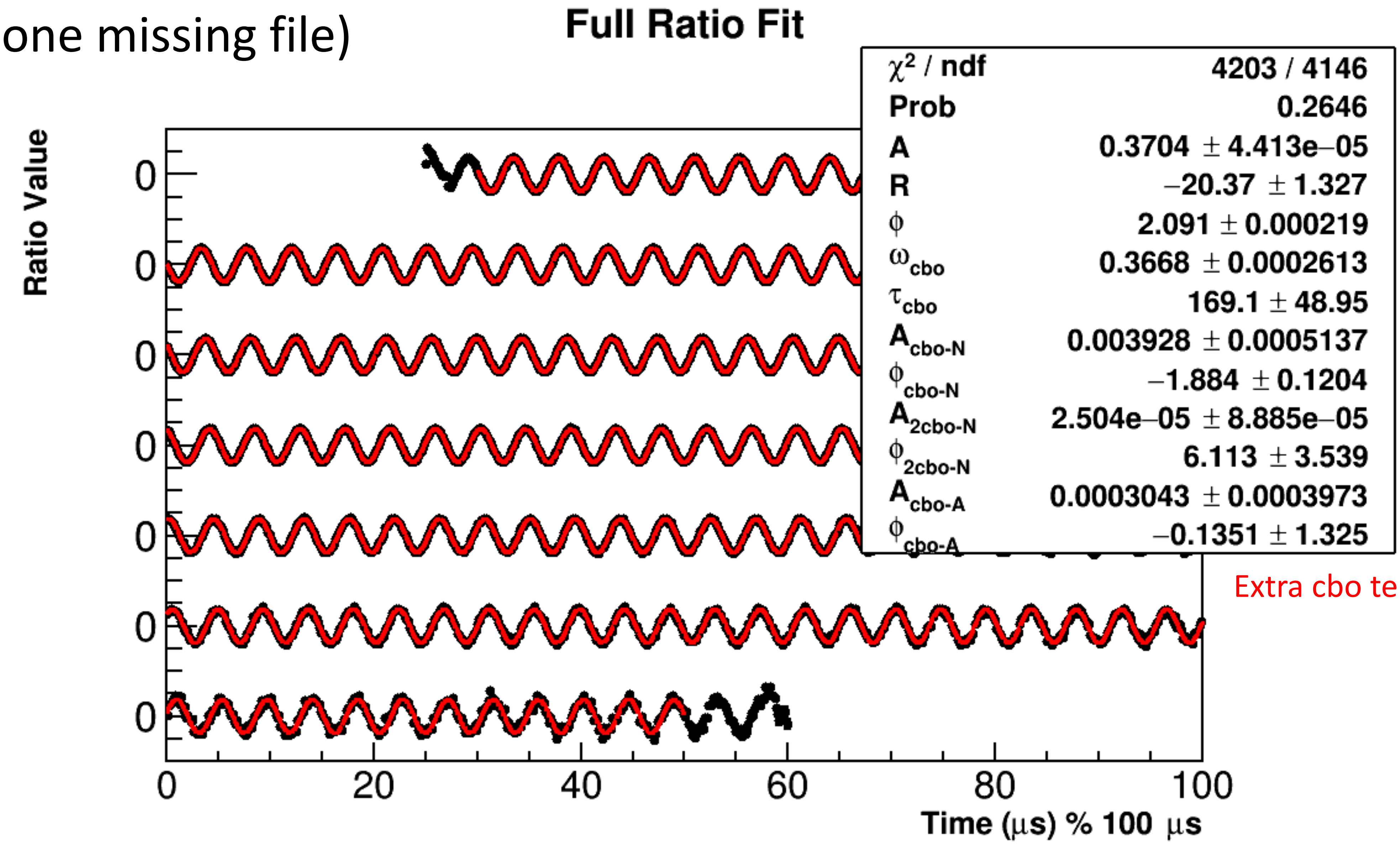
Fit - 5033D (partial)



