The Effects of Climate Change on Tradeoffs in Forest Ecosystem Services

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A thesis
submitted in partial fulfillment of the
requirements for the degree of

Master of Science

University of Washington

2016

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Program Authorized to Offer Degree:
Quantitative Ecology and Resource Management

University of Washington

Abstract

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This sample dissertation is an aid to students who are attempting to format their theses with LaTeX, a sophisticated text formatter widely used by mathematicians and scientists everywhere.

- It describes the use of a specialized macro package developed specifically for thesis production at the University. The macros customize LaTeX for the correct thesis style, allowing the student to concentrate on the substance of his or her text.[1]

- It demonstrates the solutions to a variety of formatting challenges found in thesis production.

- It serves as a template for a real dissertation.

[1] See Appendix A to obtain the source to this thesis and the class file.

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GLOSSARY

CLIMATE SCENARIO: A projection of the future climate, specifically one used by the IPCC. IMPROVE. A model of future climate that makes spatial predictions of climate variables (such as...)

ECOSYSTEM SERVICE: benefits that people receive from ecosystems, divided into four categories: supporting, provisioning, regulating and cultural [5]. Examples include food, soil formation, water purification, carbon storage, recreation, and education.

IPCC: the Intergovernmental Panel on Climate Change

CLUSTER: a set of contiguous forest stands

ACKNOWLEDGMENTS

I want to thank all those that contributed to my earning this degree.

DEDICATION

To ma femme and my family

Chapter 1

ASSESSING CHANGES IN TRADEOFFS AMONG ECOSYSTEM SERVICES IN THE DESCHUTES NATIONAL FOREST

1.1 Introduction

Forests play an important role in global ecological, social, and economic processes. They provide ecosystem services such as carbon storage, purification of water and air, wildlife habitat, recreation opportunities, and generate raw materials for goods such as food and lumber [17]. In managed forests, the extent to which forests provide these services depends in part on management practices. Optimal forest management seeks to ensure the sustained provision of these ecosystem services [1].

Like other ecosystems, forests will undergo changes as a result of the changing climate. Researchers anticipate new spatial distributions of tree species [34], increased sediment delivery to streams [30], and increasing disturbance regimes such as wildfires, drought, and insect infestation [60]. As this transformation occurs, forests' ability to provide ecosystem services will change. New growing conditions will alter timber production [25]. Novel temperatures and species compositions will alter forests' habitat suitability for inhabitants [31]. Increased frequency of disturbance regimes will impact forests' ability to store carbon [7] and provide wildlife habitat [44]. Water supplies that rely on forests' filtration capabilities may be impacted by the rising sediment levels predicted by [30].

Optimal forest management must consider the effects of the changing climate, because the time scale of forest development is of the same order as that on which climate change is predicted to operate [33]. Under alternative future climates, optimal forest management will likely differ [39]. Decisions that would once have resulted in optimal achievement of ecosystem services, now under different climatic conditions, may no longer do so. Without consideration of climate change, forest management plans may restrict forests' potential to provide ecosystem services most effectively.

Studies abound in the literature that have addressed the impact of climate change on forest ecosystem services in isolation. However, because forests provide these ecosystem services in concert with one another (see, for example, [58]), it is necessary to also understand how climate impacts the tradeoffs that exist among them. How does an increase in any one service alter our ability to acquire an amount of another? Relationships such as a marginal sacrifice in one service for substantial improvement in another may no longer exist under a new climate. To better ensure the sustained provision of ecosystem services, we must understand how these tradeoffs evolve as a function of climate.

Consider the following unfortunate, hypothetical scenario. Until recently, a particular forest served as prime habitat for bird species A. The forest's manager has also always managed the forest to generate enough revenue from timber sales to break even on property ownership. A previous analysis that ignored climate change suggested that he harvest from a small area near the northern perimeter of the forest as it was most accessible and dense with merchantable timber. Now, however, climate change is quickly reducing the area of the forest that is suitable habitat for bird species A while also increasing the timber stock of other areas of the forest. Due to the area's elevation, much of bird species A's remaining habitat are those areas on the northern perimeter of the forest that have traditionally been harvested. By not simultaneously considering the ecosystem services, the forest manager is now unknowingly sacrificing large percentages of the potential habitat for bird species A in order to maintain his timber revenues. Little does he know (bc he has not the advantage of using our research and knowing the tradeoff structure) that the tradeoff structure between these two objectives is weak. That is, they are not in strong conflict with one another. An analysis simultaneously considering both objectives would indicate that if he would shift his harvest to the eastern stands, he would sacrifice only a small amount in harvest revenues while securing the maximum possible habitat for bird species A, enabling an improved rangeshift to the more northern areas.

