# Optimization for Transportation Systems
# (M.Sc. Transportation Systems)

## Problem Set 1
## (SoSe 2024)

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# PROBLEM 1

**(a)** To maximize the total revenue, the following optimization model has been adopted:

**Decision Variables:**

- $x_A, x_B, x_C, x_D, x_E, x_F, x_G, x_H$ : The number of bicycles of types A, B, C, D, E, F, G, and H to be produced, respectively.
- $y_A, y_B, y_C, y_D, y_E, y_F, y_G, y_H$ : Binary variables indicating whether that type of bicycle is chosen for production or not

**Objective Function:**

The objective function is to maximize the total profit from selling different bicycle types. It is modeled as follows:

**Total Profit:**

**Maximize** $45x_A + 100x_B + 65x_C + 80x_D + 120x_E + 95x_F + 70x_G + 30x_H$

**Constraints:**

1. **Total Working Hour Constraint:**
   $20.5x_A + 13.5x_B + 15x_C + 16.5x_D + 24x_E + 32x_F + 25.5x_G + 19x_H \leq 200$
2. **Total Steel Mass Constraint:**
   $14.5x_A + 22x_B + 13x_C + 8x_D + 16.5x_E + 9.5x_F + 21x_G + 18.5x_H \leq 770$
3. **Total Rental Cost Constraint:**
   $1000y_A + 1200y_B + 2000y_C + 1400y_D + 2500y_E + 2100y_F + 1800y_G + 1100y_H \leq 3500$

**Additional Constraints:**

To link the two decision variables, the following equation is used:

$$x_i \leq 1000y_i$$

Where "i" is all the possible types of bicycles (A, B, C, D, E, F, G, and H).

$$y_i \leq x_i$$

(if y is 1, this indicates that at least 1 bicycle of that "i" is produced)

Variable Bounds:

The decision variables are bound as follows:

$0 \leq x_i$

$x_i \in Z, \forall i = A, B, C, D, E, F, G, H$

$0 \leq y_A, y_B, y_C, y_D, y_E, y_F, y_G, y_H \leq 1$

$y_i \in \{0,1\}, \forall i = A, B, C, D, E, F, G, H$

**Result:**

Using the above equations and using the PULP library in Python and solver CBC, we get the following output for the optimization problem:

```
Status: Optimal
Optimal production plan:
Type A: Produce 0.0 bicycles, Chosen: 0.0
Type B: Produce 14.0 bicycles, Chosen: 1.0
Type C: Produce 0.0 bicycles, Chosen: 0.0
Type D: Produce 0.0 bicycles, Chosen: 0.0
Type E: Produce 0.0 bicycles, Chosen: 0.0
Type F: Produce 0.0 bicycles, Chosen: 0.0
Type G: Produce 0.0 bicycles, Chosen: 0.0
Type H: Produce 0.0 bicycles, Chosen: 0.0
Total Profit = 1400.0
Total working hours used: 189.0 (<= 200)
Total steel mass used: 308.0 (<= 770)
Total rental costs: 1200.0 (<= 3500)
```

### (b.1)

To incorporate the condition that a maximum 3 types of bicycles can be produced only, while types C and E must be produced in any conditions, we use the following additional constraints, keeping all others same:

**Additional Constraints:**

- $y_A + y_B + y_C + y_D + y_E + y_F + y_G + y_H \leq 3$ (this enforces that a maximum 3 types of bicycles can be produced)
- $y_C + y_E = 2$ (this equation enforces that both bicycle types C and E MUST be produced)

**Results:**

Using the above equations and using the PULP library in Python and solver CBC, we get the following output for the optimization problem:

```
Status: Infeasible
No optimal solution found.
```

This is primarily because if C and E are produced, the optimization model fails in the **Total Rental Cost Constraint** since rental costs for C and E are 4500(>3500)

### (b.2)

To incorporate either the renting budget (**Total Rental Cost Constraint**) or steel mass constraint (**Total Steel Mass Constraint**), we introduce a new dummy **binary** variable **z.** Simultaneously, the updated **Total Rental Cost Constraint** and **Total Steel Mass Constraint** becomes:

**New Decision Variables:**

$\boldsymbol{z}$ **:** Either_or_variable

$\boldsymbol{z} \in \{\boldsymbol{0}, \boldsymbol{1}\}$

**New Total Steel Mass Constraint:**

$$14.5x_A + 22x_B + 13x_C + 8x_D + 16.5x_E + 9.5x_F + 21x_G + 18.5x_H \leq 770 + (\boldsymbol{1} - \boldsymbol{z}) * \boldsymbol{100000}$$

**New Total Rental Cost Constraint:**

$$1000y_A + 1200y_B + 2000y_C + 1400y_D + 2500y_E + 2100y_F + 1800y_G + 1100y_H \leq 3500 + (\boldsymbol{z}) * \boldsymbol{100000}$$

**Results:**

Using the above equations and using the PULP library in Python and solver CBC, we get the following output for the optimization problem:

```
Status: Optimal
Optimal production plan:
Type A: Produce 0.0 bicycles, Chosen: 0.0
Type B: Produce 13.0 bicycles, Chosen: 1.0
Type C: Produce 0.0 bicycles, Chosen: 0.0
Type D: Produce 0.0 bicycles, Chosen: 0.0
Type E: Produce 1.0 bicycles, Chosen: 1.0
Type F: Produce 0.0 bicycles, Chosen: 0.0
Type G: Produce 0.0 bicycles, Chosen: 0.0
Type H: Produce 0.0 bicycles, Chosen: 0.0
Total Profit = 1420.0
Total working hours used: 199.5 (<= 200)
Total steel mass used: 302.5 (<= 770)
Total rental costs: 3700.0 (<= 3500)
```

The result shows that the **New Total Steel Mass Constraint** has been **fulfilled**, while the **New Total Rental Cost Constraint** has been **discarded** while maximizing revenue.

## (b.3)

To incorporate the condition that "If bicycles of type F are produced, then bicycles of type A and G must be produced, and the type B must not be produced," we adopt the following additional constraints:

**Additional Constraints:**

- $y_A + y_G \geq 2 * y_F$ (this implies that if F is produced, A and G are produced)
- $y_F + y_B \leq 1$ (F produced implies B "NOT" produced but not vice versa)

**Results:**

Using the above equations and using the PULP library in Python and solver CBC, we get the following output for the optimization problem:

```
Status: Optimal
Optimal production plan:
Type A: Produce 0.0 bicycles, Chosen: 0.0
Type B: Produce 14.0 bicycles, Chosen: 1.0
Type C: Produce 0.0 bicycles, Chosen: 0.0
Type D: Produce 0.0 bicycles, Chosen: 0.0
Type E: Produce 0.0 bicycles, Chosen: 0.0
Type F: Produce 0.0 bicycles, Chosen: 0.0
Type G: Produce 0.0 bicycles, Chosen: 0.0
Type H: Produce 0.0 bicycles, Chosen: 0.0
Total Profit = 1400.0
Total working hours used: 189.0 (<= 200)
Total steel mass used: 308.0 (<= 770)
Total rental costs: 1200.0 (<= 3500)
```

When bicycle type F is produced, bicycle types A and G are also produced, increasing the total rental cost to 4900. This violates the **Total Rental Cost Constraint.** Hence, the model only chooses type B for production.

## (b.4)

To incorporate the condition that "If bicycles of types A and H are produced, then bicycles of type C can be produced with 20% shorter working hours while selling profit of bicycles type H can be 20% higher," we adopt three new decision variables **m,w and z**. Additionally,we make changes in the **Objective Function**, **Total Working Hour Constraint** and introduce **Additional Constraints.** These are as follows:

**New Decision Variables:**

**m**, which is an Indicator variable for both bicycle types A and H produced. "**m**" is Binary.
$m \in \{0,1\}$
**w**, which is an auxiliary variable having the equation $\boldsymbol{w = y_A * y_H * x_C}$
$w \geq 0$ and $w \in Z$
**z**, which is an auxiliary variable having the equation $\boldsymbol{z = y_A * y_H * x_H}$
$z \geq 0$ and $z \in Z$

**New Objective Function:**
**Maximize** $45x_A + 100x_B + 65x_C + 80x_D + 120x_E + 95x_F + 70x_G + 30x_H + \boldsymbol{0.2 * 30 * z}$

**New Total Working Hour Constraint:**
$20.5x_A + 13.5x_B + 15x_C + 16.5x_D + 24x_E + 32x_F + 25.5x_G + 19x_H - \boldsymbol{0.2 * 15 * w} \leq 200$

**Additional Constraints:**

To linearize the equation $\boldsymbol{m} = \boldsymbol{y_A} * \boldsymbol{y_H}$, we add the following constraints

- $y_A + y_H \leq m + 1$
- $m \leq y_A$
- $m \leq y_H$

  This implies that if bicycle types A and H are produced, m is 1, else 0.

