Bridging the Gap: Information, Returns and Choices

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Abstract

How much of the gap in choices across social groups is driven by differences in returns or the ability to predict these returns? To formalize this question, we use a decomposition exercise and a structural model to quantify the role of information quality and differences in returns in driving this gap. Focusing on the college attendance decisions of White and Hispanic high school students in Texas, we use administrative data to understand what drives their differing choices. We find that the average returns for college for Hispanics are almost zero compared to being positive for Whites. We then bound the role of these differences in returns in contributing to the gap in choices. Under reasonable assumptions, we find that information can contribute significantly to this gap. We further ask how a policymaker aiming to achieve parity in choices could accomplish this. We find that additional information should have an R-squared between 0.19 and 0.35 in explaining post-college earnings, which is challenging to achieve using standard datasets.

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1 Introduction

In social systems, where individuals' life trajectories are shaped by choices, understanding the determinants of these choices is crucial, particularly in the pursuit of equality. Standard economic models assume that individuals weigh the costs against the benefits of their decisions. However, it is rarely the case that individuals can perfectly predict the outcomes of their choices. In reality, they operate under significant uncertainty and have limited predictive capabilities about the consequences of their actions. This gap in information and prediction abilities affects the choices different people make, potentially widening or narrowing societal inequities. Therefore, it is essential to assess the extent to which these frictions contribute to differences in decision-making processes and choices.

In this paper, we focus on how differences in information and the ability to predict outcomes contribute to differences in choices. To do this, we adopt a structural approach for this analysis. Our structural model follows a basic choice framework (Roy (1951)), where individuals participate when they perceive the potential returns to be greater than their costs. We assume that individuals receive informative signals on their returns and use them to make a binary decision on whether or not to opt in or out. Using this model, we measure the quality of individuals' information. To do so, we use the ability of individuals to predict the outcome as measured by the coefficient of determination (R-Squared), which is a commonly used measure in statistics to quantify the quality of statistical models.

In this simple model, choice is driven by two parts. The first is the distribution of net returns. The second component is the quality of information individuals have for predicting their return, measured by R^2. This decomposition of the choice problem motivates us to use a decomposing method for the choice gap, inspired by the decomposition methodology of Kitagawa (1955), Blinder (1973), and Oaxaca (1973). This method breaks the choice discrepancy into two parts: the information channel and the composition channel. The information channel quantifies how much of the gap is driven by the fact that the two groups have access to different information sources. It does so by equalizing the information quality across the two groups, holding everything else constant. The residual difference, the composition channel, examines what the choice gap would be if we equalized the net returns between the two groups while maintaining their distinct information qualities on those returns.

In our empirical application and framework we focus on differences across groups in the decision to go to college. In this context, where do differences in returns and information originate? One source of these differences is the disparities that exist before the decision is made (Neal and Johnson (1996)) and arise from differences in the environments in which members of various group grow up in. Such environments can influence both returns and information. For example, differences in information can arise if members of one group are coming from affluent backgrounds, have access to more informative information on the financial outcomes of college, unlike their less fortunate counterparts. These differences in where individuals grew up can similarly drive differences in the returns, if, for example, children from richer parents can make the cost of college lower or more obtainable. Information also accumulates through learning prior to the decision. For instance, dynamic models (Cunha and Heckman (2007),Cunha et al. (2021)) of investment illustrate three ways through which early life disparities can shape future opportunities. First, inadequate early investment can limit future choices. Second, dynamic complementarity suggests that boosting investments at one stage can enhance the value of subsequent investments. Lastly, early experiences influence the information individuals possess about the value of future investments. Therefore, discrepancies in pre-choice environments can manifest as changes in the potential returns or in the returns in the information available to individuals.

Information, in the context of decision-making, is not solely a byproduct of past experiences and accumulated knowledge, but is also a function of the future environment individuals would face. This is particularly evident when we consider the challenges associated with predicting outcomes like earnings, for different social groups. For instance, earnings for different groups can vary widely due to factors like industry sector trends, geographic economic conditions, and social biases. These disparities are not only affecting the returns distribution, but also affect the ability to predict future returns. This unpredictability has real implications for individuals making life-altering decisions. Therefore, our decomposition exercise explores how disparities across various realms accumulate and influence the decision processes of different groups.

Understanding how these inequalities affect individuals choices can help policy makers to mitigate the gap. If we find that the disparities between social groups are primarily due to differences in the quality or quantity of information they possess, then addressing these informational inequalities could be a key to leveling the playing field. In such cases, policy interventions could focus on providing more comprehensive and accessible information. This approach becomes increasingly feasible with advancements in predictive algorithms and data analytics, making it easier to provide targeted, relevant information that can aid in making informed decisions. Conversely, if our analysis reveals that the differences are rooted in disparities in net returns - such as income potential or access to resources - then a different strategy is required. This would involve addressing the underlying structural issues that lead to these economic disparities. It may include reforming economic policies, improving access to education and training, or implementing measures to ensure fairer income distribution. Tackling these fundamental issues is more challenging but essential for long-term and sustainable change in bridging societal gaps.

To better understand how much information gaps can be address by a policy maker, our second measure information frictions takes a more practical approach and is motivated by the assumption that a policy maker cares to reach equality of choices. This second measure of information differences is motivated by a more practical view, where a policy maker aims to close the choice gap. They then ask, by how much should they improve or, conversely, the prediction tools of individuals, in order to achieve parity in choice. Although more information, in our setting, is always better for the individuals themselves, and therefore a policy maker who aims to maximze welfare should just provide as much information as possible, it may be a policy makers may want to balance or enhance college attendance across different social groups aiming to promoting equity, addressing inequality, and fulfilling broader policy aims. Beyond personal gains, increasing college enrollment can contribute to societal progress in research and development, stimulate technological and scientific advancement, and foster an environment conducive to lifelong learning. Additionally, a rise in the number of college-educated individuals can drive economic growth, decrease crime rates, and bolster civic involvement. Therefore, policy makers may view the equalization or augmentation of college attendance as a strategic approach to achieving these extensive societal advantages.

For our empirical exercise, we use administrative data from Texas to understand what drives the 9% gap between Hispanics and Whites in the decision to go to college at the first year after high school. We estimate a Gaussian choice model and use our decomposition approach as outlined above. As returns are not observed directly, we generate bounds on the possible set of correlation values between individual earnings if they go to college and individual earnings if they do not. We postulate different restrictions on both the correlation values themselves and what individuals may possibly know about their returns, and find that the bounds are still very wide. This implies that information could potentially have a very large effect, enough to not only close the choice gap but to turn it around. We then proceed to estimate our second measure and find that the additional precision required is also wide, implying that in order to achieve parity, the information available to individuals should be substantially improved. We then consider how a policy maker could close this choice gap by providing Hispanics with additional information. We find that providing new information, currently unknown, should be able to explain either 0.19 or 0.35 of the post-college earnings gap. These precisions are relatively high and are challenging to obtain using standard datasets. This implies that if we want to close the gap by providing information, we need to be able to generate better sources to predict earnings.

1.1 Related Literature

This paper contributes to an extensive body of literature on human capital investment decisions, anchored by the foundational work of Ben-Porath (1967). Our study intersects with research focused on the impact of monetary returns on such choices, as explored in studies by Willis and Rosen (1979), Walters (2018), Abdulkadiroğlu et al. (2020), and Freeman (1971). These papers typically make assumptions about how individuals form beliefs about returns—often measured based on observable factors—and analyze how these beliefs factor into decision-making processes. Our approach differs by examining how variations in the information available to individuals influence their choices.

Another significant aspect of our research aligns with studies that investigate the nature of individuals' beliefs, such as those by[1] Manski (2004), Wiswall and Zafar (2015), Zafar (2011), Wiswall and Zafar (2021), and Diaz-Serrano and Nilsson (2022). These works delve into systemic differences and biases in beliefs among groups defined by socio-economic status. Our paper extends this inquiry, utilizing these findings to illuminate not just the distribution of beliefs but also the quality and extent of information available to these groups.

Methodologically, our study builds upon the Marginal Treatment Literature, particularly the work of Heckman and Vytlacil (2005). This approach has previously been employed to examine the marginal

[1] See overview of the literature on beliefs elicitation in Giustinelli (2022)

treatment effects on returns to schooling, as demonstrated by Carneiro et al. (2011) and Carneiro and Lee (2009). Similar to some of these studies, we link the marginal treatment effect to beliefs. Eisenhauer et al. (2015) employed this structure to conduct a cost-benefit analysis of programs, focusing on agents' ex-post and ex-ante costs—closely paralleling our usage. Canay et al. (2020) and d'Haultfoeuille and Maurel (2013), in the context of college decisions and discrimination, demonstrate how the Roy model can identify ex-ante beliefs and preferences, aligning with our methodological approach.

Our work also resonates with recent research by Bohren et al. (2022) on systemic discrimination. Their study, akin to ours, identifies two main sources of systemic differences between social groups. The first, termed 'technological systemic discrimination', aligns with our focus on differences in return distributions and captures disparities across groups in certain outcome variables. The second, 'informational discrimination', pertains to disparities arising from varied information available to decision-makers across groups. Our research differs in its concentration not on discrimination towards individuals but on the decisions individuals make about themselves and how these systemic forces shape it, with a specific focus on the quality of information rather than its structure. We further explore a distinct measure related to this in our appendix.

While our primary focus is on educational decisions, our decomposition approach has broader applications. It can illuminate how information asymmetries contribute to decision-making disparities across various contexts. Recent studies, including those by Arnold et al. (2018), Arnold et al. (2022), and Canay et al. (2020), have explored the influence of judicial preferences and biases in decision-making. There is a growing interest in understanding how decision-making signals contribute to these disparities. Our decomposition methodology seeks to address these nuanced aspects of decision-making processes.

The remainder of the paper proceeds as follows. Section 2 describe our framework and decomposition approach. Section 3 describe the data and some descriptive statistics. Section 4 discuss the estimation results and section 5 concludes.

2 Framework

We consider a population of high school graduates, denoted by i. At the end of high school, each graduate must decide whether to attend college. The objective of individual i is to maximize earnings. Denote by Y^1 earning for an individual who attends college and by Y^0 their earnings if they do not. We assume that earnings are generated according to

$$
\begin{aligned}
Y^1 &= \alpha_1 + u_1, \\
Y^0 &= \alpha_0 + u_0
\end{aligned}
$$

where α^d, with $d \in \{0, 1\}$, is the structural component of earnings and u_d is an unpredictable component of earnings, satisfying $E[u_d|\alpha_1, \alpha_0] = 0$. Before deciding whether to attend college, each individual i receives an informative signal about the structural component of earnings. Specifically, we denote by $\boldsymbol{S}_i \in \mathcal{S}$ the vector of realized signals that individual i observes. We further assume that $\boldsymbol{S} \perp u_d|\alpha_1, \alpha_0$. Our model separates earnings into two main components. The first is a structural component, α_1 and α_0, which agents can know and form beliefs about. The second component is u_d, which is unknowable at the time of the decision. These components of earnings include idiosyncratic shocks that can only be known ex-post. Henceforth, we will treat α_1 and α_0 as earnings.

In our model, signals link outcomes to beliefs; thus, we need to establish how individuals use signals to form beliefs. We adopt the standard approach in economics and model individuals as Bayesian agents with rational expectations (Muth (1961), Lucas (1972), Sargent and Wallace (1971)). Being Bayesian means that agents observe the signal, know the correct likelihood function, and update their priors to form new beliefs over the outcomes. Our second assumption, rational expectations, implies that an individual's prior is anchored to the observed distribution of outcomes. In appendix H, we discuss how this assumption can be relaxed if a researcher believes that individuals have inaccurate priors or beliefs (Bohren et al. (2023)) and have access to data on beliefs, in addition to choice and outcome data, but our analysis from now on is restricted to Bayesian agents with rational expectations.

Finally, we assume that individuals incur some cost when attending college, that is a function of observables. Denote by X a set of observed variables, by $c(x)$ the cost of attendance and by $\mathcal{R} =$

$\alpha_1 - \alpha_0$ the structural part of the returns, then individual i's decision rule is given by

$$D = \mathbb{1}\left[E[Y^1 - Y^0|\boldsymbol{S}] \geq c(x)\right] = \mathbb{1}\left[E[\mathcal{R}|\boldsymbol{S}] \geq c(x)\right].$$

Our decision rule suggests that individuals derive risk-neutral utility from earnings but allows high school graduates to possess any utility function that strictly increases with expected returns (Vytlacil (2006)). Modeling utility as an increasing function of returns includes also the standard linear indirect utility function, that has been used in models of school and education choices (Willis and Rosen (1979), Walters (2018), Abdulkadiroğlu et al. (2020)) In our framework, we standardize this utility function to be the identity function. Therefore, $c(x)$ serves as a composite of individual preferences, known monetary and non Monterrey costs, and other barriers to college attendance, such as credit constraints, social norms, and additional limitations.

2.1 Gaussian Scalar Model

The previous setup is quite general and allows for complex signal structures. However, we will narrow our focus to the simple scalar Gaussian model to enable further discussion on our measurement of the role of information frictions and our estimation approach. Let X represent a set of observables that characterize a high school student. We begin by assuming that returns, condition on X, follow a normal distribution

$$\mathcal{R}|X = x \sim N\left(\mu_{\mathcal{R},x}, \sigma^2_{\mathcal{R},x}\right)$$

We assume that agents receive a noisy measurement on their returns, therefore their signals are distributed as

$$s|\mathcal{R}, X = x \sim N\left(\mathcal{R}, \sigma^2_{\epsilon,x}\right)$$

As the signal is normally distributed around the truth, and returns are normally distributed, individuals with signal realization s, forms the following posterior mean belief

$$E[\mathcal{R}|s, x] = \left(\frac{\mu_{\mathcal{R},x}\sigma^2_{\epsilon,x} + \sigma^2_{\mathcal{R},x}s}{\sigma^2_{\epsilon,x} + \sigma^2_{\mathcal{R},x}}\right) \tag{1}$$

As we can see, if the variance of the noise in the signal, σ_ϵ^2 is higher, agents put higher weight on their prior, and pay less attention to the information. We can now write explicitly the decision rule for individuals with cost $c(x)$ and signal realization s

$$D = \mathbb{1}\left[E[\alpha_1 - \alpha_0|s,x] \geq c(x)\right] = \mathbb{1}\left[\frac{\mu_{\mathcal{R},x}\sigma^2_{\epsilon,x} + \sigma^2_{\mathcal{R},x}s}{\sigma^2_{\epsilon,x} + \sigma^2_{\mathcal{R},x}} \geq c(x)\right]$$

From now on we subsume x, unless it adds something substantial. We can also calculate the share of individuals with cost c who choose to go to college. We first notice that as s is normally distributed and the linearity of the posterior means, the beliefs are also normally distributed with the following mean and variance

$$E[\mathcal{R}|s] \sim N\left(\mu_{\mathcal{R}}, \frac{\sigma^4_{\mathcal{R}}}{\sigma^2_{\mathcal{R}} + \sigma^2_\epsilon}\right)$$

and the share of individuals who would choose to attend college, given cost $c(x)$ is given by

$$P(D = 1|c) = \Phi\left(\frac{\mu_{\mathcal{R}} - c}{\sqrt{\frac{\sigma^4_{\mathcal{R}}}{\sigma^2_{\mathcal{R}} + \sigma^2_\epsilon}}}\right) \tag{2}$$

where Φ is the standard Gaussian Cumulative Distribution Function (CDF).

2.2 The Effect of Information and Returns on Choice

In the framework discussed, individual choice is influenced by two factors: the net returns $\mathcal{R} - c(x)$, and the individuals' knowledge of these returns. Our analysis aims to discern how these elements affect decision-making across different social groups. We begin by detailing the measurement of individuals' knowledge of their returns. Our focus is on 'information quality,' which is quantified by the coefficient of determination, often denoted by R^2 (R-Squared). This measure is commonly used in statistical literature to assess the predictive accuracy of models. Specifically, we define the quality of information as

$$R^2_{\mathcal{R}} = \frac{\text{Var}(E[\mathcal{R}|s])}{\text{Var}(\mathcal{R}|x)} \tag{3}$$

This coefficient quantifies the portion of explained variance of returns, captured by the variance of the predictor $E[\mathcal{R}|s]$, relative to the total variance in returns. In statistical contexts, we commonly use R^2 to evaluate the quality of statistical models by assessing how much of the observed variance the model can account for. In our specific context, we employ R^2 to compare the informational value held by individuals from different groups. The R^2 value ranges between 0 and 1, with a value close to one indicating that the information can explain nearly all the variance in returns. Conversely, an R^2 value close to zero suggests that the information possessed by agents is of limited utility in explaining returns.

How does the quality of information and returns affect the decision on going to college? By rearranging (3) as follows:

$$\mathrm{Var}\left(E[\mathcal{R}|s]\right) = \sigma^2_{\mathcal{R}} R^2_{\mathcal{R}}$$

We observe that as information quality increases, beliefs dispersion increases as well. Intuitively, if individuals have access to better quality, more accurate, information, then they would respond to it more, and rely on it more when updating their beliefs, this would cause an increasing belief dispersion. Therefore, better information implies higher variance in beliefs.

Whether higher beliefs dispersion implies that more individuals would attend college is contingent upon the relationship between the cost of attendance and the mean returns in the population, $\mu_{\mathcal{R}}$. Figure 1 illustrates the interaction between the prior, information quality, and cost and how they affect choices. The black line represents the cost. The two red lines represent the survival functions for priors lower than the cost, while the two blue lines represent the survival functions for priors higher than the cost. Dashed lines indicate the posterior beliefs distribution for an agent with high-quality information, and solid lines represent the posterior mean distribution with low-quality information. The figure shows that if the cost is lower than $\mu_{\mathcal{R}}$, increasing the precision of the signal—or enhancing information quality—would actually reduce college attendance. Conversely, if the mean returns exceed the cost, a reduction in information quality could prove actually increase the share of individuals who opt in to college. Moreover, the impact of supplementary information also depends on the interplay among the three factors. By taking the derivative of Equation 2 with respect to the variance of the noise, it is observed that as $\mu_{\mathcal{R}} - c$ increases, this effect becomes more pronounced.

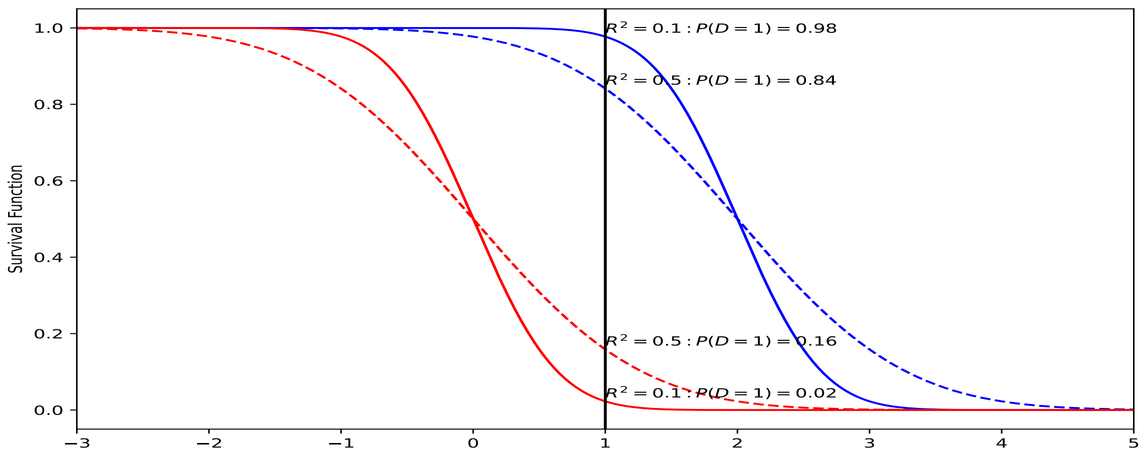

Figure 1: Cost, information and Beliefs interaction

Notes: This figure illustrates the interaction between the prior, information quality, and cost and how they affect choices. The black line represents the cost. The two red lines represent the survival functions for priors lower than the cost, while the two blue lines represent the survival functions for priors higher than the cost. Dashed lines indicate the posterior beliefs distribution for an agent with high-quality information, and solid lines represent the posterior mean distribution with low-quality information. The figure demonstrates that if the priors are higher than the cost, providing additional information reduces the share of participants from 98% to 0.84%. Conversely, if the prior is lower than the cost, improving the quality of information increases the share of individuals who opt in.

2.3 Two Measures of Information and Choice Gaps

Building on our model in the previous section we now discuss how to measure the role differences in information quality affect choices. We assume that individuals are categorized into two mutually exclusive groups, denoted by $g \in \{a, b\}$. Additionally, we posit that the proportion of college attenders in group b exceeds that in group a, formally $Pr(D = 1|b) \geq Pr(D = 1|a)$. In our empirical framework, these groups correspond to Whites and Hispanics, respectively. To understand the impact of information frictions on the choice gap, $Pr(D = 1|b) - Pr(D = 1|a)$, we introduce two metrics. The first metric explores how the dispersion in the information quality among the two groups affects their gap. The second metric assesses the level of accuracy required in the information accessible to the disadvantaged group (group a) to achieve choice parity. In the appendix we consider two more measures of the role of information. First, in section E of the appendix, we investigate an alternative measure for information's role. In this measure, we equate the signals received by individuals with the same returns in both groups. We also ensure that they update their beliefs correctly based on their own distribution of returns. We then examine how this affects the choice gap. Then in section G we consider additional decomposition method, that consider the quality of information with respect to the individuals priors and what they possibly know on the joint distribution.

2.3.1 Differences in information quality

Our first measure focuses on how much of the gap is driven by differences in the quality of information, by equating the information quality across groups. Equating information qualtiy can be thought of, in our Gaussian model, as equating the "normalized" signal dispersion, that is scaled by the variance of the underlying returns, i.e. equating the Signal-to-Noise Ratio ($SNR = \frac{\sigma^2_{\mathcal{R}}}{\sigma^2_\epsilon}$). To illustrate, note that we can write the R^2 in the Gaussian model as:

$$R^2 = \frac{\frac{\sigma^4_{\mathcal{R}}}{\sigma^2_{\mathcal{R}}+\sigma^2_\epsilon}}{\sigma^2_{\mathcal{R}}} = \frac{\sigma^2_{\mathcal{R}}}{\sigma^2_{\mathcal{R}}+\sigma^2_\epsilon} = \frac{1}{1+\frac{1}{SNR}}$$

A nice property of equating the R^2 in our model is that the two groups update beliefs similarly, while observing a with respect to the same information. To see that, using equation 1, we have

$$E[\mathcal{R}|s] = \frac{\sigma^2_\epsilon \mu_{\mathcal{R}} + \sigma^2_{\mathcal{R}} s}{\sigma^2_{\mathcal{R}}+\sigma^2_\epsilon} = \frac{\mu_{\mathcal{R}} + SNR \times s}{1+SNR}$$

This suggests that once we adjust for the signal-to-noise ratio between the two groups, each member of the group gives the same importance to the signal s when updating their information. Consequently, the impact of the same piece of information s on returns influences the beliefs of both groups equally after we equate the R^2.

