Optimizing Packaging with Origami

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Package cushioning is a major factor in the final condition of a good in the transport from manufacturer to consumer. This void filling material occupies the space between the sorted good and regular boxing absorbs the unwanted shocks applied in the transportation of goods, which otherwise would cause undue permanent damage during handling. This poster overviews the process of converting an arbitrary shape into a crease pattern which fills the negative space between the bounding volume and a polyhedra. This method is useful for minimizing cushioning inside of shipping container using a material which is recyclable and already in wide use.

The purpose of this project is that there can be a generic method for filling the void between a consumer product and arbitrary packaging container. Given a fixed product shape which must fit inside of a known but arbitrary container shape, a single method can be applied such that single (and appropriately large) sheet of paper can completely fill the space between product and container. This benefits

Representing a Folded Object

This process assumes that a closed volume in R3 is already determined: either the model already exists and dimensions are found using contact, stereoscopic, photometric methods or the model exists as a .obj from CAD modeling. In either case, a collection of data points in a point cloud is an approximate representation of the physical object: each point a vertex on a polyhedra. With the .obj file, a geometric model for origami is sent to Tachi's Origamizer to be used as a reference model.

Geometric Model of Origami

A vector representation is developed from a loosening of the restriction of a flat paper's inherent zero Gaussian area, which cannot smoothly be deformed in to a concave or convex shape without deformation. This loosening allows for deformation to occur at the creases such that any complex doubly curved surface can be approximated using a collection of polyhedra. The example of a torus is shown using Origamizer.

$$\overbrace{G_k = 2\pi - \sum_i \theta_{k,i} = 0}^{\text{Maekawa's theorem}} \qquad \overbrace{\sum_i \theta_{k,i} - \sum_i \theta^0_{k,i} = 0, \quad \sum_i \ell_{j,i} - \sum_i \ell^0_{j,i} = 0}^{\text{Boundary Conditions}}$$

$$F_k = \sum_i \sigma_{k,i}\theta_{k,i} = 0 \xrightarrow{\text{as vector eqn.}} \left[\frac{\partial \mathbf{F}}{\partial \mathbf{x}}\right] d\mathbf{x} = \mathbf{0}, d\mathbf{x} = \left(\mathbf{I} - \left[\frac{\partial \mathbf{F}}{\partial \mathbf{x}}\right]^+ \left[\frac{\partial \mathbf{F}}{\partial \mathbf{x}}\right]\right) d\mathbf{x}_0$$

The creation of a folding die is dependent on the volume of the desired shape as the desired form for a completed crease pattern does not necessarily enclose a nonzero volume but will span R3 because a sheet of paper exists in the same R3 space but has nontrivial dimensions in only 2. A method for determining a valid crease pattern is taken from Maekawa's theorem, where a major consequence is of having flat sheet of paper with 0 gaussian area is that at the intersection of creases, the number of mountain and valley creases will always differ by two

Process, Analysis, and Results

This project is not meant for direct application to for manufacturing processes. The process presented does not constitute a subsitute for formal package design by a specialized packaging engineer, who would account for the final product's optimal production speed and packaging technology by way of finite element analysis (FEM), structural analysis techniques (SAT), computer assisted design (CAD) and thermal analysis.

