elemapprox user manual

Title	elemapprox (Elementary functions approximation in ANSI C, Verilog HDL, and VHDL)
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Version	1.3.2
Rev. history	
v1.3.2	2016-02-27 Fix ASCII rendering of function plots in the documentation.
v1.3.1	2015-12-25 Documentation updated with function plot examples.
v1.3.0	2014-10-12 Added `ansicstd` version which is an alternative C port that calls the standard C library mathematical functions as defined in `math.h`. Documentation updated.
v1.2.0	2014-10-10 Added `vhdlieee` version which is compatible to the IEEE `math_real` package. Documentation updated.
v1.1.0	2014-10-07 Added VHDL version for approximating and plotting the elementary functions. Numerous documentation fixes.
v1.0.1	2014-09-24 Minor documentation corrections.
v1.0.0	2014-09-16 Initial release. This is an expanded version built upon Mark G. Arnold's Verilog Transcendental Functions paper.

1. Introduction

`elemapprox` is an ANSI C code, Verilog HDL and VHDL collection of modules (Verilog HDL) and packages (VHDL) that provide the capability of evaluating and plotting transcendental functions by evaluating them in single precision. The original work supports ASCII plotting of a subset of the functions; this version provides a more complete list of functions in addition to bitmap plotting for the transcendental functions as PBM (monochrome) image files.

elemapprox has been conceived as an extension to Prof. Mark G. Arnold's work as puhlished in HDLCON 2001. Most functions have been prefixed with the letter k in order to avoid function name clashes in both ANSI C and Verilog HDL implementations. Currently, the plain VHDL version uses unprefixed names (e.g. acos instead of kacos). An alternative VHDL version named vhdlieee is compatible to the IEEE.math_real package and features prefixed names.

The transcendental functions supported include most elementary functions (hence the name elemapprox) and the list is as follows:

Function	Description
kfabs	Floating-point absolute value (helper function).
rootof2	Calculate root-of-2 (not in vhdlieee).
kacos	Arc cosine.
kacosh	Inverse hyperbolic cosine.
kacot	Arc cotangent.
kacoth	Inverse hyperbolic cotangent.
kacsc	Arc cosecant.
kacsch	Inverse hyperbolic cosecant.
kasec	Arc secant.
kasech	Inverse hyperbolic secant.
kasin	Arc sine.
kasinh	Inverse hyperbolic sine.
katan	Arc tangent.
katan2	Two-argument (x/y) arc tangent.
katanh	Inverse hyperbolic tangent.
kcos	Cosine.
kcosh	Hyperbolic cosine.
kcot	Cotangent.
kcoth	Hyperbolic cotangent.
kcsc	Cosecant.
kcsch	Hyperbolic cosecant.
kexp	Exponential.
khypot	Hypotenuse (currently ANSI C only).
klog	Natural logarithm.
kpow	Powering function.
ksec	Secant (named secant in the plain VHDL port).
ksech	Hyperbolic secant.
ksin	Sine.
ksinh	Hyperbolic sine.
ksqrt	Square root.
ktan	Tangent.
ktanh	Hyperbolic tangent.

The reference paper and the corresponding presentation are available from the web at the following links:

- http://www.cse.lehigh.edu/~caar/marnold/papers/sanjose_hdlcon.doc
- http://www.cse.lehigh.edu/~caar/marnold/presentations/freeddy.ppt

2. File listing

The `elemapprox` C code implementation and Verilog HDL modules include the following files:

/elemapprox	Top-level directory
AUTHORS	List of authors.
LICENSE	License argeement (Modified BSD license).
README.rst	This file.
README.html	HTML version of README.
README.pdf	PDF version of README.
rst2docs.sh	Bash script for generating the HTML and PDF versions.
VERSION	Current version.
/ansic	ANSI C implementation (standalone)
clean-math-ansic.sh	Bash script for cleaning up the generated executables.
elemapprox.c	C code for the function approximations.
elemapprox.h	C header file for the above. Defines certain mathematical constants and declares function prototypes.
funcplot.c	Reference code for creating the plot data for the functions.
funcplot.h	C header file for the above.
graph.c	Collection of ASCII and PBM graphing functions.
graph.h	C header file for the above.
Makefile	GNU Makefile for building `testfunc.exe`.
plot-ansic-ascii.sh	Bash script for plotting the elementary functions as ASCII graphs using `testfunc.exe`.
plot-ansic-pbm.sh	Bash script for plotting the elementary functions as PBM images using `testfunc.exe`.
testfunc.c	Application code for testing the elementary functions. Options include PBM or ASCII image generation and function selection.
test<func>.pbm	Generated PBM image data for the function <func>.
test<func>.txt	Generated ASCII graph data for the function <func>.
test<func>-ascii.txt	Concatenation of the generated ASCII graph data for all supported functions.
/ansicstd	ANSI C implementation (based on `math.h`)
clean-math-ansic.sh	Bash script for cleaning up the generated executables.
elemapprox.c	C code for the function approximations.

