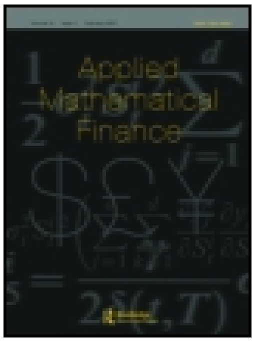

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A survey of sampling-based Bayesian analysis of financial data

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The capability of implementing a complete Bayesian analysis of experimental data has emerged over recent years due to computational advances developed within the statistical community. The objective of this paper is to provide a practical exposition of these methods in the illustrative context of a financial event study. The customary assumption of Gaussian errors underlying development of the model is later supplemented by considering Student-t errors, thus permitting a Bayesian sensitivity analysis. The supplied data analysis illustrates the advantages of the sampling-based Bayesian approach in allowing investigation of quantities beyond the scope of classical methods.

Keywords: event studies, inference, Bayesian, Markov chain Monte Carlo, Gibbs sampler

1. Introduction

Event studies as a major research tool in finance are used to test the wealth or market effect due to the occurrence of a financial event. These events can be stock splits, dividend changes, earnings announcements, etc. Using an information or 'announcement' period when the upcoming financial event is made public, the goal of the event study is to determine if public knowledge of the event has a significant impact on the security returns of an underlying firm sample. Fama *et al*. (1969) and Brown and Warner (1980, 1985) laid the foundation for much of the classical event study methodology.

Classical methods rely on two assumptions; (1) statistical testing can be performed only from quantifying the observed data, i.e., determining suitable test statistics, and (2) reliance on the idea of long-run frequency behaviour under hypothetical repetitions of similar circumstances. Inference is based on a probability statement about such repetitions of the data conditional on unknown parameters. The conclusions reached depend on experiments which may never be run, from data that will most probably never be collected.

An alternative approach is to reverse this logic and assume model unknowns, like the data, are random variables. As such these unknowns have *prior* distributions expressing our knowledge about them prior to data collection and then updated distributions conditional on the observed data. Bayesian methods consider these conditional distributions (so-called *posterior* distributions) for the

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parameters of interest. As such, the Bayesian framework presents a unifying constructive technology for inference which reduces to an examination of the joint probability distribution of all model unknowns given the data observations. Formally, to obtain this distribution requires only the use of the probability calculus. Thereafter all inference arises through features of or summarization of this *posterior* distribution.

Practical calculation of such features, including expectations, marginal distributions, quantiles, etc., provided an impediment to the wider usage of Bayesian methods until recent computational methods, through the use of iterative simulation, were introduced into the mainstream statistical literature. As a result, there has been an explosion of usage of Bayesian modelling through all areas of scientific inquiry, with model fitting implemented through these simulation methods. The objective of this paper is to provide a clear illustration of these methods for finance researchers in the well-known event study context.

This paper is organized as follows. The remainder of the introduction will review frequentist event study procedures and introduce the Bayesian paradigm. In Section 2 Markov chain Monte Carlo (MCMC) sampling methods are discussed. This serves as an introduction to a detailed discussion of the Gibbs sampler. The development of a Bayesian hierarchical model for a financial event study is then presented. Section 3 discusses the hypothesis and compares the frequentist and Bayesian results from an event study. A Bayesian sensitivity analysis for the financial event being investigated is shown. The use of such robust Bayesian methods for long-horizon event studies is also discussed. Section 4 provides a summary, conclusions and further discussion of Bayesian empirical methods for financial event studies.

1.1. *Review of event study methods*

Since the seminal work of Brown and Warner (1980, 1985), the benchmark used to judge whether 'abnormal' security returns have in fact occurred has been the market model, or

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it} \tag{1}$$

Using ordinary least squares (OLS) or maximum likelihood, the market model coefficients α_i, β_i and σ_i^2, where $\varepsilon_{it} \sim N\ (0,\sigma_i^2)$, are estimated for each security i from a vector of security returns, R_{it}, and a vector of market returns, R_{mt}, removed from the time of the event at the discretion of the researcher. Thus, a financial event implies a one-time 'abnormal return' for a specific time period '0', e.g., a single 'announcement' day, as $\varepsilon_{i0} = R_{i0} - E(R_{i0})$ where $E(R_{i0}) = \alpha_i + \beta_i R_{m0}$. Its effect can be tested for this time period for a specific security i using the statistic

$$SAR_{i0} = \frac{\hat{\varepsilon}_{i0}}{\hat{\sigma}_i} \tag{2}$$

where SAR_{i0} is the standardized abnormal or excess return for security i on the event day. The abnormal returns for a single security are assumed to be temporally independent. For a sample of N securities,

$$z_0 = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} SAR_{i0} \tag{3}$$

which is assumed to be distributed approximately as a unit normal. The abnormal returns are assumed to be comtemporaneously independent across the security sample. Of course, abnormal returns for multiple event days around the announcement date can be summed to determine a cumulative z test statistic.[1]

Over the years different adjustments and nonparametric versions of this frequentist methodology have been presented (Dodd and Warner, 1983; Corrado, 1989; Boehmer *et al.*, 1991; Corrado and Zivney, 1992; Cowan, 1992; Salinger, 1992; Giaccotto and Sfiridis, 1996).

More recently, there has been interest in long-horizon event studies, i.e., the idea that systematic long-run abnormal price reactions occur subsequent to a corporate event, e.g., initial public offerings (Loughran and Ritter, 1995), stock repurchases (Ikenberry *et al.*, 1995) and stock recommendations (Womack, 1996). For such studies abnormal returns are usually measured relative to benchmark portfolios rather than the familiar market model. A number of benchmark portfolios based on size are commonly used to determine expected returns and conduct statistical testing. Thus, whereas equal weighting of excess security returns in short-horizon market model event studies is normally used to determine test statistics, long-horizon event studies can capital weight excess returns for statistical testing. With a time horizon that can extend up to five years, test statistics formed from such abnormal or excess returns may have biases that cause misspecified test statistics (Barber and Lyon, 1997; Kothari and Warner, 1997; Cowan and Sergeant, 2001). Bayesian sampling-based empirical methods for such long-horizon event studies can largely alleviate many of these problems (Brav, 2000).

