Illustrating Good Practice in LaTeX

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1 Introduction

This document is not meant to be a guide on how to use LaTeX, since plenty of those already exist. Instead, what follows is an incoherent collection of examples that illustrate good practice. For more of my awesome LaTeX resources, visit `https://github.com/nkostin4/LaTeX-templates`.

2 Eigenvalues of a Hermitian Operator

It is not hard to show that the eigenvalues of a Hermitian operator are real. Let $|a\rangle$ be an eigenstate of the Hermitian operator A corresponding to eigenvalue a, then

$$A\,|a\rangle = a\,|a\rangle\,. \tag{2.1}$$

Then we have

$$\langle a|A|a\rangle = \langle a|a|a\rangle = a\,\langle a|a\rangle\,. \tag{2.2}$$

Taking the Hermitian conjugate of both sides of (2.1) gives

$$\langle a|\,A^\dagger = a^*\,\langle a|\,. \tag{2.3}$$

Then we also have

$$\langle a|A^\dagger|a\rangle = a^*\,\langle a|a\rangle \tag{2.4}$$

Since A is Hermitian, we have $\langle a|A^\dagger|a\rangle = \langle a|A|a\rangle$. Then comparing (2.2) and (2.4) gives

$$(a - a^*)\,\langle a|a\rangle = 0.$$

This, of course, implies that $a = a^*$, since $\langle a|a\rangle \neq 0$. Then it must be the case that a is real.

3 Unit Basis Vectors

Let $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, and $\hat{\boldsymbol{k}}$ be unit vectors parallel to the x, y, and z axes, respectively. An arbitrary vector $\boldsymbol{a}$ can be expanded in terms of these *basis vectors*:

$$\boldsymbol{a} = a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} + a_z\hat{\boldsymbol{k}}. \tag{3.1}$$

One can label a point P by its Cartesian coordinates (x, y, z), but sometimes it is more convenient to use *spherical* coordinates (r, θ, ϕ); r is the distance from the origin (the magnitude of the position vector $\boldsymbol{r}$), θ (the angle down from the z-axis) is called the *polar angle*, and ϕ (the angle around from the x-axis) is the *azimuthal angle*. The unit vectors $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, $\hat{\boldsymbol{\phi}}$ constitute an orthogonal basis set. In terms of the Cartesian unit vectors,

$$\left.\begin{aligned}
\hat{\boldsymbol{r}} &= \sin\theta\cos\phi\;\hat{\boldsymbol{\imath}} + \sin\theta\sin\phi\;\hat{\boldsymbol{\jmath}} + \cos\theta\;\hat{\boldsymbol{k}},\\
\hat{\boldsymbol{\theta}} &= \cos\theta\cos\phi\;\hat{\boldsymbol{\imath}} + \cos\theta\sin\phi\;\hat{\boldsymbol{\jmath}} - \sin\theta\;\hat{\boldsymbol{k}},\\
\hat{\boldsymbol{\phi}} &= -\sin\phi\;\hat{\boldsymbol{\imath}} + \cos\phi\;\hat{\boldsymbol{\jmath}}.
\end{aligned}\right\} \tag{3.2}$$

Finally, one can label a point P by its cylindrical coordinates (ρ, ϕ, z); ϕ has the same meaning as in spherical coordinates, and z is the same as Cartesian, but ρ is the distance to P from the z-axis. The unit vectors are

$$\left.\begin{aligned} \hat{\boldsymbol{\rho}} &= \cos\phi\ \hat{\boldsymbol{\imath}} + \sin\phi\ \hat{\boldsymbol{\jmath}}, \\ \hat{\boldsymbol{\phi}} &= -\sin\phi\ \hat{\boldsymbol{\imath}} + \cos\phi\ \hat{\boldsymbol{\jmath}}, \\ \hat{\boldsymbol{z}} &= \hat{\boldsymbol{k}}. \end{aligned}\right\} \tag{3.3}$$

4 Classical Electromagnetism

Here are the Maxwell equations:

Table 4.1: Maxwell's equations and their interpretation.

