Exam 1 Name:__________________

(open book/notes)

1) Consider surface waves in deep water, with the given profile: $\eta = a\ sin\ (Ax + Bt^2)$, where A and B are positive, real numbers.

a) Express the wavelength in terms of the given parameters

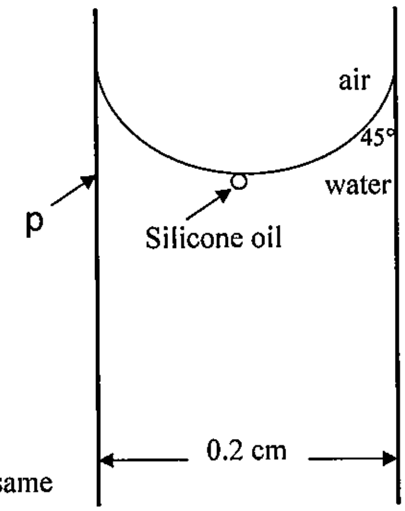
b) Express the (angular) frequency

c) Determine the magnitude and the direction (left or right-running) of the wave propagation at time $t = 5(\mathrm{s})$ if the parameters are given as $A = B = 1$

(20 points) 2) A dead cell is preserved inside a 5 micron diameter silicone oil droplet. The droplet is at the bottom of a meniscus of water in a plastic test tube, with a 1 cm diameter, and walls that have a 45 degree contact angle.

<u>Water: density 1000 kg/m^3, tension against air: 0.072 N/m</u>

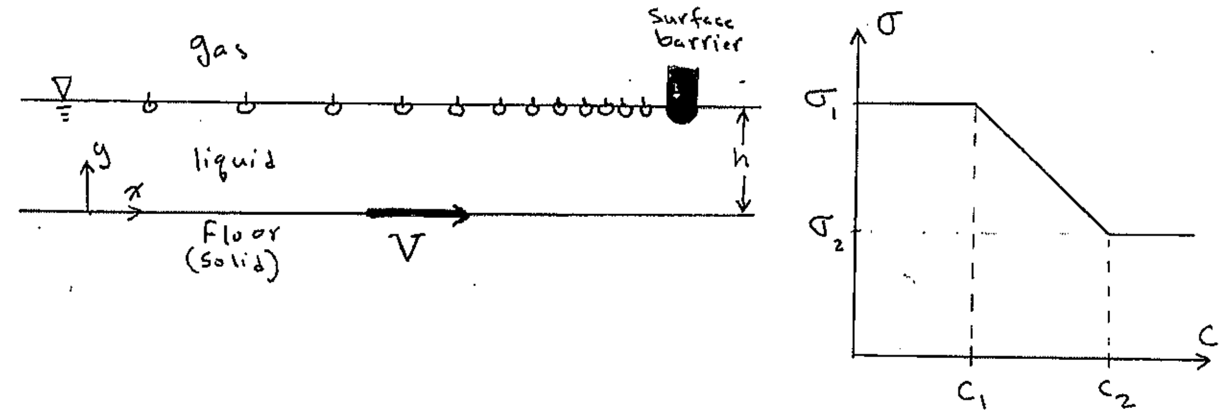
<u>Silicone oil: density ~ 800 kg/m^3, tension against air: ~ 0.02 N/m</u>



a) Compute the (gauge) pressure at the center of the silicone oil droplet. Note: you can ignore the details of the silicone oil-water contact line, i.e., at this scale it doesn't matter which configuration you think of (droplet fully submerged as shown, partially submerged, or just resting on the water surface) and you don't need to worry about the water-silicone oil interfacial tension.

b) What is the (gauge) water pressure at the test tube wall at the same depth as the droplet? (i.e. at point p)

3) Consider flow in a thin layer of liquid of density ρ and (dynamic) viscosity μ, where the depth (h) is essentially constant and small compared to the length of the channel. The flow is driven by the steady motion of the floor (located at $y = 0$) at speed V (to the right), where the fluid is supplied from the left and delivered to the right. An insoluble surfactant covers the free surface (located at $y = h$) and is trapped by a stationary surface barrier as shown, and the length of the film is very large compared to the depth of the channel. The equation of state for the surfactant is piecewise linear, as shown, and you can assume that the interfacial viscosities are negligible.



a) Sketch the velocity profile, $u(y)$, beneath the film at a large distance from its leading edge. Hint: once the surfactant-covered surface has reached steady state, its velocity is same as the surface barrier, i.e. zero.

b) Write the tangential stress balance at the gas/liquid interface, assuming that the gas side is essentially quiescent.

c) Compute the gradient in surfactant concentration gradient, $\frac{dc}{dx}$, in terms of the given parameters.

