Whitepaper Status: HLbL Lattice

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Contributors to this section (so far):
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Purpose of this talk

- Summarize current status of HLbL Lattice section of whitepaper

- Identify next steps and action items (for this workshop and possibly beyond) to bring this section to completion

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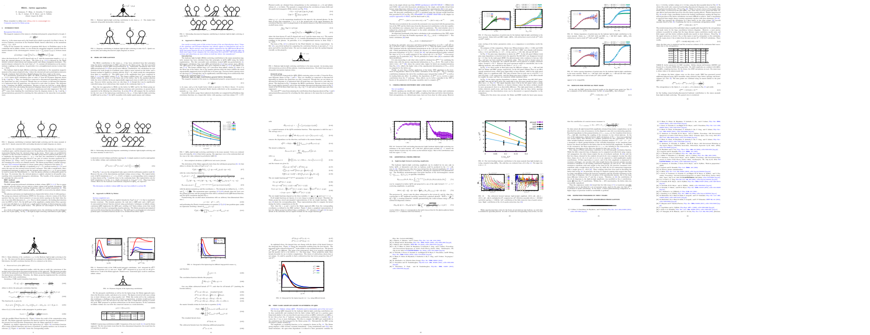
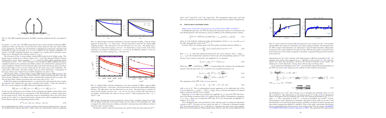
See talks on Wednesday by Luchang Jin (RBC/UKQCD) and Renwick Hudspith+En-Hung Chao (Mainz) for progress updates from individual groups.

Overview



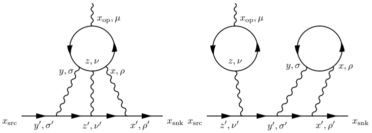


- Contributors so far:
  N. Asmussen, T. Blum, A. Gérardin, T. Izubuchi, L. Jin, C. Lehner, H. B. Meyer, and A. Nyffeler
- 25 pages, 22 figures, 38 references

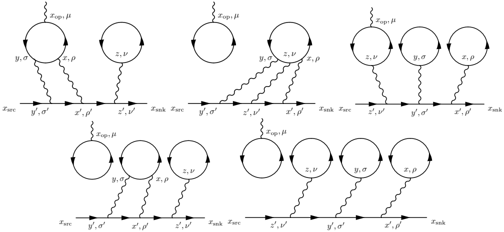
Whitepaper Outline

1. Introduction
2. HLbL on the Lattice
   - Approach to HLbL by RBC/UKQCD
   - Approach to HLbL by Mainz
3. Test case: HLbL scattering in QED
4. Pion-pole contribution
5. Cross-checks between RBC/UKQCD and Mainz
6. Results for physical pion mass
7. Additional cross-check: forward scattering amplitudes
8. Expected progress in next years
9. Summary of current knowledge from lattice

HLbL on the Lattice (I)



SU(3) leading



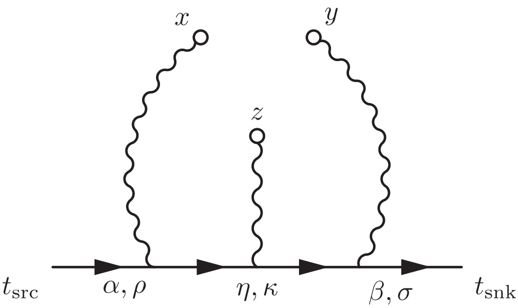
SU(3) suppressed

HLbL on the Lattice (II)

- QED non-perturbatively and momentum-space (PRL114(2015)012001)
- QED perturbatively and position-space
  - QED_L (PRD93(2016)014503, PRL118(2017)022005): $1/L^2$ finite-volume errors (with linear extent L)
  - QED_∞: exponential finite-volume errors (PRL115(2015)222003, EPJ Web Conf. 175(2018)06023), subtraction prescriptions to reduce systematic errors (PRD96(2017)034515, arXiv:1811.08320)