1.2. *The Bayesian paradigm applied to Equation (1)*

Let $R_i=\{R_1, ¼, R_T\}$ be a vector of the daily return time series for each security i of a sample of N securities. In addition there is a contemporaneous time series of market returns, R_{mt}. For the set of N securities there is a vector $\theta=\{\alpha_1, \beta_1, \sigma_1{}^2, ¼, \alpha_N, \beta_N, \sigma_N{}^2\}$ of $3N$ unknown parameters. Then, viewing R and θ as random, the joint distribution of R and θ can be expressed as

$$g\left(\alpha_i,\beta_i,\sigma_i^2,\ldots\middle|R\right) f(R)=f\left(R\middle|\alpha_i,\beta_i,\sigma_i^2,\ldots\right)\pi\left(\alpha_i,\beta_i,\sigma_i^2,\ldots\right) \tag{4}$$

Here $f\,(R|\alpha_i,\beta_i,\sigma_i{}^2, ¼)$ is the likelihood function that summarizes what is known about the data. $\pi(\alpha_i,\beta_i,\sigma_i{}^2, ¼)$ is a probabilistic statement of belief about the parameters before (prior to) obtaining data. $g(\alpha_i,\beta_i,\sigma_i{}^2, ¼|R)$ is a probabilistic statement about the parameters after (*posterior* to) obtaining data and is called the joint posterior density. The marginal density function $f\,(R)$ is a constant given the observed data.

Solving Equation (4) for g (.) is Bayes's theorem which updates the prior distribution or knowledge about θ to a *posterior* distribution for θ given the observed data R. It is this joint *posterior* density that summarizes information about the parameters after data is obtained and provides the basis for Bayesian inference. Since $f\,(R)$ is a constant, the joint posterior density is proportional to the likelihood

[1] The subjectively determined period of time around the date that the event becomes public knowledge is called the event period. It is used to eliminate any information effects from the pending event, such as when rumor or other types of information 'leakage' occur, that might contaminate a security's expected return determination from the market model prior to the actual event date. It also eliminates any lingering information effects that may exist after the event date. This event period is, thus, considered to be the time when the security's returns may be affected by the event being studied. Individual daily t-statistics or cumulative multiple-day t-statistics during this event period are used to test the null hypothesis that excess security returns are equal to zero around the time of the financial event

function times prior beliefs, or

$$g(\alpha_i,\beta_i,\sigma_i^2,\ldots|R) \propto L(\alpha_i,\beta_i,\sigma_i^2,\ldots)\pi(\alpha_i,\beta_i,\sigma_i^2,\ldots) \tag{5}$$

2. Discussion

2.1. *Markov Chain Monte Carlo methods*

In order to make inference about a particular model unknown, the marginal *posterior* distribution is required. Hence a primary objective of Bayesian analysis is the determination of the marginal posterior distribution for each parameter in the model from the joint *posterior* distribution. This is the probability distribution for the parameter of interest, conditional only on the data and requires the marginalization of all other parameters. As such, high dimensional integration is required, a task that would be impossible in most applications, including financial event studies. MCMC methods such as the Gibbs sampler have fortunately overcome this 'curse of dimensionality' by providing a means to conduct sampling-based high-dimensional integration. Thus, MCMC methods allow Bayesian data analysis of complex phenomena (see Gelfand and Smith, 1990; Gelfand *et al*., 1990).

MCMC methods acquire samples from the required marginal *posterior* distributions by constructing a suitable Markov chain. There are many ways to construct these chains, one of which is the Gibbs sampler, but most of them are derived in some fashion from the seminal work of Metropolis *et al*. (1953) and Hastings (1970).[2] The basic idea is to create a Markov chain whose stationary or equilibrium distribution is the marginal *posterior* distribution. Though draws will not initially be from the 'target' density, after sufficient iterations of the Markov chain that serve as a 'burn-in' period, the draws will essentially be from the required *posterior* distribution. Once draws from the marginal distributions are realized, arbitrarily large samples can be drawn to determine distributional features of interest or summary statistics to conduct appropriate statistical inference. For example, if samples from the marginal posterior distribution $g(\beta_i|R)$ can be obtained, then a sampling-based $E\,[\beta_i]$ can be determined. In particular, if $\beta_{il}{}^*$, $l=1,¼,L$, are L draws from this distribution, $E\,[\beta_i|R]$ can be estimated as $L^{-1}\sum_{l=1}^{L}\beta_{il}^*$.

2.2. *The Gibbs sampler*

According to Equation (5), the joint *posterior* density for θ is known up to normalizing constant, $f(R)$. The Gibbs sampler provides a convenient Markov chain whose stationary distribution is the normalized version of Equation (5). Again, after a sufficient number of iterations for *burn-in*, observations of the chain are essentially from the joint posterior distribution for θ. The full conditional density for any unknown parameter is defined as the conditional density for that parameter given all

[2] See recent books, *Bayesian Data Analysis* by Gelman *et al*. (1996) and *Markov Chain Monte Carlo* by Dani Gamerman (1997), for a detailed explanation of MCMC methods.

the other parameters and the data. For example, the β_1 parameter's full conditional distribution is

$$g(\beta_1|\alpha_1,\sigma_1^2,\alpha_2,\ldots,R)=\frac{g(\alpha_1,\beta_1,\sigma_1^2,\ldots|R)}{\int_{\beta_1} g(\alpha_1,\beta_1,\sigma_1^2,\ldots|R)\mathrm{d}\beta_1}=\frac{L(\theta)\pi(\theta)}{\int_{\beta_1} L(\theta)\pi(\theta)\mathrm{d}\beta_1} \tag{6}$$

This is the familiar result that the full conditional distribution for an unknown parameter equals the joint probability distribution divided by its marginal distribution. Thus, considering both Equations (5) and (6), the full conditional distribution for β_1 is proportional to the product of the likelihood function and the prior specification.

When implementing the Gibbs sampler algorithm, the full conditional distributions for each parameter in the model are the probability density functions which are drawn from to generate realizations of the Markov chains. It can be shown under fairly mild conditions that the Markov chains created by suitable draws from these densities will converge to the required 'target' distributions (Tierney, 1994; Smith and Roberts, 1993).

