# CMPUT 466/566, Winter 2020 Introduction to Machine learning

## Coding Assignment 2

### Problem 1 Report

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```
import struct

import numpy as np

import matplotlib.pyplot as plt
import scipy
import scipy.sparse
```

```
# sometimes despite our best efforts we will get a overflow
# Note: this was only observed on using getAccuracy on the test sets,
# this is likey just a fault of doing a summation of exponents with a
# large number of datapoints.
# Note: these errors never occured during actual training, thus, were
# assumed to be anomaly with using the test data.
np.seterr(divide='ignore', invalid='ignore', over='ignore')
```

```
{'divide': 'ignore', 'over': 'ignore', 'under': 'ignore', 'invalid': 'ignore'}
```

Load in the MNIST data for later compute.

```
def readMNISTdata():
    with open('data/t10k-images-idx3-ubyte', 'rb') as f:
        magic, size = struct.unpack(">II", f.read(8))
        nrows, ncols = struct.unpack(">II", f.read(8))
        test_data = np.fromfile(f, dtype=np.dtype(np.uint8).newbyteorder('
>'))
        test_data = test_data.reshape((size, nrows * ncols))

    with open('data/t10k-labels-idx1-ubyte', 'rb') as f:
        magic, size = struct.unpack(">II", f.read(8))
        test_labels = np.fromfile(f, dtype=np.dtype(np.uint8).newbyteorder
('>'))
        test_labels = test_labels.reshape((size, 1))

    with open('data/train-images-idx3-ubyte', 'rb') as f:
        magic, size = struct.unpack(">II", f.read(8))
        nrows, ncols = struct.unpack(">II", f.read(8))
        train_data = np.fromfile(f, dtype=np.dtype(np.uint8).newbyteorder
('>'))
        train_data = train_data.reshape((size, nrows * ncols))

    with open('data/train-labels-idx1-ubyte', 'rb') as f:
        magic, size = struct.unpack(">II", f.read(8))
        train_labels = np.fromfile(f, dtype=np.dtype(np.uint8).newbyteorde
r('>'))
        train_labels = train_labels.reshape((size, 1))

    # augmenting a constant feature of 1 (absorbing the bias term)
    train_data = np.concatenate(
        (np.ones([train_data.shape[0], 1]), train_data), axis=1)
    test_data = np.concatenate((np.ones([test_data.shape[0], 1]), test_dat
a),
                               axis=1)
    np.random.seed(314)
    np.random.shuffle(train_labels)
    np.random.seed(314)
    np.random.shuffle(train_data)

    X_train = train_data[:50000] / 256
    t_train = train_labels[:50000]

    X_val = train_data[50000:] / 256
    t_val = train_labels[50000:]

    return X_train, t_train, X_val, t_val, test_data, test_labels


X_train, t_train, X_val, t_val, X_test, t_test = readMNISTdata()

print(X_train.shape, t_train.shape, X_val.shape, t_val.shape, X_test.shap
e, t_test.shape)
```

```
(50000, 785) (50000, 1) (10000, 785) (10000, 1) (10000, 785) (10000, 1)
```

Various global configurations.

```
# ten numbers to classify: 0,1,2,3,4,5,6,7,8,9
N_class = 10

# original defined hyper parameters
# alpha = 0.1  # learning rate
# batch_size = 100  # batch size
# MaxIter = 50  # Maximum iteration
# decay = 0.  # weight decay

# original hyperparameters for the first expiremental run
alpha = 0.1  # learning rate
num_batches = 500
batch_size = 50000 // num_batches
MaxIter = 50  # Maximum iteration
decay = 0.0  # weight decay
lam =  0.00000003  # regularization loss multiplier


# spam stdout with our ML models training progress
verbose = True
```

```
def oneHotIt(Y):
    """Convert unidimensional array of labels into a one-hot variant
    where the array is size m (examples) x n (classes)."""
    m = Y.shape[0]
    Y = Y[:,0]
    OHX = scipy.sparse.csr_matrix((np.ones(m), (Y, np.array(range(m)))))
    OHX = np.array(OHX.todense()).T
    return OHX


def softmax(z):
    z -= np.max(z) + np.min(z)
    sm = (np.exp(z).T / np.sum(np.exp(z),axis=1)).T
    return sm


def getLoss(w, x, y):
    m = x.shape[0] #First we get the number of training examples
    y_mat = oneHotIt(y) #Next we convert the integer class coding into a o
ne-hot representation
    scores = np.dot(x,w) #Then we compute raw class scores given our input
and current weights
    prob = softmax(scores) #Next we perform a softmax on these scores to g
et their probabilities
    loss = (-1 / m) * np.sum(y_mat * np.log(prob)) + (lam / 2) * np.sum(w
* w) #We then find the loss of the probabilities
    grad = (-1 / m) * np.dot(x.T, (y_mat - prob)) + lam * w #And compute t
he gradient for that loss
    return loss, grad


def getProbsAndPreds(someX, w):
    probs = softmax(np.dot(someX, w))
    preds = np.argmax(probs, axis=1)
    return probs, preds


# inspired by https://medium.com/@awjuliani/simple-softmax-in-python-tutor
ial-d6b4c4ed5c16
# https://www.w3resource.com/numpy/manipulation/ndarray-flatten.php
def getAccuracy(someX, someY, w):
    prob, prede = getProbsAndPreds(someX,w)
    someY = someY.flatten()
    num_correct = np.sum(prede == someY)
    num_incorrect = np.sum(prede != someY)
    accuracy = num_correct / (num_correct + num_incorrect)
    return accuracy, num_correct, num_incorrect


def predict(x, w, t=None):
    # X_new: Nsample x (d+1)
    # W: (d+1) x K

    # TODO Your code here
```

```
# training the model
epoch_best, losses_train, acc_val, acc_best, W_best = train(X_train, t_tra
in, X_val, t_val)
```

