Usage for `genulens`

Naoki Koshimoto
Email: koshimoto.work@gmail.com
GitHub: https://github.com/nkoshimoto/genulens
Paper for the Galactic model: Koshimoto, Baba, and Bennett (2021), arXiv: 2104.03306
Paper for the code: Koshimoto and Ranc (2021), Zenodo.4784949

May 25, 2021

To use `genulens`, you need to specify input parameter values or options in the arguments.
A simple example is

```
(Example 1)
$./genulens Nsimu 1e+5 l 2.32 b -2.38 PA 60.31 vEarthlb 1.10 -6.71 VERBOSITY 2 > tmp.dat
```

This generates 10^5 (= `Nsimu`) artificial microlensing events toward the sky direction of $(l, b) = (2.32°, -2.38°)$. It uses the E+E$_\mathrm{X}$ model in Koshimoto, Baba, and Bennett (2021) [1] as the Galactic model by default. There are other three Galactic models developed in the paper, and you can use each of the E, G and G+G$_\mathrm{X}$ models by adding "`E_fg0 1`", "`G_fg0 1`", "`GXG_fg0 1`" to the arguments, respectively.

The output format can be specified by the parameter `VERBOSITY`, and Fig. 1 shows part of output lines of example 1. With "`VERBOSITY 2`", the output format is (from left to right):

- wt_j – Weight for jth line. **You need to weight each line by wt_j** when you combine them to make a probability distribution. wt_j takes mostly 1 and sometimes > 1 unless you use the "`SMALLGAMMA 1`" option.
- t_E – The Einstein radius crossing time in days.
- θ_E – The angular Einstein radius in mas.
- $\pi_\mathrm{E,N}$ – Equatorial north component of the microlens parallax vector.
- $\pi_\mathrm{E,E}$ – Equatorial east component of the microlens parallax vector.
- D_S – Distance to the source system in pc.
- $\mu_\mathrm{S,l,hel}$ – Galactic longitudinal component of the source proper motion in the heliocentric frame in mas/yr.
- $\mu_\mathrm{S,b,hel}$ – Galactic latitudinal component of the source proper motion in the heliocentric frame in mas/yr.
- i_S – Flag to indicate which component the source system belongs to. The components are:
  - 0: Thin disk (0.00 - 0.15 Gyr)
  - 1: Thin disk (0.15 - 1.00 Gyr)
  - 2: Thin disk (1.00 - 2.00 Gyr)
  - 3: Thin disk (2.00 - 3.00 Gyr)
  - 4: Thin disk (3.00 - 5.00 Gyr)
  - 5: Thin disk (5.00 - 7.00 Gyr)
  - 6: Thin disk (7.00 - 10.00 Gyr)
  - 7: Thick disk (12.0 Gyr)
  - 8: Bulge (9 ± 1 Gyr)
- i_L – Flag to indicate which component the lens system belongs to.
- f_REM – Flag to indicate evolutional stage of the lens.
  - 0: Main sequence
  - 1: White dwarf
  - 2: Neutron star
  - 3: Black hole

  Note that f_REM can be > 0 only when the "`REMNANT 1`" or "`onlyWD 1`" option is added to the argument, and can be > 1 only when the "`REMNANT 1`" option is added.

```
#----- Output of Monte Carlo simulation w/ VERBOSITY= 2 and seed= 12304357 --------
#         wtj           tE       thetaE          piEN          piEE   D_S         muSl          muSb iS iL fREM
1.789982e+00 1.026014e+01 2.893354e-01 -2.670026e-02  4.126398e-02  8159 -6.23573e+00  4.13179e+00  8  8 0
1.000000e+00 1.449606e+01 2.533772e-01  2.800766e-01  2.697542e-01 13858 -6.68344e+00 -1.68848e-01  6  3 0
1.000000e+00 6.392391e+01 2.665412e-01 -2.151892e-02 -4.385056e-02  7157 -2.54691e+00 -1.72749e+00  8  8 0
1.000000e+00 4.701406e+00 1.194560e-01  4.380464e-02  3.561815e-02  8184 -1.55973e+01  2.67340e+00  8  8 0
1.000000e+00 1.739493e+01 2.549459e-01  9.644269e-02 -4.377408e-03  8763 -6.70109e+00 -1.46376e+00  8  8 0
1.000000e+00 5.888771e+00 1.094510e-01  6.294772e-02 -2.766520e-02  7904 -8.48463e+00 -1.29959e+00  8  8 0
```

