

Statistical Correlations Between Run 1 ω_a Analyses

Nick Kinnaird

ω_a Meeting

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- This presentation serves to bring attention to the note I've written on the subject, and briefly summarize some main points
- DocDB 23621

Abstract

This note presents calculated statistical correlation coefficients between the different Run 1 ω_a analyses. The correlation coefficients were calculated with a Monte Carlo simulation taking real data as input, for which the implementation is described. The statistical correlations presented here include the effects from the different reconstructions, different analysis methods (TARQ), and different analyzer-specific analysis parameters. Depending on the combination methodology adopted, these statistical correlations may be needed in order to properly combine the results. Various versions of the statistical correlations were provided to the combination effort, and results for the combination will be detailed by a forthcoming note. These statistical correlations are the most accurate to date for Run 1, and can be used as a starting point going forward for combinations of future ω_a analyses.

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Work

- Developed a Monte Carlo to estimate the statistical correlations between the 11 different ω_a analyses in the Run 1 analysis for the 4 datasets
- Takes real data input in the form of energy binned functions, and a comparison between ReconWest and ReconEast energies
- By generating and fitting many samples of pseudo-data, the correlation coefficients can be calculated

Run 1 Analyses and Parameters							
Analysis Effort	Cornell U.	U. Washington	Europa	Shanghai Jiao Tong U.	Boston U.	U. Kentucky	
Abbreviation	CU	UW	EU	SJTU	BU	UK	
Lead Analyzer	D. Sweigart	A. Fienberg	M. Sorbara	B. Li	N. Kinnaird	T. Gorringe	
Analysis References	[2, 5]	[3, 6]	[7]	[8]	[9, 10]	[11]	
Parameter							
Analysis Methods (TARQ)	TA	TA	TA	TA	TR	Q	
Reconstruction	East	West	West	West	West	Q	
Bin Width (ns)	149.2	149.19	149.19	149.2	149.2	150	
Bin Edge (ns)	0	53.62	0	0	0	0	
T-Method Threshold (MeV)	1700–	1700–6000	1680–7020	1700–9300	1700–	-	
A-Method Threshold (MeV)	1000–3000	1000–3020	1080–3020	1000–3100	-	-	
R-Method Threshold (MeV)	-	-	-	-	1700–	-	
Q-Method Threshold (MeV)	-	-	-	-	-	300–	
Fit Start Time (μs)	30.2876	30.19	30.1364	30.2876	30.2876	30	
Fit Start Time (EndGame) (μs)	49.982	49.88308	49.8295	49.982	49.982	49.9762	
Fit End Time (μs)	649.9152	649.92526	650.021	671.4	650.0644	215.5	

Table 1: Individual analysis groups, the lead analyzers in those groups, and the methods and parameters in the respective analyses. Parameters provided via private communication and through discussion in the Run 1 Combination Task Force [12]. Note that the EndGame dataset had a different fit start time compared to the rest, hence the additional row in the table.

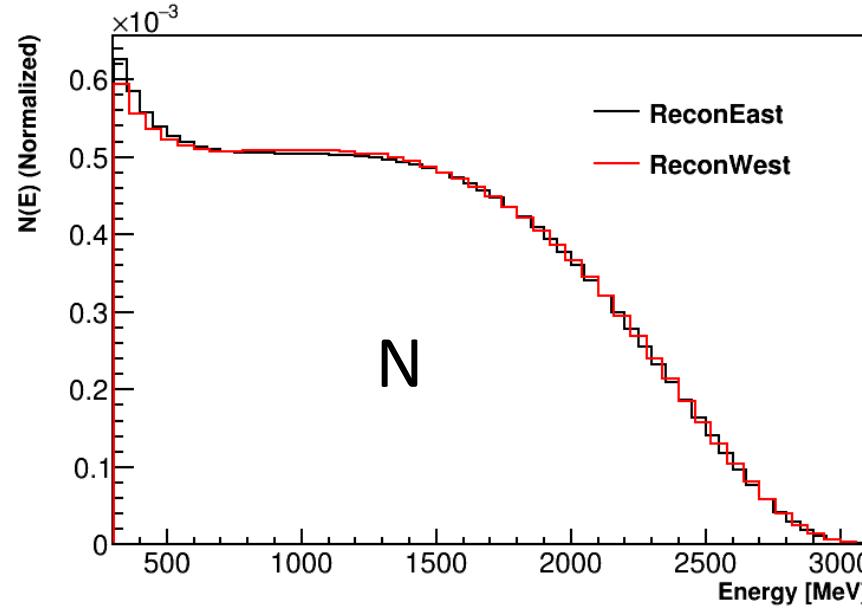
Monte Carlo

- Hits are generated with “GetRandom2()” calls on 2D time-energy functions:

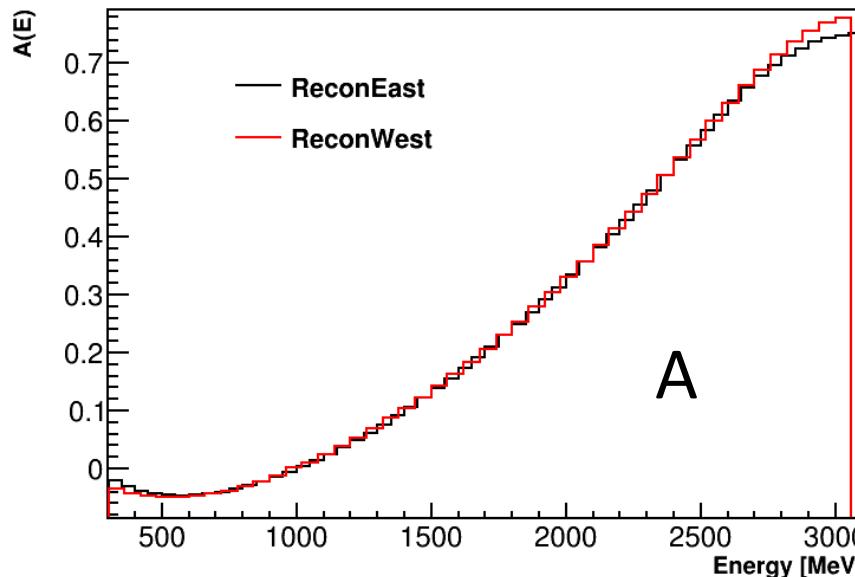
$$N(t, E) = N_0(E) \cdot e^{-t/\tau_\mu} \cdot (1 + A(E) \cos(\omega_a t + \phi(E)))$$

- These energy functions were provided either by David for ReconEast or Matteo for ReconWest
- Two sets of hits for each dataset were generated with these ReconEast or ReconWest functions

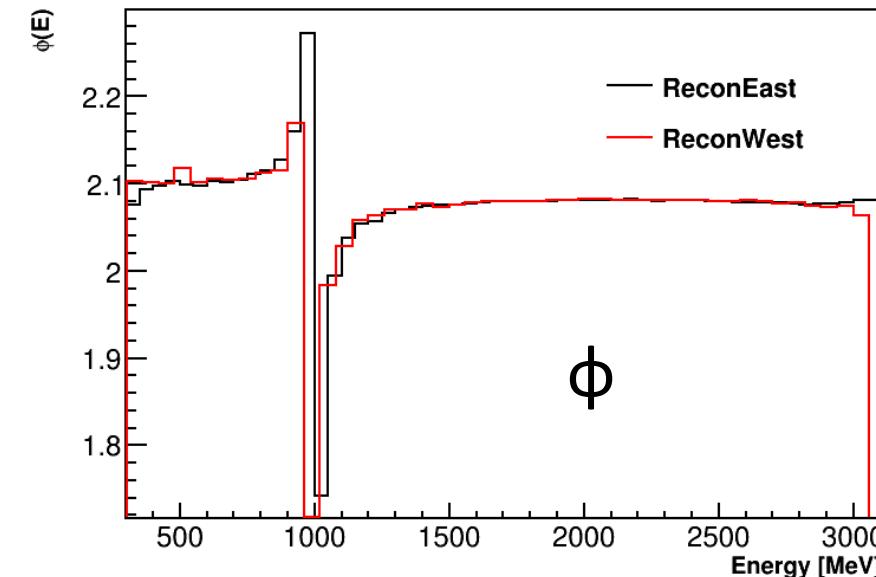
Energy binned functions:



