PHGN 200 — Electromagnetism and Optics

Block I: Electrostatics

Block I Review Slides

A Refresher on Uncertainty

Consider some function $f(x, y, z)$. The uncertainty in f due to the uncertainty in x is

$$\Delta f_x = |f(x + \Delta x, y, z) - f(x, y, z)|$$

Similarily, the uncertainty in the function f due to the uncertainty in y and z is

$$\Delta f_y = |f(x, y + \Delta y, z) - f(x, y, z)|$$

and

$$\Delta f_z = |f(x, y, z + \Delta z) - f(x, y, z)|$$

respectively. Then the total uncertainty in the function f is

$$\Delta f = \sqrt{(\Delta f_x)^2 + (\Delta f_y)^2 + (\Delta f_z)^2}$$

Worked Example — Uncertainty in the Electric Field

- Suppose a point charge is situated a distance 72 ± 2 cm from an observer. The magnitude of the charge is measured to be 4.0 ± 0.2 µC. What is the uncertainty in the observer's calculated electric field?

Solution. We know that the magnitude of the electric field created by a point charge q at a distance r is

$$E = \frac{k\ q}{r^2}$$

Note that the electric field is a function of both charge q and separation distance r.

Worked Example — Uncertainty in the Electric Field

We are given the charge and its associated uncertainty:

$$\begin{cases} q = 4.0\,\mu\mathrm{C} \\ \Delta q = 0.2\,\mu\mathrm{C} \end{cases}$$

And we are given the separation distance and its associated uncertainty:

$$\begin{cases} r = 72\,\mathrm{cm} \\ \Delta r = 2\,\mathrm{cm} \end{cases}$$

Worked Example — Uncertainty in the Electric Field

The uncertainty in the electric field due to the uncertainty in the charge is

$$\Delta E_q = \left| \frac{k \ (q + \Delta q)}{r^2} - \frac{k \ q}{r^2} \right| = 3.47 \times 10^3 \, \mathrm{N/C}$$

The uncertainty in the electric field due to the uncertainty in the distance is

$$\Delta E_r = \left| \frac{k \ q}{(r + \Delta r)^2} - \frac{k \ q}{r^2} \right| = 3.70 \times 10^3 \, \mathrm{N/C}$$

Worked Example — Uncertainty in the Electric Field

So the total uncertainty in the electric field is

$$\begin{aligned}\Delta E &= \sqrt{(\Delta E_q)^2 + (\Delta E_r)^2} \\ &= \boxed{5.07 \times 10^3\,\mathrm{N/C}} \blacktriangleleft\end{aligned}$$

Coulomb's Law for Discrete Charge Distributions

Coulomb's Law asserts that the electric field $\vec{E}$ created by a point charge q is

$$\vec{E} = \frac{k\ q}{r^3}\vec{r} = \frac{k\ q}{r^2}\hat{r}$$

where $\vec{r}$ is the vector that points from the charge to the observation point (with r being the magnitude of that vector) and k is Coulomb's constant:

$$k = \frac{1}{4\pi\epsilon_o} \approx 8.99 \times 10^9\, \frac{\mathrm{N} \cdot \mathrm{m}^2}{\mathrm{C}^2}$$

The Principle of Superposition

Electric fields obey the *principle of superposition*. Suppose there exists a collection of N discrete point charges. Then the net electric field at some location P is the sum of the contributions from each of the point charges:

$$\vec{E}_{\text{at }P} = \underbrace{\vec{E}_{\text{from }q_1} + \vec{E}_{\text{from }q_2} + \cdots + \vec{E}_{\text{from }q_N}}_{\text{sum of contributions from each charge}} = \sum_{j=1}^{N} \frac{k\ q_j}{r_j^3} \vec{r}_j$$

