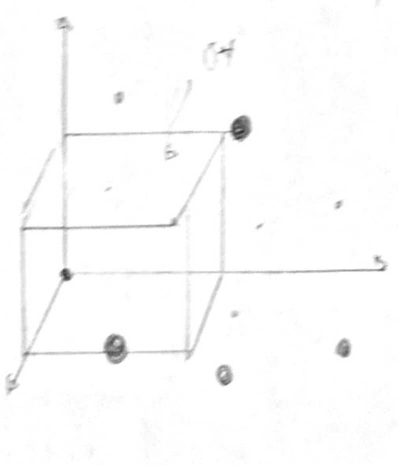
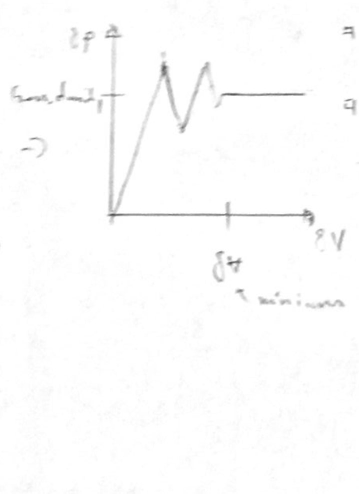
Fluid Mechanics 2019-08-29

- Fluid is a liquid-gaseous substance with medium-to-weak intermolecular forces. This class looks at ... inherently tedious. Am I gonna not enjoy it? Hopefully, I will. But I fucking doubt it, lol.
- Holy shit, he talks so slowly. Might be an issue, but manageable.
- Should I record the lectures as a sleep aid? Notes provided after class.
- Unlike the undergraduate analogue of this course, will investigate limitations of solutions.
- Exams cumulative. Homeworks are biweekly. Project assigned after Exam #1. Lit. search of active fluids research. (~5 papers) with experimental & simulation papers. Annotated papers + analysis of knowledge gaps. Recent papers (~10 years) and get an idea of how the state of the fluid area is, at the moment.
- Mid semester progress report given after proj #1.

Preliminaries: A topic of classical physics.

- Unlike linear systems, for chaotic fluid systems, approximate input does not lead to an approximate output. Inherent to nonlinear systems.
- Fluid: gas or liquid w/ weak inter-molecular forces at play and a significant random thermal movement for disorder/randomness to system. Deforms continuously when a shear stress.
  - Continuum Hypothesis: no structure, no matter the length scale.
  - λ ... mean free path b/w particles. Should be a constant for the fluid continuum.



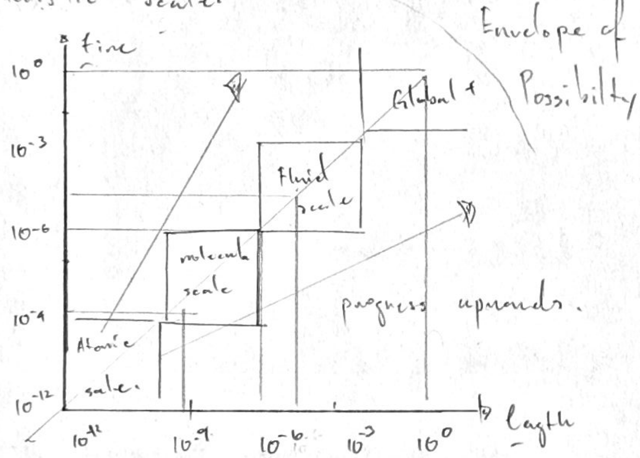


⇒ $\lambda = \sqrt[3]{\delta V}$ ... mean free path.

⇒ Scale: 10^{-7} m for (STP) air, 10^{-10} water

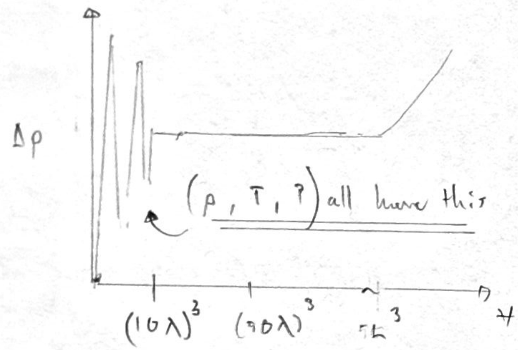
note for at edge of atmosphere $\lambda \to \sim 1$ m. which is pretty significant.

- Knudsen Number: Characterizes fluid continuum hypothesis behavior. Let "L" be the characteristic length, so that "$Kn = \frac{\lambda}{L} \ll 1$" for appropriate use. Like Biot Number for characteristic heat transfer.
- From previous, $\delta V \in (\lambda, L)$ so that the fluid element is much greater than mean free path but less than the characteristic length of volume. So continuum hypothesis also applies to the differential element.
- Similar to length, we also have a characteristic time scale $\tau \sim 10^{-8}$ s where T is the characteristic time of interest so $(\tau \ll T)$.
- Characteristic Scales



Fluid Mechanics 2014-09-03

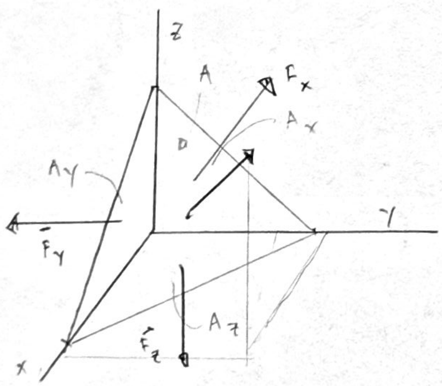
- Fluid Mech is a branch of continuum mechanics, with no structure due to thermal motion.
- Differential Fluid Element is between the scale of λ ... mean free path between molecules
- Molecular Scale deals with intermolecular, stochastic dynamics. So far boring review of Friday
- Same Relationship b/w atomic and molecular scale as molecular to fluid scale.
- Density is related to continuum hypothesis and is undefined below the mean free path, λ the differential element Δm is lost below $\sim V < \lambda^3$ for safety add an addition order
- Threshold : $\underline{(10\lambda)^3}$ for volume of differential element
- Density Plot, pictured last week



- For length scales, $\Delta\rho$ is relatively constant.
- $x \in [0, (10\lambda)^3)]$ ... undefined for mostly empty space
- $x \in [(10\lambda)^3, (50\lambda)^3]$ ... where density in element, const
- Specific Volume : volume for unit mass $v = 1/\rho$
- Geometric Specification → Newtonian

  $\cos\theta_i = \frac{A_i}{A} \to \frac{F_i}{A_i} = \frac{F}{A}$; no summation

- Pressure of Differential Element



- For hydrostatic, use Newton's law on unit tetrahedron
- Normalization by area being acted upon produces $\underline{\text{constant pressure}}$ $P = \lim_{\Delta V_i \to (10\lambda)^3} \frac{F_n}{A_n}$
- Hydrostatic pressure of non moving fluid w/ effective pressure due to dynamics. Will always return to hydrostatic.