In particular, to create the Markov chains, we might start with the OLS estimates of α_i, β_i and σ_i^2 obtained by fitting the linear regression in Equation (1) for each security. Denoting them by

$$\theta^{(0)}=\left[\alpha_1^{(0)},\beta_1^{(0)},\sigma_1^{2^{(0)}},\ldots,\alpha_N^{(0)},\beta_N^{(0)},\sigma_N^{2^{(0)}}\right] \tag{7}$$

for the first iteration, we can start with α. Thus,

$$\alpha_1^{(1)}\sim g\left(\alpha_1\middle|\beta_1^{(0)},\sigma_1^{2^{(0)}},\ldots,\sigma_N^{2^{(0)}},R\right) \tag{8}$$

Next, for β_1, we have

$$\beta_1^{(1)}\sim g\left(\beta_1\middle|\alpha_1^{(1)},\sigma_1^{2^{(0)}},\ldots,\sigma_N^{2^{(0)}},R\right) \tag{9}$$

Note that, on the right-hand side, α_1 has been updated, but not the rest of the parameters. The last parameter drawn is

$$\sigma_N^{2^{(1)}}\sim g\left(\sigma_N^2\middle|\alpha_1^{(1)},\ldots,\beta_N^{(1)},R\right) \tag{10}$$

which completes the first iteration of the Markov chain. $\theta^{(0)}$ has been revised to $\theta^{(1)}$. As noted above, after a sufficient number of iterations, say t, $\theta^{(t)}$ is approximately a draw from the joint *posterior* distribution and $\beta_i^{(t)}$, for example, is approximately a draw from the marginal posterior distribution of β_i. Though assessing when such convergence occurs for our financial event study model presents little difficulty, the issue of practical convergence diagnosis is generally a topic of ongoing research (Cowles and Carlin, 1996).

2.3. *A hierarchical Bayesian model for financial event studies*

The starting point for the great majority of event studies is the market model as given in Equation (1) where the assumption is made that the daily return of each security in the sample, R_{it}, is normally distributed with an expected value $\alpha_i+\beta_i R_{mt}$ and variance σ_i^2. These market model or regression parameters summarize the features of the data for each security. A prior specification for α_i and β_i might be that they are independent draws from normal distributions having means μ_α and μ_β and variances σ_α^2 and σ_β^2 respectively. Thus, *exchangeability* is assumed, i.e., since, *a priori*, subscripts are arbitrarily assigned to securities, the associated αs and βs should be viewed, *a priori*, as independent and identically distributed. With a normal likelihood function a convenient and flexible prior specification for the variance is an inverse gamma distribution. Using the idea of exchangeability again, the σ_i^2s are assumed independent and identically distributed draws from this inverse gamma distribution. For clarification, we say that $z \sim IG(a,b)$ if $z^{-1} \sim \text{Gamma}(a,1/b)$.

Two levels of the hierarchical model have now been specified. The first stage models the return data given θ and is represented by the likelihood function containing the market model parameters. The second stage models these parameters as indicated above. The third stage needed to complete the model specification requires a prior distribution for the 'hyperparameters', μ_α, μ_β, σ_α^2 and σ_β^2, and customarily employs normal and inverse gamma distributions. Thus, the Bayesian hierarchical model is

$$R_{it}|R_{mt} \sim N\left(\alpha_i + \beta_i R_{mt}, \sigma_i^2\right) \tag{11}$$

where $\alpha_1 \sim N\left(\mu_\alpha, \sigma_\alpha^2\right)$; $\beta_i \sim N\left(\mu_\beta, \sigma_\beta^2\right)$; $\sigma_i^2 \sim IG(c_1, c_2)$ and $\mu_\alpha \sim N\left(a, \sigma_a^2\right)$; $\mu_\beta \sim \mathrm{N}\left(\mathrm{b}, \sigma_\mathrm{b}^2\right)$; $\sigma_\alpha^2 \sim \mathrm{IG}\left(c_3, c_4\right)$; $\sigma_\beta^2 \sim IG(c_5, c_6)$. The hyperparameter specifications are typically made to be rather vague in order to let the data drive the inference about the first stage parameters. Thus, essentially non-informative or flat priors are taken on μ_α and μ_β, e.g., a normal distribution with mean zero and very large variance, such as 10^4. For σ_α^2 and σ_β^2 vague inverse gamma priors, $IG(c_3, c_4)$ and $IG(c_5, c_6)$ respectively, are also needed. However, a proper *posterior* density, i.e., a density function that is integrable, is ensured only if the inverse gamma priors are made proper. Proposition 2 of Fernandez *et al.* (1997) shows that this is accomplished if the c_3, c_4, c_5 and c_6 hyperparameters are greater than zero. One way to accomplish this is to use the previously fitted market model parameters for each security. Then, for example, the sample variance of the estimates of the α_is provides a mean for $IG(c_3, c_4)$. With a variance large relative to this mean, say three or four times the mean, moment matching can be used to solve for c_3 and c_4. Similarly, using the β estimators, values for c_5 and c_6 can be determined. In practice, for large and even moderate-sized data sets normally encountered in finance, the analysis will be insensitive to the specification used for the hyperparameters.

The Bayesian specification also requires priors for σ_i^2. A mean and variance can be easily calculated from the OLS σ_i^2 estimates for each security. Two equations in two unknowns can then be used to solve for c_1 and c_2.

Once the model has been specified, full conditional distributions for each parameter can be determined. For instance, for β_i, considering only terms having β_i,

$$g\left(\beta_i \middle| \alpha_i, \sigma_i^2, \mu_\beta, \sigma_\beta^2, \ldots, R\right) \propto \frac{1}{\left(2\pi_{\sigma_i^2}\right)^{T/2}} \exp \quad -\sum_{t=1}^{T} \frac{(R_{it} - \alpha_i - \beta_i R_{mt})^2}{2_{\sigma_i^2}} \Bigg)$$

$$\cdot \frac{1}{\left(2\pi_{\sigma_\beta^2}\right)^{1/2}} \exp\left(\frac{\left(\beta_i - \mu_\beta\right)^2}{2_{\sigma_\beta^2}}\right) \tag{12}$$

The square is completed to derive the full conditional distribution, which becomes

$$g(\beta_i|\ldots,R)\sim N\left(\frac{\sigma_\beta^2\sum_{t=1}^{T}R_{mt}(R_{it}-\alpha_i)+\mu_\beta\sigma_i^2}{\sigma_\beta^2\sum_{t=1}^{T}R_{mt}^2+\sigma_i^2},\frac{\sigma_i^2\sigma_\beta^2}{\sigma_\beta^2\sum_{t=1}^{T}R_{mt}^2+\sigma_i^2}\right) \tag{13}$$