```
running 500 batches of size 100 over data size 50000 for 50 epochs
epoch   0/50 loss:0.566486243970208192 val_acc:0.890400 correct|incorrect:8904|1096
epoch   1/50 loss:0.374267819457150985 val_acc:0.899100 correct|incorrect:8991|1009
epoch   2/50 loss:0.344192552050699652 val_acc:0.903100 correct|incorrect:9031|969
epoch   3/50 loss:0.328632189112471051 val_acc:0.906700 correct|incorrect:9067|933
epoch   4/50 loss:0.318617322573596873 val_acc:0.909300 correct|incorrect:9093|907
epoch   5/50 loss:0.311435600191826256 val_acc:0.910900 correct|incorrect:9109|891
epoch   6/50 loss:0.305933380125655796 val_acc:0.912200 correct|incorrect:9122|878
epoch   7/50 loss:0.301523825989891747 val_acc:0.913500 correct|incorrect:9135|865
epoch   8/50 loss:0.297872735350642182 val_acc:0.914300 correct|incorrect:9143|857
epoch   9/50 loss:0.294774143781366238 val_acc:0.914400 correct|incorrect:9144|856
epoch  10/50 loss:0.292093396358682478 val_acc:0.914300 correct|incorrect:9143|857
epoch  11/50 loss:0.289738288775680142 val_acc:0.915000 correct|incorrect:9150|850
epoch  12/50 loss:0.287643273384454001 val_acc:0.915400 correct|incorrect:9154|846
epoch  13/50 loss:0.285760281786000714 val_acc:0.915200 correct|incorrect:9152|848
epoch  14/50 loss:0.284053127184626053 val_acc:0.915400 correct|incorrect:9154|846
epoch  15/50 loss:0.282493949805005029 val_acc:0.915600 correct|incorrect:9156|844
epoch  16/50 loss:0.281060881199086454 val_acc:0.915500 correct|incorrect:9155|845
epoch  17/50 loss:0.279736463580185835 val_acc:0.916100 correct|incorrect:9161|839
epoch  18/50 loss:0.278506552234764837 val_acc:0.916500 correct|incorrect:9165|835
epoch  19/50 loss:0.277359535873721585 val_acc:0.916900 correct|incorrect:9169|831
epoch  20/50 loss:0.276285771522491286 val_acc:0.916400 correct|incorrect:9164|836
epoch  21/50 loss:0.275277167430250491 val_acc:0.916500 correct|incorrect:9165|835
epoch  22/50 loss:0.274326870160976177 val_acc:0.916500 correct|incorrect:9165|835
epoch  23/50 loss:0.273429026345386905 val_acc:0.916900 correct|incorrect:9169|831
epoch  24/50 loss:0.272578598821159535 val_acc:0.917000 correct|incorrect:9170|830
epoch  25/50 loss:0.271771222990210559 val_acc:0.917100 correct|incorrect:9171|829
epoch  26/50 loss:0.271003093324643840 val_acc:0.917200 correct|incorrect:9172|828
epoch  27/50 loss:0.270270872760486114 val_acc:0.916900 correct|incorrect:9169|831
epoch  28/50 loss:0.269571619670507301 val_acc:0.917400 correct|incorrect:9174|826
epoch  29/50 loss:0.268902728485058196 val_acc:0.917700 correct|incorrect:9177|823
epoch  30/50 loss:0.268261881015447567 val_acc:0.917800 correct|incorrect:9178|822
epoch  31/50 loss:0.267647006248498465 val_acc:0.918100 correct|incorrect:9181|819
epoch  32/50 loss:0.267056246904500638 val_acc:0.918400 correct|incorrect:9184|816
epoch  33/50 loss:0.266487931438921022 val_acc:0.918100 correct|incorrect:9181|819
epoch  34/50 loss:0.265940550458968139 val_acc:0.918200 correct|incorrect:9182|818
epoch  35/50 loss:0.265412736746007294 val_acc:0.918000 correct|incorrect:9180|820
epoch  36/50 loss:0.264903248242659206 val_acc:0.918100 correct|incorrect:9181|819
epoch  37/50 loss:0.264410953492637768 val_acc:0.918100 correct|incorrect:9181|819
epoch  38/50 loss:0.263934819121676045 val_acc:0.918100 correct|incorrect:9181|819
epoch  39/50 loss:0.263473899026332004 val_acc:0.918200 correct|incorrect:9182|818
epoch  40/50 loss:0.263027324999268786 val_acc:0.918100 correct|incorrect:9181|819
epoch  41/50 loss:0.262594298568636864 val_acc:0.918200 correct|incorrect:9182|818
epoch  42/50 loss:0.262174083868326913 val_acc:0.918400 correct|incorrect:9184|816
epoch  43/50 loss:0.261766001387332969 val_acc:0.918500 correct|incorrect:9185|815
epoch  44/50 loss:0.261369422471874280 val_acc:0.918500 correct|incorrect:9185|815
epoch  45/50 loss:0.260983764474609525 val_acc:0.918800 correct|incorrect:9188|812
epoch  46/50 loss:0.260608486462143851 val_acc:0.919000 correct|incorrect:9190|810
epoch  47/50 loss:0.260243085405914509 val_acc:0.919000 correct|incorrect:9190|810
epoch  48/50 loss:0.259887092792984775 val_acc:0.918900 correct|incorrect:9189|811
epoch  49/50 loss:0.259540071602753608 val_acc:0.919000 correct|incorrect:9190|810
```

The best epoch and its respective validation and test accuracies are shown below:

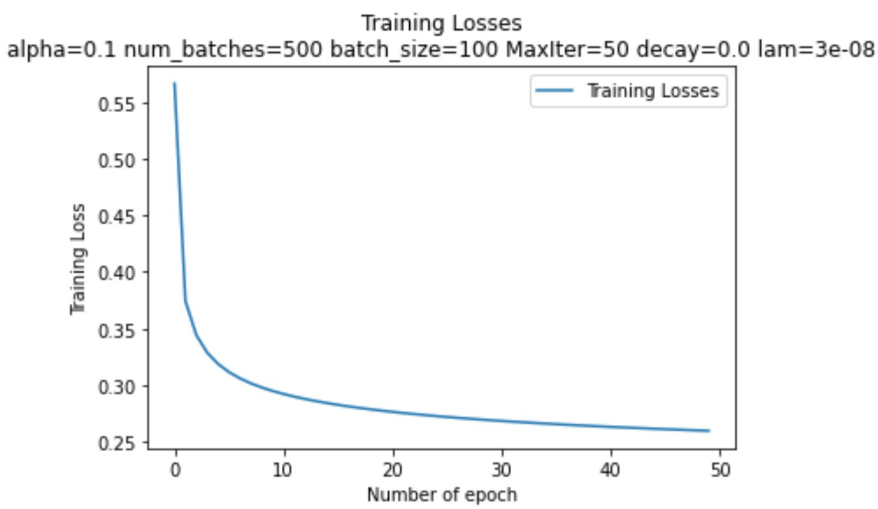
```
print(f"best epoch: {epoch_best}")
acc, num_correct, num_incorrect = getAccuracy(X_val, t_val, W_best)
print(f"val_acc:{acc:.6f} correct|incorrect:{num_correct}|{num_incorrec
t}")
acc, num_correct, num_incorrect = getAccuracy(X_test, t_test, W_best)
print(f"tst_acc:{acc:.6f} correct|incorrect:{num_correct}|{num_incorrec
t}")
```