Viscosity, usually neglected for (water, steam, air, etc.)

- Moving Fluid : $P_n = \lim_{\Delta V \to (10\lambda)^3} F_n/A_n = p + O(\mu) = \tau_n$ is the normal stress.
- Molecules changing elements is the demanded change in momentum.
- Temperature of Fluid Element from Energy :

$$\bar{E}_{kinetic} = \underbrace{\frac{1}{N}}_{\text{Per Capita}} \underbrace{\sum_N \frac{1}{2} m_N V_N^2}_{\text{Total Energy}} = \frac{3}{2} k \bar{T}^2 \to \bar{T} \text{ average temperature of average energy.}$$

↑ Boltzmann Constant = 1.38×10^{-26} J/K

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- Other Properties, Specific by unit mass
  - Internal Energy ... $u = \frac{\Delta U}{\Delta m}$; $du = C_v\, dT - \left[T \frac{\partial P}{\partial T}\Big|_{\text{Const. }\rho} - P \right] \frac{d\rho}{\rho^2}$
  - Enthalpy ... $h = u + \frac{P}{\rho}$; $dh = du + d(P/\rho)$
  - Entropy ... $T\,dS = c_v\, dT - \left(\frac{\partial P}{\partial T}\right)\Big|_{\text{Const. }\rho} \frac{d\rho}{\rho^2} = dh - \frac{dp}{\rho}$, Gibbs Eqn.
- Simple Compressible Fluid: 3 properties as a surface. so that $P = f(\rho, T)$ is the equation of state for pressure. Density and temperature are just usually the easiest to find.
- Triple Point is triple...$(\rho, P, T)_{\text{crit}}$ → $\boxed{\text{Anchors the Surface}}$
  - $\frac{\partial}{\partial \rho} P = 0$, $\frac{\partial^2}{\partial \rho^2} P = 0$ so first and second derivative
  - Liquid: $\frac{\partial P}{\partial \rho}\Big|_{T_c} \ggg 1$ very incompressible
  - Vapor: $\frac{\partial P}{\partial \rho}\Big|_{T_c} \gg 1$ kinda compressible

Fluid Mechanics 20[illegible]-04-09

- Class begins with continuum hypothesis & fluid mech. Interdependence b/w variables → $\{P, v, T\}$
- Triplet of $\{P, v, T\}_c$ are the critical triad "anchors" where 1^{st} & 2^{nd} derivative is 0
- Reduction → $(\cdot)_R = \frac{(\cdot)_R}{(\cdot)_c}$, $(\cdot) = \{P, \rho, T, v\}$, R is universal, $R = \bar{R}/\text{Mol. Wgt}$
- Z is the compressibility, $Z = \frac{P}{\rho R T} = Z(P_R, T_R)$, $Z(\lim_{P \to 0} P_R, T_R) = 1$
- Air critical is 38 atmospheres

$\frac{\partial P}{\partial \rho}$ $\frac{\partial}{\partial \rho}$

- Cases of Compressibility Analysis.
  - Case 1: Idealized for $Z = 1$ & $P = \rho R T$
  - Case 2: Van der Waals Eqn of State: $P = \frac{\rho R T}{1 - b\rho} - a\rho^2$, a & b: [illegible], b is low density reflection
  - Case 3: Redlich-Kwong and other eqns & is volume-size for high.
- Scalar Vectors should follow the above formats
  - Scalar: no preferred direction: ρ, P, T, m, h, s
  - Vector: directional preferred direction, ex $\vec{v} = |\vec{v}|\hat{e}_v = v_x \vec{e}_x + v_y \vec{e}_y + v_z \vec{e}_z = v_i \vec{e}_i$
  - Tensor: $\underline{\underline{T}}_{ij} = a_i b_j$ by dyadic of two vector, with $\underline{\underline{(\cdot)}}$ to represent 2^{nd} order
  - LHS & RHS ex. $\underline{e}_x \cdot (\underline{e}_x, \underline{e}_y)$ #
  - Del is not a vector, even if its called so here. This is because it does not obey transform rules. $\nabla = \frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y + \frac{\partial}{\partial z}\vec{e}_z$ & produces $\nabla(\cdot)$, $\nabla \cdot (\cdot)$, $\nabla \times (\cdot)$, curl or (rotor?)
- When $(\cdot) = \vec{V}$, $\nabla \times \vec{V} = \vec{\omega}$

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Fluid Mechanics 2019-09-13

- "Is this done" tally: ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| | ||||| ||||| ||||| || (10) + back

- Tensors, as well as lower order vectors/scalars show directional preference order. Scalar = no pref.

