elemapprox user manual

Title	elemapprox (Elementary functions approximation in ANSI C, Verilog HDL, and VHDL)
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Rev. history	
v1.4.0	2017-07-01 Add `verilogieee` (Verilog-2005) version. Add `common` directories for Verilog and C.
v1.3.3	2016-08-22 Add `fmin`, `fmax` for C, Verilog, and VHDL.
v1.3.2	2016-02-27 Fix ASCII rendering of function plots in the documentation.
v1.3.1	2015-12-25 Documentation updated with function plot examples.
v1.3.0	2014-10-12 Added `ansicstd` version which is an alternative C port that calls the standard C library mathematical functions as defined in `math.h`. Documentation updated.
v1.2.0	2014-10-10 Added `vhdlieee` version which is compatible to the IEEE `math_real` package. Documentation updated.
v1.1.0	2014-10-07 Added VHDL version for approximating and plotting the elementary functions. Numerous documentation fixes.
v1.0.1	2014-09-24 Minor documentation corrections.
v1.0.0	2014-09-16 Initial release. This is an expanded version built upon Mark G. Arnold's Verilog Transcendental Functions paper.

1. Introduction

elemapprox is an ANSI C code, Verilog HDL and VHDL collection of modules (Verilog HDL) and packages (VHDL) that provide the capability of evaluating and plotting transcendental functions by evaluating them in single precision. The original work supports ASCII plotting of a subset of the functions; this version provides a more complete list of functions in addition to bitmap plotting for the transcendental functions as PBM (monochrome) image files.

elemapprox has been conceived as an extension to Prof. Mark G. Arnold's work as puhlished in HDLCON 2001. Most functions have been prefixed with the letter k in order to avoid function name clashes in both ANSI C and VHDL standard-based implementations. Currently, the plain VHDL version, vhdl uses unprefixed names (e.g., acos instead of kacos). The vhdlieee version is compatible to the IEEE.math_real package and features prefixed names.

1.1 Implementations

The following table summarizes the implemented versions in C, VHDL and Verilog HDL.

Implementation	Description
ansic	ANSI C version with custom approximations
ansicstd	ANSI C version using math.h
verilog	Verilog version with custom approximations
verilogieee	Verilog 2005 version with the new arithmetic system functions
vhdl	VHDL version with custom approximations
vhdlieee	VHDL version using IEEE.math_real

1.2 Implemented functions

The transcendental functions supported include most elementary functions (hence the name elemapprox) and the list is as follows:

Function	Description
kfabs(x)	Floating-point absolute value (helper function).
kfmax(x,y)	Floating-point maximum value (helper function).
kfmin(x,y)	Floating-point minimum value (helper function).
kfloor(x)	Floating-point floor (helper function).
kceil(x)	Floating-point ceiling (helper function).
kround(x)	Floating-point round-to-nearest (helper function).
ktrunc(x)	Floating-point truncation (helper function).
kfmod(x,y)	Floating-point modulo.
rootof2(x)	Calculate root-of-2 (not in vhdlieee, verilogieee and ansicstd).
kacos(x)	Arc cosine.

`kacosh(x)`	Inverse hyperbolic cosine.
`kacot(x)`	Arc cotangent.
`kacoth(x)`	Inverse hyperbolic cotangent.
`kacsc(x)`	Arc cosecant.
`kacsch(x)`	Inverse hyperbolic cosecant.
`kasec(x)`	Arc secant.
`kasech(x)`	Inverse hyperbolic secant.
`kasin(x)`	Arc sine.
`kasinh(x)`	Inverse hyperbolic sine.
`katan(x)`	Arc tangent.
`katan2(y,x)`	Two-argument (x/y) arc tangent.
`katanh(x)`	Inverse hyperbolic tangent.
`kcos(x)`	Cosine.
`kcosh(x)`	Hyperbolic cosine.
`kcot(x)`	Cotangent.
`kcoth(x)`	Hyperbolic cotangent.
`kcsc(x)`	Cosecant.
`kcsch(x)`	Hyperbolic cosecant.
`kexp(x)`	Exponential.
`khypot(x,y)`	Hypotenuse.
`klog(x)`	Natural logarithm.
`kpow(x,y)`	Powering function.
`ksec(x)`	Secant (named `secant` in the plain VHDL port).
`ksech(x)`	Hyperbolic secant.
`ksin(x)`	Sine.
`ksinh(x)`	Hyperbolic sine.
`ksqrt(x)`	Square root.
`ktan(x)`	Tangent.
`ktanh(x)`	Hyperbolic tangent.

The reference paper and the corresponding presentation are available from the web at the following links:

- https://web.archive.org/web/20100515000000*/http://www.cse.lehigh.edu/~caar/marnold/papers/sanjose_hdlcon.doc
- https://web.archive.org/web/20100415000000*/http://www.cse.lehigh.edu/~caar/marnold/presentations/freeddy.ppt

Since the original links have been removed, the files are now only accessible from the Internet Archive.

2. File listing

The `elemapprox` C code implementation and Verilog HDL modules include the following files:

/elemapprox	Top-level directory
AUTHORS	List of authors.
LICENSE	License argeement (Modified BSD license).
README.rst	This file.
README.html	HTML version of README.
README.pdf	PDF version of README.
rst2docs.sh	Bash script for generating the HTML and PDF versions.
VERSION	Current version.
/ansic	ANSI C implementation (standalone)
clean-math-ansic.sh	Bash script for cleaning up the generated executables.
elemapprox.c	C code for the function approximations.
Makefile	GNU Makefile for building `testfunc.exe`.
plot-ansic-ascii.sh	Bash script for plotting the elementary functions as ASCII graphs using `testfunc.exe`.
plot-ansic-pbm.sh	Bash script for plotting the elementary functions as PBM images using `testfunc.exe`.
test<func>.pbm	Generated PBM image data for the function <func>.
test<func>.txt	Generated ASCII graph data for the function <func>.
test<func>-ascii.txt	Concatenation of the generated ASCII graph data for all supported functions.
/ansicstd	ANSI C implementation (based on `math.h`)
clean-math-ansic.sh	Bash script for cleaning up the generated executables.
elemapprox.c	C code for the function approximations.
Makefile	GNU Makefile for building `testfunc.exe`.
plot-ansic-ascii.sh	Bash script for plotting the elementary functions as ASCII graphs using `testfunc.exe`.
plot-ansic-pbm.sh	Bash script for plotting the elementary functions as PBM images using `testfunc.exe`.
test<func>.pbm	Generated PBM image data for the function <func>.
test<func>.txt	Generated ASCII graph data for the function <func>.
test<func>-ascii.txt	Concatenation of the generated ASCII graph data for all supported functions.
/common	Common files
/common/ansic	Common files for the C implementations
elemapprox.h	C header file for the above. Defines certain mathematical constants and declares function prototypes.
funcplot.c	Reference code for creating the plot data for the functions.
funcplot.h	C header file for the above.

graph.c	Collection of ASCII and PBM graphing functions.
graph.h	C header file for the above.
/common/scripts	Scripts for common use
diff-ascii.sh	Compare the results from ASCII plots between two different implementations (directories)
pbm2png.sh	Convert PBM to PNG images with ImageMagick's `convert`.
/common/verilog	Common files for the Verilog implementations
funcplot.v	Reference code for creating the plot data for the functions.
graph.v	Collection of ASCII and PBM graphing tasks.
testfunc.v	Application code for the elementary functions. Options include PBM or ASCII image generation and function selection.
/verilog	Verilog HDL implementation
clean-math-verilog.sh	Bash script for cleaning up the generated interpreted intermediate code (for Icarus Verilog).
constants.v	Certain mathematical constants.
elemapprox.v	Verilog HDL code for the function approximations.
plot-verilog-ascii.sh	Bash script for plotting the elementary functions as ASCII graphs using `testfunc.v`. The script Icarus Verilog' VVP interpreter which is capable of parsing command-line options.
plot-verilog-pbm.sh	Bash script for plotting the elementary functions as PBM images using `testfunc.v`.
test<func>.pbm	Generated PBM image data for the function <func>.
test<func>.txt	Generated ASCII graph data for the function <func>.
test<func>-ascii.txt	Concatenation of the generated ASCII graph data for all supported functions.
/verilogieee	Verilog 2005 implementation
clean-math-verilog.sh	Bash script for cleaning up the generated interpreted intermediate code (for Icarus Verilog).
elemapprox.v	Verilog HDL code for the function approximations.
plot-verilog-ascii.sh	Bash script for plotting the elementary functions as ASCII graphs using `testfunc.v`. The script Icarus Verilog' VVP interpreter which is capable of parsing command-line options.
plot-verilog-pbm.sh	Bash script for plotting the elementary functions as PBM images using `testfunc.v`.
test<func>.pbm	Generated PBM image data for the function <func>.
test<func>.txt	Generated ASCII graph data for the function <func>.
test<func>-ascii.txt	Concatenation of the generated ASCII graph data for all supported functions.

/vhdl	VHDL implementation
elemapprox.do	Modelsim `.do` macro file for Modelsim simulation.
elemapprox.mk	GNU Makefile for running the testbench using GHDL.
elemapprox.vhd	VHDL package code for the function approximations and related mathematical constants.
funcplot.vhd	VHDL package code for creating the plot data for the elementary functions.
graph.vhd	VHDL package code with a collection of ASCII and PBM procedures.
plot-ghdl-ascii.sh	Bash script for plotting the elementary functions as ASCII graphs using GHDL. The script generates a configuration file (`config.txt`) for controlling the simulation.
plot-ghdl-pbm.sh	Bash script for plotting the elementary functions as PBM images using GHDL. The script generates a configuration file (`config.txt`) for controlling the simulation.
plot-mti-ascii.sh	Bash script for plotting the elementary functions as ASCII graphs using Modelsim. The script generates a configuration file (`config.txt`) for controlling the simulation.
plot-mti-pbm.sh	Bash script for plotting the elementary functions as PBM images using Modelsim. The script generates a configuration file (`config.txt`) for controlling the simulation.
testfunc.vhd	VHDL testbench code for the elementary functions. Options include PBM or ASCII image generation and function selection through a configuration file.
test<func>.pbm	Generated PBM image data for the function <func>.
test<func>.txt	Generated ASCII graph data for the function <func>.
test<func>-ascii.txt	Concatenation of the generated ASCII graph data for all supported functions.
/vhdlieee	VHDL implementation compatible to `IEEE.math_real`
elemapprox.do	Modelsim `.do` macro file for Modelsim simulation.
elemapprox.mk	GNU Makefile for running the testbench using GHDL.
elemapprox.vhd	VHDL package code for the function approximations and related mathematical constants.
funcplot.vhd	VHDL package code for creating the plot data for the elementary functions.
graph.vhd	VHDL package code with a collection of ASCII and PBM procedures.

plot-ghdl-ascii.sh	Bash script for plotting the elementary functions as ASCII graphs using GHDL. The script generates a configuration file (`config.txt`) for controlling the simulation.
plot-ghdl-pbm.sh	Bash script for plotting the elementary functions as PBM images using GHDL. The script generates a configuration file (`config.txt`) for controlling the simulation.
plot-mti-ascii.sh	Bash script for plotting the elementary functions as ASCII graphs using Modelsim. The script generates a configuration file (`config.txt`) for controlling the simulation.
plot-mti-pbm.sh	Bash script for plotting the elementary functions as PBM images using Modelsim. The script generates a configuration file (`config.txt`) for controlling the simulation.
testfunc.vhd	VHDL testbench code for the elementary functions. Options include PBM or ASCII image generation and function selection through a configuration file.
test<func>.pbm	Generated PBM image data for the function <func>.
test<func>.txt	Generated ASCII graph data for the function <func>.
test<func>-ascii.txt	Concatenation of the generated ASCII graph data for all supported functions.
/refs	Reference documentation
sanjose_hdlcon.doc	MS Word document for the manuscript: M. G. Arnold, C. Walter and F. Engineer, "Verilog Transcendental Functions for Numerical Testbenches," Proceedings of the Tenth International HDL conference, Santa Clara, California, March 1, 2001.
freeddy.ppt	MS PowerPoint presentation of the above work.

