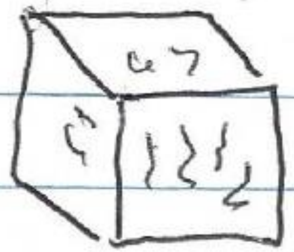
Fluid properties

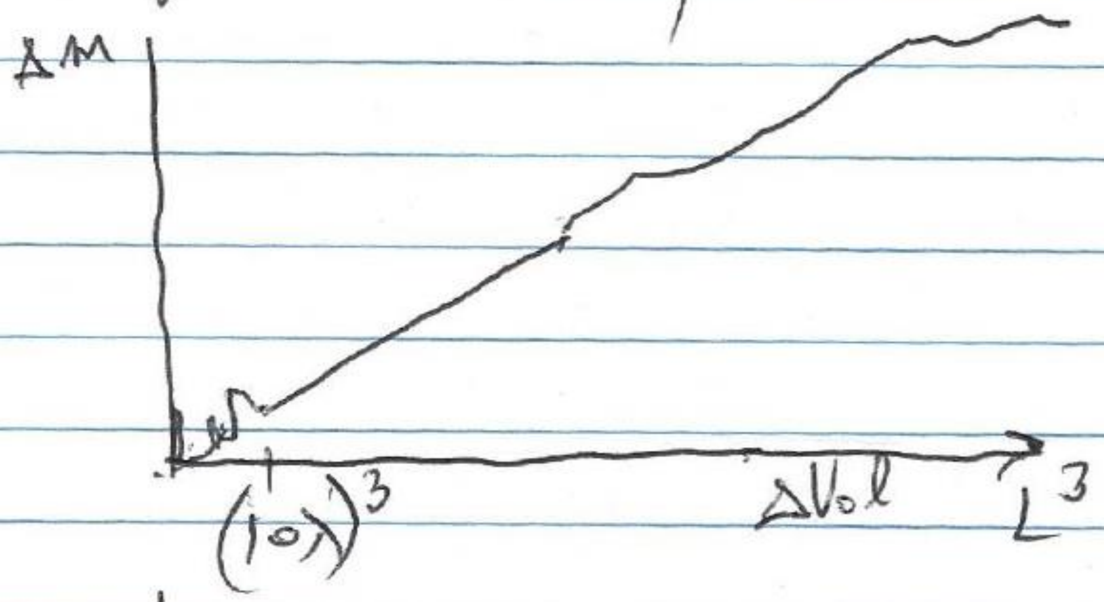
density



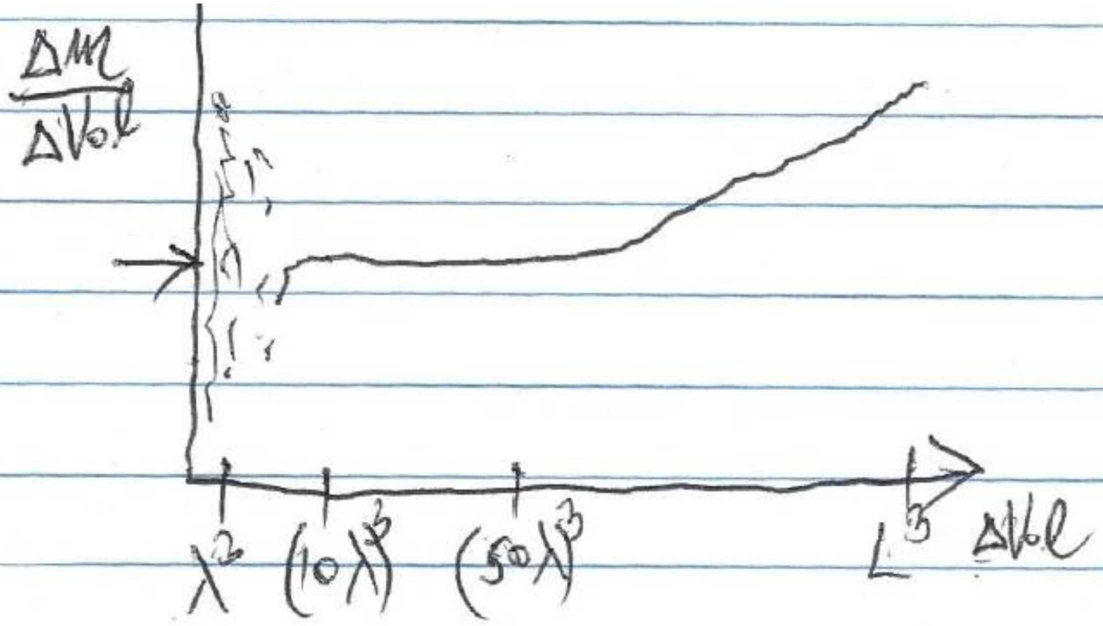
ΔVol

$\Delta m = \sum m_{molec}$ insite ΔVol

as the ΔVol goes below λ^3 Δm loses it identity



Density of a fluid element



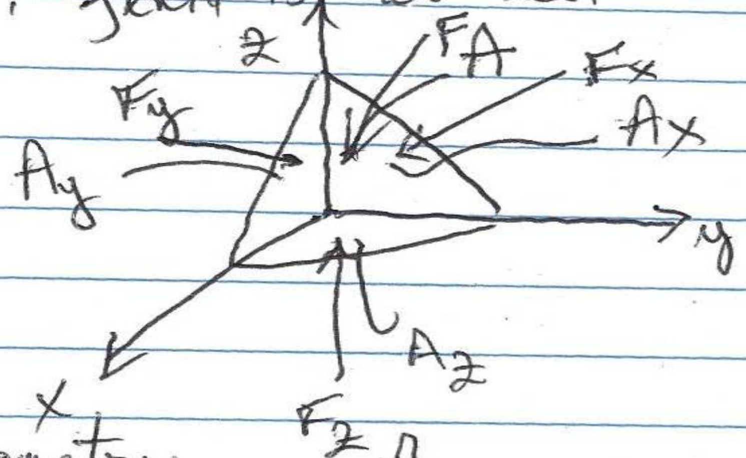
density of a fluid element: $\rho = \lim_{\Delta Vol \to (\Delta Vol)_{min}} \frac{\Delta m}{\Delta Vol}$