elemapprox.h	C header file for the above. Defines certain mathematical constants and declares function prototypes.
funcplot.c	Reference code for creating the plot data for the functions.
funcplot.h	C header file for the above.
graph.c	Collection of ASCII and PBM graphing functions.
graph.h	C header file for the above.
Makefile	GNU Makefile for building `testfunc.exe`.
plot-ansic-ascii.sh	Bash script for plotting the elementary functions as ASCII graphs using `testfunc.exe`.
plot-ansic-pbm.sh	Bash script for plotting the elementary functions as PBM images using `testfunc.exe`.
testfunc.c	Application code for testing the elementary functions. Options include PBM or ASCII image generation and function selection.
test<func>.pbm	Generated PBM image data for the function <func>.
test<func>.txt	Generated ASCII graph data for the function <func>.
test<func>-ascii.txt	Concatenation of the generated ASCII graph data for all supported functions.
/verilog	Verilog HDL implementation
clean-math-verilog.sh	Bash script for cleaning up the generated interpreted intermediate code (for Icarus Verilog).
constants.v	Certain mathematical constants.
elemapprox.v	Verilog HDL code for the function approximations.
elemapproxpp.v	Preprocessed version of the above, directly including the mathematical constants from `constants.v` and expanding all macro-definitions.
funcplot.v	Reference code for creating the plot data for the functions.
graph.v	Collection of ASCII and PBM graphing tasks.
plot-verilog-ascii.sh	Bash script for plotting the elementary functions as ASCII graphs using `testfunc.v`. The script Icarus Verilog' VVP interpreter which is capable of parsing command-line options.
plot-verilog-pbm.sh	Bash script for plotting the elementary functions as PBM images using `testfunc.v`.
testfunc.v	Application code for the elementary functions. Options include PBM or ASCII image generation and function selection.
test<func>.pbm	Generated PBM image data for the function <func>.
test<func>.txt	Generated ASCII graph data for the function <func>.
test<func>-ascii.txt	Concatenation of the generated ASCII graph data for all supported functions.

/vhdl	VHDL implementation
elemapprox.do	Modelsim `.do` macro file for Modelsim simulation.
elemapprox.mk	GNU Makefile for running the testbench using GHDL.
elemapprox.vhd	VHDL package code for the function approximations and related mathematical constants.
funcplot.vhd	VHDL package code for creating the plot data for the elementary functions.
graph.vhd	VHDL package code with a collection of ASCII and PBM procedures.
plot-ghdl-ascii.sh	Bash script for plotting the elementary functions as ASCII graphs using GHDL. The script generates a configuration file (`config.txt`) for controlling the simulation.
plot-ghdl-pbm.sh	Bash script for plotting the elementary functions as PBM images using GHDL. The script generates a configuration file (`config.txt`) for controlling the simulation.
plot-mti-ascii.sh	Bash script for plotting the elementary functions as ASCII graphs using Modelsim. The script generates a configuration file (`config.txt`) for controlling the simulation.
plot-mti-pbm.sh	Bash script for plotting the elementary functions as PBM images using Modelsim. The script generates a configuration file (`config.txt`) for controlling the simulation.
testfunc.vhd	VHDL testbench code for the elementary functions. Options include PBM or ASCII image generation and function selection through a configuration file.
test<func>.pbm	Generated PBM image data for the function <func>.
test<func>.txt	Generated ASCII graph data for the function <func>.
test<func>-ascii.txt	Concatenation of the generated ASCII graph data for all supported functions.
/vhdlieee	VHDL implementation compatible to `IEEE.math_real`
elemapprox.do	Modelsim `.do` macro file for Modelsim simulation.
elemapprox.mk	GNU Makefile for running the testbench using GHDL.
elemapprox.vhd	VHDL package code for the function approximations and related mathematical constants.
funcplot.vhd	VHDL package code for creating the plot data for the elementary functions.
graph.vhd	VHDL package code with a collection of ASCII and PBM procedures.

plot-ghdl-ascii.sh	Bash script for plotting the elementary functions as ASCII graphs using GHDL. The script generates a configuration file (`config.txt`) for controlling the simulation.
plot-ghdl-pbm.sh	Bash script for plotting the elementary functions as PBM images using GHDL. The script generates a configuration file (`config.txt`) for controlling the simulation.
plot-mti-ascii.sh	Bash script for plotting the elementary functions as ASCII graphs using Modelsim. The script generates a configuration file (`config.txt`) for controlling the simulation.
plot-mti-pbm.sh	Bash script for plotting the elementary functions as PBM images using Modelsim. The script generates a configuration file (`config.txt`) for controlling the simulation.
testfunc.vhd	VHDL testbench code for the elementary functions. Options include PBM or ASCII image generation and function selection through a configuration file.
test<func>.pbm	Generated PBM image data for the function <func>.
test<func>.txt	Generated ASCII graph data for the function <func>.
test<func>-ascii.txt	Concatenation of the generated ASCII graph data for all supported functions.
/refs	Reference documentation
sanjose_hdlcon.doc	MS Word document for the manuscript: M. G. Arnold, C. Walter and F. Engineer, "Verilog Transcendental Functions for Numerical Testbenches," Proceedings of the Tenth International HDL conference, Santa Clara, California, March 1, 2001.
freeddy.ppt	MS PowerPoint presentation of the above work.

3. Usage

Both the ANSI C and Verilog HDL versions can be used for generating graph data and depicting any of the supported transcendental functions via two similar scripts.

3.1 ANSI C

1. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate an ASCII graph for each function.

```
$ cd ansic
```

or

```
$ cd ansicstd
```

followed by

```
$ ./plot-ansic-ascii.sh
```

All generated data are also concatenated to `testfunc-ascii.txt`.

2. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate a PBM image for each function.

```
$ ./plot-ansic-pbm.sh
```

All generated data are saved in the form of PBM (monochrome bitmap) image files. Such files can be visualized using e.g. the public domain `Imagine` viewer: http://www.nyam.pe.kr/

3.2 Verilog HDL

1. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate an ASCII graph for each function.

```
$ cd verilog
$ ./plot-verilog-ascii.sh
```

All generated data are also concatenated to `testfunc-ascii.txt`.

2. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate a PBM image for each function.

```
$ ./plot-verilog-pbm.sh
```

All generated data are saved in the form of PBM (monochrome bitmap) image files.

3.3 VHDL

The VHDL version of `elemapprox` supports both GHDL (http://ghdl.free.fr) and Mentor Modelsim (http://www.model.com).

3.3.1 GHDL

1. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate an ASCII graph for each function.

```
$ cd vhdl
```

or

```
$ cd vhdlieee
```

followed by

```
$ ./plot-ghdl-ascii.sh
```

All generated data are also concatenated to `testfunc-ascii.txt`.

2. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate a PBM image for each function.

```
$ ./plot-ghdl-pbm.sh
```

All generated data are saved in the form of PBM (monochrome bitmap) image files.

3.3.2 Modelsim

1. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate an ASCII graph for each function.

```
$ cd vhdl
```

or

```
$ cd vhdlieee
```

followed by

```
$ ./plot-mti-ascii.sh
```

All generated data are also concatenated to `testfunc-ascii.txt`.

2. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate a PBM image for each function.

```
$ ./plot-mti-pbm.sh
```

All generated data are saved in the form of PBM (monochrome bitmap) image files.

4. Synthesis

The implementation code (either ANSI C, Verilog HDL or VHDL) for the transcendental functions has not been tested for high-level or RTL synthesis.

5. Prerequisites

- Standard UNIX-based tools (tested with gcc-4.6.2 on MinGW/x86) [optional if you use Modelsim].
  - make
  - bash (shell)

  For this reason, MinGW (http://www.mingw.org) or Cygwin (http://sources.redhat.com/cygwin) are suggested, since POSIX emulation environments of sufficient completeness.

- Icarus Verilog simulator (http://iverilog.icarus.com/). The Windows version can be downloaded from: http://bleyer.org/icarus/
- GHDL simulator (http://ghdl.free.fr) for VHDL. Both Windows and Linux versions can be downloaded from this site. Updated GHDL releases are available (again for multiple OSes) from: http://sourceforge.net/projects/ghdl-updates/
- Alternatively, a commercial simulator like Mentor Modelsim (http://www.model.com) can be used (however this has only been tested for the VHDL version of `elemapprox`).

A. Function plot examples

This appendix provides sample function plot visualization as raster graphics (PNG) and ASCII for the elementary functions with provided implementations.

A.1. Arc cosine (acos)

ASCII rendering.

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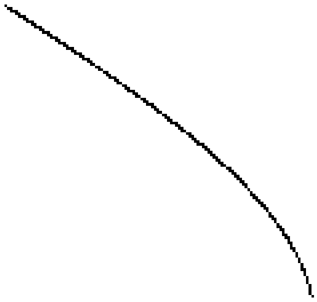
Bitmap rendering.



A.2. Inverse hyperbolic cosine (acosh)

ASCII rendering.

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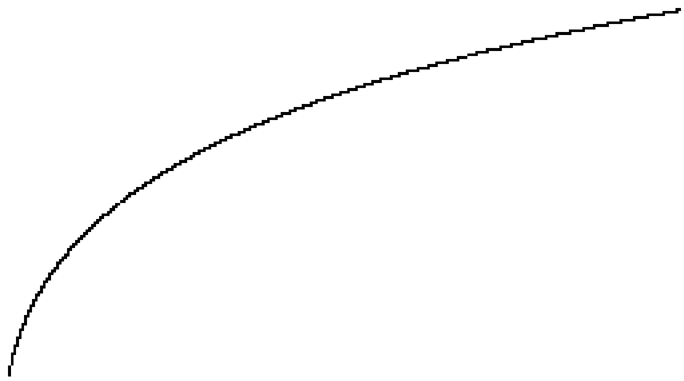

A.3. Arc cotangent (acot)

ASCII rendering.

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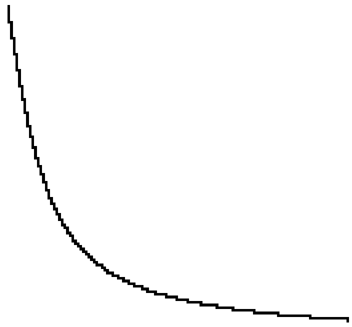
Bitmap rendering.



A.4. Inverse hyperbolic cotangent (acoth)

ASCII rendering.

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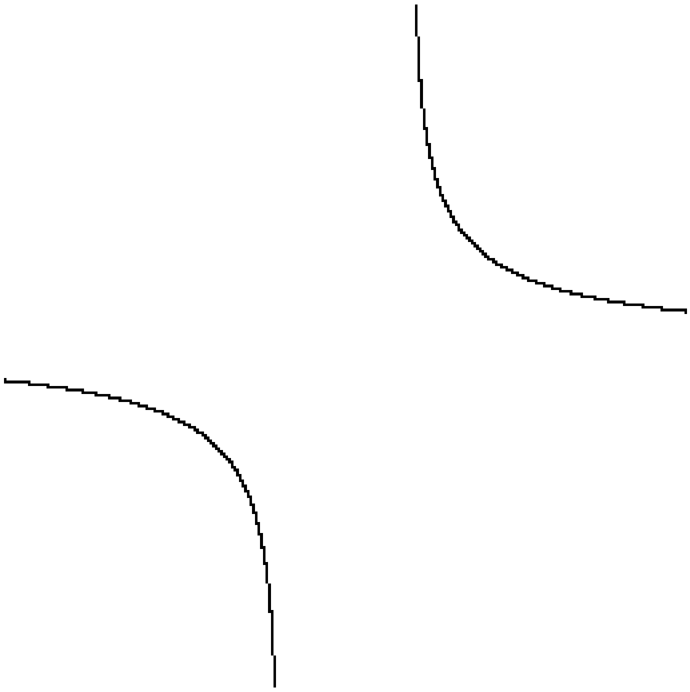
Bitmap rendering.



A.5. Arc cosecant (acsc)

ASCII rendering.

