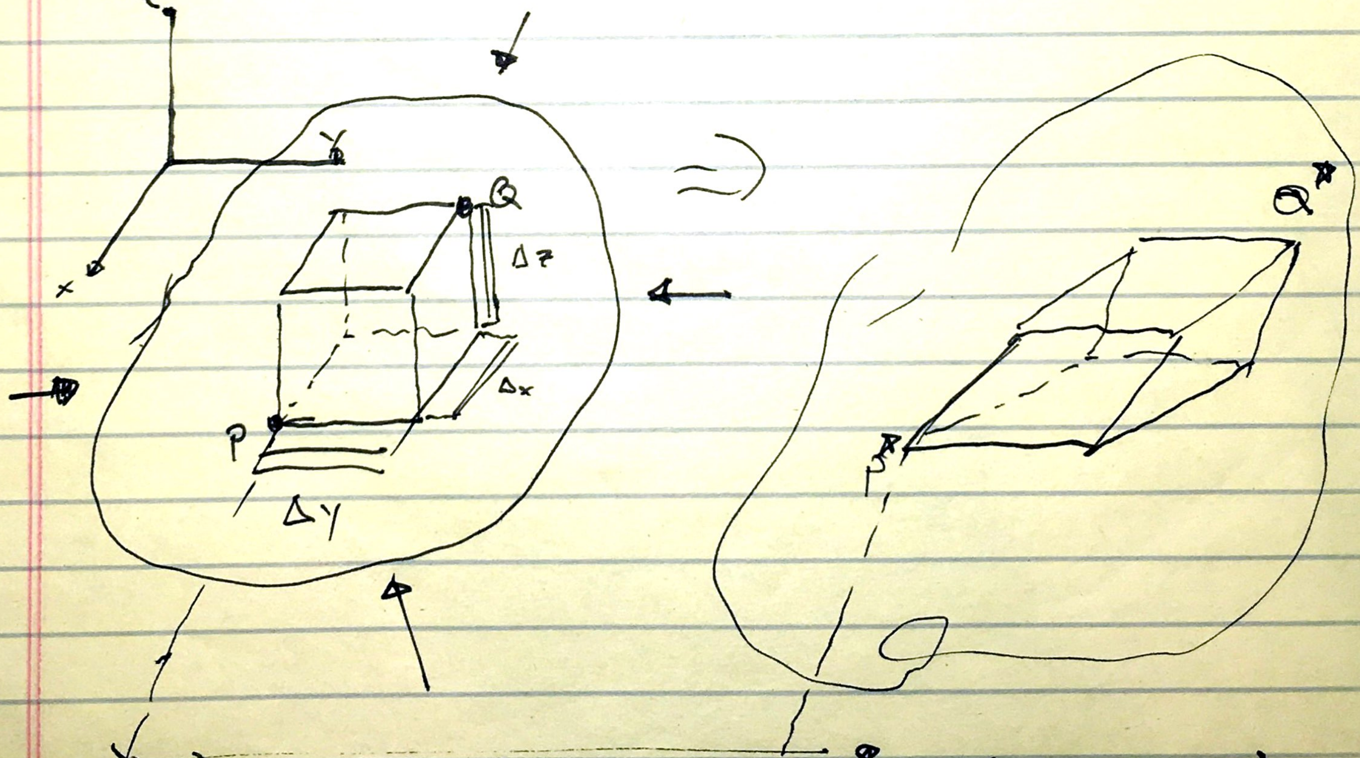
MACBOOK CHARGER

Lecture #3: Strain & Deformation

- Strain is purely geometric w/ no dependence on body material from deformation
- Displacement:
  - → Rigid Body Displacement: No Deformation
  - → Deformation → strain: depends on relative motion in body
- Deformation can be broken into
  - → Shear:  $\gamma = \cancel{\Theta} u(\pi/2 - \theta)$
  - → ~~linear~~ Normal: $\varepsilon = \Delta l / l_0$



$\vec{P} - \vec{P}^* = (u, v, w)$ & $Q - Q^* = (u^*, v^*, w^*)$

$$u^* = u + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z$$

H.O.T neglected

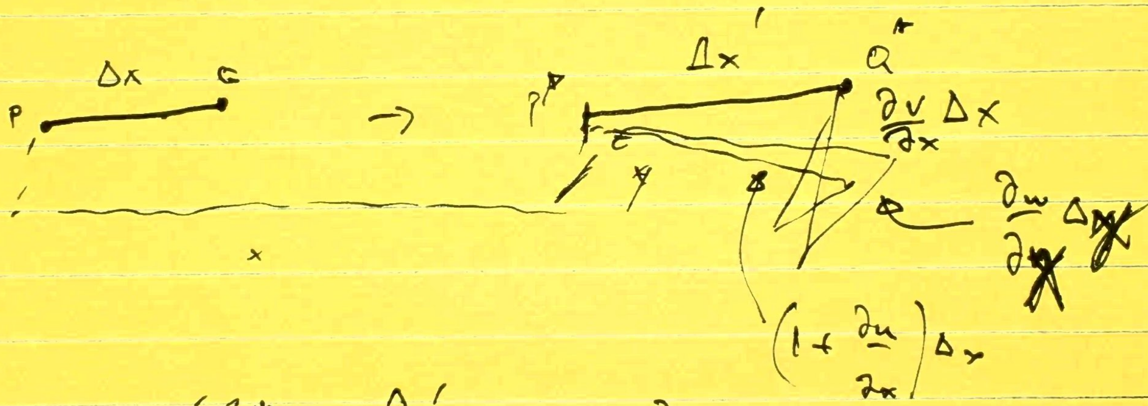
Based on Taylor Series Expansion

①
$$\left\{\begin{aligned} u^* &= u + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z \\ v^* &= v + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z \\ w^* &= w + \frac{\partial w}{\partial x}\Delta x + \frac{\partial w}{\partial y}\Delta y + \frac{\partial w}{\partial z}\Delta w \end{aligned}\right\}$$

H.O.T. neglected if small deformation and plane stay 3D.

Normal Strains :

Redraw lower



~~②~~ $\Sigma_{xx} = \cancel{\frac{\Delta x}{\Delta x}}\ \frac{\Delta x'}{\Delta x} - 1 \cong \frac{\partial u}{\partial x} + 1$

② $\Delta_x' = (1 - \Sigma_{xx})\Delta x$

③ $(\Delta_x')^2 = \left[\left(\left(1 + \frac{\partial u}{\partial x}\right)\Delta_x\right)^2 + \left(\frac{\partial u}{\partial x}\Delta x\right)^2 + \left(\frac{\partial w}{\partial x}\Delta x\right)^2\right]$

square ② -> ③

$$\left((1-\varepsilon_{xx})\underset{\Delta x}{\cancel{dx}}\right)^2 = \left[\left(1+\frac{\partial u}{\partial x}\right)\Delta x\right]^2 + \left(\frac{\partial v}{\partial x}\Delta x\right)^2 + \left(\frac{\partial w}{\partial x}\Delta x\right)^2$$

$$\downarrow$$

$$(1+\varepsilon_{xx})^2\cancel{(\Delta x)^2} = \left[1+2\frac{\partial u}{\partial y}+\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2\right]\cancel{\Delta x^2}$$