N



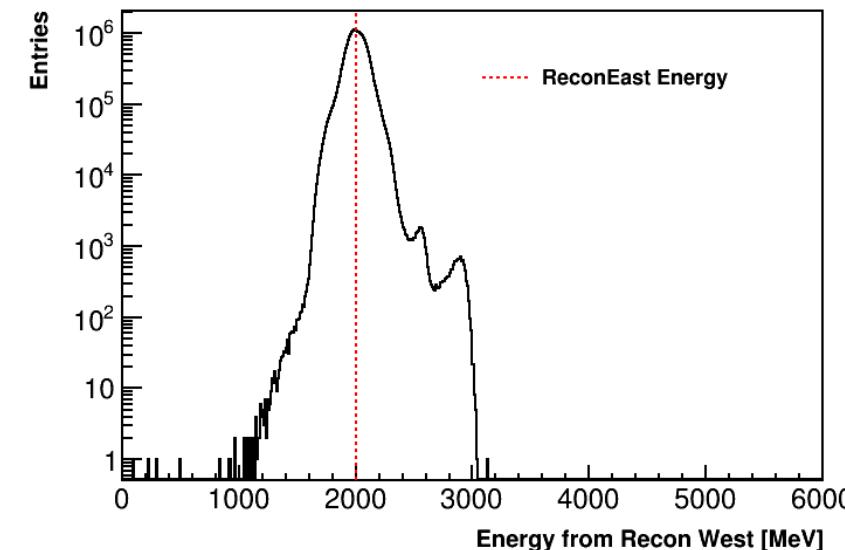
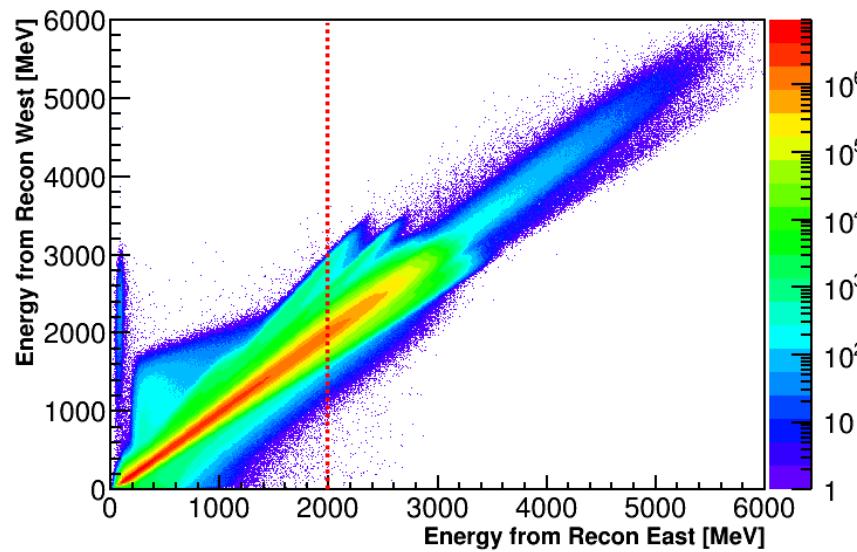
A



φ

ReconEast vs ReconWest

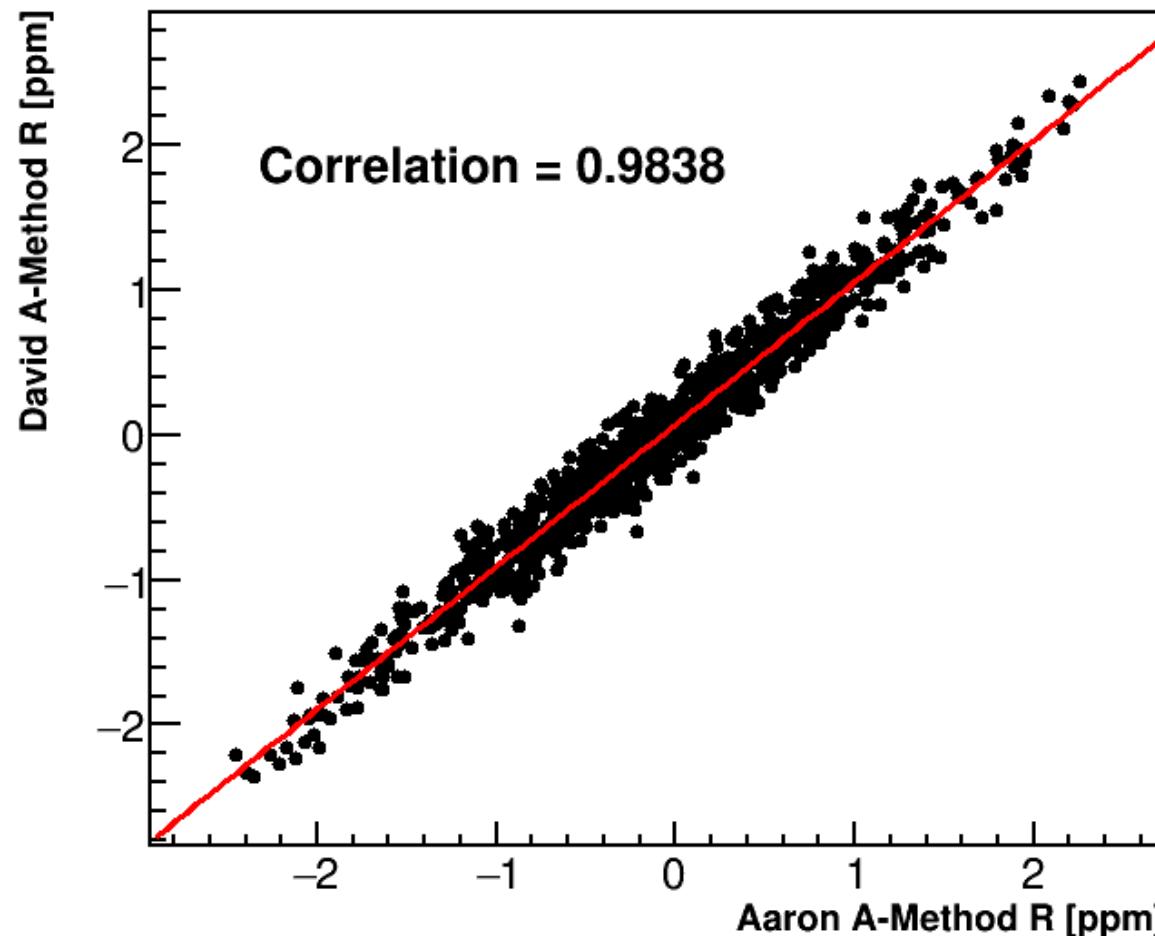
- In order to convert hits generated from one set to the other, I used Josh's east vs west comparison – DocDB 21567
- For each hit energy, I grabbed the associated energy projection, and then randomly sampled the distribution



Pseudo-Data Generation

- Generated many hits according to the statistics of the various datasets, with the associated energy binned functions and the east vs west comparison
- Submitted 1000 grid jobs for each dataset, did this twice for the East-To-West hits and the West-To-East hits
- Filled those hits into the respective analyzer histograms with the various parameters and proper weights, and then I fit all those histograms