- Everything matches up except for his weird vector identities. "Rotor" = "Curl"

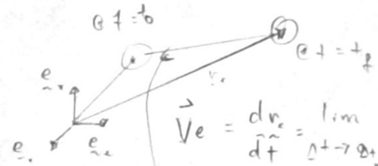
- Convective Terms of $\vec{V}\cdot(\nabla\vec{V}) = (V_x \underset{\sim}{e}_x + V_y \underset{\sim}{e}_y + V_z \underset{\sim}{e}_z)\cdot(\nabla\vec{V})$ if the spicy boi

$$= \vec{e}_x \left(V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial y} \right)$$
$$+ \vec{e}_y \left(V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} + V_z \frac{\partial V_y}{\partial z} \right)$$
$$+ \vec{e}_z \left(V_x \frac{\partial V_z}{\partial x} + V_y \frac{\partial V_z}{\partial y} + V_z \frac{\partial V_z}{\partial z} \right)$$

written more concisely as $V_i V_{j,i} \underset{\sim}{e}_j$

for any scalar field $p(\underset{\sim}{x})$, Convective Op is $tr\left((\vec{V}\cdot\nabla)(\cdot)\right) = V_i p_{,j} \delta_{ij} = V_i p_{,i}$

- Unit Identity Tensor of Rank 2: $\boxed{\underset{=}{I} = \underset{\sim}{e}_x e_x + \underset{\sim}{e}_y e_y + \underset{\sim}{e}_z \underset{\sim}{e}_z}$



$\vec{V}_e = \frac{d\underset{\sim}{r}_e}{dt} = \lim_{\Delta t \to \delta t_{min}} \frac{\Delta \underset{\sim}{r}_e}{\Delta t}$

- Fluid Element Kinematics

  - Velocity is the time dependent change in the position of the center of gravity path line or trajectory
  - δt_{min} is still significantly longer than characteristic thermal equilibrium.
  - Semicolon notation $p(t; t_0)$ with ";" a marker of choice of when to start
  - Follow along is the Lagrangian Approach. Think Lagrangian Points in orbit
  - Eulerian is a field approach. Lagrangian follows particle. Eulerian shows property evolution. maps property to a value in $\mathbb{R}$ through the entirety of the domain. Particle take value.

- Advantages of Eulerian Approach

  - Lagrangian would require direct simulation of each element being followed.
  - Each element takes on the value at that position / time.
  - Replace $x_{i_{elem}}$ for x_i to find pathlines EXCEPT for t as it is common to all.

- Example #1 : $\vec{V} = \langle xye^{-t}, yze^{-2t}, xye^{-3t} \rangle$

  - $\frac{dx_e}{dt} = x_e y_e e^{-t}$ $\quad x_e(t_0; t_0) = x_{e0}$
  - $\frac{dy_e}{dt} = y_e z_e e^{-2t}$ $\quad y_e(t_0; t_0) = y_{e0}$
  - $\frac{dz_e}{dt} = z_e x_e e^{-3t}$ $\quad z_e(t_0; t_0) = z_{e0}$

Simply write $-_e$ for each component.

Solved w/ 4th Order Runge-Kutta Method

- Streakline is to hold line to a constant. Velocity pointing at next element. A Photo
  - Streaklines are shown more in papers. Pathlines are less useful.
  - Equation for generic initial time. Hold time constant. $\frac{d\vec{r}_c}{dt} = \vec{V}(x_c(t), y_c(t), z_c(t), t < t_0)$
  - Choose our "time" t, but choose the initial relative $\boxed{t_0 \leq d = \text{fixed}}$
- Streamline is a line $\parallel$ to $\vec{V}$ at all points for fixed time t, so that $d\vec{l} \times \vec{V} = 0 \ \forall \vec{l}$
  - $\frac{dy}{dx} = \frac{V_y(x, y, z, t_0)}{V_x(x, y, z, t_0)}$; $\frac{dz}{dy} = \frac{V_z(x, y, z, t_0)}{V_y(x, y, z, t_0)}$ for fixed time $y(x_0) = y_0$; $z(x_0) = z_0$
  - least physically meaningful.

70 + ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ || = 92

$\frac{7}{2}$

Fluid Mechanics

- Classes: 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 $\to 0$
- Energy Equation ... $\rho\left(\frac{\partial e}{\partial t} + \vec{V}\cdot\nabla e\right) = \rho\vec{B}\cdot\vec{V} - \nabla\cdot(p\vec{V}) + \nabla\cdot(\underline{\underline{T}}\cdot\vec{V}) - \nabla\cdot\vec{q}$
  - $\bar{e} = \bar{u} + \frac{U_c^2}{2(C_v)_c T_c}\vec{V}\cdot\vec{V} + \frac{R}{(C_v)_c} Ma^2$; $R = \bar{R}/M$; $RT_c = a_c$ on thermodynamic prop.
  - Perfect Gas ... $\frac{R}{(C_v)_c} = \gamma - 1$, $\gamma = \frac{(C_p)_c}{(C_v)_c}$
  - [NOTE] ... write derivation of

missed this

$$\bar{\rho}\left(\frac{L}{U_c t_c}\frac{\partial \bar{e}}{\partial t} + \vec{V}\cdot\nabla e\right) = g\frac{L_c}{(C_v)_c T_c}\nabla\cdot\vec{V} - \frac{p_c}{\rho_c (C_v)_c T_c} \cdots\cdots$$

Strouhal #

missing term

$$\bar{\rho}\left(St\frac{\partial \bar{e}}{\partial t} + \vec{V}\cdot\nabla e\right) = Fr_c\frac{R}{C_v}Ma_c^2\,\bar{\rho}\,\vec{B}\cdot\vec{V} - \frac{R}{(C_v)_c}z_c\nabla\cdot(\bar{p}\vec{V})\left[???\right]$$
$$+\frac{1}{Re}\frac{R}{(C_v)_c}Ma_c^2\,\vec{\nabla}\cdot(\underline{\underline{T}}\cdot\vec{V}) - \left(\frac{P_r}{Re}\right)_c\left(\frac{C_p}{C_v}\right)_c\vec{\nabla}\cdot\vec{V}$$

- Equation of State

$$p = \rho RT \to p_c\bar{p} = \rho_c\bar{\rho}R\bar{T}T_c = p_c = \overline{\left(\frac{\rho T R}{p}\right)}\rho_c T_c = p_c = \frac{1}{Z_c}\rho_c T_c$$