3. Usage

Both the ANSI C and Verilog HDL versions can be used for generating graph data and depicting any of the supported transcendental functions via two similar scripts.

3.1 ANSI C

1. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate an ASCII graph for each function.

```
$ cd ansic
```

or

```
$ cd ansicstd
```

followed by

```
$ ./plot-ansic-ascii.sh
```

All generated data are also concatenated to `testfunc-ascii.txt`.

2. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate a PBM image for each function.

```
$ ./plot-ansic-pbm.sh
```

All generated data are saved in the form of PBM (monochrome bitmap) image files. Such files can be visualized using, e.g., the public domain `Imagine` viewer: http://www.nyam.pe.kr/ on Windows or `eog` (Eye of Gnome) on Debian-based Linux distributions (for instance, Ubuntu).

3.2 Verilog HDL

1. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate an ASCII graph for each function.

```
$ cd verilog
```

or

```
$ cd verilogieee
```

followed by

```
$ ./plot-verilog-ascii.sh
```

All generated data are also concatenated to `testfunc-ascii.txt`.

2. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate a PBM image for each function.

```
$ ./plot-verilog-pbm.sh
```

All generated data are saved in the form of PBM (monochrome bitmap) image files.

3.3 VHDL

The VHDL version of `elemapprox` supports both GHDL (http://ghdl.free.fr) and Mentor Modelsim (http://www.model.com).

3.3.1 GHDL

1. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate an ASCII graph for each function.

```
$ cd vhdl
```

or

```
$ cd vhdlieee
```

followed by

```
$ ./plot-ghdl-ascii.sh
```

All generated data are also concatenated to `testfunc-ascii.txt`.

2. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate a PBM image for each function.

```
$ ./plot-ghdl-pbm.sh
```

All generated data are saved in the form of PBM (monochrome bitmap) image files.

3.3.2 Modelsim

1. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate an ASCII graph for each function.

```
$ cd vhdl
```

or

```
$ cd vhdlieee
```

followed by

```
$ ./plot-mti-ascii.sh
```

All generated data are also concatenated to `testfunc-ascii.txt`.

2. Run the following shell script from a Unix/Linux/Cygwin command line in order to generate a PBM image for each function.

```
$ ./plot-mti-pbm.sh
```

All generated data are saved in the form of PBM (monochrome bitmap) image files.

4. Synthesis

The implementation code (either ANSI C, Verilog HDL or VHDL) for the transcendental functions has not been tested for high-level or RTL synthesis.

5. Prerequisites

- Standard UNIX-based tools (tested with gcc-4.6.2 on MinGW/x86) [optional if you use Modelsim].
  - make
  - bash (shell)

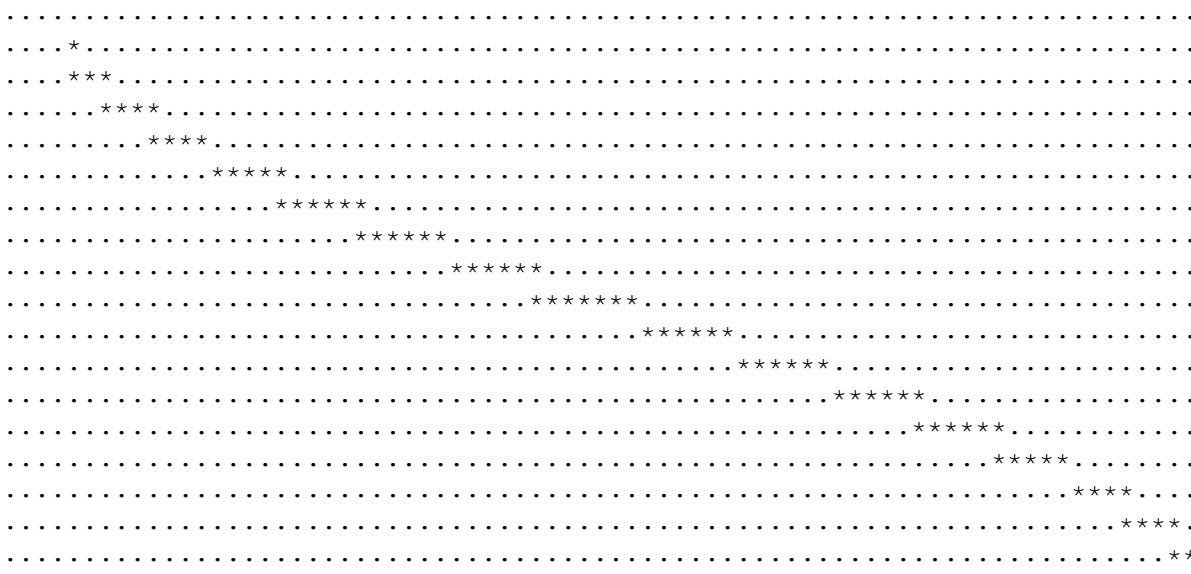
  For this reason, MinGW (http://www.mingw.org) or Cygwin (http://sources.redhat.com/cygwin) are suggested, since POSIX emulation environments of sufficient completeness.
- Icarus Verilog simulator (http://iverilog.icarus.com/). The Windows version can be downloaded from: http://bleyer.org/icarus/
- GHDL simulator (http://ghdl.free.fr) for VHDL. Both Windows and Linux versions can be downloaded from this site. Updated GHDL releases are available (again for multiple OSes) from: http://sourceforge.net/projects/ghdl-updates/
- Alternatively, a commercial simulator like Mentor Modelsim (http://www.model.com) can be used (however this has only been tested for the VHDL version of `elemapprox`).

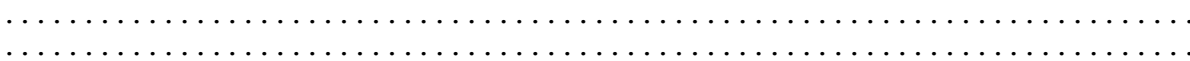
A. Function plot examples

This appendix provides sample function plot visualization as raster graphics (PNG) and ASCII for the elementary functions with provided implementations.

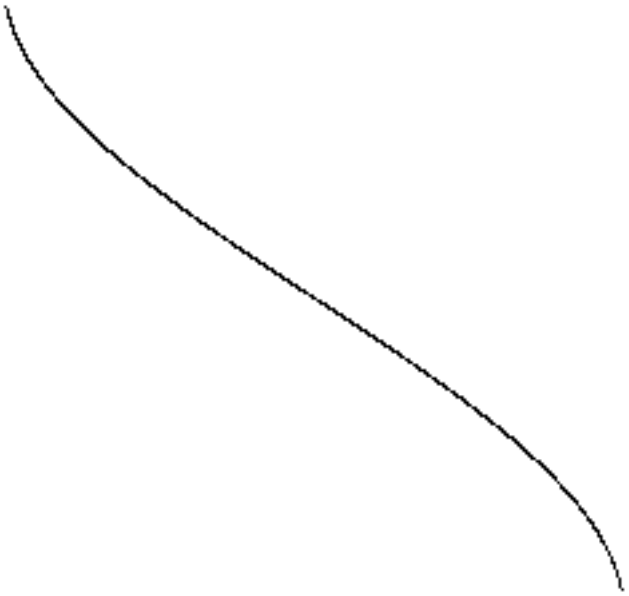
A.1. Arc cosine (acos)

ASCII rendering.



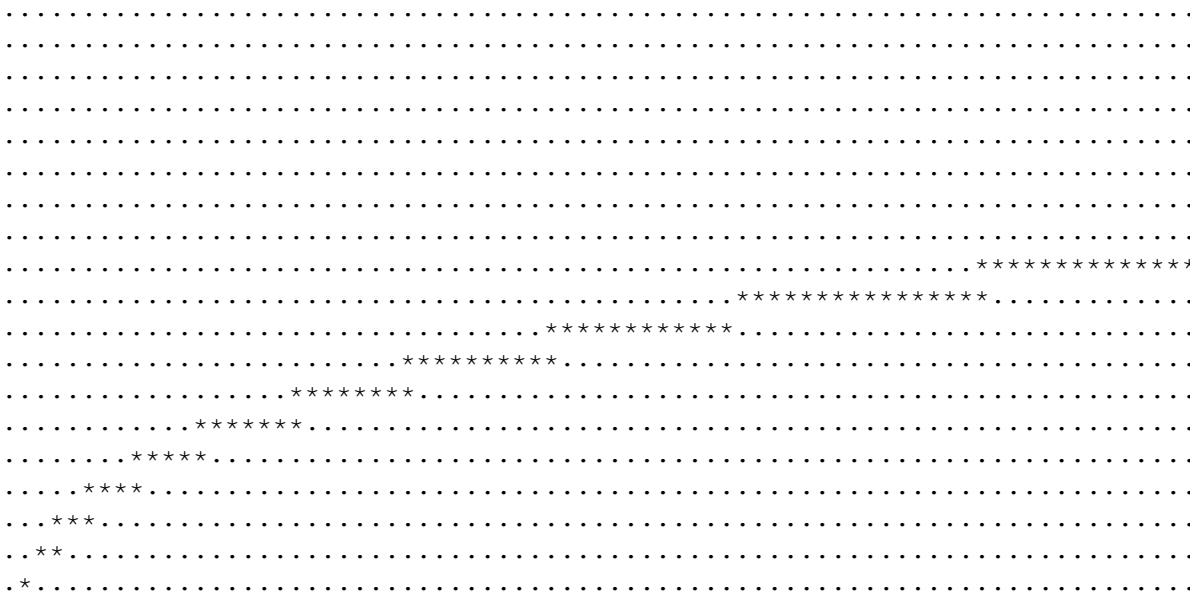


Bitmap rendering.