Similarly for the other parameters,

$$g(\alpha_i|\ldots,R)\sim N\left(\frac{T_i\sigma_\alpha^2(\bar{R}_i-\beta_i\bar{R}_m)+\mu_\alpha\sigma_i^2}{T_i\sigma_\alpha^2+\sigma_i^2},\frac{\sigma_i^2\sigma_\alpha^2}{T_i\sigma_\alpha^2+\sigma_i^2}\right) \tag{14}$$

$$g\left(\sigma_i^2|\ldots,R\right)\sim IG\quad c_1+\frac{T_i}{2},c_2+\frac{1}{2}\sum_{t=1}^{T_i}(R_{it}-\alpha_i-\beta_iR_{mt})^2\Big) \tag{15}$$

and for the hyperparameters,

$$g(\mu_\alpha|\ldots,R]\sim N\left(\frac{\sigma_a^2\sum_{i=1}^{N}\alpha_i+\sigma_\alpha^2a}{N\sigma_a^2+\sigma_\alpha^2},\frac{\sigma_a^2\sigma_\alpha^2}{N\sigma_a^2+\sigma_\alpha^2}\right) \tag{16}$$

$$g\left(\mu_\beta|\ldots,R\right]\sim N\left(\frac{\sigma_b^2\sum_{i=1}^{N}\beta_i+\sigma_\beta^2b}{N\sigma_b^2+\sigma_\beta^2},\frac{\sigma_b^2\sigma_\beta^2}{N\sigma_b^2+\sigma_\beta^2}\right) \tag{17}$$

$$g\left(\sigma_\alpha^2|\ldots,R\right)\sim IG\quad c_3+\frac{N}{2},c_4+\frac{1}{2}\sum_{i=1}^{N}(\alpha_i-\mu_\alpha)^2\Big) \tag{18}$$

$$g\left(\sigma_\beta^2|\ldots,R\right]\sim IG\quad c_5+\frac{N}{2},c_6+\frac{1}{2}\sum_{i=1}^{N}\left(\beta_i-\mu_\beta\right)^2\Big) \tag{19}$$

The full conditional candidate-generating densities for each parameter have been specified. Creation of a Markov chain follows by drawing from these distributions in sequence for each parameter as indicated above. One iteration is equivalent to a set of updates for all the parameters in the model. Thus, for a sample having N securities, there are $3N+4$ parameters to be updated for each iteration.

3. The hypothesis and empirical results

An assumption underlying the Black and Scholes (1973) option pricing model is that options are redundant assets. The future payoffs from an option can be replicated by positions in the underlying asset. Hence, the introduction of a stock option should not have an effect on the return distribution of the underlying stock. A contrary view is that options, through the use of greater leverage and lower transaction costs, may attract informed traders that prefer the options market to the stock market, affecting the mechanism by which information is incorporated into stock prices.[3] Conrad (1989), for example, shows that statistically significant abnormal returns do occur around the date of option introduction.[4] Using a 61-day event period, i.e., 30 days before and 30 days after the event date, her conclusion is based on OLS z or t-statistics for five event-period days from -3 to $+1$ where day $t=0$ is the date of option introduction for each security in the sample. Giaccotto and Sfiridis (1996) take a further look at this issue. They show that there is a variance increase in the sample's abnormal returns around the option introduction date that may bias the results of the parametric t-tests. Non-parametric rank tests and a sampling-based jackknife test statistic show that the OLS t-statistics may be biased upward. However, the main conclusion from Conrad's study is not reversed when test statistics that account for the event period variance increase are used.

Some interesting results are seen when exploratory data analysis is conducted on this data set. Using a 90% confidence interval for a two-tailed test for the market model residuals, only four out of a sample size of 29 securities[5] have significant excess returns on day -3. This day's test statistic is 4.29 in the Conrad study and is the most significant daily t-statistic. Additionally, only seven securities have significant excess returns for two or more days of a five-day cumulation period from day -3 through day $+1$. This cumulative statistic ranges from 3.64 to 5.87 in the Giaccotto/Sfiridis study conditional on the methodology used. Thus, the main conclusion from Conrad's study, that for the financial event of option introduction there is a statistically significant wealth effect for the underlying security, is upheld for a number of frequentist parametric and non-parametric tests.

A Bayesian event study using the model previously developed and applied to the Conrad data set was conducted for comparison. A parameter estimation period of 200 daily returns for each of the 29 securities in the sample is used. Assuming exchangeability across the sample of securities, i.e., each security's market model parameters are from a common underlying distribution defined by the model's hyperparameters, Bayesian smoothing will result. Extreme effects caused by security-specific 'contaminating events' during the estimation period are reduced.

Twelve hundred Gibbs iterations were used for this $91(3 \times 29 + 4)$ dimensional model. To run the FORTRAN program for the study an IBM 3094 mainframe was used. Extensive use of the IMSL Library was made to obtain random samples from required distributions.

In theory the choice of a starting value for any parameter is not of consequence since its effect on the chain is transient. A choice for the starting value in the portion of the parameter space where considerable *posterior* mass is anticipated facilitates convergence of the resulting sequence of draws to the required stationary marginal *posterior* distribution. In practice several different starting values of the parameter have been used and the resulting sample chains monitored in order to assess convergence (Gelman and Rubin, 1992). Three chains using three different starting points for one of the parameters in our model were generated to assess convergence. The three chains quickly converged. Thus, the first ten iterations for all the parameter chains in the model are arbitrarily eliminated as *burn-in*, leaving 1200 iterates for each parameter's Markov chain.

[3] See the discussion by Poon (1994).

[4] We wish to thank Professor Jennifer Conrad for sharing her data set with us.

[5] Return data for all of the securities in Conrad's data set were unavailable. There were, thus, 29 different event dates versus 30 event dates in Conrad's data set. Replication of Conrad's study with the 29-security sample resulted in single-day t-statistics that were very close to hers (see Giaccotto/Sfiridis, 1996).