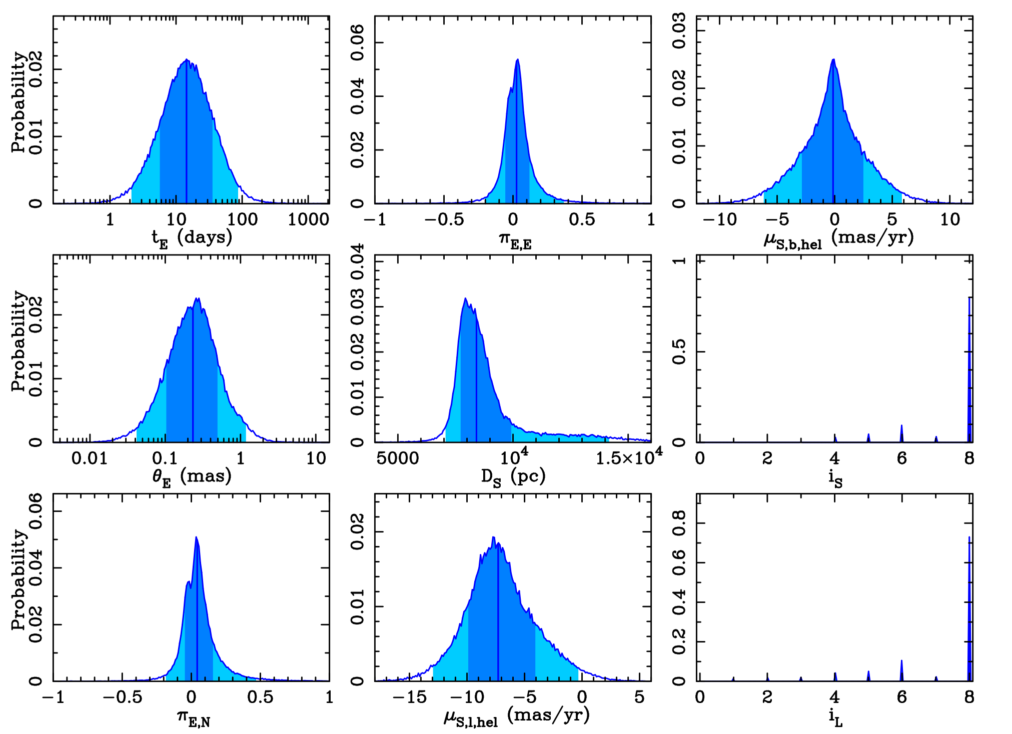

Figure 1: Part of the output lines of example 1 (top) and the probability distributions for each output parameter (bottom).

In example 1, `PA` is the position angle in degree, which is the angle between the equatorial east direction and the galactic longitudinal direction, on the sky plane. The two values after `vEarthlb` are the galactic longitudinal and latitudinal components of the Earth velocity projected onto the sky plane in km/sec. `PA` and `vEarthlb` do not have to be specified if you are just interested in the magnitude of the microlens parallax vector, and not interested in its direction. Technically, `vEarthlb` affects t_E value because it is used to convert $t_{E,hel}(= \theta_E/\mu_{rel,hel})$ into $t_E(= \theta_E/\mu_{rel,geo})$. However, in most cases, the difference in the resulted probability distributions of parameters other than the direction of the microlens parallax vector is very small between when you specify `vEarthlb` and when you do not specify it.

The output parameters with "`VERBOSITY 2`" are sufficient to calculate other (perhaps more interesting) microlensing parameters such as the lens mass and distance. Appendix A provides the formulae for the conversion.

1 Posterior Calculation

For an individual event analysis, t_E is commonly measured, and θ_E is mostly measured for a caustic-crossing event. A frequent situation is that you want to calculate the posterior probability distributions for the lens mass and distance by incorporating these constraints from observed parameters into the Galactic prior.