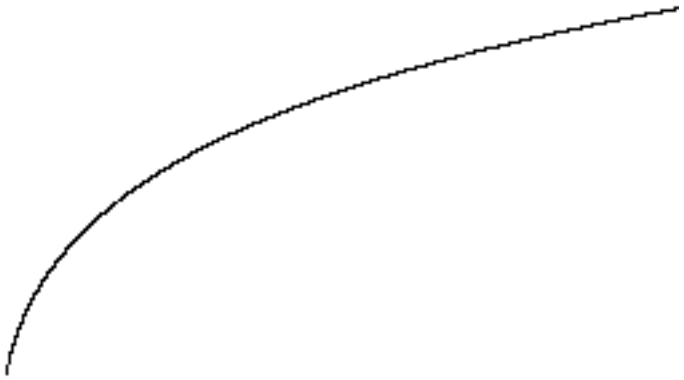


A.2. Inverse hyperbolic cosine (acosh)

ASCII rendering.



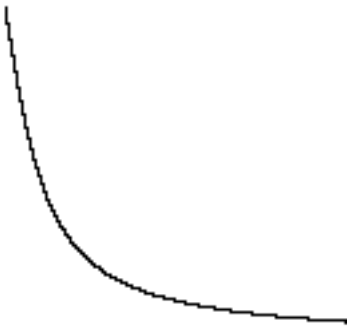
Bitmap rendering.



A.3. Arc cotangent (acot)

ASCII rendering.

Bitmap rendering.



A.4. Inverse hyperbolic cotangent (acoth)

ASCII rendering.

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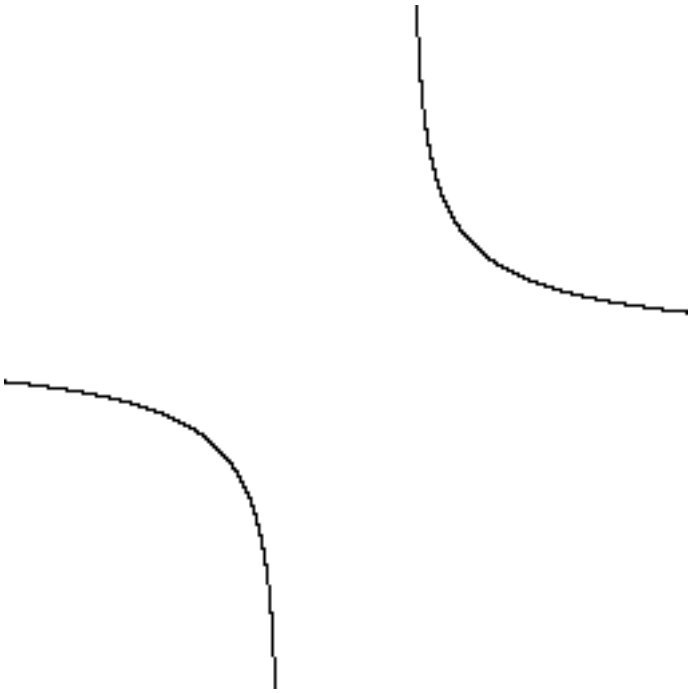
Bitmap rendering.



A.5. Arc cosecant (acsc)

ASCII rendering.

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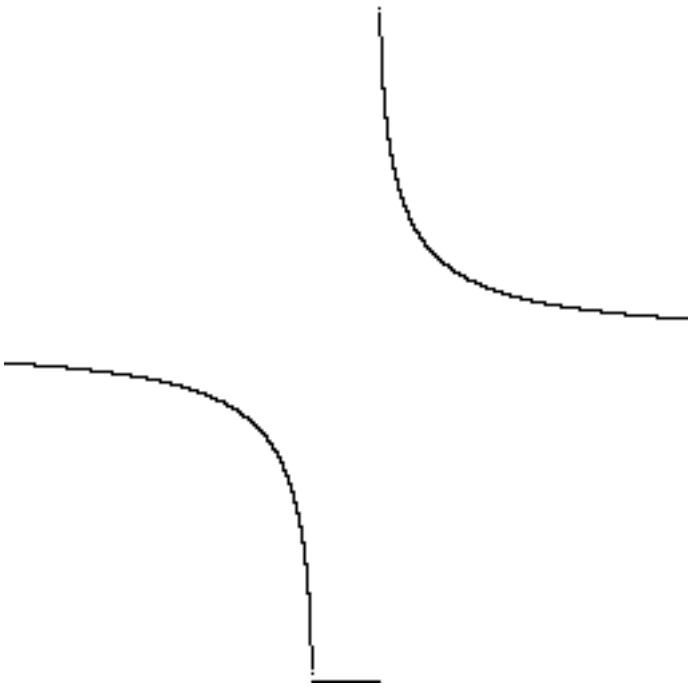
Bitmap rendering.



A.6. Inverse hyperbolic cosecant (acsch)

ASCII rendering.

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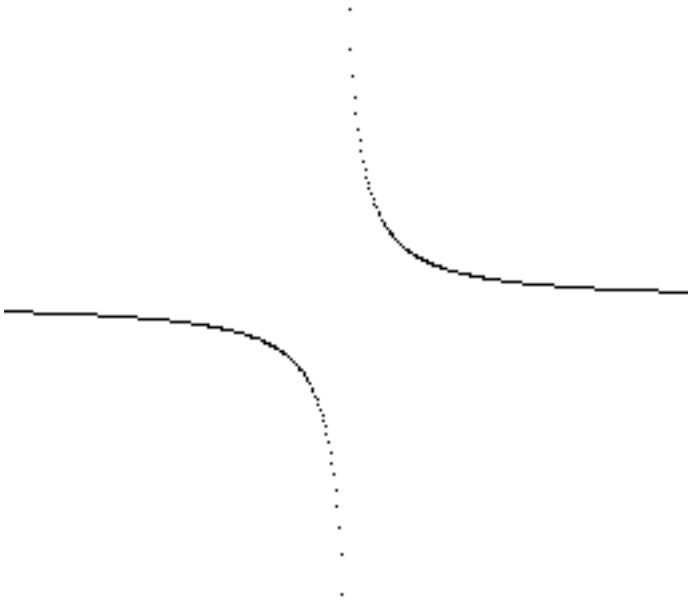
Bitmap rendering.



A.7. Arc secant (asec)

ASCII rendering.

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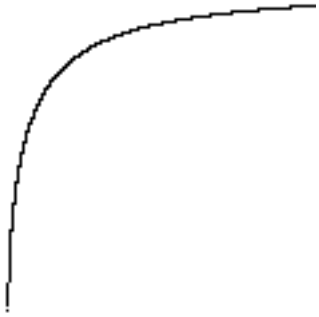
Bitmap rendering.



A.8. Inverse hyperbolic secant (asech)

ASCII rendering.

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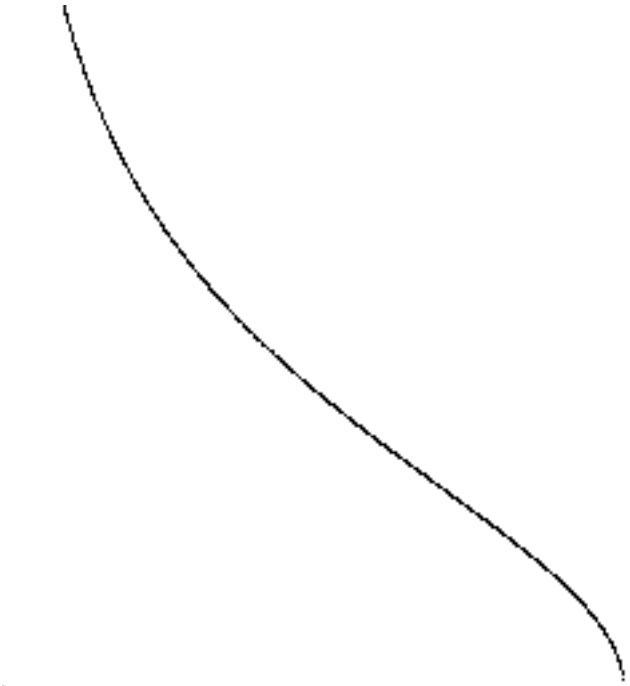
Bitmap rendering.



A.9. Arc sine (asin)

ASCII rendering.

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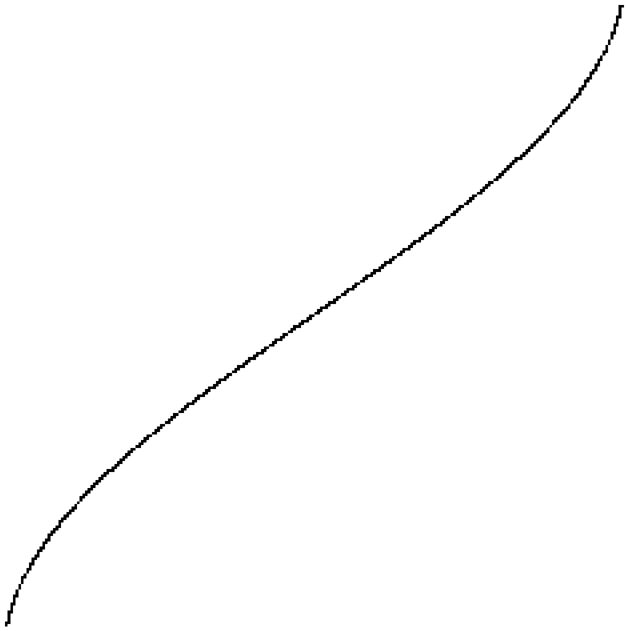
Bitmap rendering.



A.10. Inverse hyperbolic sine (asinh)

ASCII rendering.

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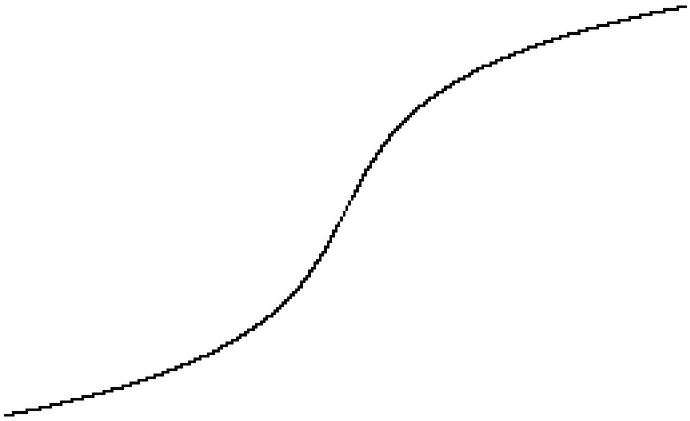
Bitmap rendering.



A.11. Arc tangent (atan)

ASCII rendering.

