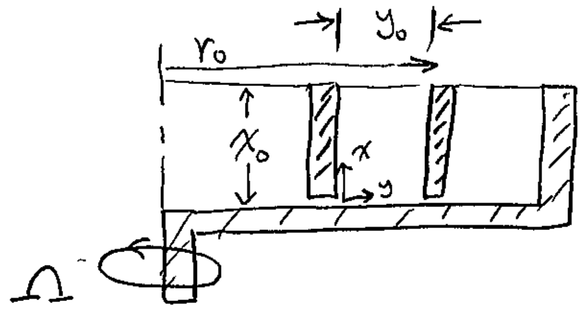
Analytical solution for deep-channel surface viscometer: following Edwards et al.



$(\delta \to 0)$ $D \equiv \frac{x_0}{y_0}$, "deep" $D >> 1$, find $D > \frac{2}{\pi}$ error is less than 1%.
Flow in absence of surfactant is obtained from $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$

when V is in the z-direction. This is N-S in the inertialess limit. BC: $\mu \frac{\partial V}{\partial x} = \mu^s \frac{\partial^2 V}{\partial y^2}$ at $x = x_0$.

For clean surface, surface velocity in the middle of channel is $V_c^* = \frac{4 V_b}{\pi \cosh(\pi D)}$, where V_b is floor speed in the middle of channel. First order approximation for circular channel:

$$V_c^* \approx \frac{4 V_b}{\pi \cosh(\pi D)} \left[1 + \frac{3}{32} \left(\frac{y_0}{r_0} \right)^2 \right].$$

Finally, get μ^s using:

$$\boxed{\frac{\mu^s \pi}{\mu y_0} = \frac{V_c^*}{V_c} - 1}$$

where V_c is the measured azimuthal speed at mid point of channel surface.

Works for $Re = \frac{\rho \Omega r_0^2}{\mu} \lesssim 100$