Let me use OGLE-2008-BLG-355 (Koshimoto et al. 2014 [2]) to show how to calculate a posterior distribution with `genulens`. You have measured $t_E = 34.0 \pm 2.2$ days and $\theta_E = 0.28 \pm 0.03$ mas for this event occurred at $(l, b) = (-0.08^\circ, -3.45^\circ)$.

1.1 Simple way: Independent Gaussian assumption

You can calculate the posterior probability distribution by

```
(Example 2)
$./genulens NlikeMIN 1e+5 l -0.08 b -3.45 tE 34.0 2.2 thetaE 0.28 0.03 VERBOSITY 3 > tmp.dat
```

and Fig. 2 shows part of the output lines (with `VERBOSITY` = 3) and the distribution for some parameters. The "`VERBOSITY 3`" option is more redundant than "`VERBOSITY 2`", and it outputs M_L, D_L, π_E, $\mu_{rel,geo}$, and I_L, in addition to the output parameters with `VERBOSITY 2`. In example 2, I do not specify `PA` or `vEarthlb` assuming that we are not interested in the direction of π_E vector now.

```
#----- Output of Monte Carlo simulation w/ VERBOSITY= 3 and seed= 12304357 --------
#          wtj          M_L   D_L   D_S          t_E      theta_E         pi_E        pi_EN         pi_EE       mu_rel        mu_Sl        mu_Sb    I_L iS iL fREM
1.090589e+00 6.073099e-01  9798 11270 3.337356e+01 2.567712e-01 5.191626e-02  1.268154e-02  5.034359e-02 2.810178e+00 -8.36728e+00  1.97348e+00  99.000  7  8 0
1.000000e+00 8.924341e-01  7467  8010 3.675385e+01 2.568369e-01 3.533856e-02 -2.433739e-02 -2.562236e-02 2.552377e+00 -4.00786e+00 -2.02992e-01  99.000  8  8 0
1.000000e+00 5.172451e-01  7701  9233 3.254831e+01 3.012192e-01 7.150788e-02  4.492107e-02  5.563698e-02 3.380217e+00 -7.02563e+00 -3.33934e-01  99.000  4  8 0
1.000000e+00 4.967772e-01  8126  9339 3.581116e+01 2.542376e-01 6.284139e-02 -1.548968e-02 -6.090247e-02 2.593054e+00 -4.58248e+00 -6.41778e-01  99.000  8  6 0
1.000000e+00 5.501188e-01  6733  7838 3.407724e+01 3.062447e-01 6.835647e-02 -4.835272e-02 -4.831792e-02 3.282422e+00 -3.57722e+00 -4.30568e+00  99.000  8  8 0
1.000000e+00 7.642087e-01  7927  8963 3.233027e+01 3.013273e-01 4.841657e-02 -3.061996e-02 -3.750443e-02 3.404233e+00 -6.64013e+00  1.77290e+00  99.000  8  8 0
```

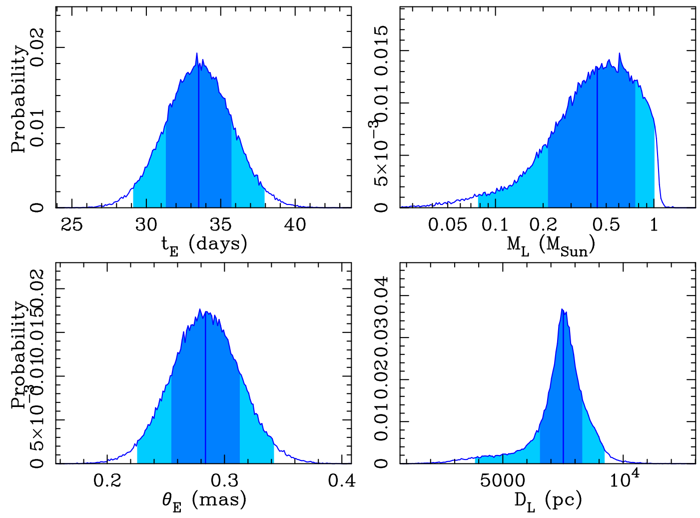

Figure 2: Part of the output lines of example 2 (top) and the probability distributions for some parameters (bottom). Note that the lens apparent magnitude in I-band, $I_{\rm L}$, only takes 99 unless the extinction value for red clump centroid $A_{I,\rm rc}$ is specified by `AIrc`.