Incompressible Flow (MANE-6560)
Exam 1

Name:________________________

1) Consider gravity waves on the surface of water of great depth, where the surface elevation is given as:

$$\eta = a\sin(kx + \omega t)$$

a) Are these waves stationary, standing, or traveling waves, and if so are they right-running or left-running?

b) If the length of the waves is λ, express the functional dependence of the pressure, in terms of the variables and parameters that govern the flow.

c) Prove that, at point h below the undisturbed surface, the pressure at the instant when the depth is $h + \eta$ bears to the undisturbed pressure at the same point the ratio $1 + \frac{\eta}{h}\exp\left(-\frac{2\pi h}{\lambda}\right):1$

2) A rigid sphere of radius a rests on a flat rigid surface, and a small amount of liquid surrounds the point of contact. If the liquid fully wets both the sphere and the flat surface (i.e. its contact angle is zero on each surface), show that there is an adhesion force of magnitude $4\pi\sigma a$ acting on the sphere (the fact that this adhesion force is independent of the volume of the liquid is noteworthy).

3) Consider small scale waves on the surface of a liquid, of the form $\eta = a\cos\ k(x - ct)$, dominated by surface tension effect, at a scale where gravity can be ignored altogether.

a) Starting from the Young-Laplace equation, write the expression for the potential energy (per unit span) for one wavelength $\lambda = \frac{2\pi}{k}$

b) Simplify the expression in the limit of linear theory ($a \ll \lambda$)

1) Consider a general wave on the surface of water: $\eta = A\, e^{iBx}\, e^{iCt}$ where A, B, and C, can in general be a complex number. Describe what each constant would have to be (i.e. real, imaginary, or complex) in order for the surface elevation function to describe:

i) a traveling wave, say to the right

ii) a standing wave

iii) a stationary wave

2) A surface disturbance of the following form is given on water with large depth:

$$\eta(x,t) = \frac{3\pi}{gx} e^{i\zeta x} + A \quad \text{, where} \quad \zeta = -\frac{4g^2}{x^2} - \frac{\pi B}{x} t$$

a) Compute the local wavenumber and frequency (as functions of the given variables x, t, and the given parameters g, A, and B.)

b) Compute the local phase velocity

3) Please answer the following questions regarding capillarity:

a) What is the relationship (ratio) between the capillary rise in a small glass tube (zero contact angle) with diameter D, vs. the capillary rise between two glass plates (again, zero contact angle) with spacing of D.

b) Consider the idea of using a large array of droplets of a liquid metal, such as gallium, with surface tension about 700 dynes/cm (0.7 N/m), in a square pattern with spacing S = 1 mm, to repel two closely spaced parallel plates. If the plates have very low surface energy and behave "superhydrophobic" against gallium with a contact angle of nearly 180°, and the droplets are initially spherical with 0.5 mm diameter,

> i) compute how close the plates would get to each other when the droplets just touch each other.
>
> ii) computer how much load per square inch (25.4 mm x 25.4 mm) the plates exert against each other in that state

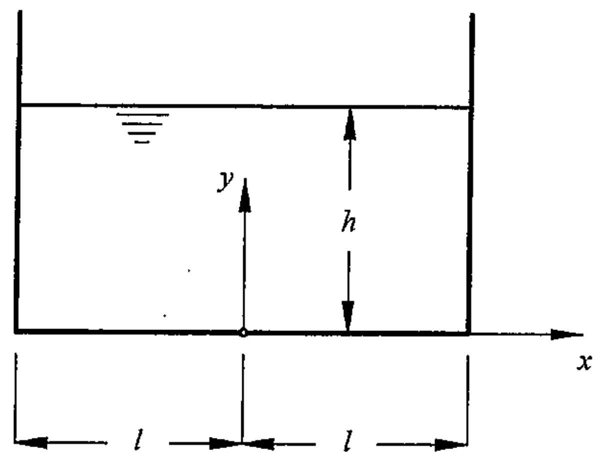
Incompressible Flow (MANE-6560)
Exam 1

Name:______________________________________

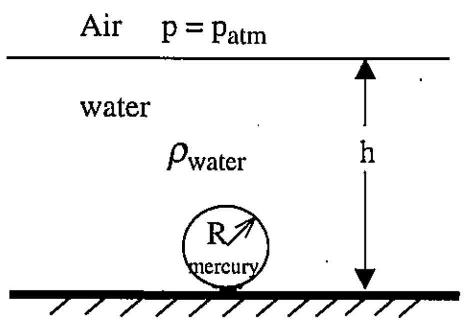
1) Consider gravity waves in a rectangular vessel defined by the figure.

