Challenging the SM with muon & electron g-2

Massimo Passera
INFN Padova

Muon g-2 Elba Physics Week
La Biodola, Isola d'Elba
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Muon g-2: a quick SM review

The muon g-2: the QED contribution

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \ 0.765857426\ (16)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann ’57; Elend ’66; MP ’04

$+ \ 24.05050988\ (28)\ (\alpha/\pi)^3$

Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP ’04;
Friot, Greynat & de Rafael ’05, Mohr, Taylor & Newell 2012

$+ \ 130.8780\ (60)\ (\alpha/\pi)^4$

Kinoshita & Lindquist ’81, … , Kinoshita & Nio ’04, ’05;
Aoyama, Hayakawa,Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Steinhauser et al. 2013, 2015 & 2016 (all electron & τ loops, analytic);
Laporta, PLB 2017 (mass independent term). COMPLETED^2!

$+ \ 750.80\ (89)\ (\alpha/\pi)^5$ **COMPLETED!**

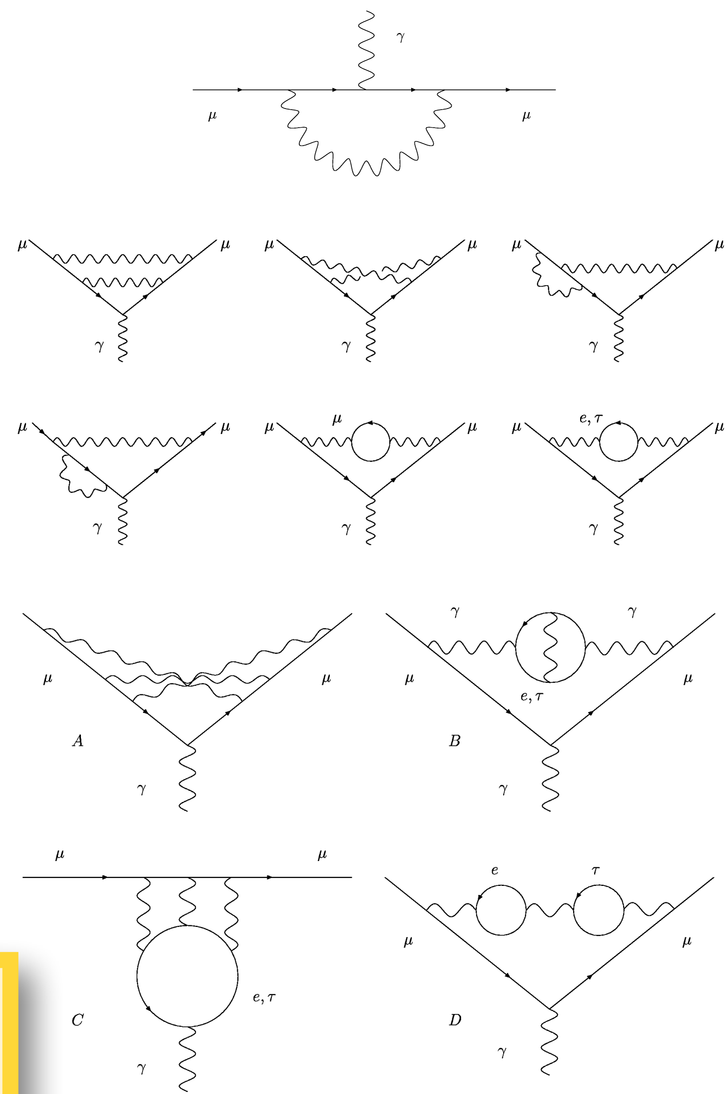
Kinoshita et al. ‘90, Yelkhovsky, Milstein, Starshenko, Laporta,…
Aoyama, Hayakawa, Kinoshita, Nio 2012 & 2015 & 2017.
Volkov 1905.08007: $A_1^{(10)}$[no lept loops] at variance, but negligible Δ.