The objective of this Bayesian event study is to determine predictive distributions for day '−3' as the most significant daily t-statistic and the cumulative 5-day period from day −3 through day +1. Henceforth day '−3' will be referred to as day '0'. For each security *i*, the return distribution can be defined as

$$f(R_{i,0}|R) = \int_{\alpha_i,\beta_i,\sigma_i^2,\ldots} f\left(R_{i,0}|\alpha_i,\beta_i,\sigma_i^2,\ldots,R_{m,0}\right) g\left(\alpha_i,\beta_i,\sigma_i^2,\ldots|R\right) d\left(\alpha_i,\beta_i,\sigma_i^2,\ldots\right) \tag{20}$$

Hence, a Monte Carlo integration for this density is

$$f(R_{i,0}|R) = \frac{\sum_{l=m+1}^{L} f\left(R_{i,0}\middle|\alpha_{i,1}^*,\beta_{i,1}^*,\sigma_{i,1}^{*\,2}\right)}{L-m} \tag{21}$$

where *m* equals the number of iterations arbitrarily eliminated as a *burn-in* period for each chain and parameters with a '*' superscript represent sample values from the appropriate Markov chain. Figures 1, 2 and 3 show the day '0' predictive return distributions along with the actual day '0' daily returns for three securities in the sample. The results show that a Bayesian event study allows examination of individual firm return behaviour. To determine a day '0' predictive return distribution for all 29 securities in the sample, the predictive distribution must be determined by averaging, or

$$R_{.,0} = \frac{\sum_{i=1}^{N=29} R_{i,0}}{29} \tag{22}$$

But

$$f(R_{.,0}|R) = \int f\left(R_{.,0}|\alpha_.,\beta_.,\sigma_.^2,\bar{R}_{m,0}\right) g\left(\alpha_.,\beta_.,\sigma_.^2|R\right) d\left(\alpha_.,\beta_.,\sigma_.^2\right) \tag{23}$$

where $\bar{R}_{m,0}$ is the day '0' mean market return for the security sample. Thus,

$$f\left(R_{.,0}|\alpha_.,\beta_.,\sigma_.^2,\bar{R}_{m,0}\right) = N\left(\alpha_. + \beta_.\bar{R}_{m,0},\sigma_.^2\right) \tag{24}$$

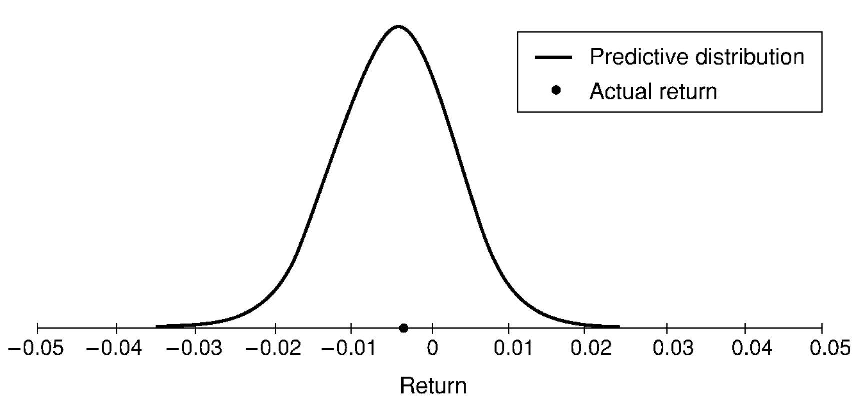

Fig. 1. Day-0 predictive return distribution: Security 5. Note: actual event-day return is statistically insignificant.

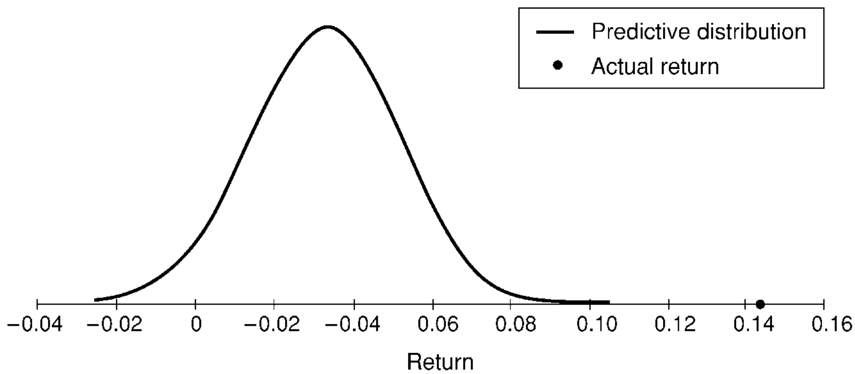

Fig. 2. Day-0 predictive return distribution: Security 23. Note: actual event-day return is statistically significant.

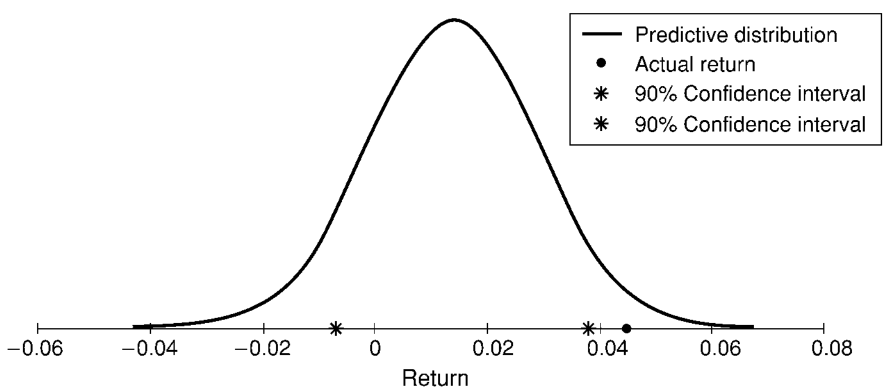

Fig. 3. Day-0 predictive return distribution: Security 15. Note: actual event-day return is statistically significant for a 90% confidence interval. Sampling-based confidence intervals are shown to allow comparison with actual event-day return.

and

$$\alpha_{.} = \frac{1}{N}\sum_{i=1}^{N=29} \alpha_i; \;\; \beta_{.} = \frac{1}{N}\sum_{i=1}^{N=29} \beta_i; \;\; \sigma^2_{.} = \frac{1}{N^2}\sum_{i=1}^{N=29} \sigma^2_i$$

Hence, if the market model parameters are aggregated for all N securities in the sample for each iteration, or

$$\alpha^*_{.1} = \frac{1}{N}\sum_{i=1}^{N} \alpha^*_{i1}; \;\; \beta^*_{.1} = \frac{1}{N}\sum_{i=1}^{N} \beta^*_{i1}; \;\; \sigma^*_{.1}2 = \frac{1}{N^2}\sum_{i=1}^{N} \sigma^*_{i1}{}^2 \tag{25}$$

a Monte Carlo integration for the day '0' predictive return distribution becomes

$$f(R_{.,0}|R) = \frac{\sum_{l=m+1}^{L} f\left(R_{.,0}\middle|\alpha^*_{.1}, \beta^*_{.1}, \sigma^{*\,2}_{.1}\right)}{L-m} \tag{26}$$

based upon the sample. Figure 4 shows the day '0' mean return predictive distribution for the sample mean return and the actual day '0' mean return. The actual return is well to the right of the predictive return distribution, indicating that the null hypothesis of no effect on the underlying stock price due to option introduction is rejected.

