Analysis Note for 60H Dataset Relative Unblinding

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High Level Summary

- Lead Analysts: Nick Kinnaird, James Mott
- Positron Reconstruction Method: Recon West
- Software Release: V9_11_00
- Dataset: gm2pro_daq_full_run1_60h_5033A_withfullDQC
- Histogramming Method: Weighted Ratio
- Gain Correction Method: Default in reconstruction
- Pileup Correction Method: Asymmetic shadow window
- Lost Muon Spectrum Extraction: put something here?
- Models for CBO and VW: Exponential envelopes, frequency from tracking analysis
- $R = -19.something \pm 1.something ppm$ (blinding with common string)
- $\chi^2/NDF = 4211/4200$something
- P value =

Final fit function:

$$R(t) = \frac{2f(t) - f_+(t) - f_-(t)}{2f(t) + f_+(t) + f_-(t)}$$

$$f_\pm(t) = f(t \pm T_a/2)$$

$$f(t) = C(t)(1 + A\cos(\omega_a t + \phi))$$

$$C(t) = 1 + A_{cbo} e^{-t/\tau_{cbo}} \cos(\omega_{cbo} t + \phi_{cbo})$$

Chapter 1

Analysis Procedures

1.1 Key parameters in reconstruction method

Find out procedures used in 60 hr production dataset

1.2 Analysis Data Preparation Procedure

- git branch: gm2analyses branch feature/KinnairdAnalyses
- Majority of code located under gm2analyses/macros/RatioMacro

1. Submit jobs to OSG to run the rootTreesAndLostMuons.fcl file which produces root trees of positron hits using the ClusterTree analyzer module and coincident MIP hits using the TestCoincidenceFinder analyzer module.
2. Submit jobs to Fermigrid to produce histograms from root trees using the ClusterTreeToHistsPileup.C macro in RatioMacro/HistMaking. Beyond standard threshold histograms this macro produces pileup and lost muon histograms all within the same root file.

1.3 Histogramming Procedure

Method: Weighted Ratio (threshold)

1. Loop through all clusters and apply an artificial deadtime (ADT) to combine hits within 6 ns into a single pulse using the same procedure and code that the pileup method uses (see below). Drop clusters with time $< 25\mu s$ or time $> 600\mu s$.
2. Histograms are constructed with ROOT's TH1F class with 149.15 ns bins from $0 - 699.96095\mu s$ corresponding to 4693 bins.
3. Randomize times by $\pm 149.15/2$ ns and fill histograms for energies > 1.7 GeV. Randomization uses ROOT's default TRandom3 class.
4. Fill one of the four histograms $\{u_+(t), u_-(t), v_1(t), v_2(t)\}$ as shown in Equation A.18 per cluster. The associated histogram is determined by generating a random number between 0 and 1, and comparing that number to the relative probabilities of the different weights.
5. Clusters filled into the $u_+(t)$ histogram have their times shifted by $t \to t - T/2$ and clusters filled into the $u_-(t)$ histogram have their times shifted by $t \to t + T/2$.

1.4 Gain Correction Procedure

Gain correction method: Default by the Italian Calibration Team

1. Long term gain is corrected using out-of-fill lasers included normalization from the Source Monitor.
2. In-fill gain is corrected using in-fill lasers including normalization from the Source Monitor.
3. Short-term double pulse (SDTP) effect is not included.

1.5 Pileup Correction Procedure

Pileup correction method: Asymmetric shadow window

1. Create a vector of clusters per calorimeter per fill. For each cluster look for a second cluster in a window from 12-18 ns after the time of the first cluster. This corresponds to a shadow dead time (SDT) of 6 ns and a shadow gap time (SGT) of 12 ns, equal to 1 and 2 times the applied ADT respectively.
2. Create shadow doublets with energies and times as:

$$E_{doublet} = E_1 + E_2$$
$$t_{doublet} = \frac{t_1 * E_1 + (t_2 - SGT) * E_2}{E_1 + E_2}$$

3. For each calorimeter construct a pileup spectrum P = doublets - singlets = D - S, where the singlets are subtracted at time $t_{doublet}$.
4. errors
5. triple pileup not included

Should this be a section at the front of the report? Or should it be in the appendix as I've place it here?

