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Experimental Mechanics

Homework # 1

I'm sorry I waste paper.

?

Problem # 1.1.a

- Derive differential equations of equilibrium in "x" or "z" directions
- Solution

$\rightarrow \varepsilon_{zz} = 1 + \frac{\partial w}{\partial z}$

$\rightarrow \Delta z' = \Delta z \left(1 + \varepsilon_{zz}\right)$

$\rightarrow (\Delta z')^2 = \Delta z^2 \left(1 + \varepsilon_{zz}\right)^2 = \left[\left(\frac{\partial u}{\partial z} \Delta z\right)^2 + \left(\frac{\partial v}{\partial z} \Delta z\right)^2 + \left(\left(1 + \varepsilon_{zz}\right) \Delta z\right)^2 \right]$

$$\varepsilon_{zz} = \sqrt{1 + 2\frac{\partial w}{\partial z} + \underbrace{\cancel{\frac{\partial^2 u}{\partial z^2}} + \cancel{\frac{\partial^2 v}{\partial z^2}}}_{\sim 0} + \frac{\partial^2 w}{\partial z^2}} - 1$$

$$\boxed{\varepsilon_{zz} = \frac{\partial u}{\partial x}}$$

×

equilibrium of forces (see solutions, or me)

0

Problem # 1.7b

- Given stress distribution, is equilibrium satisfied in absence of body forces?
- Given:

$\rightarrow \sigma_{xx} = 6x^2 - 6y^2 - 2z$

$\rightarrow \sigma_{yy} = 6y^2$

$\rightarrow \sigma_{zz} = 6x^2 + 2y - 2z + 2.5$

$\rightarrow \tau_{xy} = 2z - 12xy - 1.5$

$\rightarrow \tau_{yz} = 0$

$\rightarrow \tau_{zx} = 2x + 2y - 3$

- Solution

$\rightarrow \nabla \underset{\sim}{\sigma} = 0$

$\rightarrow$ X direction: $\frac{\partial}{\partial x}\sigma_{xx} + \frac{\partial}{\partial y}\tau_{xy} + \frac{\partial}{\partial z}\tau_{zx} = 0$

$$12x - 12x + 0 = 0$$

$\rightarrow$ Y direction: $\frac{\partial}{\partial y}\sigma_{yy} + \frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial z}\tau_{yz} = 0$

$$12y + (-12y) + 0 = 0$$

$\rightarrow$ Z direction: $\frac{\partial}{\partial z}\sigma_{zz} + \frac{\partial}{\partial x}\tau_{zx} + \frac{\partial}{\partial y}\tau_{yz} = 0$

$$-2 + 0 + 2 = 0$$

$\boxed{\text{Equilibrium Satisfied}}$

✓ 5

Problem # 1.2

- Transform Cartesian stress components

- Given :

$\rightarrow \underset{\sim}{\sigma} =$ stress tensor

$\rightarrow \sigma_{xx} = 180$ MPa

$\rightarrow \sigma_{yy} = 108$ MPa

$\rightarrow \sigma_{zz} = -72$ MPa

$\rightarrow \tau_{xy} = 54$ MPa

$\rightarrow \tau_{xz} = 90$ MPa

$\rightarrow \tau_{yz} = -72$ MPa

$$\rightarrow \underset{\sim}{r} = \begin{bmatrix} 2/3 & 2/3 & -1/3 \\ -2/3 & 1/3 & -2/3 \\ -1/3 & 2/3 & 2/3 \end{bmatrix}$$

- Solution :

$$\rightarrow \underset{\sim}{\sigma}' = r^{-1} \underset{\sim}{\sigma} \, r = \left(\frac{1}{3} \cdot \frac{1}{9} \begin{bmatrix} 2 & -2 & 1 \\ 2 & 1 & 2 \\ -1 & -2 & 2 \end{bmatrix} \right) \left(\begin{bmatrix} 180 & 54 & 90 \\ 54 & 108 & -72 \\ 90 & -72 & -72 \end{bmatrix} \right) \left(\frac{1}{3} \begin{bmatrix} 2 & 2 & -1 \\ -2 & 1 & -2 \\ -1 & 2 & 2 \end{bmatrix} \right)$$

$$= \cancel{\frac{1}{81} \begin{bmatrix} 9.7 & 12.4 \\ 13.3 & 14.6 \\ 8.6 & -10 \end{bmatrix}} \begin{bmatrix} 0 & 126 & 78 \\ 120 & 132 & -90 \\ 78 & -90 & 84 \end{bmatrix} \text{MPa}$$

X

5

Problem # 1.3

- Find associated normal strains @ (2,1,2)

(a) Assuming strains and displacements are small

(b) Not making small strain assumptions

(c) Compare (a) ; (b)

(d) Shear Strains using (a)

- Given:

→ $u = 10^{-3}\left(3x^4 + 2x^2y^2 + x + y + z^3 + 3\right)$

→ $v = 10^{-3}\left(3xy + y^3 + y^2z + z^2 + 1\right)$

→ $w = 10^{-3}\left(x^2 + xy + yz + zx + y^2 + z^2 + 2\right)$

it looks like you have the right idea, but I don't really have enough to go on...