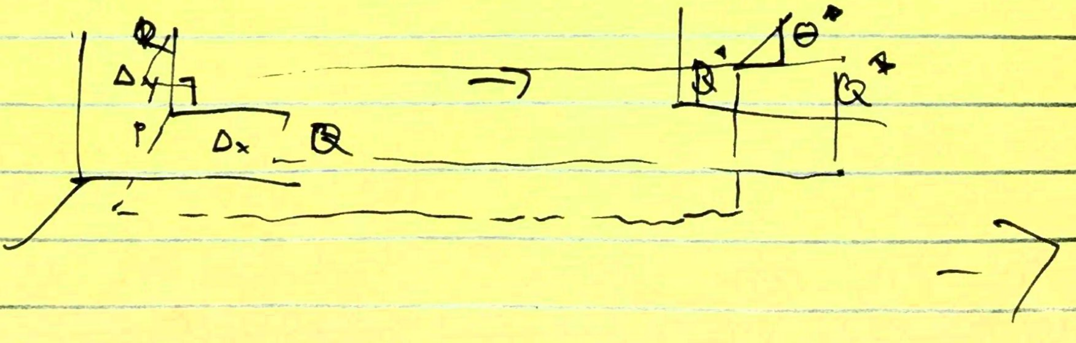
$$\boxed{\varepsilon_{xx} = \sqrt{1+2\frac{\partial u}{\partial x}+\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2}-1}$$

only works for small gague length

normal strain b/w two points in 1 direction under arbitrary forces

ε_{yy} & ε_{zz} are similar

Shear Strains



can define cosine of θ^* in deformations of
$\overline{PG}$ & $\overline{PR}$

note that $\left(\frac{\partial u}{\partial x_i}\right)^2$ will likely be v v small and can often be neglected, leading to

$$\boxed{\varepsilon_{xx} = \sqrt{1 + 2\frac{\partial u}{\partial x}} - 1} \longrightarrow$$

④

$$\boxed{\varepsilon_{xx} = \sqrt{\left(1 + \frac{\partial u}{\partial x}\right)^2} - \cancel{1} = \cancel{1} + \frac{\partial u}{\partial x}}$$

ε_{yy} & ε_{zz} are similar

$$\boxed{\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}}$$

Note: relatively simple for displacement field → strain field but not reverse

Rare to measure entire strain field experimentally

Strain Transformations are similar to stress transforms but replace

$$\sigma_{xx} \leftrightarrow \epsilon_{xx} \quad \& \quad 2\tau_{xy} \leftrightarrow \gamma_{xy}$$

must satisfy compatibility requirements

6 equations of ④ should be integrable ie strains must satisfy compatibility in order to satisfy displacements

Stress Strain Relationships

! assume linear relationship

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & \cdots & K_{16} \\ K_{21} & \ddots & & & \\ K_{31} & & \ddots & & \\ \vdots & & & \ddots & \\ \vdots & & & & \\ K_{61} & & & & K_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{xy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \end{bmatrix}$$

· K are coefficients of elasticity (36 → 21)
strain energy arguments twice reduce to 21

! · assume isotropic material (elastic constant in all all directions (21 → 2)

Elastic Constants Relationships (5 in total)

ν : Poisson's Ratio

E : Elastic Young's Modulus

μ : (G) shear modulus (torsional)

λ : Lamé's constant (mathematic)

K : Bulk modulus (used for uniform pressure)

using ν & E, can show 3D Hooke's Laws

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right]$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\left(\sigma_{xx} + \sigma_{zz}\right)\right]$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{yy}\right)\right]$$

$$\gamma_{xy} = \frac{2(1+\nu)}{E}\tau_{xy}$$

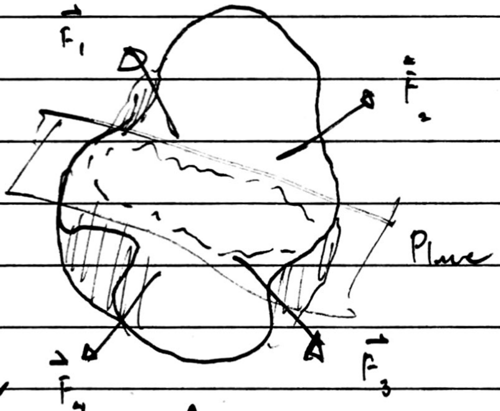
$$\gamma_{yz} = \frac{2(1+\nu)}{E}\tau_{yz}$$

$$\gamma_{xz} = \frac{2(1+\nu)}{E}\tau_{xz}$$

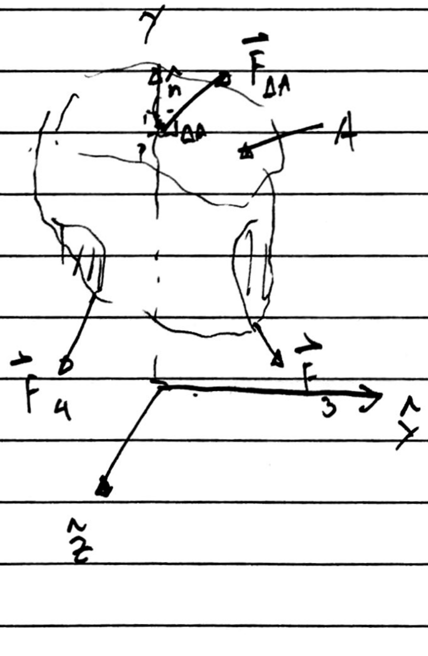
Experimental Mechanics

- Midterm: ~~prof~~ probably going to be over KZ's stuff
- ~~Fluid~~ Final: actual design of possible experimental
- Hours: 10:00 AM - 11:20 AM (not 11:50)
- What the fuck is AFM
- Soft Schedule
- Review Stress from Shigley (Chapter #1)
  - Review Machine Design Notes

!



- Under loading, find stress @ point with some types:
  - → Surface Force: contact of 2 bodies
  - → Body Forces: act on all elements in body (g, centrifugal, force field) 7

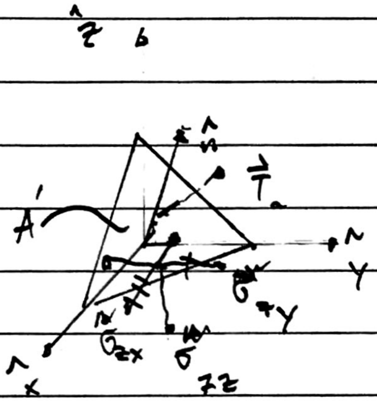


- Point P is arbitrary

$\vec{T}_P = \lim_{\Delta A \to 0} \frac{\vec{F}_{\Delta A}}{\Delta A}$; $\vec{T}_P$ is resultant stress at point P

$$\boxed{\Delta\vec{F}_n = a\,\Delta F_{n_x}\hat{i} + b\,\Delta F_{n_y}\hat{j} + c\,\Delta F_{n_z}\hat{k}}$$

but depends on $\hat{n}$



- on elementary tetrahedron

- Notation for planes

$\sigma_{(\text{plane})(\text{direction on plane})}$

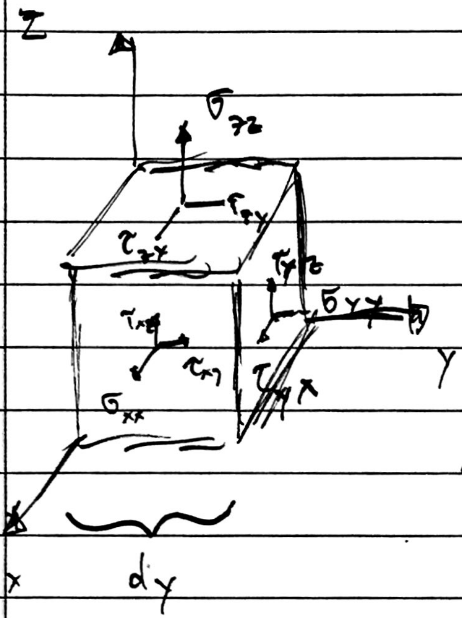
- $\vec{T}_a$ is resolved into 9 σ's

$$\Sigma F_x = 0 : \vec{T}_{nx} A - \sigma_{xx} A \cos(n,x) - \sigma_{zx} A \cos(n,z) - \sigma_{yx} A \cos(n,y) = 0$$

