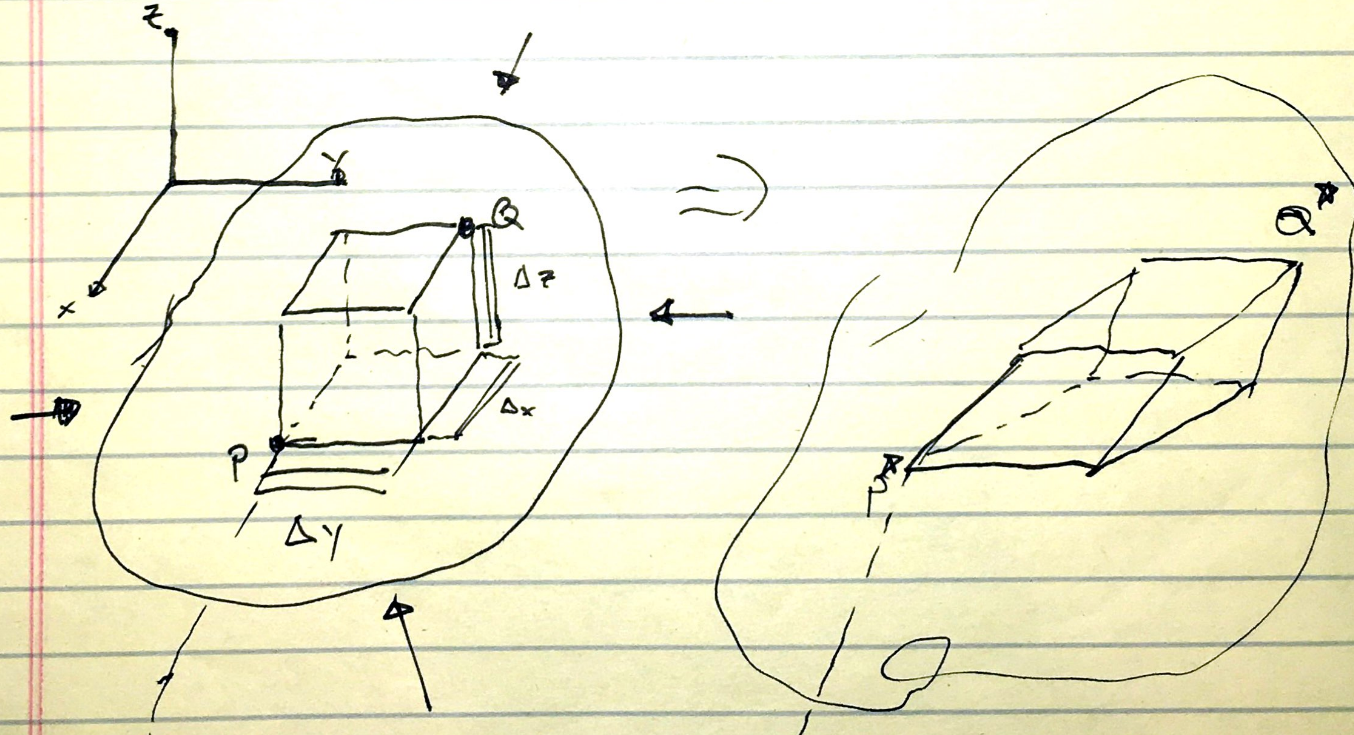
# MACBOOK CHARGER

## Lecture #3: Strain & Deformation

- Strain is purely geometric w/ no dependence on body material from deformation
- Displacement:
  - → Rigid Body Displacement: No Deformation
  - → Deformation → strain: depends on relative motion in body
- Deformation can be broken into
  - → Shear: $\gamma = \cancel{\theta}\left(\pi/2 - \theta\right)$
  - → ~~Linear~~ Normal: $\varepsilon = \Delta l / l_0$