To quantify the role of information in exploring the gap, we suggest using a decomposition method à la Kitagawa (1955), Blinder (1973), and Oaxaca (1973). In it, we break down the differences in choices into two components, stemming from the varying predictability of returns between groups and the a residual term we call composition effects. Specifically, we investigate what proportion of individuals would choose to attend college if individuals with the same cost x had access to the same quality of information. Denote $R^2_{\mathcal{R},g,x}$ the information quality on returns of individuals from group g and observables x. Similarly, denote the cost function of group g as $c_g(x)$. Within the context of the Gaussian model we can decompose the observed gap in choices to

$$
\begin{aligned}
P(D=1|b)-P(D=1|a) = & \underbrace{\int_X \Phi\left(\frac{\mu_{\mathcal{R},b,x}-c_b(x)}{\sqrt{\sigma^2_{\mathcal{R},b,x}R^2_{\mathcal{R},b,x}}}\right)dF_b(x) - \int_X \Phi\left(\frac{\mu_{\mathcal{R},b,x}-c_b(x)}{\sqrt{\sigma^2_{\mathcal{R},b,x}R^2_{\mathcal{R},a,x}}}\right)dF_b(x)}_{\text{Information Channel}} \\
& \underbrace{+\int_X \Phi\left(\frac{\mu_{\mathcal{R},b,x}-c_b(x)}{\sqrt{\sigma^2_{\mathcal{R},b,x}R^2_{\mathcal{R},a,x}}}\right)dF_b(x) - \int_c \Phi\left(\frac{\mu_{\mathcal{R},a,x}-c_a(x)}{\sqrt{\sigma^2_{\mathcal{R},a,x}R^2_{\mathcal{R},a,x}}}\right)dF_a(x)}_{\text{Composition Channel}}
\end{aligned}
$$

where we use $F_g(x)$ to denote the CDF of x for group g. The information channel quantifies the extent to which the gap in choices arises from individuals having access to information of differing quality, despite equal cost, which affects their ability to predict the outcomes of their choices. Another perspective is to consider cases where members of group a and group b employ different models to predict the outcomes of their choices. The quality of these models originates either from the information they possess or from the underlying data-generating process of outcomes. We inquire how much of the gap arises from differences in the quality of these predictive models, where quality is measured using R^2. We postulate a counterfactual world in which we equalize the quality of these two models and examine how that would impact choices. In the Gaussian model, this has a straightforward interpretation as reducing the variance of the noise in the signal observed by individuals, σ^2_ϵ. In more general settings, with unrestricted data-generating processes providing more nuisance information, equalizing R^2 does not yield a closed-form solution. In many cases, different joint distributions of signals and outcomes may produce the same R^2 but induce complex choice patterns that contribute to gaps in choices influenced by information. In section E in the appendix, we discuss another decomposition approach that equalizes the information structure across groups. This approach does not equalize the ability to predict across groups, but rather equalizes the signals that individuals with similar outcomes receive.

It is important to recognize that our analysis is a partial equilibrium exercise, where we use comparative statics to equalize the information quality between the two groups. Typically, information quality is determined endogenously within an equilibrium framework (Coate and Loury (1993), Lundberg and Startz (1983)), and is driven by choices individuals make that form the information environment and what agents can know. Furthermore, the information quality that individuals possess could be influenced by the effort they invest in acquiring it, a concept central to the standard rational inattention model (Caplin et al. (2022); Maćkowiak et al. (2023)). In this decomposition exercise, we do not delve into the underlying factors that drive these information discrepancies; rather, we accept them as given and investigate their extent of contribution to the observed disparity.

We now turn to discuss the second channel. The residual component, denoted as the composition effect, poses the inverse question: How much would the share of high school graduates from group a change if we held their information quality $R^2_{a,c}$ constant, but altered the composition of their returns and costs to match those of group b? This component informs us how much of the gap is driven by differences in the outcome distribution itself. Therefore, we interpret this component as quantifying the portion of the gap driven by the fundamentals themselves.

The two components of the distribution carry distinct policy implications. If the majority of the gap is driven by differences in predictive ability, policymakers aiming to close this gap should consider transferring the superior information or modeling techniques from group b to group a. This could involve educational interventions, information dissemination, or providing improved prediction tools for group a. Conversely, if the gap primarily stems from variations in the outcome distribution, policymakers concerned with narrowing the disparity should focus on policies that directly influence this distribution. This could include measures such as altering tax structures, providing targeted subsidies, or implementing regulatory changes that affect the underlying returns and costs for both groups. Identifying the primary driver of the gap not only enhances our understanding of its structural roots but also provides actionable insights for policymakers committed to fostering equal opportunities across different groups.

2.3.2 Quantifying Precision Needed for Outcome Equality

Our second measure takes a practical approach, focusing on how much we would need to alter the information accuracy for the disadvantaged group to achieve choice parity. If significant modifications are needed, this suggests that improving information quality may not be the most effective way to close the choice gap. Instead, policymakers aiming to reduce disparities may need to directly address differences in returns. For this measure, we find the quality of information that group a needs to have in order to achieve parity in choices with group b. Specifically we look for $R^2 \in [0, 1]$ that solves the following equation

$$P(D = 1|\mathrm{b}) = \int_X \Phi\left(\frac{\mu_{\mathcal{R},a,x} - c_a(x)}{\sigma^2_{\mathcal{R},a,x} R^2}\right) dF_a(x) \tag{4}$$

We then define the importance of information to the gap by

$$R^2 - \int_X R^2_{\mathcal{R},a,x} dF_a(x) \tag{5}$$

That captures the difference between the needed R^2 and the average current quality of information group a has. A large difference between these values suggests that policymakers would need to deliver considerably more accurate and informative signals to bridge the choice gap effectively. This metric also has ramifications in terms of policy cost. In most information cost models (Pomatto et al. (2023), Caplin et al. (2022), Maćkowiak et al. (2023)), the cost function adheres to the Blackwell order. As a result, in our Gaussian model, an elevated R^2 implies that the policymaker would incur higher costs to generate this supplemental information.

To better understand how a policymaker can influence individual choices and increase the R^2, we explore the quality of additional information needed to equate attendance rates across groups. Here, "additional information" refers to new signals that are orthogonal to an agent's existing information set; that is, we focus exclusively on previously unknown information that a policymaker could introduce. In practice, a policymaker is likely to disseminate information that correlates with what individuals already know, potentially overlapping with their private information. Therefore, in our thought exercise, we consider the case in which individuals first residualize the policymaker's signal and use only their existing information and the additional residualized information to inform their beliefs. We examine how the information quality of this additional information affects the choice gap. Specifically, we engage in three thought exercises for this purpose. First, we consider providing information that is solely informative about earnings if the individual chooses to go to college. Second, we examine the opposite scenario where the additional information is informative only about earnings if they opt not to go to college. Finally, we consider providing information that is relevant to both types of earnings. In our thought experiments, we assume that the policymaker, akin to an econometrician, can only provide information on the marginal distributions of α_1 and α_0, as she cannot know their joint distribution. For example, the policymaker could offer students a series of tests, then provide predictions on potential earnings depending on whether they attend college. To measure the precision of this additional new information, we quantify it by its ability to explain the marginals of α_1 and α_0, therefore we describe these additional signals in terms of R^2.

To formally introduce this new information, let s_n be the additional signal that a policymaker provides to Hispanics, after it has been partialled out from the agent's existing information. We assume that the signals are drawn from a Gaussian distribution and are correlated with the residual U_1 and U_0. The fact that the signal is partialled out implies that $\text{Cov}(s_n, \boldsymbol{S}) = 0$. Furthermore, as the signals and state are jointly normal, the agent's beliefs are additive. Specifically, we can write the agent's beliefs, given their current signals and the additional information, as

$$E[\alpha_1 - \alpha_0 | \mathcal{S}, s_n] = E[\alpha_1 - \alpha_0 | \mathcal{S}] + \frac{\text{Cov}(\alpha_1 - \alpha_0, s_n)}{\text{Var}(s_n)} s_n$$

As the agents are Bayesian, their mean beliefs are determined by the law of iterated expectations. Since we assume that s_n is Gaussian, we only need to derive the variance of the new beliefs with the additional information. Denote $R^2_{1,n}$ and $R^2_{0,n}$ as the information quality of the new signals on U_1 and U_0, respectively. Then note that we can express the additional component to beliefs as

$$\begin{aligned}
\text{Var}\left(\frac{\text{Cov}(s_n, U_1)}{\text{Var}(s_n)} s_n\right) &= \frac{\text{Cov}^2(s_n, U_1)}{\text{Var}(s_n)} = \sigma_1^2 R^2_{1,n} \\
\text{Var}\left(\frac{\text{Cov}(s_n, U_0)}{\text{Var}(s_n)} s_n\right) &= \frac{\text{Cov}^2(s_n, U_0)}{\text{Var}(s_n)} = \sigma_0^2 R^2_{0,n}
\end{aligned}$$

Without loss of generality, we fix $\text{Var}(s_n) = 1$ and set $\text{Cov}(s_n, U_1)^2$ to meet the required R^2 and the variance of new beliefs are distributed with the following variance

$$\begin{aligned}\mathrm{Var}(E[\mathcal{R}|\boldsymbol{S}, s_n]) &= \sigma_{\mathrm{E}}^2 + \mathrm{Cov}^2(U1, s_n) + \mathrm{Cov}^2(U_0, s_n) - 2\mathrm{Cov}(U1, s_n)\mathrm{Cov}(U1, s_n) \\ &= \sigma_{\mathrm{E}}^2 + \sigma_1^2 R_{1,n}^2 + \sigma_1^2 R_{0,n}^2 - 2\sqrt{R_{1,n}^2 R_{0,n}^2}\sigma_1\sigma_0\end{aligned}$$

We can then calculate the counterfactual share of students who would attend college if they were provided with this new information. Notice that in order to calculate the counterfactual shares we do not need to know the correlation between α_1 and α_0, as we consider how the new information is informative on the marginals, but not on the difference.

2.3.3 Other Empirical Applications

While our focus is on educational decisions, the decomposition can also shed light on how information availability shapes behavior.

Example 2.1 (Discrimination)**.** Recent research has focused on the factors contributing to disparities in decision-making across groups, largely attributing these to individual preferences. Works by Arnold et al. (2018), Arnold et al. (2022), and Canay et al. (2020), explore the role of judicial preferences and biases. There is also increasing interest in understanding how decision-making signals contribute to these disparities. The decomposition approach aims to tackle these issues.

Example 2.2 (Changes over Time)**.** The decomposition method can reveal trends in decision-making over time by comparing two different time periods, labeled as groups A and B. This enables researchers to quantify behavioral changes attributable to better information access, either through belief identification or surveys.

Example 2.3 (Policy Decomposition)**.** Researchers can use the decomposition approach to analyze the behavioral consequences of policy shifts, particularly within the rational expectations framework prevalent in economics[2]. The method quantifies the impact of policies by isolating changes in returns and information access. For example, a tax code revision that simplifies the return process (Caldwell et al. (2023)) would alter both the amounts and predictability of returns. The decomposition approach allows researchers to quantify each factor's influence on tax return applications.

[2] For instance, Bhandari et al. (2021) analyzes welfare effects due to changes in consumption information.

2.4 Model Identification and Empirical Specification

In this section we discuss how the Gaussian model can be partially identified using data on choices and outcomes. In appendix C, we show how the choice model can be identified nonparametrically with a continuous instrument and results from the Marginal Treatment effect literature (Heckman and Vytlacil (2005)) and identification of discrete choice models (Matzkin (1992),Matzkin (1993)). In what follows we briefly go over the identification of the normal model and it's important components for our analysis.

2.4.1 Identifying the Gaussian model Parameters

We assume we observe a set of covariates X, a continuous instrument Z and outcomes Y. Although it's not imperative for identification argument, we parametrize the cost function as a linear function of covariates

$$c(x, z) = Zb_z + Xb_x$$

We assume that the distribution of $\alpha_1|X, D = 1$ and $\alpha_0|D = 1, X$ is observed. In the appendix we discuss how it can be identified using Panel data and additional assumptions on the wages. For our discussion the α_1 and α_0 can be thought of as fixed effect, and are identified from panel data. We also assume that α_1 and α_0 are linear in covariates

$$\alpha_1 = X\beta_1 + U_1$$
$$\alpha_0 = X\beta_0 + U_0$$

Following our discussion on the Gaussian model, we assume that beliefs and residuals U_1 and U_0 are jointly normal, X operates only as a mean shifter and Z is independent from the potential outcomes, $Z, X \perp\!\!\!\perp U_1, U_0$.

$$\begin{pmatrix} U_1 \\ U_0 \\ E[\mathcal{R}|s,x] \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ X(\beta_1 - \beta_0) \end{pmatrix}, \begin{bmatrix} \sigma_1^2 & \rho_{1,0}\sigma_1\sigma_0 & \sigma_{1,\mathrm{E}} \\ \rho_{1,0}\sigma_1\sigma_0 & \sigma_0^2 & \sigma_{0,\mathrm{E}} \\ \sigma_{1,\mathrm{E}} & \sigma_{0,\mathrm{E}} & \sigma_{\mathrm{E}}^2 \end{bmatrix} \right)$$

The individuals decision rule is then given by

$$D = \mathbb{1}\left[E[\alpha_1 - \alpha_0 \mid s, x] \geq c(z, x)\right] = \mathbb{1}\left[E[U_1 - U_0 \mid s, x] \geq c(x, z) - X(\beta_1 - \beta_0)\right]$$

Using the fact that beliefs and U_1 and U_0 are jointly normal, we have that the choice probability is given by

$$P(D = 1|x, z) = \Phi\left(\frac{c(x, z) - X(\beta_1 - \beta_0)}{\sigma_{\mathrm{E}}}\right) \tag{6}$$

Notice that in general this is not enough to identify the cost function of parameters, as all parameters are identified up to scale. In addition, covariates can play a dual role, both affecting the outcome variable and controlling the cost. Therefore, we need to identify the scale parameter and the coefficients β_1 and β_0. To identify β_1 we use the standard Heckman Correction argument for Gaussian selection model (Heckman (1979)). Specifically, using the fact that U_1, U_0 and beliefs are jointly Gaussian, we have that

$$E[\alpha_1|D = 1, X] = E[\alpha_1 + \epsilon^1] = E[\alpha_1 + \epsilon^1] = X\beta_1 + E[U_1|D = 1, X]$$

where $E[U_1|D = 1, X] = \frac{\sigma_{1,\mathrm{E}}}{\sigma_{\mathrm{E}}} \frac{\phi\left(\frac{Xc_x - X(\beta_1-\beta_0)}{\sigma_{\mathrm{E}}(x)}\right)}{1-\Phi(\left(\frac{Xc_x - X(\beta_1-\beta_0)}{\sigma_{\mathrm{E}}(x)}\right)}$. We can follow the same argument to identify β_0, and using the fact that $E[\alpha_0|D = 0, X] = \frac{\sigma_{0,\mathrm{E}}}{\sigma_{\mathrm{E}}} \times -\frac{\phi\left(\frac{\gamma z - E[\theta]}{\sigma_\eta}\right)}{\Phi\left(\frac{\gamma z - E[\theta]}{\sigma_\eta}\right)}$. Denote the coefficient of the mills ratio as $\gamma_1 = \frac{\rho_{1,\mathrm{E}}\sigma_1}{\sigma_{\mathrm{E}}}$ and $\gamma_0 = \frac{\sigma_{0,\mathrm{E}}}{\sigma_E}$, and notice that we can identify σ_1 and σ_0 using the joint distribution of choice and earnings

$$f(D = 1, \alpha_1, z, x) = \left(1 - \Phi\left(\frac{\frac{\mu(z,x)-\mu_\eta}{\sigma_\eta} - \frac{\gamma_c^1}{\sigma_1}\left(\frac{y_1-\mu_1}{\sigma_{y_1}}\right)}{\sqrt{(1 - (\frac{\gamma_c^1}{\sigma_1})^2))}}\right)\right)\phi\left(\frac{\alpha_1 - \mu_1}{\sigma_1}\right)\frac{1}{\sigma_1} \tag{7}$$

and similarly for σ_0. Finally, in order to get σ_{E}, we can use the fact that the covariance of beliefs and returns equal to the variance of returns, $\mathrm{Cov}(\mathcal{R}, \mathrm{E}[\mathcal{R}|s, x]) = \mathrm{Var}(\mathrm{E}[\mathcal{R}|s, x])$. To see that notice that we can decompose returns as

$$\mathcal{R} = E[\mathcal{R}|s, x] + r$$

where $\mathrm{Cov}(E[\mathcal{R}|s, x], r) = 0$, and therefore $\mathrm{Cov}(E[\mathcal{R}|s, x], \mathcal{R}) = \mathrm{Var}(E[\mathcal{R}|s, x])$. Therfore, using the coefficient on the control function in the regression we have

$$\gamma_c^1 - \gamma_c^0 = \frac{\sigma_{1,\mathrm{E}} - \sigma_{0,\mathrm{E}}}{\sigma_\mathrm{E}} = \frac{\sigma_\mathrm{E}^2}{\sigma_\mathrm{E}} = \sigma_\mathrm{E}$$

Finally, the correlation between α_1 and α_0, is known to be un identified, as we can not observe both α_1 and α_0 simultaneously. Hence, our analysis explores the bounds on how this unobserved correlation affects the role of information. Then, for a given correlation parameter $\rho_{1,0}$, we can identify the R^2 needed for our decomposition by

$$\mathcal{R}^2 = \frac{\sigma_E^2}{\sigma_1^2 + \sigma_0^2 - \rho_{1,0}\sigma_1\sigma_0} \tag{8}$$

2.4.2 Testable implication of the model

In the model we assume that agents select on on outcome, therefore, this implies a testable restriction, which is that

$$\sigma_\mathrm{E} = \gamma_c^1 - \gamma_c^0 \geq 0$$

as σ_E standard error. In the section C in the section C.1.1 in the appendix we show that this restriction also hold nonparametrically, and implies that the Marginal Treatment effect (Heckman and Vytlacil (2005)) is decreasing. This is known implication of the Extended Roy Model (d'Haultfoeuille and Maurel (2013), **?**)

2.4.3 Restrictions on the correlation parameter

Our theoretical framework implies some constraints on the correlation between U_1 and U_0, that is informed by our model that implies some selection on returns. As elucidated in **?**, the variance of beliefs about returns is constrained by the actual variance of returns. This is expressed by the inequality:

$$_\mathrm{E}^2 \leq \sigma_1^2 + \sigma_0^2 - 2\rho\sigma_1\sigma_0$$

This restriction is a generalization of the known fact in the standard Roy model (Roy (1951)) with complete outcome information, where the joint distribution of potential outcomes is point-identified (Heckman and Robb (1985)). If we assume agents have complete information, the inequality becomes an equality. If we maintain that agents select based on outcomes but have incomplete information, we can use the above inequality to bound the correlation between potential outcomes.

We can further restrict the bounds using the fact that we can identify the covariance between beliefs, $\mathrm{E}[\alpha_1 - \alpha_0 | s, x]$ and U_1 and U_0. To do so we use the fact that the covariance matrix must remain positive semi-definite, we therefore restrict the set of possible $\rho_{1,0}$ to values that keep the following covariance matrix positive semi-definite.

$$\mathrm{Cov}(\boldsymbol{\alpha}, \mathbb{E}) = \begin{bmatrix} \sigma_1^2 & \rho_{1,0}\sigma_1\sigma_0 & \sigma_{1,\mathbb{E}} \\ \rho_{1,0}\sigma_1\sigma_0 & \sigma_0^2 & \sigma_{0,\mathbb{E}} \\ \sigma_{1,\mathbb{E}} & \sigma_{0,\mathbb{E}} & \sigma_{\mathbb{E}}^2 \end{bmatrix}$$

2.5 Estimation

We now turn to describe how we estimate the Gaussian choice model. As discussed in the , for non-parametric estimation, we need to employ an instrumental variable z. We therefore include it here in the discussion. We first start by estimating α_1 and α_0 by averaging wages over periods of time

$$\hat{\alpha}_{di} = \frac{1}{T-t}\sum_{a=t}^{T} Y_{i,a}^{d}$$

Then, given our $\hat{\alpha}_1$ and $\hat{\alpha}_0$, we estimate the model in three steps. In the first step, we estimate the propensity score using a probit model, the covariates X, and the instrument Z. In the second step, we use the Heckman control function approach (Heckman (1979)) to estimate β_1 and β_0. As discussed in the previous section, we obtain the standard deviation of beliefs from the coefficients on the control function. Next, we show how we can estimate the cost function. Using equation 6, we see that the probit regression coefficients, standardized by the standard deviation of beliefs, are impacted by both beliefs and costs. To adjust for this, we rescale the coefficients and add the conditional expectations, estimated using the control function approach:

$$\hat{c}(x,z) = \hat{\sigma}_\eta \times (z\hat{b}_z + X\hat{b}_x) + X(\hat{\beta}_1 - \hat{\beta}_0)$$

Finally, to get σ_1 and σ_0, we solve the maximum likelihood function as shown in equation 7.