Coulomb's Law for Continuous Charge Distributions

The differential electric field $d\vec{E}$ due to a differential amount of charge dQ is

$$d\vec{E} = \frac{k\ dQ}{r^3}\vec{r} = \frac{k\ dQ}{r^2}\hat{r}$$

where $\vec{r}$ is the vector that points from the infinitesimal charge to the observation point (with r being the magnitude of that vector and k is Coulomb's constant:

$$k = \frac{1}{4\pi\epsilon_o} \approx 8.99 \times 10^9\, \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}$$

Point of Caution — Using r^3 or r^2 in Coulomb's Law

Coulomb's Law *always* has a $1/r^2$ dependence. Always.

$$\vec{E} = \frac{k\ q}{r^2}\hat{r}$$

Note that $\hat{r}$ is a *unit* vector that specifies the direction of the electric field. In other words, $\hat{r}$ has magnitude 1 and thus the magnitude of the electric field is

$$E = \left|\vec{E}\right| = \left|\frac{k\ q}{r^2}\cancelto{1}{\hat{r}}\right| = \frac{k\ q}{r^2}$$

In practice, we usually don't like to go through the trouble of calculating the unit vector $\hat{r}$:

$$\hat{r} = \frac{\vec{r}}{r}$$

So we leave Coulomb's law in terms of the $\vec{r}$-vector:

$$\vec{E} = \frac{k\ q}{r^2}\overbrace{(\vec{r}/r)}^{\hat{r}} = \frac{k\ q}{r^3}\vec{r}$$

Infinitesimal Charge Element dQ

	3D	2D	1D
All charge distributions	$dQ = \rho\ dV$	$dQ = \sigma\ dA$	$dQ = \lambda\ d\ell$
Uniform charge distributions	$\rho = \frac{Q_{\text{total}}}{V_{\text{total}}}$	$\sigma = \frac{Q_{\text{total}}}{A_{\text{total}}}$	$\lambda = \frac{Q_{\text{total}}}{L_{\text{total}}}$
Total charge	$Q = \int \rho\ dV$	$Q = \int \sigma\ dA$	$Q = \int \lambda\ d\ell$

Remarks on Charge Distributions

Observe that the most general method to find charge is

$$Q = \int dQ$$

This works *all the time.* Always.

However, there are special circumstances in which we can simplify things. Take a look at the following example, for instance.

Worked Example — Total Charge on a Sphere

- Suppose a charge Q is uniformly distributed throughout a sphere of radius R. Find the amount of charge enclosed by a Gaussian sphere of radius r, such that $r < R$.

Solution. As we just saw, the most general method to find charge is to integrate the differential charge:

$$Q = \int \underbrace{\rho\, dV}_{dQ}$$

Worked Example — Total Charge on a Sphere

Our first step should be to build this integral. Let's start by finding what ρ is.

Recall that the charge is *uniformly distributed*. As such, the charge density is

$$\rho = \frac{\text{total charge}}{\text{total volume}} = \frac{Q}{\frac{4}{3}\pi R^3}$$

Worked Example — Total Charge on a Sphere

The infinitesimal volume element *for a sphere* is

$$dV = 4\pi r^2 \, dr$$

Where does this come from? Examine the form of the infinitesimal volume element:

$$dV = \underbrace{\left(4\pi r^2\right)}_{\text{surface area}} \; \overbrace{dr}^{\substack{\text{differential displacement in radial} \\ \text{direction}}}$$

Imagine taking a thin slice of the surface area (much like an onion peel) and extruding it outward an infinitesimal distance dr. What results is an infinitesimal volume.