-PROB DON'T NEED.What's more, the relationships between managed ecosystem services - that is, the situations in which one ecosystem service may be sacrificed in order to achieve more of another - are unknown under new climates. Under non-optimal conditions, this has the potential consequence of sacrificing some amount of an ecosystem service with no benefit in return. Under optimal conditions, sacrifices in one ecosystem service can be used to achieve more of another. In the case of being unaware of these novel tradeoff relationships, this is no longer possible. In addition to the magnitude of change in any one ecosystem service, it is also unknown how climate change will impact the relationships between ecosystem services. That is, the landscape-level simultaneous achievement of multiple ecosystem services. For instance, consider an unfortunate hypothetical situation of a forest, which up until the year 2010 was prime habitat for bird species A. Climate change is quickly reducing the area of the forest that is suitable habitat for bird species A. The forest manager has always also managed the forest in order to generate some revenue from timber sales. 10 years ago, before considering climate cvhange, he ran a multiobj model that suggested he harvest from a small area near the northern perimeter of the forest. However, climate change is now reducing the area of the forest that bird species A can live in. The only part left now that it can comfortably inhabit are those areas on the northern perimeter. The forest manager is now unknowingly sacrificing a large percentage of the potential habitat for bird species A to maintain his harvest activities. Little does he know (bc he has not the advantage of using our research and knowing the tradeoff structure) that these two objectives are not really in such strong competition. If he would shift his harvest to the eastern stands, he would only sacrifice some small amount in harvest revenues (say a percent) and would achieve the maximum possible habitat for bird species A, enabling his rangeshift to his new northern areas.

to as these are the areasin which climate change is rendering it incapable of providing habitat for long-time resident bird species A. This forest is also being managed to provide the ecosystem service of currently provides habitat for bird species Abalancingthclimate It is currently unknown how climate will alter the in favor of another. However, that other ecosystem service maypoIn forests managed for multiple ecosystem services, decisions must be made to determine the allowable levels of Consider, for example that the decision is made within a stand at a certain point in the future to forgo timber revenue in favor of habitat for species A - however, it may be that the stand under a different climate does not meet habitat requirements for species A, and so an unnecessary sacrifice has been made in forgone revenue as a result. any longer for the for a certain Forest managers today have the . Even in the best case scenario - if they do still happen to be optimal - they may be missing out on beneficial tradeoff relationships

To determine which management practices will be optimal in the future, we must first understand how climate change will impact forests' ability to provide ecosystem services. For example, how will climate change alter the ? How many acres of forest will still qualify as suitable habitat for a particular species? Many studies have considered these questions !CITE SOME PEOPLE.

However, previous studies have addressed the impact of climate change on forest ecosystem services in isolation. Because forests provide these ecosystem services in concert with one another (see, for example, [58]), we must also understand how climate impacts the tradeoffs that exist among them. Consider the simultaneous provision of wildlife habitat, carbon storage, and resistance to wildfire. How does an increase in any one service alter our ability to acquire an amount of another? Relationships such as a marginal sacrifice in one service for substantial improvement in another may no longer exist under a new climate. To better ensure the sustained provision of ecosystem services, we must understand how these tradeoffs evolve as a function of climate.

Here, I use a watershed in the Deschutes National Forest as a case study to determine how climate change impacts optimal forest management and the changes in tradeoffs among ecosystem services.

TO TEST ALL THIS STUFF, I AM USING A STUDY AREA IN THE DESCHUTES NATIONAL FOREST, KNOWN AS THE DRINK AREA. IT IS THIS BIG AND IS DI-

VIDED INTO 303 FOREST STANDS. THE AREA CONTAINS THE WATERSHED FOR THE CITIES OF BEND AND SISTERS OREGON. IT IS COMPRISED OF OLD GROWTH AND NEW GROWTH AND SOME OTHER STUFF. IT IS FLAMMABLE. WE WANT TO REDUCE THE RISK OF LONGTERM, SEVERE DEGRADATION OF THE WATER SUPPLIES TO THESE CITIES THAT WOULD RESULT FROM A HIGH SEVERITY WILDFIRE. THIS IS OUR FIRST OBJECTIVE. WE WILL DO THIS BY PERFORMING FUEL TREATMENTS. BUT THESE FUEL TREATMENTS LEAD TO SHORTERM SPIKES IN SEDIMENT CONTENT IN THE WATERSUPPLY, WHICH WE AIM TO MINIMIZE. MINIMIZING THE SEDIMENT DELIVERY TO THE WATERSHED AS A RESULT OF THE TREATMENTS IS OUR SECOND OBJECTIVE. FINALLY, THE AREA IS HOME TO THE FEDERALLY PROTECTED NORTHERN SPOTTED OWL. OUR THIRD OBJECTIVE IS ENSURING MAXIMAL HABITAT FOR THE NSO. WE WANT TO TEST OUR ABILITY TO SIMULTANEOUSLY PROCURE THESE THREE ECOSYSTEM SERVICES IN THE LONGTERM. BY LONGTERM, I MEAN I WILL STUDY IT OVER AN 80 YEAR HORIZON FROM 2015-2095. ALL MANAGEMENT ACTIVITY WILL OCCUR DURING THE INITIAL 40 YEARS. BC THE AREA GROWS SLOWLY, WE MODEL THESE 40 YEARS IN TWO 20-YEAR PLANNING HORIZONS. THE MANAGEMENT ACTIONS THAT MAY BE PRESCRIBED ARE THINNING TREATMENTS (SEE APPENDIX _ FOR TREATMENT PRESCRIPS) AND ARE DETERMINED APRIORI FOR EACH STAND AND TIME PERIOD COMBINATION. WE MEASURE THE SPIKE IN SEDIMENT DELIVERY AT THE TIME OF TREATMENT (YEARS 2025 AND 2045). WE MEASURE THE ACHIEVEMENT IN NSO HABITAT AND FIRE HAZARD AT THE END OF THE 80 YEAR PLANNING HORIZON. WE WILL DO THIS FOR EACH OF THREE DIFFERENT CLIMATE CHANGE SCENARIOS.