Adding up, I get:

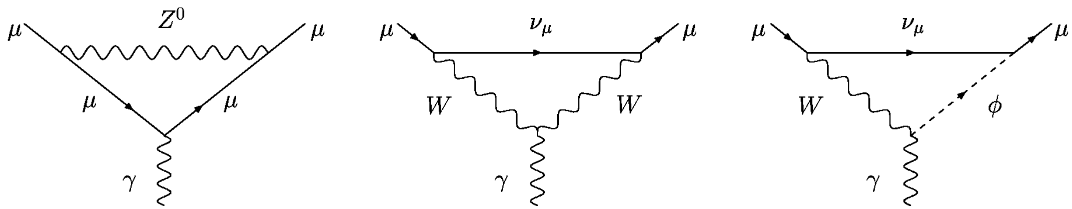
$a_\mu^{QED} = 116584718.932\ (20)(23) \times 10^{-11}$

from coeffs, mainly from 4-loop unc   from α (Cs)

with $\alpha = 1/137.035999046(27)$ [0.2ppb] 2018



- **One-loop term:**



$$a_\mu^{\mathrm{EW}}(\text{1-loop}) = \frac{5 G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1 + \frac{1}{5}\left(1 - 4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195 \times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;

Studenikin et al. '80s

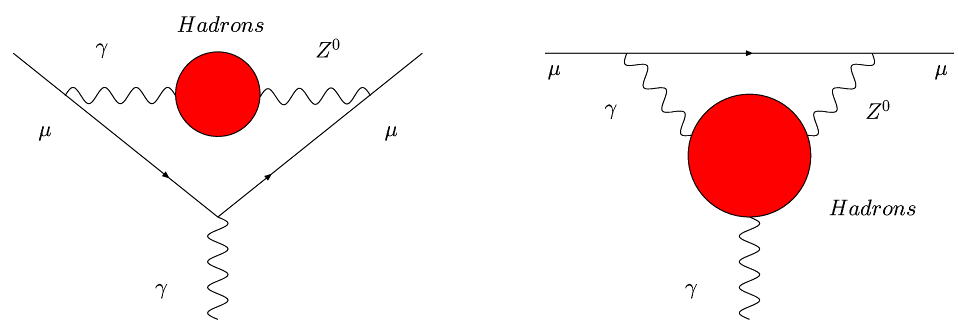
- **One-loop plus higher-order terms:**

$$\boxed{a_\mu^{\mathrm{EW}} = 153.6\ (1) \times 10^{-11}}$$

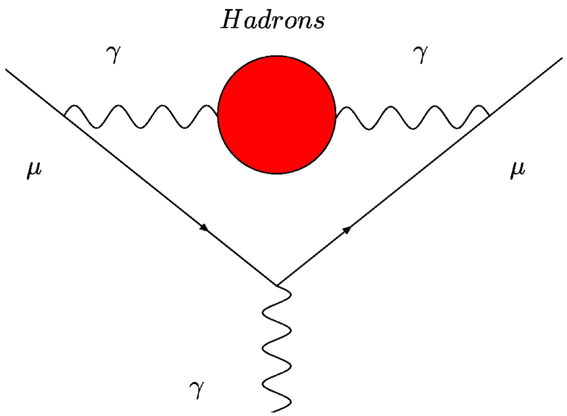
with M_{Higgs} = 125.6 (1.5) GeV

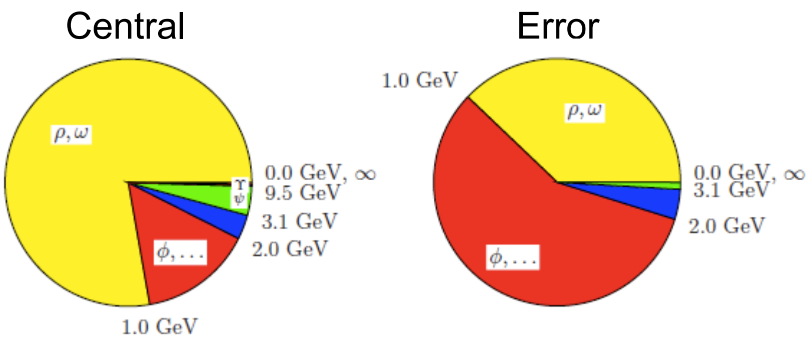
Hadronic loop uncertainties and 3-loop nonleading logs.

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.