Appendix A

Ratio Method Derivation and Fit Function

Consider the 5 parameter function:

$$N_5(t) = N_0 e^{-t/\tau}(1 + A\cos(\omega_a t + \phi)), \tag{A.1}$$

which describes some ideal dataset in histogram format. Here ϕ will be set to zero for simplicity. Now define the variables $u_+(t)$, $u_-(t)$, $v_1(t)$, and $v_2(t)$ as

$$\begin{aligned} u_+(t) &= \frac{1}{4}N_5(t + T/2) \\ u_-(t) &= \frac{1}{4}N_5(t - T/2) \\ v_1(t) &= \frac{1}{4}N_5(t) \\ v_2(t) &= \frac{1}{4}N_5(t), \end{aligned} \tag{A.2}$$

where the 1/4 out front reflects randomly splitting the whole dataset into 4 equally weighted sub-datasets, and T is the g-2 period known to high precision, $\mathcal{O}(10^{-6})$. This corresponds to a weighting of 1:1:1:1 between the datasets. To be explicit here regarding the signs, the counts that are filled into the histogram described by u_+ have their times shifted as $t \to t - T/2$, which is what the function $N_5(t + T/2)$ describes, and vice versa for u_-. To form the ratio define the variables:

$$\begin{aligned} U(t) &= u_+(t) + u_-(t) \\ V(t) &= v_1(t) + v_2(t) \\ R(t) &= \frac{V(t) - U(t)}{V(t) + U(t)}. \end{aligned} \tag{A.3}$$

Plugging in and dividing the common terms $(N_0 e^{-t/\tau}/4)$,

$$R(t) = \frac{2(1 + A\cos(\omega_a t)) - e^{-T/2\tau}(1 + A\cos(\omega_a t + \omega_a T/2)) - e^{T/2\tau}(1 + A\cos(\omega_a t - \omega_a T/2))}{2(1 + A\cos(\omega_a t)) + e^{-T/2\tau}(1 + A\cos(\omega_a t + \omega_a T/2)) + e^{T/2\tau}(1 + A\cos(\omega_a t + \omega_a T/2))}. \tag{A.4}$$

Now set $\omega_a T/2 = \delta$, and note that T is really

$$\begin{aligned} T = T_{guess} &= \frac{2\pi}{\omega_a} + \Delta T, \\ \Delta T &= T_{guess} - T_{true}. \end{aligned} \tag{A.5}$$

Being explicit,

$$\delta = \frac{\omega_a}{2} T_{guess} = \frac{\omega_a}{2}(\frac{2\pi}{\omega_a} + \Delta T) = \pi + \pi \frac{\Delta T}{T_{true}} = \pi + \pi(\delta T), \tag{A.6}$$

and δ can be redefined as

$$\delta = \pi(\delta T), \tag{A.7}$$

by flipping the sign of any cosine terms that contain δ.

Then, using the trig identity

$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b) \tag{A.8}$$

so that

$$\begin{aligned} \cos(\omega_a t \pm \delta) &= \cos(\omega_a t)\cos\delta \mp \sin(\omega_a t)\sin\delta \\ &\approx \cos(\omega_a t)(1 - \delta^2) \mp \sin(\omega_a t)\delta \\ &\approx \cos(\omega_a t), \end{aligned} \tag{A.9}$$

since $\delta \sim O(10^{-5})$, the ratio becomes

$$R(t) \approx \frac{2(1 + A\cos(\omega_a t)) - (1 - A\cos(\omega_a t))(e^{-T/2\tau} + e^{T/2\tau})}{2(1 + A\cos(\omega_a t)) + (1 - A\cos(\omega_a t))(e^{-T/2\tau} + e^{T/2\tau})}. \tag{A.10}$$

Expanding

$$e^{\pm T/2\tau} = 1 \pm \frac{T}{2\tau} + \frac{1}{2}\left(\frac{T}{2\tau}\right)^2 \pm \ldots, \tag{A.11}$$

repacing and simplifying,

$$R(t) \approx \frac{A\cos(\omega_a t) - C(1 - A\cos(\omega_a t))}{1 + C(1 - A\cos(\omega_a t))}, \tag{A.12}$$

where

$$C = \frac{1}{16}\left(\frac{T}{\tau}\right)^2 \approx 2.87 * 10^{-4}. \tag{A.13}$$