5033D (partial) T Method Fit

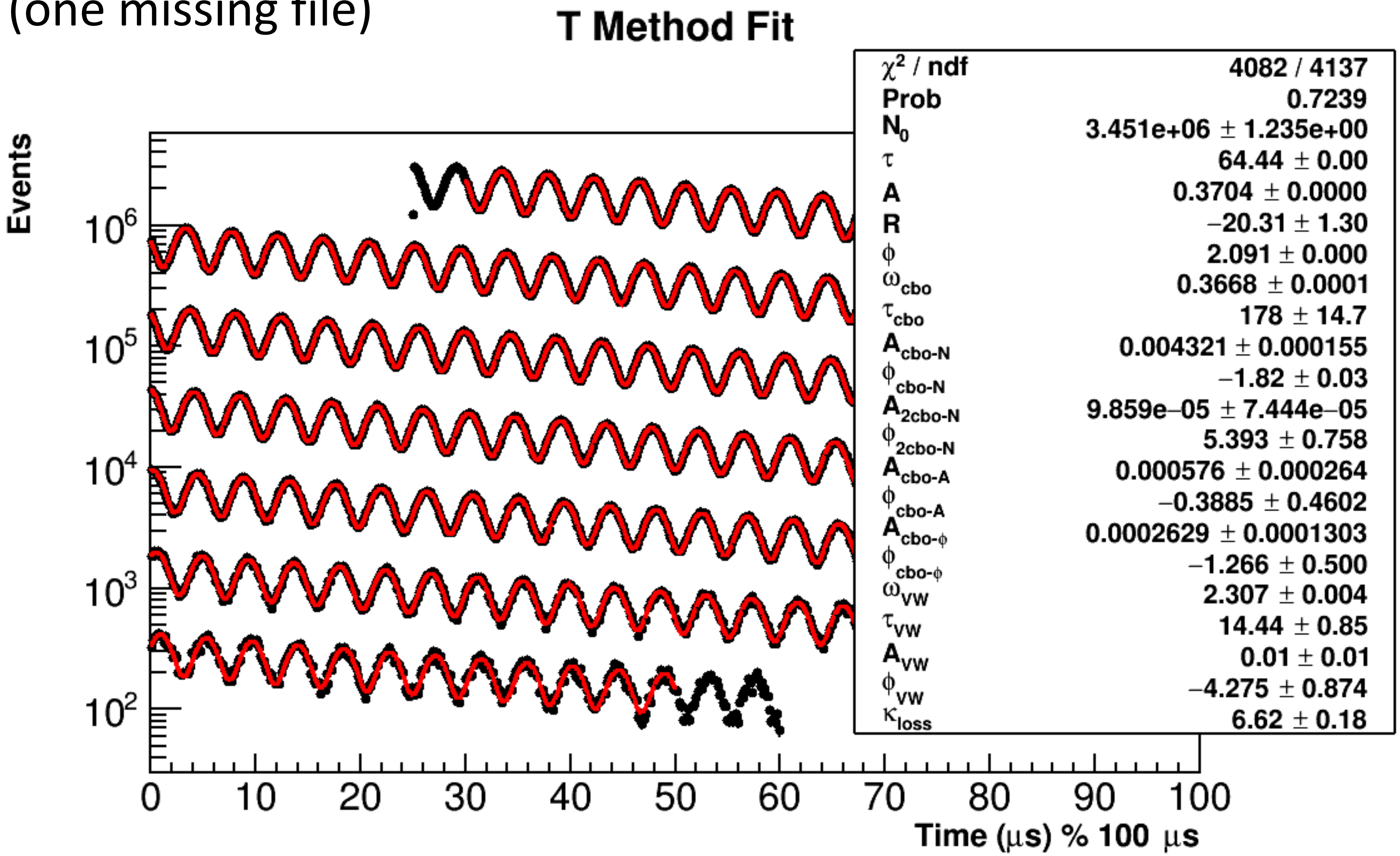


Gold Fit (one missing file)