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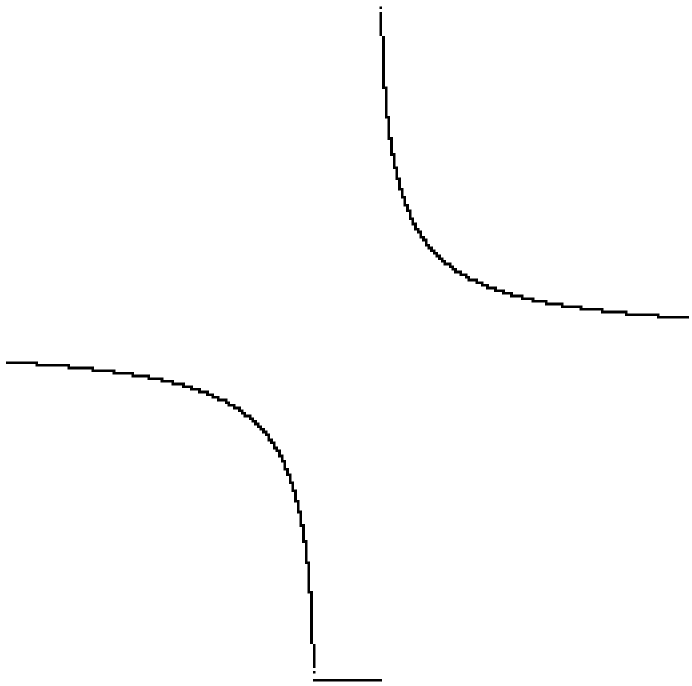
Bitmap rendering.



A.6. Inverse hyperbolic cosecant (acsch)

ASCII rendering.

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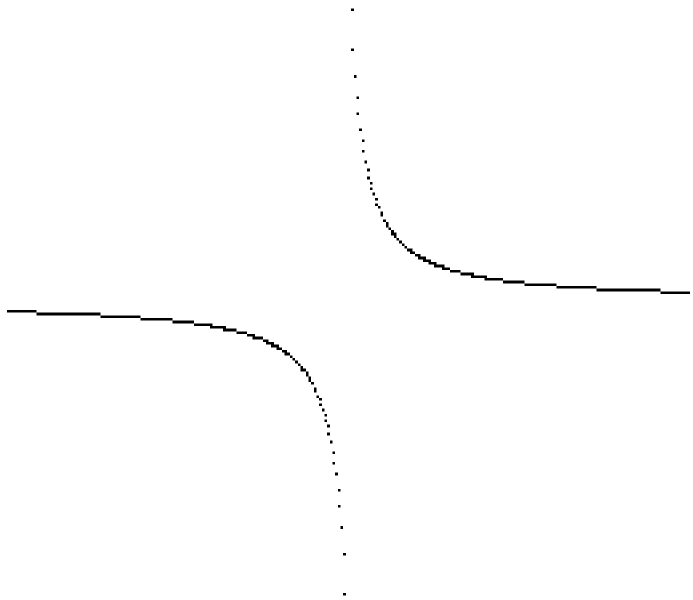
Bitmap rendering.



A.7. Arc secant (asec)

ASCII rendering.

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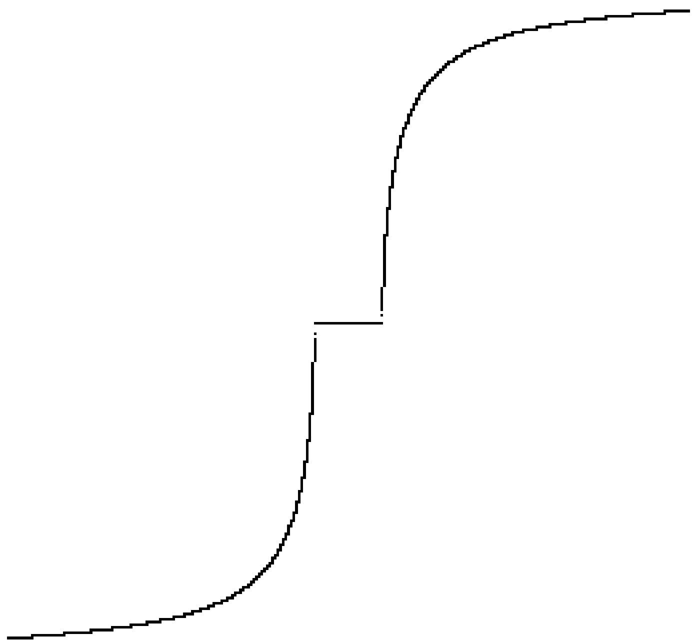
Bitmap rendering.



A.8. Inverse hyperbolic secant (asech)

ASCII rendering.

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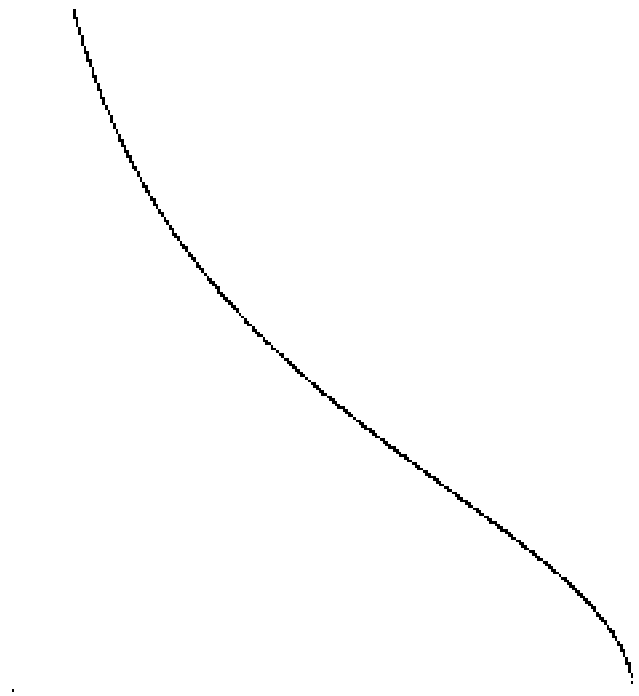
Bitmap rendering.



A.9. Arc sine (asin)

ASCII rendering.