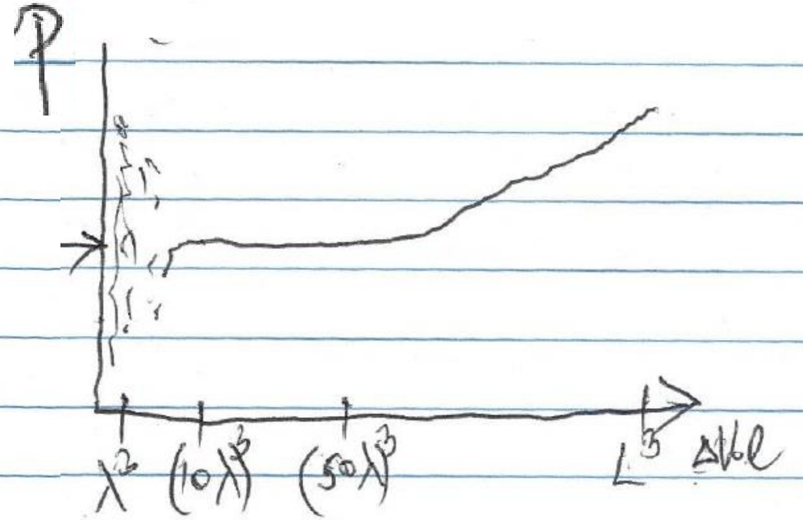
$(\Delta Vol)_{min} \sim (10\lambda)^3$

specific volume: $v = \frac{1}{\rho}$

Pressure of a fluid element

Pascal : fluid is at rest





from geometry: $\dfrac{A_x}{A} = \cos\theta_x$, $\dfrac{A_y}{A} = \cos\theta_y$, $\dfrac{A_z}{A} = \cos\theta_z$

from Newton's law for a fluid at rest:

$$\frac{F_x}{F} = \cos\theta_x, \quad \frac{F_y}{F} = \cos\theta_y, \quad \frac{F_z}{F} = \cos\theta_z$$

