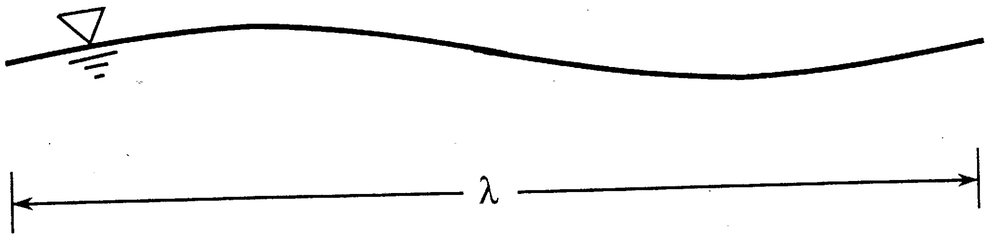
1) Under what circumstance do the wave crests in a given wave packet produced by the superposition of two waves with wavenumber k_1 and $(1 + \varepsilon)\, k_1$ (i.e. the same kind of wave packet studied in class) maintain their relative position within the packet? For water (at 20°C) waves, what is corresponding wavelength (in cm) for such waves? Explain any physical significance of this result.

2) For a linear gravity water wave with the simple representation for velocity potential (ϕ) and elevation profile (η):

$$\phi = A' \cos(kx - \omega t) e^{ky}$$

$$\eta = -\frac{A'}{c} \sin(kx - \omega t)$$



Plot the surface compression/dilation (i.e. divergence) for the cycle shown above. How would this result differ if this was a standing wave?

3) Two spherical soap bubbles of radii a_1 and a_2 are made to coalesce. Show that when the temperature of the gas in the resulting soap bubble has returned to its initial value the radius r of the bubble is given by

$$p_o r^3 + 4\sigma r^2 = p_o(a_1^3 + a_2^3) + 4\sigma(a_1^2 + a_2^2),$$

where p_o is the ambient pressure and σ is the tension of the air-liquid interface.

4) Two wetted surfaces can stick together with great strength if the liquid wets them with an angle $\theta_E < \pi/2$ (the contact angle θ_E is defined in the figure). Imagine that we mash a large drop between two plates separated by a distance H. The drop forms what is called a *capillary bridge* characterized by a radius R and a surface area πR^2. Gravity does **not** need to be considered in this problem (i.e., the Bond number, $\rho g H^2/\sigma$ is small compared to unity in this problem).

a) Compute the pressure within the drop for any liquid with surface tension σ

b) Simply the result for the case $H << R$

c) Under what condition would the force be repulsive as opposed to attractive?

d) For water ($\sigma = 0.072$ N/m) and $R = 1$ cm, $H = 5$ μm (note, a micron is 10^{-6} meters), and $\theta_E = 0$ (best case for adhesion), what is the pressure drop across the interface and the adhesion force?