MANE 6560: Homework

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For: Prof. A. Hirsa for Incompressible Flow

Problem 1: Water Jet Stability

Consider the temporal instability of a water jet in the absence of gravity the jet diameter $2R$ is $1\ [\mathrm{mm}]$ in his flowing at $1\ [\mathrm{m/s}]$ based on linear theory

Begin first by linearizing the pressure and velocity fields

- $\mathbf{u} = \mathbf{u}_s + \mathbf{u}'$
- $p = p_s + p'$

These linearized disturbance equations are subsituted into the equations of

- Continuity ... $\nabla \cdot \mathbf{u} = 0$
- Momentum ... $\dot{\mathbf{u}} + \frac{\nabla p'}{\rho} = \alpha T' g\mathbf{k} + \nu \nabla^{\mathbf{2}} \mathbf{u}$
    - the gradient of the static pressure field is identically zero
- Use the relationship of energy

Part a: Fundamental Wavelength

- use the NS and continuity to find a 6th order ODE describing the viscous momentum balance in terms of position which varies only along the length of the jet, and time
    - Use the definitions of radii of curvature from calculus
        - $R_1 = r$
        - $R_2 = \frac{-\left[1+\left(\frac{\partial r_0}{\partial x}\right)^2\right]^{3/2}}{\left(\frac{\partial^2 r_0}{\partial x^2}\right)}$
    - $\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)^2 r = -\frac{\sigma R}{2\rho}\left(\frac{1}{R^2}\frac{\partial^2 r}{\partial x^2} + \frac{\partial^4 r}{\partial x^4}\right)$
- an initial disturbance of the form
- $r = a\ \mathrm{e}^{kx-\omega t}$ much smaller than the nozzle diameter $10^{-3}R$
- Part b: Fundamental Mode
- Part c: Volume of Diameter
    - diameter of a drop is the length of the unstable wavelength found in part a
- Part d: Time to pinch off
    - initial disturbance ... 10E-3 fundamental wavelength

Problem 2: Thermal Convection for Rotating Gap

Rayleigh-Bernard convection in couette flow between rotating cylinders in the narrow gap approximation can be described by similar sets of equations. In the stress free condition, a 6^{th} order ODE suffices

$$\left(\frac{d^2}{dy^2} - (kd)^2\right)^3 V_\theta = \frac{4\kappa^2 d^4 \Omega_1 A}{\nu^2}(1+\alpha y)V_\theta \qquad (2)$$

- Taylor Number ... $T = \frac{4d^4\Omega_1 A}{\nu^2}$, which relates the rotational centrifugal force to the viscous force in a fluid