To estimate our first measure of information contribution to the gap we simply calculate the $\hat{R}^2$, as discussed in 8 for both groups. We then estimate the information channel as

$$\underbrace{\hat{P}(D=1|b)}_{\text{Observed}} - \underbrace{\frac{1}{N}\sum_i \Phi\left(\frac{X(\hat{\beta}_1 - \hat{\beta}_0) - \hat{c}(x_i, z_i)}{\sqrt{\hat{\sigma}^2_{\mathcal{R}} \hat{R}^2_a}}\right)}_{\text{Counterfactual}}$$

Where the first part is just the observed share and the second part is the counterfactual share of individuals who choose to attend, if they had the same R^2 as group a. To estimate this part we simply average over $\Phi\left(\frac{X(\hat{\beta}_1 - \hat{\beta}_0) - \hat{c}(x_i, z_i)}{\sqrt{\hat{\sigma}^2_{\mathcal{R}} \hat{R}^2_a}}\right)$ for all the observation of group b. Estimation of the composition channel is done the same. To estimate the quantity in 4, we again fix a correlation parameter and solve for the R^2 that satisfies the sample approximation of the integral in 4:

$$\sum_i \Phi\left(\frac{X_i(\hat{\beta}_1 - \hat{\beta}_0) - \hat{c}(X_i, z_i)}{\sqrt{\hat{\sigma}^2_{\mathcal{R}} R^2}}\right) = \hat{P}(D=1|b)$$

We solve the the above equation numerically using bi-section algorithm. As our empirical specification of the model assumes that $U_1, U_0 \perp\!\!\!\perp X, Z$, we then report discuss the difference between $R^2 - \hat{R}^2$, as this measure.

3 Data

Our empirical application investigates the factors contributing to the college attendance gap between Hispanic and White students. We concentrate on Texas, where there are large and comparable Hispanic and White populations, but they differ substantially in their choices. Utilizing the methods described in Sections 2.3.1 and 4, we aim to decompose the attendance choices and assess the influence of informational differences. We start by describing the data and then discuss the model results.

3.1 Data Sources and Sample Construction

Our empirical study leverages a series of confidential administrative databases from the state of Texas, the second most populous in the U.S. with a sophisticated higher education system that engages a substantial portion of its populace, including over one million high school students (Agency (2023)). Additionally, Texas boasts a significant Hispanic demographic, comprising around 12 million individuals in 2022, or about 40% of the state's total population, matched by a 40% representation of White population.

The study combines data from several Texas agencies. The primary dataset is procured from the Texas Education Agency (TEA), offering demographic details of all Texan high school students. This dataset is enriched with school characteristics from the National Center for Education Statistics (NCES), which provides a broader picture of Texas high schools. We incorporate assessments from the Texas standardized testing program, which evaluates public primary and secondary school students' competencies in various grades and subjects. Further, we integrate data concerning college enrollment decisions from the Texas Higher Education Coordinating Board (THECB), supplemented by information from the Integrated Postsecondary Education Data System (IPEDS). Finally, the Texas Workforce Commission (TWC) supplies data on post-high school earnings, completing our comprehensive dataset.

In constructing our control variables, we follow the approach used by Mountjoy (2022), utilizing three types of covariates: student-level demographics, school characteristics, and neighborhood characteristics. For student-level demographics, we include categorical variables for gender, eligibility for free or reduced price lunch as a proxy for economic disadvantage, and an indicator for graduation under one of three programs: the Distinguished Achievement Program, Recommended High School Program, or the Minimum High School Program, which reflect the various graduation tracks in Texas. In some of our analyses, we use test scores from Texas Assessment of Knowledge and Skills (TAKS) tests. We consider test scores from the exit exams in English-Language-Arts (ELA), which capture language skills, and Math test scores, these tests were held consistently across our three cohorts of interest. We then create a single measure of test scores by combining them in a one-factor model separately by cohort and normalize this factor to within-cohort percentiles. These high-stakes tests, which imply that they are likely to be indicative of student ability .Passing these exit-level test is a graduation prerequisite for Texas high school seniors in their junior and senior years.

For high school-level controls, we utilize NCES Common Core data, which incorporates the geographic locale code. This code categorizes urbanization into twelve detailed categories using Census geospatial data. Additionally, we include the distance to two-year colleges and an indicator denoting whether the school is classified as a Vocational Education School. Vocational schools are identified as those that provide formal training for semi-skilled, skilled, technical, or professional occupations to students of high school age who may opt to enhance their employment prospects, possibly instead of preparing for college admission. Controls also account for the local influence of the oil and gas industry, by measuring the long-term share of oil and gas employment at the high school level, employing NAICS industry codes from TWC workforce data. We normalize this measure of oil and gas employment by ranking it and control for its effects using a third-degree polynomial in our analysis of school characteristics.

Neighborhood characteristics include the 62 Texas commuting zones using the year-2000 mapping provided by the U.S. Department of Agriculture's Economic Research Service. We also construct an index of neighborhood quality, akin to the test score measure: We combine the tract-level Census measures of median household income and the percentage of households below the poverty line with the high school-level percentage eligible for free/reduced-price lunch into a one-factor model, then normalize this neighborhood factor to the within-cohort percentile. When controlling for neighborhood characteristics in the following discussion, we control for the third-degree polynomial of the neighborhood factor.

As outlined in section C in the appendix, nonparametric identification necessitates an instrument. We employ the measure of proximity to the nearest 4-year colleges, calculating ellipsoidal distances between the coordinates of all Texas public high schools (sourced from NCES CCD) and those of all Texas postsecondary institutions (from IPEDS). We determine the minimum distances within 4-year sectors for each high school. To supplement some missing distances, we refer to Mountjoy (2022), which involved manual collection of location data by verifying each college's institutional profile. We adopt the same methodology for the variable of distance to 2-year colleges.

We limit our sample to cohorts from 2003 to 2005 to ensure a long time horizon. This approach, leveraging our earnings data, allows us to observe outcomes 16 (for the cohort of 2003 and 2004) and 15 years (for the cohort of 2005) into the future, thus better understanding the incentives faced by these students. Additionally, the Texas Higher Education Coordinating Board (THECB) has provided data on students attending four-year colleges, including both private and public institutions, starting from 2003. We further narrow our sample to high school students who are not enrolled in special education programs, are between the ages of 17 and 18 in the 12th grade, and have graduated from high school with at least the minimum requirements. As with any study focused on a specific state, there is a risk of out-migration; however, Texas has one of the lowest out-migration rates in the U.S. (Times (2014)). Table A.7 in the appendix demonstrates the similarities in characteristics between high school students without income earnings data and the entire population. Following Mountjoy (2022), we also limit our test factor to individuals with grades below the 80s percentile. As Mountjoy (2022) discusses, high school students with a test factor higher than 80 are more likely to enroll in out-of-state colleges. Figure 14 in the appendix further illustrates that these individuals are more likely to have missing earnings data.

3.2 Summary Statistics

Table 1 presents summary statistics for the analysis cohort. The table indicates substantial disparity in socio-economic backgrounds among the groups. A significant proportion of Hispanics originate from low-income families, necessitating reduced-price or free meals. They also live in census tracts with higher unemployment rates and a greater proportion of families below the poverty line. Over 58% of Hispanics attend Title I schools, markedly more than their White counterparts. Conversely, regarding the programs offered at these schools, there is no substantial difference in the distribution. Similarly, there is no significant difference in how schools inform students about the oil industry; the proportion of high school graduates working in the oil and gas industries over the long term is similar. Geographically, Hispanics are more likely to reside in urban areas, while Whites predominantly live in suburban and rural areas. Furthermore, in terms of proximity to colleges, Hispanics tend to live nearer to both four-year and two-year colleges compared to non-Hispanic Whites.

Next, we examine the discrepancy in college attendance. The initial row of Table 1 indicates that the choice gap in the decision to attend a four-year colleges in the first year after high school graduation between Hispanics and Whites is 9%. Table A.8 in the appendix assesses the extent to which observable factors contribute to this disparity. Adjusting for individual, school, and geographical characteristics, as discussed in section 3.1, reduces the gap to between 6% and 7%. Figures 2 and 3 show that there is high dispersion for both Whites and Hispanics likelihood of attending college. The figure plots propensity scores for Hispanics and Whites attending college, estimated using a Probit model with our control set and the distance to a four-year college. The figures show that there's a large overlap between college attendance and large variation, Additionally, the distribution of propensity scores for both college attendees and non-attendees among Whites is more right-skewed than that of Hispanics. This skewness shows that Whites have a higher likelihood of college attendance than their Hispanic peers.

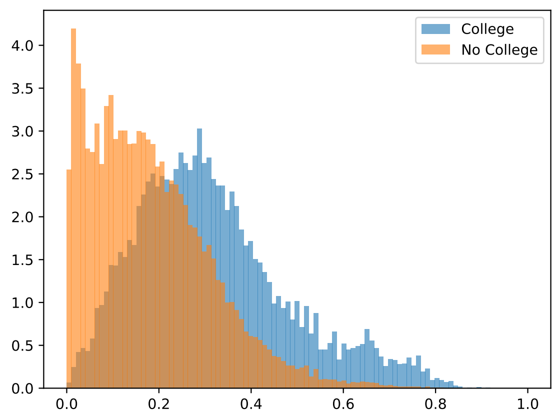

Figure 2: Raw Mean in Wages, without controls

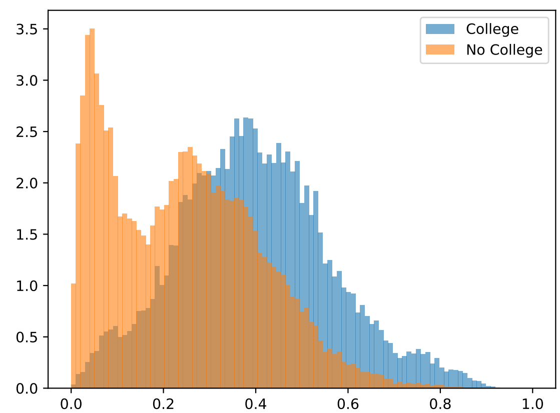

Figure 3: Raw Mean in Wages with controls

As discussed in the framework in Section 2, differences in choices can stem from variations in returns or disparities in information regarding these returns. Therefore, we delve into exploring these differences. Our exploration commences with an examination of discrepancies in test scores.

In our framework, test scores provide insights into the two key components driving decision-making. Firstly, grades serve as indicators of the distribution of human capital accumulated prior to the decision to attend or not. If we consider college education to be complementary to the accumulated human capital (Cunha and Heckman (2007), Heckman and Mosso (2014)), this suggest that differences in test scores are likely to imply also differences in college returns for Hispanics and Whites. However, grades offer more than just insights into returns; they also act as sources of information and signals available to agents. From this perspective, agents receive grades and employ them to form projections about the utility of these grades. Consequently, we also examine whether grades convey informative signals about returns and whether disparities exist between Whites and Hispanics.

Table 1 reveals a notable gap in academic readiness between Hispanics and Whites, as evidenced by exit exam grades. To what extent does this gap contribute to the overall disparity? The final row in Table A.8 in the appendix demonstrates that when we account for our measure of test scores, the gap narrows to 4.8%. Furthermore, Table 20 indicates that grades play a significant role in explaining choices. When incorporated into a Probit model that predicts choices, they enhance the Area under the Curve (AUC), increasing from 0.74 to 0.79 for Whites and from 0.73 to 0.77 for Hispanics. These results suggest that high school graduates are likely to utilize exit exam grades and their informational content in making decisions about college enrollment.

Are grades informative on returns? We explore whether grades are likely to contain information about returns. To ascertain whether grades predict earnings and returns, Figure 15 in the appendix illustrates the relationship between earnings and grades for both college attendees and non-attendees. The figure shows that for both Hispanics and Whites, higher grades correlate with increased earnings, irrespective of college attendance. Additionally, as grades increase, the earnings gap widens between those who attend college in their first year and those who do not. This is supported by the regression in Table A.9 in the appendix, which reveals that a one-unit increase in test scores raises the raw gap by approximately \$16, controlling for our set of controls. Both figures and regression table suggest that the differential informativeness of grades on the gap is relatively minor, as the gap escalates nearly proportionately with grades.

The relationship between school informativeness is further examined in Table A.10 in the appendix. This table presents the out-of-sample R^2 from a model that employs Extreme Gradient Boosting to predict earnings based on students' course-taking patterns and the pass-fail indicator for Hispanics and Whites. The R^2 values are remarkably similar for both groups. This implies that the quality of information from school performance measures is comparable for Whites and Hispanics.

These results indicate that Hispanics and Whites encounter varying distribution of returns. However, the quality of information available to them through the school system does not significantly differ. This motivates the utilization of our model to gain a deeper understanding of how these differences contribute to the choice gap.

	All	Hispanic	Whites
College Attendance	0.23 (0.42)	0.18 (0.38)	0.27 (0.44)
Test Factor Percentile	43.18 (22.02)	36.37 (22.02)	48.11 (20.67)
Math Score	45.62 (23.88)	40.29 (24.12)	49.49 (22.94)
Reading Score	47.5 (25.41)	40.11 (25.41)	52.87 (24.03)
No Disadvantage	0.7 (0.46)	0.41 (0.49)	0.91 (0.29)
Elig. Free Meals	0.22 (0.41)	0.44 (0.5)	0.06 (0.24)
Elig. Reduced Price Meals	0.06 (0.23)	0.09 (0.29)	0.03 (0.16)
Other Disadvantage	0.03 (0.16)	0.06 (0.23)	0.0 (0.05)
Distiguish	0.06 (0.24)	0.07 (0.25)	0.05 (0.23)
Minimal	0.22 (0.41)	0.19 (0.39)	0.24 (0.43)
Required	0.72 (0.45)	0.74 (0.44)	0.7 (0.46)
CT Median Income	44027.0 (21371.0)	36265.0 (15939.0)	49663.0 (22986.0)
CT Families Below Poverty Line	14.5 (10.82)	20.08 (12.19)	10.44 (7.42)
CT Share of Employed	63.21 (9.97)	59.92 (10.01)	65.6 (9.23)
Title I schools	0.34 (0.47)	0.58 (0.49)	0.17 (0.38)
No Participation in Tech Program	0.24 (0.43)	0.22 (0.41)	0.26 (0.44)
Enroll in Career Tech Elective (6-12)	0.23 (0.42)	0.2 (0.4)	0.24 (0.43)
Participate in Tech Prep Prog (9-12)	0.32 (0.47)	0.33 (0.47)	0.32 (0.47)
Participate in Tech Prep Prog	0.21 (0.41)	0.25 (0.43)	0.18 (0.38)
Share in Oil Industry	52.73 (28.53)	49.21 (29.14)	55.29 (27.79)
City	0.37 (0.48)	0.52 (0.5)	0.25 (0.44)
Suburb	0.32 (0.47)	0.24 (0.43)	0.38 (0.49)
Town	0.11 (0.31)	0.11 (0.31)	0.1 (0.31)
Rural	0.2 (0.4)	0.13 (0.34)	0.26 (0.44)
Distance to 4-Year College	19.82 (18.8)	18.19 (20.5)	21.04 (17.25)
Distance to 2-Year College	9.65 (11.49)	8.23 (11.65)	10.73 (11.24)

Table 1: Summary Statistics

Note: Columns include 12th-grade analysis cohorts from 2003-2005. NCES geographic categories are condensed into four types (city, suburb, town, rural).

We now turn to focus on earnings differences between Hispanics and Whites. Table A.2 in appendix shows the average quarterly earnings for Whites and Hispanics at various intervals post-graduation. Generally, wages are on an upward trend over time, albeit at a decreasing rate. It is also evident that the earnings of Whites are higher and more variable than those of Hispanics. We can also see a Greater variance of Whites earnings, implies that they are harder to predict, and in our Gaussian model, implies that Whites would need a more precise signal for accurate prediction. Figure 4 explores the implied differences in earnings between college attenders and non college attenders, across Hispanics and Whites. The figure plots the coefficient for attending a four-year college for both Hispanics and Whites, controlling on for cohort fixed effect. The mean differences increase over time for both groups. Notably, this difference widens in the first five years post-graduation, then stabilizes at around \$500, approximately 6% of the average quarterly earnings for Hispanics 14-16 years after graduation. Figure 5 introduces our set individuals, school level and neighborhood level controls. We see that this reduces the level, but does not affect the gap, which stays relatively similar.

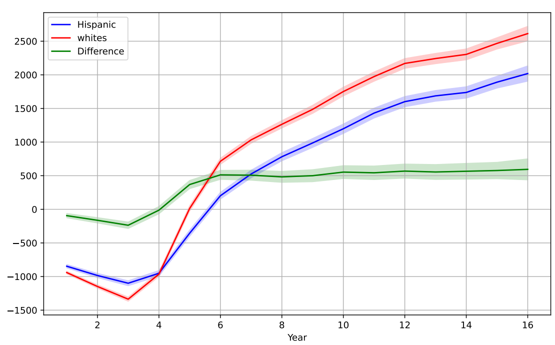

Figure 4: Raw Mean in Wages, without controls

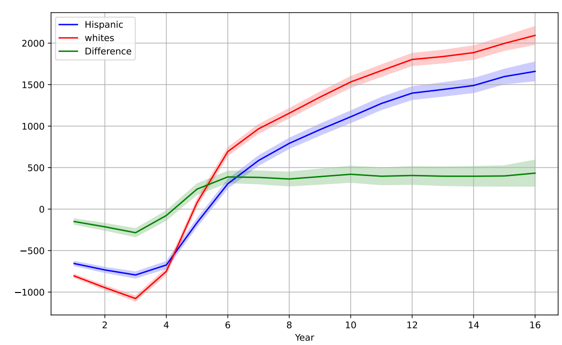

Figure 5: Raw Mean in Wages with controls

These mean differences suggest that Hispanic high school graduates may have less incentive to attend college compared to their White counterparts. However, this observed gap could be attributable to selection bias rather than reflecting the actual returns considered by the graduates. To investigate this, we utilize the distance to college from high school as an instrument in a Two-Stage Least Squares (2SLS) analysis. The use of this instrument follows the precedent set by Card (1995) and its subsequent application in works such as Carneiro and Lee (2009) Kapor (2020), Abdulkadiroğlu et al. (2020). Table A.3 in appendix, examines the correlation between the instrument and test scores. Initially, without additional controls, test scores show a significant correlation with the instrument. After including individual characteristics, this correlation persists, which might indicate that spatial sorting is non-random and likely tied to other factors that influence both outcomes and information. Subsequent rows in the table introduce more controls for school and neighborhood characteristics, which largely account for the initial correlation, rendering the coefficient on distance nearly null. Revisiting table A.3, we note that the inclusion of these controls does not markedly alter the instrument's effect.

Table A.4 in the appendix shows a strong first stage regression. The influence of distance to college on on the likelihood of attending a four-year college immediately after graduation. Controlling for our set of controls, we see that an increase of one mile in distance to college decreases the likelihood of college attendance by 0.2% for Hispanics and 0.1% for Whites. The magnitude of this effect remains relatively stable upon inclusion of different controls.

Finally, Table 2 presents the results from the 2SLS regression that instruments the treatment effect using our chosen instrument and includes all controls. It shows that, after adjusting for selection, the average effect for Hispanics is negligible, persisting up to 16 years post high school graduation. For White, on the other hand, there is a gradual effect that mirrors the earnings dynamics depicted in Figure 5, with significance emerging around 14-16 years. These findings suggest that the returns for Hispanics are generally much lower, potentially diminishing the incentive to pursue higher education. This underscores the need for a model that quantifies how these differences in returns influence decisions and contribute to the educational attainment gap.

	All	Whites	Hispanics
Avg. Wage 8-10	245.0	707.0	-1108.62
	(1194.0)	(1237.0)	(2028.0)
	245206	103198	142008
Avg. Wage 10-12	875.0	305.0	521.0
	(1436.0)	(1468.0)	(2295.0)
	239307	101284	138023
Avg. Wage 12-14	1552.0	255.0	2380.0
	(1531.0)	(1550.0)	(2370.0)
	233091	99428	133663
Avg. Wage 14-16	2605.0	377.0	5156.0
	(1632.0)	(1745.0)	(2424.0)
	149498	63271	86227

Table 2: Returns - Two Least Squares

Note:

4 Model Results

In this section, we estimate the model outlined in section 2.1 and discuss the implications of the estimated parameter for the role of information in determining the gap. Our analysis assumes that individuals are primarily concerned with their quarterly earnings 12-15 years post-graduation. As demonstrated in table 2, positive returns to college education commence approximately after 12 years. Consequently, we average the quarterly earnings within this 12-15 year period. This approach enables us to use data from our three cohorts and effectively capture the structural components, averaging over a long period.

We start our analysis by examining the relationship between the perceived cost of attending college and belief systems among Hispanic and White students. Figures 7 and 6 present histograms of the estimated costs for these groups, revealing that Hispanic students generally face lower attendance costs. As delineated in section 2, these costs encompass barriers to entry, such as credit constraints and discrimination, and are also influenced by preferences shaped by social norms, among other factors. Table 3 further shows this point, indicating that the average cost for Hispanic students corresponds to $1,199 of their quarterly earnings, compared to $2,879 for White students. Notably, while the average cost is higher for White students, the standard deviation for Hispanics, at 889, surpasses that of Whites, which is 693. This suggests a greater variability in the costs encountered by Hispanic students. These findings underscore the complexity of the cost dynamics in college attendance and its varied impact across different ethnic groups

In addition to cost analysis, Figures 6 and 7 also explore the distribution of conditional returns $\mathrm{E}[\alpha_1 - \alpha_0 | x]$, which represent the mean beliefs about returns for individuals with characteristic x. This measure captures the expected benefit of attending college, given certain attributes or circumstances. The figure a noticeable disparity: White students exhibit significantly higher expected returns than Hispanic students.

Table 3 further complements this analysis, showing that the average beliefs regarding returns[3] are lower than the actual average return for both groups. Specifically, the gap between the mean costs and mean beliefs about returns is narrower for White students (949) than for Hispanic students (2256). This implies that the share of White students would shift less in response to additional information. Following our discussion in section 2.1, this disparity suggests that, given the same quality of information, White students are more inclined on average to pursue college education compared to their Hispanic counterparts.