```
best epoch: 49
val_acc:0.919000 correct|incorrect:9190|810
tst_acc:0.268900 correct|incorrect:2689|7311
```

Noteably, we have obtained a great validation accuracy 90%! But, have also obtained a horrible test accuracy of 0.26%!. We might be overfitting in this case.

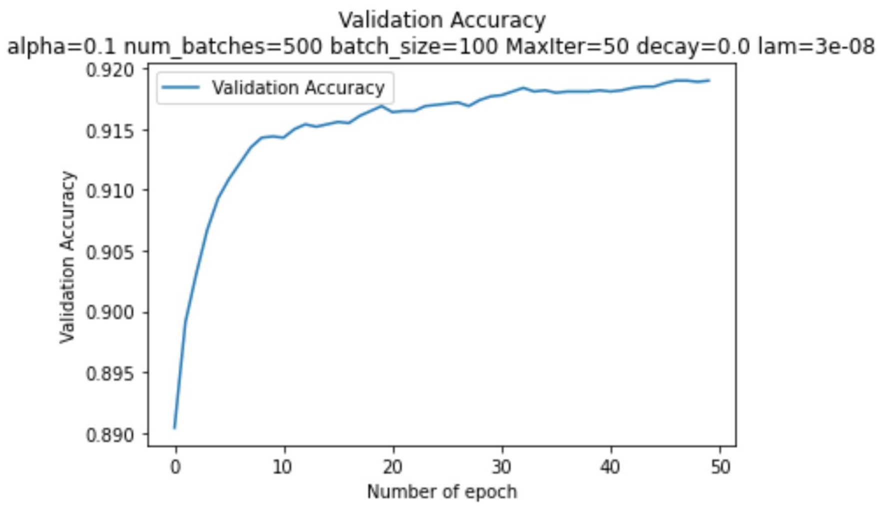
The learning curve of the training loss, where x-axis is the number of epochs, and y-axis is the training loss.

```
plot_training_losses(losses_train)
```




The accuracy curve, where x-axis is the number of epochs, and y-axis is the accuracy in decimal form (0.0-1.0).

```
plot_training_accuracy(acc_val)
```




Getting accuracy of the learned model on the various data sets (train, validation, test).

```
acc, num_correct, num_incorrect = getAccuracy(X_train, t_train, W_best)
print(f"trn_acc:{acc:.6f} correct|incorrect:{num_correct}|{num_incorrec
t}")
acc, num_correct, num_incorrect = getAccuracy(X_val, t_val, W_best)
print(f"val_acc:{acc:.6f} correct|incorrect:{num_correct}|{num_incorrec
t}")
acc, num_correct, num_incorrect = getAccuracy(X_test, t_test, W_best)
print(f"tst_acc:{acc:.6f} correct|incorrect:{num_correct}|{num_incorrec
t}")
```

```
trn_acc:0.929360 correct|incorrect:46468|3532
val_acc:0.919000 correct|incorrect:9190|810
tst_acc:0.268900 correct|incorrect:2689|7311
```

## Problem 2 Report

Ask one meaningful scientific question yourself, design your experimental protocol, present results, and draw a conclusion.

## Scientific Question

With proper tuning of the hyperparameters:

- Can we obtain a higher accuracy than 81-83% on the validation and test sets?
- Can we obtain a higher accuracy with less iterations with proper hyperparameter tuning?

## Experimental Protocol

Re-run the experiment (training the model with the MNIST data) and tuning the models hyperparameters for the best perceived validation and test accuracies.

The goal is to obtain the best validation accuracy, that still upholds an acceptable test accuracy.

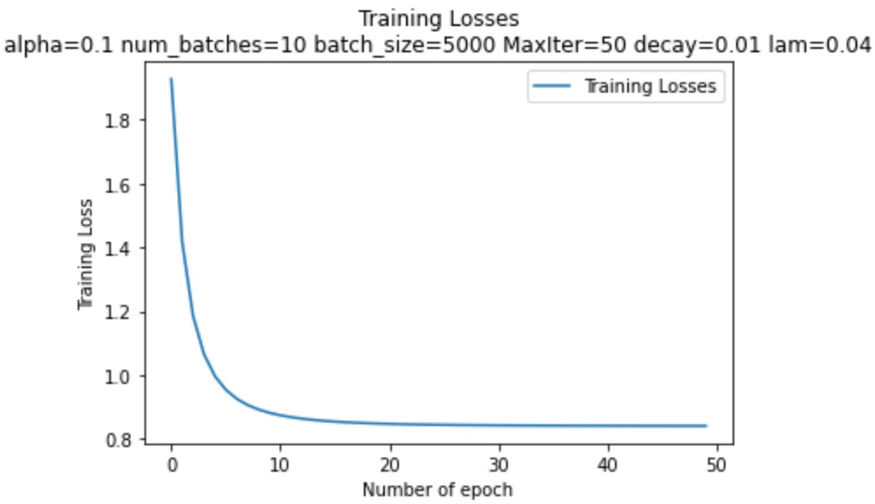
The following hyperparameters will be tuned across the experiment:

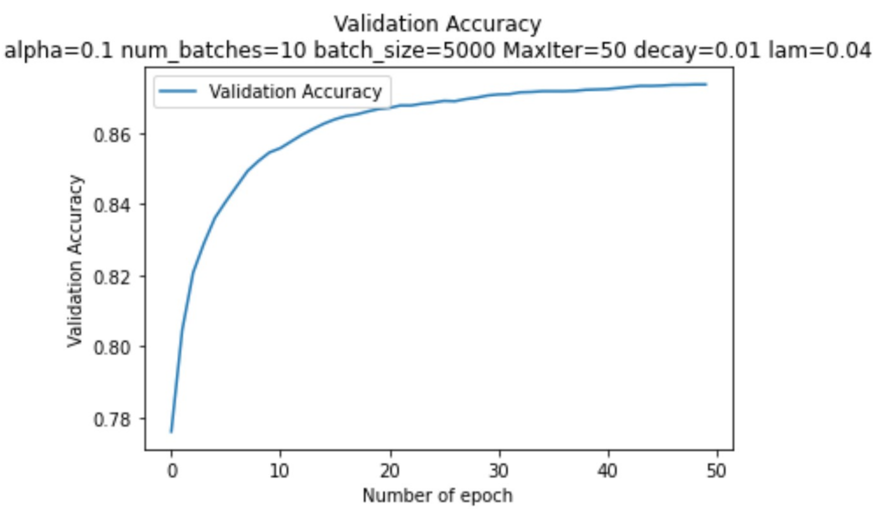
- `alpha` - learning rate
- `num_batches` - number of batches
- `batch_size` - size of batches
- `MaxIter` - number of epochs
- `decay` - weight decay
- `lam` - regularization loss multiplier

## Experimental Results