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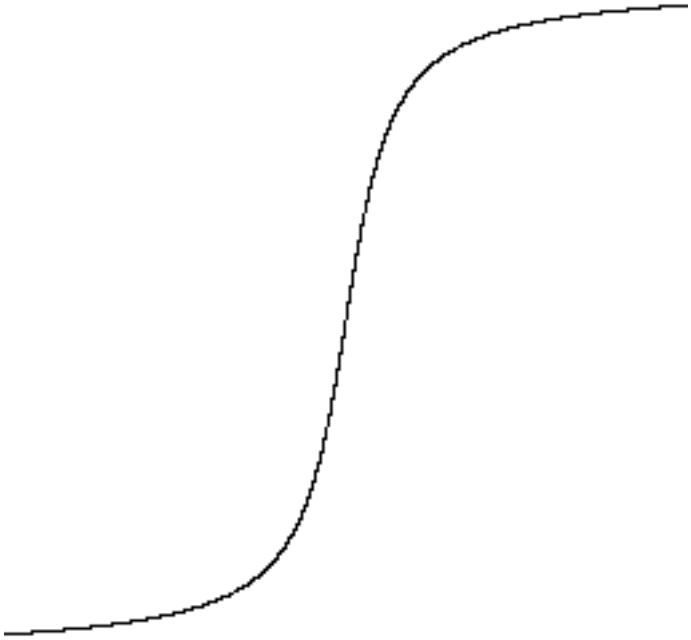
Bitmap rendering.



A.12. Inverse hyperbolic tangent (atanh)

ASCII rendering.

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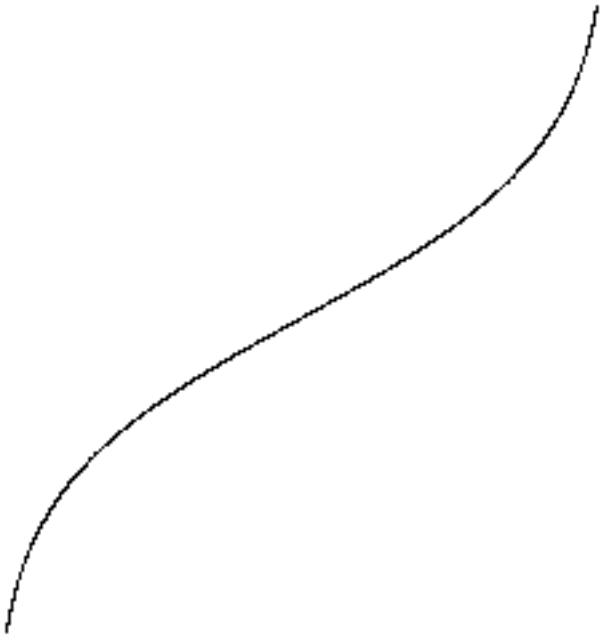
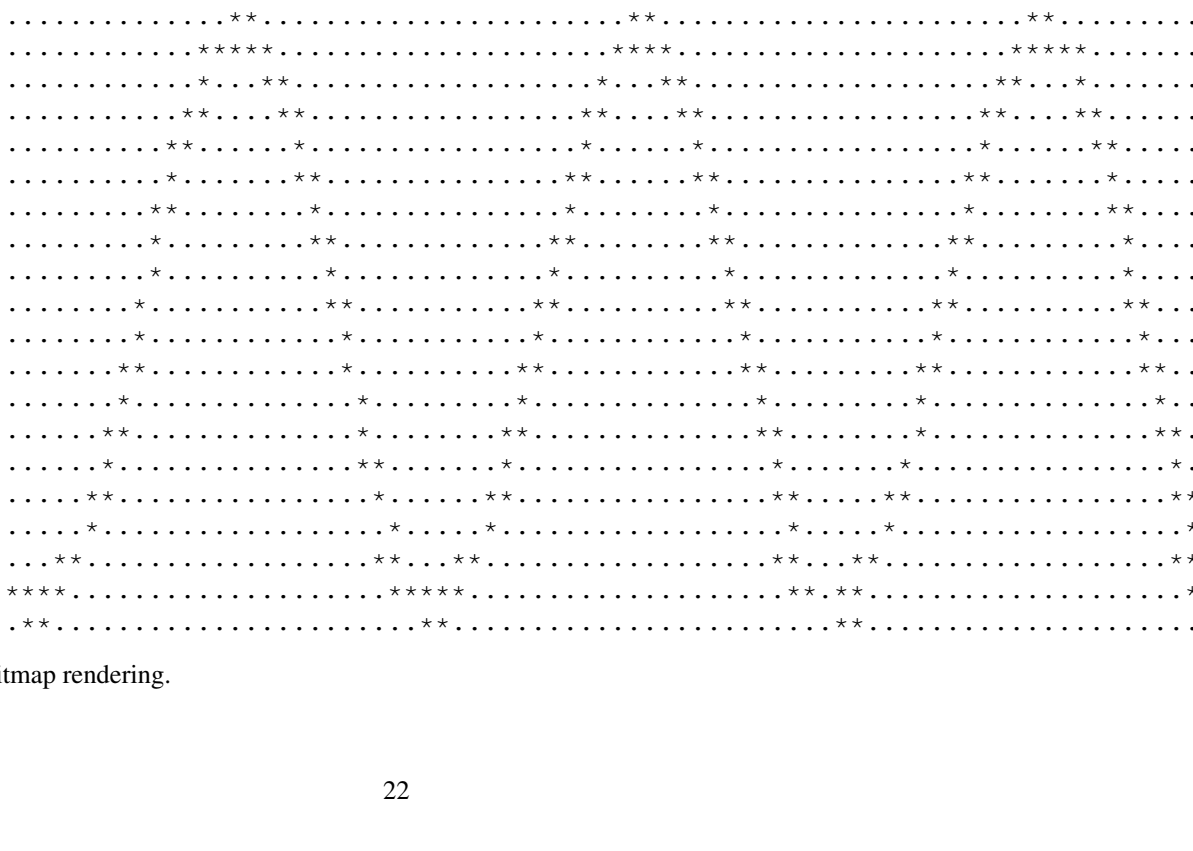
Bitmap rendering.

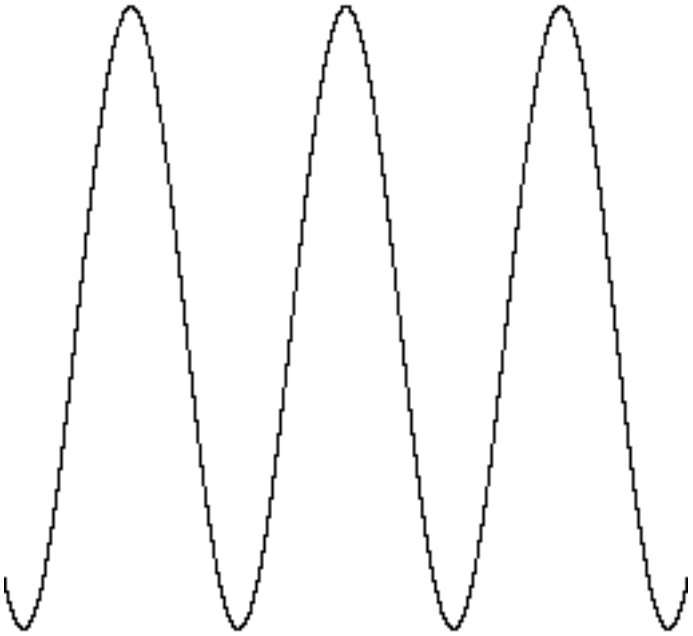


A.13. Cosine (cos)

ASCII rendering.



Bitmap rendering.



A.14. Hyperbolic cosine (cosh)

ASCII rendering.

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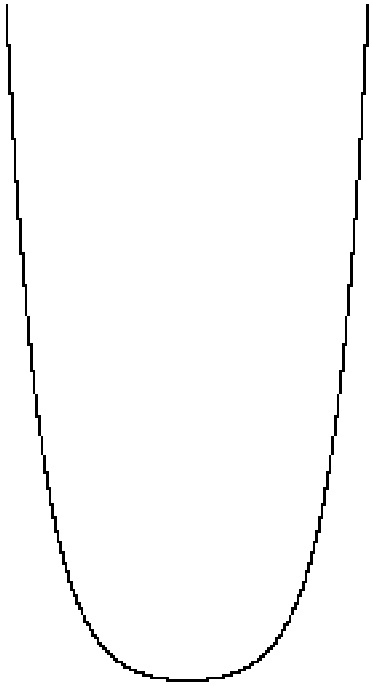
Bitmap rendering.



A.15. Cotangent (cot)

ASCII rendering.

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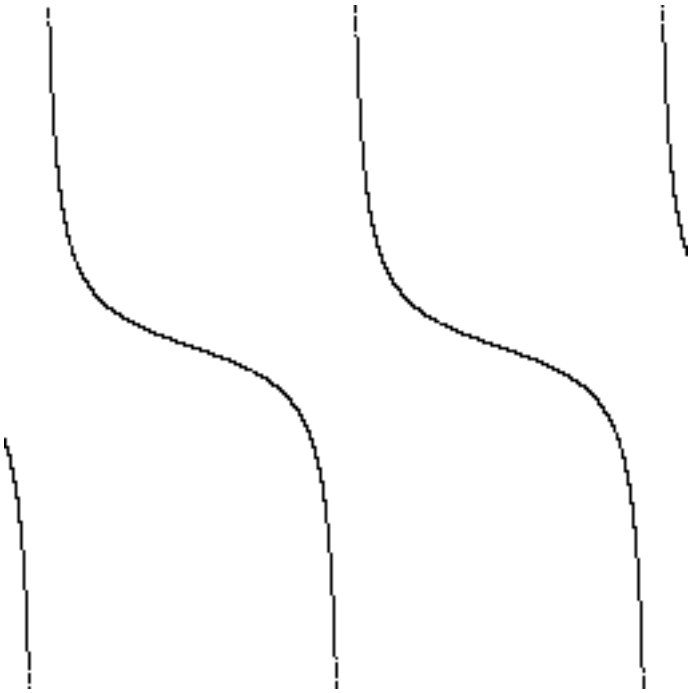
Bitmap rendering.



A.16. Hyperbolic cotangent (coth)

ASCII rendering.