- Rescale BC's as well.
- Similarity is a family of curves, used to achieve same Re, etc. by 1 soln.
- Thermodynamics
  - by $\vec{V}\cdot\nabla\vec{V} = \nabla\left(\frac{\vec{V}\cdot\vec{V}}{2}\right) - \vec{V}\times\vec{\omega}$, $\rho\left(\frac{\partial\vec{V}}{\partial t} + \overbrace{\nabla\left(\frac{\vec{V}\cdot\vec{V}}{2}\right) - \vec{V}\times\vec{\omega}}^{\vec{V}\cdot\nabla\vec{V}}\right) = \rho\vec{B} - \nabla\vec{p} + \nabla\cdot\underline{\underline{\tau}}$
  - multiply $\underline{\vec{V}\cdot(\text{eqn LHS}) = \vec{V}\cdot(\text{eqn RHS})}$ so that $()\cdot()\times() = 0$

$$\cancel{\rho\left(\frac{\partial \vec{V}}{\partial t}\cdot\vec{V}\cdot\nabla e\right) = \rho(\vec{B}\cdot\vec{V})} \quad \rho\frac{\partial u}{\partial t} + \vec{V}\cdot\nabla u$$

  - Viscous Dissipation: $\Phi = \underline{\underline{T}} : \nabla\vec{V}$
    - Doesn't need to go to heat in fluid but the fluid internal energy

Fluid Mechanics

- Eulerian to Lagrangian Recovery should be equivalent

- Counter :

$m \quad a = F$

- Newton's 2^{nd} Law: $\rho_e(t)\left(\frac{d\vec{v}}{dt}\right)_e = \dot{q}_{f_e}$ is derived from $\frac{\partial(\rho\vec{v})}{\partial t} + \vec{\nabla}\cdot(\rho\,\underline{\underline{\vec{V}\vec{V}}}) = \dot{q}_f$ for fluid element.

- Surface Integral $(\vec{V}\cdot\vec{n})$ is 1, 0, or given in the vector equation.

- Forces appear on RHS of Eqn.

  - Body Force: $\int_{V} \rho\vec{B}\,dV$

  - Surface : $\int_A \vec{\Theta}\,dS = \int_A (\underbrace{-p\vec{n}}_{\text{Hydrostatic}}^{\text{Pressure}} + \overbrace{\vec{\tau}}^{\text{Traction}})\,dS$. deviatoric. Note that $\underline{\vec{\tau} \text{ may have } \vec{n} \text{ components}}$

- Momentum Eqn.

$$\dot{Q}_t = \int_V \rho B\,dV \underbrace{- \int_A p\vec{n}\,dS + \int_A \tau\,dS}_{\text{Volume by Divergence Theorem + Greens}} \; ; \; \tau \text{ covers viscous stresses } \tau = \mu\frac{du}{dy}$$

Volume by Divergence Theorem + Greens : $\left[\int_A \vec{V}\cdot\vec{n}\,dA = \int_V \nabla\cdot\vec{V}\,dV \; ; \; \int_A a\vec{n}\,dS = \int_V \vec{\nabla}a\,dV\right]$

$\dot{q}_t = \rho\vec{B} - \nabla p + \nabla\cdot\vec{\tau}$; eleriun is $\underline{\underline{T}} = 0$ to produce Eulerian eqns of Motion.

- Navier Stokes: Viscosity; Bulk Viscosity only for Compressible, bec

  - Let $\underline{\underline{T}} = \mu(T,p)\left[\nabla\underline{v} + (\nabla\underline{v})^T - \frac{2}{3}\nabla\cdot\underline{v}\,\delta_{ij}\right] + \mu_v(T,p)\,\nabla\cdot\underline{v}\,\delta_{ij}$ defines Newtonian.

  - Bulk viscosity will generally / possibility dominate compressible flow

- Nonnewtonian Fluid, $\underline{\underline{T}} \sim f_{NN}(\nabla v)$

- Energy Equation : Let $f = e_t$ ... specific energy

$e_t = u_t + \frac{1}{2}\vec{V}\cdot\vec{V}$

- 1^{st} Law of Thermo : $\frac{d}{dt}E_t = \underbrace{\dot{Q}}_{\text{Energy into}} - \overbrace{\dot{W}}^{\text{Work done by}}$; $\dot{Q} = -\int \underline{q}\cdot\underline{n}\,dA = -\int_V \nabla\cdot\underline{q}\,dV$

- Fourier's Law : $\underline{q} = -k(T,p)\nabla T$

- Important : $\underline{\tau} = f(\nabla v)$; $q = f(\nabla T)$

Fluid Mechanics : Lecture #5 2014-04-16

- Define position as location of fluid element
- Counter: ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ | ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ ||
  @24 mins
- For the Lagrangian Approach, a property $F_e(\vec{r}_e, \vec{v}_e, p_e, \rho_e, T_e, \mu_e, h_e, s_e)$
  - property F_e has only a material / complete derivative $\frac{dF_e}{dt} = \frac{d}{dt}F_e$
- Eulerian Approach means the observer can move in spacetime, not just time.
  - F_e in this $F_e(x, y, z, t) = F_e(\vec{x}, t) \neq F_e(t)$ comes from $\frac{\partial \vec{x}}{\partial t}$
  - $dF_e = \left.\frac{\partial F_e}{\partial x}\right|_{y,z,t} dx + \left.\frac{\partial F_e}{\partial y}\right|_{x,z,t} dy + \left.\frac{\partial F_e}{\partial z}\right|_{x,y,t} dz + \left.\frac{\partial F_e}{\partial t}\right|_{x,y,z} dt \xrightarrow{\times \frac{1}{\Delta t}} \frac{\partial F}{\partial t} + \vec{V}_0 \cdot \nabla F$; $\vec{V}_0$...velocity vect.
  - Note $1/dt$ makes $\frac{\partial F_e}{\partial t}$ lonely
  $\boxed{\text{Observer's Velocity}}$
- $\vec{V}_0 = \frac{dx}{dt}\vec{e}_x + \frac{dy}{dt}\vec{e}_y + \frac{dz}{dt}\vec{e}_z$; is only ever of the observer. Independent of $\vec{V}$
- To match, make $\vec{V}_0 \equiv \vec{V}_e$ is the two approaches equivalent.