$$\therefore \quad \frac{F_x}{A_x} = \frac{F_y}{A_y} = \frac{F_z}{A_z} = \frac{F}{A}$$

$$P = \lim_{\substack{\Delta Vol \to (\Delta Vol)_{min} \\ \sim (10\lambda)^3}} \frac{F_n}{A_n}$$

aerostatic/hydrostatic pressure of a fluid element

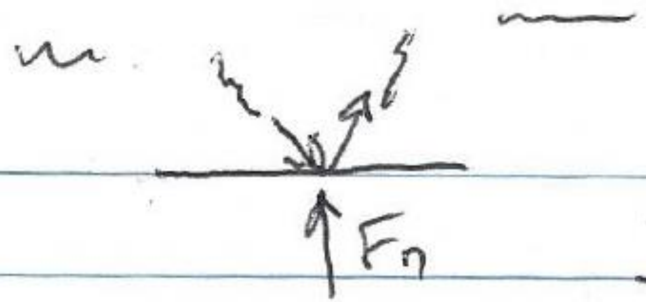
In reality, for a moving fluid,

$$p_n = \lim_{\substack{\Delta Vol \to \\ (\Delta Vol)_{min}}} \frac{F_n}{A_n} = p + O(\mu)$$

For a <u>slightly viscous fluid</u> (air, water, steam)

$O(\mu)$ is small and may be neglected

molecules



$\frac{F_n}{A_n} = \tau_n$ normal stress

Temperature of a fluid element

$$\bar{E}_{kinetic} = \frac{1}{N_{molec}} \sum_{N_{molec} \text{ inside } \Delta Vol} \frac{1}{2} m_{molec} V_{molec}^2$$

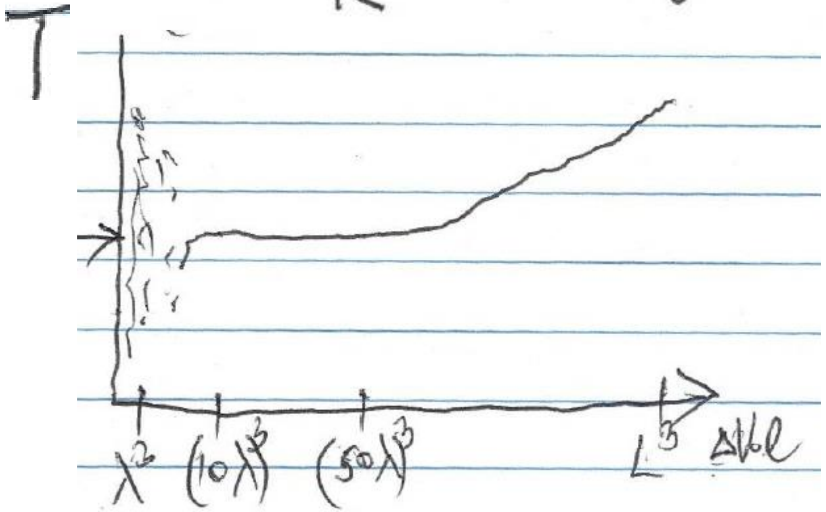
averaged kinetic energy inside the ΔVol

$$\lim_{\Delta Vol \to (\Delta Vol)_{min} \sim (10\lambda)^3} \bar{E}_k = \frac{3}{2} k T$$

absolute temperature of a fluid element

$$k = 1.38 \cdot 10^{-26} \frac{J}{K}$$

Boltzmann coefficient



Other thermodynamic properties of a fluid element

specific internal energy $u = \frac{\Delta U}{\Delta m}$ in limit $\Delta Vol \to \Delta Vol_{min}$

$$du = c_v dT - \left[T \left(\frac{\partial p}{\partial T} \right)_{\rho = const} - p \right] \frac{d\rho}{\rho^2}$$

specific enthalpy : $h = u + \frac{p}{\rho}$ or $dh = du + d\left(\frac{p}{\rho}\right)$