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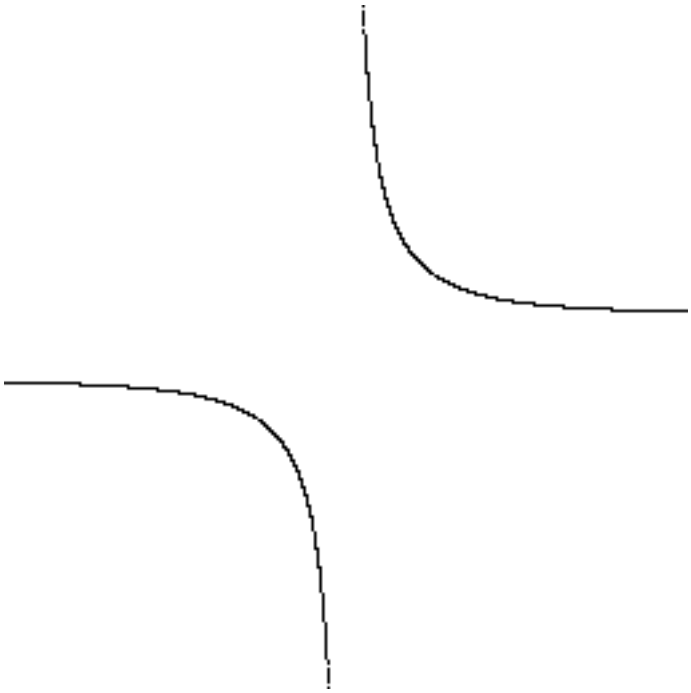
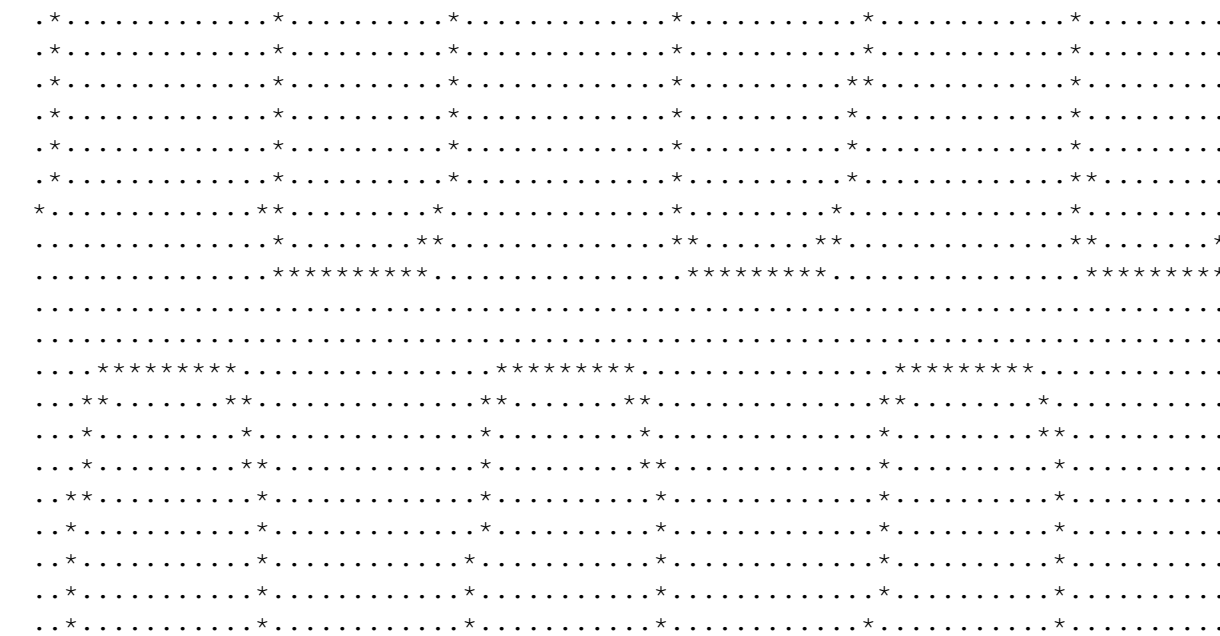
Bitmap rendering.

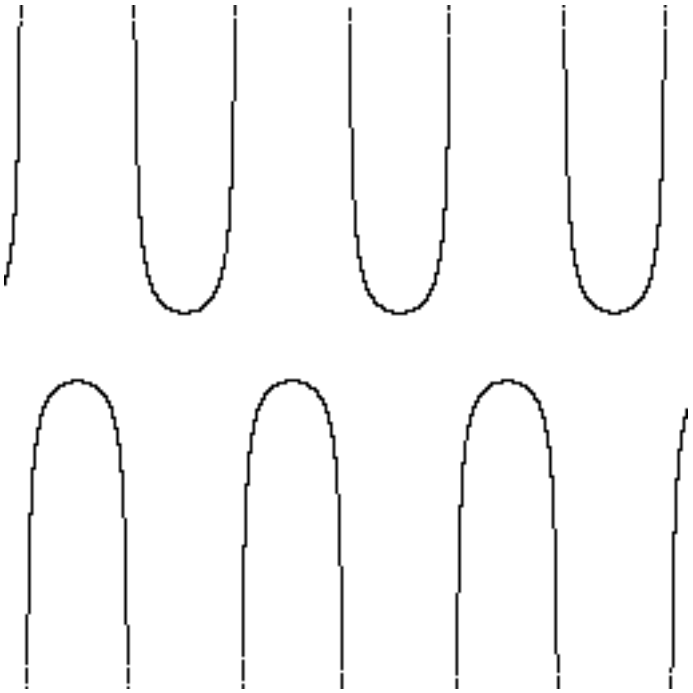


A.17. Cosecant (csc)

ASCII rendering.



Bitmap rendering.



A.18. Hyperbolic cosecant (csch)

ASCII rendering.

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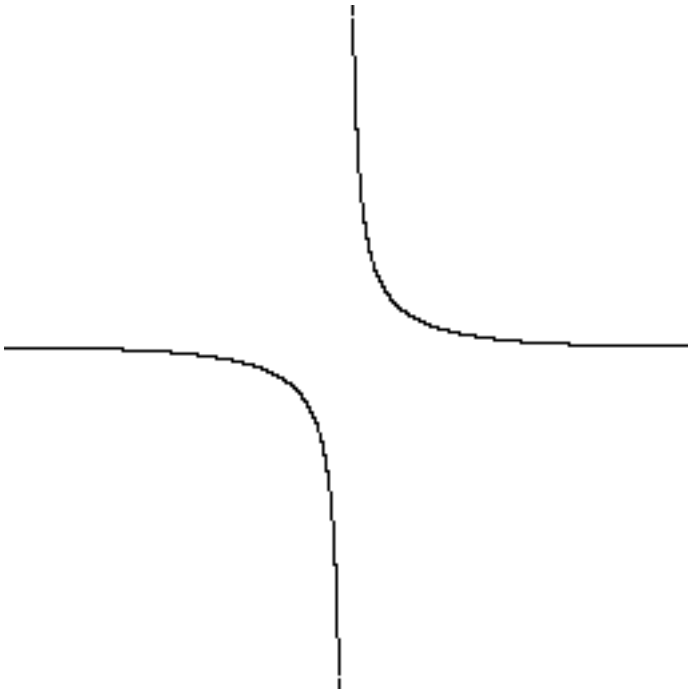
Bitmap rendering.



A.19. Exponential (exp)

ASCII rendering.

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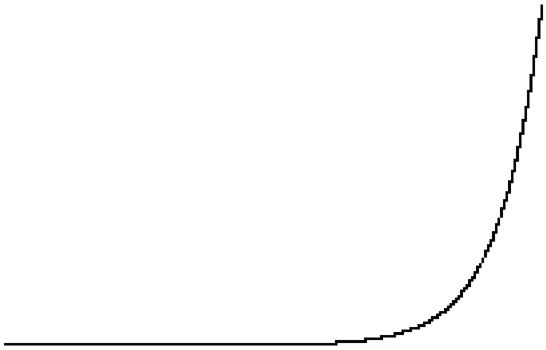
Bitmap rendering.



A.20. Natural logarithm (log)

ASCII rendering.

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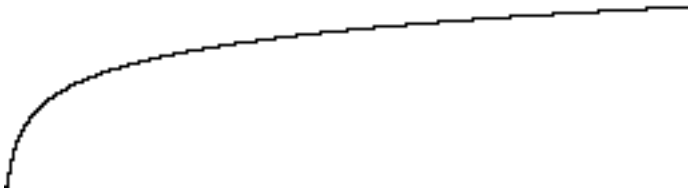

A.21. Base-10 logarithm (log10)

system-message
WARNING/2 in <stdin>, line 875
Title underline too short.

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A.21. Base-10 logarithm (log10)
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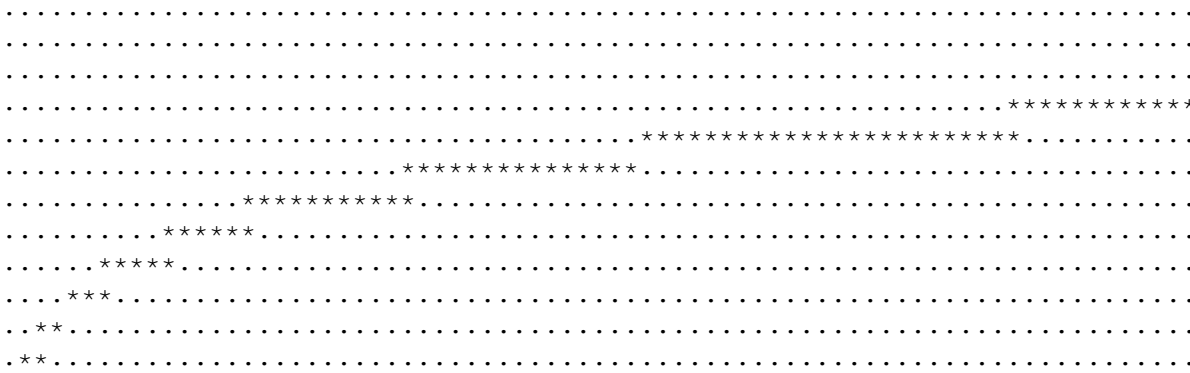
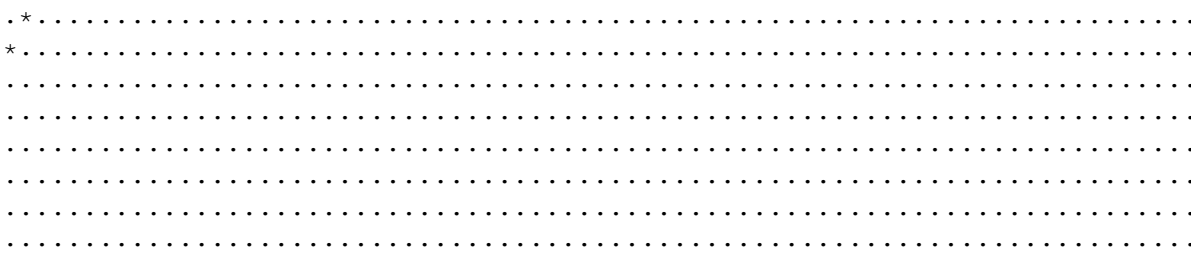
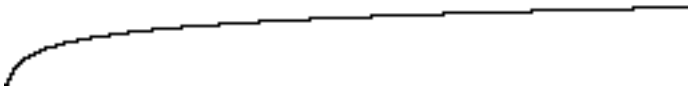
backrefs:

ASCII rendering.