$$\therefore \vec{T}_{nx} = \sigma_{xx} \cos(n,x) + \sigma_{yx} \cos(n,y) + \sigma_{zx} \cos(n,z)$$

w/ $\cos(n,x)$ ... is projection of A on planes & same can be done for $\vec{T}_{ny}$ & $\vec{T}_{nz}$

$$\text{so } \boxed{\left\| \vec{T}_n \right\| = \sqrt{|T_{nx}|^2 + |T_{ny}|^2 + |T_{nz}|^2}}$$



$$\Rightarrow \begin{array}{|ccc|l} \sigma_{xx} & \tau_{xy} & \tau_{xz} & \text{— normal in } x \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} & y \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} & z \end{array}$$

any element in body is in equilibrium

$$\boxed{\sum \vec{F} = 0}$$

Stress Equations of Equilibrium $\boxed{\text{Find it}}$

12/5/18

DEC 05 2018

Experimental Mechanics Final Presentation

- Slide Outline First
- Break outline into dendritic growth has no good analogue in the physical process of Laser Metal Project. Modulii of the steel very hard.
  - + Need Supporting Equations from some site
  - + A couple of slides?
  - + Compare scale of dendritic growth to minimum length scale of the AFM

Problem

- Set Up with: FEM Research process on pg ②
  - + Processes of Dendritic Growth linked to solidification of both unary and multicomponent liquid, examples like ice, ...
  - i) Pure Liquid
  - ii) Solid Nucleus of Spherical Shape grows into the liquid
    - ○ Begin in the free vacuum or (-free growth)
  - iii) Liquid Solid Interface grows where regions before the boundary becomes super-cooled
    - ○ There is a great visualization video in 3D → gif
    - ○ Note how the boundary is not a clean line but rather
    - ○ Relate to "Φ" or state parameter and phase field approach
    - ○ Graph of the State as a product of the environment
  - iv) Its distinctive shape comes from how the crystal anisotropy allows for solidification in energy favorable direction
    - ○ State → Propogates in shape preserving manner → "dot ~~of~~ flux?"
    - ○ Energy favorable directions
    - ○ As the dendrite growth goes on, heat energy goes into the dendrite

Experimental Mechanics : 17 Sept 2018

Notes on

- Electrical Resistance Strain Gage : (ER)
  - → $R(\epsilon, \text{material})$: resistance is a function of strain and material
  - → Wheatstone bridges used for $\Delta R \to \Delta V$
- Strain Sensitivity

- L: length
- ρ : resistivity $[\Omega \cdot m]$
- A : area

→ with increased ε, increased R

→ "$R = \rho\, L/A$", but is it from "ρ" or from "L/A"

$$R = \rho\, L/A \quad \to \quad dR = \frac{1}{A}\left(L\, d\rho + \rho\, dL - L \frac{dA}{A}\right)$$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \underbrace{\frac{dL}{L}}_{\varepsilon} - \frac{dA}{A}$$

$$A = \frac{1}{4}\pi d^2$$

Poisson's Ratio or change in transverse cross section : $\nu = -\frac{\varepsilon_T}{\varepsilon_A}$

$$d_f = d_o\left(1 - \nu \frac{dL}{L}\right)$$

$$\frac{dA}{A} = -2\nu \frac{dL}{L} + \nu^2 \overset{\emptyset}{\cancel{\left(\frac{dL}{L}\right)^2}} \cong -2\nu \frac{dL}{L}$$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon + 2\nu\varepsilon$$

$$S_A = \frac{dR/R}{\varepsilon} = \underbrace{1 + 2\nu}_{\substack{\text{change in}\\ \text{dimension}}} + \underbrace{\frac{d\rho/\rho}{\varepsilon}}_{\substack{\text{change in}\\ \text{specific resistance}}} \quad : \text{Strain Sensitivity}$$

{ dependent of the free electrons of material }

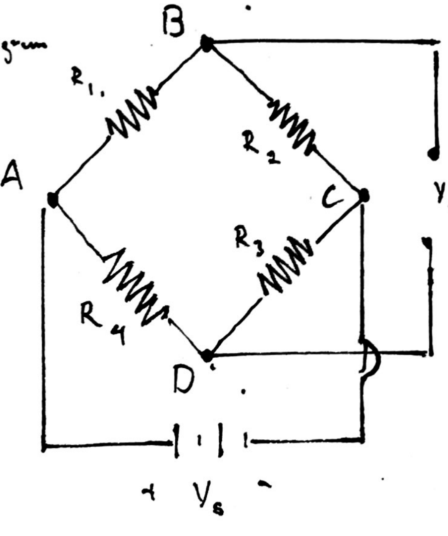
$\in [1.4, 1.7]$ $\qquad$ $\overset{?}{\in} [1, 3]$

Experimental Mechanics : 20 Sept 2018

Notes on

{ Guest Speaker
18 Oct }

- Midterm Presentation and Report
  - → Use Dr. KZ's research ~~of~~ as more familiar
  - → 5 pages use Terrall format
- Electrical Resistance Strain Gage Analysis
  - → $\frac{\Delta R}{R} = S_g \varepsilon_a$ ← Unknown strain, to be found
    - ← Gage property (see 17 Sept)
  - → Convert $\Delta R / R$ into ΔV, read by instrument called Wheatstone Bridge
- Wheatstone Bridge Equations
  - → Equations for circuit sensitivity & effective range
  - → Can go down to low resistance $\in [3, 5]\ \mu\varepsilon$
  - → Diagram



- $V_{AB} = \frac{R_1}{R_1 + R_2} V_s$
- $V_{AD} = \frac{R_4}{R_3 + R_4} V_s$

} $V_0 = V_{BD} = V_{AB} - V_{AD}$

- $V_0 = \frac{R_1(R_3 + R_4) - R_4(R_1 + R_2)}{(R_1 + R_2)(R_3 + R_4)} V_s$

- for $V_0 = 0$: $R_1 R_3 + \cancel{R_1 R_4} - \cancel{R_1 R_4} - R_2 R_4 = 0$

$$\underline{R_1 R_3 = R_2 R_4}$$

- → Place Strain Gage on any resistor, we convert $\varepsilon \rightarrow \Delta V$

$$V_0 (R_i) \rightarrow \Delta V_0 = f(R_i + \Delta R_i) \rightarrow \Delta V_0 = \frac{R_1 R_2}{(R_1 + R_2)^2} \left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} - \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4} \right) V_s$$

Neglect H.O.T.

