Modified Broyden method:

- Valid for $\omega_{-1} \ll 1$, $\omega_n \ll 1 \; \forall \, n < m$

$$|n^{m+1}\rangle = |n^m\rangle + G^0 |F^m\rangle - \sum_{n=0}^{m-1} \omega_n \gamma_{mn} |u^n\rangle$$

where

$$\gamma_{ml} = \sum_{k=0}^{m-1} c_{mk} \beta_{kl}$$

$$c_{mk} = \omega_k \langle \Delta F^k | F^m \rangle$$

$$\beta_{kn} = \left[\omega_{-1}^2 \mathbb{1} + A \right]^{-1}_{kn}$$

$$\cancel{A_{ij} = \omega_i \omega_j \langle \Delta F^j | \Delta F^i \rangle}$$

$$A_{ij} = \omega_i \omega_j \langle \Delta F^j | \Delta F^i \rangle$$

$$|u^n\rangle = G^0 |\Delta F^n\rangle + |\Delta n^n\rangle$$

$|\Delta n$

iter = 0 ; $|n^0\rangle \longrightarrow |n^0_{out}\rangle$

$$|F^0\rangle = |n^0_{out}\rangle - |n^0\rangle$$

$$|n'\rangle = |n^0\rangle + G^0|F^0\rangle$$

$|\Delta n\rangle = |n'\rangle - |n^0\rangle$

iter = 1 ; $|n'\rangle \longrightarrow |n^1_{out}\rangle$

$$|F'\rangle = |n'_{out}\rangle - |n'\rangle$$

$$|\Delta F^0\rangle = \frac{|F'\rangle - |F^0\rangle}{\left||F'\rangle - |F^0\rangle\right|}$$

sqrt$(\langle x|b\rangle)$

$$|\Delta n^0\rangle = \frac{\Delta n^0}{\left||F'\rangle - |F^0\rangle\right|}$$

$$|u^0\rangle = G^0|\Delta F^0\rangle + |\Delta n^0\rangle$$

$$A_{00} = w_0 w_0 \langle \Delta F^0|\Delta F^0\rangle$$

$$= \beta_{1\times 1} = \left[w_{-1}^2 \mathbb{1}_{1\times 1} + A_{1\times 1}\right]^{-1}$$

$$C_{10} = w_0 \langle \Delta F^0|F^1\rangle$$

$$\gamma_{00} = C_{00}\beta_{00}$$

$$|n^2\rangle = |n'\rangle + G^0|F'\rangle - w_0\gamma_{00}|u^0\rangle$$

$$|\Delta n'\rangle = |n^2\rangle - |n'\rangle$$

$$iter = 2 ; |n^2\rangle \longrightarrow |n^2_{out}\rangle$$

$$|F^2\rangle = |n^2_{out}\rangle - |n^2\rangle$$

$$|\Delta F^1\rangle = \frac{|F^2\rangle - |F^1\rangle}{\left| |F^2\rangle - |F^1\rangle \right|}$$

$$|\Delta n^1\rangle = \frac{|\Delta n^1\rangle}{\left| |F^2\rangle - |F^1\rangle \right|}$$

$$|u^1\rangle = G^0 |\Delta F^1\rangle + |\Delta n^1\rangle$$

A.resize (2x2)

$$A_{11} = \omega_1 \omega_1 \langle \Delta F^1 | \Delta F^1 \rangle$$

$$A_{01} = \omega_0 \omega_1 \langle \Delta F^1 | \Delta F^0 \rangle ; \quad A_{10} = \omega_1 \omega_0 \langle \Delta F^0 | \Delta F^1 \rangle$$

$$\beta_{2\times 2} = \left[\omega_{-1}^2 \mathbb{1}_{2\times 2} + A_{2\times 2} \right]^{-1}$$

$$C_{20} = \omega_0 \langle \Delta F^0 | F^2 \rangle$$

$$C_{21} = \omega_1 \langle \Delta F^1 | F^2 \rangle$$

$$\gamma_{2l} = \sum_{k=0}^{1} C_{2k} \beta_{kl} \quad ; l \in [0,1]$$

$$|n^3\rangle = |n^2\rangle + G^0 |F^2\rangle - \sum_{n=0}^{1} \omega_n \gamma_{2n} |u^n\rangle$$

$$|\Delta n^2\rangle = |n^3\rangle - |n^2\rangle$$