Савин Никита 492

W1

$$E_{x,y} E_{x^e} (y - a_{x^e}(x))^2 = E_{x,y} E_{x^e} (y - E(y|x) + E(y|x) - a_{x^e}(x))^2 = E_{x,y} E_{x^e} ((y - E(y|x))^2 + (E(y|x) - a_{x^e}(x))^2 + 2(y - E(y|x))(E(y|x) - a_{x^e}(x)))$$

$$= E_{x,y} E_{x^e} (y - E(y|x))^2 + E_{x,y} E_{x^e} (E(y|x) - E_{x^e} a_{x^e} + E_{x^e} a_{x^e} - a_{x^e})^2 + \cancel{2E_{x,y} E_{x^e} ((y - E(y|x))(E(y|x) - a_{x^e}(x)))} = 0$$

$$= E_{x,y} ((y - E(y|x))^2) + E_{x,y} E_{x^e} (E_{x^e} a_{x^e} - E(y|x))^2 + E_{x,y} E_{x^e} (a_{x^e} - E_{x^e} a_{x^e})^2 + 2E_{x,y} E_{x^e} (a_{x^e} - E_{x^e} a_{x^e}) \cdot (E_{x^e} a_{x^e} - E(y|x)) = \text{noise} + \text{bias} + \text{variance} + 2E_{x,y} ((E_{x^e} a_{x^e} - E(y|x)) \underbrace{E_{x^e} (-E_{x^e} a_{x^e} + a_{x^e})}_{0}) =$$

$$= \text{noise} + \text{bias} + \text{variance}$$

№2. $a(x) = \frac{1}{N}\sum_{n=1}^{N} a_n(x)$

смещение: $E_{x,y}[(E_{X^e}(\frac{1}{N}\sum_{n=1}^{N} a_{X^e}(x)) - E(y|x))^2] =$

$= E_{x,y}((\frac{1}{N}\sum_{n=1}^{N} E_{X^e}(a_{X^e}(x) - E(y|x)))^2) =$

$= E_{x,y}((E_{X^e}(a_{X^e}(x) - E(y|x)))^2) =$

$= E_{x,y}((E_{X^e}(a_{X^e}(x)) - E(y|x))^2)$.

смещение композиции совпадает со смещением одного алгоритма.

Разброс: $E_{x,y}(E_{X^e}(\frac{1}{N}\sum_{n=1}^{N} a_{X^e}(x) - E_{X^e}(\frac{1}{N}\sum_{n=1}^{N} a_{X^e}(x)))^2) = \frac{1}{N^2}\sum_{n=1}^{N}(a_{X^e} - E_{X^e}(a_{X^e}(x)))^2 +$

$+ \frac{1}{N^2}\sum_{n \neq k}((a_{X^e} - E_{X^e}(a_{X^e}(x)))(a_{X^e}(x) - E_{X^e}(a_{X^e}(x)))$

$E_{x,y}E_{X^e}[\frac{1}{N^2}\sum_{n=1}^{N}(a_{X^e} - E_{X^e}(a_{X^e}(x)))^2 +$

$+ \frac{1}{N^2}\sum_{n \neq k}((a_{X^e} - E_{X^e}(a_{X^e}(x)))(a_{X^e}(x) - E_{X^e}(a_{X^e}(x)))] =$

$= \frac{1}{N^2}E_{x,y}(E_{X^e}[\sum_{n \neq k}(a_{X^e} - Ea_{X^e})(a_{X^e} - Ea_{X^e})]) +$

$+ \frac{1}{N^2}E_{x,y}(E_{X^e}[\sum_{n=1}^{N}(a_{X^e} - Ea_{X^e})^2]) =$

$= \frac{1}{N}E_{x,y}[E_{X^e}(a_{X^e}(x) - E_{X^e}(a_{X^e}(x)))^2] +$

$+ \frac{N(N-1)}{N^2}E_{x,y}(E_{X^e}((a_{X^e} - E_{X^e}a_{X^e})(a_{X^e} - E_{X^e}a_{X^e}))) =$

$= a + b$

a – разброс базового алгоритма, дел на для комп N.

b – ковариация м/д 2 алгоритмами

№3.

$$\operatorname{corr}(\xi,\eta) = \frac{\operatorname{cov}(\xi,\eta)}{\sigma_\xi \sigma_\eta}$$

$$D(x_i; x_j) = \sigma^2; \quad \operatorname{corr}(x_i, x_j) = \rho$$

$$D\left(\frac{x_1 + \dots + x_m}{m}\right) = E\left((\dots)^2\right) - \left(E(\dots)\right)^2 =$$

$$= \frac{1}{m^2}\left(\sum_{i=1}^{m}\left(Ex_i^2\right) + m(m-1)E(x_i x_j)\right) =$$

$$- \frac{1}{m} m(Ex_i)^2 = \frac{1}{m^2}\left(m(\sigma^2 + (Ex_i)^2) + \right.$$

$$\left. + m(m-1)(\sigma^2\rho + (Ex_i)^2)\right) - (Ex_i)^2 =$$

$$= \frac{\sigma^2}{m} + \sigma^2\rho - \frac{\sigma^2\rho}{m} = \rho\sigma^2 + (1-\rho)\frac{\sigma^2}{m}.$$