MATH-4600, SPRING 2019

ADVANCED CALCULUS

Catalog description: A course emphasizing advanced concepts and methods from calculus. Topics include: multivariable integral theorems (Green's, Gauss', Stokes', Reynolds transport), extrema of multivariable functions (including Taylor's theorem and Lagrange multipliers), the calculus of variations (Euler-Lagrange equations, constraints, principle of least action), and Cartesian tensors (calculus, invariants, representations).

Instructor
A. K. Kapila
Amos Eaton 319
518-276-6894
kapila@rpi.edu
Office Hours: W, F 2:00 - 4:00 PM. When I am in, my door is usually open. You are welcome to stop by and it is seldom that I am too busy to help you.

Assistants
Erik Bergland
Amos Eaton 317
bergle@rpi.edu
Office Hours: Wed 2:00 - 3:50 PM, 317 Amos Eaton Building.

Mallory Gaspard
gaspam3@rpi.edu
Office Hours: Wed 1:00 - 2:50 PM, JROWL 1W20 (Resnick Center).

Textbook
Vector Calculus 6E
J. E. Marsden & A. Tromba: Freeman

SYLLABUS & POLICY

1. **Topics to be covered in the course**

   - Review
   - Vector fields
   - Taylor's theorem and extrema
   - Multiple integration
   - Surface integrals and integral theorems
   - Calculus of variations
   - Cartesian tensors

2. **Pre-requisites**

   A thorough knowledge of the basic facts in single and multivariable calculus and elementary linear algebra, through MATH 2010, is essential. Here is a list of topics for you to know, and if need be, review. Calculus of variations will also require facility to solve simple differential equations. Our review of these materials in class will be brief.

   - Vectors in $\mathcal{R}^2$ and $\mathcal{R}^3$: geometric and algebraic representations, basis vectors, parametric representation of curves, dot and cross products, projection, planes and distances.
   - Polar, cylindrical and spherical coordinates.

- Functions of several variables: definition, visualization, limit and continuity. Partial differentiation, differentiability, linear approximation, chain rules, directional derivative, gradient.
- Integration: Line integrals, double and triple integrals in cartesian and cylindrical coordinates.
- Elementary matrix operations.

3. **Learning Outcomes:** Functions of several variables, or multivariate functions, are the topic of study in this course. Such functions arise naturally in a variety of fields in engineering and science such as motion and deformation of solids and fluids, thermodynamics, heat and mass transfer, and electromagnetics. A successful completion of the course should enable you to

   - demonstrate a conceptual and practical understanding of multivariate functions and their rates of change,
   - compute extrema of multivariate functions,
   - demonstrate a conceptual and practical understanding of integrals of multivariate functions along paths and over surfaces and volumes,
   - demonstrate a conceptual understanding of integral theorems and their practical applications,
   - compute extrema of functionals, and
   - demonstrate a conceptual and practical understanding of the notion of Cartesian tensors.

   The primary focus will be on **understanding** the principles of multivariable calculus and learning to apply them to new, practical situations. Nevertheless, one should be aware that this is a course in mathematics, and as such, will demand a certain measure of rigor.

4. **Setup:** The class will meet twice a week. The course will employ as a managing tool the Rensselaer BlackBoard Learning Management System, which can be accessed at the following URL:

   `http://lms.rpi.edu`

   Please log in frequently to stay current and watch for frequent announcements.

5. **Policy:**

   (a) **Assignments:** Homework will be assigned regularly, collected, marked and returned. Each assignment will be scored out of 100 points, unless announced otherwise.

   - Please write your solutions clearly and coherently, with the work displayed in a sequential manner and sufficient explanation provided so that your strategy and approach are apparent to the grader. It is enough to merely summarize routine manipulations involving algebra or calculus, but concepts and techniques covered in this course must be explained in detail. Any numerical computations involving MAPLE or MATLAB must be accompanied by a copy of your code or worksheet, as appropriate. Your score in the homework will depend on both mathematical correctness and clarity of presentation.
   - Legible, handwritten solutions will be acceptable, but the use of a typesetting system such as LaTeX is strongly recommended. If you are submitting hand-written solutions, make sure that the version submitted is a neat copy of your rough work, from which all false starts have been expunged.
     Solutions that are illegible or lack sufficient explanation will be returned without grading.
   - Posted solutions will be a good template for writing your own solutions.
   - Be sure to staple the pages before submitting the homework.
   - Homework is due by 5 PM on the assigned date and can be submitted either in class or in the labelled box in Amos Eaton 301. A late submission will lose 20% of credit per day beyond the due date, and all credit once the solutions are posted on line, which may be as early as two days after the due date.
   - **It is recommended that you keep a photo copy of the homework solutions before you turn them in. This guards against accidental loss of a submission.**

(b) **Attendance:** Attendance is not mandatory. However, do not expect any special help with lecture material or homeworks if you are not a regular attendee.