[3]also known as the Average Treatment Effect in treatment effect literature

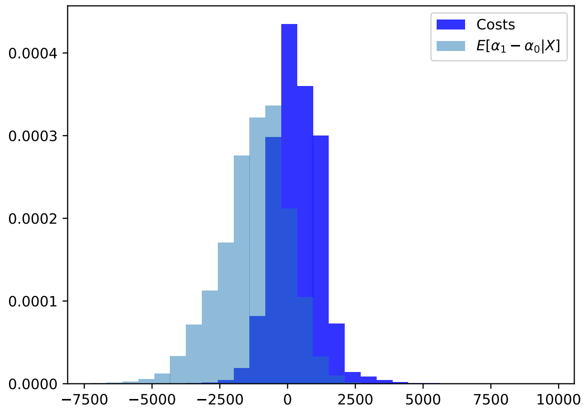

Figure 6: Hispanics Costs and Beliefs

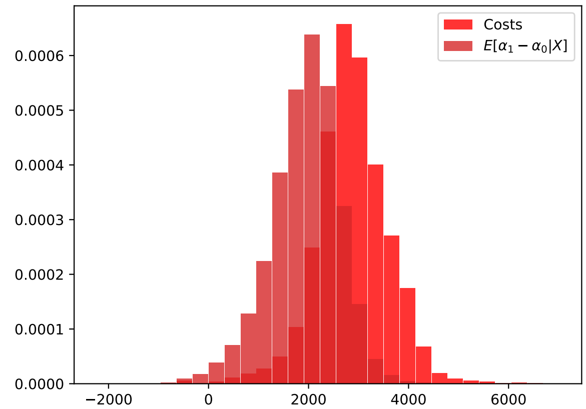

Figure 7: Whites Costs and Beliefs

We now turn to look at the estimated distributions of beliefs on $U_1 - U_0$. Table 3 presents the estimated variance of α_1, α_0 residuals, and beliefs across different wage periods for the two groups. The variance in beliefs among Hispanics is notably higher than that among Whites, and the variances for the residuals of α_1 and α_0 the means are generally higher among Whites, although we can't rule out statistically that they are equal. These differences suggest that the quality of information, as measured by R^2, is the same or lower for Whites than for Hispanics. If the residual variance of returns for Whites is higher or the same as that for Hispanics, this implies that choice outcomes are less predictable for Whites. In both cases, the quality of information on returns hinges on the covariance structure of U_1 and U_0. Figure 18 in the appendix shows plots the estimated cumulative distribution function (CDF) of the beliefs distribution, conditioned on the average covariates, and figure 19 shows the CDF where we fix all covariates and constant to zero. It is observed that for both Hispanics and Whites, the beliefs are systematically higher for the average White high school student. Concentrating on the CDF's shape when $X = 0$, we can again see that for White and Hispanics Students with the same observables, the beliefs of Whites are less dispersed.

	P(D=1)	σ_{E}	σ_1	σ_0	Avg. Cost	$E[\alpha_1]$	$E[\alpha_0]$	$E[\alpha_1 - \alpha_0]$	$Corr(E[\alpha_1 \vert x], E[\alpha_1 \vert x])$
Hispanics	0.21	2381	4490	6264	1199	6658.0	7715	-1057	0.76
		(657.0)	(125.0)	(818.0)	(889)	(1795.0)	(1843.0)	(2573.0)	(NA)
Whites	0.29	1414	5577	6316	2879	10871	8942.0	1930.0	0.95
		(873.0)	(155.0)	(491.0)	(693)	(2149.0)	(2211.0)	(3083.0)	(NA)

Table 3: Model Parameters

Note: The model parameters are estimated using different time spans of earnings after graduation. Standard errors for these parameters are provided in parentheses.

Next, we look at the implied information quality. To assess the quality of each group's information, we fix the correlation between U_1 and U_0, denoted as $\rho_{1,0}$, to different values and calculate the implied R^2.

$$R^2 = \frac{\hat{\sigma}^2_{\hat{E}}}{\hat{\sigma}^2_1 + \hat{\sigma}^2_0 - 2\rho_{1,0}\hat{\sigma}_1\hat{\sigma}_0}$$

Figure 8 displays the results of this comparison. It shows that as the correlation between U_1 and U_0 increases, the variance of returns declines, leading to higher implied information quality. For all correlation levels between U_1 and U_0, Hispanics display higher quality information on potential returns. Figure 9 preforms a similar exercise, fixing the standard errors returns, and compare the implied information quality. Despite higher information quality for Hispanics at a given correlation, the actual correlation between outcomes for Hispanics and Whites may differ. Although it is well-known that we cannot directly ascertain the correlation between the residuals of α_1 and α_0, we can make an educated guess by examining the impact of observed variables on these parameters. The last columns of Table 3 indicate that the correlation between $E[\alpha_1|X]$ and $E[\alpha_0|X]$ is notably high—ranging from 0.74 to 0.8 for Hispanics and from 0.91 to 0.97 for Whites, depending on the years used for wage measurement. This high correlation suggests that the observed variables affect both α_1 and α_0 similarly, as X increases. If unobserved variables also influence these outcomes similarly, the residuals of α_1 and α_0 would likely be highly correlated as well, implying high information quality. Notably, the correlations are much higher for Whites than for Hispanics, suggesting that while the correlation between residuals is likely positive for both groups, it is probably not identical. If we assume that the conditional expectation correlations mirror those for unobserved variables, the information quality for Whites could actually be much higher ($R^2 = 0.32$ for Hispanics with correlation 0.74 and $R^2 = 0.67$ for Whites with correlation 0.91).

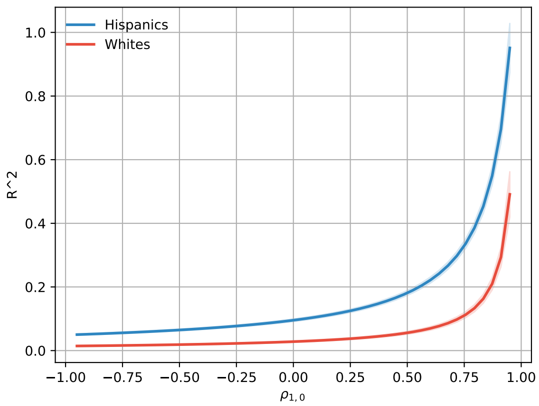

Figure 8: Information Quality

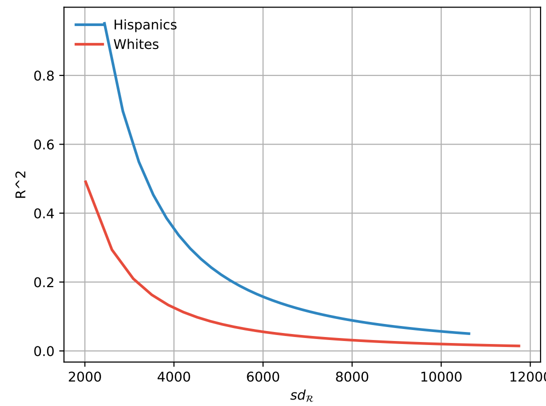

Figure 9: Information Quality

Notes: Figure 8 plots the variation in the quality of information regarding average returns to college attendance 12-15 years post high school graduation for Whites and Hispanics, as a function of the correlation between U_1 and U_0. Figure 9 plots how the quality of information fluctuates with changes in the standard deviation of $U_1 - U_0$.

4.1 The role of information Decomposition

We now turn to explore the decomposition exercise we discussed in section 2.3.1, to explain the 8% gap in college attendance ‘decisions between Hispanics and Whites. The first row in table 4 shows the bounds on the role of quality of information in explaining the choice gap. The first row shows the bounds on the proportion of White high school graduates attending college if they were to access the same level of information quality as their Hispanic counterparts, without restrictions on the possible correlations. The limits on the information's contribution to the gap are notably wide, ranging from −194%, which suggests that equipping Whites with Hispanic-level information quality could raise White college attendance by 0.15 to 0.44 percentage points, to 166%, indicating that such information could entirely eliminate the gap by reducing White attendance by 13 percentage points. These bounds are achieved at the extreme values of $\rho_{1,0}$. As previously discussed, these extreme values correspond to scenarios wherein one group has the highest possible information quality, while the other group has the lowest. Table **??** also shows that the composition channel is equally wide, capturing the residual effect, which can be interpreted as how much of the gap is affected by changing the returns and cost distribution of Hispanics to Whites. Table A.6 in the appendix shows a reverse decomposition, how much of the gap is explained by information if we gave Hispanics the information Whites have.

	Gap	Information Channel	Composition Channel
Avg. Wage 10-15	0.29-0.21=0.08		
1) No Assumptions			
LB, CF= 0.44,($\rho_w = -0.98, \rho_h = 0.9$)		-0.15 (-194.01%)	0.23 (294.0%)
UB,CF= 0.16,($\rho_w = 0.92, \rho_h = -0.96$)		0.13 (166.0%)	-0.05 (-65.8%)
2) Positive Correlation			
LB,($\rho_w = 0.01, \rho_h = 0.9$)		-0.13 (-165.18%)	0.21 (265.0%)
UB,($\rho_w = 0.92, \rho_h = 0.01$)		0.11 (137.0%)	-0.03 (-36.51%)
3) The same Correlation Parameter			
LB,($\rho_w = -0.96, \rho_h = -0.96$)		-0.03 (-42.06%)	0.11 (142.0%)
UB,($\rho_w = 0.9, \rho_h = 0.9$)		-0.01 (-12.48%)	0.09 (112.0%)
4) Max Correlation			
($\rho_w = 0.92, \rho_h = 0.9$)		0.003 (4.22%)	0.076 (95.78%)
4) $\rho_{1,0} \approx 0$			
($\rho_w = -0.01, \rho_h = -0.01$)		-0.03 (-40.12%)	0.11 (140.0%)
5) $R_1^2 = 0.1$			
LB,($\rho_w = -0.76, \rho_h = 0.88$)		-0.15 (-185.13%)	0.23 (285.0%)
UB,($\rho_w = 0.88, \rho_h = -0.9$)		0.12 (149.0%)	-0.04 (-49.28%)
6) $R_1^2 = 0.1$ and $\rho_{1,0} > 0$			
LB,($\rho_w = 0.01, \rho_h = 0.88$)		-0.13 (-159.4%)	0.2 (259.0%)
UB,($\rho_w = 0.88, \rho_h = 0.01$)		0.09 (116.0%)	-0.01 (-15.66%)

Table 4: Bounds on the role of information

Note: Bounds on the information channel, under different assumption on the correlation between α_1 and α_0

The bounds are wide as they allow for all possible correlation between α_1 and α_0. Figure 10 plots the counterfactual share of Whites endowed with Hispanic information quality, under varying values of $\rho_{1,0}$ for both Hispanics and Whites. The figure reveals that reducing the wage correlation for Hispanics, while elevating it for Whites, maximizes the proportion of Whites who would opt for college under the counterfactual information quality. Figure 11 presents the converse scenario, examining the counterfactual share of Hispanic students attending college if they possessed the same information quality as Whites. The figure indicates that as the earnings correlation for Whites rises and the earnings correlation for Hispanics declines, the proportion of Hispanic high school graduates opting for college attendance increases. The large potential effect of information shows that information is potentially an important channel in shaping the choice gap.

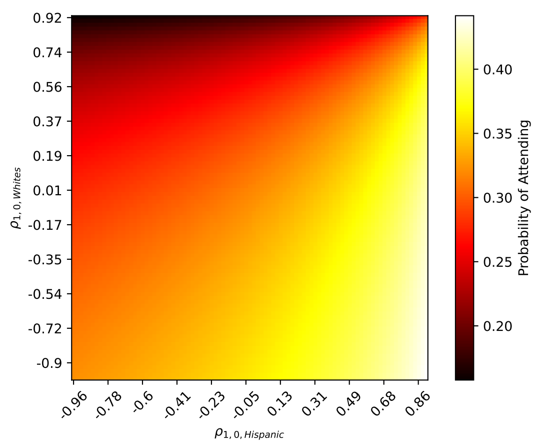

Figure 10: Counterfactuals share of Whites with Hispanic Information

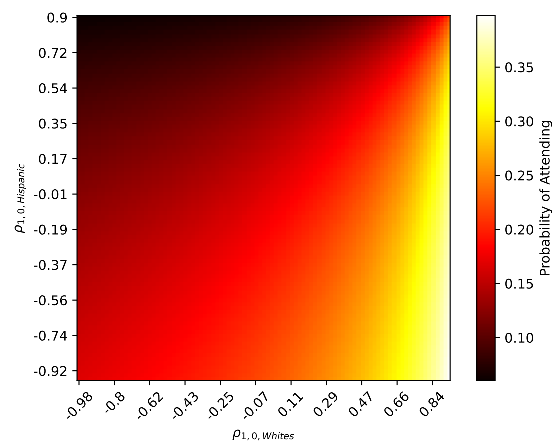

Figure 11: Counterfactuals share of Hispanics with Whites Information

The bounds we get are wide as it does not put any restrictions on correlation between α_1 and α_0 for either of the groups. We now consider how restrictions on this correlation can affect bounds. We first consider restrictions that imposing direct restrictions on the correlation between the residuals of α_1 and α_0, we then consider some restrictions on the quality of information.

The second row in table 4 considers the role of information when we focus solely on positive correlations between U_1 and U_0 residuals. This focus is motivated by our earlier estimation, which demonstrated that conditional expectations based on observables are highly and positively correlated. This restriction doesn't significantly alter the bounds, placing the role of information somewhere between

-165% and 137%. Row 3 examines the case where the correlation parameter $\rho_{1,0}$ for both Hispanics and Whites is set to be identical. This correspond to a model in which prior to making attendance decisions, the populations of Hispanics and Whites have the same distribution of skills and abilities, yet earnings disparities arise from labor market factors that affect them differently. Under this assumption, the role of information unequivocally widens the gap. In this context, most of the gap originates from differences in returns rather than information frictions. The next two rows, 4 and 5, consider two extreme scenarios. Row 4 discusses the case where the correlation is maximized for both groups. When both correlations equal one, this corresponds to a model where individuals can be characterized by a single scalar type, and earnings are an increasing function of that type. As explained in Section 2.4.3, the model can rule out the case of perfect correlation. if the implied variance of returns is lower than the estimated variance of beliefs. This row may thus capture scenarios where correlated skills have the same sign impact on earnings—e.g., ability positively correlates with earnings regardless of college attendance, while lack of conscientiousness is negatively correlate with earnings in both cases. In this scenario, the role of information is minimal, aligning with the fact that both groups have comparable value of information, as depicted in Figure 9. Row 5 explores the other extreme where U_1 and U_0 are uncorrelated. Here, the role of information in contributing to the gap is again negative, implying that equalizing the information quality of Hispanics and Whites in a world where α_1 and α_0 are uncorrelated would widen the gap.

The final two lines of the table adopt an alternative method for narrowing down possible correlations by focusing on what individuals might know prior to making a decision. Specifically, we impose assumptions on the extent to which individuals can predict the marginal distributions. While our identification strategy identifies agents' beliefs about the difference $E[\alpha_1 - \alpha_0|s, x]$, we can leverage our existing knowledge to make educated inferences about the quality of information agents possess regarding $E[\alpha_1|s, x]$. This may reduce range of plausible correlations. Section D in the appendix discuss how these restrictions affects the possible sets of correlation. Specifically, figure **??** shows that increasing the quality of information on the marginals shrings the possible set of ρ.

What are the possible values of R_1^2, and consequently R_0^2? We conduct two exercises to better understand the proportion of variance explained. First, in the appendix, we examine how well our covariates and school choice predict future earnings. Using Extreme Gradient Boosting, we forecast future income based on our set of covariates, exit exam grades, and indicators for the courses each student took and whether they passed them. This model explains approximately between 0.1 and 0.14 of the earnings variance for both college and non-college students among Hispanics and Whites. We conduct a similar exercise using NLSY97 data to predict earnings at age 30. Using a standard regression model, we explain roughly 20% of the variance. Row 6 uses these estimates to inform our bounds. We observe that the bounds remain wide, as the set of possible ρ values is also wide. In Row 7, we impose $R_1^2 = 0.1$ for both Hispanics and Whites, we can see that this does not restrict the importance of information, mainly as information mostly restrict the lower bound.

Therefore, we can see that even under more restrictive assumptions under different assumptions information quality can play a significant role in explaining the gap. Finally, in appendix G we consider similar exercise, where we equate Bayesian R^2, where R^2 the quality of information is also taking into account uncertainty of agents about the true value of $\rho_{1,0}$.

4.2 Second Measure of information gaps and Introducing Additional Signals

We now turn to our second measure of information frictions, wherein we ask how much the information quality for Hispanic high school students should be changed to induce them to attend college at the same rate as White students. Figure 12 plots, for each $\rho_{1,0}$, the implied quality of information and the requisite quality of information to achieve parity in choice. We observe that as $\rho_{1,0}$ increases, the gap between the implied and the required quality of information for Hispanics also enlarges. Intuitively, if there is less variance in the outcome variable, a larger share of it must be explained to induce sufficient variation in beliefs to prompt action. For positive values of rho it seems that the required quality of information needed to achieve parity is rather high, ranging form 0.24 in the case of indepdence, to 0.98 at the top feasible ρ, which are much higher than what we are able to produce using the detailed data data in the NLSY97.

We first consider adding information solely on either U_1 or U_0. The information quality of the additional signal on U_1 is 0.19, while for U_0, it is 0.38. Figure 13 depicts the share of attendance across varying levels of information quality $R^2_{1,n}$ and $R^2_{0,n}$. As illustrated, targeting information to one outcome effectively boosts participation. However, this strategy is constrained. To achieve parity, we need additional information with a precision of either 0.19 or 0.38. In our prior exercises to calculate R^2, we did not attain these levels, likely because the information used was already correlated with individuals' existing knowledge. Therefore, if a policymaker aims to achieve parity through the provision of accurate information, a deeper understanding of earnings formation is required.

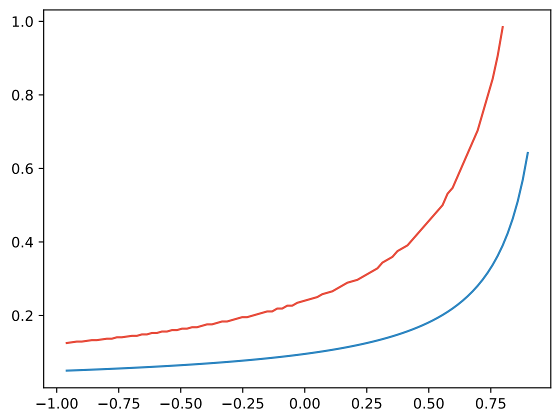

Figure 12: R^2 needed to achieve parity

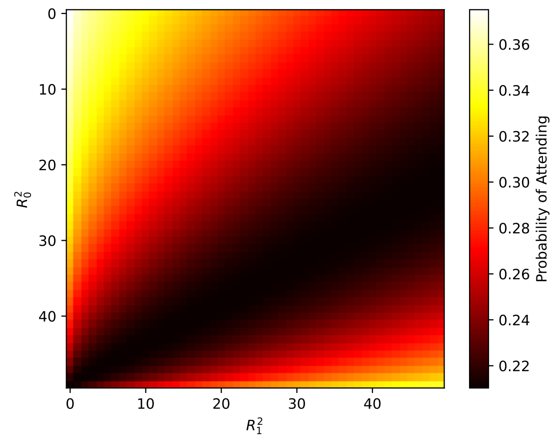

Figure 13: The effect of additional information on Earnings

In the appendix, we demonstrate how these interventions, which influence selection, also impact the wage disparities between Hispanics and Whites. This analysis is pertinent for policymakers focused on income equality rather than attendance rates. We find that the effect of these interventions on income inequality is relatively minor; providing information alone is insufficient to close the observed earnings gap.

5 Conclusions

People from diverse backgrounds have varying upbringings that influence their later life and their later life choices. These experiences not only define the constraints individual face, but also affect their opportunities, outcomes, and knowledge thereof. This project introduces a new approach to analyze how these factors impact choice disparities among different groups, by exploring how equating information across groups and introducing additional information affects choice gap. In our empirical study, the gap in college attendance between Whites and Hispanic individuals can be entirely explained by differences in information. Despite the potential significant impact of information, we find that to achieve parity in choice through policy interventions in our setup, high-precision information is required, which is not typically available in standard data sources. This suggests that while information-based initiatives may have limited effectiveness, strategies directly targeting outcomes may be more effective in the long term to achieve parity in choices.

The approach proposed in this paper could be applied to other scenarios where information about outcomes plays a critical role in creating disparities, such as cases of discrimination, healthcare, and decisions related to investing in human capital and skill development. The central idea we present here is that to comprehend the drivers of behavioral differences and choices, as well as why these disparities persist, we must describe and quantify the information individuals possess, how they acquire it, and how they utilize it. Understanding the informational environment in which people operate is essential for comprehending the existence and persistence of differences across social groups.

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A Additional Figures

Waiting for Data Release Approval

Figure 14: Demonstration of the link between grades and missing earnings

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Figure 15: Relation Between Grades and Earnings

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Figure 16: Distribution of Costs and Beliefs by College

Waiting for Data Release Approval

Figure 17: R^2 from the NLSY97

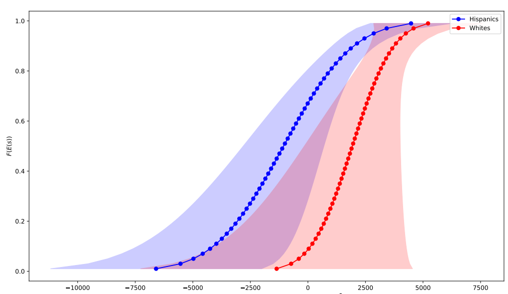

Figure 18: Counterfactuals share of whites with Hispanic Information

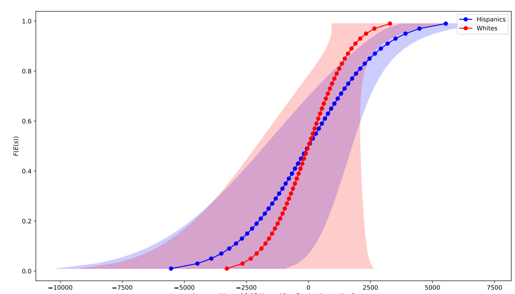

Figure 19: Beliefs CDF - Demean

B Additional Tables

		σ_{E}	σ_1	σ_0	Avg. Cost	$E[\alpha_1]$	$E[\alpha_0]$	$E[\alpha_1 - \alpha_0]$	$Corr(E[\alpha_1\|x], E[\alpha_1\|x])$
Hispanics	Avg. Wage 10-15	2271.0	4201.0	5973.0	3361.0	6150.0	7358.0	-1207.48	0.74
		(609.0)	(106.0)	(716.0)	()	(1704.0)	(1772.0)	(2459.0)	()
P(D=1)=0.21	Avg. Wage 12-14	2164.0	4455.0	6041.0	2928.0	6739.0	7619.0	-879.27	0.78
		(650.0)	(125.0)	(799.0)	()	(1716.0)	(1808.0)	(2493.0)	()
	Avg. Wage 12-15	2381.0	4490.0	6264.0	3314.0	6658.0	7715.0	-1057.44	0.76
		(657.0)	(125.0)	(818.0)	()	(1795.0)	(1843.0)	(2573.0)	()
	Avg. Wage 12-16	1759.0	4523.0	5855.0	2154.0	7399.0	7853.0	-454.56	0.8
		(770.0)	(195.0)	(1145.0)	()	(1752.0)	(1881.0)	(2570.0)	()
	Avg. Wage 14-16	2237.0	4839.0	6543.0	3150.0	7275.0	8263.0	-987.2	0.78
		(826.0)	(234.0)	(1432.0)	()	(1965.0)	(2004.0)	(2807.0)	()
Whites	Avg. Wage 10-15	1956.0	5227.0	6051.0	98.08	9588.0	8366.0	1221.0	0.91
		(795.0)	(127.0)	(409.0)	()	(2059.0)	(2077.0)	(2925.0)	()
P(D=1)=0.29	Avg. Wage 12-14	1326.0	5534.0	6217.0	-1103.21	10808.0	8816.0	1992.0	0.95
		(869.0)	(153.0)	(491.0)	()	(2065.0)	(2156.0)	(2985.0)	()
	Avg. Wage 12-15	1414.0	5577.0	6316.0	-979.32	10871.0	8942.0	1930.0	0.95
		(873.0)	(155.0)	(491.0)	()	(2149.0)	(2211.0)	(3083.0)	()
	Avg. Wage 12-16	1333.0	5627.0	6444.0	-1067.52	11055.0	9095.0	1960.0	0.96
		(1009.0)	(237.0)	(747.0)	()	(2252.0)	(2285.0)	(3209.0)	()
	Avg. Wage 14-16	1010.0	5999.0	6827.0	-2331.72	12515.0	9509.0	3006.0	0.97
		(1105.0)	(275.0)	(929.0)	()	(2366.0)	(2426.0)	(3389.0)	()

Table A.1: Model Parameters - Extended

Note: The model parameters are estimated using different time spans of earnings after graduation. Standard errors for these parameters are provided in parentheses.