The "`tE [val] [err]`" option multiplies a Gaussian probability of

$$\mathcal{L}_{t_{\rm E}} = \exp\left(-0.5\left(\frac{\texttt{val} - t_{\rm E}}{\texttt{err}}\right)^2\right) \tag{1}$$

by the Galactic prior by default, while it multiplies a uniform probability of

$$\mathcal{L}_{t_{\rm E}} = \begin{cases} 1 & \text{when } |\texttt{val} - t_{\rm E}| \leq \texttt{err} \\ 0 & \text{when } |\texttt{val} - t_{\rm E}| > \texttt{err} \end{cases} \tag{2}$$

by the Galactic prior when "`UNIFORM 1`" is added to the arguments. The same is true for the "`thetaE [val] [err]`" option which multiplies another probability of $\mathcal{L}_{\theta_{\rm E}}$.

In practice, when you add the `tE` and `thetaE` constraints, `genulens` generates many microlensing events following the Galactic model, and outputs its parameters only if the condition "$P_{\rm ran} < \mathcal{L}_{t_{\rm E}}\mathcal{L}_{\theta_{\rm E}}$" is true, where $P_{\rm ran}$ is a random number generated from the uniform distribution between 0 and 1.

Example 2 outputs $\sim 10^5$ (= `NlikeMIN`) lines that passed the condition "$P_{\rm ran} < \mathcal{L}_{t_{\rm E}}\mathcal{L}_{\theta_{\rm E}}$", which means the number of generated events are much larger than `NlikeMIN`. This is the difference between `NlikeMIN` and `Nsimu` used in example 1. If you use "`Nsimu 1e+5`" instead of "`NlikeMIN 1e+5`" in example 2, it would generate 10^5 microlensing events, and would output parameters for small fraction of them that passed the condition.

The "`NlikeMIN`" option is useful, but you need to be careful when using it because you cannot easily tell the amount of time it will take unlike the "`Nsimu`" option. I recommend that you always check the computational time with a small `NlikeMIN` value before you try a calculation with a large `NlikeMIN` value, because the computational time with a certain `NlikeMIN` value totally depends on the constraints you used for `tE`, `thetaE`, or other parameters. In the case of example 2, it takes ~ 80 secs with my laptop to generate 1.4×10^7 events and accept $\sim 10^5$ events out of them.

In general, if you find that your desired calculation would take too long, you can divide the calculation into multiple runs with different random seed values using the `seed` option. For example, you can run

```
$./genulens [options for calculation] seed 1 > tmp1.dat
$./genulens [options for calculation] seed 2 > tmp2.dat
.
.
$./genulens [options for calculation] seed 50 > tmp50.dat
```

in parallel if you have sufficient cores in your machine(s), and combine the output files to make a posterior probability distribution.

1.2 More rigorous way: Incorporation of parameter distribution from light curve modeling

In example 2, I incorporated the measured $t_{\rm E}$ and $\theta_{\rm E}$ values assuming they are independent Gaussian distributions. This assumption mostly works for events with good measurements for the parameters that you want to incorporate with the Galactic prior (i.e., $t_{\rm E}$ and $\theta_{\rm E}$ for OGLE-2008-BLG-355). The assumption becomes not good if there is a non-negligible correlation among the parameters, or the parameter distribution is heavily skewed. So, ideally, we want to convolve the Galactic prior with a parameter distribution from light curve modeling.

Currently, `genulens` only supports incorporation of the observed constraints in either a Gaussian probability form (Eq. 1) or a uniform probability form (Eq. 2), and cannot directly incorporate the parameter distribution from your light curve analysis like MCMC.