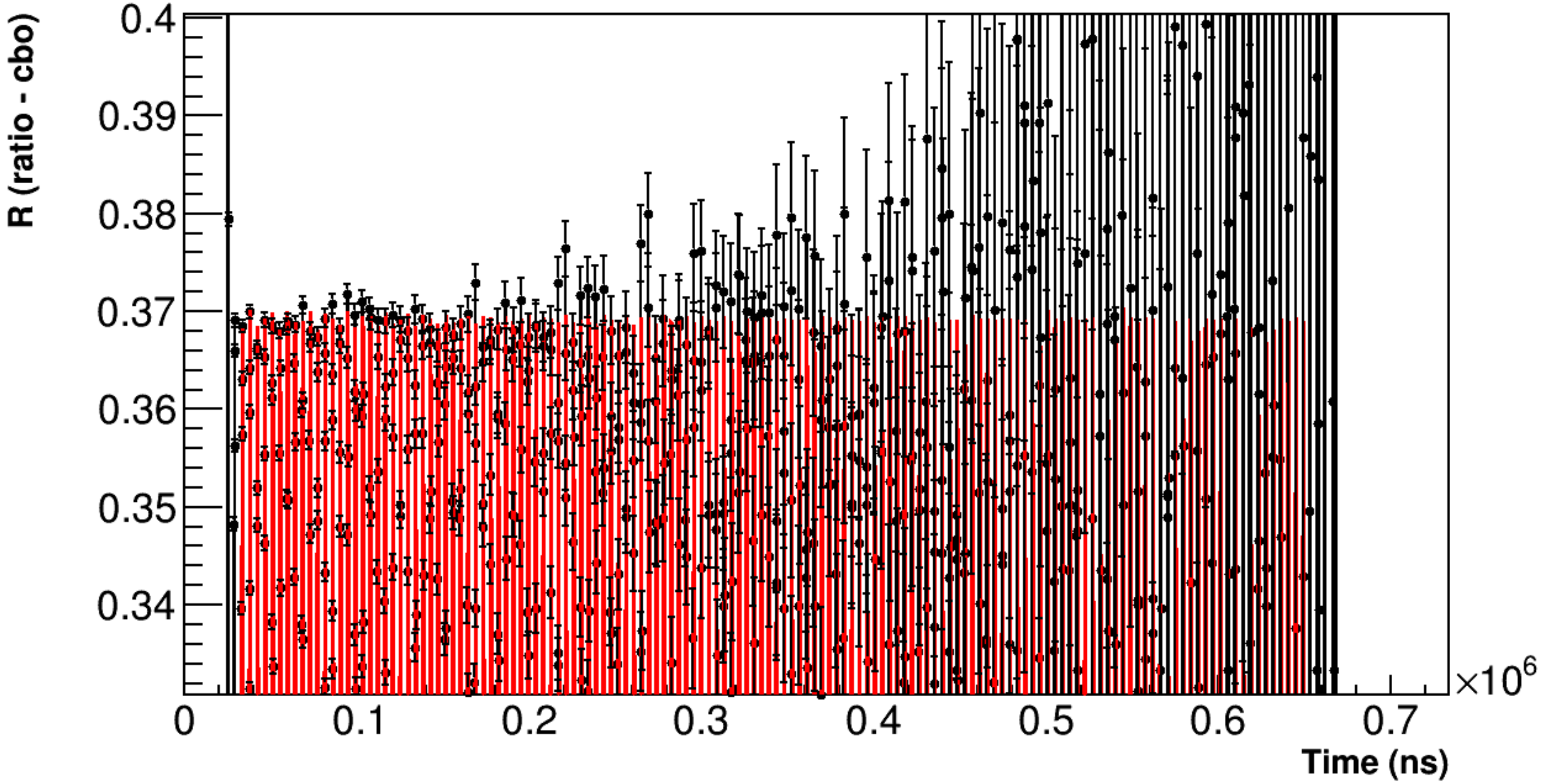


Extra cbo terms included

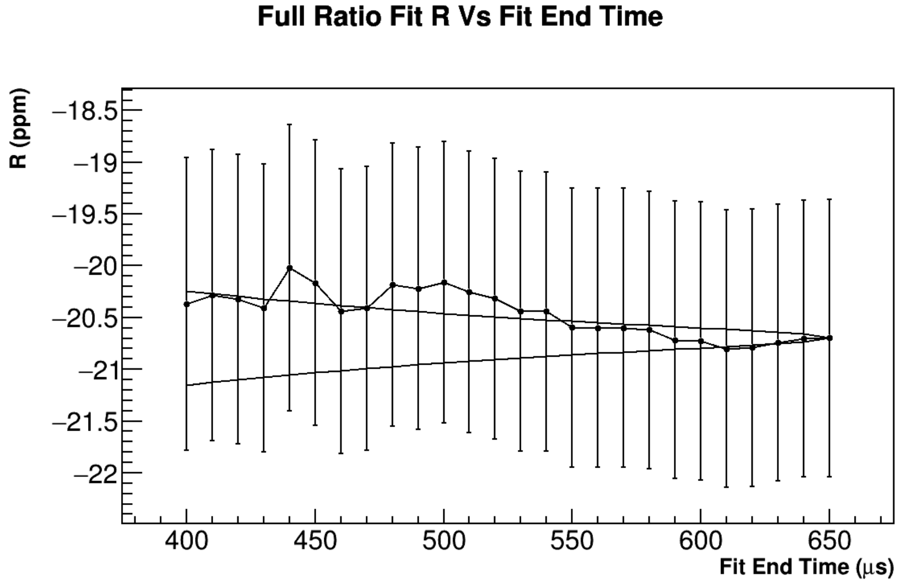
Gold Fit (one missing file)