	All	Hispanic	Whites	Difference (Whites - Hispanic)
Wage 8-10	7117.0 (4533.0)	6393.0 (3974.0)	7627.0 (4823.0)	1234.0 (6249.3)
Wage 10-12	8215.0 (5194.0)	7348.0 (4509.0)	8852.0 (5558.0)	1504.0 (7157.0)
Wage 12-14	9079.0 (5808.0)	8046.0 (4952.0)	9823.0 (6249.0)	1777.0 (7973.2)
Wage 14-16	9838.0 (6280.0)	8721.0 (5383.0)	10658.0 (6748.0)	1937.0 (8632.0)
Wage 12-15	9214.0 (5807.0)	8209.0 (4993.0)	9959.0 (6239.0)	1750.0 (7990.9)
Wage 1-16	6404.0 (3914.0)	5902.0 (3339.0)	6746.0 (4226.0)	844.0 (5385.9)

Table A.2: Wages Summary Statistics

Note: Table shows

	All	Hispanic	Whites
No Controls	-0.0156	-0.0232	-0.0436
	(0.0054)	(0.0053)	(0.0038)
Ind. Controls	-0.0277	-0.0151	-0.0392
	(0.0044)	(0.0066)	(0.0045)
+ School Char.	-0.0061	0.0074	-0.0177
	(0.004)	(0.0057)	(0.004)
+ Neighborhood Char.	-0.0014	0.0009	-0.0036
	(0.0018)	(0.0022)	(0.0021)

Table A.3: Instrument Diagnostics

	All	Hispanic	Whites
No Controls	-0.0008	-0.0007	-0.0013
	(0.0001)	(0.0002)	(0.0001)
	317278	136581	180697
Ind. Controls	-0.0008	-0.0006	-0.0011
	(0.0001)	(0.0001)	(0.0001)
	317278	136581	180697
+ School Char.	-0.0014	-0.001	-0.0019
	(0.0002)	(0.0002)	(0.0002)
	317278	136581	180697
+ Neighborhood Char.	-0.0016	-0.0023	-0.0012
	(0.0002)	(0.0003)	(0.0002)
	317278	136581	180697

Table A.4: First Stage

	All				Hispanic				Whites			
Wage Avg	8-10	10-12	12-14	14-16	8-10	10-12	12-14	14-16	8-10	10-12	12-14	14-16
No Controls	7.2078	3.1346	-0.6775	-3.4851	6.7659	4.1617	1.9655	1.4303	3.362	-3.329	-9.8946	-15.7508
	(1.5094)	(1.4327)	(1.4515)	(1.8191)	(1.5053)	(1.2071)	(1.1988)	(1.5494)	(1.2177)	(1.4513)	(1.6605)	(2.2097)
Obs.	245206	239307	233091	149498	103198	101284	99428	63271	142008	138023	133663	86227
Ind. Controls	4.9314	0.5937	-3.5101	-6.5359	5.8931	3.3429	1.2646	0.7636	4.1198	-2.1051	-8.4963	-14.2128
	(1.0258)	(0.946)	(1.0365)	(1.4211)	(1.4953)	(1.2306)	(1.1954)	(1.5131)	(1.1556)	(1.3397)	(1.5095)	(1.9959)
Obs.	245206	239307	233091	149498	103198	101284	99428	63271	142008	138023	133663	86227
+ School Char.	-1.7881	-2.9513	-4.1313	-5.7861	-1.7207	-1.0352	-0.8267	-0.6872	-1.758	-4.7777	-7.6157	-11.5689
	(1.2276)	(1.2815)	(1.3991)	(1.8907)	(1.384)	(1.4014)	(1.5289)	(2.0046)	(1.4649)	(1.7076)	(1.8884)	(2.5429)
Obs.	245206	239307	233091	149498	103198	101284	99428	63271	142008	138023	133663	86227
+ Neighborhood Char.	-0.504	-1.7866	-3.2171	-5.9322	-1.9984	-0.8557	-0.7281	-1.1675	1.6792	-0.7804	-3.5928	-8.7372
	(1.6783)	(1.9833)	(2.2052)	(2.8903)	(2.4266)	(2.8765)	(3.114)	(4.094)	(2.1061)	(2.4762)	(2.7454)	(3.6005)
Obs.	245206	239307	233091	149498	103198	101284	99428	63271	142008	138023	133663	86227

Table A.5: Reduced Form

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Table A.6: Reverse Decomposition Exercise

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Table A.7: Comparison in observables across observations with missing earnings and non-missing earnings

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Figure 20: Area Under the Curve Analysis of Predicting College Attendance Decisions

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Table A.8: College Attendance Gap

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Table A.9: Relation Between Earnings and Grades

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Table A.10: School Informativeness

	All	Hispanic	Whites
No Controls	-0.0156	-0.0232	-0.0436
	(0.0054)	(0.0053)	(0.0038)
Ind. Controls	-0.0277	-0.0151	-0.0392
	(0.0044)	(0.0066)	(0.0045)
+ School Char.	-0.0061	0.0074	-0.0177
	(0.004)	(0.0057)	(0.004)
+ Neighborhood Char.	-0.0014	0.0009	-0.0036
	(0.0018)	(0.0022)	(0.0021)

Table A.11: Caption

C Non parametric identification of the choice model

We explore the non-parametric identification of choices. First, we identify the distribution of structural components, α_1 and α_0, by leveraging panel data, an instrumental variable, and specific wage structure assumptions. Next, we establish the identification of both the cost function and the beliefs distribution. While panel data aids in identifying α_1 and α_0. This step can be skipped if if one assumes that outcomes are observed without measurement error.

In our analysis, we work under the assumption that the researcher has access to a random, independently and identically distributed sample of observations, each denoted by $(Y_{a,i}, D_i, X_i, Z_i)$. All analyses are conditional on the covariates vector X, so we omit the X notation for simplicity.

C.1 Identification of $P(\alpha_1, \mathrm{E}[\alpha_1 - \alpha_0|s])$ and $P(\alpha_0, \alpha_1 - \alpha_0|s])$ and the cost function

We impose the following assumptions on the wage data generating Process. Wages are set according to

$$Y_{i,a} = D_i(\alpha_1 + \epsilon^1_{i,a}) + (1 - D_i)(\alpha_0 + \epsilon^0_{i,a}))$$

where $Y_{i,a}$ is individual i's income at age a, D_i is a dummy variable indicating whether the HG i attended four years college or not. One can think of α_d as individual fixed effect, if that individual goes to college or not. We further impose the following assumptions on the wage process

Assumption 1. *(1) for all a we have $E[\epsilon^D_{i,a}|\alpha_D] = 0$ (2) $\alpha_1, \alpha_0 \perp\!\!\!\perp \epsilon^D_{i,a}$ and (3) there exist at least two periods a^D, a'^D for each $D \in \{0, 1\}$ such that $\epsilon^D_{i,a} \perp\!\!\!\perp \epsilon^D_{i,a'}|X$*

Denote by $P(Z) = E[D = 1|Z]$ the propensity score conditional on Z. We then employ the following assumption

Assumption 2. *The characteristic functions of the conditional distribution $\alpha_1|D = 1, P(Z) = p$, $\alpha_0|D = 1, P(Z) = p$, $\epsilon^D_{i,a}|D = 1, P(Z) = p$ and $\epsilon^D_{i,a'}|D = 1, P(Z) = p$ are non vanishing*

The first part of Assumption 1 is standard and implies that any constant is absorbed into α^D, ensuring that deviations from the structural component are independent of the fixed effects. The second restriction mandates the existence of at least two periods in which the shocks are mutually independent, given the covariates X. While this condition is restrictive, it accommodates complex correlation structures, such as finite moving averages or other forms of multi-period correlations. The Assumption 2 stipulates that the characteristic functions of the conditional distributions for $\alpha_1|D=1, P(Z)=p$, $\alpha_0|D=1, P(Z)=p$, $\epsilon^D_{i,a}|D=1, P(Z)=p$, and $\epsilon^D_{i,a'}|D=1, P(Z)=p$ are non-vanishing[4]. This is a standard assumption that is used for non parametric identification of factor models and assures us that we can use the characteristic functions to back-out the distribution of α_d.

Next, we impose restrictions on the agent information set. In the spirit of rational expectations, we assume that there are two parts to wages; a structural component, on which individuals have information on, and an unpredictable shock component that is not known to the high school gradutes.

Assumption 3 (Information Restriction)**.** *The signals individuals obtain do not contain any information on the non structural part of the wage, $\epsilon^1_{i,a}, \epsilon^0_0$.*

$$s_i \perp\!\!\!\perp \epsilon^1_{i,a}, \epsilon^0_{i,a}|\alpha_1, \alpha_0$$

This implies that individuals can only receive information on the structural component of the wage, but may not have information on time varying shocks. Finally we impose the following assumptions on the instrument Z

Assumption 4 (Instrument Restrictions)**.** *We assume that the instrumet satisfies the following conditions*

1. $\epsilon^1_{i,a}, \epsilon^0_{i,a}, \alpha_1, \alpha_0 \perp\!\!\!\perp Z$
2. $S \perp\!\!\!\perp Z|\alpha_1, \alpha_0$
3. *Z is continuously distributed on $\mathcal{Z} \subseteq \mathbb{R}$*
4. *$E[\alpha_1 - \alpha_0|s]$ continiously distributed*
5. *$c(z)$ is differentiable and covers the entire support of $E[\alpha_1 - \alpha_0|s, x]$*

[4]The non vanishing assumption can be further relaxed, as shown in Evdokimov and White (2012)

The assumptions are akin to standard Instrumental Variable (IV) assumptions (Heckman and Vytlacil (2005)), but they incorporate additional structure through the modeling of the choice equation. The first assumption establishes the instrument's independence from the outcome variables. The second dictates that information is independent of the instrument, conditioned on the structural components. Notably, these first two assumptions collectively imply that $E[\alpha_1(t_i) - \alpha_0(t_i)|s, x] \perp\!\!\!\perp Z$, aligning with standard IV assumptions where the selection variable is uncorrelated with the instrument. The final part of Assumption 4 is a technical requirement ensuring that we can recover the cost function by monitoring the derivative, as demonstrated in the proof.

Denote $E[\alpha_1 - \alpha_0|s] = \mathrm{E}$. We show the following proposition.

Proposition 1. Under assumptions (1)-(4), $P(\alpha_1, \mathrm{E})$,$P(\alpha_0, \mathrm{E})$ and the cost function $c(z)$ are identified

Proof. Let a, a' be two periods such that $\epsilon^D_{i,a} \perp\!\!\!\perp \epsilon^D_{i,a}$. We start by showing how to identify $\mathrm{P}(\alpha_d|\mathrm{E})$ First, using assumption 1, 3, and 4 we have that $\epsilon^D_{i,a} \perp\!\!\!\perp \alpha_1|p(Z) = p, D = 1$ as

$$\begin{aligned}
\mathrm{P}(\epsilon^D_{i,a}, \alpha_1|p(Z) = p, D = 1) &= \mathrm{P}(\epsilon^D_{i,a}, \alpha_1|p(Z) = p, \mathrm{E} \geq c(z)) \\
&= \mathrm{P}(\epsilon^D_{i,a}|\alpha_1, p(Z) = p, \mathrm{E} \geq c(z))\mathrm{P}(\alpha_1|p(Z) = p, \mathrm{E} \geq c(z)) \\
&= \mathrm{P}(\epsilon^D_{i,a}|p(Z) = p, \mathrm{E} \geq c(z))\mathrm{P}(\alpha_1|p(Z) = p, \mathrm{E} \geq c(z)) \\
&= \mathrm{P}(\epsilon^D_{i,a}|p(Z) = p, D = 1)\mathrm{P}(\alpha_1|p(Z) = p, D = 1)
\end{aligned}$$

where the first equality stems from the choice model, the second stems from Bayes rule, and the third equality is due to the contraction rule and the decomposition rule of conditional Independence. We have an equivalent result for α_0 and $\epsilon_{i,a'}$. Last, notice that as $\epsilon_{i,a} \perp\!\!\!\perp \epsilon_{i,a'}$, $\epsilon_{i,a}, \epsilon_{i,a'} \perp\!\!\!\perp m_{\mathcal{R}}(s)$ and $\epsilon_{i,a}, \epsilon_{i,a'} \perp\!\!\!\perp Z$ we have that $\epsilon_{i,a} \perp\!\!\!\perp \epsilon_{i,a'}|p(Z) = p, D = 1$

Therefore $\epsilon^D_{i,a}$ and $\epsilon^D_{i,a'}$ and α_D are mutually independent, conditional on D and P, and we can now utilize Kotlarski's Lemma (1967) to identify the conditional distribution of α_1 and α_0. We first show how to identify the conditional distribution of α_1. Let $\Psi(y_a, y_{a'})$ be the conditional characteristic function of $(Y_{i,a}, Y_{i,a'})$ given $(P(z) = p, D = d)$. Let $\Psi_{\alpha_1}(t), \Psi_{\epsilon_a}(t)$ and $\Psi_{\epsilon'_a}(t)$ be the conditional characteristic functions of $\alpha_1, \epsilon_{i,a}, \epsilon_{i,a'}$, given $(P(z) = p, D = d)$, then we can show that (Rao (1992), page 21 and Gilraine et al. (2020))

$$log\Psi_{\alpha_1}(t) = iE[\alpha_1|D=1,P(Z)=p]t + \int_0^t \frac{\partial}{\partial y_a}\left(log\frac{\Psi(y_a,y_{a'})}{\Psi(y_a,0)\Psi(0,y_{a'})}\right)_{y_a=0} dy_{a'}$$

Noticing that

$$\frac{\partial}{\partial y_a}\left(log\frac{\Psi(y_a,y_{a'})}{\Psi(y_a,0)\Psi(0,y_{a'})}\right)_{y_a=0} = \frac{\frac{\partial\Psi(0,y_{a'})}{\partial y_a}}{\Psi(0,y_{a'})} - iE[Y_{i,a}|D=1,P(Z)=p]t$$

and that by assumptions 1 and 3 we have $iE[Y_{i,a}|D=1,P(Z)=p]t = iE[\alpha_1|D=1,P(Z)=p]$ we then get

$$\log\Psi_{\alpha_1}(t) = \int_0^t \frac{\frac{\partial\Psi(0,y_{a'})}{\partial y_a}}{\Psi(0,y_{a'})} dy_{a'}$$

as the characteristic function fully defines the distribution and $\Psi(y_a,y_{a'})$ is observed in the data, we have identified $\mathrm{P}(\alpha_1|D=1,P(z)=p)$. Similar argument shows that we can identify $\mathrm{P}(\alpha_0|D=0,P(z)=p)$.

Next, denote by $F_{\alpha_1}(\cdot|D=1,P(Z)=p)$ the conditional CDF of α_1. Denote by $V=F_{\mathrm{E}}(\mathrm{E})$ the quantile of the beliefs in the beliefs distribution. Then following the arguments in Carneiro and Lee (2009) we have that for all k on the support of α_1 we have that

$$\begin{aligned} F_{\alpha_1}(k|P(z),D=1) = E[\mathbb{1}\{\alpha_1\leq k\}|P(Z)=p,D=1] &= E[\mathbb{1}\{\alpha_1\leq k\}|P(Z)=p,V>p(Z)] \\ &= \frac{1}{p}\int_{1-p}^1 E[\mathbb{1}\{\alpha_1\leq k\}|V=v]dv \end{aligned}$$

rewriting the equation gives us

$$pE[\mathbb{1}\{\alpha_1\leq k\}|P(Z)=p,D=1] = \int_{1-p}^1 E[\mathbb{1}\{\alpha_1\leq k\}|V=v]f(v)dv$$

Using assumption 4 we can take derivative from both sides, with respect to p, and get

$$E[\mathbb{1}\{\alpha_1\leq k\}|V=1-p] = E[\mathbb{1}\{\alpha_1\leq k\}|P(Z)=p,D=1] + p\frac{E[\mathbb{1}\{\alpha_1\leq k\}|P(Z)=p,D=1]}{\partial p}$$

Therefore we have that $\mathrm{P}(\alpha_1|V)$ is identified. Following similar steps we have that $\mathrm{P}(\alpha_0|V)$ is also identified

$$E[\mathbb{1}\{\alpha_0 \leq k\}|V = 1-p] = E[\mathbb{1}\{\alpha_0 \leq k\}|P(Z) = p, D = 0] - (1-p)\frac{E[\mathbb{1}\{\alpha_0 \leq k\}|P(Z) = p, D = 0]}{\partial p}$$

Next, observe that we can construct the probabilities $P(\alpha_1|\mathrm{E})$ and $P(\alpha_0|\mathrm{E})$ using the law of iterated expectations we have

$$e = E[\alpha_1 - \alpha_0|\mathrm{E} = e] = E[\alpha_1 - \alpha_0|F_{\mathrm{E}}(\mathrm{E}) = V] = \int \alpha_1 P(\alpha_1|V)d\alpha_1 - \int \alpha_0 P(\alpha_0|V)d\alpha_0. \tag{9}$$

Therefore we can identify the inverse $F_{\mathrm{E}}^{1}(V)$ and consequently the cumulative distribution function (CDF) of beliefs, $F_{\mathrm{E}}(e)$. As the CDF is strictly increasing by assumption 4 we can also identify $P(\alpha_1|\mathrm{E})$ and $P(\alpha_0|\mathrm{E})$ as needed. Therefore we've identified the joint $P(\alpha_1, \mathrm{E})$ and $P(\alpha_0, \mathrm{E})$. Finally, to identify the cost function, observe that

$$P(z) = \Pr(\mathrm{E} > c(z)) = 1 - F(c(z)) \implies F_{\mathrm{E}}^{-1}(1 - P(z)) = F_{\mathrm{E}}^{-1}(F(c(z))) \implies F_{\mathrm{E}}^{-1}(1 - P(z)) = c(z).$$

Finally, in order to identify the cost function, we notice that

$$P(z) = \mathrm{P}(\mathrm{E} > c(z)) \quad = 1 - F(c(z)) \implies F_{\mathrm{E}}^{-1}(1 - P(z)) = F_{\mathrm{E}}^{-1}(F(c(z)))F_{\mathrm{E}}^{-1}(1 - P(z)) = c(z)$$

which concludes the proof. $\square$

C.1.1 Testable Implication

As discussed in Canay et al. (2020), the model implies that the marginal treatment effect should be decreasing. To see that notice that we use 9 to identify the CDF of V, therefore, if we get that this is not increasing function of v, this implies that our model is mispecificed.

D Bounds on the role of information with information restrictions

In this section we consider how restrictions on what individuals may possibly know can put additional restrictions on the set of possible correlation values. Specifically, we considering imposing assumptions on the extent to which individuals can predict the marginal distributions. While our identification strategy identifies agents' beliefs about the difference $E[\alpha_1 - \alpha_0|s,x]$, we can leverage our existing knowledge to make educated inferences about the quality of information agents possess regarding $E[\alpha_1|s,x]$. This may then used to reduce range of plausible correlations.