$$\vec{P} - \vec{P^*} = (u, v, w) \quad \& \quad Q - Q^* = (u^*, v^*, w^*)$$

$$u^* = u + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z$$

H.O.T neglected

Based on Taylor Series Expansion

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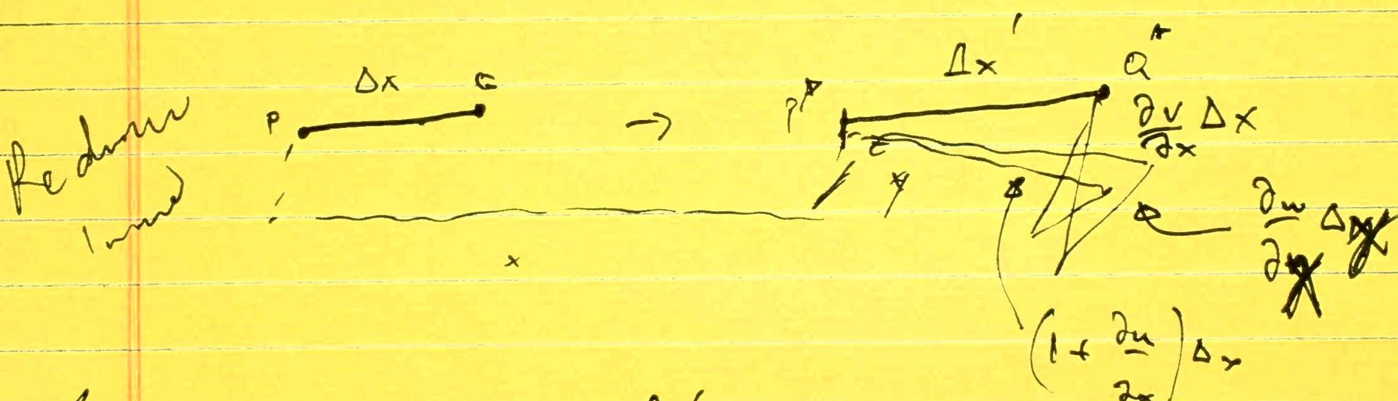
$$\left\{\begin{aligned} u^* &= u + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z \\ v^* &= v + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y + \frac{\partial u}{\partial z}\Delta z \\ w^* &= w + \frac{\partial w}{\partial x}\Delta x + \frac{\partial w}{\partial y}\Delta y + \frac{\partial w}{\partial z}\Delta w \end{aligned}\right\}$$

H.O.T. neglected if small deformation and plane stay 2D.

Normal Strains:




~~2~~ $\varepsilon_{xx} = \frac{\Delta x'}{\Delta x} - 1 \simeq \frac{\partial u}{\partial x} + 1$

2 $\Delta x' = (1 - \varepsilon_{xx})\Delta x$

3 $(\Delta x')^2 = \left[\left(\left(1 + \frac{\partial u}{\partial x}\right)\Delta x\right)^2 + \left(\frac{\partial u}{\partial x}\Delta x\right)^2 + \left(\frac{\partial w}{\partial x}\Delta x\right)^2\right]$

square ② -> ③

$$\left((1-\varepsilon_{xx})\,\cancel{\Delta x}\right)^2 = \left[\left(1+\frac{\partial u}{\partial x}\right)\Delta x\right]^2 + \left(\frac{\partial v}{\partial x}\,\Delta x\right)^2 + \left(\frac{\partial w}{\partial x}\,\Delta x\right)^2$$

$\Delta x$

$$\downarrow$$

$$(1+\varepsilon_{xx})^2\cancel{(\Delta x)^2} = \left[1 + 2\frac{\partial u}{\partial x} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right]\cancel{\Delta x^2}$$

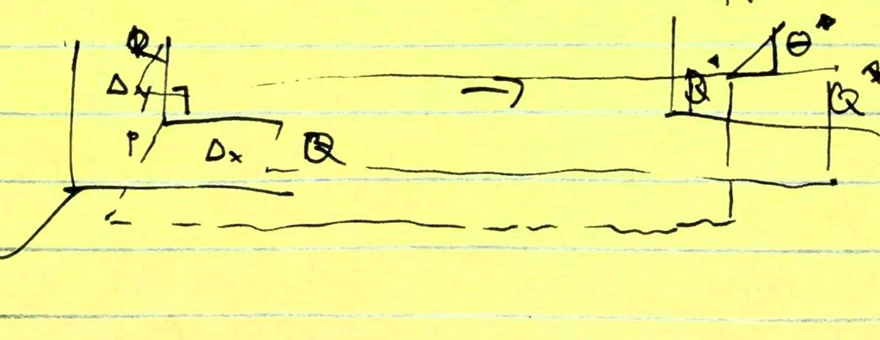
$$\boxed{\varepsilon_{xx} = \sqrt{1 + 2\frac{\partial u}{\partial x} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2} - 1}$$

