ECON 1670 Project Proposal: Factor Model with ARMA-GARCH errors

We model the return of currency y with an ARMA-GARCH error process:

$$y_t = x_t\gamma + u_t, \quad t = 1, \dots, T;$$

where x_t is a $kx1$ vector of factor returns at time t, γ is a vector of factor loadings, and u_t is an error term defined as:

$$u_t = \sum_{j=1}^{p} \phi_j u_{t-j} + \varepsilon_t + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}, \quad \varepsilon_t | \mathcal{F}_{t-1} \sim N(0, \sigma_t^2);$$

and the equation for σ_t^2 is:

$$\sigma_t^2 = \omega + \sum_{j=1}^{r} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2;$$

Define:

$$\begin{aligned} \Phi &= (\phi_1, \dots, \phi_p)^\top, \\ \Theta &= (\theta_1, \dots, \theta_q)^\top, \\ \Omega &= (\omega, \alpha_1, \dots, \alpha_r, \beta_1, \dots, \beta_s)^\top, \end{aligned}$$

and

$$\begin{aligned} A(L) &= \sum_{j=1}^{r} \alpha_j L^j, \\ B(L) &= \sum_{j=1}^{s} \beta_j L^j, \end{aligned}$$

where L is the lag operator.

Lastly, define δ as the vector of all unknown parameters:

$$\delta = (\gamma, \Phi, \Theta, \Omega)$$

The joint posterior density of unknown parameters δ is proportional to the likelihood of observing δ given data times the prior distribution on δ:

$$p(\delta \mid Y, X) \propto \ell(\delta \mid Y, X) p(\delta)$$

where $\ell(\cdot)$ is the likelihood function given by:

$$\ell(\delta \mid \text{data}) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{(y_t - x_t\gamma - \Phi(L)(y_t - x_t\gamma) - \Theta(L)\varepsilon_t)^2}{2\sigma_t^2} \right);$$

Given the joint posterior density, we derive the marginal posterior density of each parameter of interest, say ϕ_1, by integrating out all other elements in δ:

$$p(\phi_1 \mid Y, X) = \int p(\delta \mid Y, X)\, d\delta_{-\phi_1};$$

where $\delta_{-\phi_1} = (\phi_2, \dots, \phi_p, \theta_1, \dots, \theta_q, \omega, \alpha_1, \dots, \alpha_r, \beta_1, \dots, \beta_s)^\top$.

Priors

- Prior on $\underline{\gamma}$: $\gamma \sim \mathcal{N}(\mu_\gamma, \sigma_\gamma^2)$
- Prior on $\underline{\phi_i}$ for all $i \in \{1, \dots, p\}$: $\phi_i \sim \text{Unif}(-1, 1)$
- Prior on $\underline{\theta_i}$ for all $i \in \{1, \dots, q\}$: $\theta_i \sim \text{Unif}(-1, 1)$
- Prior on $\underline{\omega}$: $\omega \sim \text{Log-Normal}(\mu_\omega, \sigma_\omega^2)$
- Joint prior on $\underline{\alpha}, \underline{\beta}$: $(\alpha_1, \dots, \alpha_r, \beta_1, \dots, \beta_s) \sim \text{Dirichlet}(\theta_{\alpha_1}, \dots, \theta_{\alpha_r}, \theta_{\beta_1}, \dots, \theta_{\beta_s})$

MCMC Algorithm

In order to estimate the parameters of our ARMA-GARCH model, we will use the HMC method using the Stan software.

Sampling from the Entire Joint Posterior Distribution

Given that Stan can sample from the entire joint posterior distribution, we don't need to use the conditional distribution blocks defined above. However, if this sampling method does not converge, I will explore alternative methods which makes use of the conditional distribution blocks such as Gibbs Sampling or Hybrid MCMC.