$$L_P = \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{m}\xi_i + \left(-\sum_{i=1}^{m}\alpha_i\left(y^{(i)}(w^T x^{(i)} + w_0) - (1-\xi_i)\right)\right) - \sum_{i=1}^{m}\beta_i\xi_i \quad \text{— (1)}$$

$$\min_{w w_0}\ \max_{\alpha_i \beta_i}\ L_P = \max_{\alpha_i \beta_i}\ \min_{w w_0} L_P$$

$$\frac{\partial L_P}{\partial w_0} = 0 = -\sum_{i=1}^{m}\alpha_i y^{(i)} \Rightarrow \boxed{\sum_{i=1}^{m}\alpha_i y^{(i)} = 0} \quad \text{— (1)}$$

$$\frac{\partial L_P}{\partial w} = 0 = \frac{1}{\cancel{2}}\cancel{2}w - \sum_{i=1}^{m}\alpha_i y^{(i)} x^{(i)} = 0$$

$$\boxed{w = \sum_{i=1}^{m}\alpha_i y^{(i)} x^{(i)}} \quad \text{— (2)}$$

$$\frac{\partial L_P}{\partial \xi_i} = \cancel{C - \alpha_i - \beta_i} \quad \text{— (3)}$$

$$\boxed{C - \alpha_i - \beta_i = 0}$$

putting it in L_P

$$L_P = \frac{1}{2}\sum_{i=1}^{m}\alpha_i y^{(i)} x^{(i)T}\sum_{j=1}^{m}\alpha_j y^{(j)} x^{(j)}$$

$$-\sum_{i=1}^{m}\alpha_i\left(y^{(i)}\sum_{j=1}^{m}\alpha_j y^{(j)} x^{(j)T} x^{(i)}\right)$$

$$\cancel{-\sum_{i=1}^{m}\alpha_i w_0 y^{(i)}} + \sum_{i=1}^{m}\alpha_i - \sum_{i=1}^{m}\alpha_i\xi_i$$

$$-\sum \beta_i\xi_i + C\sum \xi_i$$

zero

$$L_D = -\frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_i \alpha_j y^{(i)} y^{(j)} x^{(i)T} x^{(j)}$$

$$+ \sum_{i=1}^{m} \alpha_i$$

max L_D

s.t $\alpha_i \geq 0$ $\quad \sum_{i=1}^{m} \alpha_i y^{(i)} = 0$

$C - \alpha_i - \beta_i = 0$

$\beta_i \geq 0$

from here

$\alpha_i \geq 0$, $\quad \beta_i = C - \alpha_i \geq 0$

$= C \geq \alpha_i$

$0 \leq \alpha_i \leq C$

$$\boxed{\begin{array}{l} \max L_D \\ \text{s.t } \sum_{i=1}^{m} \alpha_i y^{(i)} = 0 \\ 0 \leq \alpha_i \leq C \end{array}}$$

→ diff with hard margin