Math For CS 2

Assignment 1

Q1: Queueville Airlines knows that on average 5% of the people making flight reservations do not show up. (They model this information by assuming that each person independently does not show up with probability of 5%.) Consequently, their policy is to sell 52 tickets for a flight that can only hold 50 passengers. What is the probability that there will be a seat available for every passenger who shows up?

Q2: A 6-sided die is rolled n times. What is the probability all faces have appeared? (Hint: Use Principle of Inclusion and Exclusion)

Q3: $E_1, E_2, \ldots, E_n$ be n events, each with positive probability. Prove that

$$\mathbb{P}\left(\bigcap_{i=1}^{n} E_i\right) = \mathbb{P}\{E_1\} \cdot \mathbb{P}\{E_2 \mid E_1\} \cdot \mathbb{P}\{E_3 \mid E_1 \cap E_2\} \cdots \mathbb{P}\left\{E_n \mid \bigcap_{i=1}^{n-1} E_i\right\}.$$

Q4: Company A produces 10% defective products, Company B produces 20% defective products and C produces 5% defective products. If choosing a company is an equally likely event, then find the probability that the product chosen is defective.

Q5: One way to design a spam filter is to look at the words in an email. In particular, some words are more frequent in spam emails. Suppose that we have the following information:

- 50% of emails are spam;
- 1% of spam emails contain the word "refinance";
- 0.001% of non-spam emails contain the word "refinance".

Suppose that an email is checked and found to contain the word "refinance". What is the probability that the email is spam?

Q6: For the previous question (Q5), identify the probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

Q7: Prove the following set identities.

- $(L \cap R)^C = L^C \cup R^C$
- $(L \cup R)^C = L^C \cap R^C$
- $L \cup (L \cap R) = L$

Q8: Let $\mathcal{F}$ be a σ-algebra of subsets of Ω. Show that $\mathcal{F}$ is closed under countable intersections. (That is, the intersection of a countable number of sets from $\mathcal{F}$ also belongs to $\mathcal{F}$)

Q9: Given a general probability space $(\Omega, \mathcal{F}, \mathbb{P})$, prove that for all $A, B \in \mathcal{F}$

- $\mathbb{P}(A^c) = 1 - \mathbb{P}(A)$
- If $A \subseteq B$, then $\mathbb{P}(A) \leq \mathbb{P}(B)$

Q10: I pick a random number from $\{1, 2, 3, \ldots, 10\}$, and call it N. Suppose that all outcomes are equally likely. Let A be the event that N is less than 7, and let B be the event that N is an even number. Are A and B independent?