Worked Example — Total Charge on a Sphere

Here's an alternative derivation:

The volume of a sphere is

$$V = \frac{4}{3}\pi r^3$$

Taking a derivative with respect to r recovers the surface area of a sphere:

$$\frac{dV}{dr} = 4\pi r^2$$

From which it follows that the infinitesimal volume element is

$$dV = 4\pi r^2 \ dr$$

Worked Example — Total Charge on a Sphere

Putting everything together, the charge enclosed by the Gaussian surface of radius r is

$$\begin{aligned}
Q_{\text{enc}} &= \int_0^r \overbrace{\left(\frac{Q}{\frac{4}{3}\pi R^3}\right)}^{\rho} \overbrace{(4\pi r'^2 \; dr')}^{dV} \\
&= \frac{Q\ (4\pi)}{\frac{4}{3}\pi R^3} \int_0^r r'^2 \; dr' \\
&= \frac{Q \cancel{4\pi}}{\frac{\cancel{4\pi}}{3} R^3} \left(\frac{r'^3}{3}\right)\Bigg|_0^r \\
&= \frac{\cancel{3}Q}{R^3}\left(\frac{r^3}{\cancel{3}}\right) \\
&= \frac{Q\ r^3}{R^3}
\end{aligned}$$

Total Charge on a Sphere — Worked Example

Because the charge density was *uniform* (and only then), we could've avoided any integration:

$$Q_{\text{enc}} = \int \rho \, dV = \rho \int dV = \underbrace{\left(\frac{Q}{\frac{4}{3}\pi R^3}\right)}_{\rho=\text{const}} \overbrace{\left(\int dV\right)}^{\text{volume of Gaussian surface}}$$

$$= \left(\frac{Q}{\cancel{\frac{4}{3}\pi} R^3}\right) \left(\cancel{\frac{4}{3}\pi}\, r^3\right)$$

$$= \boxed{\frac{Q\ r^3}{R^3}} \blacktriangleleft$$

But the integration is totally general and works even with an *non-uniform* charge density!

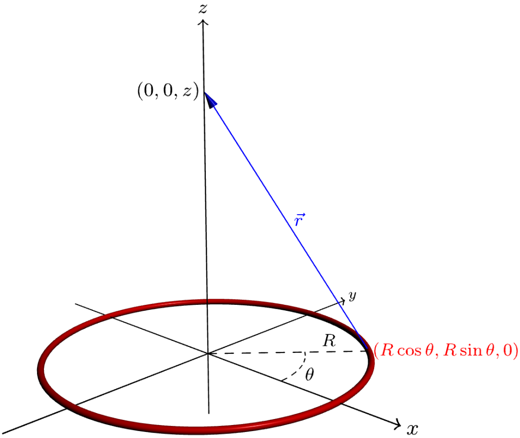
Interlude

Make sure all the steps in that last example made sense. Finding the amount of charge enclosed will come up again and again (especially when doing Gauss's law problems).

Now here's an example that pertains more to Coulomb's law.

Worked Example — Ring of Charge

Consider a ring of charge lying centered in the xy-plane. What is the electric field at some point, say z, along the z-axis?



Worked Example — Ring of Charge

Solution. We'll use Coulomb's law, which asserts that the differential electric field $d\vec{E}$ created by a differential amount of charge dQ is

$$d\vec{E} = \frac{k\ dQ}{r^3}\vec{r}$$

For a one-dimensional charge distribution, the differential charge element is $dQ = \lambda\ d\ell$. Moreover, the charge is uniformly distributed. So

$$\lambda = \frac{\text{total charge}}{\text{total length}} = \frac{Q}{2\pi R}$$

And the differential length of the ring is $d\ell = R\ d\theta$. As such, the differential charge element is

$$dQ = \underbrace{\left(\frac{Q}{2\pi\ R}\right)}_{\lambda}\underbrace{(R\ d\theta)}_{d\ell} = \frac{Q\ d\theta}{2\pi}$$

Worked Example — Ring of Charge

As always, the $\vec{r}$-vector points from the source of the electric field (the charge distribution) to the observation point:

$$\vec{r} = (0 - R\cos\theta)\,\hat{\boldsymbol{\imath}} + (0 - R\sin\theta)\,\hat{\boldsymbol{\jmath}} + (z - 0)\,\hat{\boldsymbol{k}}$$