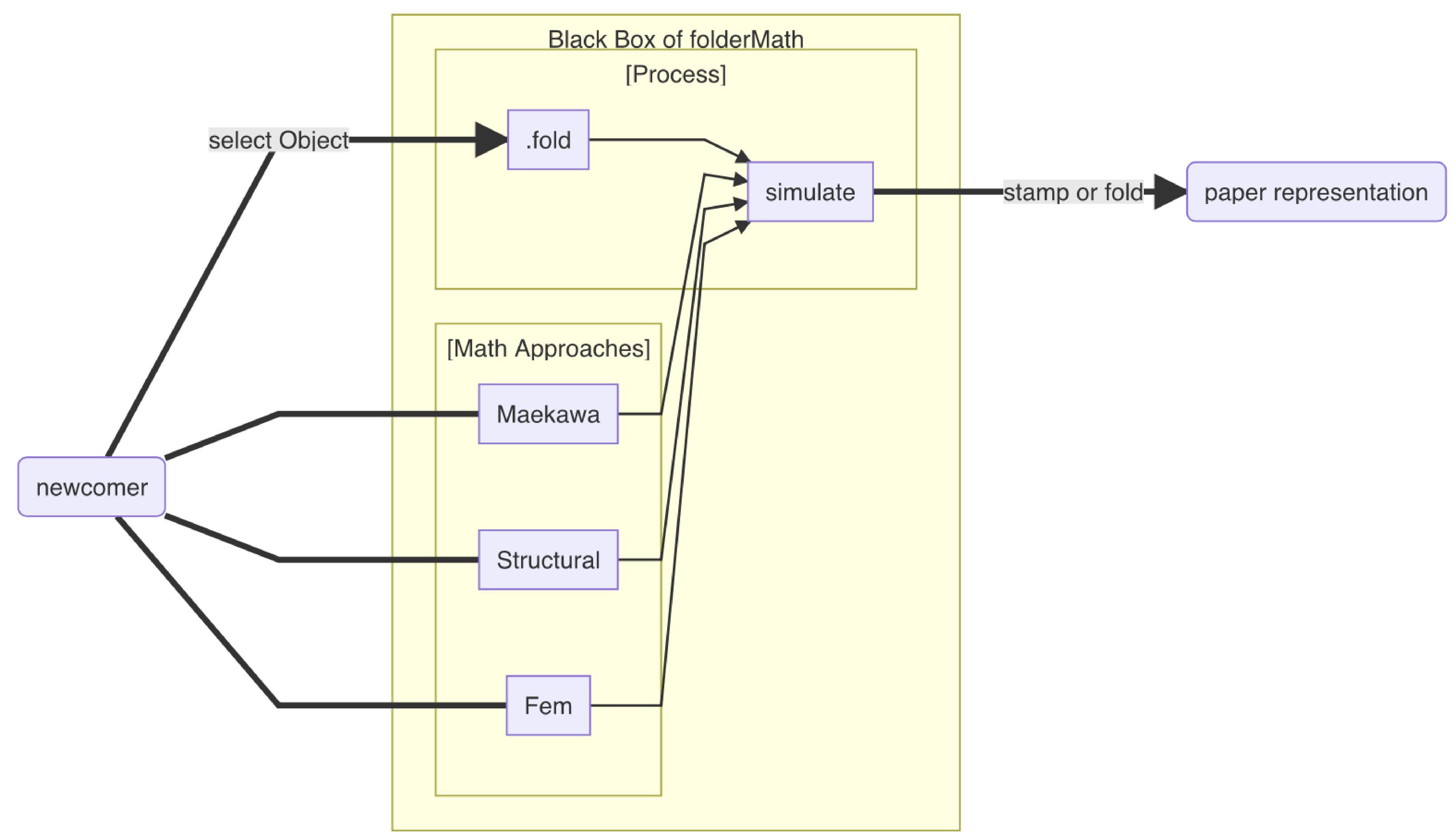

Fig. 1: Navigation through math framework to develop a process for package shipping.

Inspirations for this projects come from all fields of educational study. One which was not mentioned is the process of hydrodipping, or water transfer printing, where an unpainted consumer product recieves a graphic finish submersion into water through a prepared painted acrylic film applied on the surface of water, such that film adheres to the prepared product surface. The major difference being that though the film, like paper, conforms to the shape of the consumer product, does so continuously whereas paper, due to having Gaussian area of 0, must do so discretely. Water, being fluid, by definition cannot support a shear stress whereas a 3D printed die, such as pictured, is solid and remains rigid under loading.