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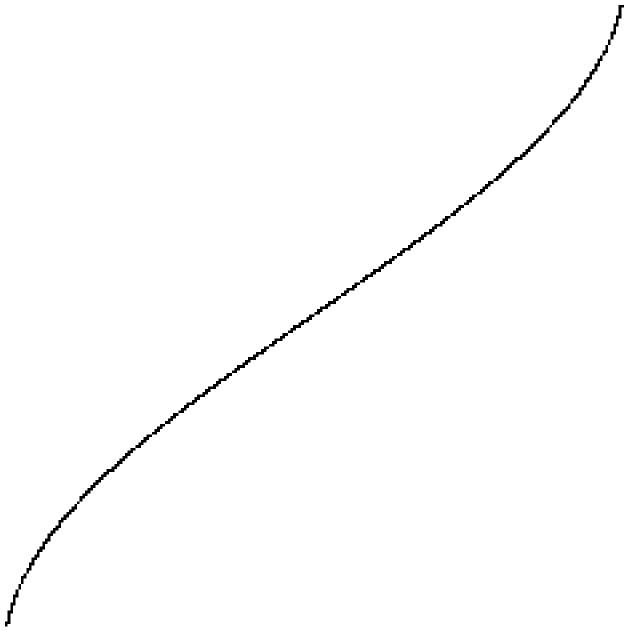
Bitmap rendering.



A.10. Inverse hyperbolic sine (asinh)

ASCII rendering.

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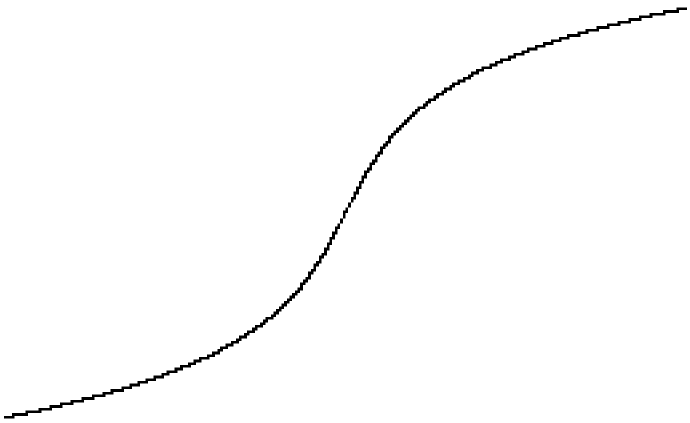
Bitmap rendering.



A.11. Arc tangent (atan)

ASCII rendering.

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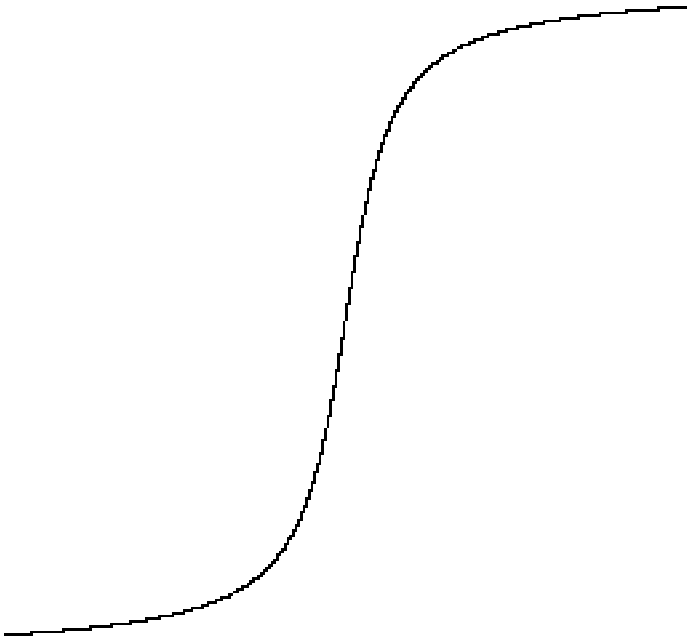

A.12. Inverse hyperbolic tangent (atanh)

ASCII rendering.

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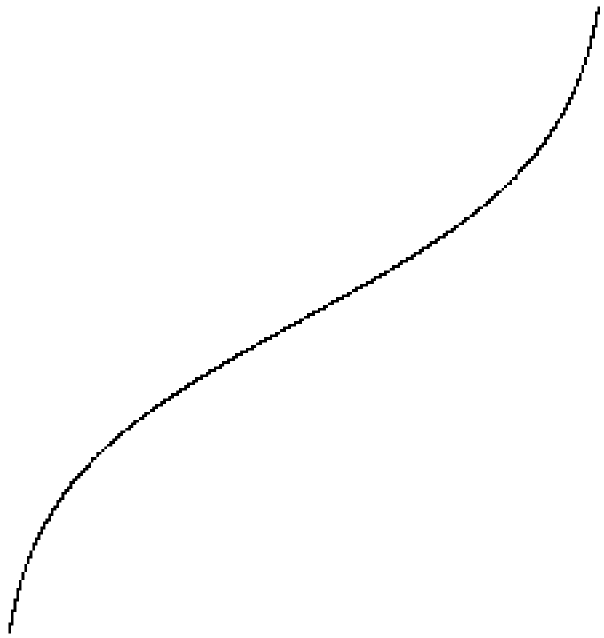

A.13. Cosine (cos)

ASCII rendering.

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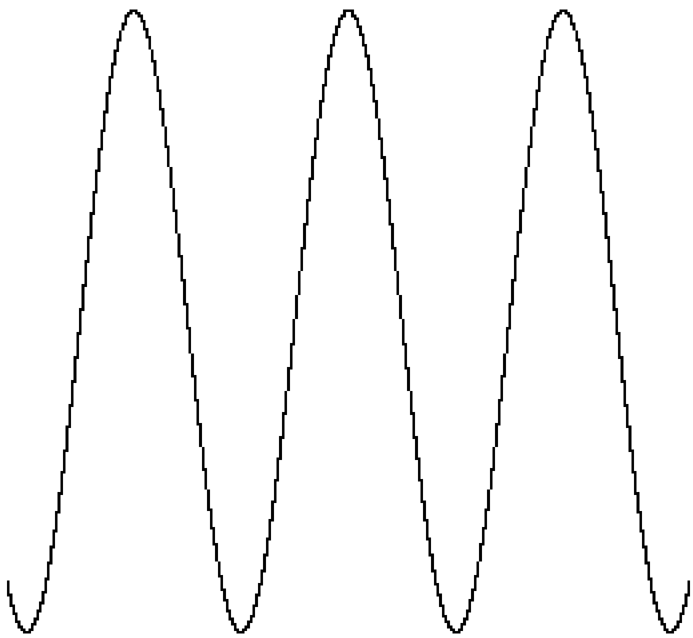
Bitmap rendering.