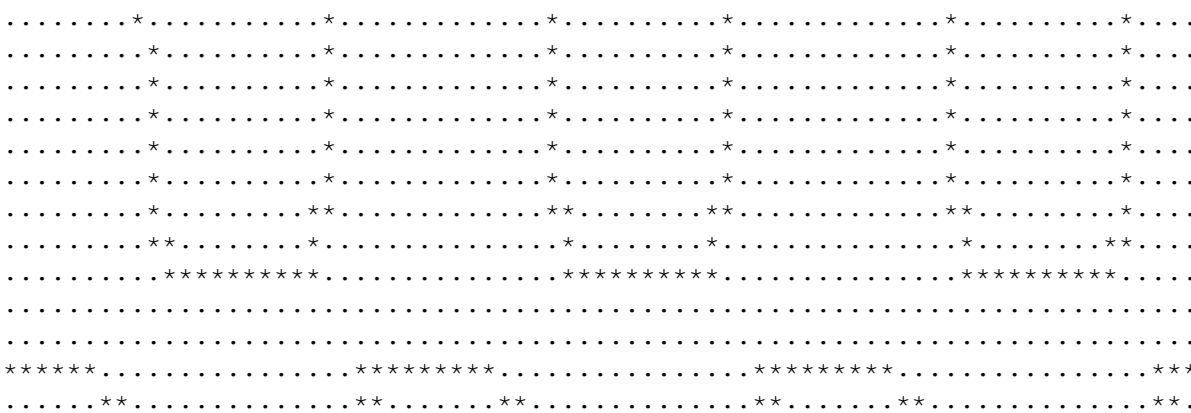


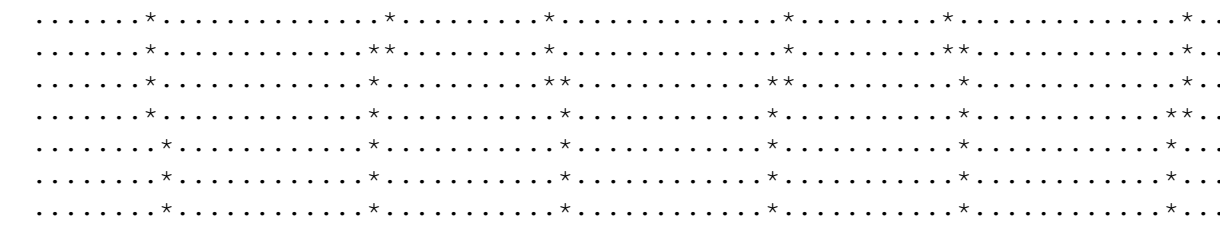
Bitmap rendering.



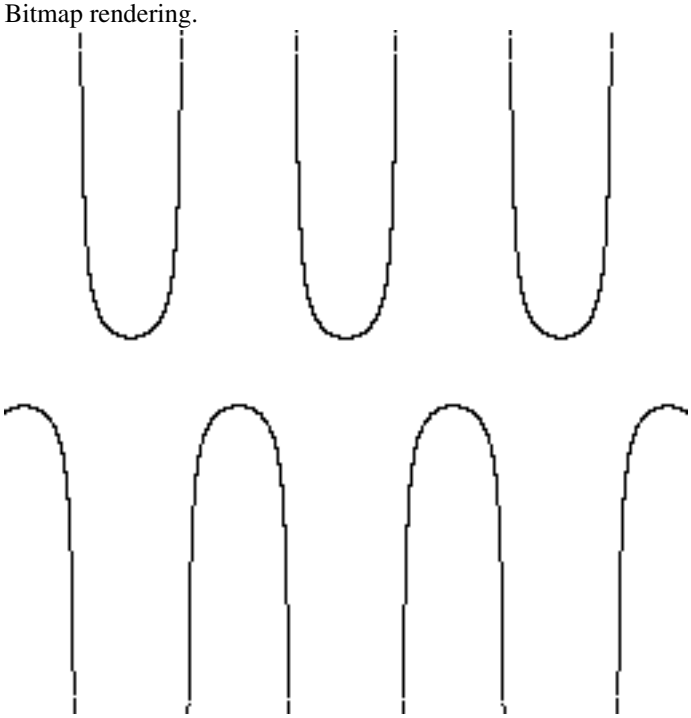
A.22. Secant (sec)

ASCII rendering.



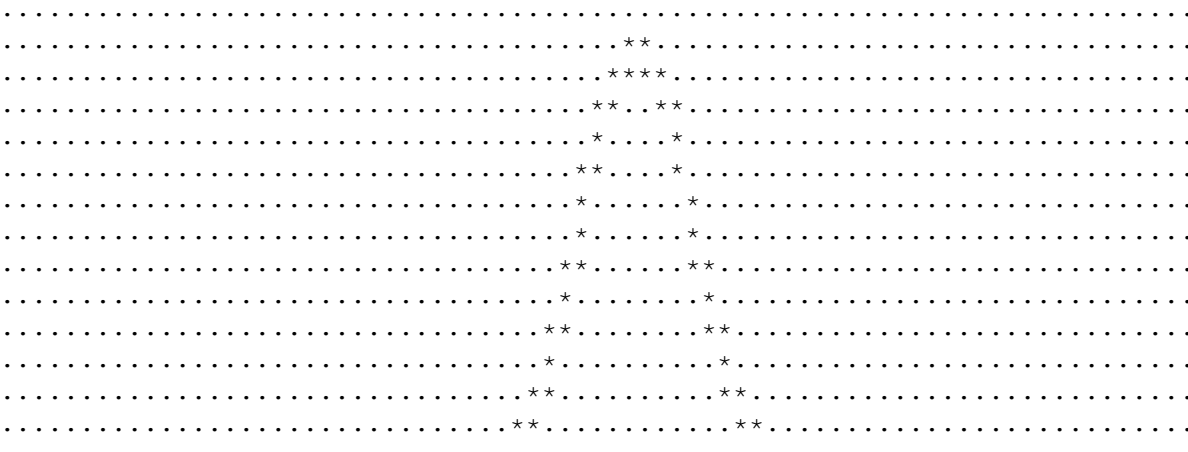


Bitmap rendering.



A.23. Hyperbolic secant (sech)

ASCII rendering.



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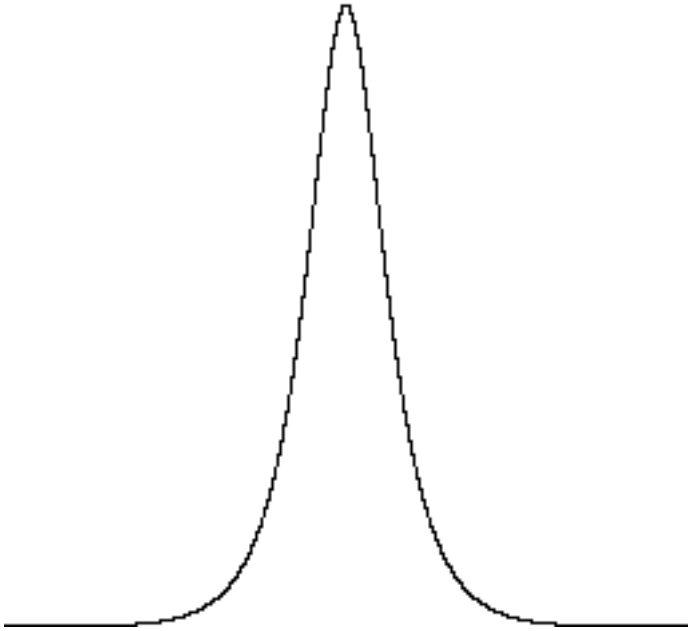

A.24. Sine (sin)

ASCII rendering.

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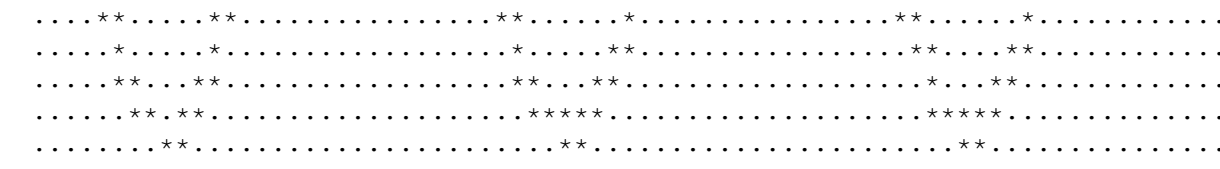
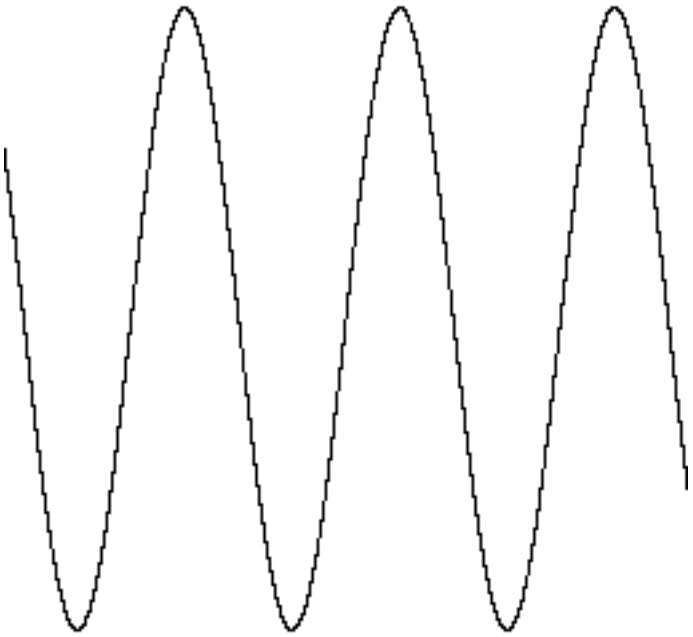
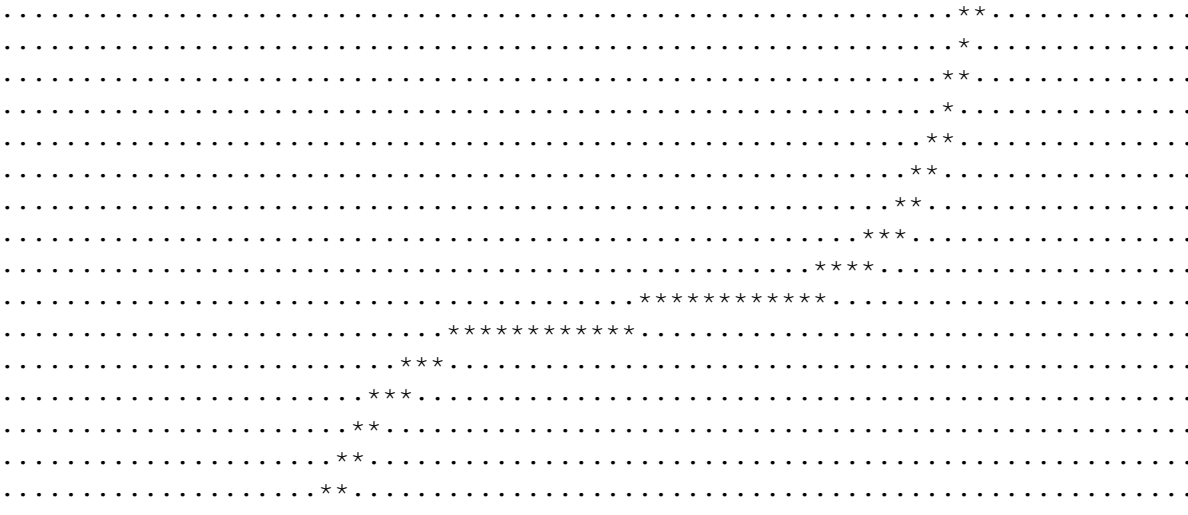


Bitmap rendering.