only works for small gague length

normal strain b/w two points in 1 direction under arbitrary forces

$\varepsilon_{yy}$ & $\varepsilon_{zz}$ are similar

Shear Strains




$\rightarrow$

can define cosine of $\theta^*$ in deformations of $\overline{PQ}$ & $\overline{PR}$

note that $\left(\frac{\partial u}{\partial x_i}\right)^2$ will likely be v v small and can often be neglected, leading to

$$\boxed{\varepsilon_{xx} = \sqrt{1 + 2\frac{\partial u}{\partial x}} - 1} \rightarrow$$

4

$$\boxed{\varepsilon_{xx} = \sqrt{\left(1 + \frac{\partial u}{\partial x}\right)^2} - \cancel{1} = \cancel{1} + \frac{\partial u}{\partial x}}$$

$\varepsilon_{yy}$ & $\varepsilon_{zz}$ are similar

$$\boxed{\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}}$$

Note: relatively simple for displacement field → strain field but not reverse

Harder to measure entire strain field experimentally

Strain Transformations are similar to stress transforms but replace

$$\sigma_{xx} \leftrightarrow \epsilon_{xx} \quad \& \quad 2\tau_{xy} \leftrightarrow \gamma_{xy}$$

must satisfy compatibility requirements

6 equations of (a) should be integrable ie strains must satisfy compatibility inorder to satisfy displacements

Stress Strain Relationships

! assume linear relationship

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & \cdots & K_{16} \\ K_{21} & \ddots & & & \\ K_{31} & & \ddots & & \\ \vdots & & & \ddots & \\ \vdots & & & & \\ K_{61} & & & & K_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \end{bmatrix}$$

- K are coefficients of elasticity (36 → 21) strain energy arguments that reduce to 21

! - assume isotropic material (elastic constant in all all directions (21 → 2)

Elastic Constants Relationships (5 in total)

$\nu$ : Poisson's Ratio

$E$ : Elastic Young's Modulus

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$\mu$ : (G) shear modulus (torsional)

$\lambda$ : Lamé's constant (mathematic)

$K$ : Bulk modulus (used for uniform pressure)

using $\nu$ & $E$, can show 3D Hooke's Laws

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right]$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})\right]$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})\right]$$

$$\gamma_{xy} = \frac{2(1+\nu)}{E}\tau_{xy}$$

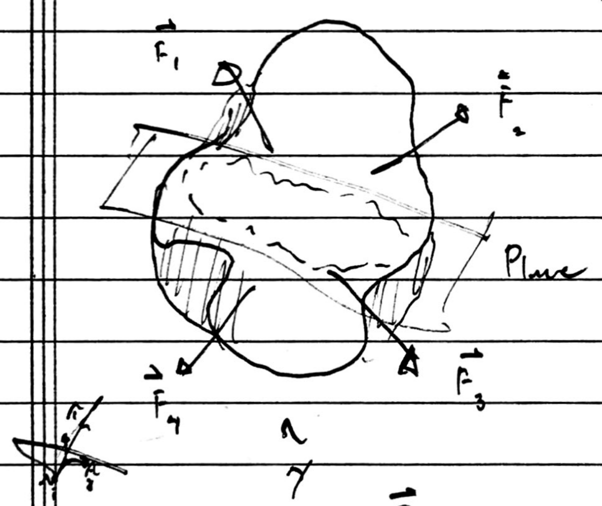
$$\gamma_{yz} = \frac{2(1+\nu)}{E}\tau_{yz}$$

