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1 Perturbation Theory — Nondegenerate Case

Perturbation theory is a useful technique to obtain approcimate solutions to eigenvalue-eigenvector problems that are too complicated to be solved exactly [1]. Suppose we want to find the eigenvalues and eigenvectors of a self-adjoint operator A which may be split up into the sum of two self-adjoint operators:

$$A = A_o + \epsilon A_1$$

We assume that we knwo something about A_o. For example, we might know some eigenvalue and its associated eigenvector. We would like to find the corresponding eigenvalue and eigenvector of A, given that the influence of A_o is known to be predominant. We have already anticipated this situation be writing the "perturbing" operator in the form of ϵA_1, where ϵ is some small parameter [1].

2 Radiation

When charges accelerate, their fields can transport energy irreversibly out to infinity — a process we call radiation [2]. Let's assume the source is localized near the orgin. Our objective is to calculate the energy it is radiating at time t_o. Imagine a gigantic sphere, out at radius r. The power passing through its surface is the integral of the Poynting vector:

$$P(r,t) = \oint \boldsymbol{S} \cdot d\boldsymbol{a} = \frac{1}{\mu_o} \oint (\boldsymbol{E} \times \boldsymbol{B}) \cdot d\boldsymbol{a}.$$

Because electromagnetic "news" travels at the speed of light, this energy actually left the source at the earlier time $t_o = t - r/c$, so the power radiated is

$$P_{\text{rad}}\left(t_o\right) = \lim_{r \to \infty} P\left(r, t_o + \frac{r}{c}\right)$$

(with t_o held constant). This is the energy (per unit time) that is carried away and never comes back [2].

References

[1] Frederick W. Byron and Robert W. Fuller. *Mathematics of Classical and Quantum Physics*. Dover Publications, Inc., 1970.

[2] David J. Griffiths. *Introduction to Electrodynamics*. Fourth. Cambridge University Press, 2017.