→ Generally only one "R" is replaced so is "active"

→ Linear function of ΔR because we neglect Higer order terms

- Bridge Sensitivity

→ "S_c" or sensitivity of bridge one by constant V_s f. single active arm

→ $S_c = \dfrac{\Delta V_0}{\Delta R_1 / R_1} = \dfrac{r}{(1+r)^2} V_s$

+ ΔV_0 for only ΔR_1

+ $r =$ ~~$R_1 R_2$~~ R_2 / R_1

$$V_s = I_T (R_1 + R_2) = I_T R_T (1+r)$$

$$V_s = \underbrace{(1+r)}_{\cancel{\text{circuit efficency}}} \underbrace{\sqrt{P_T R_T}}_{\text{physical characteristics}}$$

→ $S_c = \dfrac{r}{1+r} \sqrt{P_T R_T}$

circuit efficency
generally less than 70%

P_T : power dissapated as heat

R_T : total resistance

$$P_T R_T : \in [1, 1000]\ \mathrm{W \cdot \Omega}, \text{ commercially}$$

- Strain Analysis

→ What is known affects gage placement

→ Stress State

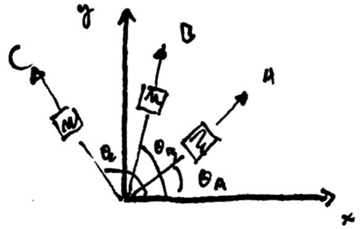
(i) Uniaxial Stress : 1 gage in stress axis

if E is known : $\sigma_{xx} = \sigma_1 = E \epsilon_{xx}$

(ii) Principal direction known but unknown magnitude : rectangular rosette in principal direction

(iii) Stress field unknown : 3 element rosettes needed

→ 3 Element Rosettes in general



• measuring ε_A, ε_B, ε_C

$\downarrow$

ε_{xx} + ε_{yy} ! by Mohr's Circle

Experimental Mechanics : 24 Sept. 2018

N Notes on Strain Analysis

- Stress transformation

$$\left.\begin{aligned} &\to \varepsilon_A = \varepsilon_{xx}\cos^2\theta_A + \varepsilon_{yy}\sin^2\theta_A + \gamma_{xy}\sin\theta_A \overset{\cos}{\cancel{\sin}}\,\theta_A \\ &\to \varepsilon_B = \varepsilon_{xx}\cos^2\theta_B + \varepsilon_{yy}\sin^2\theta_B + \gamma_{xy}\sin\theta_B \cancel{\sin}\cos\theta_B \\ &\to \varepsilon_C = \varepsilon_{xx}\cos^2\theta_C + \varepsilon_{yy}\sin^2\theta_C + \gamma_{xy}\sin\theta_C\cos\theta_C \end{aligned}\right\}$$

Principal Stress by Mohr's Circle

- Mohr's Circle or

$$\to \varepsilon_{1,2} = \frac{1}{2}\left(\varepsilon_{xx} + \varepsilon_{yy}\right) \pm \frac{1}{2}\sqrt{\left(\varepsilon_{xx} - \varepsilon_{yy}\right)^2 - \gamma_{xy}^2}$$

$$\to \tan 2\phi = \frac{\gamma_{xy}}{\varepsilon_{xx} + \varepsilon_{yy}}$$

- Correction factors of Strain Gages (Transverse Effects)

$$\to \varepsilon_a = \underset{\substack{\uparrow \\ \text{Measured Value}}}{\varepsilon_a'}\,\frac{1 - \nu K_t}{1 + K_t\left(\varepsilon_t/\varepsilon_a\right)} \quad ; \quad K_t = \frac{S_t}{S_a}$$

S_t :: transverse sensitivity

S_a :: axial sensitivity

$\varepsilon_t/\varepsilon_a$: Must be a known ratio

→ Correction Factor (S_a)

$$S_g = \frac{\Delta R/R}{\varepsilon_a} \quad ; \quad S_G^* = S_G\left(\frac{1}{C.F}\right)$$

; C.F is provided correction factor

$$\left.\begin{aligned} \to \varepsilon_{xx}' &= \frac{1}{1 - \nu K_t}\left(\varepsilon_{xx} + K_t\,\varepsilon_{yy}\right) \\ \varepsilon_{yy}' &= \frac{1}{1 - \nu K_t}\left(\varepsilon_{yy} + K_t\,\varepsilon_{xx}\right) \end{aligned}\right\}$$

Apparent related to actual strain in 2 equations

- Optical Measurements of Strain

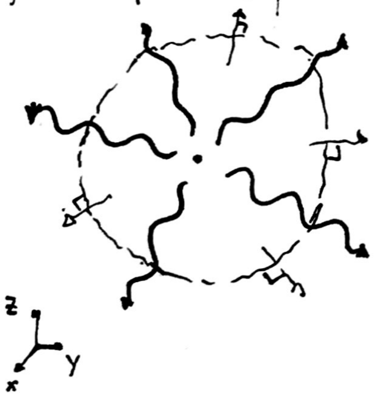
→ Topics : Wave theory of Light → Wave Equation → Superposition → Polarization . . .

Experimental Mechanics : 24 Sept 2018

Notes on Optics

- Of the $\vec{E}$-$\vec{H}$ fields, we're going to mostly neglect magnetic field
- By wave theory of light by Maxwell, $\vec{E}$ & $\vec{H}$ fields are in phase & orthogonal with waves being transverse to travel
- Parameters
  - → λ : wavelength
  - → f : frequency

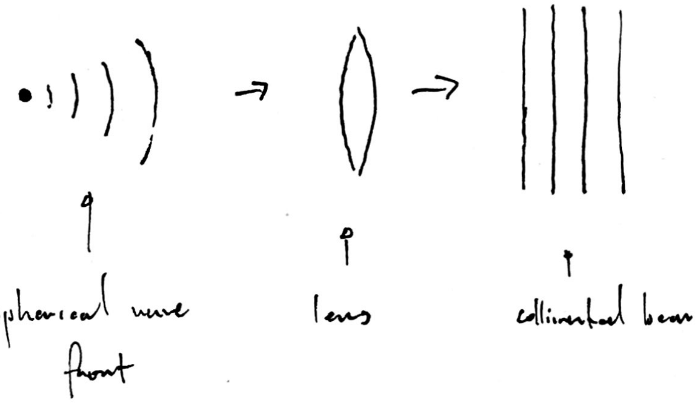
  } $\lambda f = c$; note that only monochromatic light associated to a singular λ. most a spread
- Wave Theory of light
  1) Wave front description



- The region of same phase expands as a sphere, given an optically homogenous medium
- Spherical Region is the wavefront
- Rays pass through wave front orthogonal to surface which indicates direction of energy travel

→ with travel, the wavefront surface can be approximated as planar

→ a lens can change the shape of the wave front



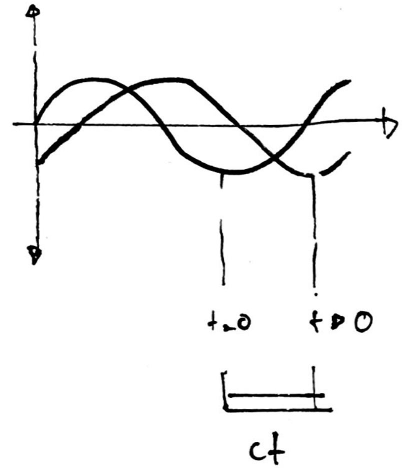
- Wave Equation

→ $E = f(z-ct) + g(z+ct)$: general as not necessarily sinusoidal

$\uparrow$ positive z $\uparrow$ z^-

magnitude of light as function of distance traveled ; t: time

→ Optical Effects



|| Amplitude: A

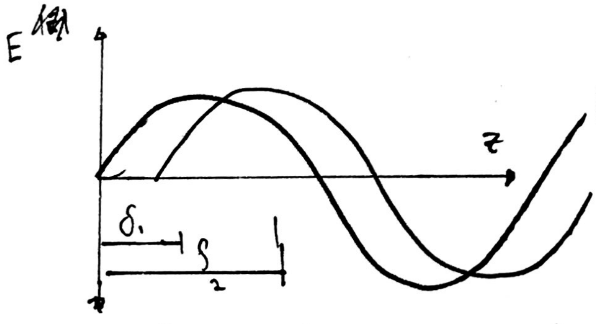
$$E = f(z-ct) = A\cos\left(\frac{2\pi}{\lambda}(z-ct)\right)$$