A.25. Hyperbolic sine (sinh)

ASCII rendering.



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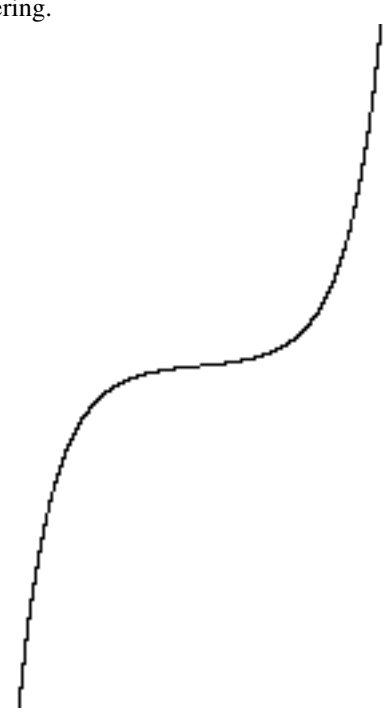
Bitmap rendering.



A.26. Square root (sqrt)

ASCII rendering.

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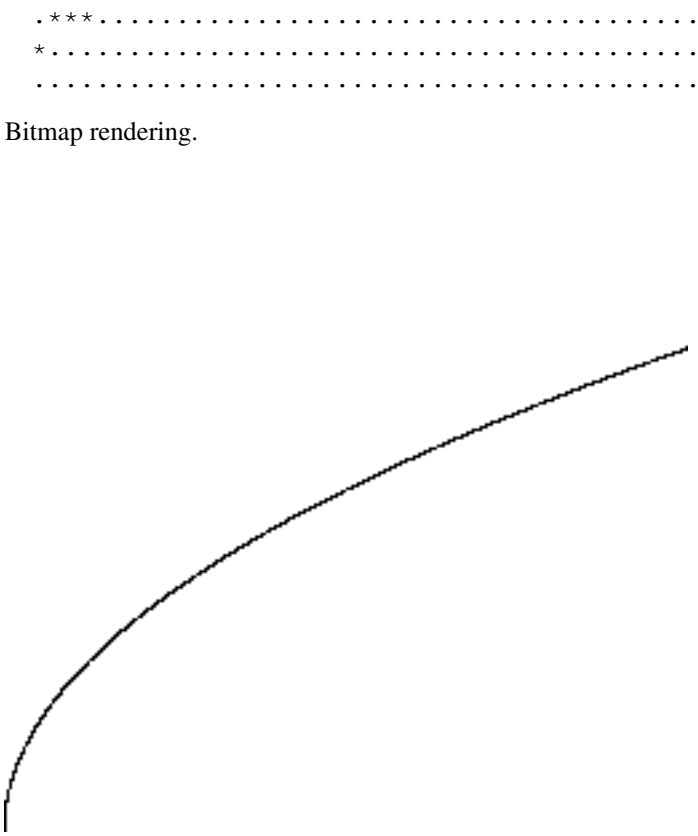
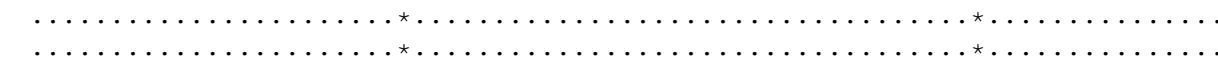


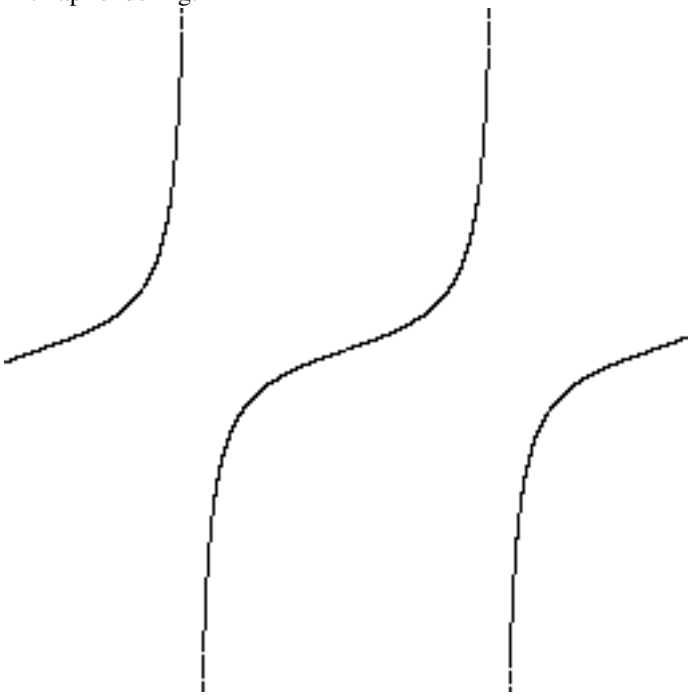
Bitmap rendering.

A.27. Tangent (tan)

ASCII rendering.

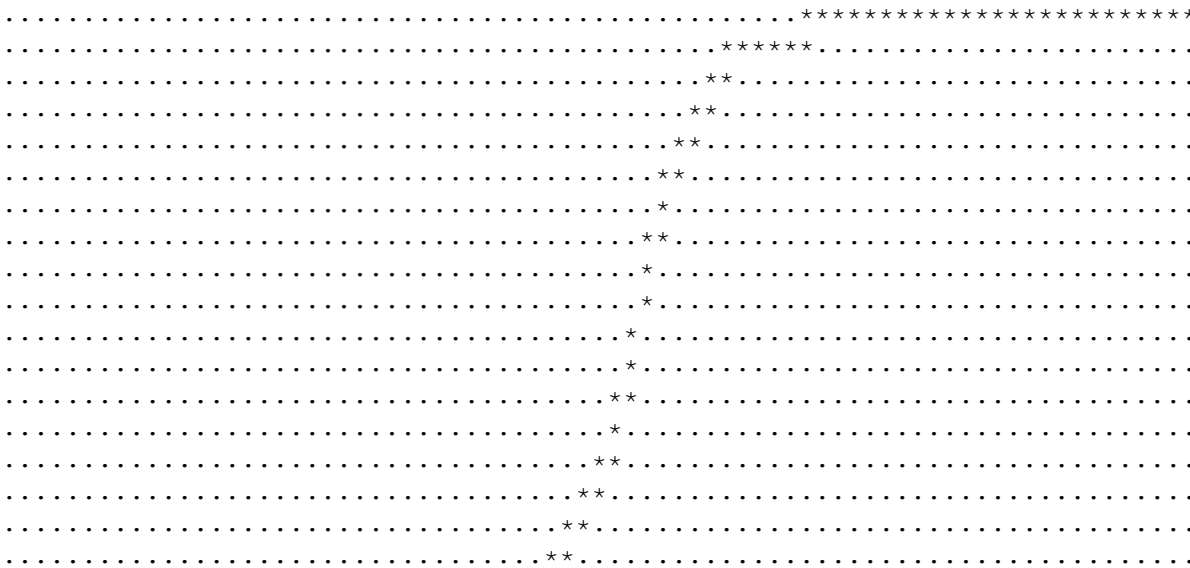


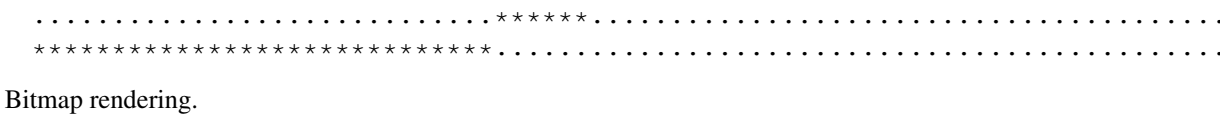
Bitmap rendering.



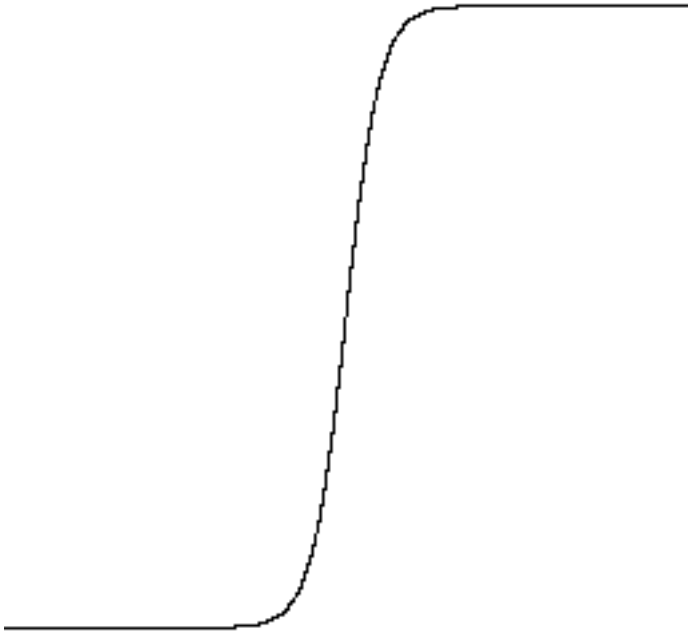
A.28. Hyperbolic tangent (tanh)

ASCII rendering.



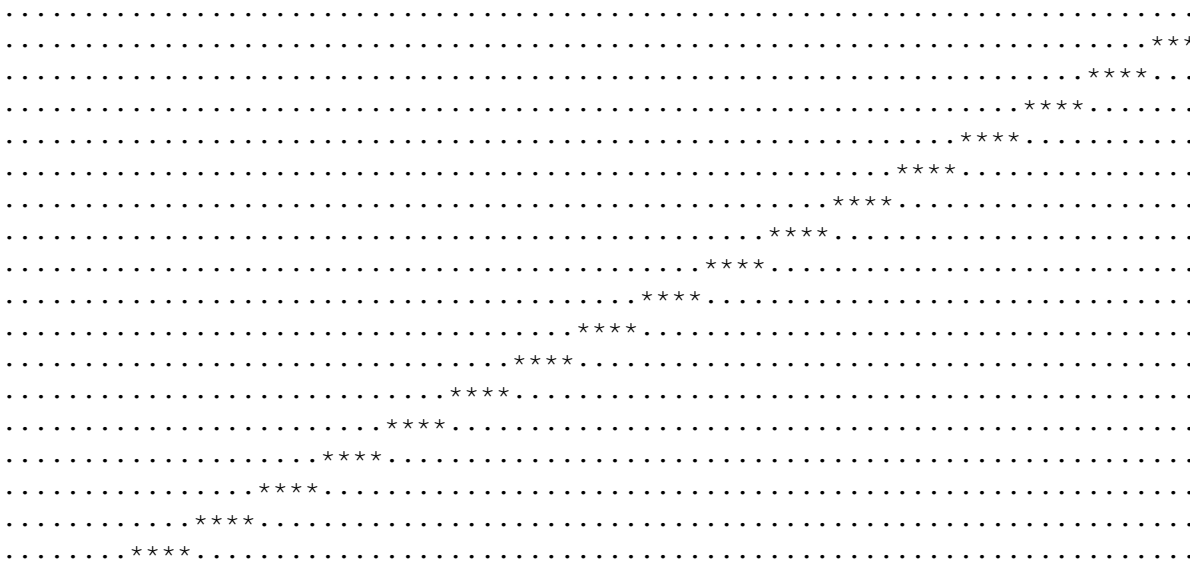


Bitmap rendering.