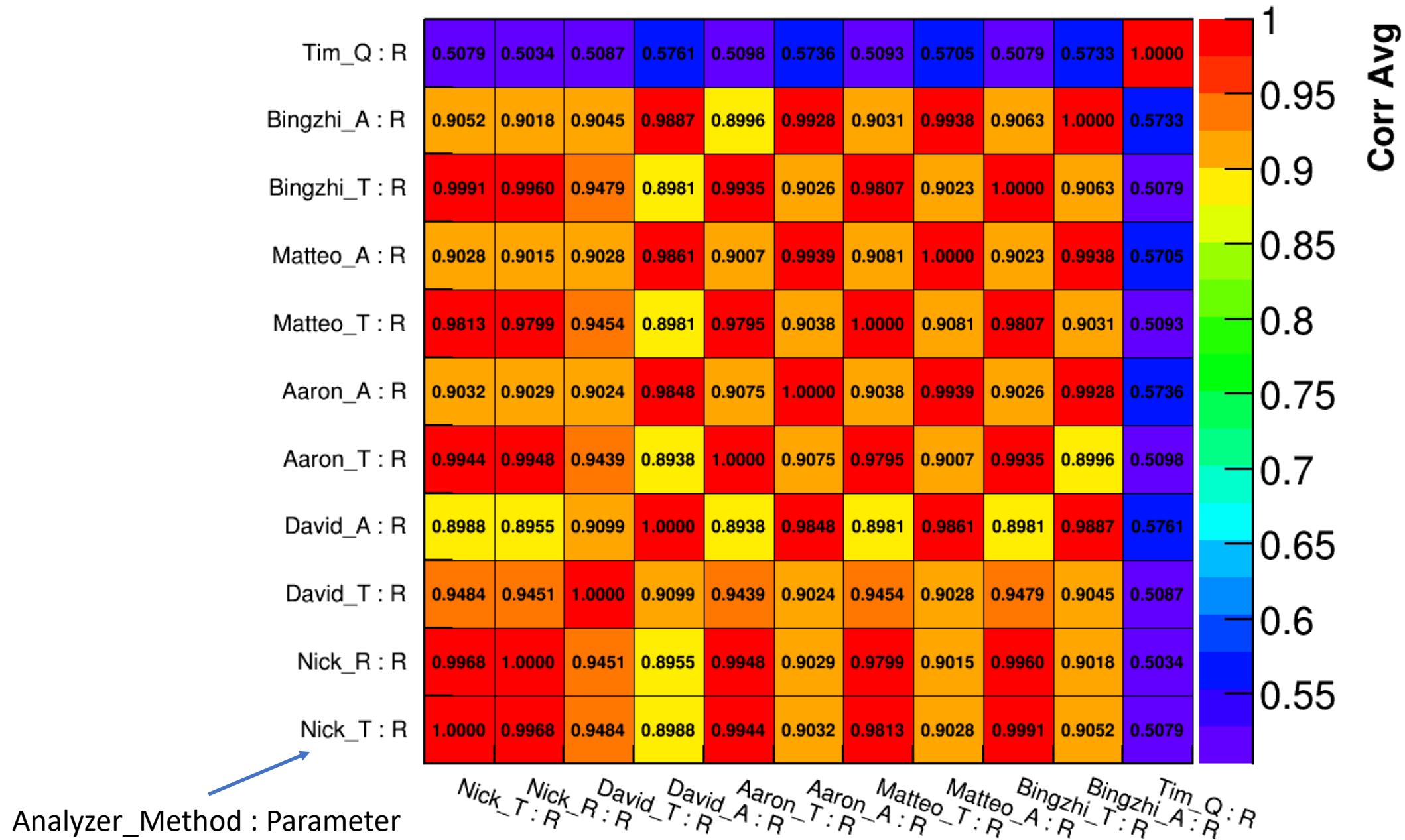
- Calculate all the correlation coefficients by plotting the fit parameters against each other for the various analyses and datasets.



Final Coefficients

- For my final coefficients I took as the values the average between the East-To-West numbers and the West-To-East numbers
- The systematic errors I called the difference between the average and either of the two numbers which feed into the average
- The statistical errors I calculated via the procedure described in Cowan's *Statistical Data Analysis* textbook (Fisher-z transform)
- Correlations across the four datasets were found to be very similar, barring some slight differences in random table entries which are usually within error of each other.

(I'll talk about some general summary points for these correlations in a moment)



EG Correlation Coefficients – Analyzer Level											
	BU T	BU R	CU T	CU A	UW T	UW A	EU T	EU A	SJTU T	SJTU A	UK Q
BU T	1.0000	0.9968	0.9484	0.8988	0.9944	0.9032	0.9813	0.9028	0.9991	0.9052	0.5079
	0.0000	0.0002	0.0040	0.0113	0.0006	0.0097	0.0013	0.0107	0.0001	0.0108	0.0275
BU R	0.9968	1.0000	0.9451	0.8955	0.9948	0.9029	0.9799	0.9015	0.9960	0.9018	0.5034
	0.0002	0.0000	0.0045	0.0122	0.0007	0.0107	0.0015	0.0115	0.0003	0.0118	0.0275
CU T	0.9484	0.9451	1.0000	0.9099	0.9439	0.9024	0.9454	0.9028	0.9479	0.9045	0.5087
	0.0040	0.0045	0.0000	0.0064	0.0042	0.0064	0.0049	0.0064	0.0038	0.0063	0.0270
CU A	0.8988	0.8955	0.9099	1.0000	0.8938	0.9848	0.8981	0.9861	0.8981	0.9887	0.5761
	0.0113	0.0122	0.0064	0.0000	0.0121	0.0011	0.0124	0.0010	0.0113	0.0010	0.0230
UW T	0.9944	0.9948	0.9439	0.8938	1.0000	0.9075	0.9795	0.9007	0.9935	0.8996	0.5098
	0.0006	0.0007	0.0042	0.0121	0.0000	0.0099	0.0017	0.0115	0.0007	0.0120	0.0274
UW A	0.9032	0.9029	0.9024	0.9848	0.9075	1.0000	0.9038	0.9939	0.9026	0.9928	0.5736
	0.0097	0.0107	0.0064	0.0011	0.0099	0.0000	0.0108	0.0006	0.0096	0.0006	0.0233
EU T	0.9813	0.9799	0.9454	0.8981	0.9795	0.9038	1.0000	0.9081	0.9807	0.9031	0.5093
	0.0013	0.0015	0.0049	0.0124	0.0017	0.0108	0.0000	0.0113	0.0014	0.0115	0.0267
EU A	0.9028	0.9015	0.9028	0.9861	0.9007	0.9939	0.9081	1.0000	0.9023	0.9938	0.5705
	0.0107	0.0115	0.0064	0.0010	0.0115	0.0006	0.0113	0.0000	0.0107	0.0005	0.0234
SJTU T	0.9991	0.9960	0.9479	0.8981	0.9935	0.9026	0.9807	0.9023	1.0000	0.9063	0.5079
	0.0001	0.0003	0.0038	0.0113	0.0007	0.0096	0.0014	0.0107	0.0000	0.0106	0.0278
SJTU A	0.9052	0.9018	0.9045	0.9887	0.8996	0.9928	0.9031	0.9938	0.9063	1.0000	0.5733
	0.0108	0.0118	0.0063	0.0010	0.0120	0.0006	0.0115	0.0005	0.0106	0.0000	0.0231
UK Q	0.5079	0.5034	0.5087	0.5761	0.5098	0.5736	0.5093	0.5705	0.5079	0.5733	1.0000
	0.0275	0.0275	0.0270	0.0230	0.0274	0.0233	0.0267	0.0234	0.0278	0.0231	0.0000

Table 8: Correlation coefficients between R values for the EG dataset, at the analyzer level. In each table cell, the top number is the correlation coefficient and the bottom number is the error on the coefficient.

EG Correlation Coefficients – Analyzer Level											
	BU T	BU R	CU T	CU A	UW T	UW A	EU T	EU A	SJTU T	SJTU A	UK Q
BU T	1.0000 0.0000	0.9968 0.0002	0.9484 0.0040	0.8988 0.0113	0.9944 0.0006	0.9032 0.0097	0.9813 0.0013	0.9028 0.0107	0.9991	0.9052	0.5079
BU R	0.9968 0.0002	1.0000 0.0000	0.9451 0.0045	0.8955 0.0122	0.9948 0.0007	0.9029 0.0107	0.9799 0.0015	0.9015 0.0115			
CU T	0.9484 0.0040	0.9451 0.0045	1.0000 0.0000	0.9099 0.0064	0.9439 0.0042	0.9024 0.0064	0.9454 0.0049	0.9028 0.0064			
CU A	0.8988 0.0113	0.8955 0.0122	0.9099 0.0064	1.0000 0.0000	0.8938 0.0121	0.9848 0.0011	0.8981 0.0124	0.9861 0.0010			
UW T	0.9944 0.0006	0.9948 0.0007	0.9439 0.0042	0.8938 0.0121	1.0000 0.0000	0.9075 0.0099	0.9795 0.0017	0.9007 0.0115			
UW A	0.9032 0.0097	0.9029 0.0107	0.9024 0.0064	0.9848 0.0011	0.9075 0.0099	1.0000 0.0000	0.9038 0.0108	0.9939 0.0006			
EU T	0.9813 0.0013	0.9799 0.0015	0.9454 0.0049	0.8981 0.0124	0.9795 0.0017	0.9038 0.0108	1.0000 0.0000	0.9081 0.0113			
EU A	0.9028 0.0107	0.9015 0.0115	0.9028 0.0064	0.9861 0.0010	0.9007 0.0115	0.9939 0.0006	0.9081 0.0113	1.0000 0.0000			
SJTU T	0.9991 0.0001	0.9960 0.0003	0.9479 0.0038	0.8981 0.0113	0.9935 0.0007	0.9026 0.0096	0.9807 0.0014	0.9023 0.0107			
SJTU A	0.9052 0.0108	0.9018 0.0118	0.9045 0.0063	0.9887 0.0010	0.8996 0.0120	0.9928 0.0006	0.9031 0.0115	0.9938 0.0005	0.9063 0.0106	1.0000 0.0000	0.5733 0.0231
UK Q	0.5079 0.0275	0.5034 0.0275	0.5087 0.0270	0.5761 0.0230	0.5098 0.0274	0.5736 0.0233	0.5093 0.0267	0.5705 0.0234	0.5079 0.0278	0.5733 0.0231	1.0000 0.0000