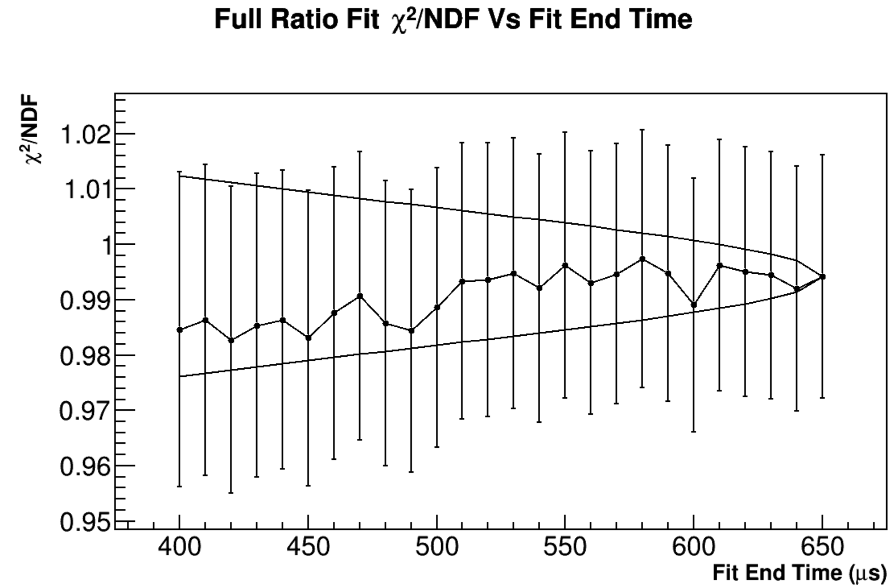


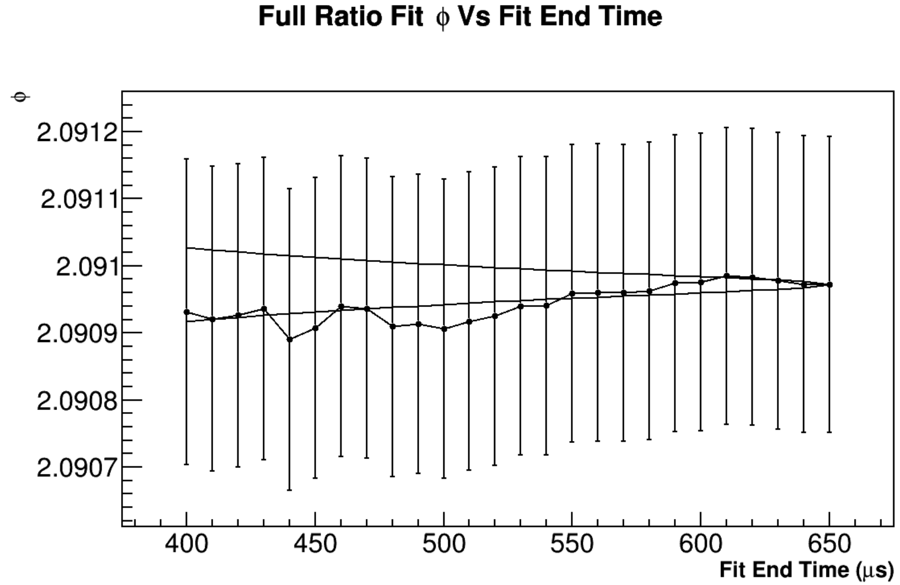
Fit end scans

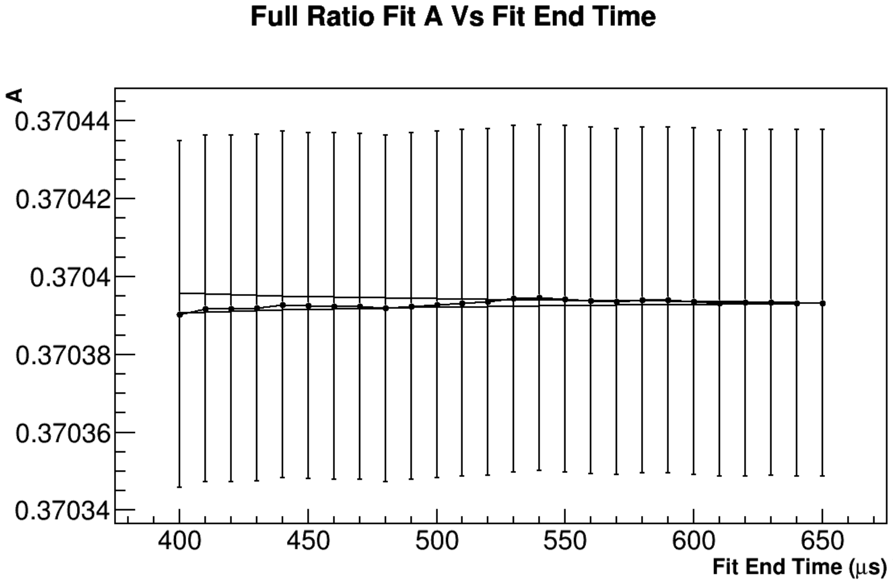


Fit end scans