A.29. Floor (floor)

ASCII rendering.



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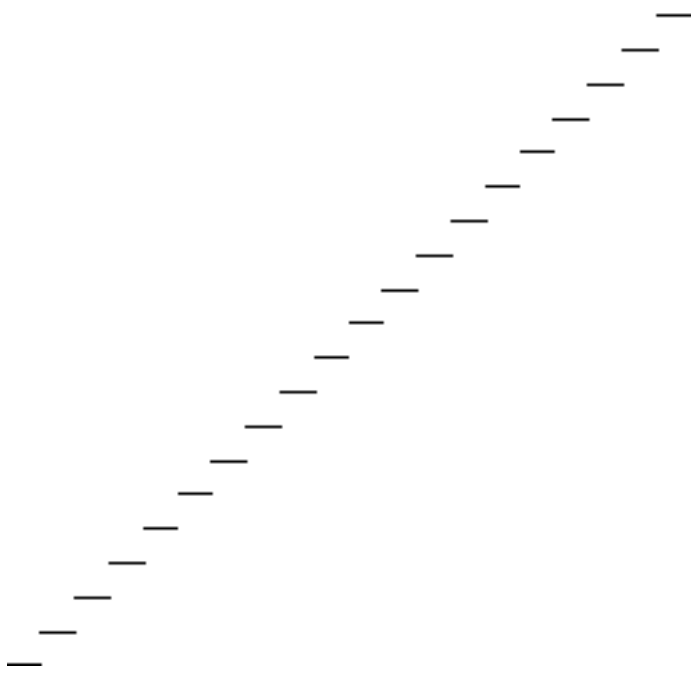
Bitmap rendering.



A.30. Ceiling (ceil)

ASCII rendering.

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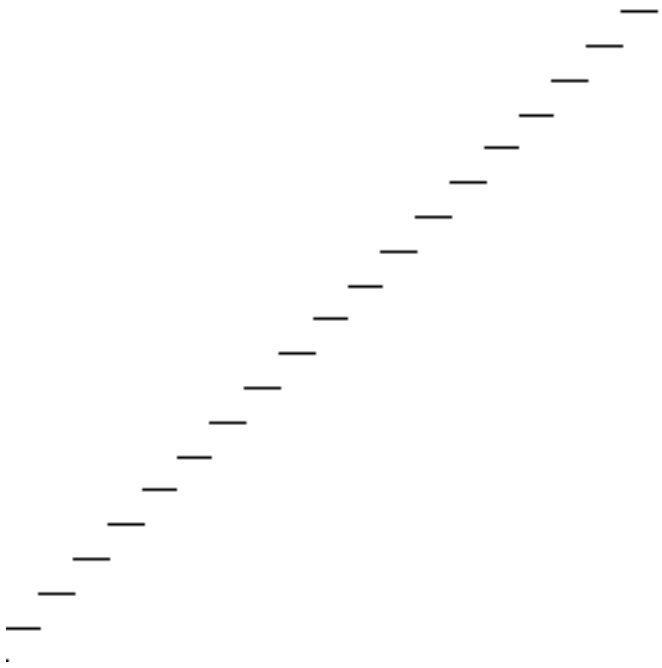
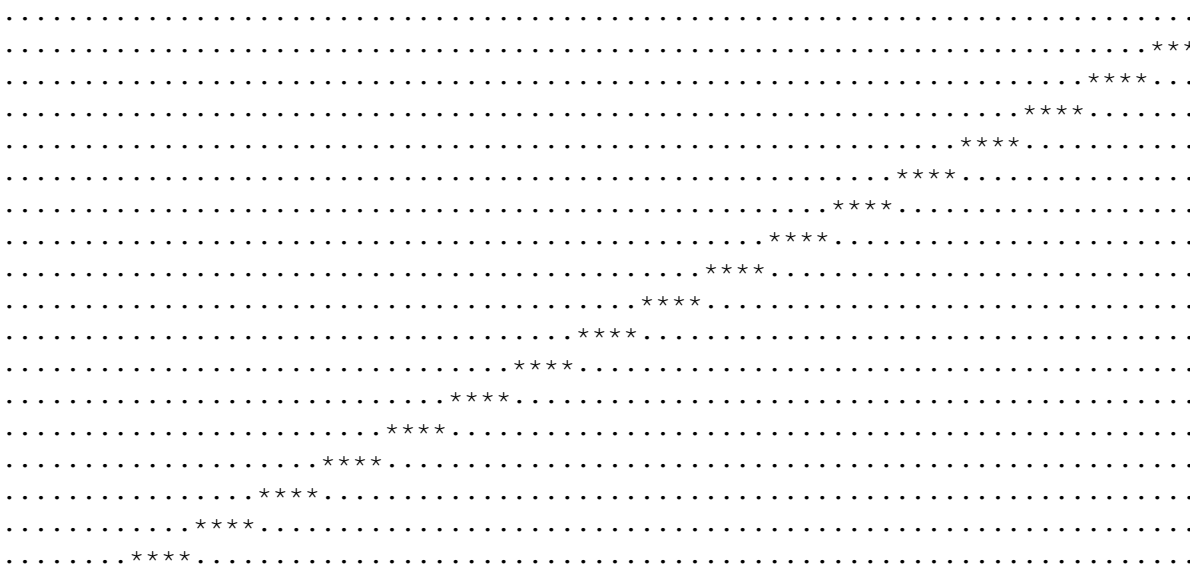
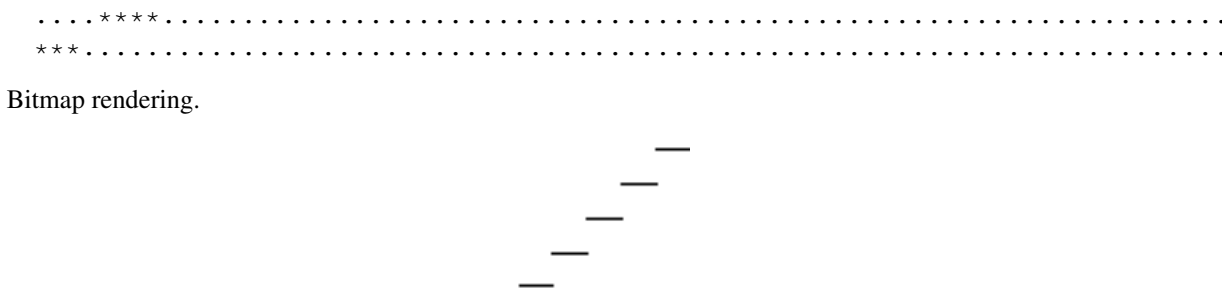
Bitmap rendering.



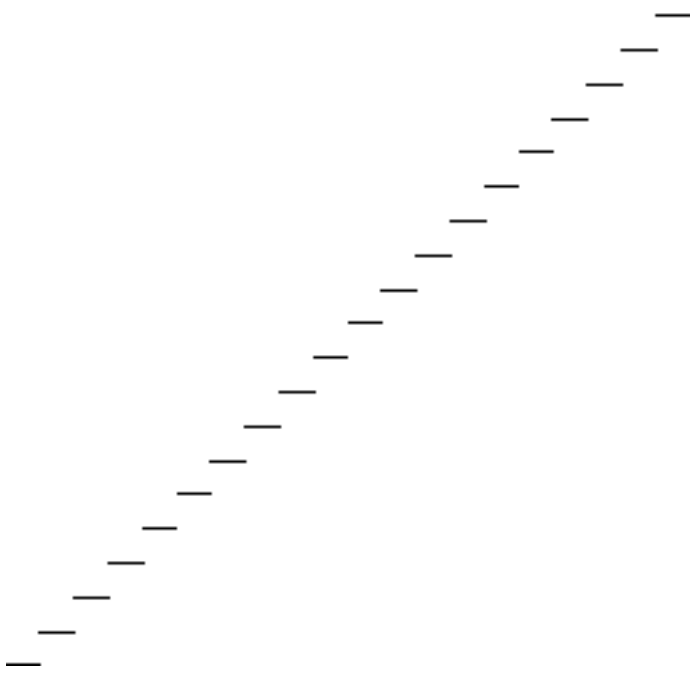
A.31. Round-to-nearest (round)

ASCII rendering.



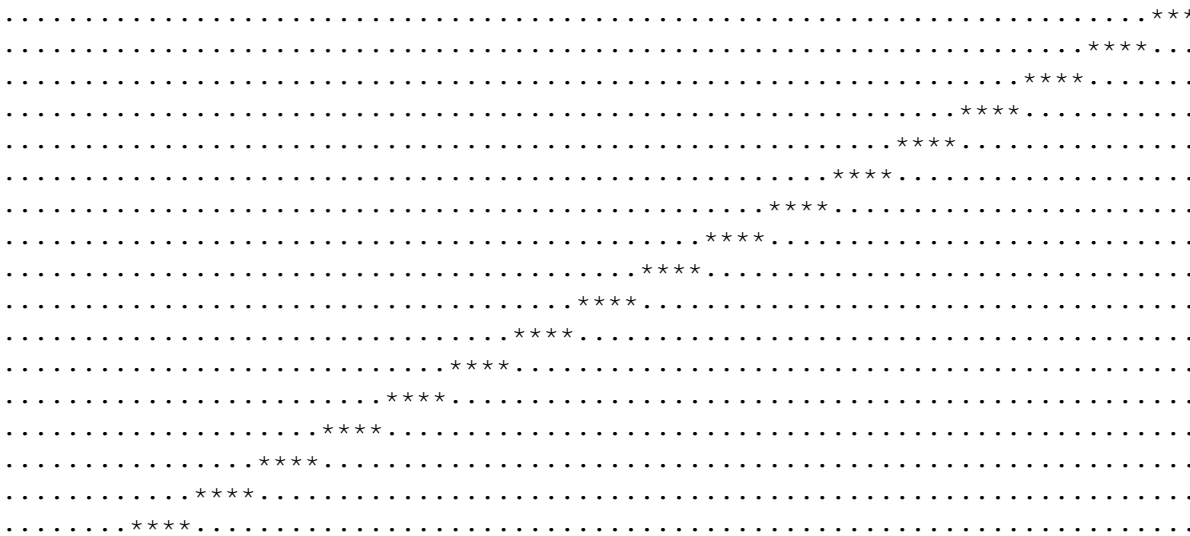


Bitmap rendering.



A.32. Truncation (trunc)

ASCII rendering.



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Bitmap rendering.