THE RESULTS WILL ENABLE US TO STUDY THE TRADEOFFS AMONG THESE THREE ECOSYSTEM SERVICES AND SEE HOW THEY VARY WITH CLIMATE CHANGE.

1.2 Methods

To determine the impacts of climate change on the tradeoff structure between ecosystem services, it is first necessary to define how the impacts of climate change are to be captured in the analysis.

1.2.1 Choosing Climate Scenarios for Comparison

I chose to capture the effects of climate change using the method employed by the IPCC, namely, through a scenario-based analysis. In a scenario analysis, multiple alternative futures are considered and no prediction is made as to which scenarios are more likely than others. There is no attempt to quantify the probability of realization of any one scenario.

The alternative futures I consider here are climate scenarios. Given the large number of potential future climates considered by the IPCC (see [24]) combined with the computational complexity involved in the study of each one, I selected a small subset of future climate scenarios for my analysis. These are “None”, “Ensemble RCP 4.5”, and “Ensemble RCP 8.5”.

The first scenario, “None”, is the assumption of no climate change. While the number of studies incorporating climate change is increasing, this is still the assumption used for many modern studies such as [51], from which this study is derived. Because it has served as the basis for many studies and assumes a static environment resembling today’s, the “None” climate scenario is the control against which I compare the other two climate scenarios.

As their names suggest, the second and third scenarios are ensembles of future climate projections. The components of the ensembles are global circulation models (GCMs) used in the IPCC’s Fifth Assessment (AR5). The USFS’s Climate-FVS [21] team selected the ensemble components and created the climate surface corresponding to the collection of these 17 GCMs. The list of the 17 scenarios included in the ensemble can be found in [13]. The climate surface contains a vector of 35 climate parameters at over 11,000 global locations for three time periods [15]. This provides a climate surface for each of the scenarios that, while temporally sparse, is spatially robust. This configuration is useful for the Drink area given its variability in elevation and slow growth.

The ensembles differ in the representative concentration pathway (RCP) assumed in the comoponent GCMs. The RCP indicates the additional radiative forcing (in W/m^2) above pre-industrial levels, with higher values of forcing indicative of more severe climate change. The GCMs in Ensemble RCP 4.5 assume a value of 4.5 W/m^2 of additional radiative forcing, and the GCMs in Ensemble RCP 8.5 assume 8.5 W/m^2 of additional radiative forcing.

I chose these three scenarios because they represent a range of predicted severity of climate change, from a $0°C$ warming by the year 2100 under the "None" scenario to a $2.6 - 4.8°C$ warming under RCP 8.5 [33].

1.2.2 Generating tradeoff relationships between ecosystem services

With this selection of climate scenarios, how can one determine the relationships between ecosystem services under each scenario? One applicable method is multi-objective mathematical optimization [56]. This approach seeks to maximize a set of objectives subject to a set of constraints. I define my objectives as the ecosystem services that the USFS prioritized for the Drink area. The set of constraints were determined through a combination of input from the USFS as well as logical constraints. The latter includes restrictions such as that one may not perform silvicultural treatments to areas that are not forested; the former includes such restrictions as how many acres may be treated in a given year.

The multi-objective model, or mathematical program, that I built and used for this analysis is provided below. The program involves linear, integer, and binary variables, making it a mixed integer program (MIP). The treatment assignment rules for the model, the acquisition of the required data, and the projection of that data into the future are described here:

Treatment scheduling and assignment rules

I simulated the 303 stands in the Drink area over the course of an 80 year planning horizon with all treatment activity occurring in the first 40 years. The activity could be selected to be completed in the first twenty year period, the second twenty year period, both, or neither. The type of treatment to be performed is dependent on silvicultural characteristics (see §B) and was determined *a priori* using the vegetation data described below. As a result, the model needed only to choose whether to perform a treatment on a stand in a given period; the model did not have to select which treatment to perform.

Acquisition and projection of data

The data required to solve the model include, for each climate scenario, each time period and each stand, a measure of fire hazard, determination of suitability for NSO habitat, and the amount of sediment deposit as a result of performing various thinning treatments.

As a measure for fire hazard, I chose the average fuel model of a stand according to the Anderson fuel model rating system [4]. This fuel model rating is an integer 1-13, that describes the fuel characteristics of an area, with larger fuel models corresponding to larger fuel loads, making it a suitable proxy for fire hazard. To determine the initial fuel models of each stand, I obtained the GNN structure map for map year 2012 (`http://lemma.forestry.oregonstate.edu/data/structure-maps`) from Oregon State University's Landscape Ecology, Modeling, Mapping & Analysis (LEMMA) group. The LEMMA group provides this data in a format compatible with the USFS's Forest Vegetation Simulator (FVS). I used FVS's database extension to import this data into FVS and then used FVS's Fire and Fuels Extension[49] (FFE) to compute the average fuel model for each stand. I then used Climate-FVS to project the stands' vegetation forward 80 years until the end of the planning horizon under each of the climate scenarios.