The muon g-2: the Hadronic LO contribution (HLO)





F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$K(s) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)} \qquad a_\mu^{\mathrm{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds \, K(s) \, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s) R(s)$$

a_μ^{HLO} = 6894.6 (32.5) x 10^{-11} F. Jegerlehner, arXiv:1711.06089

= 6931 (34) x 10^{-11} Davier, Hoecker, Malaescu, Zhang, arXiv:1706.09436

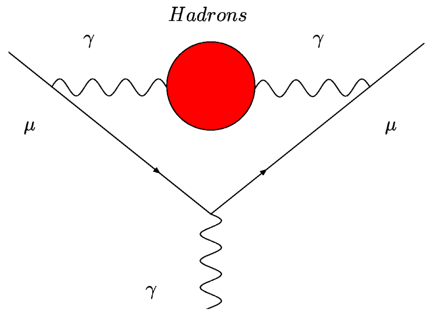
= 6932.7 (24.6) x 10^{-11} Keshavarzi, Nomura, Teubner, arXiv:1802.02995

- **Radiative Corrections are crucial.** S. Actis et al, Eur. Phys. J. C66 (2010) 585
- **Lots of progress in lattice calculations.** Muon g-2 Theory Initiative

See Colangelo's talk

New space-like proposal for HLO: MUonE

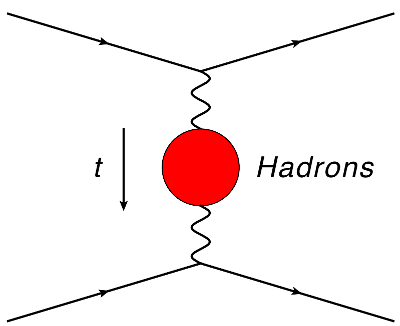
- **Time-like formula for the leading hadronic contribution a_μ^{HLO}:**



$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^0_{\rm had}(s)$$

$$K(s) = \int_0^1 dx\, \frac{x^2(1-x)}{x^2+(1-x)\left(s/m_\mu^2\right)}$$

- **Alternatively, exchanging the x and s integrations in a_μ^{HLO}**



$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

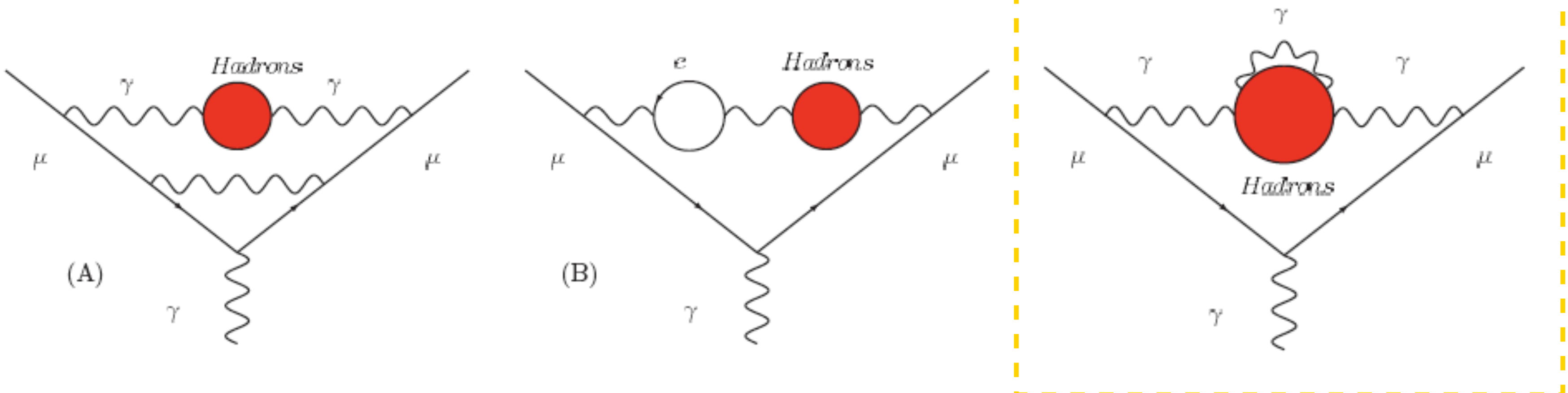
$\Delta\alpha_{had}(t)$ is the hadronic contribution to the running of α in the space-like region. It can be extracted from mu-e scattering data!