```
# don't spam stdout
verbose = False

hyperparameter_tests = [
    {  # 88 test accuracy
        "alpha": 0.1,
        "num_batches": 10,
        "MaxIter": 50,
        "decay": 0.01,
        "lam": 0.04
    },
    { # 89.25 test accuracy
        "alpha": 0.3,
        "num_batches": 10,
        "MaxIter": 100,
        "decay": 0.002,
        "lam": 0.00000003
    },
    {
        "alpha": 0.4,
        "num_batches": 20,
        "MaxIter": 100,
        "decay": 0.0025,
        "lam": 0.00000003
    },
    { # feels like over fitting
        "alpha": 0.3,
        "num_batches": 10,
        "MaxIter": 100,
        "decay": 0.002,
        "lam": 0.00000003
    },
    {
        "alpha": 0.36,
        "num_batches": 10,
        "MaxIter": 50,
        "decay": 0.014,
        "lam": 0.00000003
    },
    {
        "alpha": 0.37,
        "num_batches": 16,
        "MaxIter": 200,
        "decay": 0.01,
        "lam": 0.00000003
    },
    {
        "alpha": 0.37,
        "num_batches": 16,
        "MaxIter": 75,
        "decay": 0.01,
        "lam": 0.00000003
    },
    {
```

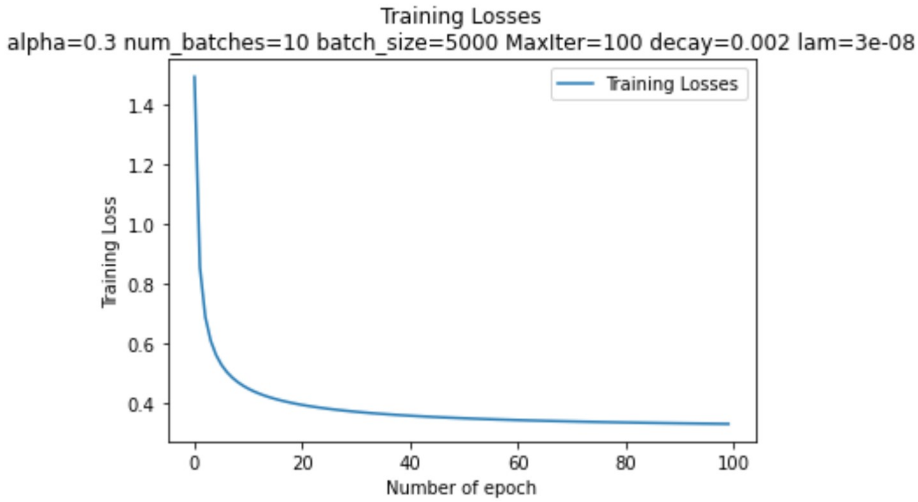
```
(50000, 785) (50000, 1) (10000, 785) (10000, 1) (10000, 785) (10000, 1)
alpha=0.1 num_batches=10 batch_size=5000 MaxIter=50 decay=0.01 lam=0.04
running 10 batches of size 5000 over data size 50000 for 50 epochs
```

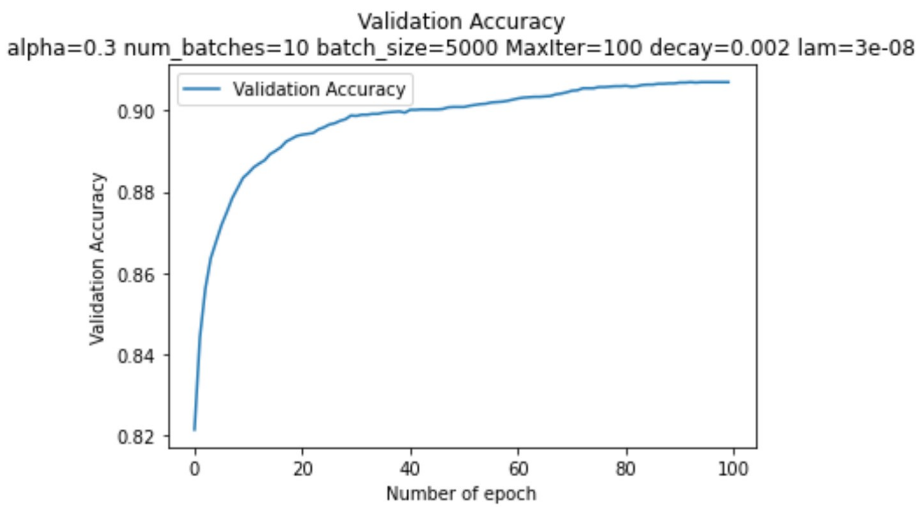