The magnitude of the $\vec{r}$-vector is

$$\begin{aligned} r &= \sqrt{(-R\cos\theta)^2 + (-R\sin\theta)^2 + (z)^2} \\ &= \sqrt{R^2\cos^2\theta + R^2\sin^2\theta + z^2} \\ &= \sqrt{R^2\cancel{\left(\cos^2\theta + \sin^2\theta\right)} + z^2} \\ &= \sqrt{R^2 + z^2} \end{aligned}$$

Worked Example — Ring of Charge

Putting everyting together, the differential electric field is

$$d\vec{E} = \frac{k\ \left(\frac{Q\ d\theta}{2\pi}\right)}{\left(R^2+z^2\right)^{(3/2)}}\left[(-R\cos\theta)\,\hat{\boldsymbol{\imath}} + (-R\sin\theta)\,\hat{\boldsymbol{\jmath}} + (z)\hat{\boldsymbol{k}}\right]$$

Worked Example — Ring of Charge

The x and y components of the electric field are

$$dE_x = \frac{k\left(\frac{Q\ d\theta}{2\pi}\right)(-R\cos\theta)}{(R^2+z^2)^{(3/2)}}$$

and

$$dE_y = \frac{k\left(\frac{Q\ d\theta}{2\pi}\right)(-R\sin\theta)}{(R^2+z^2)^{(3/2)}}$$

respectively. Both of these vanish (integating $\sin\theta$ or $\cos\theta$ over its period gives zero). Why does this make sense physically?

Worked Example — Ring of Charge

That leaves us with

$$d\vec{E} = \frac{k \left(\frac{Q\ d\theta}{2\pi}\right) z}{\left(R^2 + z^2\right)^{(3/2)}}\ \hat{\boldsymbol{k}}$$

Integrating from 0 to 2π gives

$$\boxed{\vec{E} = \frac{k\ Q\ z}{\left(R^2 + z^2\right)^{(3/2)}}\hat{\boldsymbol{k}}} \blacktriangleleft$$

Does this look familiar, from, say the equation sheet?

Gauss's Law

Gauss's Law asserts that the net electrix flux through a closed surface is proportional to the amount of charge enclosed:

$$\text{Net Flux } = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_o}$$

Electric Flux

The most general equation for electric flux is

$$\text{Flux } = \int \vec{E} \cdot d\vec{A}$$

If the surface is closed, then the net flux through that closed surface is

$$\text{Net Flux } = \oint \vec{E} \cdot d\vec{A}$$

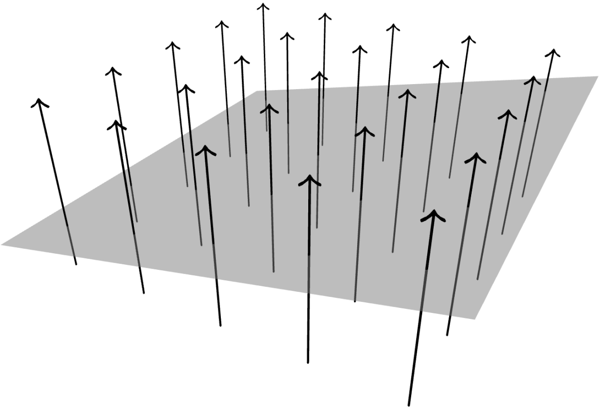
If the electric field is *uniform* and the area flat, the flux becomes

$$\text{Flux } = \vec{E} \cdot \vec{A} = EA\cos\theta$$

where θ is the angle between the electric field and the area vector.

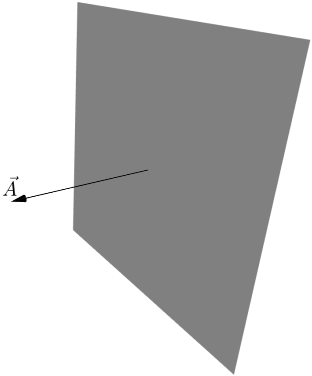
Visualizing Flux

Visually, we can think of flux as the number of field lines passing through a surface.



