

# The Vertical Waist in the Ratio Method Analysis

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## Abstract

This note details the handling of the vertical waist (VW) effect in the Ratio Method analysis. The 60h and Endgame datasets lie on resonances where  $\omega_{VW} \approx 10 \cdot \omega_a$ , which in combination with the fast rotation (FR) effect leads to inflated VW amplitudes in the Ratio Method, where one might originally assume that the VW effect can be neglected from the analysis. In the 9d dataset (not on a resonance) it was found that the Ratio Method flattened out the VW amplitudes as a function of calorimeter, leading to greater cancellation and a systematically lower calorimeter sum VW amplitude as compared to the T-Method. The solution used to eliminate these problems was to randomize out the VW (in tandem with the FR) in the data so that the effect can be acceptably ommitted from the fit.

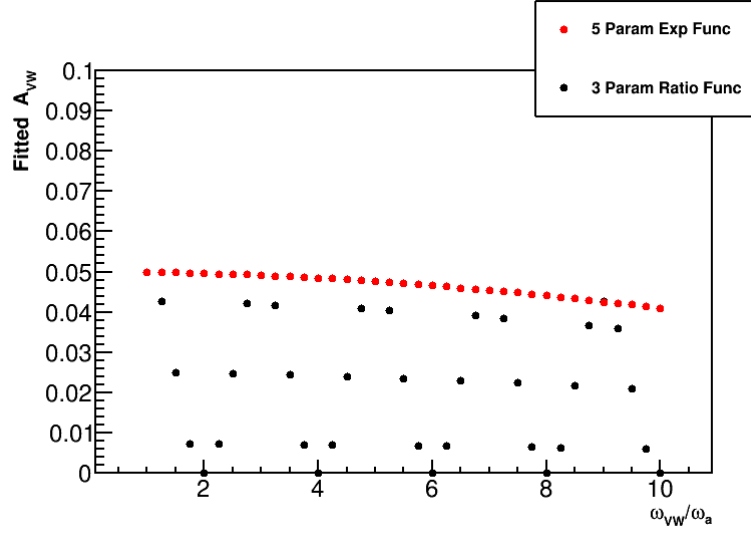
## 1 Introduction

In the Ratio Method, effects with frequencies which are an even multiple of  $\omega_a$  divide out completely, as shown in Figure 1. The VW frequency is given by

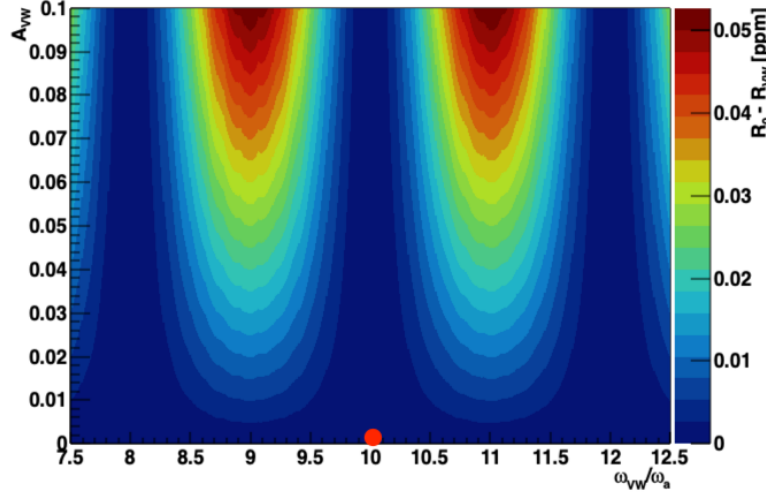
$$f_{VW} = f_c - 2 \cdot f_y, \quad (1)$$

$$= f_c - 2 \cdot \kappa_{VW} \cdot f_{cbo} \sqrt{2f_c / (\kappa_{VW} \cdot f_{cbo}) - 1}, \quad (2)$$