Equation	Interpretation
$\nabla \cdot \boldsymbol{E} = \dfrac{\rho}{\epsilon_o}$	$\boldsymbol{E}$-fields with divergence come from charges (electric monopoles).
$\nabla \cdot \boldsymbol{B} = 0$	$\boldsymbol{B}$-fields with divergence don't exist (no magnetic monopoles).
$\nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t}$	$\boldsymbol{E}$-fields with curl come from time-varying $\boldsymbol{B}$-fields — and those *only*.
$\nabla \times \boldsymbol{B} = \mu_o \boldsymbol{J} + \mu_o \epsilon_o \dfrac{\partial \boldsymbol{E}}{\partial t}$ $= \mu_o \left(\boldsymbol{J} + \boldsymbol{J}_D\right)$	$\boldsymbol{B}$-fields with curl come from currents (moving electric monopoles) and from time-varying $\boldsymbol{E}$-fields. Sometimes we refer to $\epsilon_o \frac{\partial \boldsymbol{E}}{\partial t}$ as displacement current and bundle the J's together.

In regions of space where there is no charge or current, Maxwell's equations read

$$\left.\begin{aligned} \nabla \cdot \boldsymbol{E} &= 0, \qquad & \nabla \times \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t}, \\ \nabla \cdot \boldsymbol{B} &= 0, \qquad & \nabla \times \boldsymbol{B} &= \mu_o \epsilon_o \frac{\partial \boldsymbol{E}}{\partial t}. \end{aligned}\right\}$$

The Lorentz force law reads

$$\boldsymbol{F} = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right).$$

Finally, most experts agree that

$$\boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t}, \qquad \text{and} \qquad \boldsymbol{B} = \nabla \times \boldsymbol{A}.$$

5 Matrix Multiplication

Consider some $A \in \mathbb{R}^{p \times q}$ and $B \in \mathbb{R}^{q \times p}$. We can represent these matrices as

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1q} \\ a_{21} & a_{22} & \cdots & a_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ a_{p1} & a_{p2} & \cdots & a_{pq} \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & a_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{q1} & b_{q2} & \cdots & b_{qp} \end{pmatrix}$$

respectively. Then the product AB can be written

$$AB = \begin{pmatrix} a_{11}b_{11} + \cdots + a_{1q}b_{q1} & a_{11}b_{12} + \cdots + a_{1q}b_{q2} & \cdots & a_{11}b_{1p} + \cdots + a_{1q}b_{qp} \\ a_{21}b_{11} + \cdots + a_{2q}b_{q1} & a_{21}b_{12} + \cdots + a_{2q}b_{q2} & \cdots & a_{21}b_{1p} + \cdots + a_{2q}b_{qp} \\ \vdots & \vdots & \ddots & \vdots \\ a_{p1}b_{11} + \cdots + a_{pq}b_{q1} & a_{p1}b_{12} + \cdots + a_{pq}b_{q2} & \cdots & a_{p1}b_{1p} + \cdots + a_{pq}b_{qp} \end{pmatrix}.$$

6 Sinusoidal Waves

A sinusoidal wave assumes the form

$$g(z,t) = A\cos\left[k\left(z - vt\right) + \delta\right].$$

Figure (6.1) shows this function at $t = 0$. The amplitude of the wave is A. The argument of the cosine is called the phase, and δ is the phase constant. At $z = vt - \delta/k$, the phase is zero; this is known as the "central maximum." Observe that δ/k is the distance by which the central maximum is "delayed." Finally, k is the wave number; it is related to the wavelength λ by the equation

$$\lambda = \frac{2\pi}{k},$$

for when z advances by $2\pi/k$, the cosine executes one complete cycle.

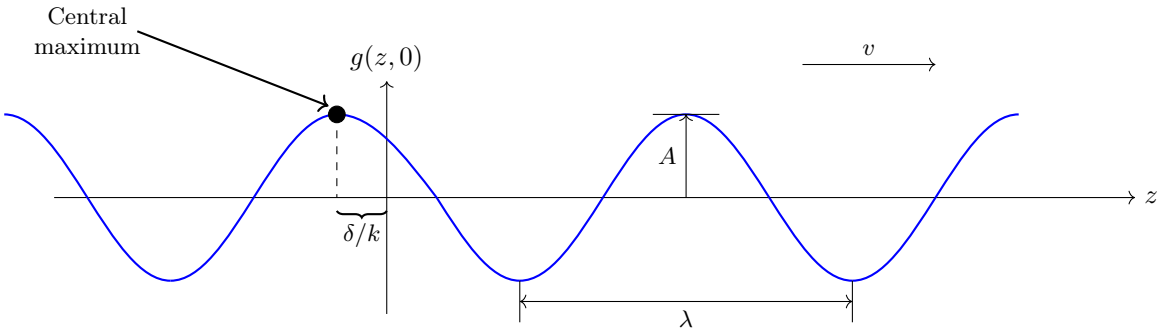


Figure 6.1: A sinusoidal wave train. With the passage of time, the entire wave train proceeds to the right.