(c) **Academic Integrity:** Student-teacher relationships are built on trust. For example, students must trust that teachers have made appropriate decisions about the structure and content of the courses they teach, and teachers must trust that the assignments that students turn in are their own. Acts, which violate this trust, undermine the educational process. The **Rensselaer Handbook of Student Rights and Responsibilities** defines various forms of Academic Dishonesty and you should make yourself familiar with these.

Working together on homework assignments in small groups is encouraged, and you should feel free to discuss among the group how to approach a problem or how to carry out certain calculations. However, detailed sharing of solutions is prohibited, and work submitted for grading is expected to be your own rather than a copy of some one else's work. Finding a definition on line is fine, but handing in the entire solution to a problem found on line is plagiarism unless the source is properly cited. The HW is intended to help you learn the subject, and is not a test of your web-surfing ability. The first occurrence of any violation of the above policy will lead to a grade of zero for the HW concerned. A second occurrence will lead to a report to the Dean of Students.

Collaborating on examinations is not permitted either. If cheating is suspected, an explanation will be sought. An unsatisfactory explanation will result in a grade of zero and a report to the Dean of students.

(d) **Tests:** There will be two in-class tests and a final examination; see the course outline below for a tentative schedule. Syllabus for each test, and any change in a test date, will be announced well in advance of the test. **It will be prudent to wait making any end-of-term travel plans until after the Final Examination Schedule has been announced.**

(e) **Makeups:** A missed test cannot be made up, except when the student has made prior arrangement with me, or can demonstrate that he or she was too ill to sit for the examination. I *may* make further exceptions in individual cases should circumstances warrant them.

(f) **Grades:** Prior to the final examination you will receive an advance numerical score for the course. This score is based on a weighted average of performance in homework solutions (60%) and the two tests (40%). The course score is determined by whichever of the following weighting results in a better score.

- Course score = 75% advance score + 25% score on the final
- Course score = 25% advance score + 75% score on the final

Thus a strong achievement in the final examination improves your score, while a weak performance does not penalize you excessively.

Your course grade will be determined entirely by your numerical score. Your *need* to get a good grade (to stay in school or keep your financial aid, for example) does not play a role. Numerical scores will be changed into letter grades at the end of the term according to the following scheme:

$A(93-100)$, $A^-(89-92)$
$B^+(85-88)$, $B(81-84)$, $B^-(77-80)$
$C^+(72-76)$, $C(68-71)$, $C^-(64-67)$
$D(50-63)$, $F(0-49)$

The grading scheme is independent of the class average.

(g) **Appealing Grades:** If you are not satisfied with your grade in an assignment or test, feel free to discuss it with me within one week of the day on which the assignment or test is returned to you. If the matter is unresolved, you may appeal to the department head.

6. **Text:** The subject matter of the course is classical and is covered in numerous books. We shall roughly follow the Marsden-Tromba text. You may also consult *Advanced Calculus* by Susan Colley which has been used in recent years. The text does not cover two of the topics: calculus of variations and cartesian tensors. I shall provide notes on these topics and suggest other references when we come to cover the material in class.

A final word: This course demands a considerable amount of effort. Our pace will not be slow, so manage your time well and try not to fall behind. Above all, do see me right away if you are experiencing difficulties. You are responsible for learning the material, of course, but the TA and I are here to help you learn. I encourage you strongly to use our office hours.

Welcome back, and best wishes for a very pleasant Spring term.

COURSE OUTLINE

Advanced Calculus

WEEK	CHAPTER	COMMENTS
Week 1, Jan 10	Review (Chapters 1 and 2)	
Week 2, Jan 14 - 18	Review (Chapters 1 and 2)	
Week 3, Jan 21 - 25	Chapter 4	MLK holiday, Monday class on Tuesday.
Week 4, Jan 28 - Feb 1	Chapter 4, 3	
Week 5, Feb 4 - 8	Chapter 3	
Week 6, Feb 11- 15	Chapter 5	Test I
Week 7, Feb 18 - 22	Chapter 5	No classes on Monday.
Week 8, Feb 25 - March 1	Chapters 6,7	
Week 9, March 4 - 8		Spring Break
Week 10, March 11 - 15	Chapter 8	
Week 11, March 18 - 22	Chapter 8, Calculus of variations	
Week 12, March 25 - 29	Calculus of variations	GM Week
Week 13, April 1 - 5	Calculus of variations	Test II
Week 14, April 8 - 12	Cartesian tensors	
Week 15, April 15 - 19	Cartesian tensors	
Week 16, April 22 - 26	Cartesian tensors	Classes end on Friday.
Week 17, April 30 - May 4		Final exams.

NOTES:

1. The Chapter numbers refer to the Marsden & Tromba text.
2. The outline is not set in stone. We may make some changes as we go along.
3. Tests are tentatively scheduled for the weeks of Feb 11 and April 1. Any changes will be announced in class well in advance of each test.