  $\frac{DF_e}{Dt} = \frac{dF_e}{dt} = \left(\frac{\partial F}{\partial t}\right)_e + \vec{V}_e(t) \cdot (\nabla F)_e$

- $\boxed{\text{Step 1 will be find the velocity field } \vec{V} \text{ from the Eulerian Approach}}$
- Specific Interest : $F_e = \vec{V}$, so that $\vec{a} = \frac{D\vec{v}_e}{Dt} = \frac{d\vec{v}_e}{dt} = \left(\frac{\partial \vec{V}}{\partial t}\right)_e + \vec{V}_e(t) \cdot (\underline{\underline{\nabla \vec{V}}})_e$
  - Deformation : $\underline{\underline{D}} = \frac{1}{2}(\nabla F + (\nabla F)^T)$
    $V(x+dx, y+dy, z+dz, t)$:
  - Spin : $\underline{\underline{\Omega}} = \frac{1}{2}(\nabla F - (\nabla F)^T)$; $\boxed{\text{Vorticity} = 2 \times \langle\text{spin}\rangle}$
- Fluid Flow Eqns. by Control Volume Analysis (Volume Fixed in Time)
  - Extensive Properties depend on volume: mass, volume, net momentum / energy, ...

    $(F_{II})_{t+\frac{\Delta t}{2}} + (F_{III})_{t+\frac{\Delta t}{2}} = (F_I)_{t-\frac{\Delta t}{2}} + (F_{II})_{t-\frac{\Delta t}{2}} + \dot{Q}_F \Delta t$; $\dot{Q}_F$ is flow production

  - In the limit $\Delta t \to 0$, factor dt to one side.

    $\frac{d}{dt}(F_{II}) = \dot{F}_{inlet} - \dot{F}_{outlet} + \dot{Q}_F \to$ Set up for Reynolds Transport

- From Reynolds Transport ... $\frac{d}{dt}(F_{II}) = \dot{F}_{in} - \dot{F}_{out} + \dot{Q}_F$; $F_{II} = \int_{\forall} \rho f \, d\forall$ defined as density, normalized makes sense for extensive properties. $\langle f \rangle$ is the specific $\langle \_ \rangle$
  - $e = u + \frac{1}{2} \vec{V}\vec{V}$ also has the internal energy term
  - Notice the sign naturally changes as dot is b/w ± 1
  - General Form: $\dot{Q}_f = \underbrace{\frac{d}{dt} \int_{\forall} \rho f \, d\forall}_{\text{Rate of Internal}} + \underbrace{\int_A \rho f (\vec{V} \cdot \vec{n}) \, dA}_{\text{Flow out}}$ Area Integrals Use Divergence Theorem to create volume integral

    Integral Eqn. of Balance
  - Term Simplification: $\int_A \rho f (\vec{V} \cdot \vec{n}) \, dA = \int_{\forall} \nabla \cdot (\rho f \vec{V}) \, d\forall$; $\dot{Q} = \int_{\forall} \dot{q} \, d\forall$
  - Differential by freezing time: $\int_{\forall} \Big[\underbrace{\frac{\partial}{\partial t}(\rho f) + \nabla \cdot (\rho f \vec{V})}_{\text{this always } 0} \Big] d\forall = 0$; $\int_{\forall} (\cdot) \, d\forall \neq 0$

    (ad $\dot{q}_f$ as needed) Eulerian Frame
    - for limits $\lim_{V \to V_{min}} \rho f + \nabla \cdot (\rho \vec{V} f) = \dot{q}_f$ differential form of conservation.

- Conservation Equation
  - Continuity $(f = 1)$: $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0$ [Mass]
    - Regular Form: $\rho \left(\frac{\partial f}{\partial t} + \vec{V} \cdot \nabla (f) \right) = \dot{q}_f$ for $\boxed{\rho \text{ is } \underline{\text{NOT}} \text{ need to be const. bec. Chain Rule}}$
    - Regular Form makes <u>no</u> assumptions
  - Forms: (Integral v/s Differential) / (Conservation v/s Regular)
  - Solving Pathline Eqn. for Lagrangian

    $\frac{d\vec{r}_e}{dt} = \vec{V}(\vec{r}_e(t), t)$ w/ $\vec{r}_e(t_0) = \vec{r}_0$ $\Bigg\}$ $\left(\frac{df}{dt}\right)_e = \left(\frac{\partial f}{\partial t}\right)_e + \vec{V}_e(t) \cdot (\nabla f)_e$

    $\rho_e \left(\frac{df}{dt}\right)_e = (\dot{q}_F)_e$
  - Regular Form can also be $\rho \Big(\underbrace{\frac{\partial f}{\partial t} + \vec{V} \cdot \nabla f}_{\frac{df}{dt}} \Big)$ $\frac{df}{dt}$ or the total derivative.

    $\frac{1}{\rho}\left(\frac{\partial f}{\partial t} + \vec{V} \cdot \nabla \rho \right) = -(\vec{\nabla} \cdot \vec{V})$

    $\frac{1}{\rho_e} \frac{d\rho_e}{dt} = -(\nabla \cdot \vec{V})_e$ = relative density change
  - Lagrangian Elmt.: $(\ )_e = (\ )\Big|_{x = x_e(t),\ y = y_e(t),\ z = z_e(t)}$

Fluid Mechanics 2019-09-26

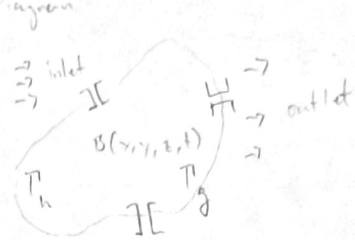
- Domain of Interest ~~||||~~ ~~||||~~ ~~||||~~ ~~||||~~ || · Diagram

$$\frac{1}{\rho_e(t)}\frac{\partial \rho}{\partial t} = -(\nabla\cdot\vec{v})_e$$

$$\rho_e(t)\left(\frac{d\vec{v}}{dt}\right)_e = \rho_e B_e - (\nabla p)_e + (\nabla T)_e$$

$$e_{(t)} = u + \frac{1}{2}\vec{v}\cdot\vec{v}$$

$$\underset{=}{T} = f(\nabla\vec{v}) \; ; \; \underset{-}{q} = f_2(\nabla T)$$

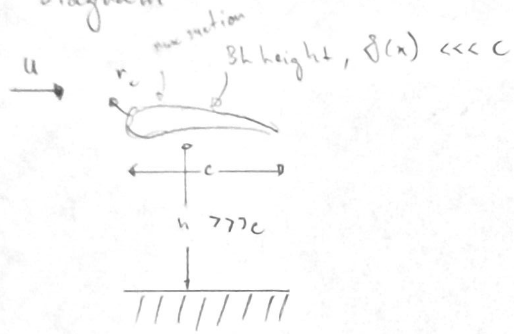


$$\Gamma_h \cup \Gamma_g = \Gamma$$

- Within the domain, the inlet is a generation term, outlet corresponds to a sink of flow
- Generation to inlet BC