- Solution:

(a)/(b)/(c) $\varepsilon_{xx} = \sqrt{1 + 2\left(\underbrace{10^{-3}\left(12x^3 + 4xy^2 + 1\right)}_{\frac{\partial u}{\partial x}}\right) + \underbrace{10^{-3}\left(36x^2 + 4y^2\right)}_{\left(\frac{\partial u}{\partial x}\right)^2 ?} + \underbrace{\cancel{10^{-3}(0)}}_{\left(\frac{\partial v}{\partial x}\right)^2 ?} + \underbrace{10^{-3}(2)}_{\left(\frac{\partial w}{\partial x}\right)^2 ?}} - 1 \Big|_{x=2,y=1}$

? ? ? or $\frac{\partial v}{\partial x}$?

$= \sqrt{1 + 10^{-3}\left(4y^2 + 4xy^2 + 12x^3 + 36x^2 + 4\right)} - 1 \Big|_{2,1,2}$

$= \sqrt{1 + 10^{-3}\left(4(1)^2 + 4(2)(1)^2 + 12(2)^3 + 36(2)^2 + 4\right)} - 1 \Big|_{2,1,2}$

$\simeq 10^{-3}\left(12x^3 + 4xy^2 + 1\right)$ ✓ @ (2,1,2) = ?

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$\varepsilon_{yy} = \sqrt{1 + 2\left(10^{-3}\left(3x + 3y^2 + 2yz\right)\right) + 10^{-3}\left(\left(6y + 2z\right) + \left(4x^2\right) + (2)\right)} - 1 \Big|_{2,1,2}$

$\simeq 10^{-3}\left(3x + 3y^2 + 2yz\right)$ ✓ @ (2,1,2) = ?

$\varepsilon_{zz} = \sqrt{1 + 2\left(10^{-3}\left(y + x + 2z\right)\right) + 10^{-3}\left(2 + 6z\right)} - 1 \Big|_{2,1,2}$

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$\simeq 10^{-3}\left(y + x + 2z\right)$ ✓ @ (2,1,2) = ?

(c) Approximations should not be good as x, y, z are greater than 1.

(b) $\gamma_{xz} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$ ✓

$$= 10^{-3}\left(\left(4x^2y + 1\right) + \left(3y\right)\right)\Big|_{2,1,2} = 10^{-3}\left(\left(4(2)^2(1) + 1 + 3(1)\right)\right)$$

$$= 10^{-3}(12) = \boxed{12 \times 10^{-2}}$$ X?

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 10^{-3}\left(\left(y^2 + 2z\right) + \left(x + 2y + z\right)\right)\Big|_{2,1,2}$$ ✓

$$= 10^{-3}\left(\left(1^2 + 2(2)\right) + \left(2 + 2(1) + 2\right)\right)$$

$$= \boxed{1.1 \times 10^{-2}}$$ ✓

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$$\gamma_{zx} = \cancel{\frac{\partial \theta}{\partial z}} \; \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 10^{-3}\left(\left(2x + y + z\right) + 3z^2\right)\Big|_{2,1,2}$$ ✓

$$= \boxed{1.4 \times 10^{-2}}$$ ✓

Problem 1.4

- Determine strain ε_{zz}

- Given : plane stress condition of aluminum

$\rightarrow E = 71$ GPa $\quad \rightarrow$ ~~$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0$~~ $\quad \rightarrow \varepsilon_{yy} = 600 \mu\varepsilon$

$\rightarrow \nu = 0.33 \quad \rightarrow \varepsilon_{xx} = 800\ \mu\varepsilon \quad \rightarrow \sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$

- Solution

$$\rightarrow \varepsilon_{zz} = \frac{1}{E}\left[\cancel{\sigma_{zz}}^{0} - \nu(\sigma_{xx} + \sigma_{yy})\right] = -\frac{\nu}{E}(80.6 + 69.8)\,\text{MPa} = \boxed{-6.9 \times 10^{-4}}$$

should such a strain be positive or negative (just by knowledge of elastic material behavior... $\uparrow \nu$)

absolute mag. ok ✓

$$\rightarrow \varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \cancel{\sigma_{zz}}^{0})\right] \rightarrow \sigma_{xx} = E\varepsilon_{xx} + \nu\sigma_{yy}$$

$$\rightarrow \varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{xx} + \cancel{\sigma_{zz}}^{0})\right] \rightarrow \sigma_{yy} = E\varepsilon_{yy} + \nu\sigma_{xx}$$

$\sigma_{xx} = 80.6$ MPa

$\sigma_{yy} = 69.8$ MPa

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