November 18, 2019

MANE 6960-01 Fluid Mechanics
Fall Semester 2019
HW #5
Due: December 5, 2019

1. Consider the Rayleigh problem, where the plate is moved with the velocity $V_x(y = 0; t) = t^{n}$, where the exponent n is a parameter. By looking for similarity solutions show that the velocity may be written as $V_x(y; t) = t^{\epsilon} g(y\, t^{\beta})$. What are the values of β and ϵ? Write the ordinary differential equation that g must satisfy with the corresponding boundary conditions. Solve this equation for n=1/2. Comment on how the diffusion length scales with time?

2. Use the 4th order Runge-Kutta numerical integration scheme to solve the boundary layer eq $2\frac{d^3f}{d\eta^3} + f\frac{d^2f}{d\eta^2} = 0$ with the boundary conditions $f(0) = \frac{df}{d\eta}(0) = 0, \frac{df}{d\eta}(\eta \to \infty) = 1$. Using iterations until all conditions are satisfied, determine $\frac{d^2f}{d\eta^2}(0)$ and plot the functions $f, \frac{df}{d\eta}, \frac{d^2f}{d\eta^2}$. For numerical computations apply the far-field condition at $\eta = 10$. You can use the Matlab procedure ODE45.

3. Use the 4th order Runge-Kutta numerical integration scheme to numerically solve the Falkner-Skan equation $\frac{d^3 f}{d\eta^5} + f\frac{d^2 f}{d\eta^2} - \frac{2m}{m+1}[(\frac{df}{d\eta})^2 - 1] = 0$ with boundary conditions $f(0) = \frac{df}{d\eta}(0) = 0, \frac{df}{d\eta}(\infty) = 1$ (for a boundary layer with a far-field axial velocity U(x) = $U_1\, x^m$) when (i) $m = 1$, (ii) $m = 0$, and (iii) $m = -0.0904$. For each m, using iterations until all conditions are satisfied, determine $\frac{d^2 f}{d\eta^2}(0)$ and plot the functions $f, \frac{df}{d\eta}, \frac{d^2 f}{d\eta^2}$. For numerical computations apply the far-field condition at $\eta = 10$. You can use the Matlab procedure ODE45.

4. Consider the steady, incompressible, viscous and axisymmetric Burgers vortex that is given in a cylindrical coordinate system. For this flow the radial velocity is: $V_r = -ar$ where a is a constant.

(a) From the continuity eq in axisymmetric cylindrical coordinates, determine the axial velocity V_z.

(b) Then use the azimuthal momentum eq in axisymmetric cylindrical coordinates to determine the azimuthal velocity V_θ(r). Assume the centerline condition $V_\theta(0)=0$ and the far-field relation $V_\theta(r>>1) = \Gamma/(2\pi r)$ where Γ is a constant. Hint: assume $2\pi r\, V_\theta/\, \Gamma = f(\eta)$ where $\eta = \frac{r}{\sqrt{\nu/2a}}$ and derive a differential eq for f. Use the example of a decaying vortex in time to analytically solve f(η) and determine V_θ(r).

(c) Apply the 4th order Runge-Kutta numerical integration scheme to numerically solve the differential eq for f with $f(0)=0$ and $f(\eta >>1)=1$. Using iterations until all conditions are satisfied, determine $\frac{df}{d\eta}(0)$ and plot the functions $f, \frac{df}{d\eta}$. For numerical computations apply the far-field condition at $\eta = 10$. You can use the Matlab procedure ODE45.