- Inlet Boundary only mimics the flow at the inlet section of domain
- Lagrangian Expressions are ODE wrt. time.
- Variation in the boundary condition significantly affect the flow behaviour.
- As for the outlet, whats going on? Also equally important
- Methods
  - All Velocity, State parameters: $\vec{v}(\underset{\sim}{x}, t)$, $P(\underset{\sim}{x}, t)$, $T(\underset{\sim}{x}, t)$
  - 2 components of $\underset{\sim}{v}$ and 1 component of $\underset{\sim}{\omega} = \nabla\times\underset{\sim}{v}$, relaxes problem slightly
    - Choice of $\underset{\sim}{\omega}$ allows for information of flow conditions
- Outlet conditions
  - $\frac{\partial\vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v} = \frac{\partial P}{\partial t} + \vec{v}\cdot\nabla p = \frac{\partial T}{\partial t} + \vec{v}\cdot\nabla T = 0$ is called non-reflective $\rightarrow$ are to use for analytic
  - If same as inlet, periodic outlet BC, requires active source
  - No pressure "drop" (linear active) but,
- Notes are terrible but this has been $\boxed{\text{my favorite lecture}}$ vortex stability
  - No penetration: $(\vec{v}\cdot\vec{n})\big|_{BC} = 0$; $\vec{n} = \frac{\nabla B}{|\nabla B|}$
  - Porous media: $(\vec{v}\cdot\vec{n})\big|_{BC} = f(\vec{v}\cdot\vec{n})/\text{Given}$ on $B(\underset{\sim}{x}, t) = 0$

- Nondimensional equations use normalization by characteristic properties, by choice
- Counter:
- Example of Airfoil Flow
  - Diagram



  - Parameters
    - $\delta(x)$ ... BL height along length
    - c ... chord length, or position
    - $u(x)\big|_{x=c}$ can be 2/3x U free stream
    - r_c ... radius of curvature at $x=0$

      $r_c <<< c$

  - Choice of Length Scale depends on quantity of interest (POV's)
    - c → fluid dynamical scale
    - h → '$h >>> c$' is for finding noise of the flow around
    - r → local values to find local velocity & stall of the wing
    - δ → BL height along length of chord
  - Numeric element scale should be atleast 2 orders of magnitude smaller than char.
  - No useful CFD code to find the resonant freq. of airfoils
- Pick the best length scale for the physics problem aspect to be highlighted.
- Time scales are important as well
  - Case 1 ... c/U or convection time (tip to tail)
  - Case 2 ... $f_{separate}$ / f_{turb} / f_{shed}
  - Case 3 ... $f_{\text{Control Surface}}$ on ~ min / sec.
- Example
  - Given ... $\underbrace{L_x, L_y, L_z}_{\text{Length Scale}}$; $\underbrace{U_x, U_y, U_z}_{\text{Velocity Scale}}$; $\underbrace{\rho_c, p_c, T_c}_{\text{Thermo. Conditi.}}$

$$\left\{\begin{array}{l} \bar{x} = x/L_x \;;\; \bar{y} = y/L_y \;;\; \bar{z} = z/L_z \\ \bar{V}_x = V_x/U_x \;;\; \bar{V}_y = V_y/U_y \;;\; \bar{U}_z = V_z/U_z \\ \rho\text{, const} \\ \bar{T} = T/T_c \\ \tau = \mu \frac{U}{c} \bar{\tau} \end{array}\right.$$

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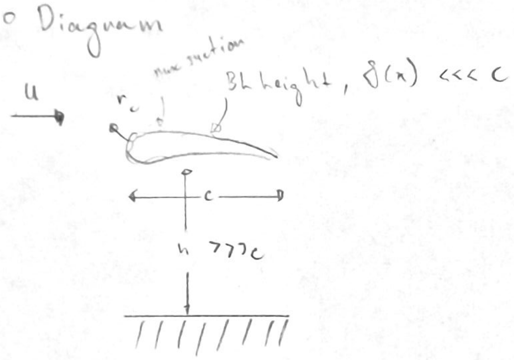

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- Continuity to Strohal

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \underset{\sim}{V}) = 0 \rightarrow \frac{L_x}{U_x t_c}\left(\frac{\partial \bar{\rho}}{\partial \bar{t}}\right) + \frac{\partial}{\partial \bar{x}}\left(\bar{\rho}\bar{V}_x\right) + \frac{L_x}{U_x}\frac{U_y}{L_y}\frac{\partial}{\partial \bar{y}}\left(\bar{\rho}\bar{V}_y\right) + \frac{L_y}{U_x}\frac{U_z}{L_z}\frac{\partial}{\partial_z}\left(\bar{\rho}\bar{V}_z\right)$$

  - Strouhal ... $St = \frac{L_x}{U_x t_c}$ / $\frac{L_x}{U_x}\frac{U_y}{t_c} \sim \frac{L_x}{U_x}\frac{U_z}{t_c} \sim 1$
  - Regular Case ... $St\frac{\partial \bar{\rho}}{\partial t} + \bar{\nabla}\cdot(\bar{\rho}\underset{\sim}{\hat{V}}) = 0$, Length scales internally similar

    $\bar{\nabla}$ is gradient to dimensionless length
  - Strouhal = $t_{char} / t_{convective}$, $\mathcal{O}(Sr) = \mathcal{O}(1)$ or less
- Conservation of Motion, $\rho\left(\frac{dV}{dt} + \underset{\sim}{V}\cdot\nabla\underset{\sim}{v}\right) = \rho\underset{\sim}{B} - \nabla p + \nabla\cdot\underset{=}{\tau}$, $\vec{B} = g\vec{B}$