Through previous conversations with the USFS, it was determined that any area meeting the following characteristics would be considered ideal NSO habitat:

1. elevation less than 1830 m

2. the presence of trees with DBH no less than 76 cm

3. canopy closure of at least 60%

4. greater than 200 ha in size

I attained a digital elevation model from the US Department of Agriculture's GeoSpatial Data Gateway to compute average stand elevation and check for the first criterion. I checked the second and third criteria using the vegetation data produced by FVS. If the first three criteria are met but the area is not 200 ha in size, it is still classified as NSO habitat but is penalized by a factor of $e = 0.5$. Since stands were generally less than 200 ha in size, the last criterion required the enumeration of all clusters of stands whose combined contiguous area exceeded 200 ha. The model checks whether all stands in such a cluster meet the first three criteria to determine whether the penalization is required.

I retrieved data on sediment delivery using the Watershed Erosion Prediction Project (WEPP) online GIS tool [27]. This tool takes as input soil textures, treatment types, years of simulation, and custom climate data. I obtained soil texture data for the area from the USDA's Soil Survey Geographic (SSURGO) database. Treatment types are those specified in §B, and the years of simulation correspond to the planning horizon of the model. The custom climate data was obtained through the Climate-FVS climate data server [14]. Using the climate data provided by Climate-FVS ensured consistency of climate parameters in the predictions for both sediment and vegetation data.

The Multi-objective MIP

The first objective is to minimize the average fuel model at the end of the 80-year planning horizon:

$$Minimize \quad F = \sum_{i \in I} \sum_{r \in R} F_{i,r} x_{i,r} \tag{1.1}$$

In equation (1.1), I sum over all stands $i \in I$ and all treatment prescriptions $r \in R$ to obtain a cumulative fire hazard metric F, which measures the total fire hazard at the area at the end of the planning horizon. The coefficients $F_{i,r}$ are the area-weighted fuel models of each stand $i \in I$ at the end of the planning horizon if stand i is assigned to treatment prescription $r \in R$. The possible treatment prescriptions $r \in R$ are treat in the first period ($r = 1$), treat in the second period ($r = 2$), treat in both periods ($r = 3$), or do not treat ($r = 0$).

The second objective is to minimize the peak short-term sediment delivery that results from performing treatments in either period one (S_1) or period two (S_2):

$$\textit{Minimize} \quad S = \max\{S_1, S_2\} \tag{1.2}$$

The last objective is to maximize the area of suitable northern spotted owl habitat at the end of the planning horizon.

$$\textit{Maximize} \quad O = \sum_{i \in I_\omega} \left(a_i p_i + e a_i \left(\sum_{j \in R_i} x_{i,j} - p_i \right) \right) \tag{1.3}$$

The set of stands in the sum $i \in I_\omega$ are those that meet the first three criteria for NSO habitat under at least one treatment prescription $j \in R_i$, where R_i is the set of treatment prescriptions for stand i such that it meets the first three NSO habitat criteria at the end of the planning horizon. If a stand i does not meet these criteria under any treatment prescriptions (if the set $R_i = \{\emptyset\}$), then $i \notin I_\omega$. If the model assigns a stand $i \in I_\omega$ a treatment prescription $j \in R_i$, then stand i meets the first three NSO habitat criteria at the end of the planning horizon, and the variable $x_{i,j} = 1$. If, in addition, the stand i is part of a cluster of stands all meeting the first three NSO habitat criteria and whose combined contiguous area is greater than 200 ha, then the variable $p_i = 1$. Notice that when $p_i = 0$, the stand's contribution is discounted by $e = 0.5$, and when $p_i = 1$ it is not.

The objectives are subject to the following constraints. First, I define accounting variables for the sediment delivery that results from the performance of the prescribed management actions.

$$\sum_{i \in I} \sum_{r \in 1,3} s_{i,1} x_{i,r} = S_1 \tag{1.4}$$

$$\sum_{i \in I} \sum_{r \in 2,3} s_{i,2} x_{i,r} = S_2 \tag{1.5}$$

The coefficients $s_{i,t}$ are the amount of sediment (in tonnes) that would result from treating stand i in time period t.

In order to control the trigger variables p_i indicating a stand's inclusion in a 200 ha cluster of NSO habitat, I used the following two constraints:

$$\sum_{i \in D_c} \sum_{j \in R_i} x_{i,j} - |c| q_c \geq 0 \qquad \forall c \in C \tag{1.6}$$

$$\sum_{c \in C_i} q_c - p_i \geq 0 \qquad \forall i \in I_\omega \tag{1.7}$$

$c \in C$ are the clusters of stands whose combined area is greater than 200 ha. A cluster c contains the set of stands $i \in D_c$. Equation (1.6) specifies that all stands $i \in D_c$ within a cluster $c \in C$ must be assigned a management prescription such that they meet all NSO habitat criteria in order for the cluster trigger variable q_c to take value 1.

Equation (1.7) specifies that if no cluster $c \in C_i$ - the set of clusters that contain site i - meets NSO qualifications, then the trigger variable p_i must equal 0. If some cluster $c \in C_i$ does meet NSO qualifications, then the objective function (1.3) will draw up the value of the variable p_i to 1.

I also impose the restriction that each stand may be assigned to at most one treatment prescription.

$$\sum_{r \in R} x_{i,r} = 1 \qquad \forall i \in I \tag{1.8}$$

Next, I ensured that the area treated in each time period is less than a prespecified maximum area A:

$$\sum_{i \in I} \sum_{r \in 1,3} a_i x_{i,r} = H_1 \tag{1.9}$$

$$\sum_{i \in I} \sum_{r \in 2,3} a_i x_{i,r} = H_2 \tag{1.10}$$

$$H_1 \leq A \tag{1.11}$$

$$H_2 \leq A \tag{1.12}$$

where the first two equations define the accounting variables for the areas treated in time periods 1 and 2, H_1 and H_2, and the second two equations impose the upper bound.