A.14. Hyperbolic cosine (cosh)

ASCII rendering.

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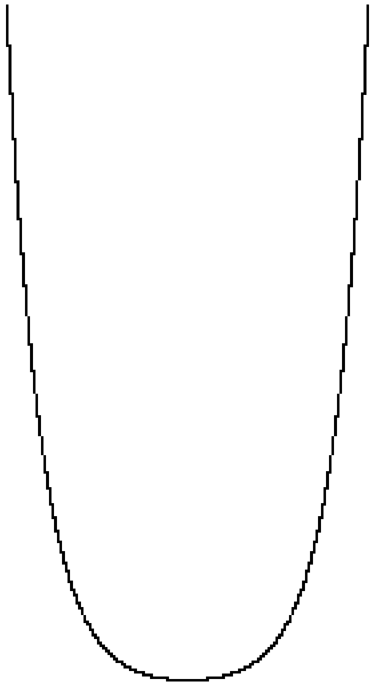

A.15. Cotangent (cot)

ASCII rendering.

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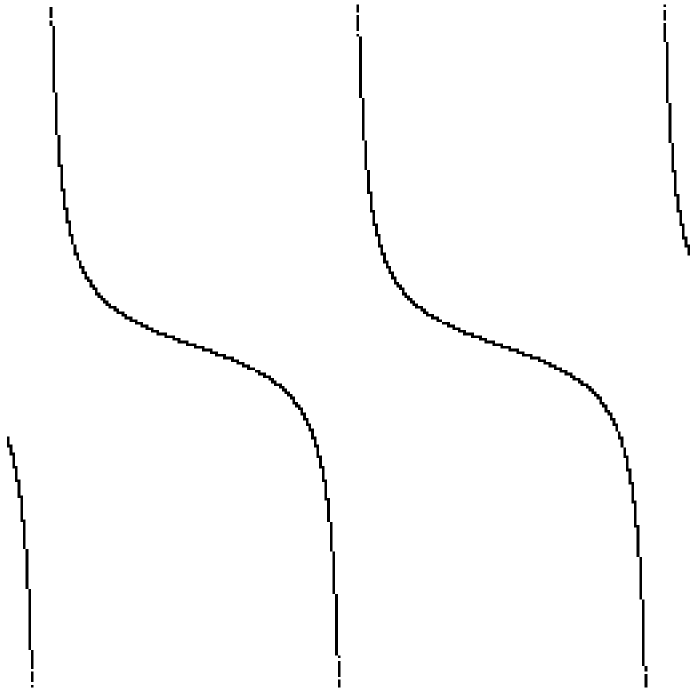

A.16. Hyperbolic cotangent (coth)

ASCII rendering.

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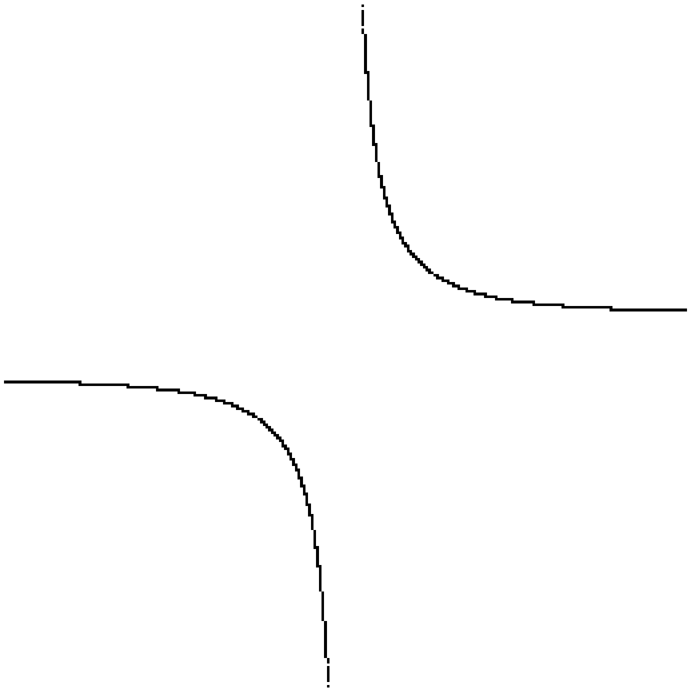

A.17. Cosecant (csc)

ASCII rendering.

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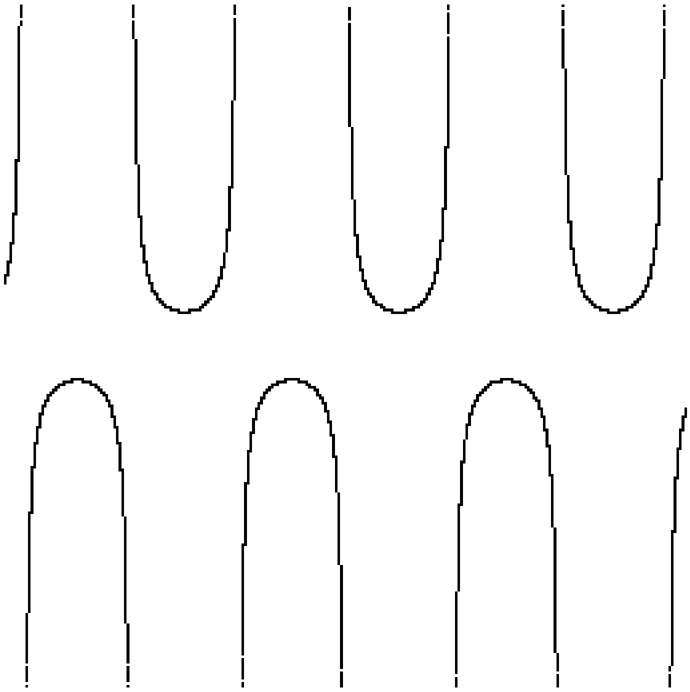