```
trn_acc:0.873720 correct|incorrect:43686|6314
val_acc:0.873600 correct|incorrect:8736|1264
tst_acc:0.881500 correct|incorrect:8815|1185
```

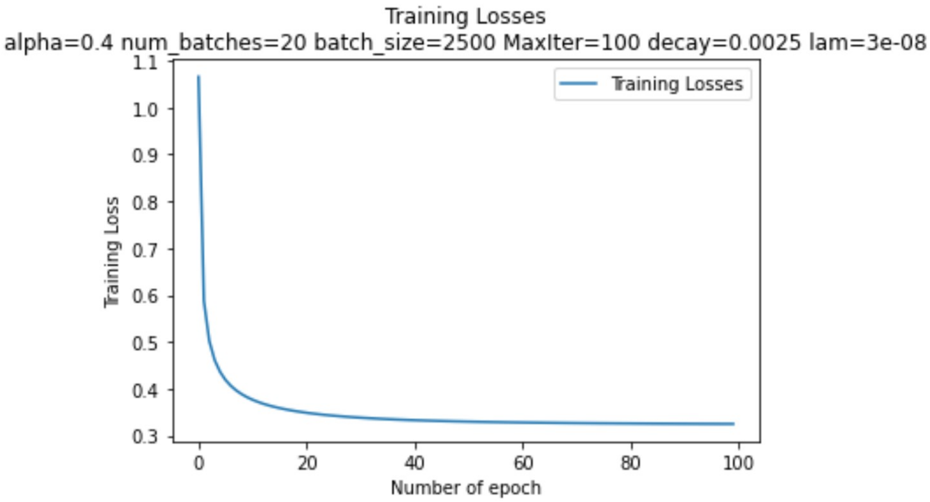
```
(50000, 785) (50000, 1) (10000, 785) (10000, 1) (10000, 785) (10000, 1)
alpha=0.3 num_batches=10 batch_size=5000 MaxIter=100 decay=0.002 lam=3e-08
running 10 batches of size 5000 over data size 50000 for 100 epochs
```

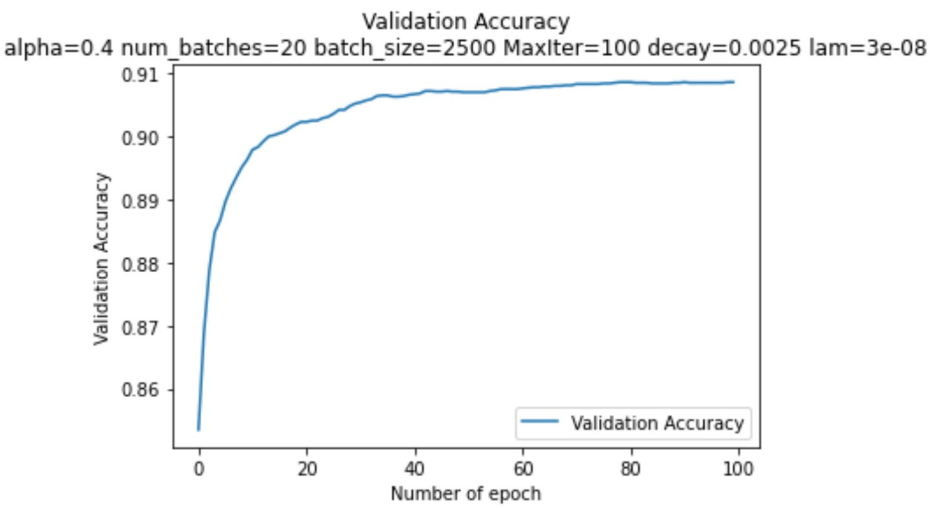


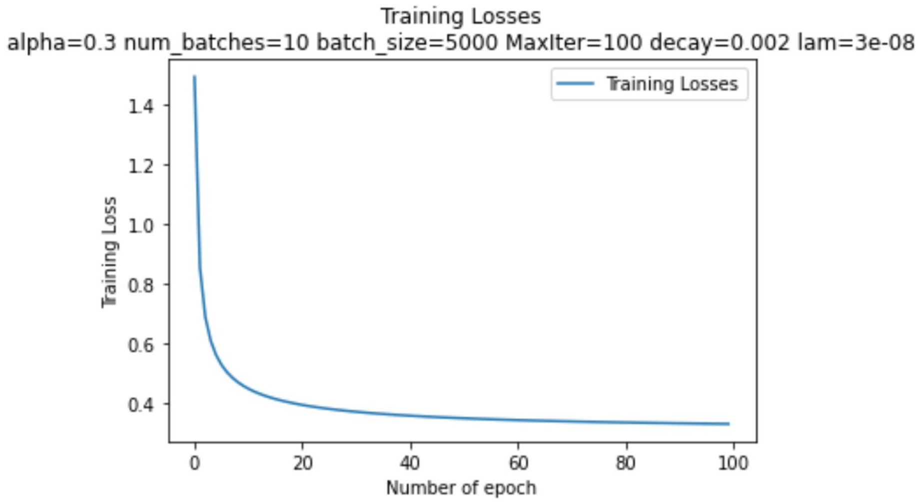

