PRACTICE FINAL EXAM
MATH 202

Name: ______________________________________

e-mail: ____

Problem	Score	Out of
1		20
2		25
3		24
4		24
5		25
6		30
7		6
8		36
Style		10
Total		200

You have 3 hours. Questions are printed on both sides of the pages. If you run out of paper for any of the problems, ask for extra paper. Notes, textbooks, calculators and other material are prohibited.

For maximum style points, your work should be legible, easy to follow, and explained when necessary. Be sure to read all questions carefully. Show all your work, except where instructed otherwise.

Bonus points will be awarded to the first person to point out a mistake in a question.

Problem 1 (4+4+4+4+4 points)**.** State whether each of the following statements is true or false. **You do not need to justify your answers.** For each part, you get 4 points for the correct answer, 0 points for an incorrect answer, and 2 points if you leave it blank.

(a) If (r, θ) are the polar coordinates for (x, y), then $\theta = \tan^{-1}(\frac{y}{x})$.

(b) If an alternating series is absolutely convergent, then it is conditionally convergent.

(c) If an alternating series $a_1 + a_2 + a_3 + \cdots$ satisfies $|a_{n+1}| \leq |a_n|$ for all n, then it converges.

(d) $1 - i = \sqrt{2}e^{\frac{7\pi i}{4}}$ where $i^2 = -1$.

(e) If a complex number z satisfies $\bar{z} = z$, then z is a real number.

Problem 2 (10+5+10 points)**.**

(a) Suppose we cut a thin slice of thickness Δh through a sphere of radius R at a height h above the center of the sphere. What is the approximate volume of this slice in terms of h, R and Δh?

(b) Write an integral that gives the volume of that portion of the sphere that is between height H and the top of the sphere.

(c) Determine the percentage of the volume of the Earth that is above the Arctic Circle. The Arctic Circle is at a latitude of 66.55° North (1.162 radians North). Assume that the Earth is spherical.

Your final answer should just have numbers in it (no variables), but it is OK to have several numbers in your answer and you do ***not*** *have to do the arithmetic to write your answer as a single number.*

Problem 3 (12+12 points)**.** Do the following series diverge or converge? **State clearly which test you are using and show that all the conditions of the test are satisfied.**

(a)

$$\sum_{n=1}^{\infty} \frac{3^n}{5^n + \sqrt{n}}$$

(b)

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{\frac{3}{2}}}$$

Problem 4 (12+12 points). Evaluate each of the following integrals.

(a)

$$\int_0^3 \frac{t}{\sqrt{t+1}}dt$$

(b)

$$\int \frac{10x-2x^2}{(x-1)^2(x+3)}dx$$

Problem 5 (10+5+10 points).

(a) Sketch the slope field for the differential equation $\frac{dy}{dx} = (x+1)e^{-x}$. Do not worry too much about the exact slope at each point, but your sketch should make it clear where the slope is positive, negative, and zero, and where the slope is constant, increasing or decreasing as x and y change.

(b) On your sketch, draw the solution curve that goes through the origin.

(c) Find an exact expression for y in the solution curve from (b) by solving the differential equation exactly.

Problem 6 (15+15 points).

(a) On the same axes, sketch the curves $r = 3\sin\theta$ and $r = 1 + \sin\theta$.

(b) Find the area of the region that lies both inside $r = 3\sin\theta$ and outside $r = 1 + \sin\theta$.

Problem 7 (6 points)**.** Suppose a particle follows the trajectory $(4\sin 5t, 6\cos 5t)$ over time. What is the speed of the particle at time $t = \frac{\pi}{4}$?

Problem 8 (12+12+12 points)**.** The Taylor series for some function $f(x)$ of the form $\ln(b+cx)$ about $x=a$ is

$$\ln(5) + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}2^n(x-1)^n}{5^n n}$$

(a) What is a and what is $f(x)$?

(b) Find the interval of convergence for the Taylor series, making sure to check the endpoints.

(c) Find an upper bound for the error when the first three terms of this Taylor series are used to approximate $\ln 7$.