↳ $T = \lambda/c = 1/f$; $f = \frac{\omega}{2\pi}$

↳ Wave Number $\xi = 2\pi/\lambda$

ξ should be "xi"

- Wave Interaction



$$\left.\begin{aligned} E_1 &= a_1\cos\left(\frac{2\pi}{\lambda}(z-ct)\right) \\ E_2 &= a_2\cos\left(\frac{2\pi}{\lambda}(z-ct)\right) \end{aligned}\right\} \text{for } a_1 = a_2$$

$\left.\begin{aligned} &\delta_1 : \text{initial phase of } E_1 \\ &\delta_2 : \text{initial phase of } E_2 \end{aligned}\right\}$ δ is the linear phase difference : $\delta = \delta_2 - \delta_1$

Use of superposition to measure is basis of Moiré, photoelasticity, and interferometry

$$\left.\begin{aligned} E_1 &= a_1\cos\left(\frac{2\pi}{\lambda}(z_0+\delta_1+ct)\right) = a_1\cos(\phi_1-\omega t) \\ E_2 &= a_2\cos\left(\frac{2\pi}{\lambda}(z_0+\delta_2+ct)\right) = a_2\cos(\phi_2-\omega t) \end{aligned}\right\} \phi_i = \frac{2\pi}{\lambda}(z_0-\delta_i)$$

→ if in same plane, we can write

$$\left.\begin{aligned} &E = E_1 + E_2 = a\cos(\phi - \omega t) \\ &\cdot\ a^2 = a_1^2 + a_2^2 + 2a_1a_2\cos(\phi_2 - \phi_1) \\ &\cdot\ \phi = \tan^{-1}\left(\frac{a_1\sin\phi_1 + a_2\sin\phi_2}{a_1\cos\phi_1 + a_2\cos\phi_2}\right) \end{aligned}\right\}$$

Resultant "E" has the same frequency but $a(a_1, a_2, \phi_1, \phi_2)$ and $\phi(a_1, a_2, \phi_1, \phi_2)$

→ Special Case of $a_1 = a_2$: $a^n = \sqrt{4 a_0^2 \cos^2 \frac{\pi\delta}{\lambda}}$; $a \propto \sqrt{I}$; I : intensity

$$I \propto a^2$$

$$\propto 4a^2\cos^2\left(\frac{\pi\delta}{\lambda}\right)$$

so that as intensity changes its a function of the phase difference only

I_{max} is when $\{\delta = n\lambda,\ n \in \mathbb{I}\} \rightarrow I_{max} = 4a^2$

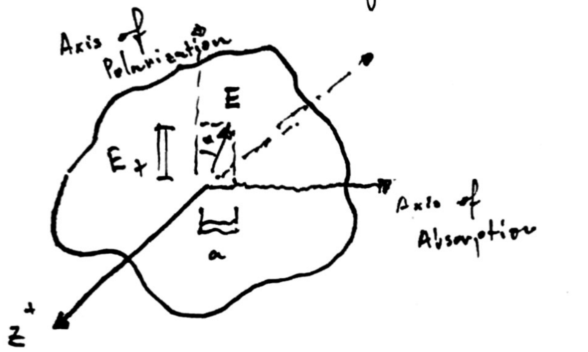
I_{min} $\{\delta = \frac{2n+1}{2}\lambda,\ n \in \mathbb{I}\} \rightarrow \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \ldots = 0$

+ These equations are the basis of interference methods

Experimental Mechanics: 1 Oct 2017

Notes on Birefringence

- Polariscope: use linear polarizers through a transparent model material
- Plane polarizer for plane wave; circularly with wave plane
- Linear or Plane Polarizer
  - → Thin material between light source and continues



$\vec{E}$: light vector

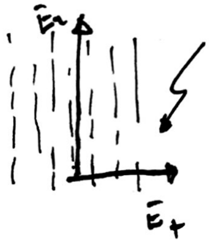
E_t: transmitted light

E_a: absorbed light

α: angle b/w $\vec{E}$ & axis of polarization

- → Equation for Light Vector: $\vec{E} = a\cos\frac{2\pi}{\lambda}(z_0 - ct) \rightarrow a\cos(\omega t)$
  - + initial phase (δ) of light can be neglected
  - + $f = c/\lambda$; $\omega = 2\pi f$
- → Break into E_t and E_a
  - + $E_a = a\cos(\omega t)\sin\alpha$
  - + $E_t = a\cos(\omega t)\cos\alpha$

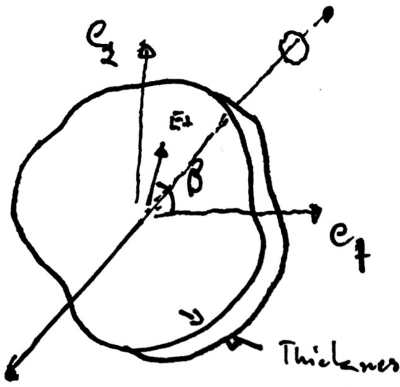
- Polaroid Filters



Iodoquinine crystals in nitrocellulose } J sheet: older & replaced w/ H Sheets

- Wave Plate: transmits 2 orthogonal light vectors with different velocities

→ Wave plate <u>after</u> linear polarizer



Thickness "h"

- β : angle
- e_1 : index of refraction axis (#1) / fast axis
  - → n_1 : index of refraction
  - → c_1 : velocity of propagation
- Labeled so that $c_1 > c_2$

+ $E_{t_1} = E_1 \cos\beta = a\cos(\omega t)\cos\alpha\cos\beta = k\cos\omega t\cos\beta \; \}\; k = a\cos\alpha$
+ $E_{t_2} = E_2 \sin\beta = k\cos\omega t\sin\beta$
+ δ : phase shift de to $c_1 \neq c_2$; n w.r.t. air

$$\left.\begin{aligned} \delta_1 &= hn_1 - hn \\ \delta_2 &= hn_2 - hn \end{aligned}\right\} \delta = h(n_2 - n_1);$$

relative linear phase difference

$$\Delta = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda}h(n_2 - n_1)$$: Angular ~~h~~ phase shift

→ Classification

+ Quarter wave : $\Delta = \pi/2$
+ Half wave : $\Delta = \pi, 2\pi, \ldots$

→ Results in

$$\left.\begin{aligned} &+ E_{t,1}' : k\cos\beta\cos\omega t \\ &+ E_{t,2}' : k\sin\beta\cos(\omega t - \Delta) \end{aligned}\right\} E_t' = \sqrt{E_{t_1}^2 + E_{t_2}^2} \Rightarrow \cancel{|E_T| = k\cos}$$

→ Angle Relative to e_1 : $\tan\gamma = \dfrac{E_{t_2}'}{E_{T_1}'} = \dfrac{\cos(\omega t - \Delta)}{\cos(\omega t)}\tan\beta$