The way we can do it is that if we have a guess for the quality of information individuals have on the marginals. Let R_1^2 and R_0^2 represent the quality of information individuals have on U_1 and U_0 respectively. Using the covariance matrix between $E[\alpha_1|s,x]$, $E[\alpha_0|s,x]$, α_1, and α_0, we can derive bounds on $\rho_{1,0}$. To see this, first, notice that we can write the identified beliefs variance, σ_E^2, as:

$$
\begin{aligned}
\sigma_{\mathrm{E}}^2 &= \mathrm{Var}(E[\alpha_1 - \alpha_0|s,x]) \\
&= \mathrm{Var}(E[\alpha_1|s,x]) + \mathrm{Var}(E[\alpha_0|s,x]) - 2\mathrm{Cov}(E[\alpha_1|s,x], E[\alpha_0|s,x]) \\
&= \sigma_1^2 R^2 + \sigma_0^2 R^2 - 2\mathrm{Cov}(E[\alpha_1|s,x], E[\alpha_0|s,x])
\end{aligned}
$$

Then, using our guess on R_1^2 and R_0^2, we can solve for $\mathrm{Cov}(E[\alpha_1|s,x], E[\alpha_0|s,x])$. Similar to our identification discussion in Section 2.4, we know that the covariance between α_d and the beliefs on it $\mathrm{E}[\alpha_d|s,x]$ satisfies

$$
\mathrm{Cov}(\alpha_1, \mathrm{E}[Y_1|s,x]) = \mathrm{Var}(E[\alpha_1|s,x]) = \sigma_1^2 R^2
$$

and similarly for $\mathrm{Cov}(y_0, E[y_0|s])$. Finally, we can also derive $\mathrm{Cov}_{0,\mathrm{E}_1} = \mathrm{Cov}(\alpha_0, E[\alpha_1|s,x])$ and $\mathrm{Cov}_{0,\mathrm{E}_0} = \mathrm{Cov}(\alpha_0, E[\alpha_1|s,x])$ using our identified $\mathrm{Cov}(E[\alpha_1 - \alpha_0|s,x], \alpha_d)$. Specifically, we have that

$$
\begin{aligned}
\gamma_1 \times \sigma_{\mathrm{E}} &= \mathrm{Cov}(E[\alpha_1 - \alpha_0|s,x], \alpha_1) \\
&= \mathrm{Cov}(E[\alpha_1|s,x], \alpha_1) - \mathrm{Cov}(E[\alpha_0|s,x], \alpha_1) \\
&= \sigma_1^2 R_1^2 + \mathrm{Cov}_{1,\mathrm{E}_0}
\end{aligned}
$$

which allows us to solve for $\mathrm{Cov}_{1,\mathrm{E}_0}$. Using a similar argument, we can solve for $\mathrm{Cov}_{0,\mathrm{E}_1}$. We can further restrict the set of free parameters, by noticing that in the Gaussian model the posterior mean is simply a linear function of the signals, therefore, we have that

$$\mathrm{Cov}(\alpha_1, \mathrm{E}[\alpha_0|s,x]) = \mathrm{Cov}(\mathrm{E}[U_1|s,x] + \nu, \mathrm{E}[U_0|s,x]) = \mathrm{Cov}(\mathrm{E}[U_1|s,x], \mathrm{E}[U_0|s,x])$$

where ν is the uncorrelated residual of projecting U_1 on s, and therefore is uncorrelated with s. Using this insights we can solve for R_0^2 in terms of R_1^2.

$$R_0^2 = \frac{\mathrm{Cov}_{\mathrm{E}_1,\mathrm{E}_0} - \gamma_0\sigma_\eta}{\sigma_0^2}$$

With the above calculations, for a guess of R_1^2, we can derive the following covariance matrix between beliefs and the U_1 and U_0

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_0 & \sigma_1^2 R_1^2 & \mathrm{Cov}_{\mathrm{E}_1,\mathrm{E}_0} \\ \rho\sigma_1\sigma_0 & \sigma_0^2 & \mathrm{Cov}_{\mathrm{E}_0,\mathrm{E}_1} & \sigma_0^2 R_0^2 \\ \sigma_1^2 R_1^2 & \mathrm{Cov}_{\mathrm{E}_0,\mathrm{E}_1} & \sigma_1^2 R_1^2 & \mathrm{Cov}_{\mathrm{E1,E0}} \\ \mathrm{Cov}_{\mathrm{E}_1,\mathrm{E}_0} & \sigma_0^2 R_0^2 & \mathrm{Cov}_{\mathrm{E1,E0}} & \sigma_0^2 R_0^2 \end{pmatrix}$$

where all parameters are known beside ρ. Feasible values for ρ are values between -1 and 1 that assure that the covariance matrix is positive semi-definite, that allows us to restrict the possible quality of information on the returns.

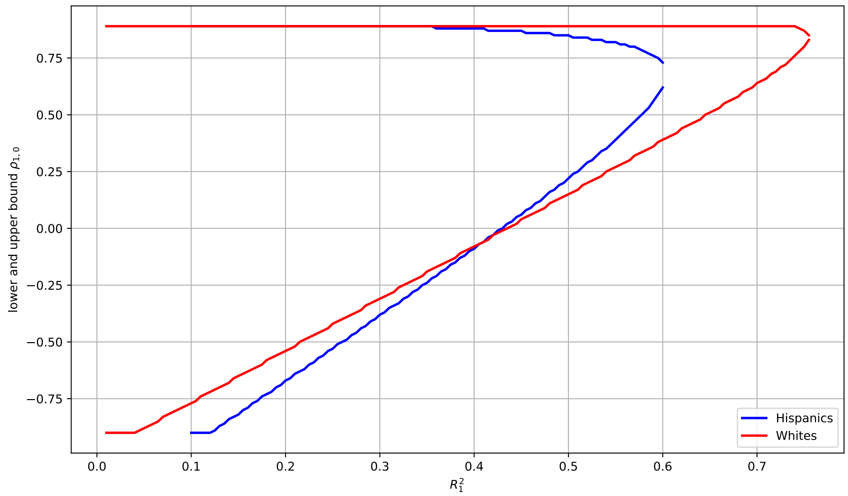

Figure 21: Lower and Upper bound for $\rho_{1,0}$

Notes: The figure plots the upper and lower bound of possible $\rho_{1,0}$ for both Hispanics and whites for residuals from the residual of average income between 12-15 years after high school graduations for both Hispanics and Whites.

Figure 21 plots the lower and upper bounds for $\rho_{1,0}$ for a fixed R_1^2 for both Hispanics and whites. The figure shows that increasing the R_1^2, shrinks the set of possible $\rho_{1,0}$. To understand the intuition, consider the extreme case in which agents have perfect knowledge of U_1. In this situation, $\rho_{1,0}$ cannot equal one because that would imply agents have perfect knowledge of $U_1 - U_0$. This is only possible if $\sigma_{\mathrm{E}}^2 = \sigma_1^2 + \sigma_0^2 - 2\sigma_1\sigma_0$, which is not the case. The covariance matrix restricts values to ensure that some s can generate consistent beliefs. The figure also indicates that we can bound the maximum information each group has on its marginals. In general, it appears that the amount of information whites can have is much lower than that of Hispanics.

E Measuring the Role of Information by Equating Information Structure Across Groups

In the main text we considered two way to measure the role of information frictions on choice. We now consider an additional one that aims to equate the information structure across groups. Information strucutres is widely used in economics and captures the relation between the state variables and the information individuals have on them XXXXX

Another way to understand this measure, is to ask what if idnvidiausl from both groups, we had the same returns set would also have the same information set - i.e. they have access to the same type of signals. We emphsize that equating the information set does not in general implies that we equate the interepetation of these signals. For example, consider the case where

To formalize these notion we consider the following decomposition, which is very similar to our information quality decomposition, discussed in in section **??**. We again decompose the choice gap to differences in choice that are attributed to differences arising from differences in the information structure. We again suggest the following decomposition à la Kitagawa (1955), Blinder (1973), and Oaxaca (1973).

$$
\begin{aligned}
&\underbrace{P(D=1|\text{Group } b) - P(D=1|\text{Group } a)}_{\text{Total Effect}} = \\
&\underbrace{P(D=1|\text{Group } b) - P(D=1|\text{Group } b \text{ with information structure of Group } a)}_{\text{Information Channel}} \\
&+ \underbrace{P(D=1|\text{Group } b \text{ with information structure of Group } a) - P(D=1|\text{Group } a)}_{\text{Composition Channel}} = \\
&\underbrace{\int_{\mathcal{R}\times c} \mathcal{P}(E_b(s) \geq c|\mathcal{R}, c, b)\pi_b(\mathcal{R}, c) d\mathcal{R}dc - \int_{\mathcal{R}\times c} \mathcal{P}(E_{a,b}(s) \geq c|\mathcal{R}, c, a)\pi_b(\mathcal{R}, c) d\mathcal{R}dc}_{\text{Information Channel}} \\
&+ \underbrace{\int_{\mathcal{R}\times c} \mathcal{P}(E_{a,b}(s) \geq c|\mathcal{R}, c, a)\pi_b(\mathcal{R}, c) - \int_{\mathcal{R}\times c} \mathcal{P}(E_a(s) \geq c|\mathcal{R}, a)\pi_a(\mathcal{R}, c) d\mathcal{R}dc}_{\text{Composition Channel}}
\end{aligned}
$$

where

$$E_{a,a}(s) = \int_{\mathcal{R}} \mathcal{R} \frac{P(s|\mathcal{R}, a) \times \pi_a(\mathcal{R})}{\int_{\tilde{\mathcal{R}}} P(s|\tilde{\mathcal{R}}, a) \times \pi_a(\tilde{\mathcal{R}}) d\tilde{\mathcal{R}}} d\mathcal{R}$$

and

$$E_{a,b}(s) = \int_{\tilde{\mathcal{R}}} \tilde{\mathcal{R}} \frac{\overbrace{P(s|\tilde{\mathcal{R}}, a)}^{\text{Information}} \times \overbrace{\pi_b(\tilde{\mathcal{R}})}^{\text{earnings}}}{\int P(s|\tilde{\mathcal{R}}, a) \pi_b(\tilde{\mathcal{R}}) d\tilde{\mathcal{R}}} d\tilde{\mathcal{R}}$$

The information channel captures how much would the gap in choices change if the information structure that both group have would be the same as those of group a. Therefore, the information channel quantifies how much of the gap in choices stems from the fact that individuals with the same returns to receive different information on their returns. Notice that we require from the agent to update their beliefs according to the new data generating process. i.e. we ask how would they update their beliefs knowing that the distribution of of signals that they receive is drawn from a new distribution. The thought experiment we preform here is equating the information set of individuals with the same returns, while still requiring that they update the their beliefs correctly, given the how the labor market operates.

The residual component, which we denote as the composition effect, asks the reverse question: how much of the gap is driven by the fact that agents have different net returns from choosing to attend college. Specifically, we fix the information of group a and change the costs and returns to match those of group b. Where we update beliefs accordingly.

In the scalar normal case we can write this these terms explicitly

$$\begin{aligned} P(D=1|b) - P(D=1|a) = & \underbrace{\int_c \Phi\left(\frac{\mu_{\mathcal{R},b,c} - c}{\sqrt{\frac{\sigma^4_{\mathcal{R},b,c}}{\sigma^2_{\mathcal{R},b,c} + \sigma^2_{\epsilon,b,c}}}}\right) \pi_b(c)dc - \int_c \Phi\left(\frac{\mu_{\mathcal{R},b,c} - c}{\sqrt{\frac{\sigma^4_{\mathcal{R},b,c}}{\sigma^2_{\mathcal{R},b,c} + \sigma^2_{\epsilon,a,c}}}}\right) \pi_b(c)dc}_{\text{Information Channel}} \\ & + \underbrace{\int_c \Phi\left(\frac{\mu_{\mathcal{R},b,c} - c}{\sqrt{\frac{\sigma^4_{\mathcal{R},b,c}}{\sigma^2_{\mathcal{R},b,c} + \sigma^2_{\epsilon,a,c}}}}\right) \pi_b(c)dc - \int_c \Phi\left(\frac{\mu_{\mathcal{R},a,c} - c}{\sqrt{\frac{\sigma^4_{\mathcal{R},a,c}}{\sigma^2_{\mathcal{R},a,c} + \sigma^2_{\epsilon,a,c}}}}\right) \pi_a(c)dc}_{\text{Composition Channel}} \end{aligned}$$

Therefore, we find that in the scalar normal case, equating information structure across two groups essentially means equalizing the level of uncertainty surrounding true returns. Contrary to popular belief, reduced uncertainty doesn't necessarily imply that more individuals would opt for college. For instance, if returns are generally low but high for a few, policymakers might prefer to obfuscate information. The analysis above simply quantifies the change in attendance gaps if both groups had identical information structure.

In general, information structure captures the type of information accessible to individuals with similar outcomes. These disparities can stem from various environmental factors. For example, if members of group b have more academically inclined parents than those in group a, they are more likely to receive accurate information about college benefits, offering more precise signals about potential returns. Additionally, if the social networks of group b members are closely tied to a specific industry requiring information, this could lead to discrepancies in individuals' ability to predict returns.

The following two examples show that differences in the information structure, and therefore also in the information quality, can arise in equilibrium, from the environment in which both groups operate.

Example E.1 (Occupation and Earnings)**.** The informational content of the signals individuals might be more dependent on the structure of the economy itself. For instance, consider the case where the earnings of non-college-goers are zero for both members of groups a and b, and there are two occupations in the economy: lawyers and accountants. Both lawyers and accountants are paid either a high or low wage, $H > 0 > L$, with equal probability. Prior to deciding to go to college, individuals receive an informative signal on their potential returns if they end up being lawyers. Denote these signals as $\tilde{H}_{\text{law}}$ and $\tilde{L}_{\text{law}}$. The distributions of occupations, earnings, and the signal for each group are given below.

Group b

		H	**L**
Lawyer	$\tilde{H}_{law}$	$\frac{6}{20} \times \frac{5}{6}$	$\frac{6}{20} \times \frac{1}{6}$
	$\tilde{L}_{law}$	$\frac{6}{20} \times \frac{1}{6}$	$\frac{6}{20} \times \frac{5}{6}$
Accountant	$\tilde{H}_{law}$	$\frac{4}{20} \times \frac{1}{2}$	$\frac{4}{20} \times \frac{1}{2}$
	$\tilde{L}_{law}$	$\frac{4}{20} \times \frac{1}{2}$	$\frac{4}{20} \times \frac{1}{2}$

Group a

		H	**L**
Lawyer	$\tilde{H}_{law}$	$\frac{4}{20} \times \frac{5}{6}$	$\frac{4}{20} \times \frac{1}{6}$
	$\tilde{L}_{law}$	$\frac{4}{20} \times \frac{1}{6}$	$\frac{4}{20} \times \frac{5}{6}$
Accountant	$\tilde{H}_{law}$	$\frac{6}{20} \times \frac{1}{2}$	$\frac{6}{20} \times \frac{1}{2}$
	$\tilde{L}_{law}$	$\frac{6}{20} \times \frac{1}{2}$	$\frac{6}{20} \times \frac{1}{2}$

Table A.12: Demonstration of Information Structure

In this economy, the share of high earners and low earners is $\frac{1}{2}$ for both groups. The share of individuals in both groups with signals $\tilde{H}_{\text{law}}$ and $\tilde{L}_{\text{law}}$ is also $\frac{1}{2}$. Moreover, for both groups, individuals who end up as lawyers and received a high signal have a $\frac{5}{6}$ probability of having high earnings. The only difference between the two groups is the share of individuals who end up being lawyers, versus those ending up being accountants. This difference implies that the signals each individual from each group receives have different information content, generating differences in the distribution of beliefs. For members of group b, the information structure is given by

$$P(\tilde{H}|H) = P(\tilde{H}|\text{Lawyer}, H)P(\text{Lawyer}|H) + P(\tilde{H}|\text{Acc}, H)P(\text{Acc}|H) = \frac{7}{10} \tag{10}$$

$$P(\tilde{H}|L) = P(\tilde{H}|\text{Lawyer}, L)P(\text{Lawyer}|L) + P(\tilde{H}|\text{Acc}, L)P(\text{Acc}|L) = \frac{3}{10} \tag{11}$$

Similarly, for members from group a we have

$$P(\tilde{H}|H) = P(\tilde{H}|\text{Lawyer}, H)P(\text{Lawyer}|H) + P(\tilde{H}|\text{Acc}, H)P(\text{Acc}|H) = \frac{19}{30} \tag{12}$$

$$P(\tilde{H}|L) = P(\tilde{H}|\text{Lawyer}, L)P(\text{Lawyer}|L) + P(\tilde{H}|\text{Acc}, L)P(\text{Acc}|L) = \frac{11}{30} \tag{13}$$

which implies that even when the marginal distribution of the signal and returns is the same, the implied beliefs given the same signal are different

$$m_{R_{\langle b,b\rangle}}(\tilde{H}) = H \times \frac{7}{10} + L \times \frac{3}{10} \tag{14}$$

$$m_{R_{\langle a,a\rangle}}(\tilde{H}) = H \times \frac{19}{30} + L \times \frac{11}{30} \tag{15}$$

Therefore, although the marginal distribution of signals and returns is the same in the economy, the information structure is different, and the same signal would be interpreted differently in both cases. What does it mean to switch the information structure between group a and group b in this environment? In the thought experiment we perform here, we ask what would be the observed behavior if we provided a signal with the same informational content on the returns as the other group. In this sense, our decomposition approach is "reduced form" in spirit, as we do not describe what drives the differences in information. Instead, we explore the ways in which systemic differences in information on earnings are provided to individuals and how they affect the observed gaps in behavior. These differences can arise from various channels, some due to the way the economy is structured, others might be due to differences in individuals, such as the ability to process information or the financial ability to acquire information.

Example E.2 (Knowledge of some structural components)**.** In some cases, individuals may know specific parts of the data-generating process of earnings. For example, assume that the earnings are determined by a function with a known component to the decision-maker, x, and some unknown component ν_d:

$$\alpha^1 = m_1(x, \nu_1) \tag{16}$$

$$\alpha^0 = m_0(x, \nu_0) \tag{17}$$

Here, x could represent known ability, latent cost of effort, or parental connections in the labor market. In this case, the information structure is simply the probability of observing x, given the earnings $P(x|\alpha^1, \alpha^0)$. This assumption is common in economic models where we believe that some variables affecting the outcomes are known to the decision-makers while making choices, and they use them to form beliefs about the outcomes. Therefore, in our thought experiment of switching the information structure between groups, we separate the two roles of x. Specifically, we fix the distribution of x in the population, thereby keeping the distribution of earnings fixed. But we ask what would happen if the agent did not know x, but instead had access to a similar information environment as group a, and how that would change choice patterns.

We now proceed to investigate the second component of decomposition. As articulated by Dinardo (1996), we can express the composition channel as:

$$\underbrace{P_{\langle a|b\rangle} - P_{\langle a|a\rangle}}_{\text{Composition Channel}} = \int_{\alpha^1,\alpha^0} P(m_{R_{\langle a,b\rangle}}(s) \geq 0|\alpha^1, \alpha^0, a)\frac{\pi(\alpha^1, \alpha^0|b)}{\pi(\alpha^1, \alpha^0|a)}\pi(\alpha^1, \alpha^0|a) \tag{18}$$

$$- \int_{\alpha^1,\alpha^0} P(m_{R_{\langle a,a\rangle}}(s) \geq 0|\alpha^1, \alpha^0, a)\pi(\alpha^1, \alpha^0|a)d\alpha^1 d\alpha^0 \tag{19}$$

In the composition channel, we maintain the information structure of group a, yet re-weight the population of group a to align with the distribution of group b. This thought experiment explores how the share of college attenders from group a would change if we modified the composition of the group, so that their distribution of earnings would align with that of group b. In this counterfactual, we are not breaking the connection between information and earnings, as we did in example 2.3, but merely shifting the proportion of individuals at certain earnings levels, ensuring that they take the change into account while forming their beliefs. As we alter the distribution of earnings, while keeping the information structure fixed, we also modify the marginal distribution of signals within the population. This means that if, for instance, we increased the proportion of potential students with high α_1, α_0, we are also enlarging the population's share of those receiving signals tied to higher earnings. Consequently, maintaining the information structure fixed means that we are transforming the distribution of signals in the population, but keep it's meaning.

Example E.3 (Knowledge of some structural components-Continued)**.** In this example, the composition channel involves adjusting the share of members in group a with specific earnings levels, to align with those from group b. It's important to note that we are not necessarily equalizing the share of variable x between the two groups. If x represents, for example, ability, and the function $m(., \nu)$ varies between groups, our hypothetical scenario doesn't balance the share of high and low ability across both groups. If m differs, matching the share of high and low ability could result in significantly different distributions. Since the HGs are not concerned with ability itself but as an indicator of their returns, aligning individual parts across groups doesn't provide insight into how the distribution of outcomes influences choice.

These disparities in beliefs and signal informativeness have been documented among students from different social strata (Manski (2004), **?**, **?**). Studies have also shown that grades provide different information on returns and that investment and outreach vary among social groups (**?**,**?**,**?**).

It's crucial to understand that our analysis offers a partial equilibrium perspective on changing information structure. This technology could change endogenously through two channels. First, individuals may exert effort to generate better information in response to the distribution of returns. Second, differences in information could arise due to selection and equilibrium effects. For example, if information influences labor market selection patterns, and employers respond to these patterns, our counterfactuals won't address this. Our analysis assumes that the existing information structure is a given and demonstrates further details in the appendix.

In the following parts we discuss how this decomposition measure can identificed under different assumption on the data or the type of fundamentals and information.

E.1 Non-parametric point Identification of the Decomposition Components

Fix two groups $g \in \{a, b\}$. In the subsequent sections, we demonstrate how to identify the quantity

$$P_{\langle a,b\rangle} = \int_{\mathcal{R}\times c} \mathcal{P}(E_{a,b}(s) \geq c|\mathcal{R}, c, a)\pi_b(\mathcal{R}, c) \tag{20}$$

required for decomposition. As outlined in Section C, the primary challenge lies in constructing the distribution of posterior means that incorporates both the counterfactual distribution of signals and returns. This must be achieved despite having access only to the conditional expectations distribution, rather than the complete information structure available to agents. We first establish conditions for point identification, then extend our analysis to more general cases for identifying this quantity.

E.1.1 Point identification under increasing beliefs function

We start by showing that if we are willing to assume that the information is scalar, and that beliefs are increasing function of that signal, then the quantity in 20 is identified.

Proposition 2. Let $E[\mathcal{R}|s]$ be a strictly increasing function of s, then equation 2 is identified.

Proof. The claim follows trivially from the fact that a strictly monotonic transformation is merely a renaming of the signal but does not alter its information content. Therefore, individuals update beliefs in the same manner, using either the information structure's likelihood functions $P(s|\mathcal{R})$ with support $\mathcal{S}$ or $P(E[\mathcal{R}|s]|\mathcal{R})$ with support given by the posterior means, for any prior. To illustrate this in our continuous density of signals case, we have

$$E_{a,b}(s) = \int \mathcal{R} \frac{P_a(s|\mathcal{R})\pi_b(\mathcal{R})}{\int P_a(s|\mathcal{R})\pi_b(\mathcal{R})d\mathcal{R}} d\mathcal{R} = \int \mathcal{R} \frac{\left|\frac{1}{\frac{\partial E_a}{\partial s}}\right| P_a(E_a|\mathcal{R})\pi_b(\mathcal{R})}{\left|\frac{1}{\frac{\partial E_a}{\partial s}}\right| \int P_a(E_a|\mathcal{R})\pi_b(\mathcal{R})d\mathcal{R}} d\mathcal{R} = E[\mathcal{R}|E_a(s); b]$$

where E_a denotes the beliefs of group a, with their information structure and prior, and $E[\mathcal{R}|E_a(s); b]$ is the belief induced by observing the signal $E_a(s)$ and prior π_b. As demonstrated in section C, for a given correlation between α_1 and α_0, we can identify the joint distribution of $E[\mathcal{R}|s]$ and $\mathcal{R}$ for groups a and b. Therefore, as each signal corresponds to a unique belief, we can calculate the implied counterfactual beliefs distribution directly from the identified distribution of beliefs. Consequently, $P(E_{a,b}(s)|\mathcal{R})$ is identified, and equation 20 is trivially identified. $\square$

Under what conditions can we expect the conditional expectation to be a strictly increasing function of returns? A sufficient condition for this is that the joint distribution of $\mathcal{R}$ and s satisfies the Monotone Likelihood Ratio Property (MLRP). The following corollary formalizes this claim.

