ASSIGNMENT 3

3. If μ= 55, σ_{4a}= 4, $\sigma_{4\beta}$=10, σ_4 =c 15, In this which is better?

1.Introduction

Statistics is used to analyse data and make meaningful decisions. When comparing different datasets that have the same mean (average), we cannot judge them only by the mean. We must also examine how spread out the data values are.

This spread is measured using standard deviation (σ).

Given Information:

- Mean (μ) = 55
- $\sigma_a = 4$
- $\sigma_\beta = 10$
- $\sigma c = 15$

All three distributions have the same mean (55).
The only difference between them is their **standard deviation.**

The objective of this assignment is to clearly determine which distribution is better and explain why with proper reasoning.

2. Understanding Mean (μ)

- **Definition**
  The mean is the average value of a dataset.
  It represents the centre or balance point of the distribution.
- **Formula:**

$$\mu = \frac{\sum X}{N}$$

  Where:

  ΣX = Sum of all observations

  N = Number of observations

- **Interpretation in This Case**
  Since: $\mu = 55$
  All three distributions are cantered at 55.
- **This means:**
  - ✓ The middle value is the same
  - ✓ The central tendency is identical
  - ✓ The difference lies only in the spread

3.Understanding Standard Deviation (σ)

- **Definition**

  Standard deviation measures how far data values are from the mean.

  It shows the degree of variation or dispersion.

- **Formula**

$$\sigma = \sqrt{\frac{\sum(X - \mu)^2}{N}}$$

This formula calculates:

1. The difference between each value and the mean
2. Squares those differences
3. Finds the average
4. Takes the square root

- **Meaning of Standard Deviation**

  Small σ → Values are very close to mean

  Large σ → Values are widely scattered

4.Concept of Variability and Consistency

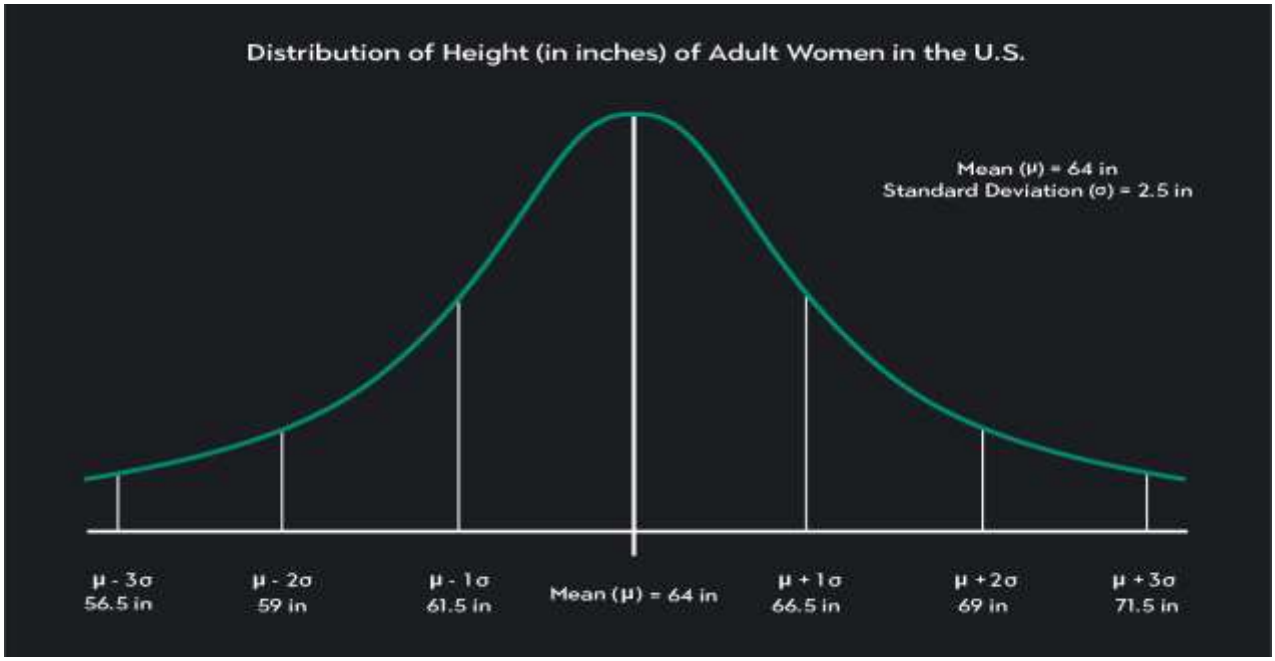
Variability shows how consistent the data is.