Using the expansion

$$f(x) = \frac{1}{1+x} = 1 - x + x^2 - \ldots, \quad |x| < 1, \tag{A.14}$$

and since C is small, the denominator can be manipulated such that

$$\begin{aligned} R(t) &\approx (A\cos(\omega_a t)) - C(1 - A\cos(\omega_a t)))(1 - C(1 - A\cos(\omega_a t))) \\ &\approx A\cos(\omega_a t) - C + CA^2\cos^2(\omega_a t), \end{aligned} \tag{A.15}$$

after dropping terms of $\mathcal{O}(C^2)$ and higher. In practice the last term is ommitted since it has a minimal effect on the fitted value of ω_a [**cite**], and one arrives at

$$R(t) \approx A\cos(\omega_a t) - C, \tag{A.16}$$

the conventional 3 parameter ratio function.

In order to avoid approximations one can instead weight the counts in the histograms as

$$u_+(t) : u_-(t) : v_1(t) : v_2(t) = e^{T/2\tau} : e^{-T/2\tau} : 1 : 1, \tag{A.17}$$

so that

$$\begin{aligned} u_+(t) &= \frac{e^{T/2\tau}}{2 + e^{T/2\tau} + e^{-T/2\tau}} N_5(t + T/2) \\ u_-(t) &= \frac{e^{-T/2\tau}}{2 + e^{T/2\tau} + e^{-T/2\tau}} N_5(t - T/2) \\ v_1(t) &= \frac{1}{2 + e^{T/2\tau} + e^{-T/2\tau}} N_5(t) \\ v_2(t) &= \frac{1}{2 + e^{T/2\tau} + e^{-T/2\tau}} N_5(t). \end{aligned} \tag{A.18}$$

(These factors out front aren't so far off from 1/4 since $e^{\pm T/2\tau} \approx e^{\pm 4.35/2*64.4} \approx 1.034, .967.$) Then instead $R(t)$ becomes

$$R(t) = \frac{2(1 + A\cos(\omega_a t)) - (1 - A\cos(\omega_a t + \delta)) - (1 - A\cos(\omega_a t - \delta))}{2(1 + A\cos(\omega_a t)) + (1 - A\cos(\omega_a t + \delta)) + (1 - A\cos(\omega_a t + \delta))}, \tag{A.19}$$

where the $e^{\pm T/2\tau}$ terms out front now cancel. Using Equation A.9 again and this time avoiding approximations in δ,

$$R(t) = \frac{2A\cos(\omega_a t)(1 + \cos\delta)}{4 + 2A\cos(\omega_a t)(1 - \cos\delta)}, \tag{A.20}$$

after simplifying. In the limit that

$$\delta = \pi(\delta T) \to 0 \tag{A.21}$$

since δT is small,

$$R(t) \approx A\cos(\omega_a t), \tag{A.22}$$

with the only approximation being made at $\mathcal{O}(\delta^2) \sim \mathcal{O}(10^{-10})$.

Finally, while the 3 parameter ratio function suffices for fits to data containing slow modulations, it does not suffice for faster oscillation features. In that case it is more useful to fit with the non-approximated or simplified version of the ratio,

$$\begin{aligned} R(t) &= \frac{v_1(t) + v_2(t) - u_+(t) - u_-(t)}{v_1(t) + v_2(t) + u_+(t) + u_-(t)}, \\ &= \frac{2f(t) - f_+(t) - f_-(t)}{2f(t) + f_+(t) + f_-(t)}, \end{aligned} \tag{A.23}$$

where

$$\begin{aligned} f(t) &= C(t)(1 + A\cos(\omega_a t + \phi)) \\ f_\pm(t) &= f(t \pm T_a/2), \end{aligned} \tag{A.24}$$

and $C(t)$ can encode any other effects in the data that need to be fitted for, such as the CBO,

$$C(t) = 1 + A_{cbo}e^{-t/\tau_{cbo}}\cos(\omega_{cbo}t + \phi_{cbo}). \tag{A.25}$$

Additionally, any other fit parameters such as A or ϕ can be made a fuction of t. Using the non-approximated form for the final fit function gives greater confidence in the fit results for the high precision ω_a extraction necessary for the experimental measurement.

Appendix B

Ratio Method Errors