Birthday Fairy Scheduling Assistant

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Motivation

The Birthday Fairy brings joy to all, deciding when desserts will be had and who shall prepare them. But God help the Birthday Fairy if there's a dessert drought. People need cake, and if forced to go without, Birthday Fairy heads will roll.

The model here tries to keep Birthday Fairy heads in place by minimizing the number of consecutive dessert-less soups.

Model Notation

Parameters

- K: set of soup dates
- P: set of participants
- D_j: set of acceptable dates for the birthday celebration for participant $j \in P$, which is assumed to mean "somewhere in the vicinity of either their birthday or half-birthday"

Decision variables

$$x_{ijk} = \begin{cases} 1 & \text{if participant } i \text{ is assigned to bring dessert for participant } j \text{ on date } k \\ 0 & \text{otherwise} \end{cases}$$

Indicator variables

$$w_{k,k+1} = \begin{cases} 1 & \text{if no dessert is provided on consecutive dates } k \text{ and } k+1 \\ 0 & \text{otherwise} \end{cases}$$

$$y_k = \begin{cases} 1 & \text{if dessert is provided on date } k \\ 0 & \text{otherwise} \end{cases}$$

Model Formulation

Objective function:

$$Minimize \quad \sum_{k \in K} w_{k,k+1} \tag{1}$$

Subject to:

$$\sum_{j \in P, j \neq i} \sum_{k \in K} x_{ijk} = 1 \qquad \forall i \in P \tag{2}$$

$$\sum_{i \in P, i \neq j} \sum_{k \in K} x_{ijk} = 1 \qquad \forall j \in P \tag{3}$$

$$\sum_{i \in P, i \neq j} \sum_{k \in K \setminus D_j} x_{ijk} = 0 \qquad \forall j \in P \tag{4}$$

$$\sum_{i \in P} \sum_{j \in P, j \neq i} x_{ijk} \geq y_k \qquad \forall k \in K \tag{5}$$

$$1 - y_k - y_{k+1} \leq w_{k,k+1} \qquad \forall k \in K \tag{6}$$

$$x_{ijk}, y_k, w_{k,k+1} \in \{0,1\} \qquad \forall i, j \in P, i \neq j, \forall k \in K \tag{7}$$

Model Description

Equation (1) is the objective function minimizing the number of consecutive weeks without dessert.

Equation (2) requires that each participant $i \in P$ must bring a dessert.

Equation (3) requires that each participant $j \in P$ must receive a dessert.

Equation (4) requires that each participant $j \in P$ must receive cake on a "good day" for them.

Equation (5) controls the value of the trigger variable y_k, taking value 0 if no desserts are assigned to be brought on date k.

Equation (6) controls the value of the trigger variable $w_{k,k+1}$, taking value 0 if no desserts are assigned to be brought on either date k or date $k+1$.