a) Write the equation that governs the velocity potential and show how each of the 3 components of the velocity vector in a Cartesian system (u, v, w) are related to the potential

b) Write the appropriate boundary conditions for the mathematical problem (in the context of inviscid theory)



2 a) Consider a spherical drop of mercury, with radius R, at the bottom of a pool of water. Write an expression for the pressure at the **center** of the mercury drop, in terms of the appropriate parameters. Note, the surface tension between air and water is denoted as $\sigma_{\text{air-water}}$ and between water and mercury is $\sigma_{\text{water-mercury}}$, and the density of mercury is ρ_{mercury}



2 b) Consider the annular region between concentric cylinders of radii a_1 and a_2, filled with water with surface tension σ. Consider what would happen if the region is overfilled, but only slightly so that water does not wet the top of the cylinders, and only bulges up slightly. Use appropriate analysis to <u>**determine which of the three scenarios will occur**</u>:

i) point of maximum height occurs at the radius midway between the cylinders, that is:

$$r = 0.5\,(a_1 + a_2)$$

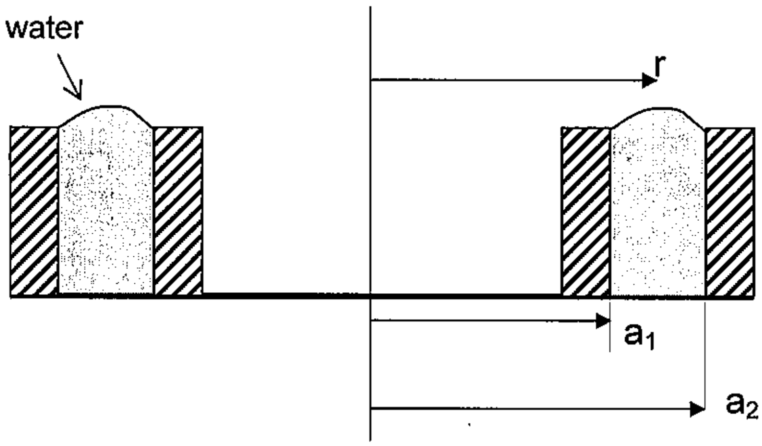
ii) point of maximum height occurs at the radius between the inner cylinder and the midway, that is:

$$a_1 < r < 0.5\,(a_1 + a_2)$$

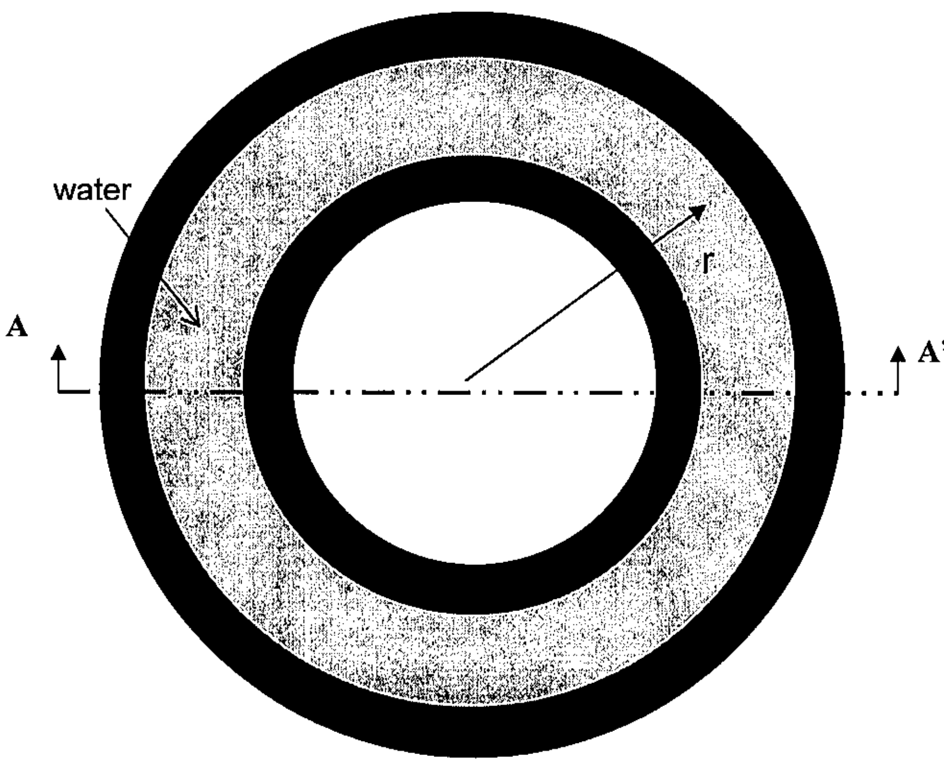
iii) point of maximum height occurs between the mid-point and the outer cylinder, that is:

$$0.5\,(a_1 + a_2) < r < a_2$$

A-A' Cross-sectional (side) view



Top view



3) A jet skier traveling at 50 knots cuts perpendicular across the wake of a large ship traveling at 25 knots (1 knot ≈ 1.688 ft/s ≈ 0.51 m/s).