At any fixed point z, the string vibrates up and down, undergoing one full cycle in a period

$$T = \frac{2\pi}{kv}.$$

7 Electric Fields on Pointy Things

The pointy bits on a conducting surface should produce greater electric fields then the flat bits. Let's show that numerically. Laplace's equation, in two dimensions, reads

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0. \tag{7.1}$$

We can solve Laplace's equation numerically using a finite difference method for a rectangular region. That is, we partition a rectangle into a lattice of grid points. If the spacing in the x-direction is Δx, the second derivative $\frac{\partial^2 V}{\partial x^2}$ is

$$\frac{\partial^2 V}{\partial x^2} = \frac{V(x + \Delta x, y) - 2V(x,y) + V(x - \Delta x, y)}{\Delta x^2}. \tag{7.2}$$

Similarly, if the spacing in the y-direction is Δy, the second derivative $\frac{\partial^2 V}{\partial y^2}$ is

$$\frac{\partial^2 V}{\partial y^2} = \frac{V(x, y+\Delta y) - 2V(x,y) + V(x, y-\Delta y)}{\Delta y^2}. \tag{7.3}$$

For convenience, let's say $\Delta x = \Delta y = h$. The Laplacian operator in two-dimensions becomes

$$\frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial y^2} = \frac{V(x+h, y) + V(x-h, y) + V(x, y+h) + V(x, y-h) - 4V(x,y)}{h^2}. \tag{7.4}$$

Then Laplace's equation becomes

$$V(x+h, y) + V(x-h, y) + V(x, y+h) + V(x, y-h) - 4V(x,y) = 0 \tag{7.5}$$

$$\implies \boxed{V(x,y) = \frac{V(x+h, y) + V(x-h, y) + V(x, y+h) + V(x, y-h)}{4}}. \tag{7.6}$$

That is, the potential $V(x,y)$ is the average of its four neighboring points on the lattice. Let's make this concrete by drawing a diagram.