- Conditioned Light using wave plates and linear polarizers
  - → 3 defined cases
  - → Plane Polarized Light : $\beta = 0$; Δ has no restriction

$$E_t' = k\cos(\omega t)$$

$$\gamma = 0$$

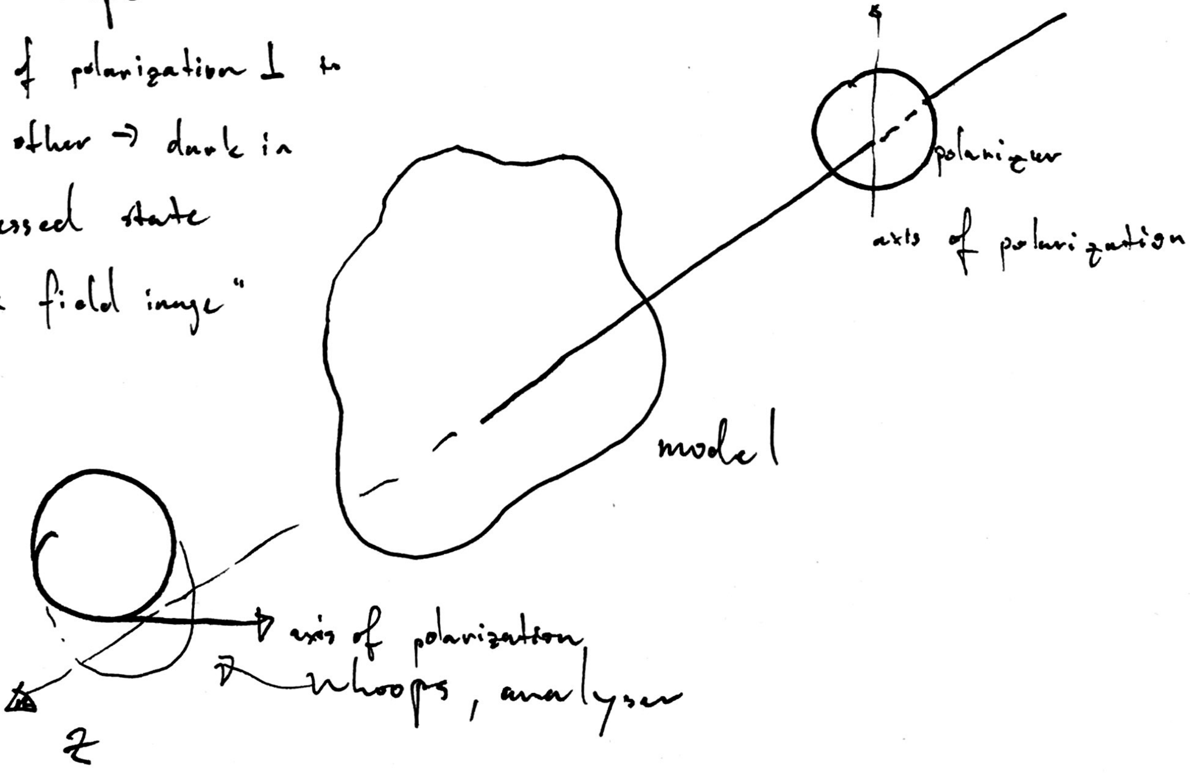
  - → Circularly Polarized Light : $\Delta = \pi/2$; $\beta = \pi/4$ for left ; $\beta = 3\pi/4$ for right

$$\left.\begin{aligned} E_t' &= \frac{\sqrt{2}}{2}k \\ \gamma &= \omega t \end{aligned}\right\} \text{Constant Magnitude, helical shape}$$

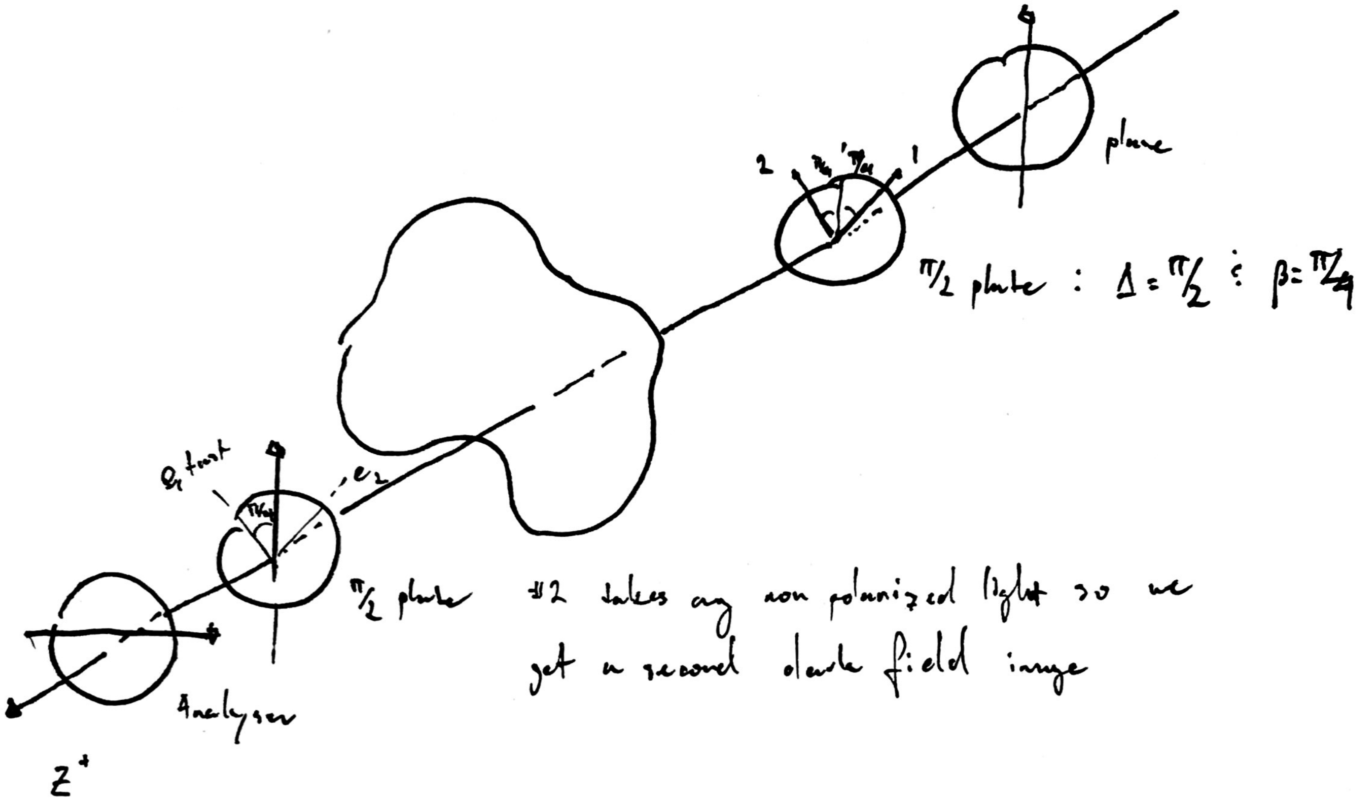
  - → Elliptically Polarized : quarter wave plate ($\Delta = \pi/2$)

$$\left\{ \beta \neq \frac{n\pi}{2} \; , \; n \in \mathbb{I} \right\}$$

- Plane Polariscope
  - → Axis of polarization ⊥ to each other → dark in unstressed state "dark field image"

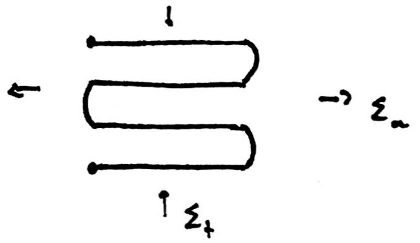


- Circular Polarizer



#2 takes any non polarized light so we get a second dark field image

- Most commonly used is advance or constantan w/ $S_A \sim 2.1$
  - → 45% Ni, 55% Cu is nice because of linear for elastic ε and even into plastic
  - → High "ρ" and insensitive to ΔT
- Gage Construction
  - → needs to be relatively long (~ 0.1 m) but we want ε at a specific location
  - → Put onto grid pattern