  - Normalization ... $\bar{\rho}\left(\underbrace{\frac{L_c}{U_c t_c}}_{\text{Strouhal}}\frac{\partial \underset{\sim}{\bar{V}}}{\partial \bar{t}} + \overset{\text{mistake}}{\cancel{\frac{U_c^2}{L_c}}}\underset{\sim}{\bar{V}}\cdot\nabla\underset{\sim}{\bar{v}}\right) = \underbrace{\frac{gL_c}{U_c^2}}_{\text{Froude measures body force effect}}\bar{\rho}\underset{\sim}{\bar{B}} - \overset{\text{Euler \#}}{\frac{p_c}{\rho_c U_c^2}}\bar{\nabla}p + \overset{1/Re}{\frac{\mu_c}{\rho_c U_c L_c}}\bar{\nabla}\cdot\underset{=}{\tau}$

- Euler Number

  - Liquid ... $Eu_c = \frac{p_c}{\rho_c U_c^2}$; Gases ... $\frac{p_c RT_c}{\rho_c U_c^2 RT_c} = Z_c\frac{RT_c}{U_c^2} = Z_c\frac{a_c^2}{U_\infty^2} = \frac{Z_c}{Ma_c^2}$
  - $Z_c = \frac{p_c}{\rho_c RT_c}$; $RT_c = a_c^2$; Mach #, $Ma_c = U_c/a_c$; isothermal char. speed sound

- Reynolds Number ... Re ... $\frac{\rho_c U L_c}{\mu_c} = \frac{\rho_c U_c^2}{\mu U_c/L_c} = \frac{(\text{dynamic Pressure})}{(\text{viscous Stress})}$

  $Re_c = \frac{\rho U_c^2}{}$

  $= \frac{L_c}{(\mu_c/\rho_c U_c)} = \frac{(\text{charistic len.})}{(\text{viscous length})}$ = # viscous scales in problem

$$\bar{\rho}\left(St_c\frac{\partial \underset{\approx}{\bar{v}}}{\partial t} + \underset{\sim}{\hat{V}}\cdot(\nabla\underset{\sim}{\bar{v}})\right) = Fr_c\,\bar{\rho}\,\underset{\sim}{\bar{B}} - \underbrace{Eu_c}_{\text{Gas: } \frac{Z_c}{Ma_c^2}}\nabla p + \frac{1}{Re}\left(\nabla\cdot\underset{\approx}{T}\right)$$

Fluid Mechanics by Joel 2019-10-07

- Thermodynamics of Fluid Element, Class # ... |||| |||| |||| |||| |||| ||||

  - Eqn. #1 ... $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$ | Cons. of Mass, Momentum
  - " #2 ... $\rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v}\right) = \rho \vec{B} + \nabla \cdot \underline{\underline{\tau}} - \nabla p$ | and Energy (#1, 2, 3)
  - #3 ... $\rho\left(\frac{\partial e}{\partial t} + \vec{v} \cdot \nabla e\right) = \rho \vec{B} \cdot \vec{v} - \nabla \cdot (p \vec{V}) + \nabla \cdot (\vec{v} \cdot \underline{\underline{\tau}})$ | used here

- [NOTE] ... Heat Conduction not apparent in Cons. of Energy.

  Internal Energy un affected by body force '$\vec{B}$'

- Viscous effect always increases internal energy, so '$\Delta T > 0$', always.

- Example ... $\rho\left(\frac{\partial}{\partial t}\left(\frac{p}{\rho}\right) + \vec{v} \cdot \nabla\left(\frac{p}{\rho}\right)\right) \Rightarrow \quad p(\nabla \cdot \vec{v}) = \rho\left(\frac{\partial}{\partial t}\left(\frac{p}{\rho}\right) + \vec{v} \cdot \nabla\left(\frac{p}{\rho}\right)\right) - \left(\frac{\partial p}{\partial t} + \vec{v} \cdot \nabla p\right)$

  - Remember that enthalpy (specific) ... $h = u + \frac{p}{\rho}$ = <internal energy> + <??>
  - So that ... $\rho_e (t) \left(\frac{dh}{dt}\right)_e - \left(\frac{dp}{dt}\right)_e = \Phi_e - (\nabla \cdot \vec{q})_e$ ... Eulerian Balance of Spec. Enthalpy
  - Enthalpy is a mathematical concept. 'u', internal energy is a physical one.

- Combine Lecture Eqns #5 & #7

$$\rho\left(\frac{d}{dt}\left(h + \frac{\vec{v} \cdot \vec{v}}{2}\right) + \vec{v} \cdot \nabla\left(h + \frac{\vec{v} \cdot \vec{v}}{2}\right)\right) - \frac{\partial p}{\partial t} - \cancel{\vec{v} \cdot \nabla p} = \Phi - \nabla \cdot \vec{q} + \rho \vec{B} \cdot \vec{v} - \cancel{\vec{v} \cdot \nabla p} + \vec{v} \cdot (\nabla \cdot \underline{\underline{\tau}})$$

$$\rho\left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla\right)\left(h + \frac{\vec{v} \cdot \vec{v}}{2}\right) - \frac{\partial p}{\partial t} = \Phi - \nabla \cdot \vec{q} + \rho \vec{B} \cdot \vec{v} + \vec{v} \cdot (\nabla \cdot \underline{\underline{\tau}})$$

  - Assume invariant w/ time body force '$\vec{B}$' is given by potential

$$\text{?const?} = \frac{\partial p}{\partial t} + \Phi + \vec{v} \cdot (\nabla \cdot \underline{\underline{\tau}}) - \nabla \cdot \vec{q} = ??, \quad \vec{B} = -\nabla \phi_B, \quad \frac{\partial \vec{B}}{\partial t} = 0, \text{ in substitution.}$$

- Specific Total Enthalpy ... $h_t = h + \frac{\vec{v} \cdot \vec{v}}{2} + \phi_B$ is two new terms