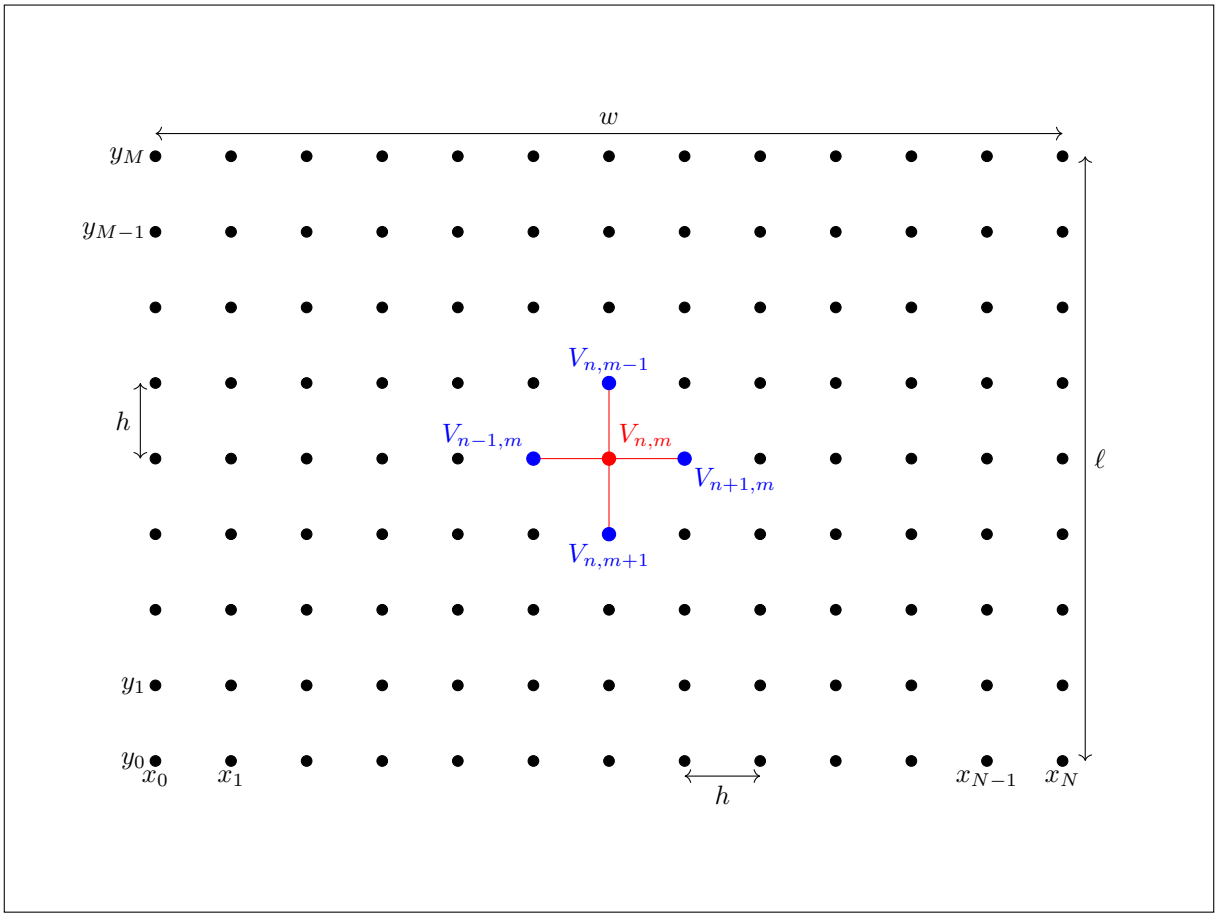


Figure 7.1: A two-dimensional lattice of points.

As shown in Fig. (7.1), we partition the interval $0 < x < w$ into $N+1$ evenly spaced points. That is,

$$x_n = nh, \qquad \text{for} \quad n \in \{0, 1, \ldots, N\}. \tag{7.7}$$

Likewise, we divide the interval $0 < y < \ell$ into $M+1$ evenly spaced points. That is,

$$y_m = mh, \qquad \text{for} \quad m \in \{0, 1, \ldots M\}. \tag{7.8}$$

What follows is the implementation of this numerical method.

```
"""
Created on Mon Feb 03 13:12:20: 2020
Author: Nicholas D. Kostin
Description: Numerical solution to Laplace's equation for a rectangle
"""

import numpy as np, matplotlib.pyplot as plt

plt.rcParams['mathtext.fontset'] = 'stix'
plt.rcParams['font.family'] = 'STIXGeneral'

# Physical constants
eps0 = 8.85e-12 # permittivity of free space

# Dimensions of region
region_width = 120 # Horizontal length of region
region_length = 80  # Vertical length of region

# Dimensions of conucting rectangle
rectangle_width = 60  # Horizontal length of conducting rectangle
rectangle_length = 40  # Vertical length of conducting rectangle

# Location of bottom-left corner of conducting rectangle
rect_corner = 20

# Set potential at boundaries of region to be zero
boundary_potential = 0

# Dirichlet Boundary conditions on conducting rectangle
# All sides at the same potential
V_box = 9

# Set color interpolation and color map
color_interpolation = 20
color_map = plt.cm.coolwarm # dope-ass colors

# Create lattice of points
X, Y = np.meshgrid(np.arange(0, region_width), np.arange(0, region_length))

# Set array size and the interior value with some guess
V = np.ones((region_length, region_width))
V.fill(0)

# Set potential at boundaries of region
V[1, :] = boundary_potential # potential at bottom
V[region_length - 1, :] = boundary_potential # potential at top
V[:, 1] = boundary_potential # potential at left
V[:, region_width - 1] = boundary_potential # potential at right


# Set potential at boundaries of conducting rectangle
V[rect_corner, rect_corner:rect_corner + rectangle_width] = V_box # potential at bottom
V[rect_corner + rectangle_length, rect_corner:rect_corner + rectangle_width] = V_box #
    potential at top
V[rect_corner:rect_corner + rectangle_length, rect_corner] = V_box # potential at left
V[rect_corner:rect_corner + rectangle_length, rect_corner + rectangle_width] = V_box #
    potential at right

# Implement 1000 iterations (and hope that's enough for convergence)
for iteration in range(0,1000):
    for i in range(1, region_length - 1, 1):
        for j in range(1, region_width - 1, 1):
            # Conducting surfaces are equipotentials
            if rect_corner <= i <= rect_corner + rectangle_length and rect_corner <= j <=
    rect_corner + rectangle_width:
                V[i, j] = V[i, j]
            # Implement finite difference method
```

```
            else:
                V[i, j] = (1/4) * (V[i - 1][j] + V[i + 1][j] + V[i][j - 1] + V[i][j + 1])

# Create array for electric field
E = np.zeros((region_length, region_width, 2))

# Compute electric field
for i in range (region_length):
    for j in range (region_width):
        # Electric field is zero inside a conductor
        if rect_corner <= i <= rect_corner + rectangle_length and rect_corner <= j <=
    rect_corner + rectangle_width:
            E[i, j] = [0, 0]
        # Electric field is derivative of potential otherwise
        elif i != 0 and i != region_length - 1 and j != 0 and j != region_width - 1:
            E[i, j] = [(V[i + 1, j] - V[i - 1, j])/(2), (V[i, j + 1] - V[i, j - 1])/(2)]

# Create array for magnitude of the electric field
E_magnitude = np.zeros((region_length, region_width))

# Compute the magnitude of the elctric field
for i in range (region_length):
    for j in range (region_width):
        E_magnitude[i, j] = (E[i, j, 0]**2 + E[i, j, 1]**2)**(1/2)

# Plot a contour map of the potential
plt.figure(1)
plt.title("Contour of Potential")
plt.contourf(X, Y, V, color_interpolation, cmap=color_map)
plt.colorbar() # Set colorbar
plt.show() # Display the color map

# Plot a contour map of the electric field magnitude
plt.figure(2)
plt.title("Contour of Magnitude of Electric Field")
plt.contourf(X, Y, E_magnitude, color_interpolation, cmap=color_map)
plt.colorbar() # Set colorbar
plt.show() # Display the color map
```