- The T-A-Methods correlation is typically around 91%, T-Q around 50%, T-R around 99.6%, A-R around 90%, and A-Q around 57%, regardless of reconstruction.
- The T-Method correlations between ReconWest analyzers is typically around 98–99%, the A-Method correlations between ReconWest analyzers is typically around 99%.
- I'll talk about the ReconEast-ReconWest correlations in a bit

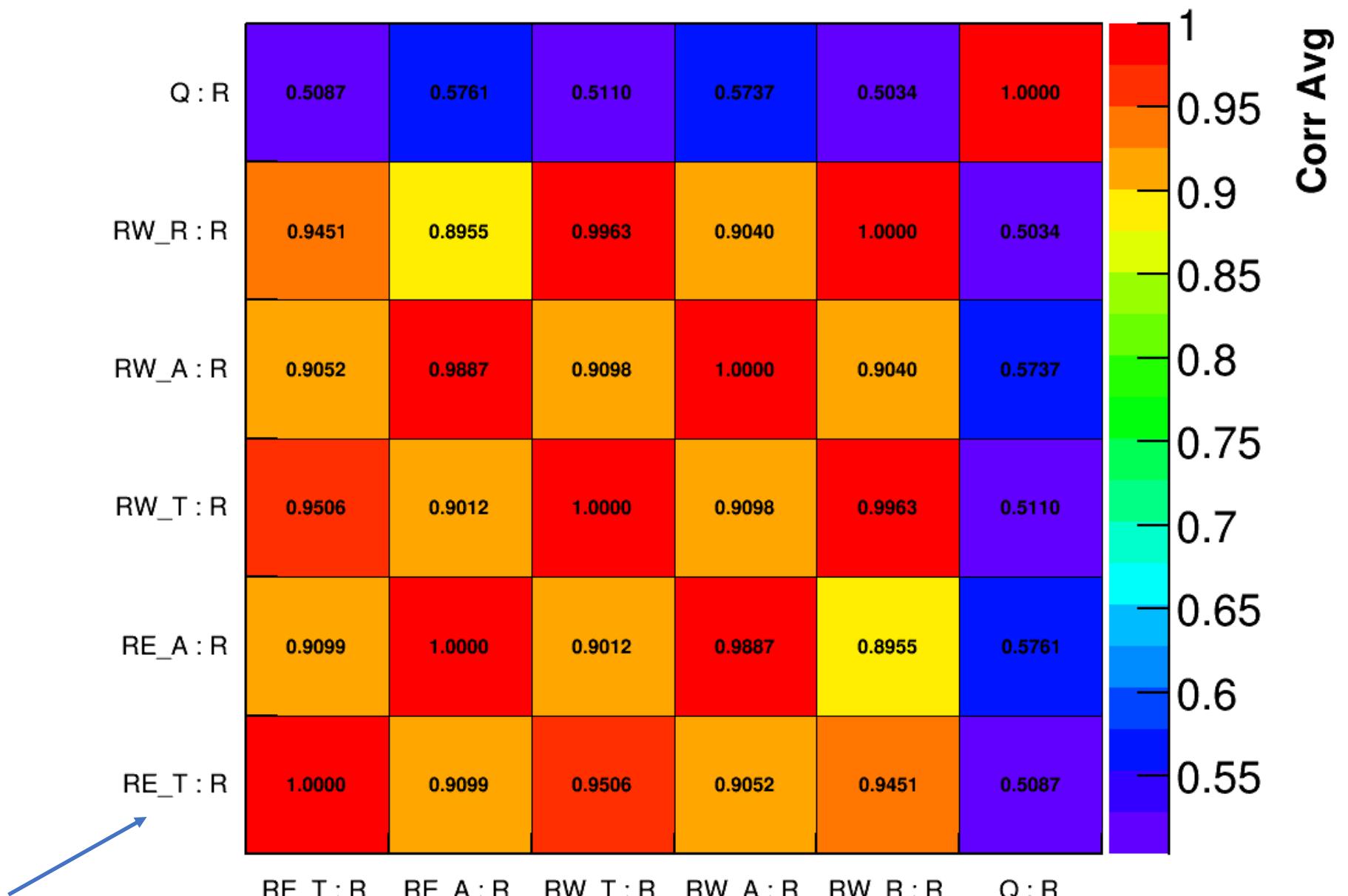
Table 8: Correlation coefficients between R values for the EG dataset, at the analyzer level. In each table cell, the top number is the correlation coefficient and the bottom number is the error on the coefficient.

EG Correlation Coefficients – Analyzer Level											
	BU T	BU R	CU T	CU A	UW T	UW A	EU T	EU A	SJTU T	SJTU A	UK Q
BU T	1.0000	0.9968	0.9484	0.8988	0.9944	0.9032	0.9813	0.9023	1.0000	0.9063	0.5079
	0.0000	0.0002	0.0040	0.0113	0.0006	0.0097					
BU R	0.9968	1.0000	0.9451	0.8955	0.9948	0.9029	0.9795	0.9038	0.9000	0.9063	0.5079
	0.0002	0.0000	0.0045	0.0122	0.0007	0.0107					
CU T	0.9484	0.9451	1.0000	0.9099	0.9439	0.9024	0.9454	0.9848	0.9000	0.9063	0.5079
	0.0040	0.0045	0.0000	0.0064	0.0042	0.0064					
CU A	0.8988	0.8955	0.9099	1.0000	0.8938	0.9848	0.9881	0.9023	1.0000	0.9063	0.5079
	0.0113	0.0122	0.0064	0.0000	0.0121	0.0011					
UW T	0.9944	0.9948	0.9439	0.8938	1.0000	0.9075	0.9795	0.9038	0.9000	0.9063	0.5079
	0.0006	0.0007	0.0042	0.0121	0.0000	0.0099					
UW A	0.9032	0.9029	0.9024	0.9848	0.9075	1.0000	0.9031	0.9938	0.9000	0.9063	0.5079
	0.0097	0.0107	0.0064	0.0011	0.0099	0.0000					
EU T	0.9813	0.9799	0.9454	0.8981	0.9795	0.9038	0.9807	0.9023	1.0000	0.9063	0.5079
	0.0013	0.0015	0.0049	0.0124	0.0017	0.0108					
EU A	0.9028	0.9015	0.9028	0.9861	0.9007	0.9939	0.9031	0.9938	0.9000	0.9063	0.5079
	0.0107	0.0115	0.0064	0.0010	0.0115	0.0006					
SJTU T	0.9991	0.9960	0.9479	0.8981	0.9935	0.9026	0.9807	0.9023	1.0000	0.9063	0.5079
	0.0001	0.0003	0.0038	0.0113	0.0007	0.0096					
SJTU A	0.9052	0.9018	0.9045	0.9887	0.8996	0.9928	0.9031	0.9938	0.9000	0.9063	0.5079
	0.0108	0.0118	0.0063	0.0010	0.0120	0.0006					
UK Q	0.5079	0.5034	0.5087	0.5761	0.5098	0.5736	0.5093	0.5705	0.5079	0.5733	1.0000
	0.0275	0.0275	0.0270	0.0230	0.0274	0.0233					

Table 8: Correlation coefficients between R values for the EG dataset, at the analyzer level. In each table cell, the top number is the correlation coefficient and the bottom number is the error on the coefficient.

East vs West Correlations

- I also calculated correlation coefficients after averaging the RW T and A method results
- (I also determined coefficients after doing various averaging of different methods with different weights, but won't talk about that here)



Reconstruction_Method : Parameter

EG Correlation Coefficients – Recon. Level						
	RE T	RE A	RW T	RW A	RW R	Q
RE T	1.0000 0.0000	0.9099 0.0064	0.9506 0.0039	0.9052 0.0062	0.9451 0.0045	0.5087 0.0270
RE A	0.9099 0.0064	1.0000 0.0000	0.9012 0.0115	0.9887 0.0008	0.8955 0.0122	0.5761 0.0230
RW T	0.9506 0.0039	0.9012 0.0115	1.0000 0.0000	0.9098 0.0104	0.9963 0.0003	0.5110 0.0273
RW A	0.9052 0.0062	0.9887 0.0008	0.9098 0.0104	1.0000 0.0000	0.9040 0.0112	0.5737 0.0232
RW R	0.9451 0.0045	0.8955 0.0122	0.9963 0.0003	0.9040 0.0112	1.0000 0.0000	0.5034 0.0275
Q	0.5087 0.0270	0.5761 0.0230	0.5110 0.0273	0.5737 0.0232	0.5034 0.0275	1.0000 0.0000

The ReconEast-ReconWest correlations are around 95% for the T-Methods, and around 98.9% for the A-Methods. The errors on these correlations are around 0.4 and 0.1% respectively.

Table 12: Correlation coefficients between R values for the EG dataset, at the reconstruction level, after the ReconWest T-Method and A-Method R values were averaged among the different analyzers. In each table cell, the top number is the correlation coefficient and the bottom number is the error on the coefficient.