where  $f_c$  is the cyclotron frequency,  $f_y$  is the vertical betatron frequency, and  $f_{cbo}$  is the CBO frequency. The time-dependence of the VW effect is excluded and the fit parameter is  $\kappa_{VW}$ , a percent level adjustment factor to the theoretical frequency as found by both the tracking and  $\omega_a$  analysis [1]. Table 1 gives the VW frequencies for the Run 1 datasets. As shown the VW frequency for both the 60h and Endgame datasets are very close to an even multiple of  $\omega_a$ . This implies that the VW effect should be divided out in the data, unobservable, and therefore unnecessary in the fit function. However, when looking at the FFT of ratio fit residuals in Figure 2, a VW peak can be seen at early times in the 60h dataset. Figure 3 shows a fit start scan for the p value of the fit for the Endgame dataset, where the p value rises rapidly at early times. These two pieces of evidence point to the necessary inclusion of the VW in the ratio fit, even for even multiple frequency datasets.

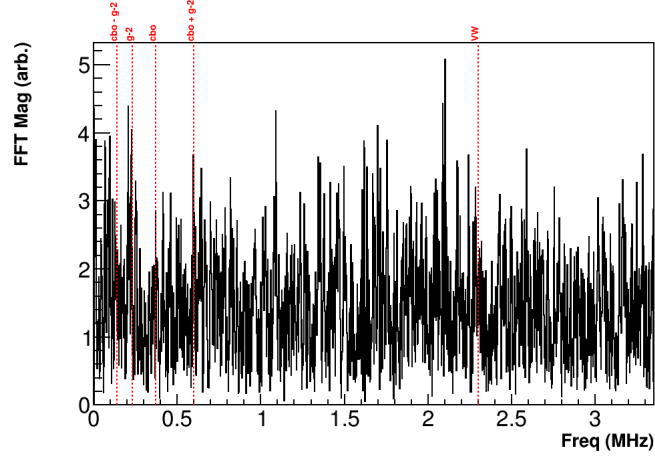


(a) Fitted VW amplitude with a five parameter function in red and a three parameter ratio function in black. The input amplitude was 0.05. The slight fall off of the red points is due to the high frequencies interacting with the bin widths; performing an integral fit removes this trend.

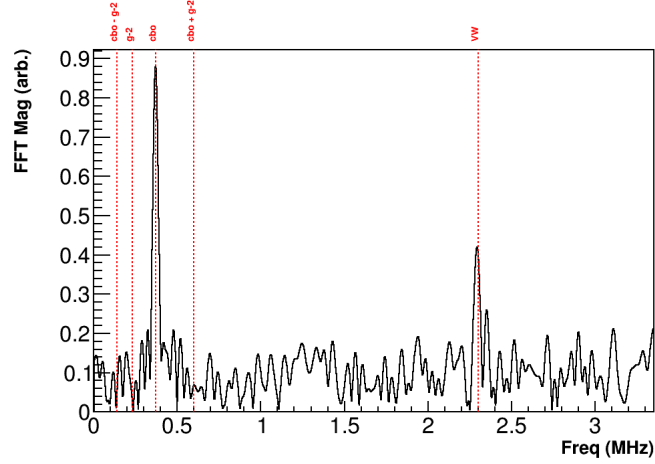


(b) The difference in the maximum value of the ratio (not the fit parameter  $\mathbf{R}$ ), as a function of VW frequency and amplitude. The difference in the ratio is approximately  $5 \times 10^{-6}$  at  $30 \mu\text{s}$ .

Figure 1: Two separate Toy MC simulations showing the division of a VW effect as a function of it's frequency in units of  $\omega_a$ . For even multiple frequencies the effect dies away while for odd multiples it is preserved.



(a) All times, 30.2  $\mu$ s to 650  $\mu$ s.



(b) Early times, 30.2  $\mu$ s to 60.2  $\mu$ s.

Figure 2: FFT of fit residuals using a 3 parameter ratio function to fit the 60h dataset. Peaks can be seen when looking at fit residuals over early times as opposed to all times. The size of the VW peak greatly depends on the choice of random seed. The CBO peak can also be seen.

Run 1 Dataset VW Frequencies			
Name	$n$ Value	$f_{VW}$ (MHz)	Multiple of $\omega_a$
60h	0.108	2.3	10.04
9d	0.120	2.04	8.87
Endgame	0.108	2.3	10.04

Table 1:  $n$  values and VW frequencies for the three Run 1 datasets analyzed for this note. (The Highkick dataset has the same parameters as the 9d.) While technically the VW frequency is changing, the frequencies given here are those determined from the peak of the VW peak in the FFT of the residuals from a five parameter fit to the data, so the numbers are close enough.