Finally, I specified fluctuation constraints to bound the differences in the area treated in between time periods:

$$\ell H_1 - H_2 \leq 0 \tag{1.13}$$
$$-uH_1 + H_2 \leq 0 \tag{1.14}$$

I define a maximum of 20% fluctuation between time periods. That is, $\ell = 0.8$ and $u = 1.2$.

Together with the binary specifications on our variables (equation (1.15)), the complete model is

$$\begin{aligned} &\mathit{Minimize} \\ &\qquad F = \sum_{i \in I} \sum_{r \in R} F_{i,r} x_{i,r} \\ &\qquad S = \max\{S_1, S_2\} \\ &\mathit{Maximize} \\ &\qquad O = \sum_{i \in I_\omega} \left(a_i p_i + e a_i \left(\sum_{j \in R_i} x_{i,j} - p_i \right) \right) \end{aligned}$$

Subject to:

$$
\begin{aligned}
\sum_{i\in I}\sum_{r\in 1,3} s_{i,r}x_{i,r} &= S_1 \\
\sum_{i\in I}\sum_{r\in 2,3} s_{i,r}x_{i,r} &= S_2 \\
\sum_{i\in D_c}\sum_{j\in R_i} x_{i,j} - |c|q_c &\geq 0 \qquad \forall c \in C \\
\sum_{c\in C_i} q_c - p_i &\geq 0 \qquad \forall i \in I_\omega \\
\sum_{r\in R} x_{i,r} &= 1 \qquad \forall i \in I \\
\sum_{i\in I}\sum_{r\in 1,3} a_i x_{i,r} &= H_1 \\
\sum_{i\in I}\sum_{r\in 2,3} a_i x_{i,r} &= H_2 \\
H_1 &\leq A \\
H_2 &\leq A \\
\ell H_1 - H_2 &\leq 0 \\
-uH_1 + H_2 &\leq 0 \\
x_{i,r}, p_i, q_c \in \{0,1\} &\quad \forall i \in I, r \in R, c \in C
\end{aligned}
\tag{1.15}
$$

1.2.3 Model solution

Solving a bounded and non-degenerate multi-objective optimization problem with N objectives produces a set of objective vectors (also called "solutions") $\mathbf{z} \in Z$ where $\mathbf{z} = \langle z^1, \ldots, z^N \rangle$. The set of solutions Z is referred to as the Pareto-optimal frontier or efficient frontier or, simply, frontier. The solutions comprising an efficient frontier have the special relationship such that no component of a solution $\mathbf{z}^i$ can be improved upon without one of the other components $\mathbf{z}^j$ $(j \neq i)$ degrading. For example, this relationship in the current problem means that further reducing fire hazard would require either additional sediment deposits, the sacrifice of NSO habitat, or both.

Thus the efficient frontier provides information on the tradeoff structure that exists between ecosystem services. Parameterizing and solving the model for each of the climate scenarios generates three frontiers: Z_{None}, $Z_{4.5}$, and $Z_{8.5}$. As climate is the driver of the differences in these frontiers, the comparison of frontiers provides insight into how climate impacts the tradeoff structures between the ecosystem services.

To solve the models, I wrote my own implementation of Tóth's Alpha-Delta algorithm [55] that is generalized for any multi-objective problem with $N \geq 2$ objectives. The Alpha-Delta algorithm finds the optimal set Z by iteratively slicing the N-dimensional objective space with a tilted $N-1$ dimensional plane. To derive the frontiers, I used an alpha parameter of $\alpha = .01$ and delta parameters of $\delta_O = 1$ ha and $\delta_S = 0.5$ tonnes for the NSO habitat and sediment delivery objectives, respectively.

1.2.4 Comparing Tradeoffs under each Climate Change Scenario

No standardized procedure exists for comparing frontiers or measuring the conflict between objectives within a frontier. To address the former, I draw on methods used in the field of evolutionary multi-objective optimization (EMO). To address the latter, I apply methods used in objective pruning in many-objective optimization.

Comparing frontiers

Researchers in the field of EMO develop algorithms to generate a set of non-dominated solutions that best represent the true Pareto-optimal frontier [18]. To test their algorithms, they compare their resulting frontiers to a known Pareto front for benchmark multi-objective optimization problems [37]. There is no assurance of optimality of the solutions derived using these algorithms, so they require a means of comparing the resulting frontiers to determine if one algorithm produces a “better” non-dominated frontier than another. Zitzler et al. provide a review of comparison methods in [65]. These methods aim to quantify certain traits about a frontier that can be used to measure their success in approximation of the true frontier.

My motivation in comparing frontiers is different from EMO in that, rather than comparing non-dominated sets produced by identical models, I aim to compare frontiers generated by models with the same structure but different parameterizations. As a result, not all comparison methods are applicable, such as the indicator for the number of Pareto points contained in the frontier (all points on my frontiers are Pareto-optimal). However, other comparison methods still have value in our analysis. I chose a subset of these methods: the binary epsilon and binary hypervolume indicators, and the unary distance, unary hypervolume, and unary spacing indicators.