```
trn_acc:0.912060 correct|incorrect:45603|4397
val_acc:0.906900 correct|incorrect:9069|931
tst_acc:0.793300 correct|incorrect:7933|2067
```

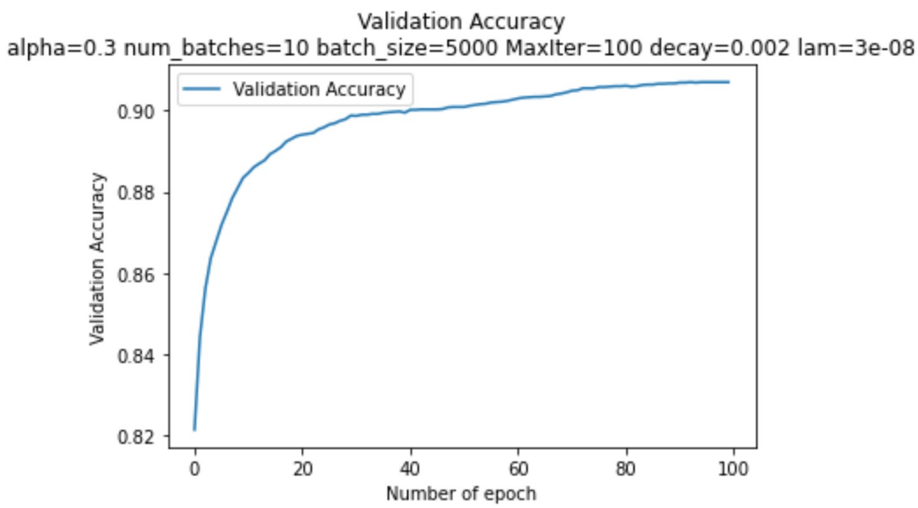
```
(50000, 785) (50000, 1) (10000, 785) (10000, 1) (10000, 785) (10000, 1)
alpha=0.4 num_batches=20 batch_size=2500 MaxIter=100 decay=0.0025 lam=3e-08
running 20 batches of size 2500 over data size 50000 for 100 epochs
```



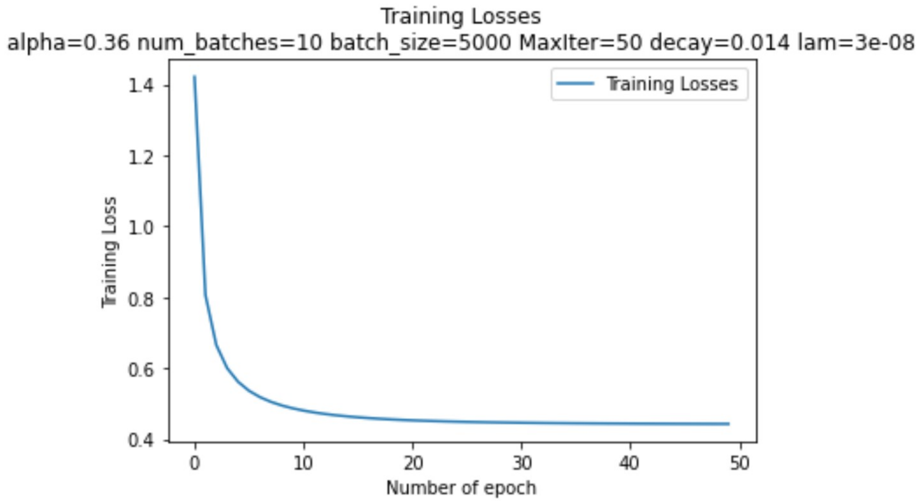



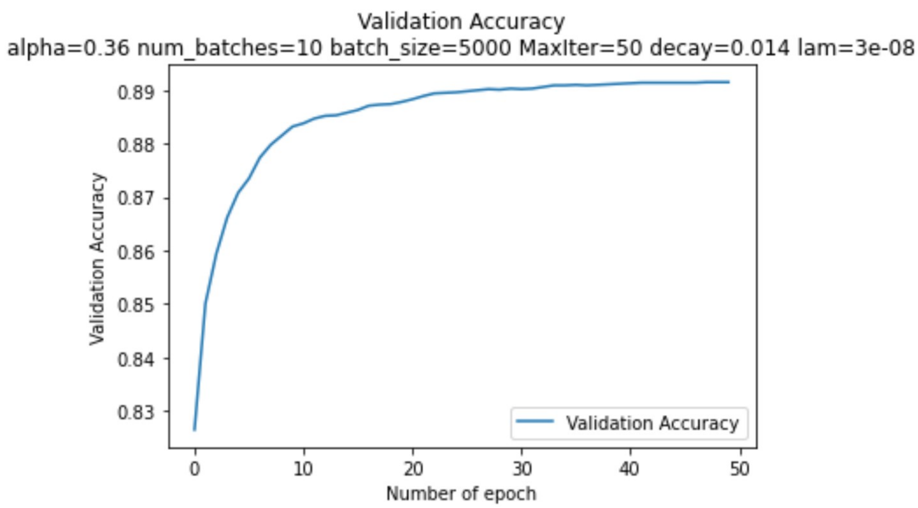

```
trn_acc:0.914320 correct|incorrect:45716|4284
val_acc:0.908600 correct|incorrect:9086|914
tst_acc:0.790500 correct|incorrect:7905|2095

(50000, 785) (50000, 1) (10000, 785) (10000, 1) (10000, 785) (10000, 1)
alpha=0.3 num_batches=10 batch_size=5000 MaxIter=100 decay=0.002 lam=3e-08
running 10 batches of size 5000 over data size 50000 for 100 epochs
```



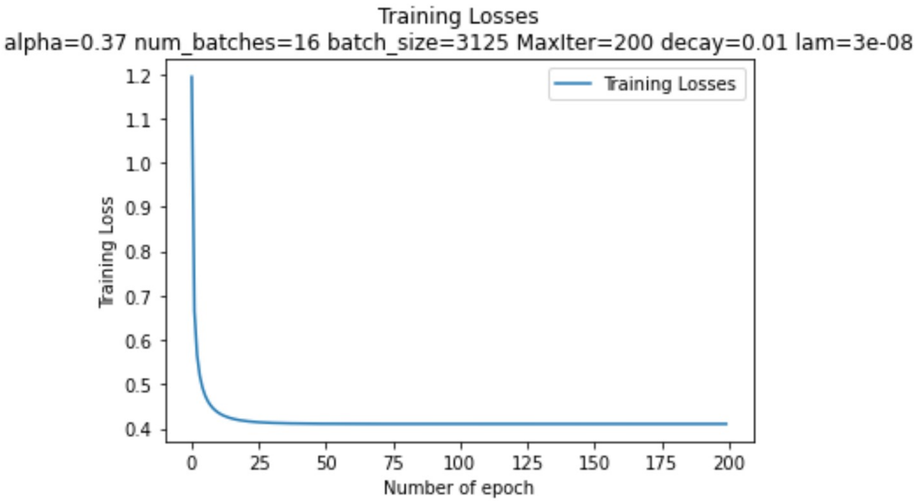

```
trn_acc:0.912060 correct|incorrect:45603|4397
val_acc:0.906900 correct|incorrect:9069|931
tst_acc:0.793300 correct|incorrect:7933|2067