To linearize the equation $\boldsymbol{w} = \boldsymbol{m} * \boldsymbol{x_C}$ , we add the following constraints

- $w \leq x_C - 1000 * (1 - m)$ (this implies that when m=1 [i.e., A and H are BOTH produced], w =$x_C$ else when m =0 [i.e., A or H or None are produced] w is negative)
- $w \geq 0$

To linearize the equation $\boldsymbol{z} = \boldsymbol{m} * \boldsymbol{x_H}$ , we add the following constraints

- $z \leq x_H - 1000 * (1 - m)$ (this implies that when m=1 [i.e. A and H are BOTH produced], z =$x_H$ , else when m =0 [i.e., A or H or None are produced] z is negative)
- $z \geq 0$

**Results:**

Using the above equations and using the PULP library in Python and solver CBC, we get the following output for the optimization problem:

```
Status: Optimal
Optimal production plan:
Type A: Produce 1.0 bicycles, Chosen: 1.0
Type B: Produce 11.0 bicycles, Chosen: 1.0
Type C: Produce 0.0 bicycles, Chosen: 0.0
Type D: Produce 0.0 bicycles, Chosen: 0.0
Type E: Produce 0.0 bicycles, Chosen: 0.0
Type F: Produce 0.0 bicycles, Chosen: 0.0
Type G: Produce 0.0 bicycles, Chosen: 0.0
Type H: Produce 1.0 bicycles, Chosen: 1.0
Total Profit = 1181.0
Total working hours used: 188.0 (<= 200)
Total steel mass used: 275.0 (<= 770)
Total rental costs: 3300.0 (<= 3500)
```

**(b.5)**

To incorporate the condition that ". If bicycles of types B and G are produced, then minimum one of the types C and F should be produced, however, total personnel requirements should not exceed 180 hours, then" we adopt a new decision variable "p" along with additional constraints and also change the working hours constraint. These are as follows:

**New Decision Variables:**

**p**, which is an indicator variable for both bicycle types B and G produced. "p" is **Binary**.

$$p \in \{0,1\}$$

**New Total Working Hour Constraint:**

$$20.5x_A + 13.5x_B + 15x_C + 16.5x_D + 24x_E + 32x_F + 25.5x_G + 19x_H \leq \mathbf{200 - 20} * \boldsymbol{p}$$

**Additional Constraints:**

- $y_C + y_F \geq y_B + y_G - 1$ (this enforces that when BOTH bicycle types B and G are made, then a <u>minimum</u> of one type of bicycle C and F are produced)

To linearize the equation $\boldsymbol{p} = \boldsymbol{y_B} * \boldsymbol{y_G}$, we add the following constraints

- $y_B + y_G \leq p + 1$
- $p \leq y_B$
- $p \leq y_G$
  This implies that if bicycle types B and G are produced, p is 1, else 0.

**Results:**

Using the above equations and using the PULP library in Python and solver CBC, we get the following output for the optimization problem:

```
Status: Optimal
Optimal production plan:
Type A: Produce 0.0 bicycles, Chosen: 0.0
Type B: Produce 14.0 bicycles, Chosen: 1.0
Type C: Produce 0.0 bicycles, Chosen: 0.0
Type D: Produce 0.0 bicycles, Chosen: 0.0
Type E: Produce 0.0 bicycles, Chosen: 0.0
Type F: Produce 0.0 bicycles, Chosen: 0.0
Type G: Produce 0.0 bicycles, Chosen: 0.0
Type H: Produce 0.0 bicycles, Chosen: 0.0
Total Profit = 1400.0
Total working hours used: 189.0 (<= 200)
Total steel mass used: 308.0 (<= 770)
Total rental costs: 1200.0 (<= 3500)
```

With the condition that "if bicycle types B and G are produced, at least one of types C or F must also be produced," the total rental cost increases to either 5000 or 5100, respectively. This violates the Total Rental Cost Constraint. Consequently, the model only selects type B for production.