8 Infinite Square Well Potential

Suppose we have a quantum particle with mass m in an infinite square well potential:

$$V(x) = \begin{cases} 0, & 0 \leqslant x \leqslant \infty \\ \infty & \text{otherwise} \end{cases}.$$

The expectation value of the position $\hat{x}$ of the particle measured in the energy eigenstate $|\psi_n\rangle$ that is represented by $\psi_n(x) = \sqrt{\frac{2}{\pi}} \sin(nx)$, we use

$$\langle \psi_n \,|\, \hat{x} \,|\, \psi_n \rangle = \int_0^\pi \psi_n^*(x)\; x\; \psi_n(x)\; dx = \frac{2}{\pi} \int_0^\pi x\; \sin^2(nx)\; dx = \frac{\pi}{2}. \tag{8.1}$$

The expectation value of the momentum $\hat{p}$ in the eigenstate $|\psi_n\rangle$ is given by

$$\langle \psi_n \,|\, \hat{p} \,|\, \psi_n \rangle = \int_0^\pi \psi_n^*(x) \left(-i\hbar \frac{d}{dx}\right) \psi_n(x)\; dx = 0. \tag{8.2}$$

9 Commuting Hermitian Matrices

Let $A, B \in \mathbb{C}^{p \times p}$ be Hermitian matrices. Prove that AB is Hermitian if and only if A and B commute (*i.e.* they don't work from home).

Proof. A matrix $\mathscr{A} \in \mathbb{C}^{p\times p}$ is Hermitian if it satisfies

$$\mathscr{A}^\dagger = \mathscr{A}.$$

That is, a matrix $\mathscr{A} \in \mathbb{C}^{p\times p}$ is Hermitian if it is its own conjugate transpose. Moreover, for $\mathscr{A} \in \mathbb{C}^{p\times q}$ and $\mathscr{B} \in \mathbb{C}^{q\times r}$, we have the identity

$$(\mathscr{A}\mathscr{B})^\dagger = \mathscr{B}^\dagger\mathscr{A}^\dagger.$$

We will start by proving the forward direction. That is, we will show that if A and B commute, then AB must be Hermitian. Using the identity above to take the conjugate transpose of AB we get

$$(AB)^\dagger = B^\dagger A^\dagger.$$

But since we assumed A and B to both be Hermitian, we have that $B^\dagger = B$ and $A^\dagger = A$. Explicitly,