  - Balance, Eulerian ... $\left[\rho\left(\frac{\partial h_t}{\partial t} + \vec{v} \cdot \nabla h_t\right)\right]_e = \left[\frac{\partial p}{\partial t} + \Phi + \vec{v} \cdot (\nabla \cdot \underline{\underline{\tau}}) - \nabla \cdot \vec{q}\right]_e$ along elmt
  - If neglect, $\nabla \underline{\underline{\tau}}$ & $\nabla \underline{\underline{v}}$, $\rightarrow \sim \mathcal{O}(1)$ or "order of 1" in Big $\mathcal{O}$ notation.
    - $\rho_e (t) \left(\frac{\partial h_t}{\partial t}\right)_e = \left(\frac{\partial p}{\partial t}\right)_e$ is produced ... see the relation b/w (ρ, p, h) if v & T are constant, like. Recover the newtonian description.
  - Steady Flow ... $\frac{d}{dt}(\cdot) = 0$, $(\cdot) =$ Cnst. ... Constant, Outside of BL!
  - Varies in BL or across shockwaves. as $\nabla V >>> 1$ due to velocity grad

▫ Btw, shockwaves explored by Grant + Hilbert, ~1940 of Euler Equations

Compressible Flow!

▫ Possibily, supersonic flow (high Re), $(\rho / p / T)$ can jump over small $(< 10^{-6} m)$ or $(< 10^{-7} m)$ regions, or "shock-wave" layer. Visualization of fluid elmts. is hard but important as they happen on a molecular basis as 'λ' is on same scale as the shockwave "thickness". Good to ~1.1 or ~1.2 of c.

▫ Thickness $\propto$ 1/⟨prop. velocity⟩, N-S Eqns limited to $\sim 10^{-6} m$ (λ)

▫ Need Kinetic Theory to really model shockwaves / molecular dynamics → DNS

▫ Lead to adiabatic, inviscid shockwaves ... $\underset{\text{inviscid}}{\Phi} + \underbrace{\vec{V}\cdot(\nabla\cdot\underline{\underline{T}}) - \nabla\cdot\vec{q}}_{\text{cancel in SW}}$

• Entropy Balance by Gibbs (~1850's)

▫ Gibb's Eqn... $T ds = du + p dv$, $v = 1/\rho$ ... specific volume.

▫ $dh = d(u + \frac{P}{\rho}) \rightarrow \rho T\left(\frac{\partial s}{\partial t} + \vec{v}\cdot\nabla s\right) = \rho\left(\frac{\partial h}{\partial t} + \vec{v}\cdot\nabla h\right) - \left(\frac{\partial p}{\partial t} + \vec{V}\cdot\nabla p\right) \rightarrow \rho T\left(\frac{\partial s}{\partial t} + \vec{V}\cdot\nabla s\right) = \Phi - \nabla\cdot\vec{q}$

▫ Newton did not have entropy because he didnt have viscosity

▫ Lagrangian Specific Entropy Balance ... $\boxed{\rho_e(t)\, T_e(t) \left(\frac{ds}{dt}\right)_e = \Phi_e - (\nabla\cdot\vec{q})_e}$

▫ The Total derivative is used so any increment in $(t, x, y, z, etc...)$

▫ Entropy is an attempt to simplify the application of total enthalpy... 'h_t'

▫ Eulerian Specific Entropy Balance ... $\boxed{\rho T\left(\frac{\partial s}{\partial t} - \underset{\text{Convection Term}}{\underline{V}\cdot\nabla s}\right) = \Phi - \nabla\cdot\vec{q}}$

▫ ⟨Inviscid⟩ + ⟨Heat Conduction⟩ ~ 0,

▫ "$\left(\frac{ds}{dt}\right)_e = 0$, $s_e = \text{const}$" or Isentropic

▫ Again, 'Φ_e' is $\mathbb{R}^+ \;\forall\; \Omega$

▫ Finally, "$$\boxed{\begin{aligned} &\left[\rho\left(\frac{\partial s}{\partial t} + \vec{V}\cdot\nabla s\right) + \nabla\cdot\left(\frac{\vec{q}}{T}\right) = \frac{\Phi}{T} + k\frac{|\nabla T|^2}{T^2}\right] \geq 0 \\ &\left[\rho_e(t)\left(\frac{ds}{dt}\right)_e + \nabla\cdot\left(\frac{\vec{q}}{T}\right)_e = \frac{\Phi_e}{T_e} + \left(k\frac{|\nabla T|^2}{T^2}\right)_e\right] \geq 0 \end{aligned}}$$" Eulerian / Lagrangian

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Fluid Mechanics by Joel. 2019-10-07

- Thermodynamics of Fluid Element, Class #... ~~卌卌卌卌卌卌~~

  - Eqn. #1 ... $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$ | Cons. of Mass, Momentum and Energy (#1, 2, 3) used here
  - " #2 ... $\rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v}\right) = \rho \vec{B} + \nabla \cdot \underline{\underline{\tau}} - \nabla p$
  - #3 ... $\rho\left(\frac{\partial e}{\partial t} + \vec{v} \cdot \nabla e\right) = \rho \vec{B} \cdot \vec{v} - \nabla \cdot (p \vec{v}) + \nabla \cdot (\vec{v} \cdot \underline{\underline{\tau}})$

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  - Remember that enthalpy (specific), ... $h = u + \frac{p}{\rho}$ = <internal energy> + <??>
  - So that ... $\rho_e(t)\left(\frac{dh}{dt}\right)_e - \left(\frac{dp}{dt}\right)_e = \Phi_e - (\nabla \cdot \vec{q})_e$ ... Eulerian Balance of Spec. Enthalpy
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  - Again, 'Φ_e' is $\mathbb{R}^+ \; \forall \; \Omega$
- Finally, "

$$\boxed{\begin{aligned} &\left[\rho \left(\frac{\partial s}{\partial t} + \vec{V} \cdot \nabla s \right) + \vec{\nabla} \cdot \left(\frac{\vec{q}}{T} \right) = \frac{\Phi}{T} + k \frac{|\nabla T|^2}{T^2} \right] \geq 0 \\ &\left[\rho_e(t) \left(\frac{ds}{dt} \right)_e + \nabla \cdot \left(\frac{\vec{q}}{T} \right)_e = \frac{\Phi_e}{T_e} + \left(k \frac{|\nabla T|^2}{T^2} \right)_e \right] \geq 0 \end{aligned}}$$

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