A.18. Hyperbolic cosecant (csch)

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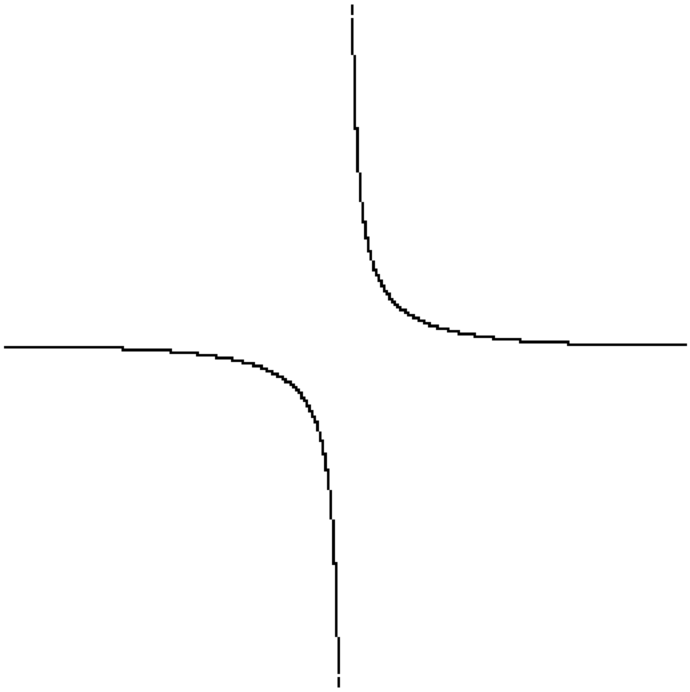

A.19. Exponential (exp)

ASCII rendering.

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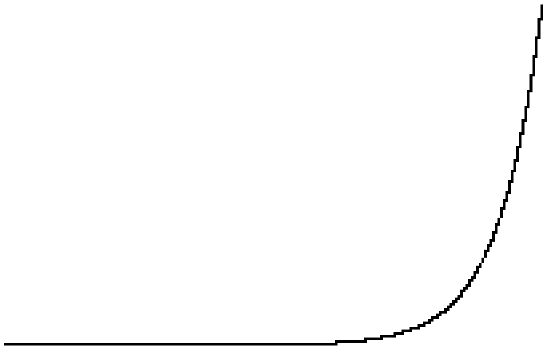

A.20. Natural logarithm (log)

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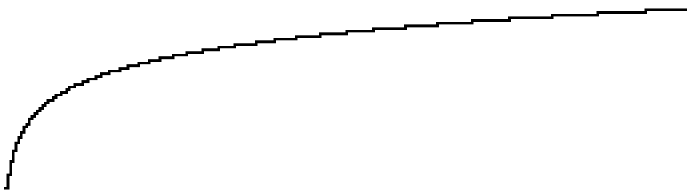

A.21. Secant (sec)

ASCII rendering.

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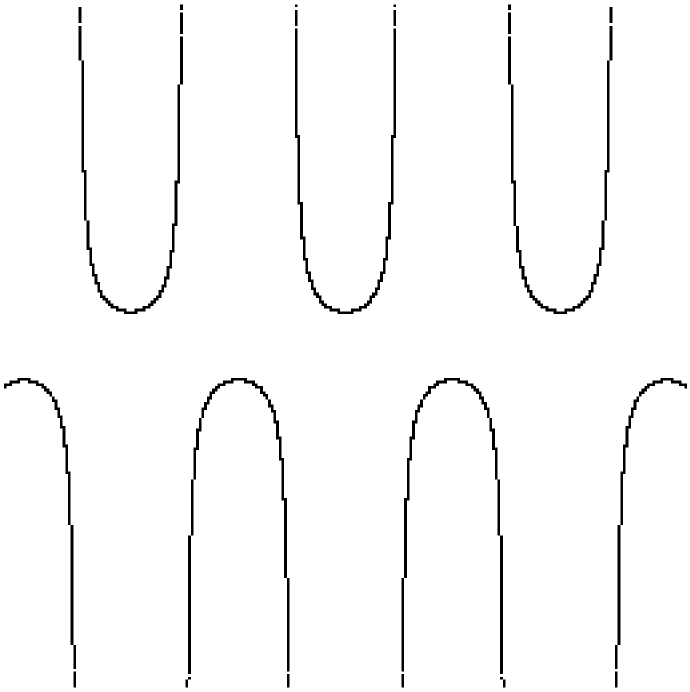

A.22. Hyperbolic secant (sech)

ASCII rendering.