Carloni Calame, MP, Trentadue, Venanzoni, 1504.02228; Abbiendi et al, 1609.08987

See Venanzoni's talk

HNLO: Vacuum Polarization

Already included in a_μ^{HLO}

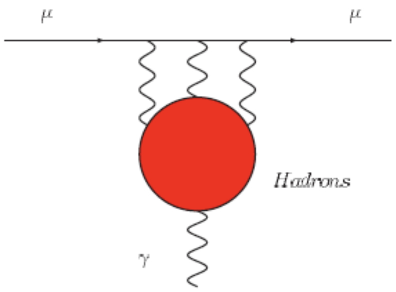


O(α³) contributions of diagrams containing hadronic vacuum polarization insertions:

$$a_\mu^{HNLO}(vp) = -99.27\ (67) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. 2011, Jegerlehner 2017

HNLO: Light-by-light contribution



- This term had a troubled life! Nowadays:

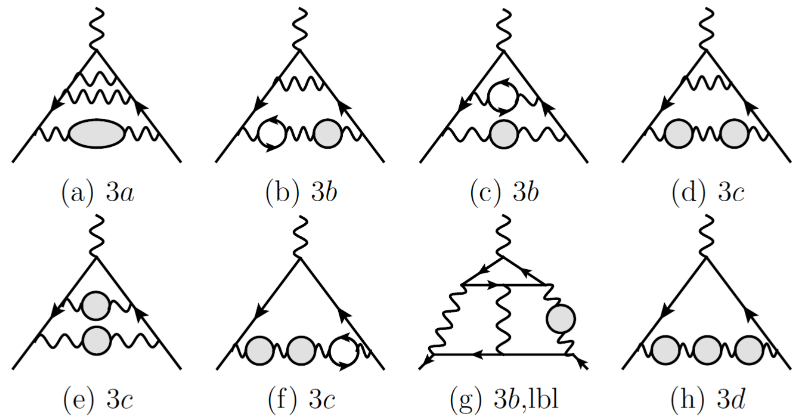
$a_\mu^{HNLO}(lbl) = +80\ (40) \times 10^{-11}$	Knecht & Nyffeler '02
$a_\mu^{HNLO}(lbl) = +136\ (25) \times 10^{-11}$	Melnikov & Vainshtein '03
$a_\mu^{HNLO}(lbl) = +105\ (26) \times 10^{-11}$	Prades, de Rafael, Vainshtein '09
$a_\mu^{HNLO}(lbl) = +100\ (29) \times 10^{-11}$	Jegerlehner, arXiv:1705.00263

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

- Improvements expected in the π^0 transition form factor A. Nyffeler 1602.03398
- The HLbL contribution can be expressed in terms of observables in a dispersive approach. Colangelo et al, 2014-15-17; Vanderhaeghen et al, 2014.
- Lots of progress on the lattice.

See Colangelo's talk

HNNLO: Vacuum Polarization



$O(\alpha^4)$ contributions of diagrams containing hadronic vacuum polarization insertions:

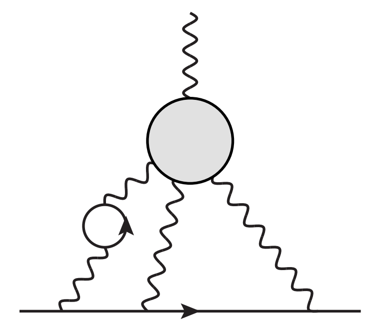
$$a_\mu^{\mathrm{HNNLO}}(\mathrm{vp}) = 12.4\ (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

HNNLO: Light-by-light

$$a_\mu^{\mathrm{HNNLO}}(\mathrm{lbl}) = 3\ (2) \times 10^{-11}$$

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014



Comparisons of the SM predictions with the measured g-2 value:

$a_\mu^{EXP} = 116592091\ (63) \times 10^{-11}$

E821 – Final Report: PRD73 (2006) 072 with latest value of $\lambda = \mu_\mu/\mu_p$ from CODATA'10