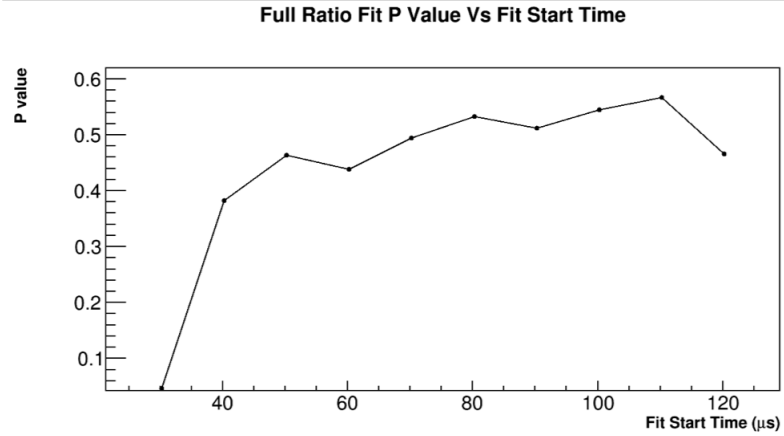
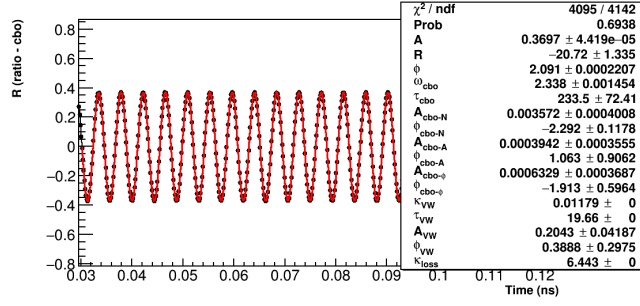
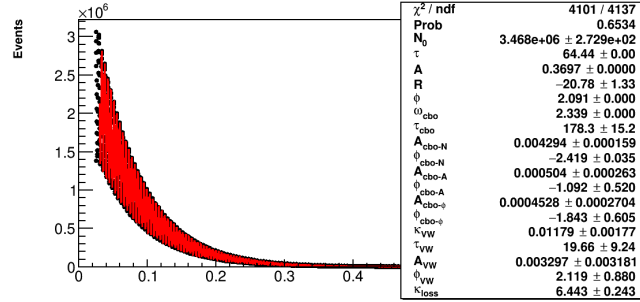


Figure 3: P value vs fit start time for the Endgame dataset. A sharp rise can be seen at early times. A similar trend, though less severe, can be seen for the 60h dataset.

## 2 On a resonance in the 60h and Endgame datasets



(a) Ratio fit results with VW included of 60h dataset.



(b) T method fit results of 60h dataset.

Figure 4

My plan would then be to include the VW in the ratio fit, with those VW parameters to which the ratio fit can't fit effectively, the frequency, lifetime, and sometimes phase, fixed from a T method fit, and the amplitude allowed to float. What then happens strangely enough however is that the VW amplitude in the ratio fit floats to unphysically high values, as seen in Figure 4a where the amplitude is 0.2043 compared to the T method VW fitted amplitude in Figure 4b at 0.003297, a factor of 62 increase. This factor difference varies wildly from seed to seed, but always remains very high. (Here it can be seen that the VW phase is floated in the fits and varies between the two, probably influencing the final fitted values.) If I include the VW and fix all parameters to the results from the T method fit, then what happens is R changes by nothing (less than 1 ppb) and the VW peak is unchanged. It seems the fit really does prefer such a high amplitude, which I still don't understand.