Color code: Mainz group, RBC/UKQCD group

Approach to HLbL by RBC/UKQCD (I)

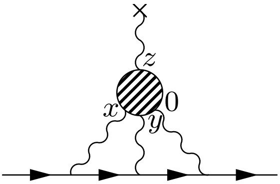


- Importance sampling: sample x and y and use them as sources for point-source propagators; exact sampling strategy optimized for diagram and ensemble
- Sum over z and external photon is exact
- Moments method: compute $F_2(q^2 = 0)$ directly

Approach to HLbL by RBC/UKQCD (II)

- Use of domain-wall fermions with $O(a^2)$ discretization errors
- For QED$_L$, can perform muon/photon part efficiently using convolutions (TODO: add to whitepaper)
- Results all in finite-volume QED$_L$
- RBC/UKQCD's version of QED$_\infty$ discussed below as part of next section

Approach to HLbL by Mainz (I)



- Use QED_∞ and O(4) symmetry: master formula

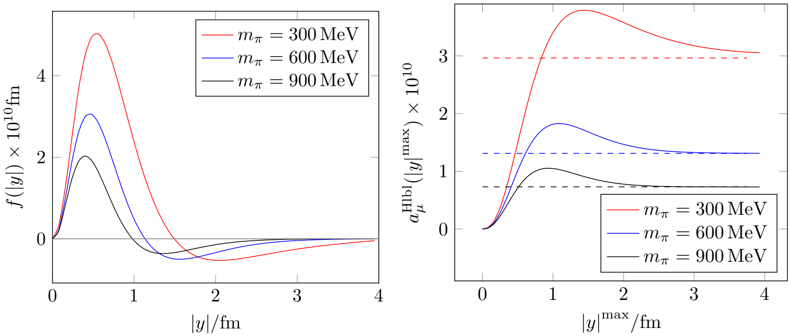
$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} \int_y \int_x \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)\, i\,\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y)\,,$$

$$i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) = -\int_z z_\rho \left\langle j_\mu(x) j_\nu(y) j_\sigma(z) j_\lambda(0) \right\rangle$$

- The kernel function $\bar{\mathcal{L}}$ is precomputed semi-analytically and stored for multiple use
- Use improved Wilson fermions with $O(a^2)$ discretization errors

Approach to HLbL by Mainz (II)

Numerical study of the VMD model pion-pole contribution for $\widehat{\Pi}$



(left: integrand after integration over $|x|$ and relative angle, right: integrated in $|y|$ compared to continuum, infinite-volume known result)

Long tails and large volume needed for physical pion mass!

Approach to HLbL by Mainz (III)

- RBC/UKQCD proposed to perform subtractions on kernel function using

$$j_\mu(x) = \partial_\nu(x_\mu j_\nu(x)) \tag{1}$$

and therefore

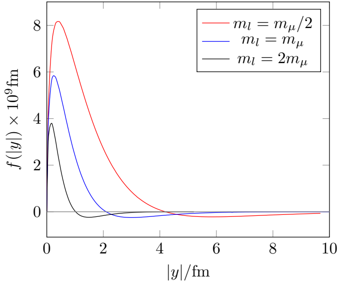
$$\int_x j_\mu(x) = 0\,. \tag{2}$$

- Subtraction can shift long to short distances and affect discretization and finite-volume errors
- Different versions of this have been explored by both Mainz and RBC/UKQCD, see below.