$a_\mu^{\rm SM} \times 10^{11}$	$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM}$	σ
116 591 783 (44)	$308\ (77) \times 10^{-11}$	4.0 [1]
116 591 820 (45)	$271\ (77) \times 10^{-11}$	3.5 [2]
116 591 821 (38)	$270\ (74) \times 10^{-11}$	3.7 [3]

with the hadronic light-by-light $a_\mu^{HNLO}(lbl) = 100\ (29) \times 10^{-11}$ of F. Jegerlehner arXiv:1705.00263, and the hadronic leading-order of:

[1] F. Jegerlehner, arXiv:1711.06089.
[2] Davier, Hoecker, Malaescu, Zhang, arXiv:1706.09436.
[3] Keshavarzi, Nomura, Teubner, arXiv:1802.02995.

Δa_μ: could it be errors in the hadronic cross section?

- Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$
$\Delta\alpha_{had}^{(5)} \rightarrow$

$$\boxed{\begin{aligned} a &= \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\ s_u < M_Z^2, \\ b &= \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)}, \end{aligned}}$$

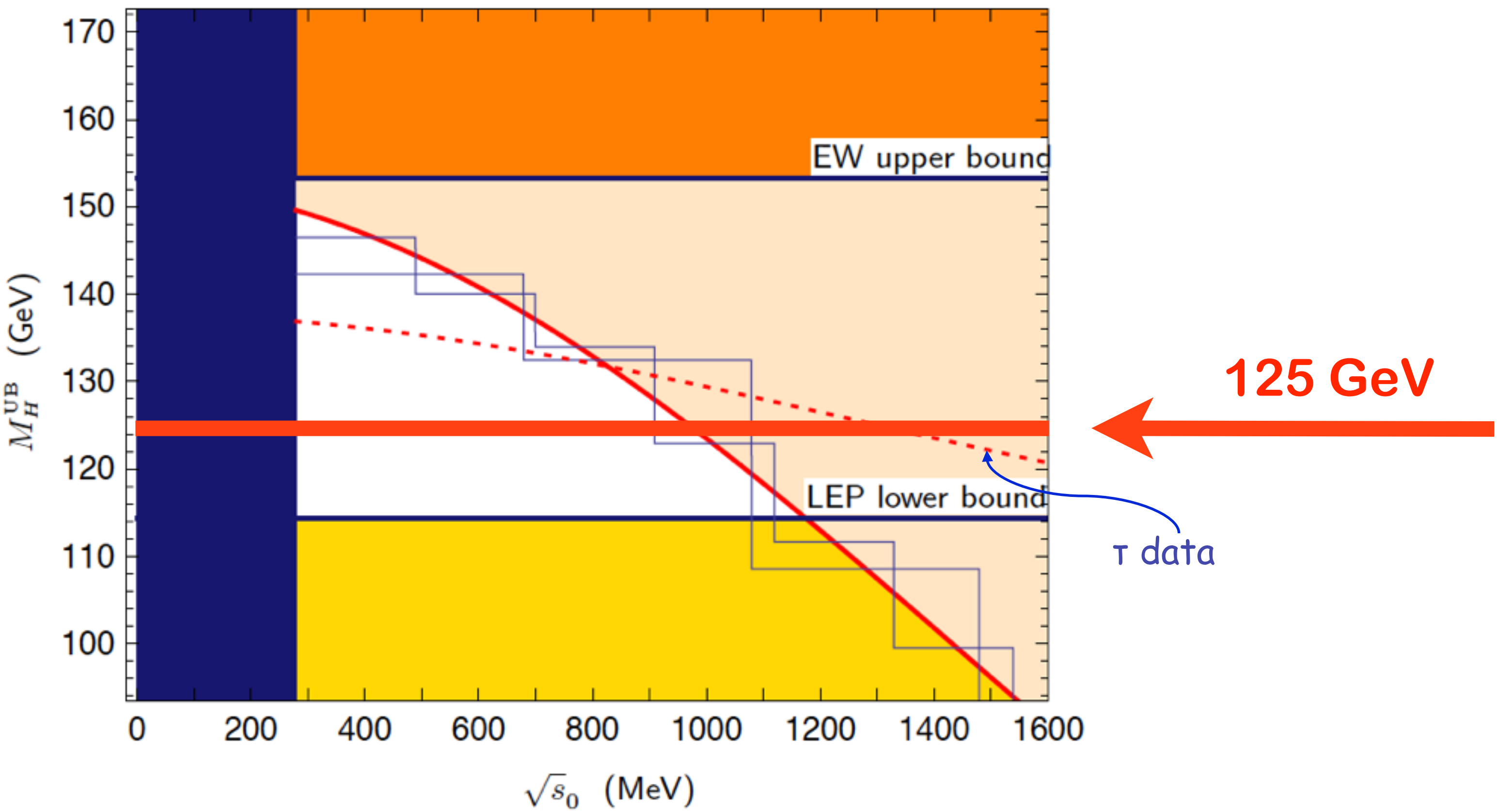
and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

