

Name: _____

NetId: _____

legible please!

Recitation section:

☐ **005** - 09:45 - 209 - Zhuang Su

☐ **008** - 09:45 - 303 - Vlad Margarint

☐ **006** - 11:15 - 209 - Zhuang Su

☐ **009** - 11:15 - 303 - Vlad Margarint

☐ **007** - 14:45 - 209 - Zhuang Su

☐ **010** - 14:45 - 303 - Vlad Margarint

Calculus Fall 2021 – Exam 01

DO NOT OPEN YET

...and wait until the proctor announces that it is time to start.

In the mean time, please write your name and NetID **legibly**, mark your lecture and recitation sections, and read the instructions below carefully.

- * Do not unstaple nor remove pages from this booklet.
- * Write your solution within the corresponding box.
- * **All the important steps must be justified.**
- * All the space outside boxes is scratch paper, and will be ignored when grading.
- * The back side of each page is also scratch paper.

Good luck!

Exercise	Grade
1	/10
2	/10
3	/10
4	/10
5	/10
Total	/50

Exercise 1. Find the finite or infinite limit, or explain why it does not exist. 3+4+3 points

$$\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 3x - 1}}{2x - 1}$$

$$\lim_{x \rightarrow -1^-} \frac{x^2 - 1}{2x^2 - 6x - 8}$$

$$\lim_{x \rightarrow 3^+} \frac{x^2 + x - 6}{x^2 - 7x + 12}$$

Scratch paper

Exercise 2. Find the finite or infinite limit, or explain why it does not exist. 3+3+4 points

$$\lim_{x \rightarrow +\infty} \tan^{-1}(\log_{10}(x))$$

$$\lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{x^2 - 4x + 3}$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 + x - 1}}{2x + 10}$$

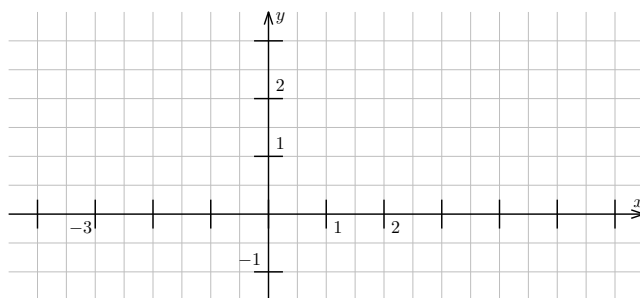
Scratch paper

Exercise 3.

4+3+3 points

Draw the graph of a function f satisfying:

- (a) $f(3)$ is undefined (e) $\lim_{x \rightarrow -2^-} f(x) = 1$
(b) $\lim_{x \rightarrow 0^+} f(x) = 2$ (f) $f(0) = 1$
(c) $\lim_{x \rightarrow 0^-} f(x) = 0$ (g) $\lim_{x \rightarrow -2^+} f(x) = -1$
(d) $f(-2) = -1$ (h) $\lim_{x \rightarrow 3} f(x) = -1$



For what values of the constants A and B is the function

$$f(x) = \begin{cases} 4 - 2x + A, & x < 1, \\ 1, & x = 1 \\ 2B + x^2, & x > 1. \end{cases}$$

continuous at the point $a = 1$? Justify.

Show that the equation $x^{10} + 2x = 2^x$ has a solution in $(0, 1)$.

Scratch paper

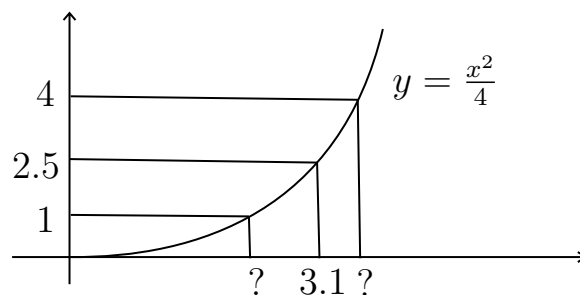
Exercise 4.

5+5 points

Find the finite or infinite limit, or explain why it does not exist: $\lim_{x \rightarrow 3} |x - 3| \cos \left(-\frac{1}{x - 3} \right)$.

Use the given graph of $f(x) = \frac{x^2}{4}$ to find a number $\delta > 0$ such that

$$\text{if } |x - 3.1| < \delta \text{ then } \left| \frac{x^2}{4} - 2.5 \right| < 1.5$$



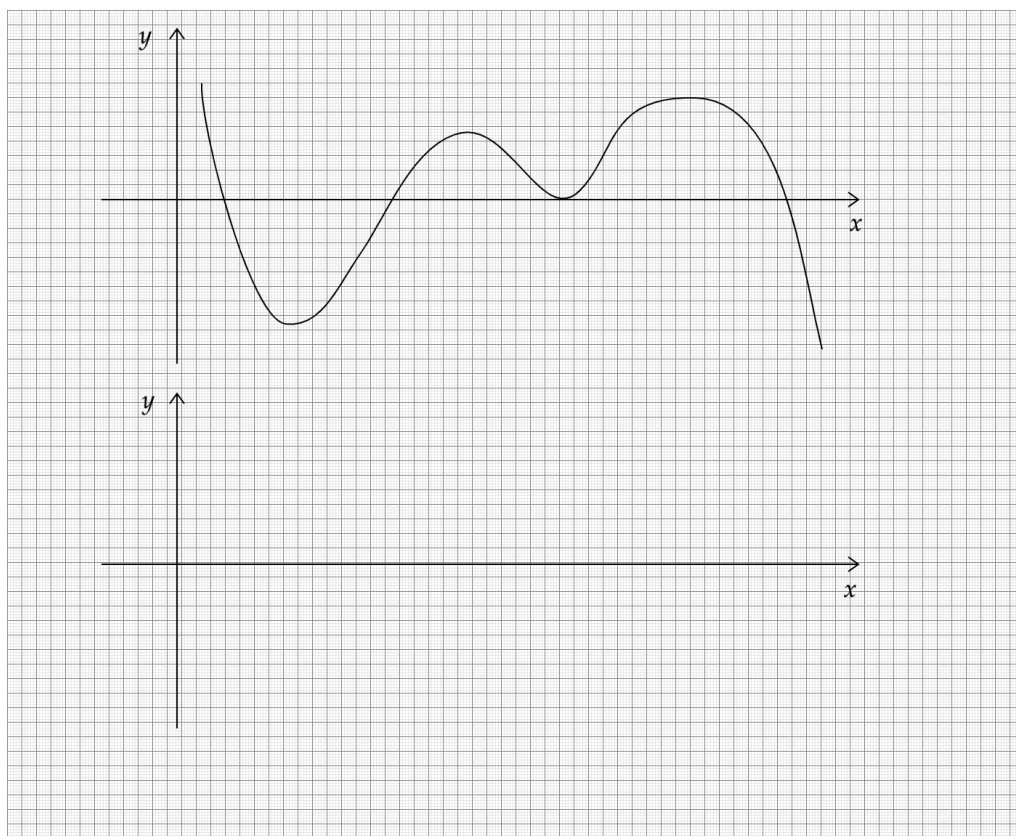
Scratch paper

Exercise 5.

5+5 points

Find the derivative (or show that it does not exist) of the function $f(x) = x^2 + |x^3|$ at the point $a = 0$.

Use the given graph of the function f to sketch the graph of its derivative f' .



Please do not write here