Test case: HLbL scattering in QED (I)

Mainz:

Replace quark loop by lepton loop with mass m_l:



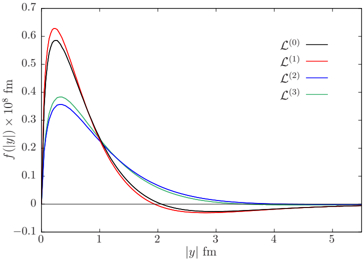
m_l/m_μ	$a_\mu^{\mathrm{LbL}} \times 10^{11}$ (exact)	$a_\mu^{\mathrm{LbL}} \times 10^{11}$	Precision	Deviation
1/2	1229.07	1257.5(6.2)(2.4)	0.5%	2.3%
1	464.97	470.6(2.3)(2.1)	0.7%	1.2%
2	150.31	150.4(0.7)(1.7)	1.2%	0.06%

First uncertainty from 3d integration, second from small $|y|$ extrapolation; good agreement

Test case: HLbL scattering in QED (II)

Mainz:

Study of subtraction kernels for $m_l = m_\mu$:

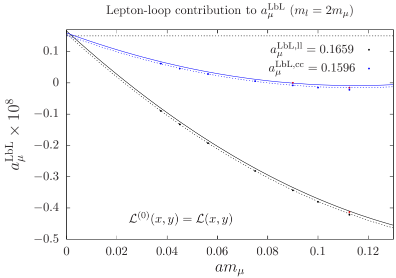


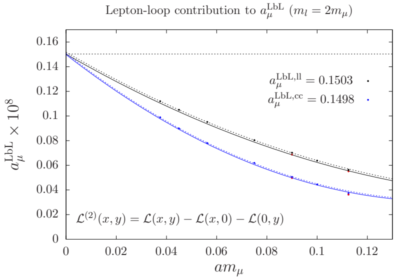
$$
\begin{aligned}
\mathcal{L}^{(0)}(x,y) &= \bar{\mathcal{L}}(x,y)\,, \quad \text{(standard kernel)}\\
\mathcal{L}^{(1)}(x,y) &= \bar{\mathcal{L}}(x,y) - \frac{1}{2}\bar{\mathcal{L}}(x,x) - \frac{1}{2}\bar{\mathcal{L}}(y,y)\,,\\
\mathcal{L}^{(2)}(x,y) &= \bar{\mathcal{L}}(x,y) - \bar{\mathcal{L}}(0,y) - \bar{\mathcal{L}}(x,0)\,,\\
\mathcal{L}^{(3)}(x,y) &= \bar{\mathcal{L}}(x,y) - \bar{\mathcal{L}}(0,y) - \bar{\mathcal{L}}(x,x) + \bar{\mathcal{L}}(0,x)\,,
\end{aligned}
$$

Shift from long to short distances observed

Test case: HLbL scattering in QED (III)

Mainz:

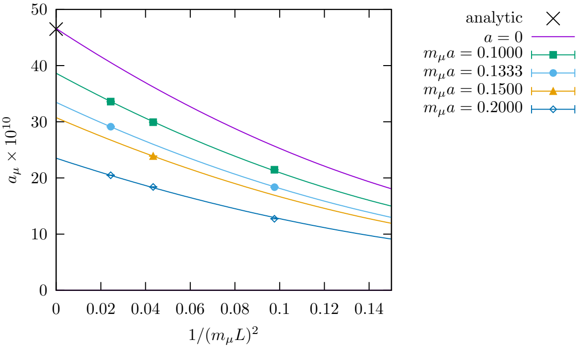




Study $a \to 0$ limit with different discretized currents (conserved/local) and subtractions; note change of y axis range

Test case: HLbL scattering in QED (IV)

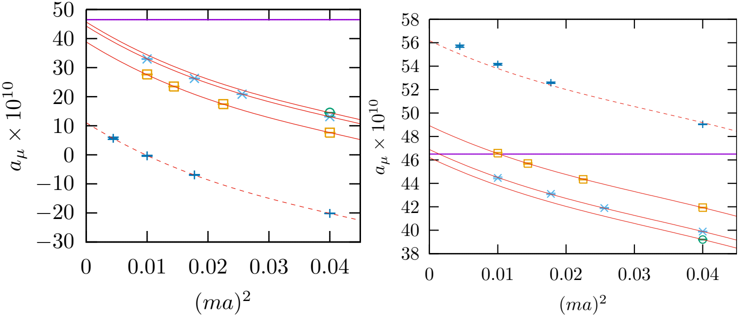
RBC/UKQCD:



QED_L extrapolation for $m_\mu = m_l$

Test case: HLbL scattering in QED (V)

RBC/UKQCD:



left: unsubtracted, right: subtracted

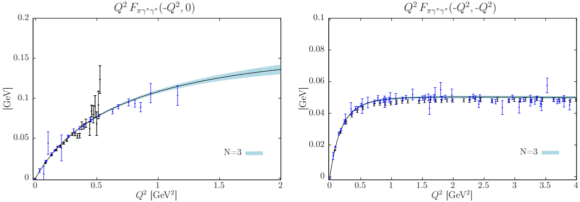
$$\mathfrak{G}^{(2)}_{\sigma,\kappa,\rho}(y,z,x) = \mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(y,z,x) - \mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(z,z,x) - \mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(y,z,z)$$

Volumes: mL=3.2, 4.8, 6.4,9.6

Pion-pole contribution (I)

Mainz: (see also talk by A. Nyffeler on Thursday)

$$i \int \mathrm{d}^4x \, e^{iq_1\cdot x} \, \langle \Omega | T\{J_\mu(x) J_\nu(0)\} | \pi^0(p) \rangle = \epsilon_{\mu\nu\alpha\beta} \, q_1^\alpha \, q_2^\beta \, \mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$$



200 MeV pion mass, black: pion rest frame, blue: moving frame; z-expansion fit described in whitepaper

Pion-pole contribution (II)

Mainz: arXiv:1903.09471

Result with $q_1 = q_2 = 0$ from lattice:

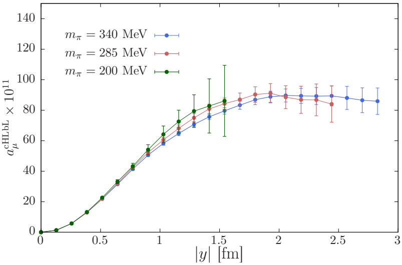
$$a_\mu^{\mathrm{HLbL};\pi^0} = (59.7 \pm 3.4 \pm 0.9 \pm 0.5) \times 10^{-11} = (59.7 \pm 3.6) \times 10^{-11}$$

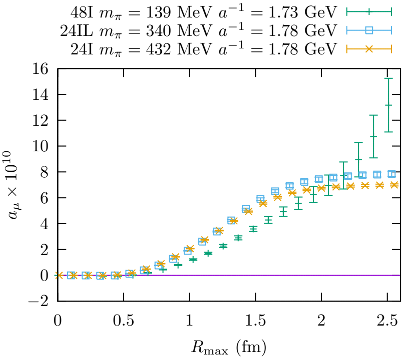
Result with $q_1 = q_2 = 0$ from experiment:

$$a_\mu^{HLbL\ \pi^0} = (62.3 \pm 2.3) \times 10^{-11}$$

Cross-checks between RBC/UKQCD and Mainz (I)

Comparison for different pion masses, QED_∞ at non-zero lattice spacing and different discretizations for Mainz (left) and RBC/UKQCD (right)

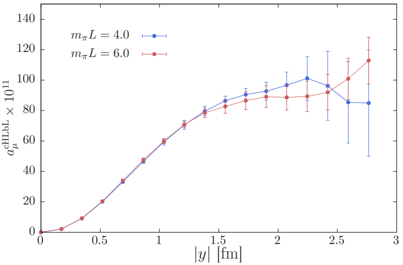


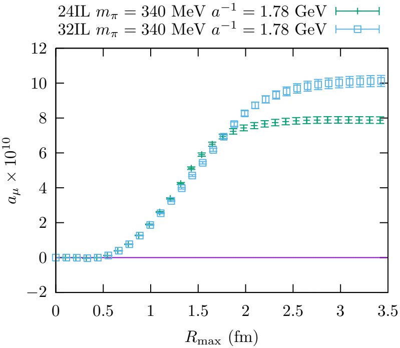


Agreement for 340 MeV pion mass could be accident (L$\to \infty$, $a \to 0$ not taken; different regulators used)