Area Vectors

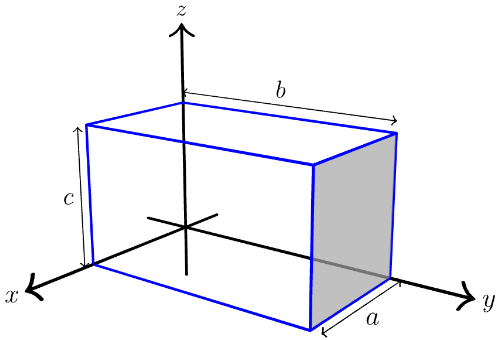
Area vectors are always perpendicular to the surface they describe.



The direction of an area vector is ambiguous for an *open* surface.

Worked Example — Rectangular Box Flux

- Suppose there's an electric field $\vec{E} = w_1 y^2\ \hat{\boldsymbol{\imath}} + w_2 z^2\ \hat{\boldsymbol{\jmath}} + w_3 x^2\ \hat{\boldsymbol{k}}$, where w_1, w_2, and w_3 are constants somewhere in space. There's also a rectangular prism sitting as shown.



Find flux through the shaded side.

Solution. Flux is given by

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

The electric field is given as $\vec{E} = w_1 y^2 \ \hat{\boldsymbol{\imath}} + w_2 z^2 \ \hat{\boldsymbol{\jmath}} + w_3 x^2 \ \hat{\boldsymbol{k}}$. The differential area vector has magnitude $dz \ dx$ (since that shaded side is parallel to the zx-plane). Additionally, since area vectors are normal to their surface, the area vector points in the $\hat{\boldsymbol{\jmath}}$-direction. As such,

$$d\vec{A} = dz \ dx \ \hat{\boldsymbol{\jmath}}$$

So the dot product is

$$\vec{E} \cdot d\vec{A} = \left(w_1 y^2 \ \hat{\boldsymbol{\imath}} + w_2 z^2 \ \hat{\boldsymbol{\jmath}} + w_3 x^2 \ \hat{\boldsymbol{k}}\right) \cdot (dz \ dx \ \hat{\boldsymbol{\jmath}}) = w_2 z^2 \ dz \ dx$$

Worked Example — Rectangular Box Flux

Integrating to find the flux,

$$\begin{aligned}\Phi_E = \int \vec{E}\cdot d\vec{A} &= \int_0^a \int_0^c w_2 z^2 \ dz \ dx \\ &= w_2 \underbrace{\left(\int_0^a dx\right)}_{a} \underbrace{\left(\int_0^c z^2 \ dz\right)}_{c^3/3} \\ &= \frac{w_2 \ a \ c^3}{3}\end{aligned}$$

Worked Example — Rectangular Box Flux

Alternatively, we could've avoided a double integral by defining the area vector as

$$d\vec{A} = a\ dz\ \hat{\boldsymbol{\jmath}}$$

All that matters is that we integrate with respect to z, since the y-component of the electric field (the only component that matters) is non-uniform with respect to z. Then the dot product becomes

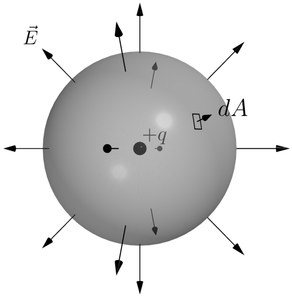
$$\vec{E} \cdot d\vec{A} = \left(w_1 y^2\ \hat{\boldsymbol{\imath}} + w_2 z^2\ \hat{\boldsymbol{\jmath}} + w_3 x^2\ \hat{\boldsymbol{k}} \right) \cdot (a\ dz\ \hat{\boldsymbol{\jmath}}) = w_2 a z^2\ dz$$

And the flux is

$$\Phi_E = \int \vec{E} \cdot d\vec{A} = \int_0^c w_2 a z^2\ dz = \boxed{\frac{w_2\ a\ c^3}{3}} \blacktriangleleft$$

Area Vectors — What if the Area ain't Flat?