One thing I considered was whether the ratio method and fit really does somehow artificially raise the VW amplitude. Seems very unlikely but I made a Toy MC with a large VW effect and checked what came out, the first results of which are shown in Figure ???. That figure used the results from a 3 parameter ratio function, or

$$R(t) \approx A \cos(\omega_a t), \quad (3)$$

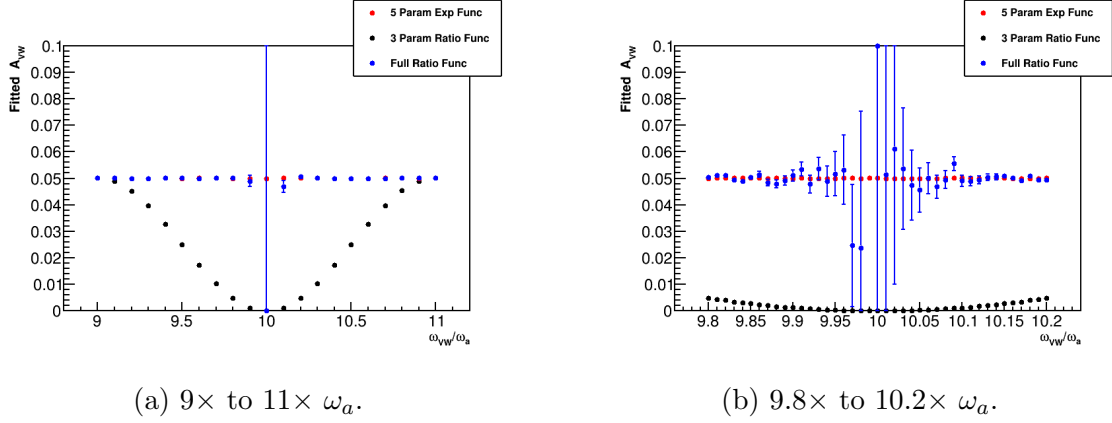


Figure 5: Fitted VW amplitude as a function of frequency and with different types of fit functions.

whereas I'm really fitting with the function

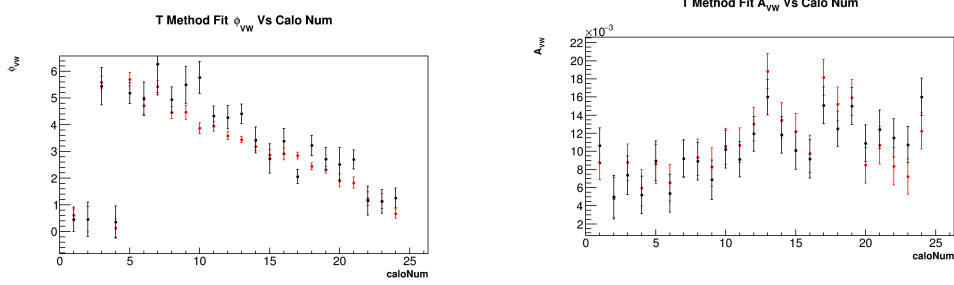
$$R(t) = \frac{2f(t) - f_+(t) - f_-(t)}{2f(t) + f_+(t) + f_-(t)}, \quad (4)$$

$$f_{\pm}(t) = f(t \pm T_a/2), \quad (5)$$

$$f(t) = V(t) \cdot (1 + A \cdot \cos(\omega_a t + \phi)), \quad (6)$$

or as I call it the full ratio function, with  $V(t)$  being the VW effect, and without other background terms which are unnecessary in the Toy MC. Fitting with this function yields the results shown in Figure 5. As shown, the fitted VW amplitude dies away at even frequencies for the 3 parameter ratio function, however there is some interesting behaviour with the results from the full ratio function. The results do in fact make perfect sense when considering Equation 6 more carefully. What  $f(t)$  represents in that function is the five parameter function, barring the  $N_0$  and  $\tau_\mu$  terms which divide out. In that case the fitted amplitude between the five parameter function and the T method function should be the exact same. This is the case between the red and blue points until the frequency gets very close to 10 times  $\omega_a$ . As seen the fit becomes unstable, with largely differing amplitudes in some cases with large error bars. There are also some points with values different from the real value, and with small, inconsistent errors. What is happening here makes sense in that the VW effect in the ratio has almost completely divided out, but yet I am trying to fit it with a function where the value of  $A_{VW}$  does not particularly matter. In essence I believe I am using a term in the fit function which isn't particularly well motivated.