Note that use of some comparison methods for the frontiers requires the normalization of the objective space. This is because the climate scenarios may significantly alter the bounds on the achievable values of the ecosystem services, resulting in frontiers whose objective spaces will not necessarily overlap and with incomparable distributions of solutions within. The chosen normalization of each frontier is the unit hypercube, with each objective bounded between 0 and 1 and the frontier bounded by $[0,1]^N$. Without loss of generality, I convert all objectives to maximization, define the nadir solution to be at the origin and the ideal solution to be at the point $\vec{\mathbf{1}}$. The nadir solution $\mathbf{z}_{\text{nad}}$ of a frontier of points $z \in Z$ is defined as the objective vector with components

$$\mathbf{z}_i^{\text{nad}} = \inf_z \{z_i\} \quad \forall 1 \leq i \leq N \tag{1.16}$$

and the ideal solution is the objective vector with components

$$\mathbf{z}_i^{\text{ideal}} = \sup_z \{z_i\} \quad \forall 1 \leq i \leq N \tag{1.17}$$

Binary epsilon indicator I_ϵ Given two frontiers, Z_1 and Z_2, the binary epsilon indicator is defined as [65]

$$I_\epsilon(Z_1, Z_2) = \inf_{\epsilon \in \mathbb{R}} \{ \forall \mathbf{z}_2 \in Z_2 \exists \mathbf{z}_1 \in Z_1 : \mathbf{z}_1 \succeq_\epsilon \mathbf{z}_2 \} \tag{1.18}$$

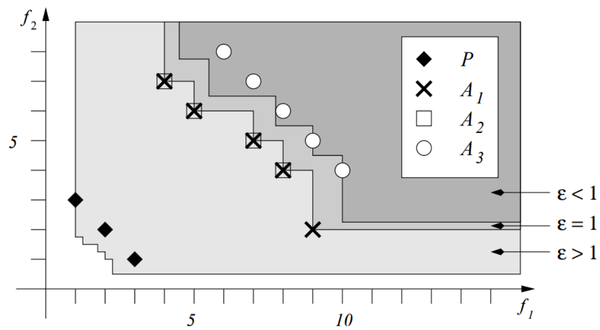

Figure 1.1: Any frontier Z with solutions in the light gray area (closest to origin) would have $I_\epsilon(Z, A_1) > 1$; $I_\epsilon(A_1, A_1) = I_\epsilon(A_2, A_1) = 1$; any frontier Z existing only in the darker gray areas such as A_3 would have $I_\epsilon(Z, A_1) < 1$

where $\succeq$ is the ϵ-dominance relationship:

$$\mathbf{z}_1 \succeq_\epsilon \mathbf{z}_2 \iff \forall 1 \leq i \leq N : \epsilon \mathbf{z}_1^i \geq \mathbf{z}_2^i \tag{1.19}$$

That is, ϵ is the minimum factor by which all points in one frontier Z_1 must be multiplied such that all solutions in Z_1 at least weakly dominate all solutions in the other frontier Z_2.

Unary hypervolume indicator I_{H1} and binary hypervolume indicator I_{H2} For a frontier Z comprised of solutions $\mathbf{z} = \langle z^1, \ldots, z^N \rangle$ and with the objective space defined such that the origin is the nadir point, then the volume of a single solution $\mathbf{z}_i$ is the volume of the hyperrectangle r_i whose diagonal corners are the origin and the solution $\mathbf{z}_i$. The hypervolume of the frontier is the volume of the union of the hyperrectangles corresponding to the solutions in the frontier:

$$I_{H1}(Z) = \text{vol}\left(\bigcup_{i=1}^{|Z|} r_i\right) \tag{1.20}$$

Then define the binary hypervolume indicator of two frontiers Z_1 and Z_2 as [64]

$$I_{H2}(Z_1, Z_2) = I_{H1}(Z_1 + Z_2) - I_{H1}(Z_2) \tag{1.21}$$

Relation	Solutions		Frontiers	
Strictly dominates	$\mathbf{z}^1 \succ\succ \mathbf{z}^2$	$\mathbf{z}_i^1$ is better than $\mathbf{z}_i^2 \quad \forall 1 \leq i \leq N$	$Z_1 \succ\succ Z_2$	$\exists \mathbf{z}^1 \in Z_1 \succ succ \mathbf{z}^2 \quad \forall \mathbf{z}^2 \in Z_2$
Dominates	$\mathbf{z}^1 \succ \mathbf{z}^2$	$\exists 1 \leq i \leq N : \mathbf{z}_i^1$ is better than $\mathbf{z}_i^2$, and $\mathbf{z}_i^1$ is not worse than $\mathbf{z}_i^2 \quad \forall 1 \leq i \leq N$	$Z_1 \succ Z_2$	every $\mathbf{z}^2 \in Z_2$ is dominated by at least one $\mathbf{z}^1 \in Z_1$
Better			$Z_1 \rhd Z_2$	every $\mathbf{z}^2 \in Z_2$ is weakly dominated by at least one $\mathbf{z}^1 \in Z_1$ and $Z_1 \neq Z_2$
Weakly dominates	$\mathbf{z}^1 \succeq \mathbf{z}^2$	$\mathbf{z}^1$ is at least as good as $\mathbf{z}^2$ in all objectives	$Z_1 \succeq Z_2$	All solutions in $\mathbf{z}^2 \in Z_2$ are weakly dominated by a solution $\mathbf{z}^1 \in Z_1$
Incomparable	$\mathbf{z}^1 \| \mathbf{z}^2$	Neither $\mathbf{z}^1$ nor $\mathbf{z}^2$ weakly dominates the other	$Z_1 \| Z_2$	Neither Z_1 nor Z_2 weakly dominates the other