Non-planar geometries (such as spheres and cylinders) can still be described with *infinitesimal* area vectors. What we have here, is a Gaussian surface enclosing a point charge.



The sphere can be described with infinitesimal area vectors that are *locally* normal to the infinitesimal patch of area that they describe. The outward direction is ascribed positive signage for a closed surface (so the sign of the area vectors isn't ambiguous for a closed surface).

Using Gauss's Law to Solve for Electric Fields

Gauss's law is a law of nature. It is *always* true that the net flux through a closed surface is proportional to the charge enclosed.

Sometimes, there exist certain nice symmetries (planar, spherical, and cylindrical) which allow us to use Gauss's Law to solve for the electric field.

Worked Example — Nonuniformly Charged Cylinder

- An infinitely long solid cylindrical conductor of radius R has a non-uniform charge density of $\rho = Cr^2$ with r being the distance from the center of the axis of the cylinder and C being some constant. Calculate the magnitude of the electric field a distance r from the axis of the cylinder.

Solution. Gauss's law reads

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_o}$$

Worked Example — Nonuniformly Charged Cylinder

Let's start with the right-hand side of Gauss's Law. We want the charge enclosed:

$$Q = \int dQ = \int \rho \, dV$$

We are given the charge density as $\rho = Cr^2$. As before, dV is the infinitesimal volume element.

Worked Example — Nonuniformly Charged Cylinder

The infinitesimal volume element *for a cylinder* is

$$dV = 2\pi r L \, dr$$

Where does this come from? Examine the form of the infinitesimal volume element:

$$dV = \underbrace{(2\pi r L)}_{\text{surface area}} \overbrace{\quad dr \quad}^{\substack{\text{differential displacement in radial} \\ \text{direction}}}$$

Again, imagine taking a tiny slice of surface area (like an onion peel) and extruding it outward in the radial direction by some amount dr.

Worked Example — Nonuniformly Charged Cylinder

Here's an alternative derivation:

The volume of a cylinder is

$$V = \pi r^2 L$$

Taking a derivative with respect to r recovers the surface area of the tube:

$$\frac{dV}{dr} = 2\pi r L$$

From which it follows that the infinitesimal volume element is

$$dV = 2\pi r L \; dr$$

Worked Example — Nonuniformly Charged Cylinder

Putting everything together, the total charge enclosed by the Gaussian surface of radius r is

$$\begin{aligned} Q_{\text{enc}} &= \int_0^r \overbrace{\left(Cr'^2\right)}^{\rho} \overbrace{\left(2\pi r' L \, dr'\right)}^{dV} \\ &= 2\pi LC \int_0^r r'^3 \, dr' \\ &= 2\pi LC \left(\frac{r'^4}{4}\right) \Bigg|_0^r \\ &= 2\pi LC \left(\frac{r^4}{4}\right) \end{aligned}$$

Worked Example — Nonuniformly Charged Cylinder

Gauss's Law reads

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\rm enc}}{\epsilon_o}$$

We already took care of the right-hand side. The left-hand side of Gauss's law is a surface integral over the *Gaussian surface.* Can we play our symmetry tricks?

Worked Example — Nonuniformly Charged Cylinder

Yes! We can use our symmetry tricks! Why? Because trix are for kids.

Note that $\vec{E}$ is parallel to $d\vec{A}$. Then $\vec{E} \cdot d\vec{A} = E\,dA$ and we have

$$\oint \vec{E} \cdot d\vec{A} = \oint E\,dA$$

Observe that the electric field is *uniform* on the Gaussian surface. Then

$$\oint E\,dA = E \oint dA = E \underbrace{(2\pi r L)}_{\text{surface area}}$$

Worked Example — Nonuniformly Charged Cylinder

Observe that our charged cylinder is *infinite* in length, but our Gaussian cylinder has length L. So then why don't we consider the caps of the cylinder when evaluating the left-hand side of Gauss's Law?

