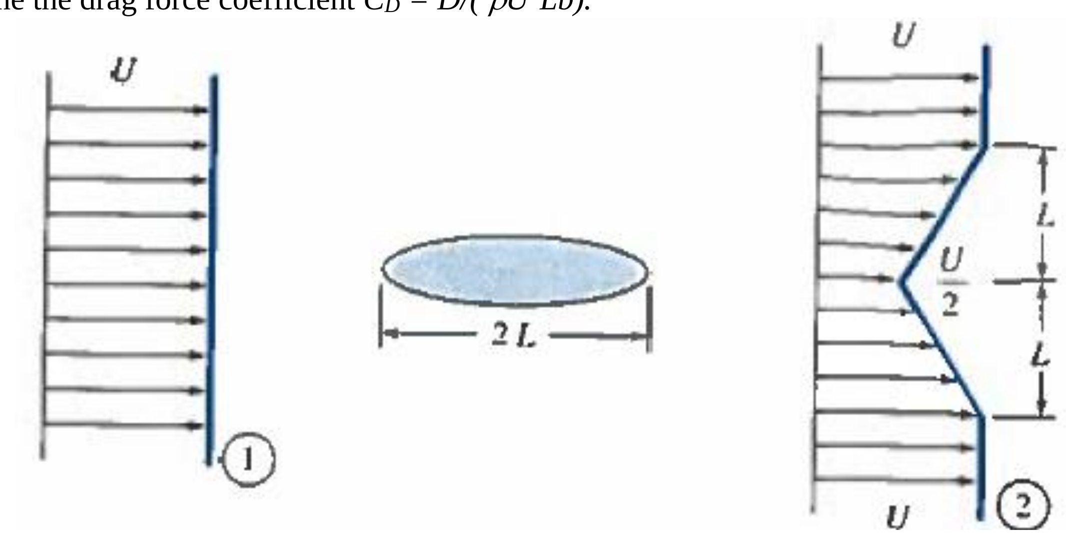
October 21, 2019

MANE 6520-01 Fluid Mechanics
Fall Semester 2019
Problems set #3
Due: November 4, 2019

1. (i) a challenging problem:
   Show that the vorticity transport equation for a compressible flow of a Newtonian fluid is:
$$\frac{d\vec{\omega}}{dt} = \vec{\omega} \bullet \nabla \vec{V} - \vec{\omega}\left(\nabla \bullet \vec{V}\right) + \nabla T \times \nabla s + \mu\left[\frac{1}{\rho}\nabla^2 \vec{\omega} - \nabla\left(\frac{1}{\rho}\right) \times \left(\nabla \times \vec{\omega}\right)\right]$$
$$+\left(\mu_v + \frac{4}{3}\mu\right)\nabla\left(\frac{1}{\rho}\right) \times \nabla\left(\nabla \bullet \vec{V}\right).$$
   (ii) Show that when the flow is inviscid:
$$\frac{d\left(\vec{\omega}/\rho\right)}{dt} = \frac{\vec{\omega}}{\rho} \bullet \nabla \vec{V} + \frac{1}{\rho}\nabla T \times \nabla s.$$
   Here, T is temperature. Assume viscosity and bulk viscosity are contant.
2. Determine the pressure in the troposphere where $0 < z < 11{,}000$ m and
   $T(z) = 288$ K – [(288-217)/11000] z.

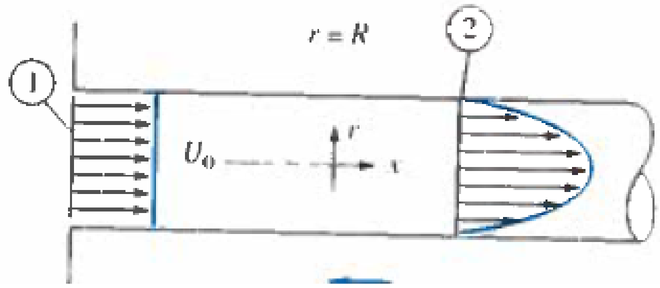
3. When a uniform stream with an upstream axial velocity $u_1 = U$ flows past a cylindrical body, it creates behind the body a low-speed wake axial that may be idealized as a V-shape profile, as shown in the figure below. Assume that the flow is steady, two-dimensional (with width b normal to the paper), and incompressible (with constant density ρ). Also, the pressures p_1 and p_2 far ahead and behind the body are equal. Use the integral equations for the conservation of mass and balance of momentum to derive a formula for the drag force D (axial force) exerted on the body. Also, determine the drag force coefficient $C_D = D/(\rho U^2 L b)$.



4. Consider a steady, axisymmetric and incompressible flow (with constant density ρ) in a circular pipe of radius R. The inlet (section 1) axial velocity is uniform, $u_1 = U_0$. The flow at the pipe exit (section 2) is fully developed (parallel axial flow). Use the integral equations for the conservation of mass and balance of momentum to determine u_{max} in terms of R and to find the wall drag force F in terms of p_1, p_2, U_0, ρ, R if the flow at section 2 is

(*a*) Laminar: $u_2 = u_{max}\left(1 - \dfrac{r^2}{R^2}\right)$

(*b*) Turbulent: $u_2 \approx u_{max}\left(1 - \dfrac{r}{R}\right)^{1/7}$



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