An alternative way, in the case of OGLE-2008-BLG-355, is to conduct a similar calculation to example 2, but with the "`UNIFORM 1`" option and larger error bars for $t_{\rm E}$ and $\theta_{\rm E}$, e.g.,

```
(Example 3)
$./genulens NlikeMIN 1e+6 l -0.08 b -3.45 tE 34.0 9.0 thetaE 0.28 0.12 VERBOSITY 2
UNIFORM 1 > tmp.dat
```

The error bars should be taken so that it covers major parameter space. Note that "`NlikeMIN 1e+6`" is probably too small to have a good resolution, this is just an example.

Separately, you should be able to calculate a joint probability distribution of $t_{\rm E}$ and $\theta_{\rm E}$, $P_{\rm LC}(t_{\rm E}, \theta_{\rm E})$, from your light curve analysis. By multiplying $P_{\rm LC}(t_{\rm E}, \theta_{\rm E})$ by ${\rm wt}_j$ in each line of the output of example 3, you can calculate a posterior probability distribution for $t_{\rm E}$ and $\theta_{\rm E}$ (and other output parameters) including the correlation between $t_{\rm E}$ and $\theta_{\rm E}$, and non-Gaussian shape of them.

This is just my idea to incorporate it, and maybe there is a better way. Please let me know if you find a better idea.

2 Input Parameters

Coming soon.

A Formulae for Conversion

The following summarizes formulae to convert the output parameters with `VERBOSITY` = 2 into other common parameters used in microlensing studies.

$$\pi_{\rm E} = \sqrt{\pi_{\rm E,N}^2 + \pi_{\rm E,E}^2} \tag{3}$$

$$M_{\rm L} = \frac{\theta_{\rm E}}{\kappa \pi_{\rm E}} \quad (\kappa = 8.144 \text{ mas } M_\odot^{-1}) \tag{4}$$

$$D_{\rm L} = \frac{\rm AU}{\pi_{\rm E}\theta_{\rm E} + {\rm AU}/D_{\rm S}} \tag{5}$$

$$\boldsymbol{\mu}_{\rm rel,geo} = \frac{\theta_{\rm E}}{t_{\rm E}} \frac{\boldsymbol{\pi}_{\rm E}}{\pi_{\rm E}} \tag{6}$$

$$\text{or} \quad \begin{pmatrix} \mu_{\rm rel,geo,N} \\ \mu_{\rm rel,geo,E} \end{pmatrix} = \frac{\theta_{\rm E}}{t_{\rm E}\, \pi_{\rm E}} \begin{pmatrix} \pi_{\rm E,N} \\ \pi_{\rm E,E} \end{pmatrix} \tag{7}$$

$$\text{or} \quad \begin{pmatrix} \mu_{\rm rel,geo,b} \\ \mu_{\rm rel,geo,l} \end{pmatrix} = \begin{pmatrix} \cos {\rm PA} & -\sin {\rm PA} \\ \sin {\rm PA} & \cos {\rm PA} \end{pmatrix} \begin{pmatrix} \mu_{\rm rel,geo,N} \\ \mu_{\rm rel,geo,E} \end{pmatrix} \tag{8}$$

$$\boldsymbol{\mu}_{\rm rel,hel} = \boldsymbol{\mu}_{\rm rel,geo} + \frac{\pi_{\rm E}\, \theta_{\rm E}}{\rm AU} \boldsymbol{v}_{\oplus,\perp} \tag{9}$$

$$\boldsymbol{\mu}_{\rm L,hel} = \boldsymbol{\mu}_{\rm rel,hel} + \boldsymbol{\mu}_{\rm S,hel} \tag{10}$$

$$\boldsymbol{v}_{\rm L,\perp} = D_{\rm L}\, \boldsymbol{\mu}_{\rm rel,hel} + \boldsymbol{v}_{\odot,\perp} \tag{11}$$

References

[1] Koshimoto, N., Baba, J., & Bennett, D. P. 2021, arXiv:2104.03306

[2] Koshimoto, N., Udalski, A., Sumi, T., et al. 2014, ApJ, 788, 128

[3] Nataf, D. M., Gould, A., Fouqué, P., et al. 2013, ApJ, 769, 88. doi:10.1088/0004-637X/769/2/88