$$\gamma_{xz} = \frac{2(1+\nu)}{E}\tau_{xz}$$

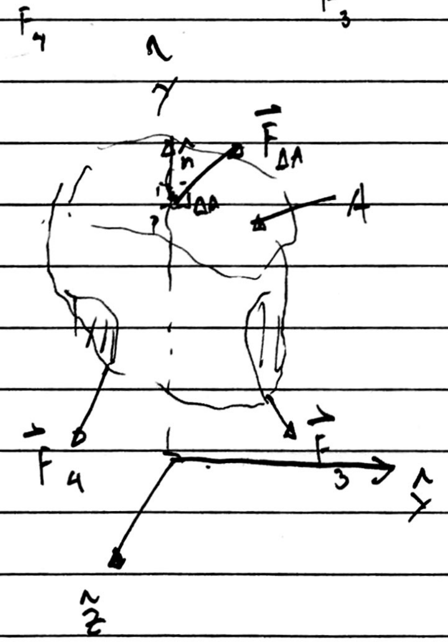
## Experimental Mechanics

- Midterm: ~~prof~~ probably going to be over KZ's stuff
- ~~Fluid~~ Final: actual design of possible experimental
- Hours: 10:00 AM - 11:20 AM (not 11:50)
- What the fuck is AFM
- Soft Schedule
- Review Stress from Shigley (Machine #1)

! - Review Machine Design Notes




- Under loading, find stress @ point with some types:
  - → Surface Force: contact of 2 bodies
  - → Body Forces: act on all elements in body ($g$, centrifugal, force field)

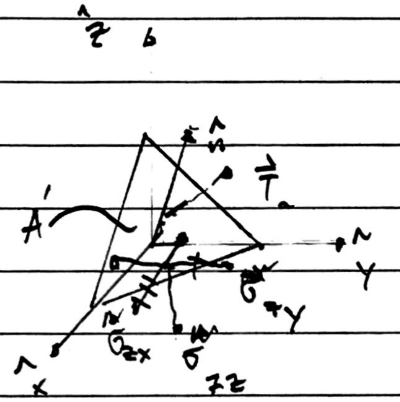



- Point P is arbitrary

$$\vec{T}_P = \lim_{\Delta A \to 0} \frac{\vec{F}_{\Delta A}}{\Delta A}$$ ; $\vec{T}_P$ is resultant stress at point P

$$\boxed{\Delta \vec{F}_n = a\,\Delta F_{n_x}\,\hat{i} + b\,\Delta F_{n_y}\,\hat{j} + c\,\Delta F_{n_z}\,\hat{k}}$$

but depends on $\hat{n}$




· on elementary tetrahedron

· Notation for planes

$$\sigma_{(\text{plane})(\text{direction on plane})}$$

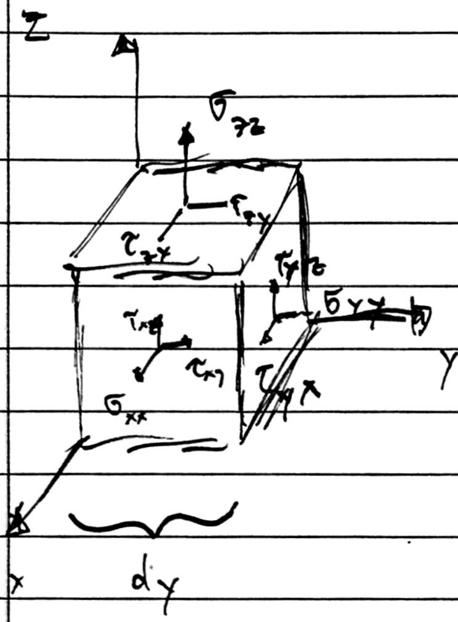
· $\vec{T}_n$ is resolved into 9 $\sigma$'s

$$\Sigma F_x = 0 : \quad \vec{T}_{nx} A - \sigma_{xx} A \cos(n,x) - \sigma_{zx} A \cos(n,z) - \sigma_{yx} A \cos(n,y) = 0$$

$$\therefore \vec{T}_{nx} = \sigma_{xx} \cos(n,x) + \sigma_{yx} \cos(n,y) + \sigma_{zx} \cos(n,z)$$

w/ $\cos(n,x)$ ... is projection of A on planes & same can be done for $\vec{T}_{ny}$ & $\vec{T}_{nz}$

so $$\boxed{\left\|\vec{T}_n\right\| = \sqrt{|T_{nx}|^2 + |T_{ny}|^2 + |T_{nz}|^2}}$$




$$\Rightarrow \begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{vmatrix}$$

— normal in x
y
z

any element in body is in equilibrium

$$\boxed{\sum \vec{F} = 0}$$

Stress Equations of Equilibrium $\boxed{\text{Find it}}$