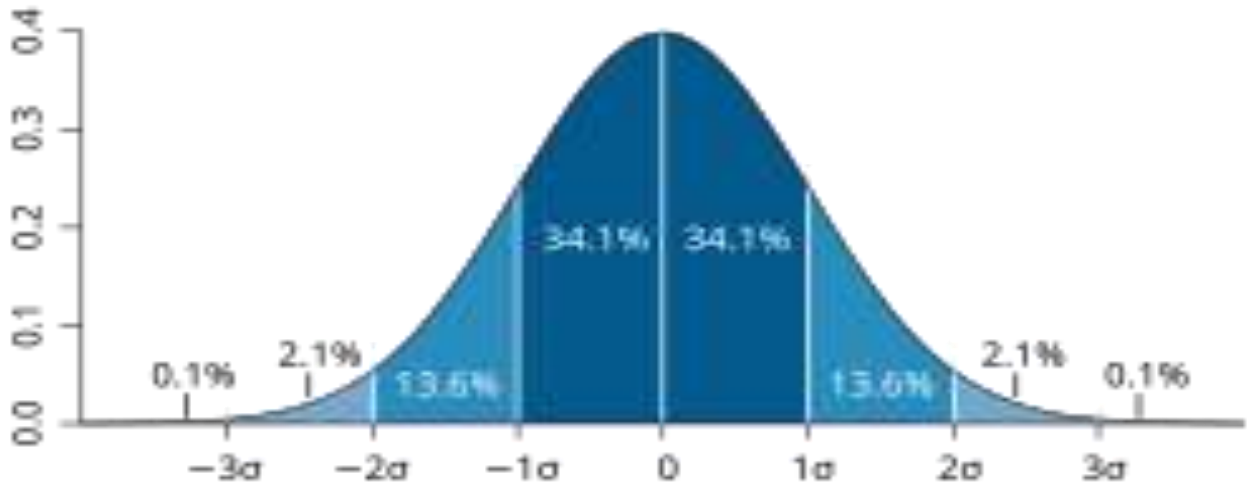
Standard Deviation	Variability Level	Consistency Level	Reliability
$\sigma_a = 4$	Very Low	Very High	Excellent
$\sigma_\beta = 10$	Moderate	Moderate	Average
$\sigma c = 15$	High	Low	Poor

Important Principle:

Smaller variation = Higher consistency = Better performance

5. Visual Representation (Normal Distribution Curves)





a. Distribution A (σ = 4)

- Curve is narrow and tall
- Most values lie very close to 55
- Very little fluctuation
- Highly consistent

Meaning: Results are stable and predictable.

b. Distribution B (σ = 10)

- Curve is moderately wide
- Values are moderately spread
- Acceptable level of variation

Meaning: Average consistency.

c. Distribution C (σ = 15)

- Curve is wide and flat
- Values are spread far from mean
- Large variation

Meaning: Unstable and less predictable results.

6. Empirical Rule (68–95–99.7 Rule)

This rule applies to normal distribution.

It states:

- 68% of data lies within ±1σ
- 95% lies within ±2σ
- 99.7% lies within ±3σ

For $\sigma_a = 4$

- 68% → 55 ± 4 → (51, 59)
- 95% → 55 ± 8 → (47, 63)
- 99.7% → 55 ± 12 → (43, 67)

Data remains very close to mean.

For $\sigma_\beta = 10$

- 68% → (45, 65)
- 95% → (35, 75)
- 99.7% → (25, 85)

Moderate spread.

For $\sigma_C = 15$

- 68% → (40, 70)
- 95% → (25, 85)
- 99.7% → (10, 100)

Very wide range**.**

Observation:

- As σ increases, range increases.
- More spread means less stability.

7.Step-by-Step Logical Comparison

Step 1: Compare Means

All are equal → No difference in centre.

Step 2: Compare Standard Deviations

$$4 < 10 < 15$$

Step 3: Analyse Spread

- σ = 4 → Smallest spread
- σ = 10 → Moderate spread
- σ = 15 → Largest spread

Step 4: Decide Based on Statistical Principle

Smaller standard deviation indicates:

- Less risk
- More control
- More precision
- Better quality

8. Real-Life Application Example

Imagine three factories producing bolts of length 55 mm.

- Factory A (σ = 4) → Almost all bolts are very close to 55 mm
- Factory B (σ = 10) → Some bolts vary moderately
- Factory C (σ = 15) → Large variation in bolt sizes

Which factory is better?

Factory A, because products are consistent.

9.Comparison Summary Table

Distribution	Mean	Standard Deviation	Spread	Stability	Quality
A	55	4	Very Small	Very High	Best
B	55	10	Medium	Moderate	Average
C	55	15	Large	Low	Poor

10.Mathematical Justification

Given:

$$\mu = 55$$
$$\sigma_A = 4$$
$$\sigma_B = 10$$
$$\sigma_C = 15$$

Since:

$$4 < 10 < 15$$

The smallest standard deviation gives the most concentrated data around the mean.

11.Conclusion

In this comparison, all three distributions have the same mean ($\mu = 55$), which means their central value is identical. Therefore, the decision depends entirely on the standard deviation, which measures the spread or variability of the data.

Among the three values (**$\sigma = 4, 10,$ and 15**), the smallest standard deviation is 4. A smaller standard deviation indicates that the data points are closely clustered around the mean, showing higher consistency, stability, and reliability. As the standard deviation increases from 4 to 15, the spread of the data becomes wider, leading to greater variability and less predictability.

Since **$4 < 10 < 15$**, the distribution with $\sigma = 4$ has the least variation and the highest level of consistency. Therefore, Distribution A ($\sigma = 4$) is the best distribution, while $\sigma = 10$ is moderate and $\sigma = 15$ is the least consistent.