Table 1.1: Definitions of dominance relations between solutions and frontiers [65]

	Name of indicator	Relation					
		$\succ\succ$	$\succ$	$\rhd$	$\succeq$	$=$	$\|$
	I_ϵ	$I_\epsilon(Z_1, Z_2) < 1$	-	$I_\epsilon(Z_1, Z_2) \leq 1$ $I_\epsilon(Z_2, Z_1) > 1$	$I_\epsilon(Z_1, Z_2) \leq 1$	$I_\epsilon(Z_1, Z_2) = 1$ $I_\epsilon(Z_2, Z_1) = 1$	$I_\epsilon(Z_1, Z_2) > 1$ $I_\epsilon(Z_2, Z_1) > 1$
	I_{H2}	-	-	$I_{H2}(Z_1, Z_2) > 0$ $I_{H2}(Z_2, Z_1) = 0$	$I_{H2}(Z_1, Z_2) \geq 0$ $I_{H2}(Z_2, Z_1) = 0$	$I_{H2}(Z_1, Z_2) = 0$ $I_{H2}(Z_2, Z_1) = 0$	$I_{H2}(Z_1, Z_2) > 0$ $I_{H2}(Z_2, Z_1) > 0$
	I_d	-	-	-	-	-	-
	I_s	-	-	-	-	-	-

Table 1.2: Tests using indicators to determine dominance relationships between frontiers [65]. While general tests of dominance relationships may not be avaiable for some metrics (any cell with '-'), conclusions may still be drawn. For instance, $I_d(Z_1) < I_d(Z_2) \Rightarrow Z_2 \not\rhd Z_1$.

where $I_{H1}(Z_1+Z_2)$ is the unary hypervolume indicator of the merged frontier consisting of all solutions from frontiers Z_1 and Z_2. The binary hypervolume indicator provides the volume of frontier Z_1 that is not contained within frontier Z_2. Larger values of I_{H1} correspond to frontiers occupying larger fractions of the objective space, indicating less conflict between the objectives. For frontiers Z_1 and Z_2 in comparable scales (that is, in normalized objective spaces), if $I_{H2}(Z_1, Z_2) > I_{H2}(Z_2, Z_1)$ this indicates less conflict between objectives in Z_1 than in Z_2. I_{H2} can also be used to determine if one frontier is "better" than another (see Tables 1.2.4 and 1.2.4).

I developed a custom algorithm to solve for the unary hypervolume idicator. The details of the algorithm may be found in §A.

Unary distance indicator I_d The unary distance indicator used for the analysis is analogous to the unary distance indicator described in [16], but instead of computing the distance to a reference Pareto frontier I measure the average distance from the frontier to the ideal solution:

$$I_d = \frac{\sum_{\mathbf{z}\in Z} ||\mathbf{z}^{\text{ideal}} - \mathbf{z}||}{N} \tag{1.22}$$

Smaller values of I_d correspond to frontiers that are closer to the ideal solution, which implies less conflict between the objectives.

Unary Spacing Indicator I_s The unary spacing indicator, or Schott's spacing metric[50] computes the standard deviation of the distance between points in the frontier, defined as

$$I_s = \sqrt{\frac{1}{N-1}\sum_{\mathbf{z}\in Z}(d_z - \bar{d})^2} \tag{1.23}$$

where

$$d_z = \min_{\mathbf{y}\in Z, \mathbf{y}\neq\mathbf{z}} ||\mathbf{z} - \mathbf{y}|| \tag{1.24}$$

and $\bar{d}$ is the average of all d_z. In EMO, the spacing indicator provides a measure of an algorithm's ability to search the frontier space uniformly. In the current analysis, the spacing metric provides a measure of the flexibility afforded to the decision maker under each climate scenario. Larger spacing metrics imply larger sacrifices between decisions and less flexibility.

Quantifying conflict between objectives within a frontier

The above methods provide frontier-level metrics of conflict and tradeoffs. To determine the degree of conflict between two objectives within a single frontier, we employ an approach used in many-objective optimization. Given the increased difficulty in solving many-objective optimization problems [36], researchers in this field seek to reduce the number of objectives considered in the model. To determine which objectives most strongly influence the shape of the frontier, they compute the correlation between each pair of objectives [19]. The objective pairs with the most negative correlation are most in conflict. To rank the relative conflict between objectives in each climate scenario, I computed their Pearson correlation coefficients:

$$\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma(X)\sigma(Y)} \tag{1.25}$$

where, for some objectives x and y, X and Y are

$$X = \{\mathbf{z}_x^1, \mathbf{z}_x^2, \ldots, \mathbf{z}_x^{|Z|}\} \tag{1.26}$$

$$Y = \{\mathbf{z}_y^1, \mathbf{z}_y^2, \ldots, \mathbf{z}_y^{|Z|}\} \tag{1.27}$$

1.3 Results and Discussion

This sample thesis was produced by the LaTeX document class it describes and its format is consonant with the Graduate School's electronic dissertation guidelines, as of November, 2014, at least. However, use of this package does not guarantee acceptability of a particular thesis.