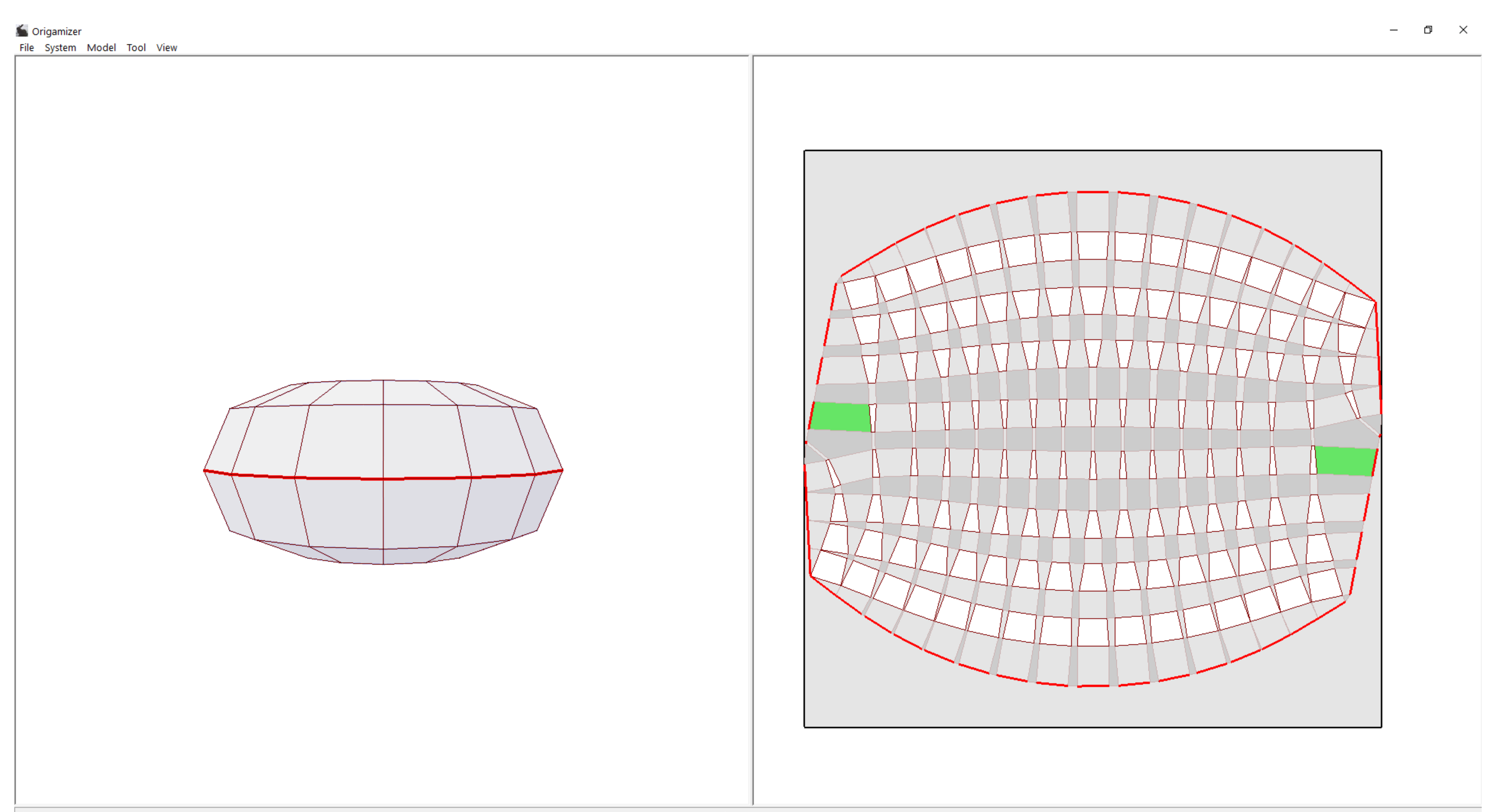

Fig. 2: A doubly curved surface with a single hole.

Comparison

Once a crease pattern is applied to a sheet of paper, the local region of the crease undergoes significant plastic deformation, which is permanent and irreversible. The angle of fold need not be steep so long as the deformation is localized to a small area around the infinitesimally thin creaseline. This is leveraged when the crease pattern is created into a solid die such that the process of folding is made repeatable.

Figured below are three images of the die generated, printed, and a possible application in art. Using a fold file which was simulated using Origami Simulator by Amanda, a crease pattern is exported in a partially folded state in .obj format which is readily accepted by the open source software Blender.

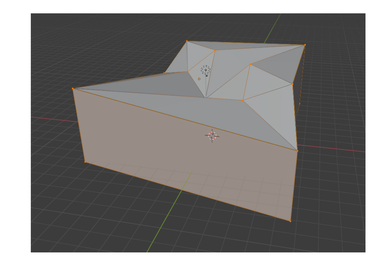

Fig. 3: Object extruded in blender

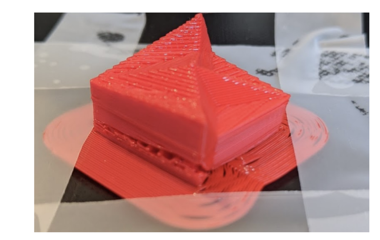

Fig. 4: 3D printed die for rapid fabrication.

Fig. 5: Possible application to folding cranes.

Remarks

As this project is not meant as a practical implementation for mass application in consumer packaging. Rather it is more akin to an art project resulting from the first global pandemic of COVID-19 and the resulting unprecedented loss of life which is inherently a commentary on the human condition.

Akin to how an infinitesimally small particle at the interface of two moving fluids, trapped in surface waves between a buoyant fluid and a depressive one is exactly analogous to a lone sailor adrift at sea, the Navier-Stokes equations describes water just as readily as it does air. The Euler-Lagrange equations describing the path traced and Noether's Theorem informing intuition, indeed from the Earth's surface to the ocean floor, world full of solid and fluids, is math all the way down.

Since air is just as fluid as water, a hydrofoil is geometrically similar to an aerofoil. The only separation between the two being which fluid they are designed for. But from a more optimistic perspective, the world is math all the way up as well. The same forces which drown are same ones which enable one to swim and to fly. Learning to see this is a gift in its own right because recognition of where one is the first step necessary in learning how to move and how to move on.

References

The QR code in the upper righthander corner will direct to the local paper hosted on gitbook which holds a complete list of resources. Those mentioned in this poster are Origamizer by Prof. Tomohiro Tachi, Origami Simulator by Amanda Gahessi, and Robert Lang's Mathematical Methods for Origami Design.