a) compute the wavelength of waves at the center of the ship's wake
b) Estimate the frequency at which the jet skier will encounter the waves in the center
c) Estimate the frequency at which the jet skier will encounter the waves at the edge of the Kelvin wake

Incompressible Flow
Exam 1

Name:____________________________________

1) Calculate the average potential energy (in Joules) in one wavelength of a deep-water gravity wave with surface elevation given (in meters):

$$\eta = 0.2\ \sin\ \{3.1416\ (x - 1.7671\ t)\}$$

where x is also in meters and t is in seconds.

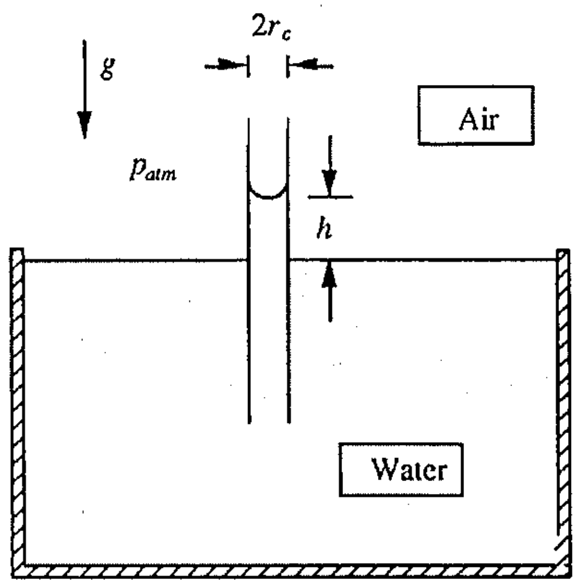
2) Consider gravity waves of small amplitude in a liquid layer of finite depth h. If the potential for such waves is given as: $\phi = B \cosh k(y + h) \cos k\,(x - ct)$

a) What are the units of B?
b) Determine the corresponding surface elevation profile η

3 a) Determine the equilibrium height h of water ($\sigma = 73$ dyne/cm, $\rho = 1$ g/cm^3) that will rise through a narrow circular-cylindrical capillary of radius r_c=100 microns under the influence of gravity (see the figure below). Assume the water perfectly wets the walls of the capillary, hence, the shape of the meniscus is hemispherical. Here, $g = 980$ cm/s^2.

b) Determine the capillary rise if instead of a circular tube, the water was rising between two glass slides with gap equal to the diameter of the tube.

Explain which will be the larger h, the axisymmetric geometry posed in (a) or the planar system in (b).



4) The waves which are directly following a ship 220 ft (67.06 m) long are observed to overtake it at intervals of 16.5 seconds, and it takes a crest 6 seconds to run along the ship. Compute the length of the waves and the speed of the ship.

Incompressible Flow
Exam-1

Name:_______________________

(30 Points) 1 For a small-amplitude gravity-capillary wave in deep water with surface elevation:

$$\eta\,(x,t) = a \sin\,(k\,x - \omega\,t)$$

a) find the velocity potential ϕ as a function of variables x, y, t and parameters a, ω, and k.

b) Find the perturbation pressure, p, as a function of variables x, y, t and parameters a, ρ, g, σ, ω, and k.

(30 points) 2. Waves run up on a shore with a period of 12 s.

a) Determine their phase velocity and wavelength in deep water well away from the shore (take density to be 1000 kg/m^3)

b) Estimate the time elapsed since the waves were generated in a storm occurring 800 km out to sea.

c) Estimate the depth at which the waves begin to be sensibly influenced (say changed by 1%) by the sea bed as they approach the shore.

(20 points) 3. A train of simple harmonic waves of length λ passes over the surface of water of great depth. Prove that, at a point whose depth below the undisturbed surface is h, the pressure at the instants when the disturbed depth of the point is $h + \eta$ bears to the undisturbed pressure at the same point the ratio

$$1 + \frac{\eta}{h} \exp\left(-\frac{2\pi h}{\lambda}\right) : 1$$