(ε>0), in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$$

- How much does the M_H upper bound from the EW fit change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to accommodate Δa_μ ?



W.J. Marciano, A. Sirlin, MP, 2008 & 2010

The muon g-2: connection with the SM Higgs mass (2)

- Given the quoted exp. uncertainty of $\sigma(s)$, the possibility to explain the muon g-2 with these very large shifts $\Delta\sigma(s)$ appears to be very unlikely.

- Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energy (below ~ 1 GeV) where $\sigma(s)$ is precisely measured.

- Vice versa, assuming we now have a SM Higgs with M_H = 125 GeV, if we bridge the M_H discrepancy in the EW fit decreasing the low-energy hadronic cross section, the muon g-2 discrepancy increases.

W.J. Marciano, A. Sirlin, MP, 2008 & 2010

Testing the SM with the electron g-2

The QED prediction of the electron g-2

e

a_e^{QED} = + $(1/2)(\alpha/\pi)$ - $0.328\,478\,444\,002\,55(33)\,(\alpha/\pi)^2$

Schwinger 1948   Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; CODATA Mar '12

$A_1^{(4)} = -0.328\,478\,965\,579\,193\,78\ldots$ $O(10^{-18})$ in a_e

$A_2^{(4)}\,(m_e/m_\mu) = 5.197\,386\,68\,(26) \times 10^{-7}$

$A_2^{(4)}\,(m_e/m_\tau) = 1.837\,98\,(33) \times 10^{-9}$

+ $1.181\,234\,016\,816\,(11)\,(\alpha/\pi)^3$ $O(10^{-19})$ in a_e

Kinoshita; Barbieri; Laporta, Remiddi; … , Li, Samuel; MP '06; Giudice, Paradisi, MP 2012

$A_1^{(6)} = 1.181\,241\,456\,587\ldots$

$A_2^{(6)}\,(m_e/m_\mu) = -7.373\,941\,62\,(27) \times 10^{-6}$

$A_2^{(6)}\,(m_e/m_\tau) = -6.5830\,(11) \times 10^{-8}$

$A_3^{(6)}\,(m_e/m_\mu,\, m_e/m_\tau) = 1.909\,82\,(34) \times 10^{-13}$

- $1.9113213917\,(12)\,(\alpha/\pi)^4$ $O(10^{-20})$ in a_e

Kinoshita & Lindquist '81, … , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio 2015 & 2017; Kurz, Liu, Marquard & Steinhauser 2014. Laporta, arXiv:1704.06996 (mass independent term)

+ $6.67\,(19)\,(\alpha/\pi)^5$ **Complete Result! (12672 mass indep. diagrams!)**

Aoyama, Hayakawa, Kinoshita, Nio, 2012, 2018. Volkov 1905.08007: $A_1^{(10)}$[no lept loops] at variance.