Cross-checks between RBC/UKQCD and Mainz (II)

$m_\pi = 285$ MeV (left, Mainz) and $m_\pi = 340$ MeV (right, RBC/UKQCD) volume dependence:

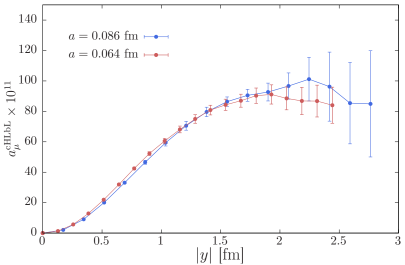


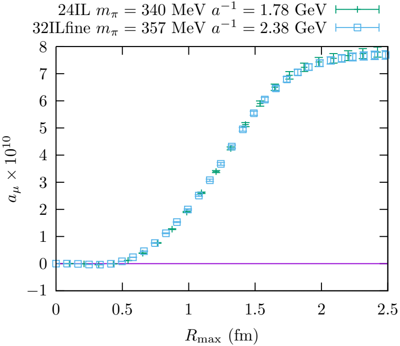


Consistency within statistical errors

Cross-checks between RBC/UKQCD and Mainz (III)

Lattice spacing dependence from Mainz (left) and RBC/UKQCD (right):

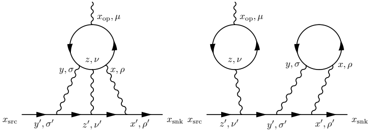


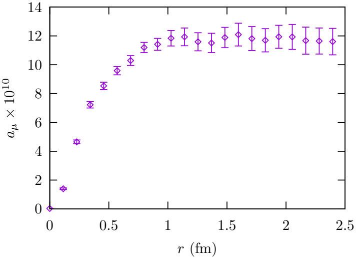


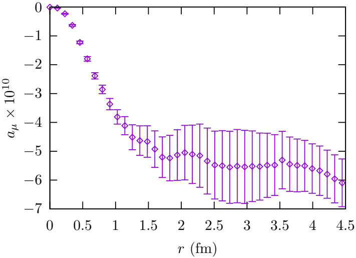
Effects seem minor $\Rightarrow$ Mainz and RBC/UKQCD results for heavy pion mass compatible

Results for physical pion mass (I)

So far only result (RBC/UKQCD, QED_L): PRL118(2017)022005







$$a_\mu^{\mathrm{HLbL}} = 5.35(1.35) \times 10^{-10}$$

Results for physical pion mass (II)

Continuum limit:

$$a_\mu^{\mathrm{HLbL}} = 5.35(1.35) \times 10^{-10}$$

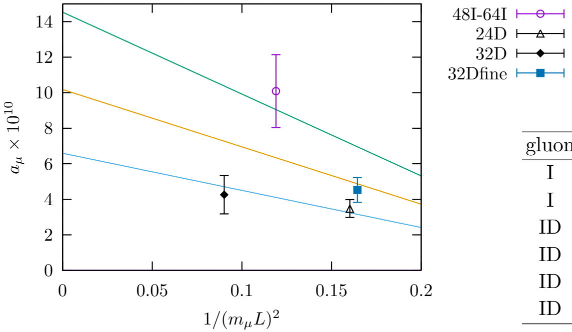
$m_\pi = 139$ MeV, $m_\pi L \approx 4$, $a^{-1} = 1.73$ GeV

$$a_\mu^{\mathrm{HLbL}} = 4.63(4.44) \times 10^{-10}.$$

$m_\pi = 139$ MeV, $m_\pi L \approx 4$, $a^{-1} = 1.73$ GeV, 2.36 GeV
Extrapolation using $a_\mu^{\mathrm{HLbL}}(a) = a_\mu^{\mathrm{HLbL}}(a = 0) + c_1 a^2$