(50000, 785) (50000, 1) (10000, 785) (10000, 1) (10000, 785) (10000, 1)
alpha=0.36 num_batches=10 batch_size=5000 MaxIter=50 decay=0.014 lam=3e-08
running 10 batches of size 5000 over data size 50000 for 50 epochs
```

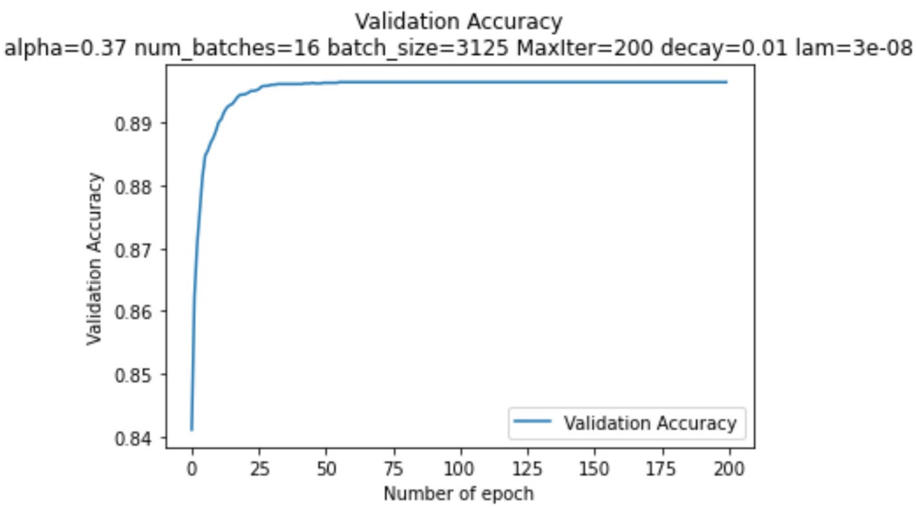


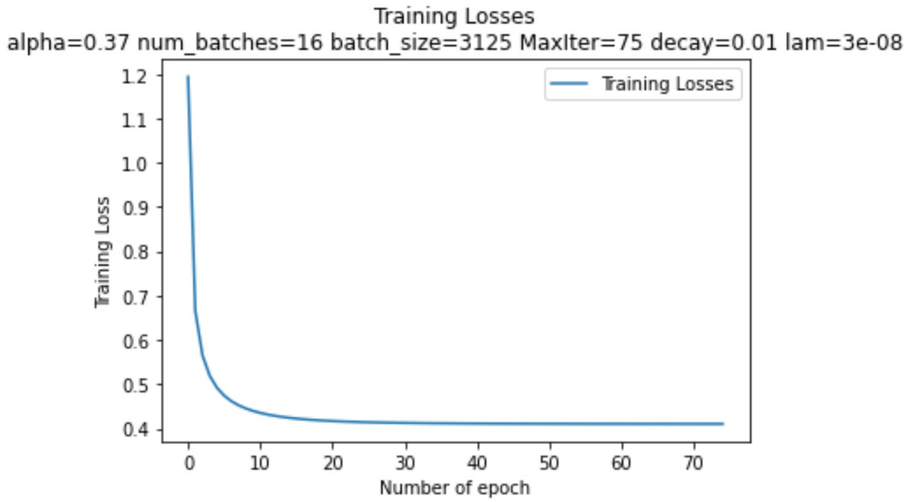

```
trn_acc:0.894380 correct|incorrect:44719|5281
val_acc:0.891500 correct|incorrect:8915|1085
tst_acc:0.894400 correct|incorrect:8944|1056
```

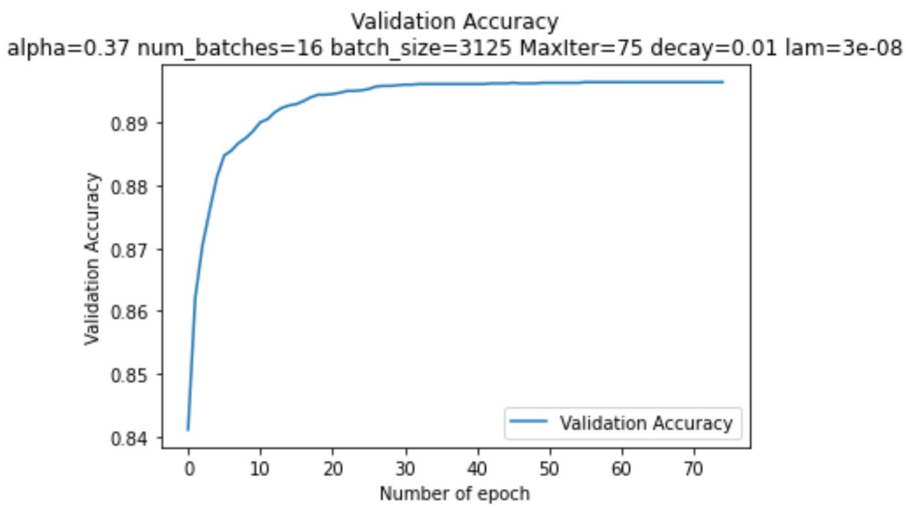
```
(50000, 785) (50000, 1) (10000, 785) (10000, 1) (10000, 785) (10000, 1)
alpha=0.37 num_batches=16 batch_size=3125 MaxIter=200 decay=0.01 lam=3e-08
running 16 batches of size 3125 over data size 50000 for 200 epochs
```



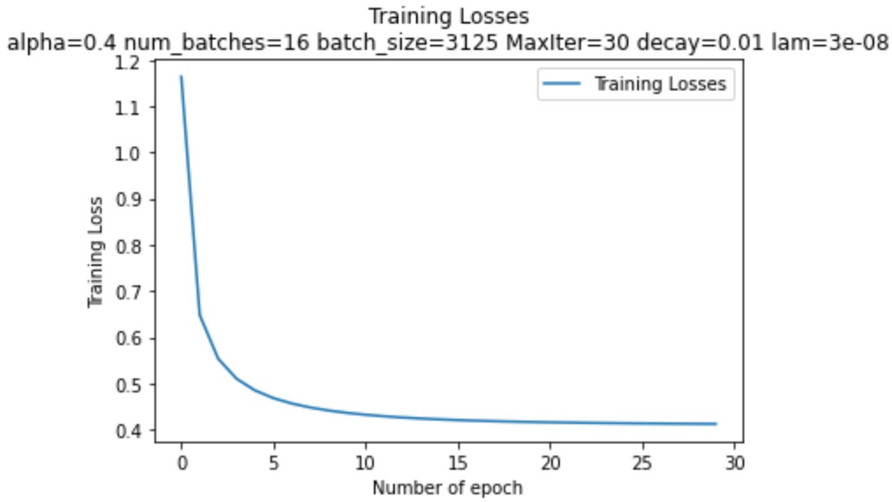



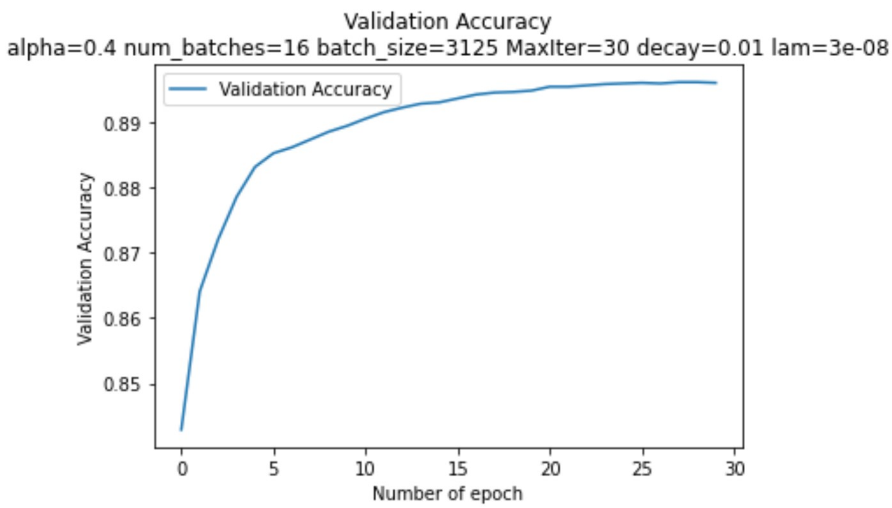