$1.3\ 10^{-14}$ in a_e NB: $(\alpha/\pi)^6 \sim O(10^{-16})$

- The SM prediction is:

$$a_e^{SM}(\alpha) = a_e^{QED}(\alpha) + a_e^{EW} + a_e^{HAD}$$

- The EW (1&2 loop) term is: Czarnecki, Krause, Marciano ’96, Jegerlehner 2017

$$a_e^{EW} = 0.3053\,(23) \times 10^{-13}$$

- The Hadronic contribution, at LO+NLO+NNLO, is:

Nomura & Teubner ’12, Jegerlehner 2017; Krause’97; Kurz, Liu, Marquard & Steinhauser 2014

$$a_e^{HAD} = 16.93\,(12) \times 10^{-13}$$

$$a_e^{HLO} = +\,18.490\,(108) \times 10^{-13}$$

$$a_e^{HNLO} = [-2.213(12)_{vac} + 0.37(5)_{lbl}] \times 10^{-13}$$

$$a_e^{HNNLO} = +\,0.28\,(1) \times 10^{-13}$$

- Which value of α should we use to compute a_e^{SM}?

$(g\text{-}2)_e$ no longer gives the best value of α

- The 2008 measurement of the electron g-2 is:

  $a_e^{EXP} = 11596521807.3\ (2.8) \times 10^{-13}$ Hanneke et al, PRL100 (2008) 120801

  vs. old (factor of 15 improvement, 1.8σ difference):

  $a_e^{EXP} = 11596521883\ (42) \times 10^{-13}$ Van Dyck et al, PRL59 (1987) 26

- Equate $\boxed{a_e^{SM}(\alpha) = a_e^{EXP}}$ → "g_e-2" determination of alpha:

$$\alpha^{-1} = 137.035\,999\,149\,(33) \quad [0.24\text{ ppb}]$$

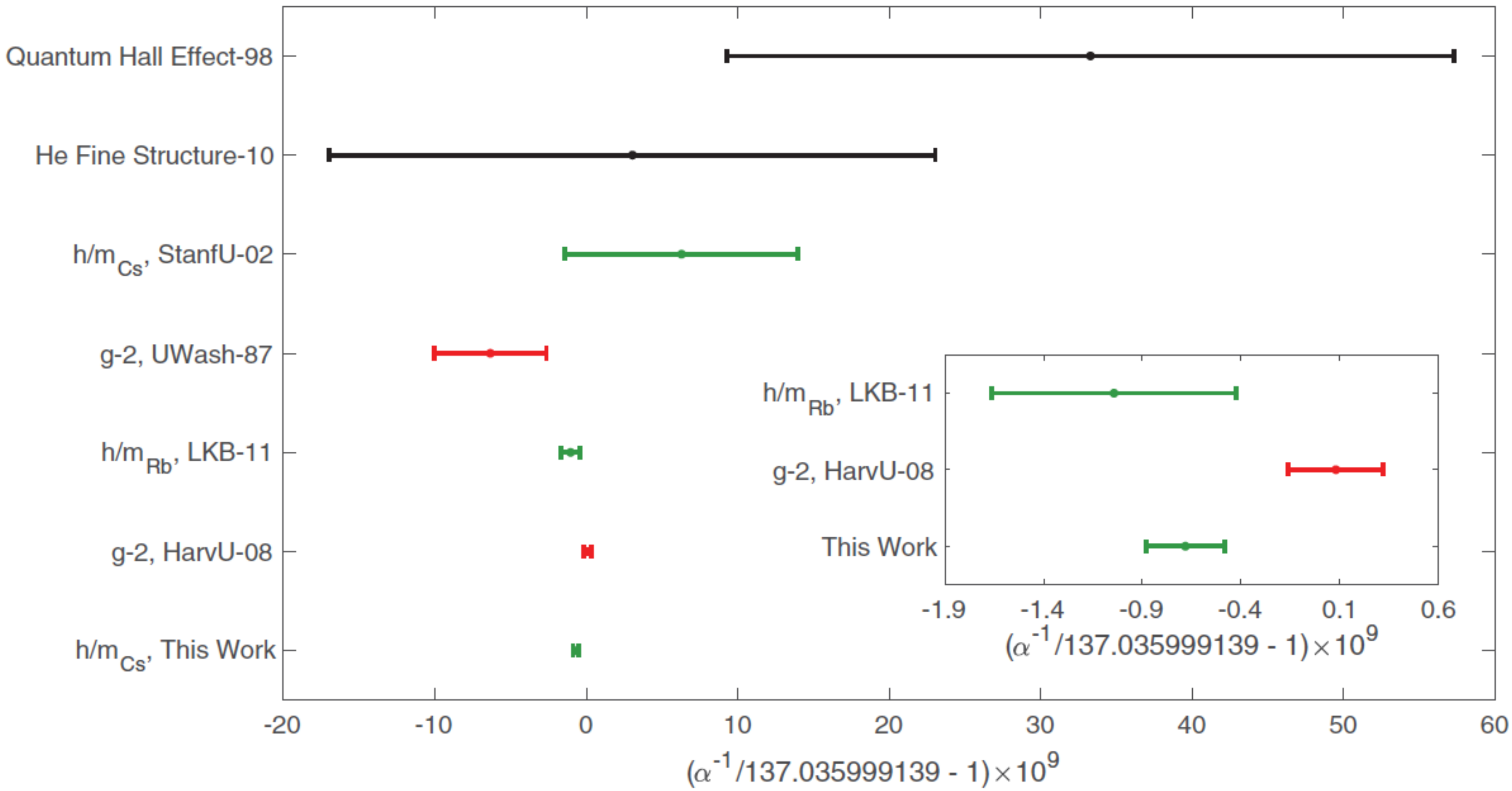
- Compare it with the best determination of alpha:

$$\alpha^{-1} = 137.036\,999\,046\,(27)\ \ [0.20\text{ ppb}]$$ Science 360 (2018) 191 (Cs)

(was α^{-1} =137.035 998 995 (85) [0.62 ppb] PRL106 (2011) & CODATA 2016)

2.4 sigma discrepancy

Determinations of alpha



Richard H. Parker, Chenghui Yu, Weicheng Zhong, Brian Estey, Holger Müller
Science 360 (2018) 191

- Using α = 1/137.036 999 046 (27) [Cs 2018], the SM prediction for the electron g-2 is:

$$a_e^{SM} = 115\,965\,218\,16.0\,(0.1)\,(0.1)\,(2.3) \times 10^{-13}$$

δC_5^{qed} δa_e^{had} from $\delta\alpha$

- The (EXP - SM) difference is:

$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -\,8.7\,(3.6) \times 10^{-13}$$

i.e. 2.4 sigma difference. Note the negative sign!
(the 5-loop contrib. to a_e^{QED} is 4.5×10^{-13})

- The present sensitivity is $\delta\Delta a_e = 3.6 \times 10^{-13}$, ie ($10^{-13}$ units):

$$\underbrace{(0.1)_{\rm QED5}, \quad (0.1)_{\rm HAD}}_{(0.2)_{\rm TH}}, \quad (2.3)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\rm EXP}}$$

- The $(g\text{-}2)_e$ exp. error may soon drop below 10^{-13} and work is in progress to further reduce the error induced by $\delta\alpha$ →

sensitivity below 10^{-13} may be reached with ongoing exp work

- In a broad class of BSM theories, contributions to a_l scale as

$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}}\right)^2$$

This Naive Scaling leads to:

$$\boxed{\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}; \qquad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.8 \times 10^{-6}}$$

Giudice, Paradisi & MP, JHEP 2012

Testing new physics with the electron g-2 (2)

e

- The sensitivity in Δa_e may soon be 10^{-13} or better! This will bring a_e to play a pivotal role in probing new physics in the leptonic sector.

- NP scenarios exist which violate Naive Scaling. They can lead to larger effects in Δa_e and contributions to EDMs, LFV or lepton universality breaking observables.

Giudice, Paradisi & MP, JHEP 2012
Crivellin, Hoferichter, Schmidt-Wellenburg, PRD 2018

- One real scalar with a mass of ~ 250−1000 MeV could explain the deviations in a_μ and a_e, through one- and two-loop processes, respectively.

Davoudiasl & Marciano, PRD 2018

Conclusions

- **Muon g-2**: $\Delta a_\mu \sim 3.5 - 4\ \sigma$. Please let us know asap!

- **Electron g-2**: $\Delta a_e = -\ 2.4\ \sigma$. NP sensitivity limited only by the exp uncertainties in α and a_e. May soon play a pivotal role in probing NP in the leptonic sector.