Results for physical pion mass (III)

Infinite-volume limit:



gluon	m_π (MeV)	size	a^{-1} (GeV)
I	139.2	48^3	1.730
I	139.2	64^3	2.359
ID	139.2	24^3	1.015
ID	139.2	32^3	1.015
ID	139.2	32^3	1.4
ID	139.2	48^3	1.015

$$a_\mu^{\mathrm{HLbL}}(a, L) = a_\mu^{\mathrm{HLbL}}(1 + b_1 a^2)\left(1 + c_1 \frac{1}{m_\mu^2 L^2}\right)$$

Update: see talk by Luchang

Additional cross-check: forward scattering amplitudes (I)

Mainz: PRD98(2018)074501

$$\Pi_{\mu_1\mu_2\mu_3\mu_4}(P_4; P_1, P_2) \equiv \int_{X_1,X_2,X_4} e^{-i\sum_a P_a\cdot X_a} \Big\langle J_{\mu_1}(X_1) J_{\mu_2}(X_2) J_{\mu_3}(0) J_{\mu_4}(X_4) \Big\rangle$$

Total of eight $\gamma^*\gamma^* \to \gamma^*\gamma^*$ amplitudes in particular transversely polarized photon amplitude

$$\mathcal{M}_{\mathrm{TT}}(-Q_1^2, -Q_2^2, -Q_1\cdot Q_2) = \frac{e^4}{4} R_{\mu_1\mu_3} R_{\mu_2\mu_4} \Pi_{\mu_1\mu_3\mu_4\mu_2}(-Q_2; -Q_1, Q_1)$$

$R_{\mu\nu}$ project onto the plane orthogonal to Q_1, Q_2

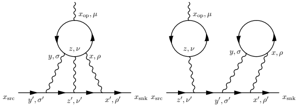
Additional cross-check: forward scattering amplitudes (II)

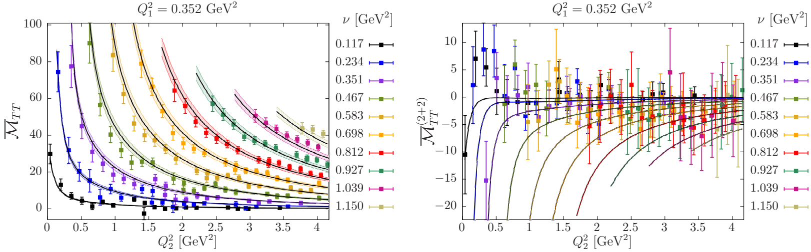
Define subtracted amplitudes:

$$\overline{\mathcal{M}} \equiv \mathcal{M}_{\mathrm{TT}}(q_1^2, q_2^2, \nu) - \mathcal{M}_{\mathrm{TT}}(q_1^2, q_2^2, 0)$$
$$= \frac{2\nu^2}{\pi} \int_{\nu_0}^{\infty} d\nu' \frac{\sqrt{\nu'^2 - q_1^2 q_2^2}}{\nu'(\nu'^2 - \nu^2 - i\epsilon)} (\sigma_0 + \sigma_2)(\nu')$$

σ_J is total cross-section for $\gamma^*\gamma^* \to$ hadrons with total helicity J

Additional cross-check: forward scattering amplitudes (III)





At moderate spacelike virtualities can be described by set of resonance poles (solid lines above)

Expected progress in next years

This section still needs to be written

Summary of current knowledge from lattice

This section still needs to be written

Status summary / Action items

- Overall in good shape, delivered on comparison of groups at $m_\pi \approx 300$ MeV promised in 2018 plenary workshop
- Update progress made in last year
- Missing sections on expected progress and summary
- Editorial work to remove redundant explanations
- Note: working group session on Wednesday afternoon to coordinate and implement remaining steps