$$(AB)^\dagger = B^\dagger A^\dagger = BA.$$

But since we assumed A and B commute, then $BA = AB$. And we have

$$(AB)^\dagger = B^\dagger A^\dagger = BA = AB$$

Thereby AB satisfies the definition of a Hermitian matrix. Now we will show the reverse direction: that if AB is Hermitian then A and B must necessarily commute. If AB is Hermitian, then

$$(AB)^\dagger = AB$$

But we already showed that we can rewrite the left-hand side of the equation above as $(AB)^\dagger = B^\dagger A^\dagger = AB$, since we assumed A and B to both be Hermitian. Then we have

$$BA = AB.$$

And clearly, A and B must necessarily commute.

■

10 Blind Text

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper.

11 Some Results from Real Analysis

Here we have a *theorem*.

Bolzano-Weierstrass Theorem

Every bounded sequence has a convergent subsequence.

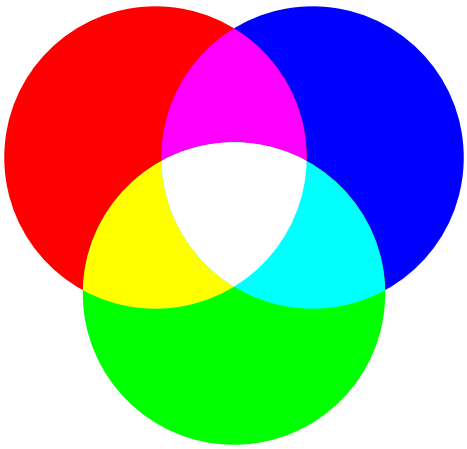
What follows is a *lemma*.

Lemma

Cauchy sequences are bounded.

12 Computer Colors

HTML color codes are hexadecimal triplets representing the colors red, green and blue.



13 Awesome Circled Numbers and Highlighting

Here are some cool circled numbers. The first argument is whatever is to be circled, and the second argument is the background color. The third argument is the outline color, and the fourth argument is color of the first argument.

0 1 2 3 4 5 6 7 8 9

Isn't this cool?

These circled numbers work in equations too!

$$\int_{a}^{b} f(x)dx + \int_{b}^{a} f(x)dx = 0$$

Don't forget, however, that too much color is cringe, so be exercise caution!

14 Consider a Graph

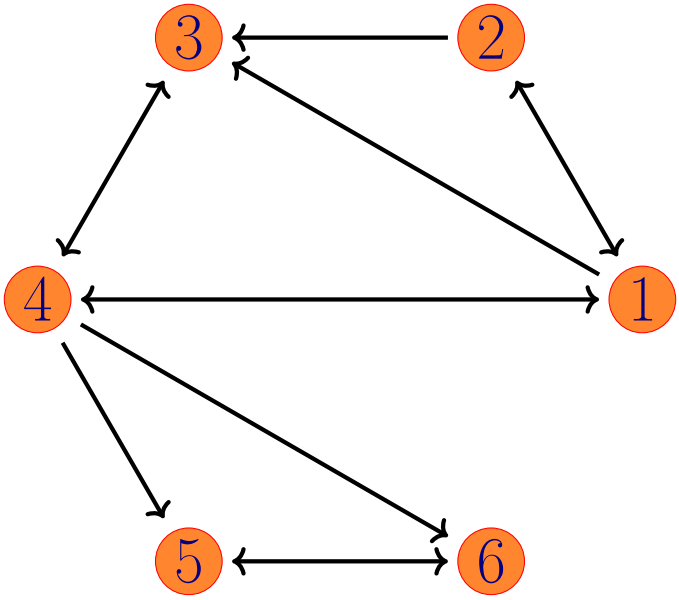
Consider a graph with vertices

$$V = \{1, 2, 3, 4, 5, 6\}$$

and edges

$$E = \{1 \to 2, 1 \to 3, 1 \to 4, 2 \to 1, 2 \to 3, 3 \to 4, 4 \to 1, 4 \to 3, 4 \to 5, 4 \to 6, 5 \to 6, 6 \to 5\}$$

The directed graph for this vertex set V and edge set E is



15 A Particle in 3D

A particle in 3D has mass m and is in the potential $V(x, y, z) = \frac{1}{2}m\omega^2\left(x^2 + y^2 + z^2\right)$.