For a 5-day cumulative predictive return distribution,

$$R_{.,\mathrm{cum}} = \sum_{t=-3}^{T=+1} R_{.,t} \tag{27}$$

which, given θ and R, is distributed as

$$f\left(R_{.,\mathrm{cum}}\middle|\alpha_{.}, \beta_{.}, \sigma^2_{.}, \bar{R}_{m,\mathrm{cum}}\right) = N\left(5\alpha_{.} + \beta_{.} \sum_{t=-3}^{T=+1} \bar{R}_{mt}, 5\sigma^2_{.}\right) \tag{28}$$

A Monte Carlo integration for the predictive distribution $f\ (R_{.,cum}|R)$ follows analogously to Equation (26) with $f\ (R_{.,0}|\alpha_{.l}{}^*,\ \beta_{.l}{}^*,\ \sigma^*{}_{.l}{}^2)$ replaced by $f\ (R_{.,cum}|\alpha_{.l}{}^*,\ \beta_{.l}{}^*,\ \sigma^*{}_{.l}{}^2)$. Figure 5 shows this cumulative 5-day predictive return distribution and the actual average cumulative return. Once again, the actual return is well to the right of the predictive return distribution and the null hypothesis of no stock price effect on the underlying stock on the date of option introduction is rejected. Both of the above conclusions are consistent with frequentist results (Conrad (1989) and Giaccotto and Sfiridis (1996)), but are routinely handled in a unified fashion within the Bayesian framework. Since there is a large amount of data available and non-informative priors are used, i.e., no subjective prior information is being incorporated into the model, Bayesian and frequentist results are expected to be in agreement.

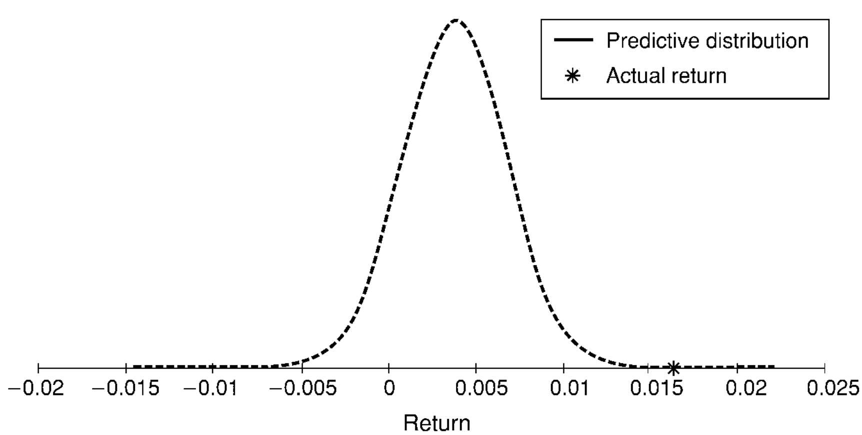

Fig. 4. Predictive return distribution for day-0 mean return.

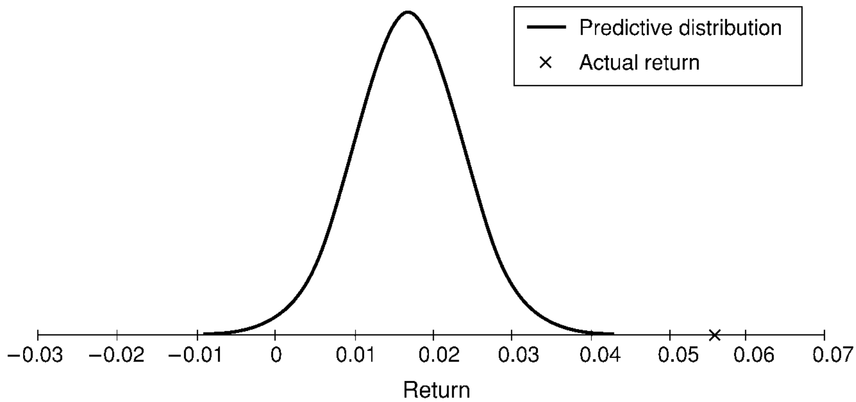

Fig. 5. Predictive return distribution for cumulative 5-day mean return.

Last, the issue of parameter 'shrinkage' is considered. By this is meant the smoothing of the OLS estimates of, for example, the β_i's from the individually fitted models. It may be very reasonable to assume, *a priori*, that firms face similar systematic risk in the economy at any point in time and, hence, individual betas are independent draws from the same underlying market-wide beta distribution. The Bayesian *posterior* estimate of β_i accounts for this by shrinking it away from the simple OLS estimate and towards the sample's grand average, μ_β. Figure 6 shows the Bayesian beta versus the OLS beta for each security in the sample. As expected, variability of the Bayesian βs is less than that for the OLS estimates.

The Bayesian paradigm has great potential for handling more complicated return scenarios. For example, the assumption of cross-sectional independence is violated if the announcement day is common to the entire security sample. This is a common occurrence is many event studies and can be

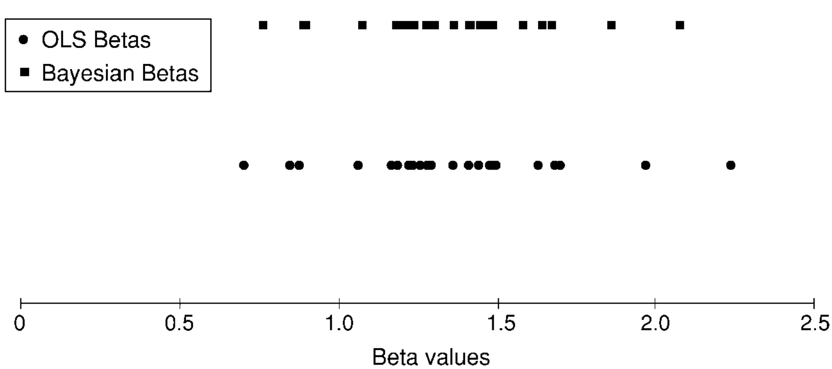

Fig. 6. OLS versus Bayesian beta estimates. Note: illustration of shrinkage of the sample Bayesian estimates compared to those for OLS.

readily handled using MCMC methods. The problem is now thrust from the univariate to the multivariate environment where security returns are distributed multivariate normal with a mean vector of dimension N and an $N \times N$ covariance matrix. Sampling will then be from multivariate normal and inverse Wishart distributions.[6] The Wishart is the multivariate equivalent of the gamma distribution.