Corollary 1. Let $P(\mathcal{R}, s)$ satisfy the strict Monotone Likelihood Ratio Property,

$$\forall s > s', x > x' \quad P(\mathcal{R}|s)P(\mathcal{R}'|s') > P(\mathcal{R}'|s)P(\mathcal{R}|s') \tag{21}$$

then the quantity in equation 2 is identified.

Proof. The corollary follows from the preceding proposition and the fact that MLRP implies First-Order Stochastic Dominance,

$$F_s(\mathcal{R}) \leq F_{s'}(\mathcal{R}) \tag{22}$$

which implies that the conditional expectation is strictly increasing,

$$E[\mathcal{R}|s] = \int_{\mathcal{R}} (1 - F_s(\mathcal{R}))\, d\mathcal{R} > \int_{\mathcal{R}} (1 - F_{s'}(\mathcal{R}))\, d\mathcal{R} = E[\mathcal{R}|s'].$$

Here, F_s denotes the CDF of $\mathcal{R}$, conditional on s. □

E.1.2 Identification under the general Gaussian Model

We first introduce a general Gaussian model with a finite number of signals. Throughout the discussion, we fix the cost c and make the identification argument conditional on c. We assume that α_1 and α_0 are jointly normally distributed. Further, we assume that individuals observe a scalar signal S, and the structural components of earnings α_1, α_0 are drawn from a joint normal distribution.

$$\begin{pmatrix} \boldsymbol{S} \\ \alpha_1 \\ \alpha_0 \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \boldsymbol{\mu}_s \\ \mu_1 \\ \mu_0 \end{pmatrix}, \begin{bmatrix} \Sigma_{\boldsymbol{S}}, \Sigma_{\boldsymbol{S},1}, \Sigma_{\boldsymbol{S},0} \\ \Sigma_{\boldsymbol{S},1}, \sigma_1, \sigma_{1,0} \\ \Sigma_{\boldsymbol{S},0}, \sigma_{1,0}, \sigma_0 \end{bmatrix} \right)$$

Using the properties of the normal distribution, we can write the joint distribution of the signals and the returns, where $\Delta = \alpha_1 - \alpha_0$, as

$$\begin{pmatrix} \boldsymbol{S} \\ \alpha_1 - \alpha_0 \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \boldsymbol{\mu}_s \\ \mu_1 - \mu_0 \end{pmatrix}, \begin{bmatrix} \Sigma_{\boldsymbol{S}} & \Sigma_{\boldsymbol{S},\Delta} \\ \Sigma_{\boldsymbol{S},\Delta}^T & \sigma_1^2 + \sigma_0^2 - 2\sigma_{1,0} \end{bmatrix} \right)$$

Where $\Sigma_{\boldsymbol{S},\Delta} = \Sigma_{\boldsymbol{S},1} - \Sigma_{\boldsymbol{S},0}$. Given a signal realization $\boldsymbol{S}$, the information structure, $\Pr(\boldsymbol{S}|\theta)$, is then given by

$$\Pr(\boldsymbol{S}|\mathcal{R}) = \mathcal{N}\left(\mu_{\boldsymbol{S}} + \Sigma_{S,\mathcal{R}}\sigma_{\mathcal{R}}^{-2}(\mathcal{R} - \mu_{\mathcal{R}}), \Sigma_{\boldsymbol{S}} - \Sigma_{\boldsymbol{S},\mathcal{R}}\sigma_{\mathcal{R}}^{-2}\Sigma_{\boldsymbol{S},\mathcal{R}}^T\right)$$

An individual with signal realization $\boldsymbol{S}$ forms the following posterior mean:

$$E[\Delta|\boldsymbol{S}] = \mu_{\mathcal{R}} + \Sigma_{S,R}^T \Sigma_{\boldsymbol{S}}^{-1}(\boldsymbol{S} - \mu_S)$$

This implies that individuals i with cost c and signal realization $\boldsymbol{S}$ would choose to go to college if

$$D = \mathbb{1}\left[E[\alpha_1 - \alpha_0|\boldsymbol{S}] \geq c\right] = \mathbb{1}\left[\mu_{\mathcal{R}} + \Sigma_{S,R}^T \Sigma_{\boldsymbol{S}}^{-1}(\boldsymbol{S} - \mu_S) \geq c\right]$$

We can calculate the share of students who attend college with cost c. First, we note that the beliefs distribution is given by

$$E[\mathcal{R}|\boldsymbol{S}] \sim \mathcal{N}\left(\mu_{\mathcal{R}}, \Sigma_{S,\mathcal{R}}^T \Sigma_S^{-1} \Sigma_{S,\mathcal{R}}\right)$$

Therefore, the share of individuals who would go to college is given by

$$P(D = 1|c) = \Phi\left(\frac{\mu_\Delta - c}{\Sigma_{S,\Delta}^T \Sigma_S^{-1} \Sigma_{S,\Delta}}\right)$$

Now, again, we assume that individuals are divided into two groups $g \in \{a, b\}$. Fixing a copula parameter between α_1 and α_0 for each group, and using results from section C, we know we can identify the joint distribution of returns and beliefs for groups a and b, $P_a(\mathcal{R}, E(s))$ and $P_b(\mathcal{R}, E(s))$. We now show that this is sufficient to identify the quantity in 2 and solve for the decomposition.

Given the information structure of group a, we can derive the counterfactual joint distribution of signals and returns as follows[5]

$$\begin{pmatrix} \boldsymbol{S}_a \\ \mathcal{R}_b \end{pmatrix} \sim N\left(\begin{pmatrix} k_a + m_a \mu_{\boldsymbol{S}_a} \\ \mu_{\mathcal{R}_b} \end{pmatrix}, \begin{bmatrix} m_a \sigma_b^2 {m_a}^T + \Sigma_{\boldsymbol{S}_a} - m_a \Sigma_{\boldsymbol{S}_a,\mathcal{R}_b}^T & m_a \sigma_{\mathcal{R}_b}^2 \\ {m_a}^T \sigma_{\mathcal{R}_b}^2 & \sigma_{\mathcal{R}_b}^2 \end{bmatrix}\right)$$

where $k_a = \mu_{S_a} - \Sigma_{\boldsymbol{S}_a,\mathcal{R}_b} \sigma_{\mathcal{R}_b}^{-2} \mu_{\mathcal{R}_b}$ and $m_a = \Sigma_{\boldsymbol{S}_a,\mathcal{R}_b} \sigma_{\mathcal{R}_b}^{-2}$ and subscript $g \in \{a, b\}$ indicates that the parameters are from the distribution of group g.

[5] We slightly abuse notation here setting $\mathcal{R}_b$ to denote that it's distributed as in group b

We can now derive the counterfactual posterior mean belief, given a signal realization $\boldsymbol{S}$.

$$E_{a,b} = \mu_b + m_a^T \sigma_{\mathcal{R}_b}^2 \left(\Sigma_{\boldsymbol{S}_a,\mathcal{R}_a} \sigma_{\mathcal{R}_a}^{-2} \sigma_{\mathcal{R}_b}^2 \sigma_{\mathcal{R}_a}^{-2} \Sigma_{\boldsymbol{S}_a,\mathcal{R}_a}^T + \Sigma_{\boldsymbol{S}_a} - \Sigma_{\boldsymbol{S}_a,\mathcal{R}_a} \sigma_{\mathcal{R}_a}^{-2} \Sigma_{\boldsymbol{S}_a,\mathcal{R}_a}^T \right)^{-1} \left(\boldsymbol{S}_a - k_a - m_a \mu_{\boldsymbol{S}_a} \right)$$

and the counterfactual belief distribution is given by

$$E_{a,b} \sim N\left(\mu_b, \sigma_{\mathcal{R}_b}^4 m_a^T \left(\left(m_a \sigma_b^2 m_a^T + \Sigma_{\boldsymbol{S}_a} - m_a \Sigma_{\boldsymbol{S}_a,\mathcal{R}_a}^T \right)^{-1} \right)^T m_a \right)$$

Denote by OV_a the identified variance of beliefs for group a

$$OV_a = \Sigma_{S_a,\mathcal{R}_a}^T \Sigma_{S_a}^{-1} \Sigma_{S_a,\mathcal{R}_a}$$

The following proposition assert that we can identify the variance of the counterfactaul beliefs distribution

Proposition 3. Let $\mathcal{R}$ and signal vector $\boldsymbol{S}$ be jointly Gaussian-distributed, conditional on the cost c, for members of both group a and b. Then we we can point identify the counterfactual quantity

$$\int_{\mathcal{R}\times c} \mathcal{P}(E_{a,b}(s) \geq c|\mathcal{R}, c, a) \pi_b(\mathcal{R}|c) p(c) d\mathcal{R} dc$$

Proof. The proof follows from the following derivation:

$$
\begin{aligned}
\mathrm{Var}(E_{a,b}) &= \sigma^4_{\mathcal{R}_b} m_a^T \left(\left(\Sigma_{\boldsymbol{S}_a,\mathcal{R}_a} \sigma^{-2}_{\mathcal{R}_a} \sigma^2_{\mathcal{R}_b} \sigma^{-2}_{\mathcal{R}_a} \Sigma^T_{\boldsymbol{S}_a,\mathcal{R}_a} + \Sigma_{\boldsymbol{S}_a} - \Sigma_{\boldsymbol{S}_a,\mathcal{R}_a} \sigma^{-2}_{\mathcal{R}_a} \Sigma^T_{\boldsymbol{S}_a,\mathcal{R}_a} \right)^{-1} \right)^T m_a \\
&= \frac{\sigma^4_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} \Sigma^T_{\boldsymbol{S}_a,\mathcal{R}_a} \left(\Sigma_{\boldsymbol{S}_a,\mathcal{R}_a} \Sigma^T_{\boldsymbol{S}_a,\mathcal{R}_a} \left(\frac{\sigma^2_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} - \frac{1}{\sigma^2_{\mathcal{R}_a}} \right) + \Sigma_{\boldsymbol{S}_a} \right)^{-1} \Bigg)^T \Sigma_{\boldsymbol{S}_a,\mathcal{R}_a} \\
&= \frac{\sigma^4_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} \Sigma^T_{\boldsymbol{S}_a,\mathcal{R}_a} \left(\Sigma^{-1}_{S_a} - \frac{(\frac{\sigma^2_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} - \frac{1}{\sigma^2_{\mathcal{R}_a}}) \Sigma^{-1}_{S_a} \Sigma_{S_a,R_a} \Sigma^T_{S_a,R_a} \Sigma^{-1}_{S_a}}{1 + (\frac{\sigma^2_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} - \frac{1}{\sigma^2_{\mathcal{R}_a}}) \Sigma^T_{S_a,R_a} \Sigma^{-1}_{S_a} \Sigma_{S_a,R_a}} \right)^T \Sigma_{\boldsymbol{S}_a,\mathcal{R}_a} \\
&= \frac{\sigma^4_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} \left(OV_a - \frac{(\frac{\sigma^2_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} - \frac{1}{\sigma^2_{\mathcal{R}_a}}) OV_a^2}{1 + (\frac{\sigma^2_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} - \frac{1}{\sigma^2_{\mathcal{R}_a}}) OV_a} \right) \\
&= \frac{\sigma^4_{\mathcal{R}_b}}{\sigma^2_{\mathcal{R}_b} + \frac{\sigma^2_{\mathcal{R}_a}(\sigma^2_{\mathcal{R}_a} - OV_a)}{OV_a}}
\end{aligned}
$$

where in the third row we used the Sherman-Morrison formula and the definition of OV_b. □

Remark. Notice that in the normal case, where both the returns distribution and signals are normally distributed, there is no loss of generality in assuming that high school graduates receive only a scalar noise of the form

$$s = \mathcal{R} + \epsilon$$

where $\epsilon \sim \mathcal{N}(0, \sigma^2_\epsilon)$. Following the same steps as before, we can show that the observed variance of beliefs is given by

$$OV = \frac{\sigma^4_{\mathcal{R}}}{\sigma^2_{\mathcal{R}} + \sigma^2_\epsilon}$$

which implies that the information structure $P(S|\mathcal{R}) = \mathcal{N}(\mathcal{R}, \sigma^2_\epsilon)$ is identified by

$$\sigma^2_\epsilon = \frac{\sigma^2_{\mathcal{R}}(\sigma^2_{\mathcal{R}} - OV)}{OV}$$

Given the information structure, the counterfactual distribution is simply given by

$$\frac{\sigma^4_a}{\sigma^2_a - \frac{\sigma^2_{\mathcal{R},b}(\sigma^2_{\mathcal{R},b} + OV_b)}{OV_b}}$$

which aligns with the counterfactual quantity when agents have a richer signal structure.

E.1.3 Identification of the the information structure Decomposition with data on the full belief distribution

In some cases researchers may hope to elicit information on the the probabilities that an agent put on each outcome realization (Manski (2004), Wiswall and Zafar (2015),Zafar (2011),Wiswall and Zafar (2021) Diaz-Serrano and Nilsson (2022)). We now turn to show that this information is sufficient for point identification our choice gap decomposition, with respect to the information structure.

We assume that individuals from group b have earnings distribution π_b and access to the information structure $(\mathrm{P}(S|b, \boldsymbol{\alpha}), \mathcal{S})$, and for group a have returns distribution π_a and access to the information structure $(\mathrm{P}(S|a, \boldsymbol{\alpha}), \mathcal{S})$. Denote by $q_{s,g} \in \Delta(\boldsymbol{\alpha})$ the posterior beliefs induced by a signal realization $s \in S$ and prior π_g. We let $q_{s,g}(\boldsymbol{\alpha})$ be the assigned density that this posterior puts on state $\boldsymbol{\alpha}$. Furthermore, we assume that we observe for each group the joint distribution, $\phi(\boldsymbol{\alpha}, q_s)$, of returns $\boldsymbol{\alpha}$ and the posterior beliefs q_s.

We start by noting that within the framework, knowing beliefs allows us to identify a richer notion of costs. Specifically, denote by $B_i = \int_{\mathcal{R}} \mathcal{R} q_i(\mathbb{R}) d\mathcal{R}$ the measured posterior mean for individuals with beliefs q_i and notice that

$$P(D = 1|x, B) = E[\mathbb{1}[B_i \geq c(x, \nu)]]$$

where ν is additional cost heterogeneity, that is not included in our identifying discussion in section 2.4. Under some regularity conditions and the assumption that $B \perp\!\!\!\perp \nu|X$ we can identify the distribution of $c(x, \nu)$ for each x and B, using variation in B. The identicification here relies on B as being a "special regressor" that is needed for identification, as discussedi in (Lewbel (2012)). We therefore, assume that we know the joint distribution of $P(q_i, c_i|x)$.

To identify the outcomes distribution we can use two approaches. The first is to use the measured beliefs and simply integrate over beleifs,i.e.

$$\pi_g(\mathcal{R}) = \int_i q_i(\mathcal{R}) di$$

Under the assumption on rational expectations is held, then this should would provide us the initial prior[6]

We start by showing the following lemma that shows that for a fixed information structure, there's a mapping from the posterior, given prior π'_g to a posterior under a different prior. Let π_g and $\pi_{g'}$ be two priors with the same support, then for each s, information structure $P(s|\alpha)$ prior π_g and implied posterior q_s, the counterfactual posterior with prior $\pi_{g'}$ is given by $q_{s,g'} = \frac{\frac{q_s(\boldsymbol{\alpha})}{\pi(\boldsymbol{\alpha})}\pi_{g'}(\boldsymbol{\alpha})}{\int_{\boldsymbol{\alpha}} \frac{q_s(\boldsymbol{\alpha})}{\pi(\boldsymbol{\alpha})}\pi_{g'}(\boldsymbol{\alpha})d\boldsymbol{\alpha}}$

Proof.

$$\begin{aligned}
q_{s,g'}(\boldsymbol{\alpha}) &= \frac{P(s|\boldsymbol{\alpha})\pi_{g'}(\boldsymbol{\alpha})}{\int_{\boldsymbol{\alpha}} p(s|\boldsymbol{\alpha})\pi_{g'}(\boldsymbol{\alpha})d\boldsymbol{\alpha}} \\
&= \frac{\frac{P(s)q_s(\boldsymbol{\alpha})}{\pi(\boldsymbol{\alpha})}\pi_{g'}(\boldsymbol{\alpha})}{\int_{\boldsymbol{\alpha}} \frac{P(s)q_s(\boldsymbol{\alpha})}{\pi(\boldsymbol{\alpha})}\pi_{g'}(\boldsymbol{\alpha})d\boldsymbol{\alpha}} \\
&= \frac{\frac{q_s(\boldsymbol{\alpha})}{\pi(\boldsymbol{\alpha})}\pi_{g'}(\boldsymbol{\alpha})}{\int_{\boldsymbol{\alpha}} \frac{q_s(\boldsymbol{\alpha})}{\pi(\boldsymbol{\alpha})}\pi_{g'}(\boldsymbol{\alpha})d\boldsymbol{\alpha}}
\end{aligned}$$

□

Lemma E.1.3 demonstrates that the counterfactual posterior can be calculated from the known posterior π_g and the counterfactual distribution $\pi_{g'}$, without requiring explicit knowledge of the information structure. Given the counterfactual posteriors, one can also derive the counterfactual means and thus identify all components of the decomposition. We proceed to establish that all parts of the decomposition are identified.

Fix some cost c, and recall that for our decomposition we needed to identify the distribution of counterfactual posterior mean, if the returns were drawn according to group b, information according to group a and updated correctly in this new counterfactual world

$$P_{\langle a,b\rangle} = \int_{\boldsymbol{\alpha}} \mathcal{P}(E_{a,b}(s) \geq 0|\boldsymbol{\alpha}, a), \pi_b(\boldsymbol{\alpha})d\boldsymbol{\alpha}$$

[6] If ones have also access to to outcomes data, maintain the common prior assumption, and assume that agents are Bayesian with inaccurate beliefs (Bohren et al. (2023)).

Proposition 4. Assume we know $\phi_a(q_{s,a}, c, \boldsymbol{\mathcal{R}})$ and $\phi_b(q_{s,a}, c, \boldsymbol{\mathcal{R}})$ then the conditional distribution $\mathcal{P}(E_{a,b}(s)|\boldsymbol{\alpha}, a)$ is identified and so is $P_{\langle a,b\rangle}$ in 20

Proof. The proof follows from Lemma E.1.3. Notice that according to Lemma E.1.3, every two signals that generate the same posterior for group a, also generate the same posteriors in the counterfactual case where $\boldsymbol{\alpha}$ is distributed according to π_b; therefore, it's enough to know the posterior without requiring the information structure. Further, using Lemma 3, we can identify the distribution of the counterfactual posteriors by calculating the implied distribution of the composition $\frac{\left(\frac{q_S(\boldsymbol{\alpha})}{\pi(\boldsymbol{\alpha})}\right)\pi_{g'}(\boldsymbol{\alpha})}{\int_{\boldsymbol{\alpha}}\left(\frac{q_S(\boldsymbol{\alpha})}{\pi(\boldsymbol{\alpha})}\right)\pi_{g'}(\boldsymbol{\alpha})d\boldsymbol{\alpha}}$. Finally, to obtain $P(E_{a,b}|\boldsymbol{\alpha})$, we only need to map each posterior to its implied mean. As $P(E_{a,b}|\boldsymbol{\alpha})$ is identified, $P_{\langle a,b\rangle}$ is trivially identified, along with the decomposition XXX. □

One implication of Proposition 4 is that in the case where we have binary outcomes $Y \in \{1, 0\}$, and we know the joint distribution of $\phi(E[Y|s], Y)$, the decomposition is point identified using simply the conditional mean beliefs.

Corollary 2. If outcomes are binary $Y \in \{1, 0\}$ and we observe the joint $\phi(E[Y|s], Y)$, then $P_{\langle a,b\rangle}$ in 20 is point identified

Proof. Simply follows from proposition 4 and the fact that in the bianry case $E[Y|s]$ is the posterior distribution. □

The case of binary outcomes is prevalent in many applications within the discrimination literature. For instance, in bail decisions, judges are often modeled as agents attempting to predict the likelihood of reoffense (e.g., Arnold et al. (2018)), Researchers may wish to quantify the extent to which disparities in decisions made for Black and White defendants are driven by the information available to judges or by the underlying distribution of reoffending rates. The above corollary demonstrates that we can decompose this gap and precisely identify the role each component plays. Similar arguments can be extended to other contexts, such as hiring decisions (Bertrand and Mullainathan (2004),Kline et al. (2022)) or treatment allocation in medical settings (Chan et al. (2022)).

E.1.4 Partial Identification

Results based on Bergemann et al. (2022). tbw shortly.

F The effect of information on Earnings Inequality

In this section, we explore how adding additional information on potential returns would affect the observed wage gaps. Specifically, we examine how the differences

$$\mathrm{E}[Y|Whites] - \mathrm{E}[Y|Hispanics]$$

alter as we introduce more precise information on either U_1 or U_0. As discussed in the text, we focus on additional information that has been residualized from other information individuals may possess. Since the posterior mean of a normal distribution is linear, the covariance matrix between $(s_n, E[Y_1 - Y_0|s, x], U_1, U_0)$ is as follows:

$$\begin{pmatrix} \sigma^2_{s_n} & 0 & \sigma_{s_n,U_1} & \sigma_{s_n,U_0} \\ 0 & \sigma^2_{\mathrm{E}} & \sigma_{\mathrm{E},U_1} & \sigma_{\mathrm{E},U_0} \\ \sigma_{s_n,U_1} & \sigma_{\mathrm{E},U_1} & \sigma^2_1 & \rho\sigma_1\sigma_0 \\ \sigma_{s_n,U_0} & \sigma_{\mathrm{E},U_0} & \rho\sigma_1\sigma_0 & \sigma^2_0 \end{pmatrix}$$

As seen in section 4.2, the most substantial changes in selection patterns occur when we introduce a signal on either U_1 or U_0. Consequently, we confine our focus to these signals.