High Correlation Regime

- Something like a third or so of the correlations were above the "high-correlation" regime cutoff, where

$$r_{ij} > \sigma_i / \sigma_j, \quad \sigma_j > \sigma_i$$

- Some of the correlations were within error of the cutoff, others were not
- Perhaps with further improvements to the Monte Carlo to make it more like real data all the coefficients would drop below the cutoff
 - Hard to get systematic effects into the Monte Carlo which would have a big effect

Deviations Between Analyses

- Beyond the correlations it's of interest to see the deviations between the different results

Analysis	Run 1 Commonly Blinded Results							
	60h		HK		9d		EG	
	R	σ_R	R	σ_R	R	σ_R	R	σ_R
BU T	-28.8023	1.3582	-27.0442	1.1561	-27.9171	0.9301	-27.7020	0.7584
BU R	-28.9668	1.3598	-27.2093	1.1574	-27.9218	0.9327	-27.7654	0.7576
CU T	-28.2111	1.3377	-27.2093	1.1336	-28.0202	0.9126	-27.7152	0.7474
CU A	-28.2288	1.2079	-26.9466	1.0234	-27.5532	0.8240	-27.5902	0.6758
UW T	-28.6199	1.3308	-27.0049	1.1277	-27.8989	0.9079	-27.7144	0.7437
UW A	-28.6373	1.2184	-26.9657	1.0302	-27.5723	0.8305	-27.6694	0.6799
EU T	-28.8848	1.3327	-27.0806	1.1203	-27.8890	0.9067	-27.8772	0.7435
EU A	-28.4813	1.1938	-27.0213	1.0120	-27.5998	0.8146	-27.7276	0.6679
SJTU T	-28.7398	1.3314	-27.0019	1.1281	-27.8935	0.9084	-27.6658	0.7441
SJTU A	-28.4228	1.2061	-27.0910	1.0223	-27.7440	0.8224	-27.6945	0.6729
UK Q	-29.2062	2.0585	-24.9464	1.7478	-26.2794	1.4032	-27.9905	1.2690

Table 2: Best-fit results for R and the respective statistical errors for the different analyses of the Run 1 datasets, in units of ppm. R is commonly blinded, and contains both a software and hardware blinding offset.

$$1\sigma_{\text{allowed}} = \sqrt{\sigma_i^2 + \sigma_j^2 - 2r_{ij}\sigma_i\sigma_j}$$

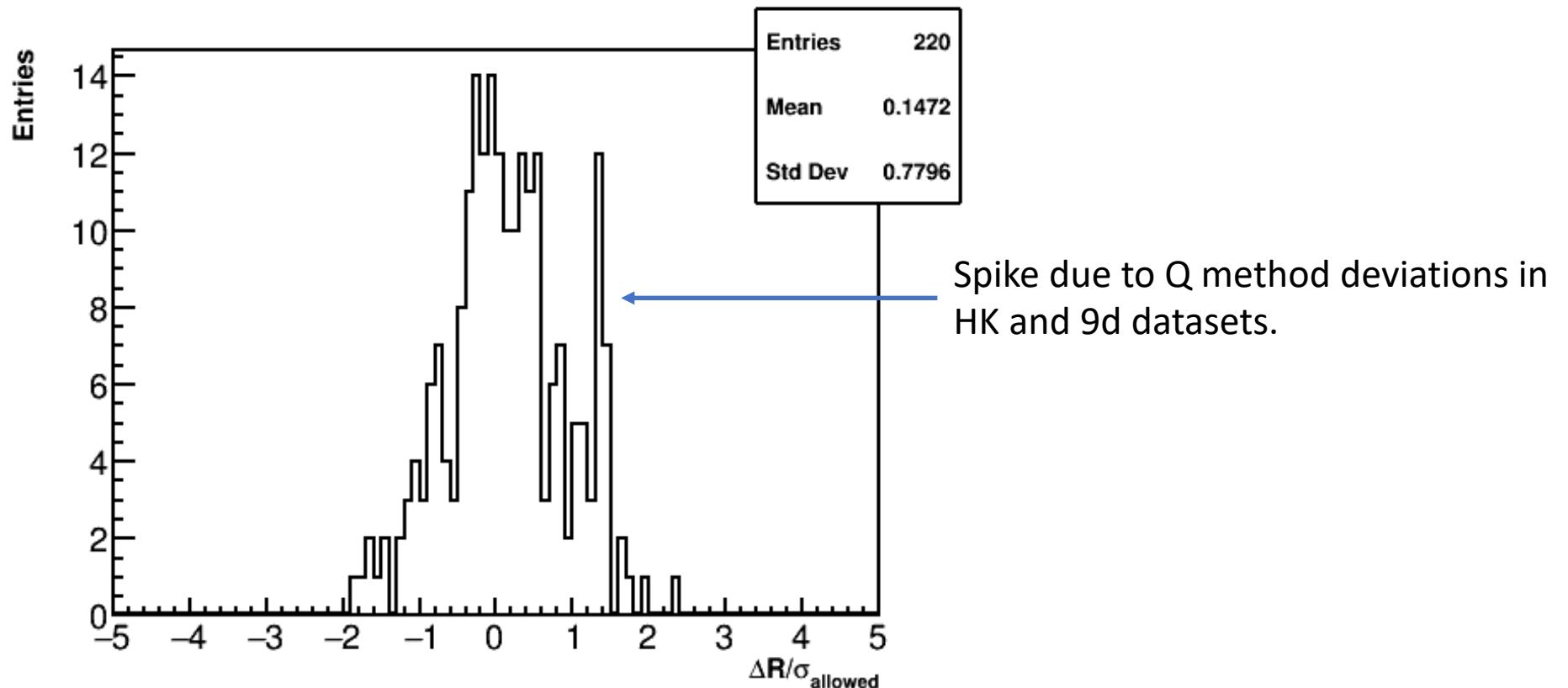
EG Analysis Differences											
	BUT	BUR	CUT	CUA	UWT	UWA	EUT	EU A	SJTUT	SJTUA	UK Q
BUT	0.0000	0.0602	0.2421	0.3325	0.0806	0.3256	0.1461	0.3266	0.0346	0.3226	1.0990
	+0.00	-1.05	-0.05	+0.34	-0.15	+0.10	-1.20	-0.08	+1.04	+0.02	-0.26
BUR	0.0602	0.0000	0.2495	0.3372	0.0781	0.3257	0.1513	0.3282	0.0687	0.3276	1.1029
	+1.05	+0.00	+0.20	+0.52	+0.65	+0.29	-0.74	+0.12	+1.45	+0.22	-0.20
CUT	0.2421	0.2495	0.0000	0.3100	0.2497	0.3221	0.2464	0.3215	0.2409	0.3188	1.0972
	+0.05	-0.20	+0.00	+0.40	+0.00	+0.14	-0.66	-0.04	+0.21	+0.07	-0.25
CUA	0.3325	0.3372	0.3100	0.0000	0.3337	0.1183	0.3270	0.1122	0.3274	0.1012	1.0387
	-0.34	-0.52	-0.40	+0.00	-0.37	-0.67	-0.88	-1.22	-0.23	-1.03	-0.39
UWT	0.0806	0.0781	0.2497	0.3337	0.0000	0.3125	0.1505	0.3230	0.0851	0.3249	1.0960
	+0.15	-0.65	-0.00	+0.37	+0.00	+0.14	-1.08	-0.04	+0.57	+0.06	-0.25
UWA	0.3256	0.3257	0.3221	0.1183	0.3125	0.0000	0.3182	0.0751	0.3204	0.0816	1.0406
	-0.10	-0.29	-0.14	+0.67	-0.14	+0.00	-0.65	-0.77	+0.01	-0.31	-0.31
EUT	0.1461	0.1513	0.2464	0.3270	0.1505	0.3182	0.0000	0.3115	0.1460	0.3193	1.0964
	+1.20	+0.74	+0.66	+0.88	+1.08	+0.65	+0.00	+0.48	+1.45	+0.57	-0.10
EU A	0.3266	0.3282	0.3215	0.1122	0.3230	0.0751	0.3115	0.0000	0.3208	0.0746	1.0437
	+0.08	-0.12	+0.04	+1.22	+0.04	+0.77	-0.48	+0.00	+0.19	+0.44	-0.25
SJTUT	0.0346	0.0687	0.2409	0.3274	0.0851	0.3204	0.1460	0.3208	0.0000	0.3145	1.0977
	-1.04	-1.45	-0.21	+0.23	-0.57	-0.01	-1.45	-0.19	+0.00	-0.09	-0.30
SJTUA	0.3226	0.3276	0.3188	0.1012	0.3249	0.0816	0.3193	0.0746	0.3145	0.0000	1.0411
	-0.02	-0.22	-0.07	+1.03	-0.06	+0.31	-0.57	-0.44	+0.09	+0.00	-0.28
UK Q	1.0990	1.1029	1.0972	1.0387	1.0960	1.0406	1.0964	1.0437	1.0977	1.0411	0.0000
	+0.26	+0.20	+0.25	+0.39	+0.25	+0.31	+0.10	+0.25	+0.30	+0.28	+0.00

1 σ
Allowed Deviation
(ppb)

σ Deviation

Table 16: Differences in results for the EG dataset between the different analyses. The top number is the allowed ppb difference in R , σ_{allowed} , as calculated from the correlation coefficients and analysis errors. The bottom number is the calculated deviation between analyses, calculated as $\sigma_{\text{dev}} = (R_{\text{column}} - R_{\text{row}})/\sigma_{\text{allowed}}$, where R_{column} and R_{row} come from Table 2 for the respective analyses.