- 1930's Ruge / Simmons : wire on paper
- 1950's Sanders & Roe : photo etched on metallic foil usually bonded to plastic for durability & insulation

  - → need special appreciation for bonding to ~~specimine~~ specimin
  - → $L_{gage} \in [200\,\mu m,\ 100\,mm]$
- Arrays
  - → Linear Array :  for capturing a line of ε
  - → Rosettes
    - + 90° Rosette :  if we know principal strain
    - + 45° Rosette :  & 60° 
- Gage Sensitivity and Gage Factor
  - → using a grid adds transverse component "$\varepsilon_\perp$" which we don't want to measure
  - → $S_A = \frac{dR/R}{\varepsilon} \approx \frac{\Delta R/R}{\varepsilon}$ must be transformed for biaxial strain
  - → $\Delta R/R = \underbrace{S_a \varepsilon_a}_{\text{Axial Sensitivity}} + \underbrace{S_t \varepsilon_t}_{\text{Transverse Sensitivity}} + \underbrace{\cancelto{0}{S_s \gamma_{at}}}_{\text{Shear sensitivity}} = S_a(\varepsilon_a + K_t \varepsilon_t)$; $\varepsilon_t = -\nu \varepsilon_a$

    "$K_t = S_t/S_a$" or transverse sensitivity factor
  - → S_g = Gage factor

$$\Delta R/R = S_g \varepsilon_a = S_a \varepsilon_a (1 - \nu K_t) \ ; \ S_g = S_a(1-\nu K_t) \quad w/ \ [S_g, S_a, K_t] \in \begin{bmatrix} 1.96 \to 2.13, \\ 1.93 \to 2.14 \\ -4.2 \to 1.8 \end{bmatrix}$$

- Error comes from only considering axial strain, knowing there is some transverse strain

$$\Delta R/R = \frac{S_g \varepsilon_a}{1-\nu K_t}\left(1 + K_t \frac{\varepsilon_t}{\varepsilon_a}\right) \rightarrow \varepsilon_a = \frac{\Delta R/R}{S_g} \frac{1-\nu K_t}{1+K_t(\varepsilon_t/\varepsilon_a)}$$ is true axial strain

apparent $$\varepsilon_a' = \frac{\Delta R/R}{S_g} \rightarrow \varepsilon_a = \varepsilon_a' \frac{1-\nu K_t}{1+K_t(\varepsilon_t/\varepsilon_a)}$$

so that % error from neglecting transverse component

$$\mathcal{E} = \frac{\varepsilon_a - \varepsilon_a'}{\varepsilon_a}(100) = \cancel{\mathcal{E}} \; \frac{K_t(\varepsilon_t/\varepsilon_a + \nu)}{1-\nu K_t}$$?

$\varepsilon_t/\varepsilon_a$ is a prescribed value from problem; independent of ν

Experimental Mechanics

- Stress-Optics Laws
- Isoclinic/Isochromatic Fringe Patterns

Stress Optics

- Linear elastic model material
  - Stress free → η_0: index of refraction
  - stressed state → temporary birefringence
    - $\eta_1, \eta_2, \eta_3 \propto \sigma$

$\eta \to n$, oops

  - Related by
    - $\eta_1 - n_0 = c_1 \sigma_1 + c_2 (\sigma_2 + \sigma_3)$
    - $n_2 - n_0 = c_1 \sigma_2 + c_2 (\sigma_3 + \sigma_1)$
    - $n_3 - n_0 = c_1 \sigma_3 + c_2 (\sigma_1 + \sigma_2)$
  - Generally 2D for simplification : $\sigma_3 = 0$
    - $n_1 - n_0 = c_1 \sigma_1 + c_2 \sigma_2$
    - $n_2 - n_0 = c_1 \sigma_2 + c_2 \sigma_1$
- Retardation Effect comes from material behavior in waveplate like manner, rewrite w/o n_0

$$n_2 - n_1 = (c_2 - c_1)(\sigma_1 - \sigma_2) = c\,(\sigma_1 - \sigma_2) \;;\; c = c_2 - c_1$$

$$n_3 - n_2 = C(\sigma_2 - \sigma_3)\,;\quad n_1 - n_3 = C(\sigma_3 - \sigma_1)$$

- Brewsters $[\text{Pa}^{-1}]\, 10^{-12}$

- $\sigma_1 \geq \sigma_2 \geq \sigma_3 \rightarrow n_3 \geq n_2 \geq n_1$

- $\Delta = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda} h\,(n_2 - n_1)$ Substitute

Waveplate height

$$\cancel{\Delta_{12}} = \frac{2\pi h C}{\cancel{C}} = \frac{2\pi h c}{\lambda}(\sigma_2 - \sigma_1)$$

Magnitude of relative retardation

$$\sigma_1 - \sigma_2 = \frac{N f_\sigma}{h}\,;\quad N = \frac{\Delta}{2\pi}$$

- N : fringe order ; cycles retardation

normalizing relative retardation

- f_σ : material "fringe value" property

$f_\sigma(\lambda)$ or λ dependent ; calibration constant

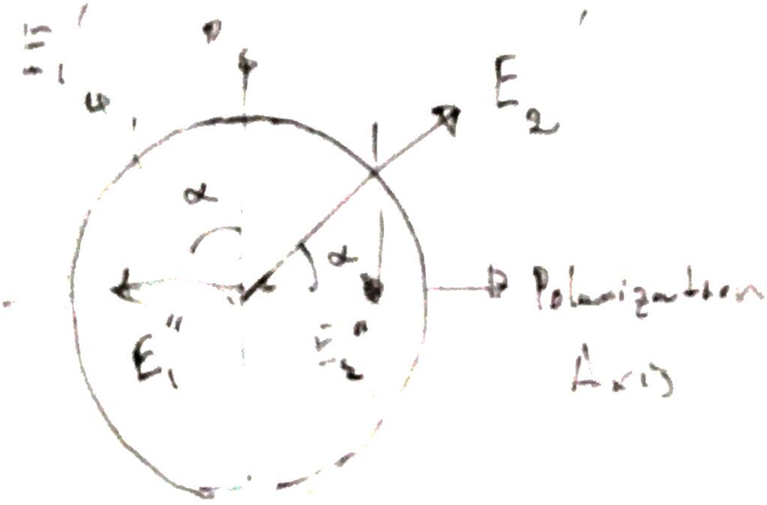
$E_{t(p)}$: light entering

$E_{py} = k \cos \omega t$

$$E_1 = k \cos \alpha \cos \omega t \qquad \Rightarrow \qquad E_1' = k \cos \alpha \cos(\omega t - \Delta_1)$$

$$E_2 = k \overset{\sin}{\cancel{\cos}} \alpha \cos \omega t \qquad\qquad E_2' = k \overset{\sin}{\cancel{\cos}} \alpha \cos(\omega t - \Delta_2)$$

$$\Delta_1 = \frac{2\pi h}{\lambda}(n_1 - 1) \; ; \; \Delta_2 = \frac{2\pi h}{\lambda}(n_2 - 1)$$



$$E_{ax} = E_2'' - E_1''$$

$$= E_2' \cos\alpha - E_1' \sin\alpha$$

$$= k\sin\alpha\cos(\omega t - \Delta_2)\cos\alpha$$

$$- k\cos\alpha\cos(\omega t - \Delta_1)\sin\alpha$$

$$= k\sin\alpha\cos\alpha\left[\cos(\omega t - \Delta_2) - \cos(\omega t - \Delta_1)\right]$$

$$= k\sin 2\alpha \underbrace{\sin\left(\frac{\Delta_2 - \Delta_1}{2}\right)}_{\text{Amplitude}} \underbrace{\sin\left(\omega t - \frac{\Delta_2 + \Delta_1}{2}\right)}$$

$\frac{\Delta_2 + \Delta_1}{2}$; average angular phase shift wrt circuit, not really useful

$$\Delta_2 - \Delta_1 = \Delta = \frac{2\pi h}{\lambda}(n_2 - n_1) = \frac{2\pi hc}{\lambda}(\sigma_1 - \sigma_2)$$

Relative retardation $\propto$ Intensity !