1.4 Conclusion

Here's a conclusion.

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Appendix A
COMPUTING A FRONTIER'S HYPERVOLUME INDICATOR

Given a set of Pareto optimal solutions $\mathcal{P}$ to a multi-objective mathematical programming model with a set of objectives O of cardinality $N := |O|$, this algorithm computes the volume V of the objective space bounded by the Pareto frontier defined by the solutions $x \in \mathcal{P}$. The objectives are assumed to be normalized so that the objective space is the N-dimensional unit hypercube with the origin and the point $\vec{\mathbf{1}}$ defining the nadir objective vector and the ideal objective vector, respectively. That is, all objectives are assumed to be maximized with bounds $[0, 1]$.

The algorithm projects the objective space into $N - 1$ dimensions by eliminating the dimension associated with an (arbitrarily-chosen) objective $p \in O$. The set of objectives is $\overline{O} := O \backslash \{p\}$. It is assumed that $x \in \mathcal{P}$ are sorted in descending order according to p. The algorithm proceeds by sequentially adding solutions to the $(N - 1)$-dimensional space, and calculating the contribution to the frontier volume as a product of the volume contribution in $N - 1$ dimensions and its achievement in objective p.

Let $\overline{V_x}$ be the $(N - 1)$-dimensional volume contribution of solution x and x_p be the achievement of solution x in objective p. Further, let F be the set of non-dominated solutions in $N - 1$ dimensions. I compute the N-dimensional volume of the frontier V as follows.

Figure A.1: Algorithm to compute the volume of a Pareto frontier

$V \leftarrow 0$
$\overline{V} \leftarrow 0$
$F \leftarrow \emptyset$
for all $x \in \mathcal{P}$ **do**
$\overline{V}_x \leftarrow \prod_{o \in \overline{O}} x_o - \overline{V}$
for all $f \in F$ **do**
if $f_o < x_o \forall o \in \overline{O}$ **then**
$F \leftarrow F \backslash \{f\}$
end if
end for
for all $o \in \overline{O}$ **do**
$F_{x,o} := \{\, f \in F : f_o > x_o \,\}$
Sort $f \in F_{x,o}$ in ascending order by their oth component, f_o
$v_i \leftarrow x_o$
for all $f \in F_{x,o}$ **do**
$v_t \leftarrow f_o$
$\delta_o := v_t - v_i$
$\overline{V}_x \leftarrow \overline{V}_x + \delta_o \prod_{\sigma \in \overline{O} \backslash \{o\}} f_\sigma$
$v_i \leftarrow v_t$
end for
end for
$F \leftarrow F \cup \{x\}$
$\overline{V} \leftarrow \overline{V} + \overline{V}_x$
$V \leftarrow V + x_p \overline{V}_x$
end for

Appendix B

TREATMENT SPECIFICATION FOR THE DRINK AREA

TREATMENT SPECIFICATION FOR THE DRINK PLANNING AREA

Stand density index[1]	Crown bulk density, kg/m^3	Number of trees <18 cm dbh per ha	Fuel model	Combined basal area of mountain hemlock and white fir >46 cm dbh, (m^2)	Treatment
Lodgepole pine plant association					
<87	NA	NA	NA	NA	**Prescribed burning[2]**
>=87	>0.037	>49	>=10	NA	**Thin & Pile and burn slash and fuels[3]**
			<10	NA	**Thin & Pile and burn slash**
Mixed conifer wet or Mountain Hemlock plant associations					
<87	NA	NA	NA	NA	**Prescribed burning**
>=87	>0.037	>49	=10	>7.5	**Thin & Pile and burn slash and fuels & Prescribed burning**
				<=7.5	**Thin & Pile and burn slash and fuels**
			>=11	NA	**Thin & Pile and burn slash and fuels**
			<10	NA	**Thin & Pile and burn slash**
		<=49	=10	>=7.5	**Prescribed burning**
	<=0.037	NA	=10	>=7.5	**Prescribed burning**
	NA	NA	=6,8,9 or 10	NA	**Prescribed burning** (if prescribed burning occurred in period 1; the treatment applies to second period only)
Mixed Conifer dry plant association					
<87	NA	NA	NA	NA	**Prescribed burning**
>=87	>0.037	>49	=10 or 11	NA	**Thin & Pile and burn slash and fuels & Prescribed burning**
			>=12	NA	**Thin & Pile and burn slash and fuels**
			<10	NA	**Thin & Pile and burn slash**
		<=49	=10 or 11	NA	**Prescribed burning**
	<=0.037	NA	=10 or 11	NA	**Prescribed burning**
	NA	NA	=6,8,9 or 10	NA	**Prescribed burning** (if prescribed burning occurred in period 1; the treatment applies to second period only)

Unless otherwise specified, vegetation conditions are assessed at the beginning of each planning period. If the conditions are met a corresponding combination of treatments is scheduled. No treatment is applied if none of the criteria is met.

[1] The stand density index is in metric units; conversion to imperial units is need for simulations using Forest Vegetation simulator.

[2] Prescribed burning is an application of prescribed fire, a fire ignited by management actions to meet specific objectives (Glossary of Wildland Fire Terminology, 2012).

[3] Pile and burn slash assumes removal of the cut trees only, while pile and burn slash and fuels involves removal of the materials that were on the ground before thin (Wall, Powers, 2012; personal communication).