- I put all the deviations from the four datasets into a histogram. (From the top side of the diagonal.)
- It's pretty reasonable all things considered. (Note that it shouldn't be a perfect Gaussian since the table entries are correlated.)

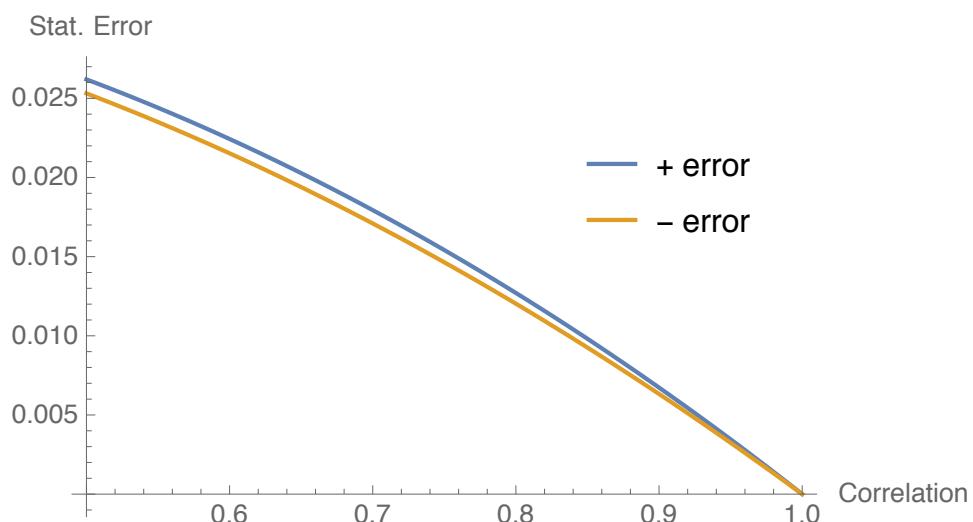


Conclusions

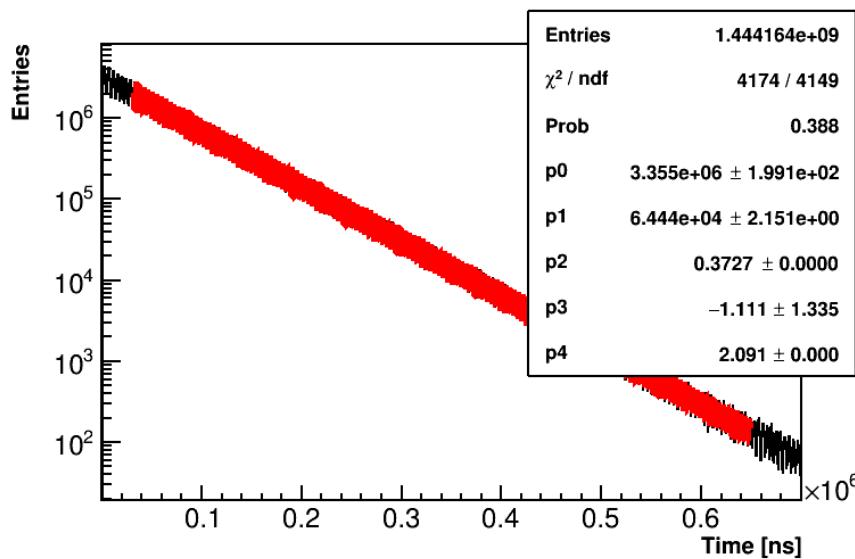
- I provided all correlation coefficients and different versions of them to David so that he could run them through his combination machinery, and compare the differences
- Going forward there are a few different ways the Monte Carlo could be improved for future combinations (if necessary and desired), listed in the note
- My note can serve as a starting point for such work
- Feel free to give the note a read and send any questions my way if you'd like