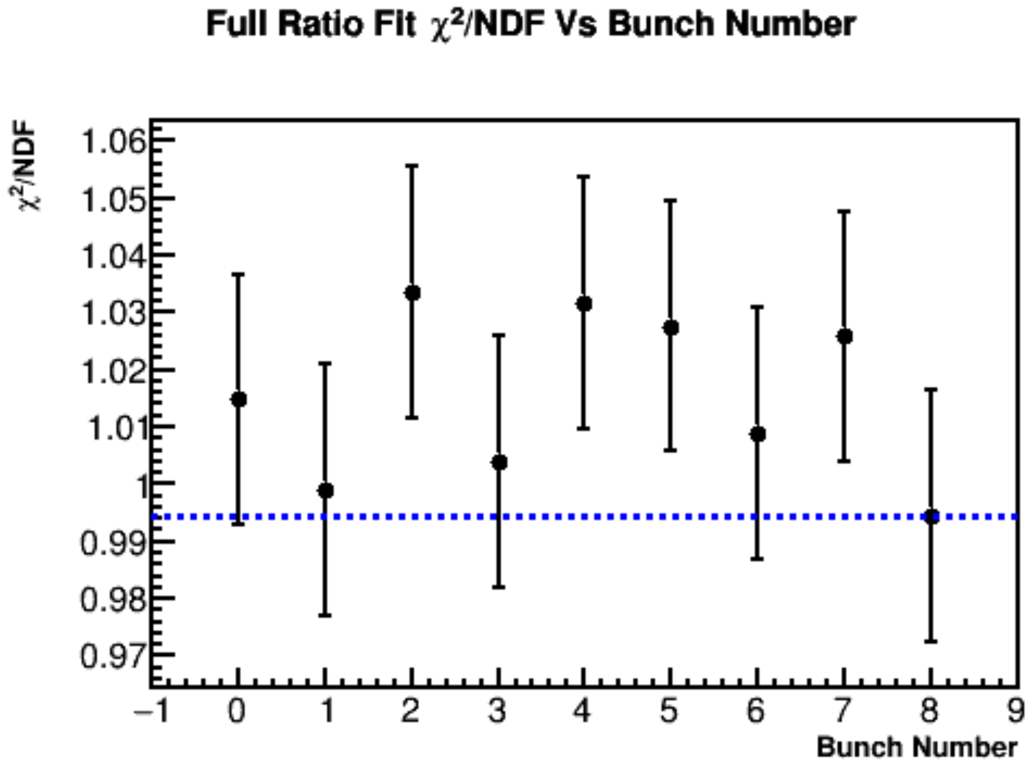




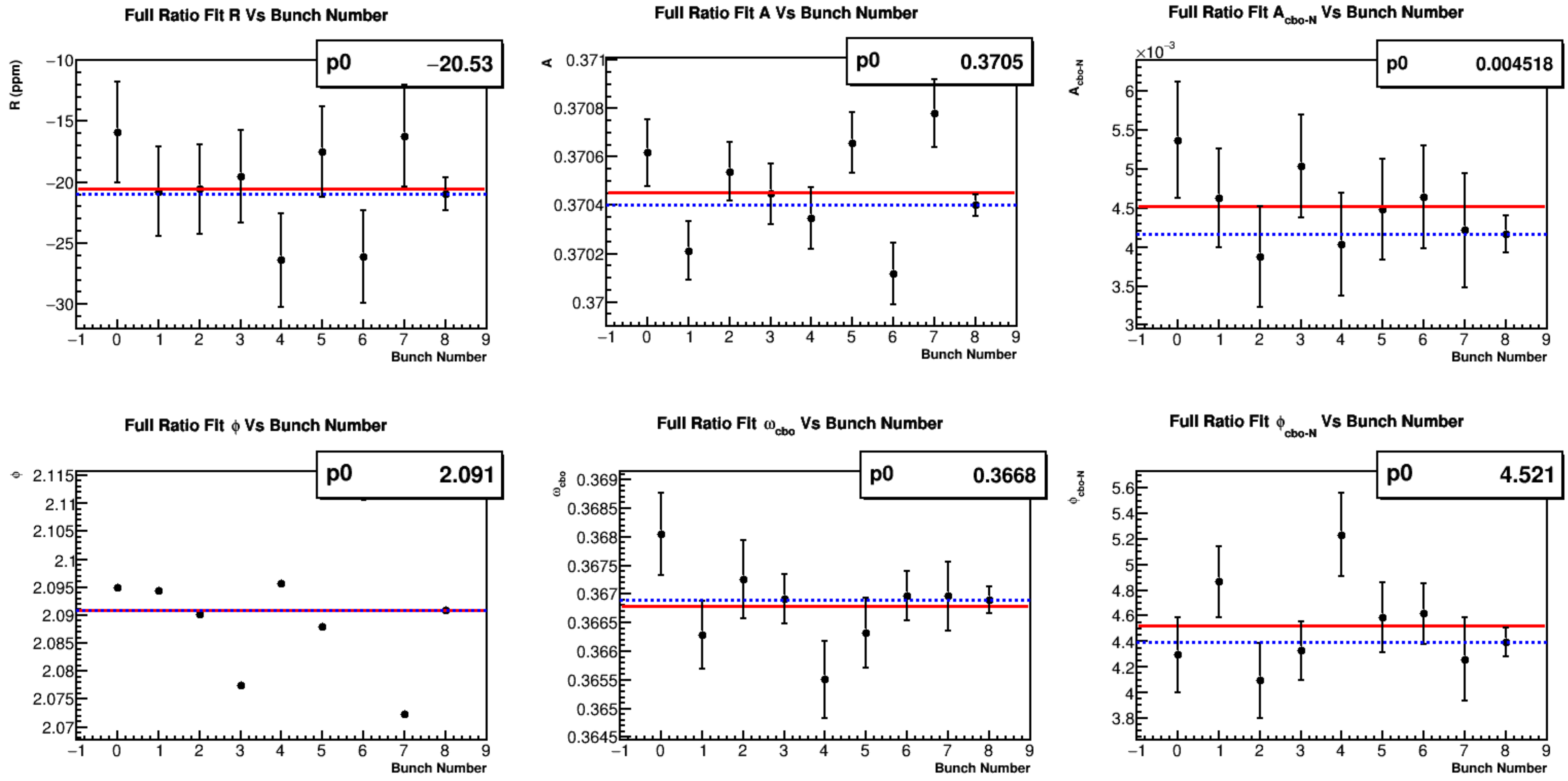




Fit Results Vs Bunch Number



Bunch 8 is all bunches added together, the dotted blue line is set to the value at bunch 8.



Bunch 8 is all bunches added together, red line is a fit to bunches 0 -7, the dotted blue line is set to the value at bunch 8.