3.1. *Bayesian sensitivity analysis*

For response variables studied in finance symmetric errors having a Gaussian distribution about the mean are assumed. However, research indicates that daily security return data are non-normal, i.e., excessive skewness and/or kurtosis may be present (Aggarwal and Aggarwal, 1993; Harvey and Zhou, 1993). Aggrawal and Aggrawal document when significant departures from normality of return distributions may occur. For example, for daily return data, smaller portfolios consisting of NASDAQ-traded stocks will tend to have greater departures from normality. Thus, a more realistic specification for the error distribution may be Student-t where fatter tails allow for a greater probability that residuals will be further from the mean.

For long-horizon event studies abnormal returns show significant departures from normality for a number of reasons. Cowan and Sergeant (2001) and Brav (2000) discuss skewness and cross-sectional dependence of the abnormal returns as two of the most difficult problems. The first arises from the compounding of single-period abnormal returns over long horizons. A skewing to the right of the return distribution thus occurs. The second problem arises because over long horizons there is increased overlapping in calendar time of abnormal returns for the security sample. Thus, the assumption of contemporaneous independence of returns across firms is violated. Cowan and Sergeant (2001) show that the non-normality of security abnormal returns persists regardless of sample size and holding-period duration. Brav (2000) shows how such biases can be alternatively specified and analysed within the foregoing Bayesian sampling-based approach.

Table 1 shows the statistical moments of the estimation period normalized return residuals for each security in the option sample used in our short-horizon event study. The table indicates that on average the error distributions are symmetric, but exhibit significant leptokurtosis, i.e., there is less probability mass near the mean and more in the tails. The central limit theorem is usually appealed to after cross-sectional aggregation is accomplished. However, if sample sizes are not large, how good will such an approximation be?

The normal distribution can be considered a special case of the Student-t distribution where the Student-t approaches the normal for high degrees of freedom, ν, equal to 30. However, at lower degrees of freedom the Student-t has more mass in the tails, approaching a Cauchy distribution for $\nu=1$. At $\nu=4$, the kurtosis of the Student-t distribution is 4.5–5.0, very close to the average seen for the security sample.

Sampling from a Student-t distribution can be accomplished if one interprets a $t_\nu(\mu,\sigma^2)$ as a continuous mixture of normal distributions having a common mean with variances distributed as draws from an inverse gamma distribution having shape parameter $a=\nu/2$ and scale parameter

[6] Gelman *et al.*, (1996) Chapter 3.

Table 1. Security sample statistical moments using standardized return residuals

Security	*Mean*	*Variance*	*Skewness*[a]	*Kurtosis*[b]
1	0	1.0	−0.175	3.297
2	0	1.0	−0.459	4.161
3	0	1.0	0.564	5.485
4	0	1.0	0.470	2.806
5	0	1.0	0.070	3.596
6	0	1.0	0.275	4.047
7	0	1.0	−0.370	4.094
8	0	1.0	−0.802	6.582
9	0	1.0	0.247	4.047
10	0	1.0	0.514	3.827
11	0	1.0	0.152	3.363
12	0	1.0	0.108	4.857
13	0	1.0	−0.197	4.252
14	0	1.0	0.197	4.237
15	0	1.0	0.628	6.564
16	0	1.0	0.344	3.982
17	0	1.0	−0.247	3.986
18	0	1.0	0.444	4.515
19	0	1.0	0.117	6.650
20	0	1.0	0.716	5.208
21	0	1.0	0.114	4.761
22	0	1.0	0.952	5.467
23	0	1.0	−0.011	4.313
24	0	1.0	0.250	3.458
25	0	1.0	0.049	4.328
26	0	1.0	0.315	4.968
27	0	1.0	0.321	3.281
28	0	1.0	−0.221	3.793
29	0	1.0	0.645	4.192
Average =	0	1.0	0.173	4.34

[a]95% confidence interval is −0.326 to 0.326.
[b]95% confidence interval is 2.307 to 3.693.

$b = \nu\sigma^2/2$.[7] That is, R_{it} will be distributed as $t_\nu(\mu_i, \sigma_i^2)$ if Λ^*_{i1} is drawn from $IG\left(\frac{\nu}{2}, \frac{\nu\sigma_i^2}{2}\right)$ and then R_{it} is drawn from $N(\mu_i, \Lambda_i)$. Within the Gibbs sampler this merely entails drawing $N\Lambda^*_{il}$s in addition to the previous $3N+4$ components of θ. In particular, all of the above predictive return distributions can then be determined as shown in Section 2 with Λ^*_{i1} replacing $\sigma^*_{i1}{}^2$.

Two versions of the event study were run modelling the kurtosis of the return residuals for each

[7] Gelman *et al.* (1996) Chapter 12.

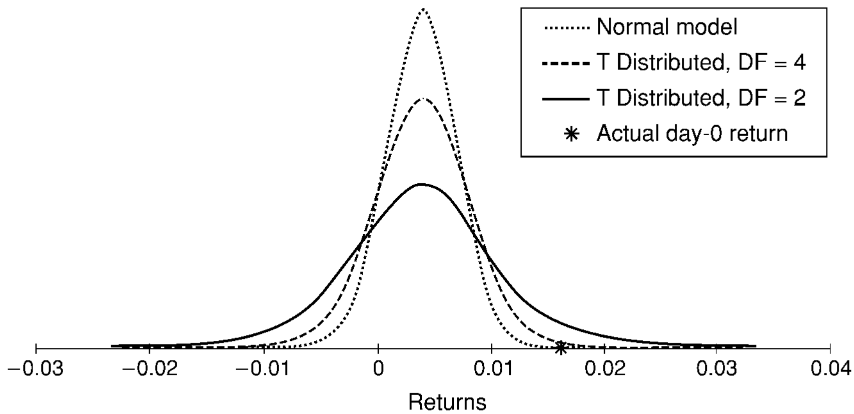

Fig. 7. Day-0 mean return predictive distribution under different distributional assumptions.