Backup

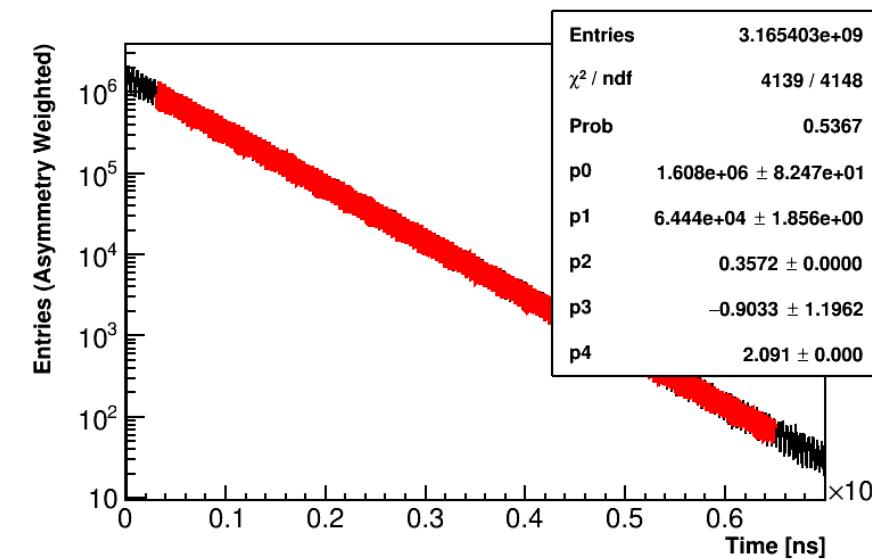
Statistical Error



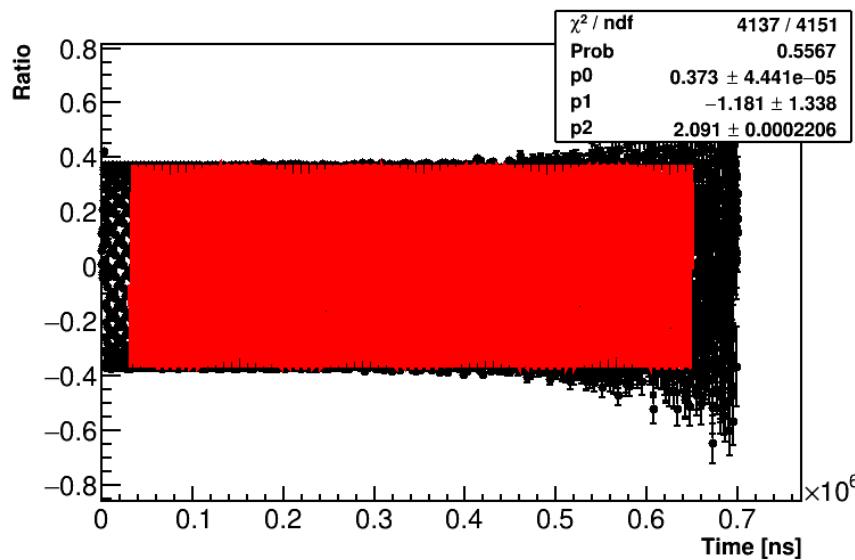
T



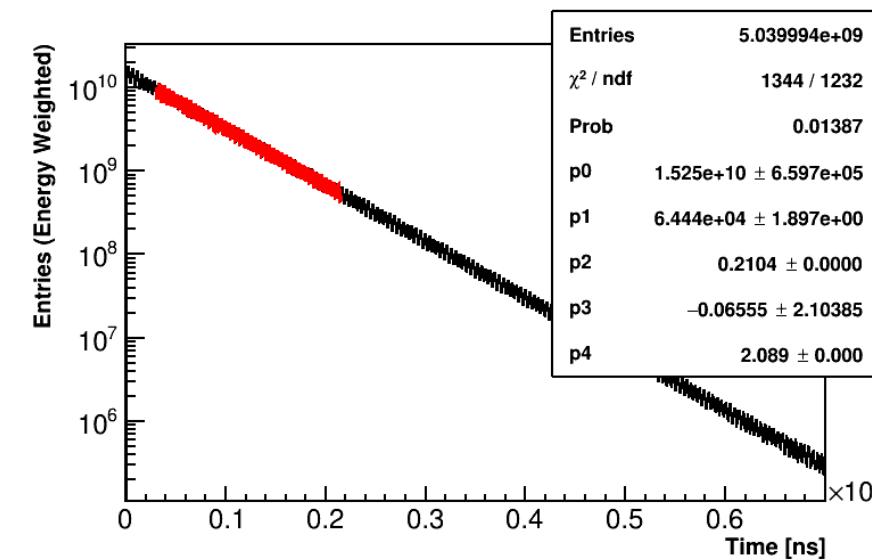
A

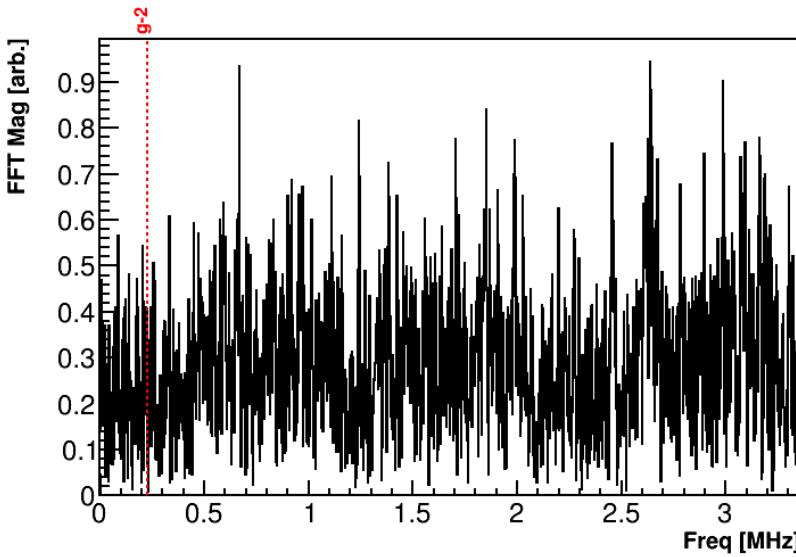
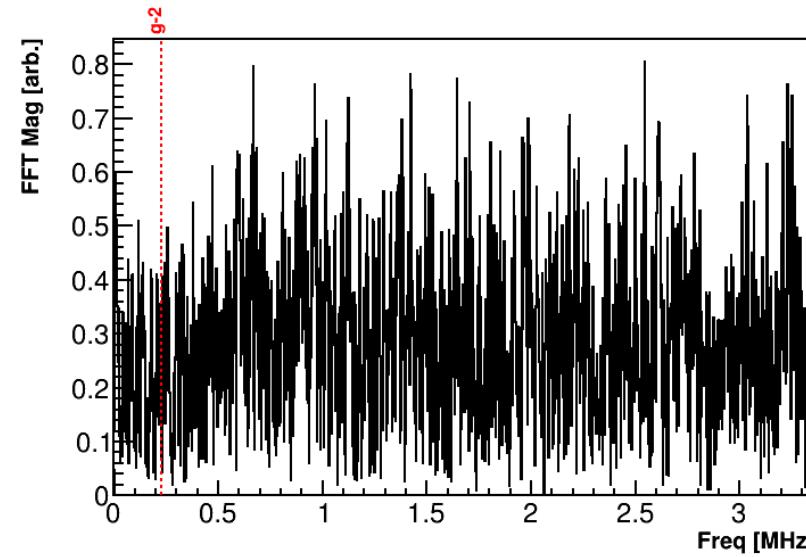
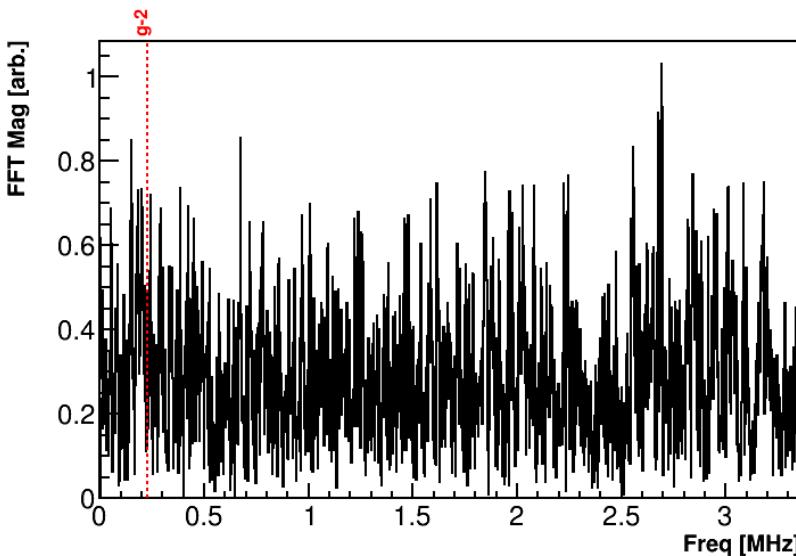
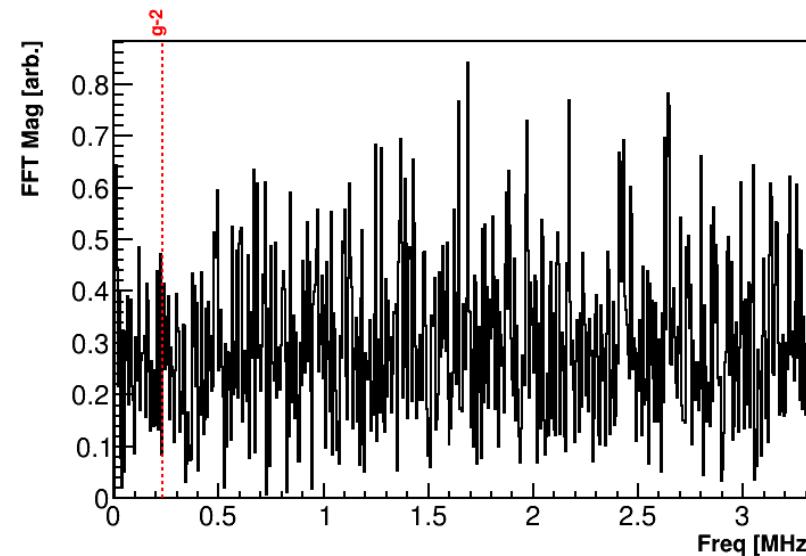