```
trn_acc:0.899320 correct|incorrect:44966|5034
val_acc:0.896500 correct|incorrect:8965|1035
tst_acc:0.892700 correct|incorrect:8927|1073

(50000, 785) (50000, 1) (10000, 785) (10000, 1) (10000, 785) (10000, 1)
alpha=0.37 num_batches=16 batch_size=3125 MaxIter=75 decay=0.01 lam=3e-08
running 16 batches of size 3125 over data size 50000 for 75 epochs
```



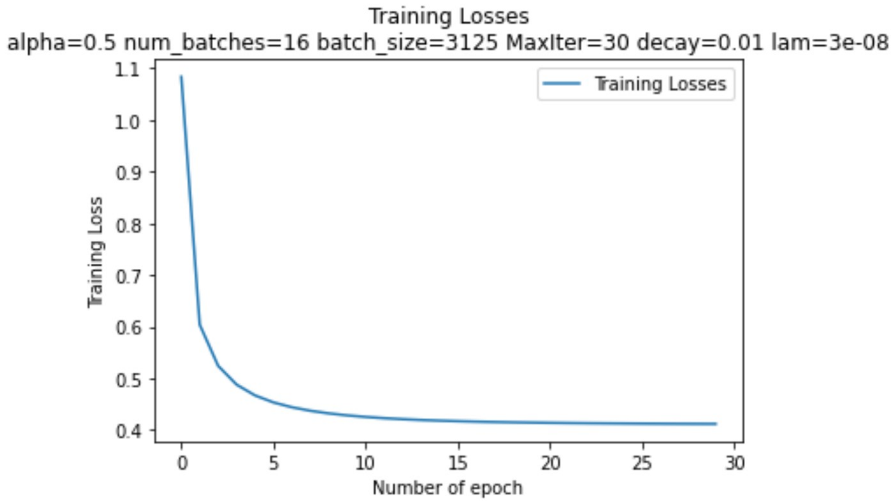



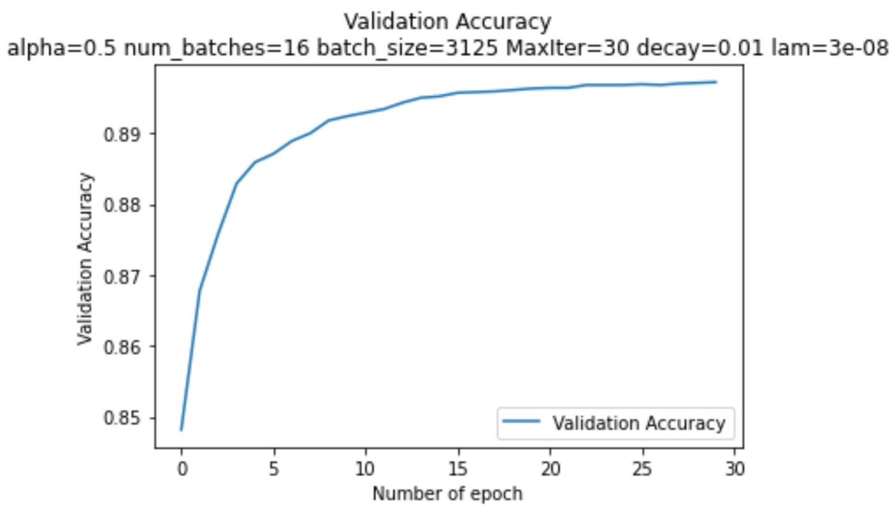

```
trn_acc:0.899320 correct|incorrect:44966|5034
val_acc:0.896500 correct|incorrect:8965|1035
tst_acc:0.892700 correct|incorrect:8927|1073

(50000, 785) (50000, 1) (10000, 785) (10000, 1) (10000, 785) (10000, 1)
alpha=0.4 num_batches=16 batch_size=3125 MaxIter=30 decay=0.01 lam=3e-08
running 16 batches of size 3125 over data size 50000 for 30 epochs
```




```
trn_acc:0.898560 correct|incorrect:44928|5072
val_acc:0.896200 correct|incorrect:8962|1038
tst_acc:0.891500 correct|incorrect:8915|1085

(50000, 785) (50000, 1) (10000, 785) (10000, 1) (10000, 785) (10000, 1)
alpha=0.5 num_batches=16 batch_size=3125 MaxIter=30 decay=0.01 lam=3e-08
running 16 batches of size 3125 over data size 50000 for 30 epochs
```




```
trn_acc:0.898900 correct|incorrect:44945|5055
val_acc:0.897200 correct|incorrect:8972|1028
tst_acc:0.891500 correct|incorrect:8915|1085
```

### Conclusion

Despite my primitive hyperparameter tuning method, tuning the hyperparameters as shown within the experimental results noted a increase in validation and test accuracy from ~82% to ~89%.

The final tuned hyperparameters I would recommend using for a production deployment of this ML model would be the following:

- `alpha=0.5` - learning rate
- `num_batches=16` - number of batches
- `batch_size=3125` - size of batches
- `MaxIter=30` - number of epochs
- `decay=0.01` - weight decay
- `lam=0.00000003` - regularization loss multiplier

Also we were able to optimize the number of iterations from 50 to 30 without losing much accuracy. This was possible by increasing the learning rate from `alpha=0.1` to `alpha=0.5` and by visual inspection of the validation accuracy graphs showning that the logrithmic curve was still not appoaching its limit.

One take away from this expirement is how useful the training losses and validation accuracy plots were for addressing potential issues with the training of the model. I wonder if there is any other methods for visualizing the training of ML models that could help assist in human fine-tining and troubleshooting of said models.