15.1 Energies of the Lowest Four Eigenstates

Suppose we need to find the energies of the four lowest eigenstates of the Hamiltonian and their corresponding degeneracies. We can start by expanding the potential as

$$V(x, y, z) = \underbrace{\frac{1}{2}m\omega^2x^2}_{V_x(x)} + \underbrace{\frac{1}{2}m\omega^2y^2}_{V_y(y)} + \underbrace{\frac{1}{2}m\omega^2z^2}_{V_z(z)}.$$

And the total eigenenergy is

$$E_{n_xn_yn_z} = E_{n_x} + E_{n_y} + E_{n_z} = \hbar\omega\left(n_x + \frac{1}{2}\right) + \hbar\omega\left(n_y + \frac{1}{2}\right) + \hbar\omega\left(n_z + \frac{1}{2}\right) \tag{15.1}$$

where $n_x, n_y, n_z = 0, 1, 2, \ldots$ are independent from each other. The ground state should have $n_x = n_y = n_z = 0$, so its energy is given by

$$E_{000} = 3\hbar\omega\left(\frac{1}{2}\right) = \frac{3}{2}\hbar\omega.$$

The first excited state has energy

$$\underbrace{E_{100} = E_{010} = E_{001}}_{\text{degeneracy 3}} = \frac{\hbar}{\omega}\left(\frac{3}{2} + \frac{1}{2} + \frac{1}{2}\right) = \frac{5}{2}\hbar\omega.$$

The second excited state has energy

$$\underbrace{E_{200} = E_{020} = E_{002} = E_{110} = E_{101} = E_{011}}_{\text{degeneracy 6}} = \frac{\hbar}{\omega}\left(\frac{5}{2} + \frac{1}{2} + \frac{1}{2}\right) = \frac{7}{2}\hbar\omega.$$

And finally, the third excited state has energy

$$\underbrace{\left\{\begin{array}{l} E_{300} = E_{030} = E_{003} = E_{111} = E_{102} \\ E_{120} = E_{201} = E_{021} = E_{210} = E_{012} \end{array}\right\}}_{\text{degeneracy 10}} = \frac{\hbar}{\omega}\left(\frac{7}{2} + \frac{1}{2} + \frac{1}{2}\right) = \frac{9}{2}\hbar\omega.$$

These results are summarized in the table below.

Table 15.1: Energies and corresponding degeneracies of the 3D QHO

State	Energy	Degeneracy
Ground state	$\frac{3}{2}\hbar\omega$	1
First excited state	$\frac{5}{2}\hbar\omega$	3
Second excited state	$\frac{7}{2}\hbar\omega$	6
Third excited state	$\frac{9}{2}\hbar\omega$	10

15.2 Degeneracy of an Arbitrary Excited State

Now suppose we need to find the degeneracy g_n of the n^{th} excited state as a function of n. For our three-dimensional harmonic oscillator, we can define

$$n = n_x + n_y + n_z.$$

It follows from Eq. (15.1) that the energy is strictly a function of n. That is, any combination of n_x, n_y, and n_z, as long as they produce the same n, will yield the same energy. Such states are said to be *degenerate*. Our goal is to find how many possible combinations of n_x, n_y, and n_z exist for a given n.
To explicitly determine the degeneracy g_n for a given state, we fix the value of n. Because of the restriction imposed by setting $n = n_x + n_y + n_z$, we really have two free parameters (say, n_x and n_y). We let n_x assume any allowable integer value:

$$n_x \in \{0, 1, \ldots, n\}.$$

Then n_y can assume any integer value given the sum $n_x + n_y$ does not exceed n:

$$n_y \in \{0, 1, \ldots, n - n_x\}.$$

In particular, for any value of n_x, there are $n - n_x + 1$ unique choices for n_y (note the size of the set of permissible n_y values). Finally, n_z is automatically fixed:

$$n_z = n - n_x - n_y.$$

For each permissible value of n_x (from 0 to n), there are $n - n_x + 1$ choices for n_y. Summing the number of these choices for all allowable values of n_x yields

$$\sum_{n_x=0}^{n} (n - n_x + 1) = (n+1) + n + (n-2) + \cdots + \cancelto{0}{(n-n)}$$
$$= \frac{(n+1)(n+2)}{2}.$$

The series above is well known. Thus, we find that the degeneracy is

$$g_n = \frac{(n+1)(n+2)}{2}.$$