Experimental Mechanics

- Stress - Optics Laws
- Isoclinic / Isochromatic Fringe Patterns

Stress Optics

- Linear elastic model material
  - Stress free → η_0 : index of refraction
  - stressed state → temporary birefringence
    - $\eta_1, \eta_2, \eta_3 \propto \sigma$
    - Related by

$\eta \to n$, oops

- $n_1 - n_0 = c_1 \sigma_1 + c_2 (\sigma_2 + \sigma_3)$
- $n_2 - n_0 = c_1 \sigma_2 + c_2 (\sigma_3 + \sigma_1)$
- $n_3 - n_0 = c_1 \sigma_3 + c_2 (\sigma_1 + \sigma_2)$

- Generally 2D for simplification : $\sigma_3 = 0$
  - $n_1 - n_0 = c_1 \sigma_1 + c_2 \sigma_2$
  - $n_2 - n_0 = c_1 \sigma_2 + c_2 \sigma_1$
- Retardation effect comes from material behavior in waveplate like manner, rewrite w/o n_0

$$n_2 - n_1 = (c_2 - c_1)(\sigma_1 - \sigma_2) = c(\sigma_1 - \sigma_2) \; ; \; c = c_2 - c_1$$

$$n_3 - n_2 = C(\sigma_2 - \sigma_3)\ ;\ n_1 - n_3 = C(\sigma_3 - \sigma_1)$$

- Brewsters $[Pa^{-1}]\ w^{-12}$
- $\sigma_1 \geq \sigma_2 \leq \sigma_3 \rightarrow n_3 \geq n_2 \geq n_1$
- $\Delta = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda} h\,(n_2 - n_1)$

Substitute

Waveplate height

$$\Delta_{12} = \frac{2\pi h \cancel{c}}{\cancel{c}} = \frac{2\pi h c}{\lambda}(\sigma_2 - \sigma_1)$$

Magnitude of relative retardation

$$\sigma_1 - \sigma_2 = \frac{N f_\sigma}{h}\ ;\ N = \frac{\Delta}{2\pi}$$

- N : fringe order ; cycles retardation

normalizing relative retardation

- f_σ : material "fringe value" property

$f_\sigma(\lambda)$ or λ dependent ; calibration constant

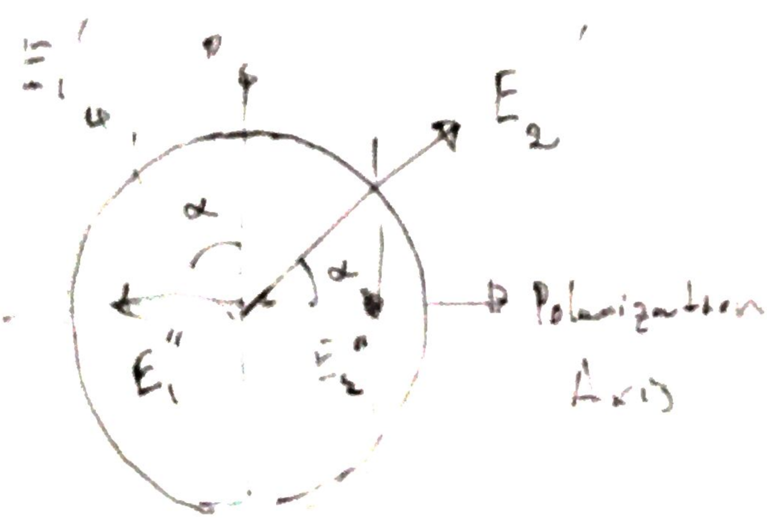
$E_{t(p)}$: light entering

$E_{py} = k \cos \omega t$

$$E_1 = k \cos \alpha \cos \omega t \quad \Rightarrow \quad E_1' = k \cos \alpha \cos(\omega t - \Delta_1)$$

$$E_2 = k \cancel{\cos}\overset{\sin}{} \alpha \cos \omega t \qquad\quad E_2' = k \cancel{\cos}\overset{\sin}{} \alpha \cos(\omega t - \Delta_2)$$

$$\Delta_1 = \frac{2\pi h}{\lambda}(n_1 - 1) \;;\; \Delta_2 = \frac{2\pi h}{\lambda}(n_2 - 1)$$



$$E_{ax} = E_2'' - E_1''$$

$$= E_2' \cos\alpha - E_1' \sin\alpha$$

$$= k \sin\alpha \cos(\omega t - \Delta_2) \cos\alpha$$

$$- k \cos\alpha \cos(\omega t - \Delta_1) \sin\alpha$$

$$= k \sin\alpha \cos\alpha \left[\cos(\omega t - \Delta_2) - \cos(\omega t - \Delta_1)\right]$$

$$= \underbrace{k \sin 2\alpha \sin\left(\frac{\Delta_2 - \Delta_1}{2}\right)}_{\text{Amplitude}} \sin\Big(\omega t - \underbrace{\frac{\Delta_2 + \Delta_1}{2}}\Big)$$

$\frac{\Delta_2 + \Delta_1}{2}$; average angular phase shift

at circuit, not really useful

$$\Delta_2 - \Delta_1 = \Delta = \frac{2\pi h}{\lambda}(n_2 - n_1) = \frac{2\pi hc}{\lambda}(\sigma_1 - \sigma_2)$$

Relative retardation $\propto$ Intensity !

$I = \frac{1}{2} \sin^2 2\alpha \sin^2\left(\frac{\Delta}{2}\right)$

- Extinction of Light (i.e. $I = 0$)

i) $\sin^2 2\alpha = 0$: principal stress direction

if $\alpha = \frac{n\pi}{2}$, $n \in \mathbb{N}$

- no restriction of location → full field
- fringe : loci of points w/ σ coincident with polarization angle
- isoclinic Case

ii) $\sin^2\left(\frac{\Delta}{2}\right) = 0$: $\frac{\Delta}{2} = n\pi$

$$\Delta = \frac{2\pi hc}{\lambda}(\sigma_1 - \sigma_2)$$

$$n = \frac{\Delta}{2\pi} = \frac{hc}{\lambda}(\sigma_1 - \sigma_2)$$

Order of Extinction

$I = 0$ for $\forall\ \sigma_1 = \sigma_2$