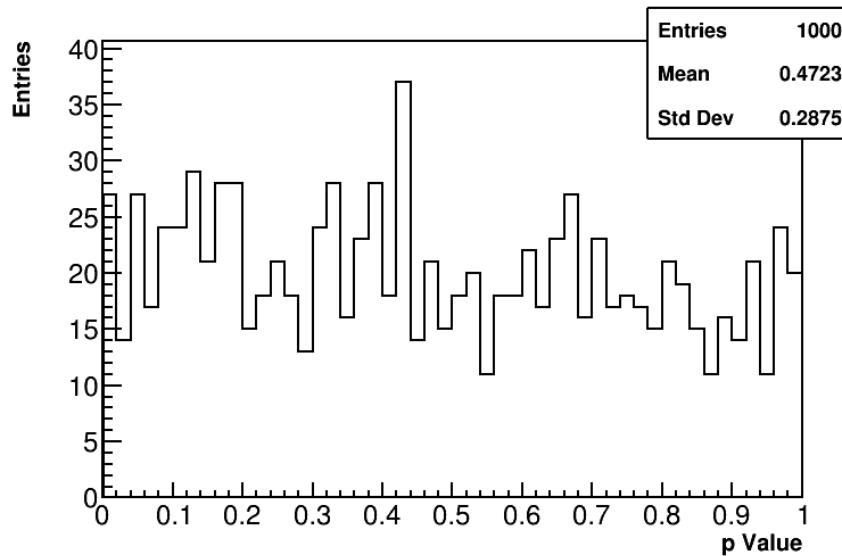
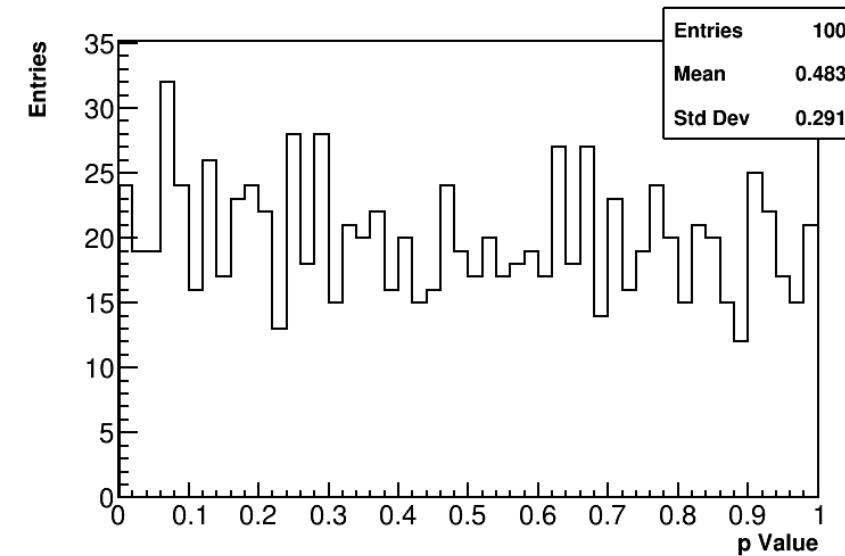
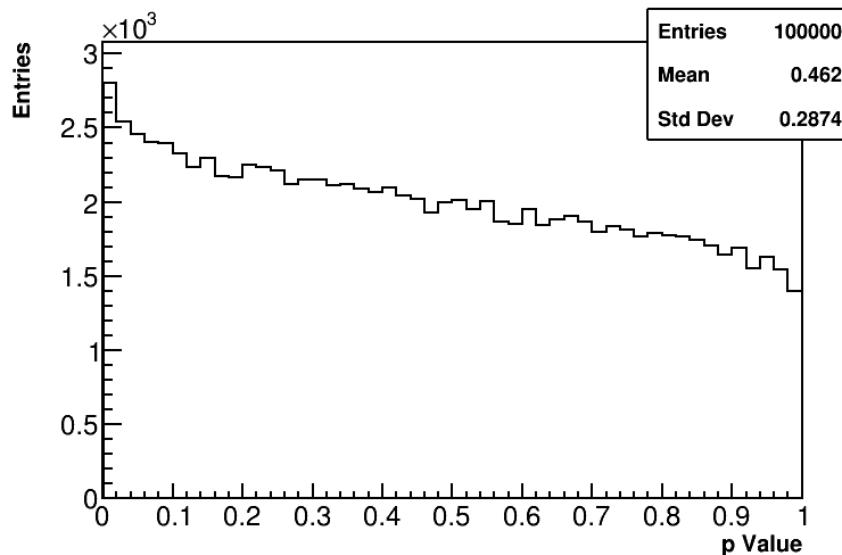
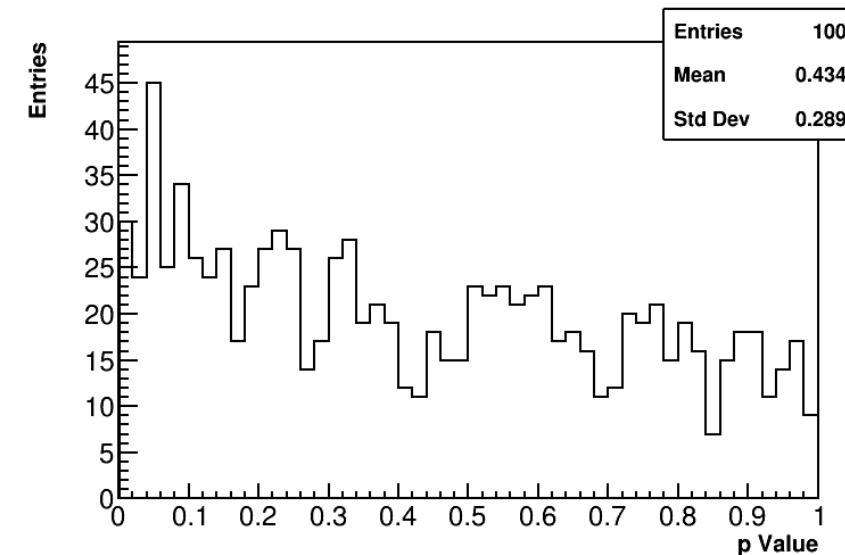
R

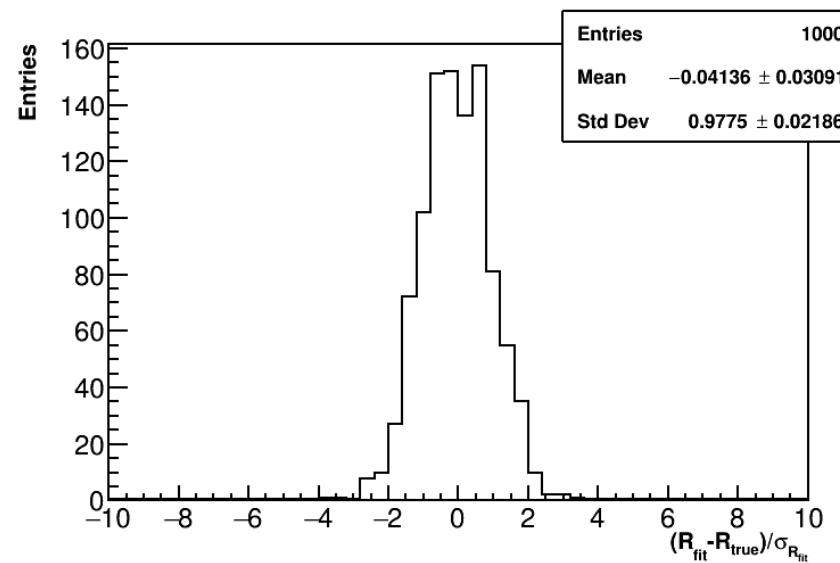
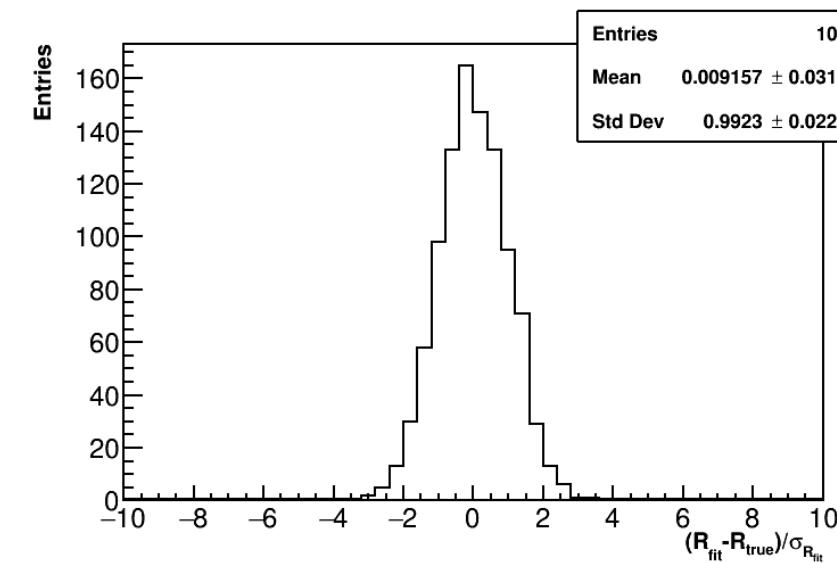
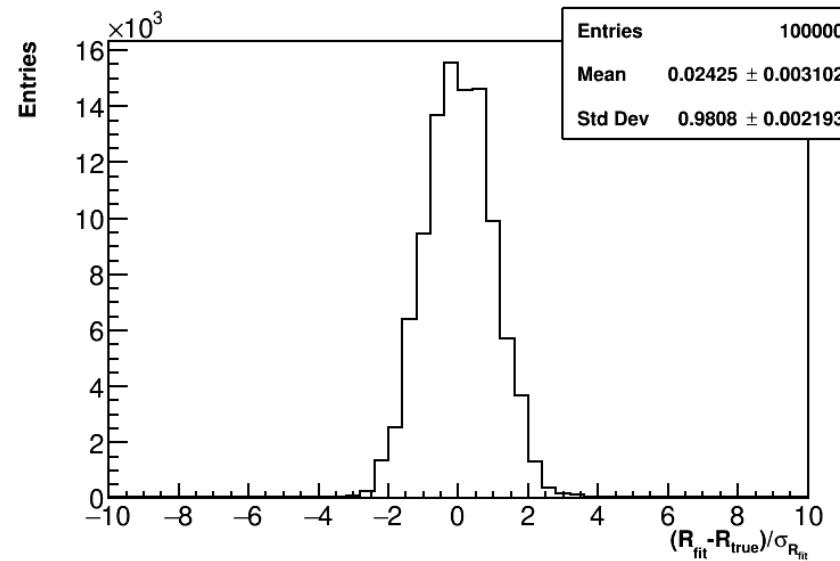


Q



T**A****R****Q**

T**A****R****Q**

T**A****R****Q**