Recall that the electric field points radially outward from the cylinder. The caps of the Gaussian cylinder are normal to the axis of the cylinder. So the electric field is perpendicular to the area vectors and

$$\text{Flux } = \int \vec{E} \cdot d\vec{A} = 0$$

So the total electric flux for the cylinder is

$$\oint \vec{E} \cdot d\vec{A} = \cancelto{0}{\int_{\text{top}} \vec{E} \cdot d\vec{A}} + \int_{\text{tube}} \vec{E} \cdot d\vec{A} + \cancelto{0}{\int_{\text{bottom}} \vec{E} \cdot d\vec{A}}$$

Worked Example — Nonuniformly Charged Cylinder

Putting everything together gives us

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_o}$$

$$E\left(\cancel{2\pi} r \cancel{L}\right) = \frac{\cancel{2\pi} C \cancel{L} r^4}{4\epsilon_o}$$

$$\boxed{E = \frac{Cr^3}{4\epsilon_o}} \blacktriangleleft$$

Electric Potential

The electric potential V tells us how much potential energy something has per unit charge:

$$U = qV \implies \boxed{\Delta U = q\Delta V}$$

For a pair of point charges, the potential energy is

$$U = \frac{k\ q_1 q_2}{r}$$

This implies that the voltage set up by a single point charge is

$$V = \frac{k\ q}{r}$$

Electric Potential for Continuous Charge Distributions

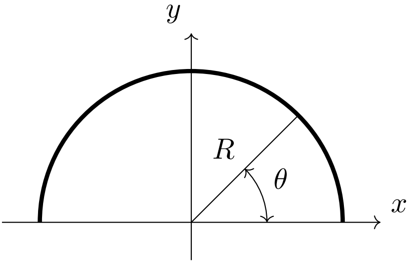
Voltages obey the superposition principle:

$$V = \sum_{i=1}^{N} \frac{k\ q_i}{r_i} \quad \Longleftrightarrow \quad V = \int \frac{k\ dQ}{r}$$

That is, we can find the voltage created by a charged object by paritioning it into infinitesimal charges dQ and integrating.

Worked Example — Potential of a Nonuniform Arc

▶ A semi-circular arc with radius R has a non-uniform linear charge density given by $\lambda = a \sin\theta$, where a is some positive constant and θ is measured from the positive x-axis.



Assuming the potential is zero at infinity, what is the total electric potential at the origin due to this charge distribution?

Solution. The total electric potential at the origin can be found by integrating:

$$V = \int \frac{k\ dQ}{r}$$

We have a one-dimensional charge distribution, so

$$dQ = \lambda\ d\ell = \overbrace{(a \sin\theta)}^{\lambda}\overbrace{(R\ d\theta)}^{d\ell}$$

The magnitude of the $\vec{r}$-vector that appears in the integral above is simply R (since that's the distance from the semicircle of charge to the origin).

Worked Example — Potential of a Nonuniform Arc

Putting everything together gives us

$$V = \int \frac{k\ dQ}{r} = \int_0^{\pi} \frac{k\ \overbrace{(a\ \sin\theta)\,(R\ d\theta)}^{dQ}}{R}$$

$$= ka \int_0^{\pi} \sin\theta\ d\theta$$

$$= \boxed{2ka} \blacktriangleleft$$

Relationship Between Voltage and Field

Electric potential and electric field are related by a derivative:

$$E_x = -\frac{dV}{dx}$$

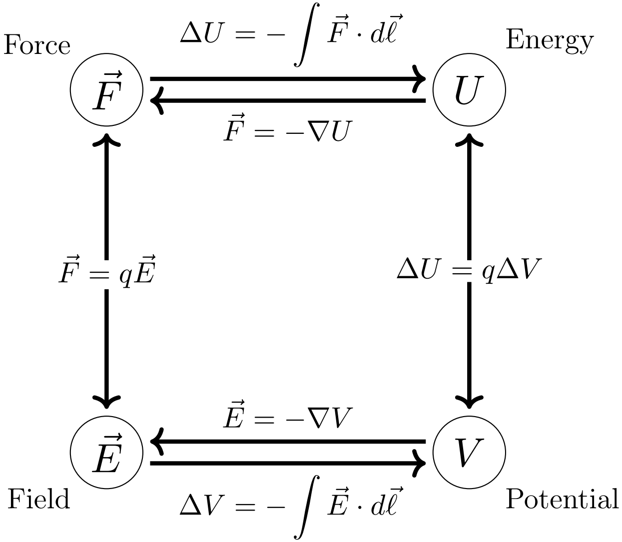
More generally,

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{\boldsymbol{\imath}} - \frac{\partial V}{\partial y}\hat{\boldsymbol{\jmath}} - \frac{\partial V}{\partial z}\hat{\boldsymbol{k}}$$

For example, we know the potential of a point charge is $V = \frac{kq}{r}$. So the radial component of the electric field is

$$E_r = -\frac{\partial V}{\partial r} = -\left(-\frac{k\ q}{r^2}\right) = \frac{k\ q}{r^2}$$

General Relationships Among Energy, Potential, Field, and Force



Worked Example — Field From Potential

▶ Suppose the electric potential in some region of space is given as $V(x, y, z) = 2.0x^5y^3 + 4.0e^{5z}$. What is the z-component of the electric field?

Solution. Recall that

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{\boldsymbol{\imath}} - \frac{\partial V}{\partial y}\hat{\boldsymbol{\jmath}} - \frac{\partial V}{\partial z}\hat{\boldsymbol{k}}$$

So the z-component of the electric field is

$$E_z = -\frac{\partial V}{\partial z} = -20.0e^{5z}$$

evaluated at the appropriate location. ◀

Visualizing Electric Fields and Potentials

Here's a term that's used all the time in electrostatics: *equipotentials*. Recall that an equipotential line denotes a line where voltage is constant (much like contour lines on a topographical map).

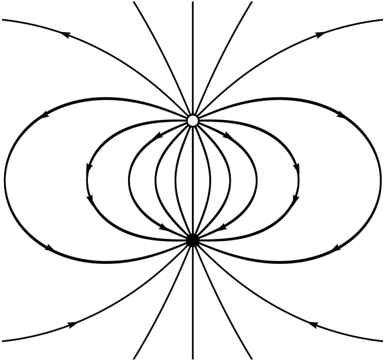
There's a phrase that helps me out a ton when thinking about potential and fields:

Electric field lines point downhill with respect to voltage.

But that's a little too abstract. Let's see what that means in real life.

Electric Field and Potential Visualized

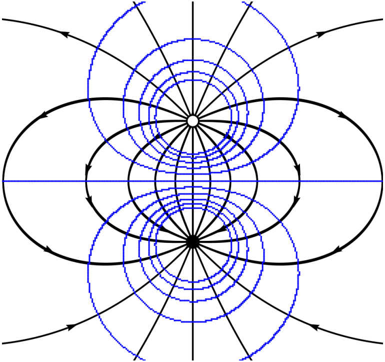
Suppose a have a *dipole*. (This is nothing more than a positive and negative point charge separated some distance.)



This dipole system will make a field like that depicted above. But what about the potential? What does that look like?

Electric Field and Potential Visualized

The equipotential lines are *normal* to the electric field lines.



We can think of the positive charge kind of like a hill, and the negative charge kind of like a valley. Now observe how the electric field lines flow *downhill* with respect to the equipotential lines. (This is the negative sign in the equation $\vec{E} = -\nabla V$.)