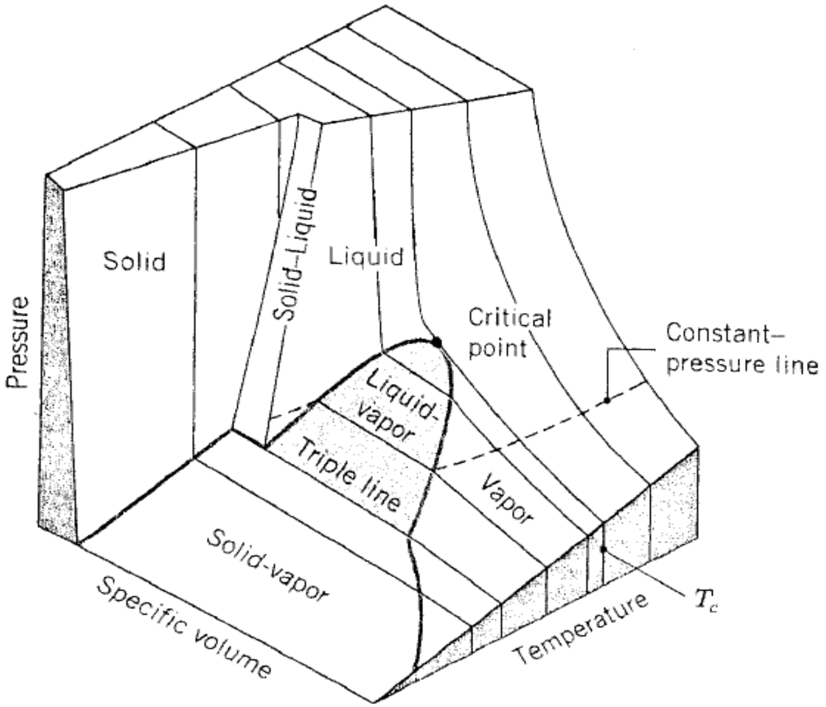
specific entropy : $T ds = c_v dT - \left(\frac{\partial p}{\partial T} \right)_\rho \frac{d\rho}{\rho^2}$

we can show that $T ds = dh - \frac{dp}{\rho}$ (Gibbs eq)

A simple compressible fluid → a fluid where 2 properties define the rest.

For example, $P = f(\rho, T)$ → eq of state

pressure is a surface above ρ, T plane.



Phase diagram: Solid, liquid, vapor

The critical point (P_c, ρ_c, T_c) characterizes a substance

At the critical point: $\left(\frac{\partial P}{\partial \rho}\right)_T = 0$ & $\left(\frac{\partial^2 P}{\partial \rho^2}\right)_T = 0$

The triple line - co existence of solid-liquid-vapor

Liquid is characterized by $\left(\frac{\partial P}{\partial \rho}\right)_T$ = very high

i.e. there is a need for large pressure changes to measure small specific volume changes. For systems with T = const and small pressure changes $\rho \simeq$ const.

A liquid may be idealized as an incompressible fluid with ρ = const.

Note that $\left(\frac{\partial T}{\partial \rho}\right)_P$ is not high.

Vapor (gas) is characterized by $\left(\frac{\partial P}{\partial \rho}\right)_T$ that is moderate, i.e. small changes of ρ result in small changes of P.

For gases,

$P_R = \frac{P}{P_c}$ reduced pressure

$\rho_R = \frac{\rho}{\rho_c}$ reduced density or $v_R = \frac{v}{v_c}$ reduced specific volume

$T_R = \frac{T}{T_c}$ reduced temperature

$Z = \frac{P}{\rho R T}$ $\quad R = \frac{\mathbb{R}}{M}$ is the specific gas constant

$\mathbb{R}$ is the universal gas constant

Z is a function of P_R and T_R

We observe:

(1) as $p_R \to 0$: $Z \to 1$ for T_R

$\therefore$ For low pressure and low density (or high specific volume) conditions $p = \rho R T$

eq of state for a perfect (thermodynamically perfect) gas (ideal gas)

(2) Van der Waals eq of state

$$p = \frac{\rho R T}{1 - b\rho} - a\rho^2$$

(3) Redlich-Kwong eq of state