$I = \frac{1}{4} \sin^2 2\alpha \sin^2\left(\frac{\Delta}{2}\right)$

- Extinction of Light (i.e. $I = 0$)

i) $\sin^2 2\alpha = 0$: principal stress direction

if $\alpha = \frac{n\pi}{2}$, $n \in \mathbb{N}$

- no restriction of location → full field
- fringe : loci of points w/ σ coincident with polarization angle
- isoclinic Case

ii) $\sin^2\left(\frac{\Delta}{2}\right) = 0$: $\frac{\Delta}{2} = n\pi$

$$\Delta = \frac{2\pi h c}{\lambda}(\sigma_1 - \sigma_2)$$

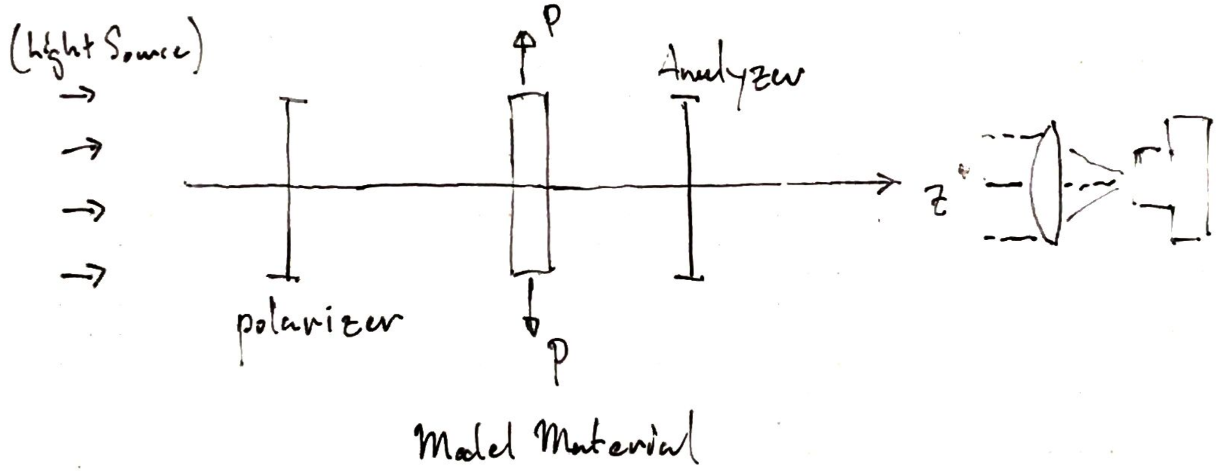
$$\underset{\uparrow}{n} = \frac{\Delta}{2\pi} = \frac{hc}{\lambda}(\sigma_1 - \sigma_2)$$

Order of Extinction

$I = 0$ for if $\sigma_1 = \sigma_2$

Experimental Mechanics: 9 October 2018

- Lecture #11: Photoelasticity & Fringes (Isoclinic vs. Isochromatic)
  - Photoelastic Materials
  - Analysis of Fringe Patterns
- Linear Polariscope



$$I = K \underbrace{\sin^2\left(\frac{\Delta}{2}\right)}_{0} \underbrace{\sin^2(2\alpha)}_{0} = 0$$

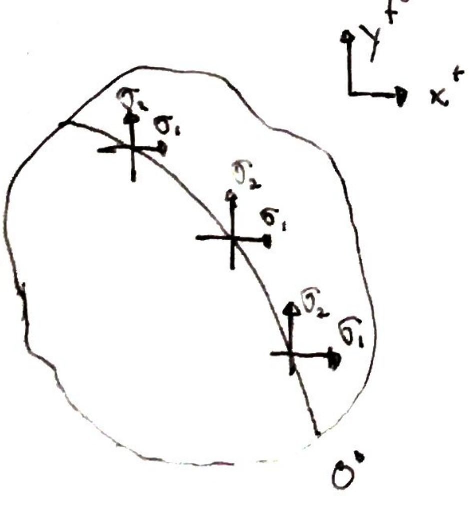
or

- if $\sin^2 2\alpha$: Isoclinic ; or principal stress inline w/pol.
- " $\sin^2 \frac{\Delta}{2}$: Isochromatic ; $\frac{\Delta}{2}$

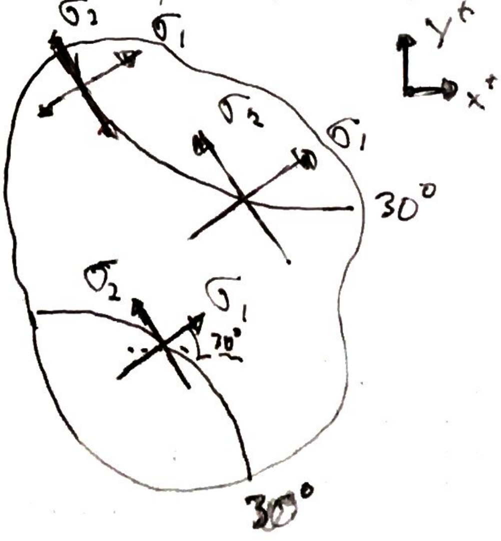
- Isoclinic : $I = 0$ because $\sin^2 2\alpha = 0$

$$2\alpha = n\pi , \quad n = 1, 2, 3 \ldots$$

  - Principal stress in axis of polarizer
  - Isoclinic parameter: rotate polarizer to varied orientation to create a full field map of stress
  - Overlay of outline of isoclinic fringe create 2D stress direction
  - Isoclines must pass through isotropic or singular point; $\sigma_1 = \sigma_2$
  - Isoclinic of atleast one parameter will have coincided with axis of symmetry.
  - Stress/loaded as singular ∴ isoclinic of all parameters



a) Polarizer set to 0°



b) Polarizer set to 30°

- Isochromatic: $I = 0$ because $\sin^2 \frac{\Delta}{2} = 1$

$$\Delta/2 = n\pi \; ; \; n = 1, 2, 3 \ldots$$

$$n = \frac{\Delta}{2\pi} = \frac{hC}{\lambda}(\sigma_1 - \sigma_2)$$

  - n = fringe order
  - Δ = relative retardation ($\propto [\sigma_1 - \sigma_2]$) & is an integer multiple of λ of light
  - Give lines of principal stress difference $\sigma_1 - \sigma_2$, we know $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$ by Mohr's Circle

Quarter Wave Plates before & after model material @ ±45° isoclinics will be removed

- Ultimately, want to know n^{th} order of fringe order
  - Might be done by tracking loading lines
  - Through movement of line under loading.
  - With white light, 0^{th} order is black and successive are λ
  - Isotropic ($\sigma_1 = \sigma_2$) will always be black

- Sample Pieces has been "model material", as few materials are isotropic and stress induced birefringence
- Still, can be loaded to see a full field stress distribution by applying stressed geometries.

· Generally a polycarbonate:

+ Homolite 100
+ Polycarbonate

Low Creep

+ Epoxy Resin
+ Urethane Rubber

Older Materials