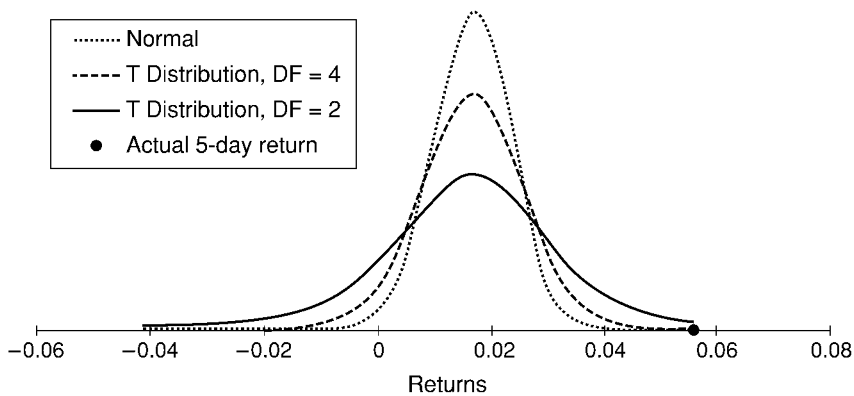

Fig. 8. Cumulative 5-day mean predictive return distribution under different distributional assumptions.

security with Student-t distributions having $\nu=4$ and 2. The former number corresponds to kurtosis very close to the average for the sample and is a robust test of the inference determined in the earlier event study. The latter corresponds to a kurtosis value of approximately 11, greater than for our sample, but valuable nonetheless for illustrating the effect of kurtosis on the resulting inference. Results for the day '0' and cumulative 5-day mean returns are shown in Figs 7 and 8 respectively. Both figures indicate that the inference for the original event study is also valid for predictive distributions assuming a Student-t distribution with $\nu=4$ at each of the customary significance levels, i.e., 10%, 5% or 1%. However, a Student-t distribution with $\nu=2$ permits such heavy tails that previous inference no longer emerges.

Table 2 gives the sampling-based confidence intervals for the day '0' and cumulative 5-day tests.

Table 2. Sampling-based confidence intervals for day-0 and cumulative 5-day mean returns for three residual return specifications

Panel 1. Day-0 mean return[a]			
Normal	90%	95%	99%
Minimum	−0.0003	−0.0010	−0.0030
Maximum	0.0086	0.0095	0.0107
T Distribution, DF=4			
Minimum	−0.0024	−0.0040	−0.0071
Maximum	0.011	0.0123	0.0143
T Distribution, DF=2			
Minimum	−0.0084	−0.0117	−0.0785
Maximum	0.0162	0.0184	0.0282
Panel 2. Cumulative 5-day mean return[b]			
Normal	90%	95%	99%
Minimum	0.0074	0.0058	0.0016
Maximum	0.0275	0.0297	0.0322
T Distribution, DF=4			
Minimum	−0.0032	−0.0010	−0.0084
Maximum	0.0323	0.0356	0.0399
T Distribution, DF=2			
Minimum	−0.0103	−0.0178	−0.1563
Maximum	0.0443	0.0497	0.0719

[a]Actual day-0 mean return = 0.0162.
[b]Actual cumulative 5-day mean return = 0.0559.

These were done by determining a predictive return for each of the 1200 Gibbs iterations, retaining every third return to diminish the problem of serial correlation of the Gibbs iterates, and then sorting the 400 predictive returns. Any desired confidence interval for a one or two-tailed test can then be determined simply by elimination of the correct number from each end of the sorted returns. Highest probability density estimates, or what is better known as HPD estimates, are thus determined. Under the confidence intervals specified in Table 2, the day '0' mean return is statistically significant for the normal and Student-t ($\nu = 4$) distributions. However, again for the Student-t ($\nu = 2$) distribution the null hypothesis of no statistically significant return around the event date cannot be rejected at all three significance levels. For the cumulative 5-day mean return again the null hypothesis cannot be rejected only at the 1% significance level under the Student-t ($\nu = 2$) distribution. These differing results are due solely to the likelihood specification for the residual distributions.

4. Summary and conclusions

Recent computational advances have allowed a dramatic increase in model fitting capability within a Bayesian framework. A fully Bayesian analysis of financial event study data was performed where Gibbs sampling was used to construct Markov chains that provide sampling distributions of model parameters. Though this presentation is primarily illustrative and pedagogic, it has been shown to be a useful alternative to existing frequentist methods, allowing implementation of event studies in situations where the return residuals would not be satisfactorily modelled assuming normality. Thus, it can lead to more appropriate and hopefully clarifying inference. Additionally, it can illuminate the price reaction to a financial event at the individual firm level. It is noteworthy that, though the Gibbs sampler uses OLS or maximum likelihood parameter estimates as starting points, it is only those values of the Markov trajectories that are drawn after convergence to the required stationary marginal distribution occurs that provide the inference for the model parameters.

Bayesian MCMC sampling methods can be useful in other ways for the implementation of financial event studies. For example, the choice of the time period for parameter estimation is usually at the discretion of the researcher. Implementation of a Bayesian change-point model allows determination of when the return distribution may have changed due to the event (Gelfand and Sfiridis, 1996). Short-horizon market model event studies normally use equal weightings to determine sample averages as shown in Equations (22)–(26). However, a Bayesian event study could be extended to use capital weighting as a size proxy for each security in the sample. This new N-dimensional weighting vector would become part of the overall parameter vector of the analysis and its marginal *posterior* distribution determined by an appropriate MCMC algorithm. This might very well not be the Gibbs sampler depending upon the prior distribution chosen to model the weights. Lastly, in the illustrative application the full conditional distributions used to implement the Gibbs sampler have a known functional form that can readily be sampled from. However, this is not always possible, such as when individual security variances are specified through stochastic volatility (Jacquier *et al*., 1994). In this case other MCMC algorithms, such as the Metropolis–Hastings algorithm, can be implemented to acquire samples from the required marginal posterior distributions (Chib and Greenberg, 1995). Thus, Bayesian sampling-based empirical methods can increase the ability to more effectively investigate financial market behaviour.

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