If my fit results to the real data returned values with large error bars consistent with 0, or consistent with the T method results, then I believe there wouldn't be a problem, and I could do one or the other. Conceptually, I think the 'right' answer would be that the T method values are right, and I should just fix the values in the ratio method to them and call it a day. However, going back up to a point in an earlier paragraph, this doesn't improve the fit and the VW signal still appears



(a) VW phases per calo in the 9d dataset. (b) VW amplitudes per calo in the 9d dataset.

Figure 6: Red points are T method results and black points are ratio method results. In the ratio fits, the VW frequencies and lifetimes are fixed to those from the T method results.

in the data unchanged. So at the end of the day I have this problem, which seems contradictory in that I don't think I should be able to see the VW but can, and that my fit shouldn't prefer a specific amplitude but does, and that amplitude is an unphysically large value.

I was going to leave it out in the 60h dataset and call it a day, until the Endgame results showed a much larger change in the p value that seems un-ignorable.

Probably show said change or picture here or something - look up these results again

### 3 9d Dataset - a different problem

While in the 60h and Endgame datasets the VW frequency is nearly 10 times  $\omega_a$ , in the 9d the frequency is closer to an odd multiple, or  $\omega_{VW} \approx 8.87 \cdot \omega_a$  at  $t = \infty$ . With what I've shown so far, I expect the VW signal to be easily fittable with an amplitude equal to that from the T method results. Looking at per calo fit results in Figure 6, the VW phases and amplitudes are consistent between the T method and ratio method results. As shown there are no observable differences. When I look at the fits to the sum of the calorimeter data however, I see a systematically smaller VW amplitude in the ratio fit results as compared to the T method results. This is true for all random seeds as shown in Figure 7. While I expect a reduction in the VW amplitude from per calo fits to the calorimeter sum fit, as the phases go from  $0-2\pi$  around the ring, I still expect the amplitudes to be consistent between the T method and ratio method. If I take calorimeters adjacent to each other around the ring versus calorimeters separated from each other, then I see the behaviour. So what seems to be happening is that there is a cancellation when adding the calorimeters together in the ratio method that doesn't occur in the T method. That is something I don't understand, and can't come up with a reason why that should happen. I'm not sure if this is at all related to the previously discussed problem or not, whether it's another symptom or something else strange.



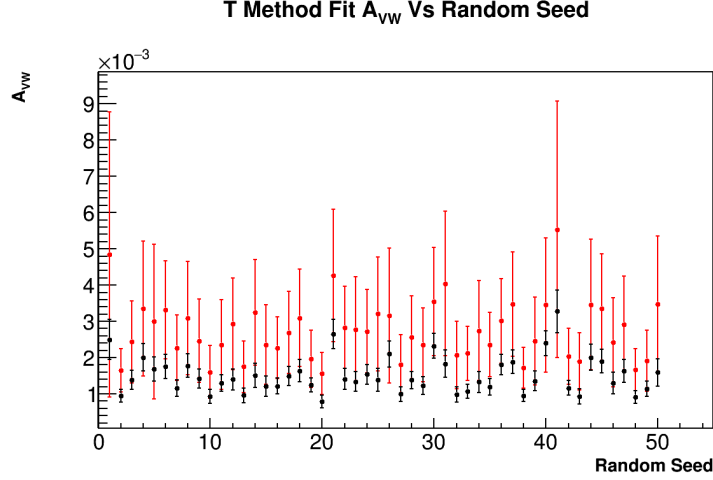


Figure 7: VW amplitudes as a function of random seed for T method fits compared to ratio method fits. T method fits are in red and ratio fits are in black. The error bars are smaller on the ratio fits because the VW frequencies and lifetimes are fixed to those from the T method fits. There is a consistently smaller VW amplitude in the ratio fits as compared to the T method fits.

## 4 Correlations and fit start scans

## 5 Randomizing out the effect...

[2]

## References

- [1] J. Mott et al. *CBO Frequency Change*. Muon g-2 DocDB 14208. 2018.
- [2] N. Kinnaid. *Vertical Waist and Fast Rotation in the Ratio Method*. Muon g-2 DocDB 19210. 2019.