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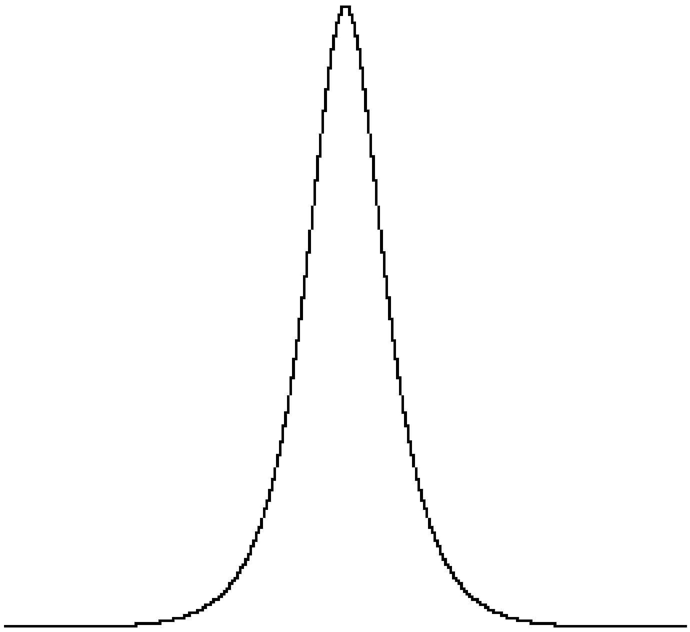

A.23. Sine (sin)

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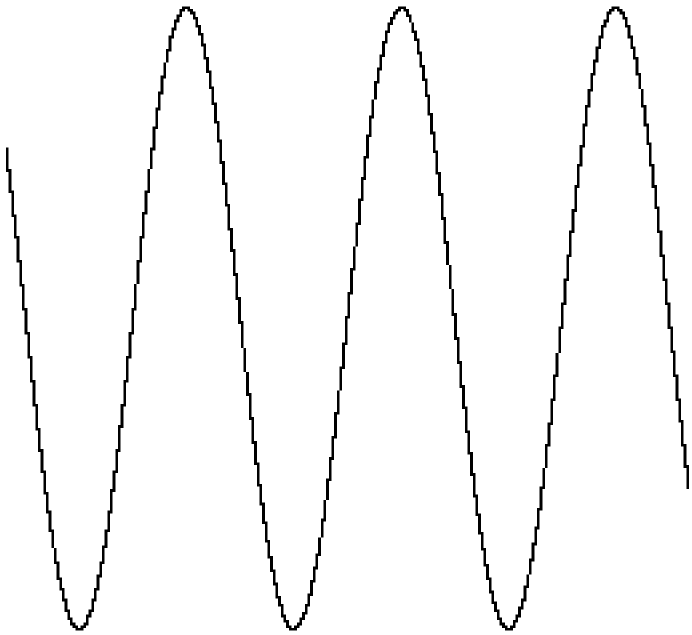

A.24. Hyperbolic sine (sinh)

ASCII rendering.

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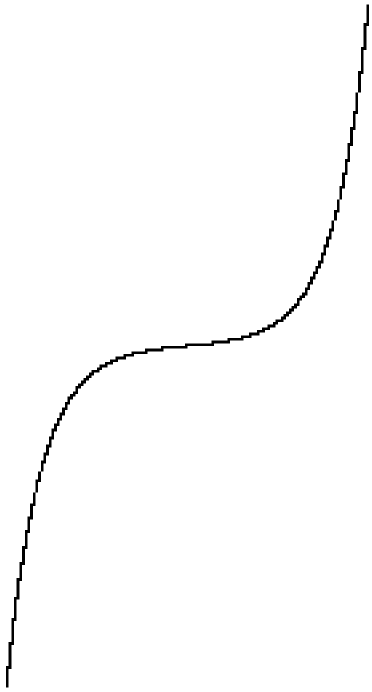

A.25. Square root (sqrt)

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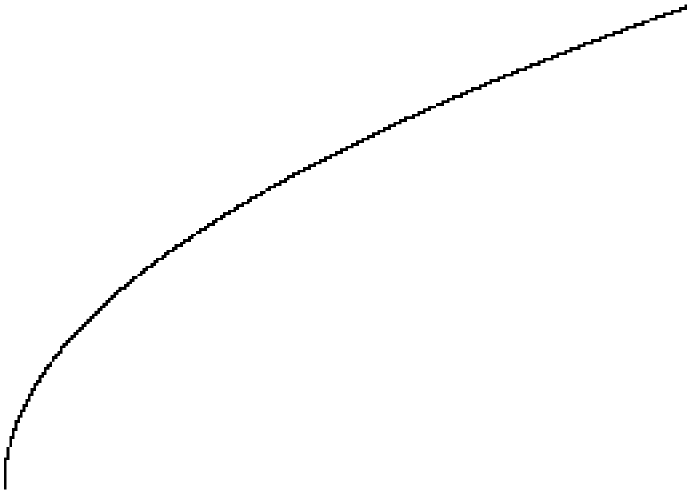

A.26. Tangent (tan)

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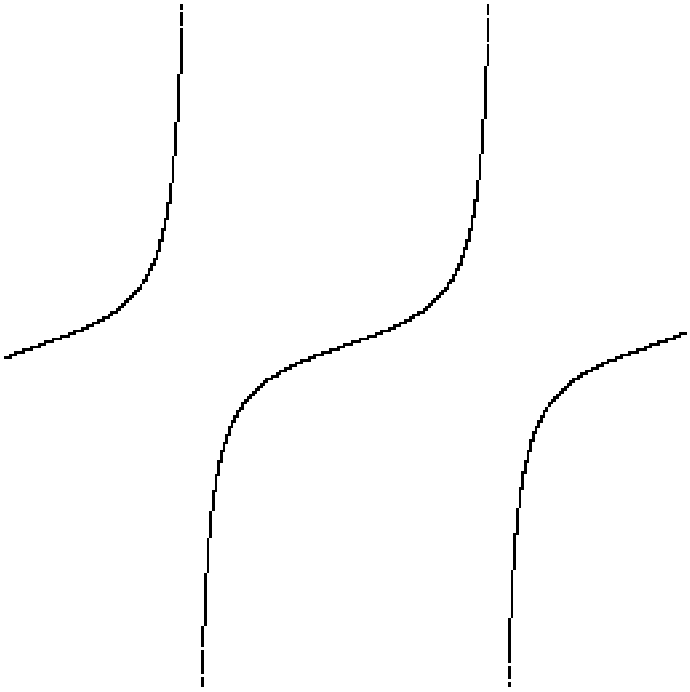

A.27. Hyperbolic tangent (tanh)

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