

Master Worky theorem.

$$T(n) \equiv aT\left(\frac{n}{b}\right) + f(n) \quad T(1) = c$$

Definition

$$T(n) = \begin{cases} O(n^d) & \text{if } a < b^d \\ O(n^d \log n) & \text{if } a = b^d \\ O(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

$$1) \quad T(n) \equiv 9T\left(\frac{n}{3}\right) + O(n^2)$$

$$a \equiv 9 \quad b \equiv 3 \quad d \equiv 2$$

$$\text{Case 2: } a = b^d \quad (9 = 3^2)$$

$$\therefore O(n^2 \log n)$$

$$2) \quad T(n) \equiv 2T\left(\frac{n}{4}\right) + O(n)$$

$$a \equiv 2 \quad b \equiv 4 \quad d \equiv 1$$

$$\text{Case 1: } a < b^d \quad (2 < 4^1)$$

$$\therefore O(n)$$

$$3) \quad T(n) \equiv 2T\left(\frac{n}{4}\right) + O(1)$$

$$a \equiv 2 \quad b \equiv 4 \quad d \equiv 0$$

$$\text{Case 3: } a > b^d \quad (2 > 4^0)$$

$$\therefore O(\sqrt{n})$$