1 Question 4

Due to linearity of the trace operation:

$$\mathrm{tr}(\mathcal{F}^T\mathcal{F} - \mathbf{I}_{2\times 2}) = \mathrm{tr}(\mathcal{F}^T\mathcal{F}) - \mathrm{tr}(\mathbf{I}_{2\times 2}) = \mathrm{tr}(\mathcal{F}^T\mathcal{F}) - 2 \tag{1}$$

With the definition of matrix multiplication:

$$\mathrm{tr}(\mathcal{F}^T\mathcal{F}) = \sum_i \left[\mathcal{F}^T\mathcal{F}\right]_{ii} = \sum_i \left(\sum_j \mathcal{F}^T_{ij}\mathcal{F}_{ji}\right) = \sum_{i,j} \mathcal{F}^2_{ij} \tag{2}$$

And finally employing the definition of the Frobenius norm:

$$\sum_{i,j} \mathcal{F}^2_{ij} = \|\mathcal{F}\|^2_F \tag{3}$$

We arrive at the desired result

$$\mathrm{tr}(\mathcal{F}^T\mathcal{F} - \mathbf{I}_{2\times 2}) = \|\mathcal{F}\|^2_F - 2 \tag{4}$$