September 26, 2019

MANE 6520-01 Fluid Mechanics
Fall Semester 2019
Problem Set #2
Due: October 10, 2019

1. Given the flow velocity field: $u(x,y,z,t) = 2xt$, $v(x,y,z,t) = 4y$, and $w(x,y,z,t)=0$.
   a) Find the element's path line and velocity vector as function of time t only for an element that passed through the point $x=1, y=1, z=1$ at $t=1$. Construct a relationship $x_e = f_1(y_e)$ for the element's path line.
   b) Find the equation $x=f_2(y)$ which describes the streamline that passes through the point $x=1, y=1, z=1$ at $t=1$.
   c) Find the equation $x=f_3(y)$ for the streak line at time $t=1$ constructed by all fluid elements that have passed through the point $x=1, y=1, z=1$.
   d) Plot all the above lines in a diagram x *vs.* y and compare between the lines.

2. The temperature on the surface of a lake is given by the field $T(\boldsymbol{x},t)$. Find the rate of change of temperature recorded on a thermometer that is dragged through the surface along a trajectory given by $\boldsymbol{x} = \boldsymbol{r}(t)$. Write the answer in terms of T and $\boldsymbol{r}$ and their derivatives.

3. Consider the plane (2D) stagnation flow given by $u = cx$, $v = - cy$, $w=0$. Find the expressions for the vorticity vector $\vec{\omega}$, the rate of strain tensor $\nabla\vec{V}$, its symmetric ($\mathcal{S}$) and skew-symmetric ($\mathcal{T}$) components, and the rate of volumetric expansion for this flow.

4. Use the identities given in class to prove that: $\boldsymbol{V} \bullet \nabla\boldsymbol{V} = \frac{1}{2} \nabla (\boldsymbol{V} \bullet \boldsymbol{V}) - \boldsymbol{V} \times \boldsymbol{\omega}$, where $\boldsymbol{\omega} = \nabla \times \boldsymbol{V}$.

5. Determine the size of the diameter of a cylinder and the Knudsen number (Kn) of the flow of air characterized by speed of sound of 340 m/s and kinematic viscosity $\nu = 1.7 \times 10^{-5}\ m^2/s$:
   a) with Ma = 0.5 and Re = 10;
   b) with Ma = 0.5 and Re = 10^6;
   c) with Ma = 0.001 and Re = 100;
   d) with Ma = 0.001 and Re = 10^6.

   Discuss the differences between the various cases and the implications with respect to the validity of fluid mechanics theory for each case.