September 12, 2019

MANE 6520-01 Fluid Mechanics
Fall Semester 2019
Problem Set #1
Due: September 26, 2019

1) At the thermodynamic critical point the pressure first and second derivatives with specific volume vanish.

a) Prove that a gas behaving according to the perfect gas equation of state does not have a thermodynamic critical point.

b) Derive the formulas for the parameters *a* and *b* in the Van der Waals equation of state as a function of the critical pressure and temperature of the gas, and the specific gas constant. Determine the value of the compressibility factor of the Van Der Waals gas.

c) The Redlich-Kwong equation of state is given by

$$p = RT / (v-b) - a\, T^{-1/2} / [v(v+b)]$$

Determine the values of *a* and *b* as a function of the critical pressure and temperature of the gas, and the specific gas constant. Determine the value of the compressibility factor of the Redlich-Kwong gas at the critical point.

2) Which of the following expressions are allowed in index notation?
1. $x = a_i b_i$. 2. $u_j = T_{ij} v_j$ 3. $u_i = T_{ij} v_j$ 4. $e_i = A_{ij} v_j + t_i$
5. $a_{kl} = t_{lk} + v_{kwl}$ 6. $b_i = a_{ij} c_j + t_i$ 7. $c_i = \epsilon_{ijk} b_{jck}$ 8. $t = a_{ii} + g_{jj} + e_{kk}$

3) Like a matrix, a tensor $\boldsymbol{A}=(a_{ij})$ of second order is symmetric when $a_{ij}=a_{ji}$ and skew-symmetric when $a_{ij}=-a_{ji}$. Also, the transpose tensor, $\boldsymbol{A}^t=(a_{ji})$.
Consider the tensor of second order:

$$\mathbf{T} = 1\, \mathbf{e_x}\, \mathbf{e_x} + 2\, \mathbf{e_x}\, \mathbf{e_y} + 3\, \mathbf{e_x}\, \mathbf{e_z} + 4\, \mathbf{e_y}\, \mathbf{e_y} + 5\, \mathbf{e_y}\, \mathbf{e_y} + 6\, \mathbf{e_y}\, \mathbf{e_z} + 7\, \mathbf{e_z}\, \mathbf{e_x} + 8\, \mathbf{e_z}\, \mathbf{e_y} + 9\, \mathbf{e_z}\, \mathbf{e_z}$$

Write its symmetric, $\boldsymbol{S}=(T+T^t)/2$, and skew-symmetric, $\boldsymbol{R}=(T-T^t)/2$ tensors.

4) Let $\boldsymbol{S}$ be a symmetric tensor and $\boldsymbol{R}$ be a skew-symmetric tensor of $\boldsymbol{T}$.
Show that $\boldsymbol{S} : \boldsymbol{R}=0$.

5) Use tensor analysis developed in class to prove the identity:
$\boldsymbol{V} \bullet \nabla \boldsymbol{V} = \frac{1}{2} \nabla (\boldsymbol{V} \bullet \boldsymbol{V}) - \boldsymbol{V} \times \boldsymbol{\omega}$, where $\boldsymbol{V}$ is the velocity vector and $\boldsymbol{\omega} = \nabla \times \boldsymbol{V}$.