Assuming information is provided solely on U_1, it follows that $\mathrm{Cov}(s_n, U_0) = 0$. The derivation for the case with information only on U_0 is analogous. Let $\tilde{E}$ and $\tilde{P}(d = 1|x, z)$ represent the counterfactual conditional mean earnings, given the new signal, and the counterfactual propensity score, respectively. Denote by $\sigma_{\tilde{E}} = \sigma^2_{\mathrm{E}} + \sigma^2_{s_n,1}$, the counterfactual variance in beliefs. Then, the new mean earnings are given by

$$\tilde{E}[Y|g] = \int_X [\tilde{E}[Y|x, z, d = 1, g]\tilde{P}(d = 1|x, z, g) + \tilde{E}[Y|x, z, d = 0, g]\tilde{P}(d = 0|x, z, g)]f(x, z)d(x, z)$$

where f is the density of X, Z. We can further simplify $E[Y_1|x, z, D = 1, g]$ and $E[Y_1|x, z, D = 0, g]$

$$E[Y|x, z, D = 1, g] = x\beta_1^g + \frac{\sigma_{\tilde{E},1}}{\sigma_{\tilde{E}}} \frac{\phi\left(\frac{\mu_x - c(x,z)}{\sigma_{\tilde{\mathrm{E}}}}\right)}{1 - \Phi\left(\frac{\mu_x - c(x,z)}{\sigma_{\tilde{\mathrm{E}}}}\right)}$$

$$E[Y|x, z, D = 0, g] = x\beta_0^g - \gamma_0 \frac{\phi\left(\frac{\mu_x - c(x,z)}{\sigma_{\tilde{\mathrm{E}}}}\right)}{\Phi\left(\frac{\mu_x - c(x,z)}{\sigma_{\tilde{\mathrm{E}}}}\right)}$$

where we can use γ_0, as our additional signal is not correlated with U_0.

Figure 22 plots the implied mean earnings of Hispanics, given a signal's effect on average earnings for different information quality R_1^2. As seen in the table, the earnings gap between Hispanics and whites stands at \$1750. As the signal quality increases, Hispanic income also increases. At the extreme possible case, we provide only information on U_1 and $R_1^2 = 0.97$, the earnings are \$9572, which are still lower than the measured average earnings of whites. The main takeaway from this figure is that providing even highly accurate information does not substantially mitigate inequality. This implies that policymakers wishing to address inequality by relying on selection[7] need to provide a more complex signal. This would deviate from the normative model and push into college those individuals who would benefit most, and push out those with negative returns. Generally, this would require leaving the Gaussian model which we are using here, and constructing a more complicated signal.

[7]Under the assumption that large earnings are invariant to significant changes in selection patterns

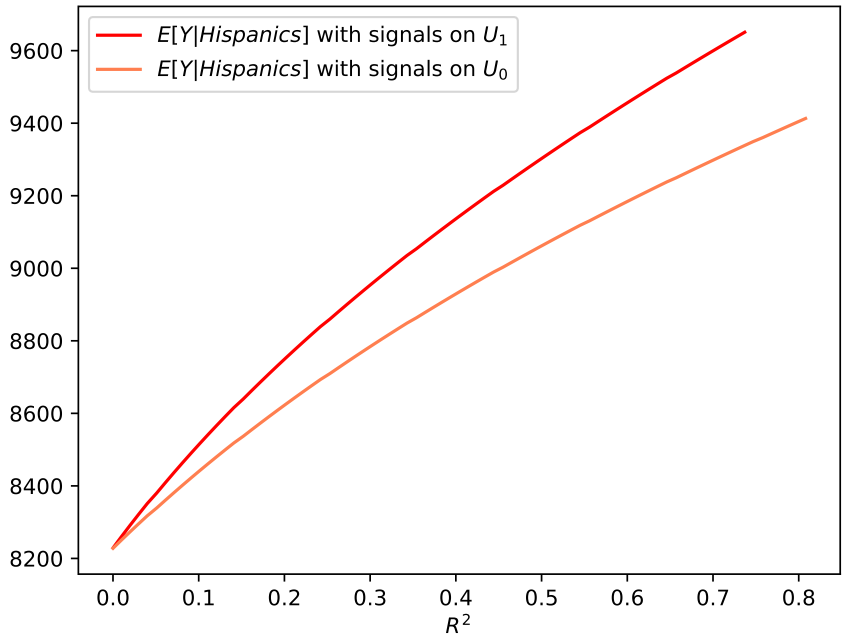

Figure 22: Cost, information and Beliefs interaction

Note: This figure plots the effect on the observed mean earnings of Hispanics when an additional informative signal with information quality R_d^2 is introduced, informing on either U_1 or U_0. The red line depicts the change in mean earnings 12 to 15 years after high school graduation, following the introduction of additional information, with quality R^2, on earnings for those who choose to attend college the first year after high school graduation. The orange line conducts the same exercise showing the changes in mean earnings, due to introduction of additional information with information quality R^2 on earnings for individuals who do not attend college.

??

G Quality of Information from the Bayesian Perspective of Individuals with Non-Informative Priors

In our analysis in Section 2.3.1, we examine the quality of information with regard to the true underlying variance of $U_1 - U_0$. This approach yields wide bounds due to the necessity of considering all possible correlation values between U_1 and U_0. In this section, we employ a different approach to redefine the quality of information. We do this by considering the expected explained variance under various correlation values, taking into account the individual prior beliefs regarding the correlation parameter. Specifically, we assume that agents cannot determine the correlation between U_1 and U_0, treating it as part of the uncertainty they encounter. We posit that agents hold a non-informative prior over the feasible correlation values, distributed as $\rho_{1,0} \sim U(\rho_{min}, \rho_{max})$. We quantify information quality as the proportion of explained variance relative to the total variance of the outcome, which encompasses the variance resulting from uncertainty over $\rho_{1,0}$. Let

$$R^2(\rho) = \frac{\text{Var}(E[\mathcal{R}|s])}{\sigma_1^2 + \sigma_0^2 - 2\rho\sigma_1\sigma_0}$$

and $f(\rho)$ denotes the prior distribution over the set of possible correlation parameters, we then define the the Bayesian R-Squared as the mean R^2, given the prior

$$BR^2 = \int_\rho R^2(\rho) f(\rho) d\rho \tag{23}$$

This approach is in line with the discussion by Gelman et al. (2019), which considered a similar measure for assessing model performance under uncertainty of the parameters. Within our framework, we limit the model uncertainty of agents exclusively to the correlation parameter, maintaining the the assumption that agents are confident in their predictions and know the likelihood functions $P(S|\mathcal{R})$.

Notice that we can write the identified variance of beliefs using (23) as

$$\text{Var}(E[\mathcal{R}|s]) = BR^2 \div \int_\rho \frac{1}{\sigma_1^2 + \sigma_0^2 - 2\rho\sigma_1\sigma_0} f(\rho) d\rho = \frac{BR^2}{\int_\rho \sigma_{\mathcal{R}}^{-1}(\rho) f(\rho) d\rho}$$

where we let $\sigma_{\mathcal{R}}^{-1}(\rho)$ denote the inverse of the implied variance of returns for a given correlation value ρ. Denote by BR_g^2 the Bayesian R^2 of group g. We can now define our alternative information quality decomposition, similar to (2.3.1), as

$$P(D=1|b) - P(D=1|a) = \int_X \Phi\left(\frac{\mu_{\mathcal{R},b,x} - c_b(x)}{\sqrt{\frac{BR_b^2}{\int_\rho \sigma_{\mathcal{R},b,x}^{-1}(\rho) f(\rho) d\rho}}}\right) dF_b(x) - \int_X \Phi\left(\frac{\mu_{\mathcal{R},b,x} - c_b(x)}{\sqrt{\frac{BR_a^2}{\int_\rho \sigma_{\mathcal{R},b,x}^{-1}(\rho) f(\rho) d\rho}}}\right) dF_b(x)$$
$$+ \int_X \Phi\left(\frac{\mu_{\mathcal{R},b,x} - c_b(x)}{\sqrt{\frac{BR_a^2}{\int_\rho \sigma_{\mathcal{R},b,x}^{-1}(\rho) f(\rho) d\rho}}}\right) dF_b(x) - \int_X \Phi\left(\frac{\mu_{\mathcal{R},a,x} - c_a(x)}{\sqrt{\frac{BR_a^2}{\int_\rho \sigma_{\mathcal{R},a,x}^{-1}(\rho) f(\rho) d\rho}}}\right) dF_a(x)$$

Therefore, our decomposition asks how much of the gap is driven by differences in information quality, taking into account fundamental model uncertainty on the correlation, from the agents perspective.

We next discuss how we determine ρ_{min} and ρ_{max}. We consider a set of feasible priors. The first is the set of feasible values, as discussed in section 2.4.3. Although these are restrictions implied by the models, if individuals observe additional signals they can use them to restrict the feasible set of ρs even further, and therefore, from their perspective the average value of information may be different. Unfortunatly, we do not observe the signals, but we only know the $\text{Var}(E[U_1 - U_0|s, x]$. In section D we discuss how, given a guess on the quality of information on the marginals R_1^2, we can derive the covariance matrix of $U_1, U_0, E[U_1|s], E[U_0|s]$. The next claim shows, that knowing this is sufficient to identify the set of possible correlation values agents infer are feasible, given their signals. Let $\boldsymbol{S}$ denote the vector of signals agents observe. Let the matrix

$$A = \begin{bmatrix} \sigma_{S_1,S_1} & \cdots & \sigma_{S_1,S_n} & \sigma_{S_1,U_1} & \sigma_{S_1,U_0} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \sigma_{S_n,S_1} & \cdots & \sigma_{S_n,S_n} & \sigma_{S_n,U_1} & \sigma_{S_n,U_0} \\ \sigma_{U_1,S_1} & \cdots & \sigma_{U_1,S_n} & \sigma_{U_1,U_1} & \sigma_{U_1,U_0} \\ \sigma_{U_0,S_1} & \cdots & \sigma_{U_0,S_n} & \sigma_{U_0,U_1} & \sigma_{U_0,U_0} \end{bmatrix}$$

be the covariance matrix between the signals S, U_1 and U_0. And let

$$B = \begin{bmatrix} \sigma^2_{\mathrm{E}_1} & \sigma_{\mathrm{E}_1,\mathrm{E}_0} & \sigma_{\mathrm{E}_1,U_1} & \sigma_{\mathrm{E}_1,U_0} \\ \sigma_{\mathrm{E}_0,\mathrm{E}_1} & \sigma^1_{\mathrm{E}_0} & \sigma_{\mathrm{E}_0,U_1} & \sigma_{\mathrm{E}_0,U_0} \\ \sigma_{U_1,\mathrm{E}_1} & \sigma_{U_1,\mathrm{E}_0} & \sigma^2_1 & \rho\sigma_1\sigma_0 \\ \sigma_{U_0,\mathrm{E}_1} & \sigma_{U_0,\mathrm{E}_0} & \rho\sigma_1\sigma_0 & \sigma^2_0 \end{bmatrix}$$

be the covariance matrix between $\mathrm{E}[U_1|s]$, $\mathrm{E}[U_0|s]$, U_1 and U_0.

The following proposition shows that the set of ρ that keep A Positive Semi-Definite (PSD) is the same is the set of ρ that keeps matrix B PSD.

Proposition 5. A correlation value ρ is feasible in matrix A if and only if it is feasible in matrix B

Proof. Without loss of generality, we restrict attention to the case where signals are independent. This is without loss as we can always residualized signals, and as conditional distribution in the Gaussian case is linear, we do not change the information. The first direction says that if the covariance matrix beliefs and state variables reject than

We start by showing the first direction. We consider the reverse case and show that if ρ is in-feasible under matrix A, then it's infeasible under matrix B as well. Fix correlation value ρ, and assume that it makes matrix A non PSD. If it so, then there exists a vector t such that $t'At < 0$. Denote t_{s_i} the value in vector t that corresponds to signal s_i. and by t_1 and t_0 the value in vector t that correspond to U_1 and U_0. We can then write

$$t'At = \sum_i t^2_{s_i} + t_1\left(\sum \sigma_{s_i,1}t_{s_i}\right) + t_0\left(\sum \sigma_{s_i,0}t_{s_i}\right) + \sigma^2_1 t^2_1 + \sigma^2_0 t^2_0 + 2\rho\sigma_0\sigma_1 t_1 t_0 < 0 \tag{24}$$

We now show that there must exists a vector k, such that $k'Bk < 0$. Denote k_{E_d}, k_1 and k_0, similar to before, then

$$\begin{aligned} k'Bk &= 2k_1(\sigma^2_{\mathrm{E}_1}k_{\mathrm{E}_1} + \sigma_{\mathrm{E}_1,\mathrm{E}_0}k_{\mathrm{E}_0}) \\ &\quad + 2k_0(\sigma^2_{\mathrm{E}_0}k_{\mathrm{E}_0} + \sigma_{\mathrm{E}_1,\mathrm{E}_0}k_{\mathrm{E}_1}) \\ &\quad + (2\sigma_{1,0}k_{\mathrm{E}_1}k_{\mathrm{E}_0} + \sigma_{\mathrm{E}_1}k^2_{\mathrm{E}_1} + \sigma_{\mathrm{E}_0}k^2_{\mathrm{E}_0}) \\ &\quad + \sigma^2_1 k^2_1 + \sigma^2_0 k^2_0 + 2\rho\sigma_1\sigma_0 k_1 k_0 \end{aligned}$$

As we assumed that signals are uncorrleated, and the conditional distribution of Gaussian model is linear function of signals, we can rewrite these expressions as

$$
\begin{aligned}
k'Bk &= 2k_1(k_{\mathrm{E}_1}\sum_{s_i}\sigma^2_{s_i,1} + k_{\mathrm{E}_0}\sum_{s_i}\sigma_{s_i,1}\sigma_{s_i,0}) \\
&+ 2k_0(k_{\mathrm{E}_0}\sum_{s_i}\sigma^2_{s_i,0} + k_{\mathrm{E}_1}\sum_{s_i}\sigma_{s_i,1}\sigma_{s_i,0}) \\
&+ (2k_{\mathrm{E}_1}k_{\mathrm{E}_0}\sum_{s_i}\sigma_{s_i,1}\sigma_{s_i,0}) + k^2_{\mathrm{E}_1}\sum_{s_i}\sigma^2_{s_i,1} + k^2_{\mathrm{E}_0}\sum_{s_i}\sigma^2_{s_i,0}) \\
&+ \sigma_1^2k_1^2 + \sigma_0^2k_0^2 + 2\rho\sigma_1\sigma_0k_1k_0 \\
&= 2k_1(\sum_{s_i}\sigma_{s_i,1}(\sigma_{s_i,1}k_{\mathrm{E}_1} + \sigma_{s_i,0}k_{\mathrm{E}_0})) \\
&+ 2k_0(\sum_{s_i}\sigma_{s_i,0}(\sigma_{s_i,0}k_{\mathrm{E}_0} + \sigma_{s_i,1}k_{\mathrm{E}_1})) \\
&+ \sum_{s_i}(\sigma_{s_i,1}k_{\mathrm{E}_1} + \sigma_{s_i,0}k_{\mathrm{E}_0})^2 \\
&+ \sigma_1^2k_1^2 + \sigma_0^2k_0^2 + 2\rho\sigma_1\sigma_0k_1k_0
\end{aligned}
$$

We now show how to find values of the vector k that makes this expression negative. We set $k_1 = t_1$ and $k_0 = t_0$. We use the additional two values of k to equate the remaining values such that $k'Bk = t'At < 0$. To do so, we notice we have two equation for two parameters

$$
\begin{aligned}
&\sum_{s_i}(\sigma_{s_i,1}k_{\mathrm{E}_1} + \sigma_{s_i,0}k_{\mathrm{E}_0}))(k_1\sigma_{s_i,1} + k_0\sigma_{s_i,0}) \\
&= \sum_{s_i}t_{s_i}(k_1\sigma_{s_i,1} + k_0\sigma_{s_i,0})
\end{aligned}
$$

and

$$
\sum_{s_i}(\sigma_{s_i,1}k_{\mathrm{E}_1} + \sigma_{s_i,0}k_{\mathrm{E}_0})^2 = \sum_{i}t^2_{s_i} \tag{25}
$$

We can then solve for k_{E_1} which gives us

$$
k_{\mathrm{E}_1} = \frac{\sum_{s_i}t_{s_i}(k_1\sigma_{s_i,1} + k_0\sigma_{s_i,0}) - k_{\mathrm{E}_0}(k_1\sigma_{s_i,1} + k_0\sigma_{s_i,0})}{\sum_{s_i}\sigma_{s_i,1}}
$$

plug this back into equation 25, we see that we have continuous function of k_{E_0}. This function goes from from 0 to infinity, the right hand side is a finite and positive expression, then by the Intermediate value theorem there exists a solution, which implies that there exists a vector for which $k'Bk < 0$ and B is not PSD. To show the reverse direction, follows the same steps in the reverse direction, which concludes the proof. $\square$

G.1 Results of the Bayesian Information Quality Decomposition

Table A.13 shows the result of the effect of equating BR^2 as we just discussed. The first row shows the effect with restricting the set of possible ρ according to the discussion we had in section 2.4.3. We can see that equating the information quality as such, would result in increasing the gap by 7 percentage points. This happens as the information quality of Hispanics is better than the information quality of Hispanics. Therefore, the differences in information are pushing the choice gap down. The next row shows the quality of information, restricting the set of possible values of ρ to positive ones.

	Gap	Information Channel	Composition Channel
1) Feasible ρ			
($\rho_{min} = -0.96, \rho_{max} = 0.9$)		-0.073 (-91.41%)	0.152 (191.0%)
2) Feasible ρ, $\rho \geq 0$			
($\rho_{min} = 0, \rho_{max} = 0.9$)		-0.052 (-65.59%)	0.131 (166.0%)

Table A.13: Information Decomposition - Average quality

H Discussion on Rational Expectations

Our analysis in the main text rests on the assumption that individuals adhere to Bayesian principles and have rational expectations. We incorporate this into our model and identification strategy by assuming that (1) individuals interpret signals accurately using the correct likelihood function, and (2) their prior distribution on returns is accurate. Maintaining the Bayesian perspective, our model of belief formation could be violated in two ways[8]. Agents might employ incorrect likelihoods or hold erroneous priors, or exhibit both inaccuracies. We recognize that generally, any deviation from rational expectations can be viewed as model mis-specification. Specifically, our model posits that agents' beliefs about $\mathcal{R}$, or an increasing function thereof, are formed correctly. However, if agents are Bayesian but derive an incorrect posterior, this too is indicative of a mis-specified utility function[9]. To illustrate, denote $\tilde{\mathrm{E}}(s)$ as the subjective belief and $\tilde{q}(\mathcal{R}|s)$ as the subjective posterior given signal s, and let $q(\mathcal{R}|s)$ represent the accurate posterior conditional on signal s. We then have[10]

$$\tilde{E}(s) = \int_{\mathcal{R}} \mathcal{R}\tilde{q}(\mathcal{R}|s)d\mathcal{R} = \int_{\mathcal{R}} \mathcal{R}\frac{\tilde{q}(\mathcal{R}|s)}{q(\mathcal{R}|s)}q(\mathcal{R}|s)d\mathcal{R} = \mathrm{E}\left[\mathcal{R}\frac{\tilde{q}(\mathcal{R}|s)}{q(\mathcal{R}|s)}|s\right]$$

Thus, a violation of the rational expectations assumption essentially represents a re-weighting of returns, analogous to a misclassified utility function. It's important to note that as long as this reweighting implies an increasing relation with the true posterior mean, i.e. we can write $E[\mathcal{R}|s]$ as some increasing function of $\tilde{E}(s)$, then this is not an issue for identification, as we discussed in section 2.

Disentangling mis-specified utility, beliefs, and priors presents a significant challenge, obscuring the influence of information, as captured by signals, on decision-making. We thus focus on a specific violation of rational expectations where agents hold an incorrect prior but interpret signals accurately. This concept, referred to as inaccurate beliefs[11]. As discussed in C, we use the accurate beliefs assumption identify the distribution of beliefs. With a more comprehensive data set encompassing both outcomes and surveyed beliefs, researchers can point identify systematic biases in belief formation and presence of common priors.[12] For clarity, we postulate that researchers have access to the joint

[8] We presuppose a population with a shared prior and access to the same information structure. The notion of a common prior is restrictive, which we acknowledge but do not explore alternatives to this here.

[9] This concept aligns with discussions in Bohren et al. (2023) regarding identification issues, and with the concept of omitted payoff bias in Kleinberg et al. (2018).

[10] We assume that individuals know correct support, such that their posterior might update incorrectly, but it has the same support as the correct posterior

[11] Inaccurate beliefs, as discussed in Bohren et al. (2023), particularly in the context of discrimination, reflect how agents may use biased priors that lead to erroneous belief updating.

[12] Vatter (2022) examines an alternative method for identifying posterior distributions, using known changes in available signals. Specifically, it utilizes changes over time in quality metrics and demonstrates that adjustments to the threshold can be used to identify the prior distribution, before observing the signal, under linear preferences assumption.

distribution $P(\tilde{E}, \mathcal{R})$, where $\tilde{E}$ denote a subjective belief not necessarily derived from rational expectations[13]. Additionally, we assume that researchers can estimate or access a common the agents mis-specified prior. As explicated in section **?**, one might deduce the subjective common prior either by eliciting and averaging the full belief distributions or through a structural method that assumes the subjective prior is some function of the underlying observed distribution of outcomes.

Two components are essential for the decomposition exercise. The first concerns how a mis-specified prior might change if we alter the underlying distribution of returns. One of the benefits of rational expectations arises from its linkage of true outcomes to beliefs. Should this link be disrupted, the researcher must posit how these elements interact. We proceed under the assumption that mis-specified priors remain constant with changes in the distribution of $\mathcal{R}$. Other assumptions by researchers could be made, and the framework of analysis would remain unchanged. The second aspect to consider is the value of information in the presence of biased beliefs. Here, we assume that the value of information is the R^2 that could would be implied if an agent had the same information structure and used the correct prior.

We start by looking at the way we measure information issues, as we talked about in section E. Here, we can use the ideas from that section but modify the IC constraints. This rule makes sure the information setup meets the inaccurate beliefs agents have :

$$\begin{aligned}
&\forall \tilde{E}, \mathrm{E}_{cf} \quad \int_{\mathcal{R}} \pi(\mathcal{R}, \tilde{E}, \mathrm{E}_{cf}) \frac{\pi_{IP,a}(\mathcal{R})}{\pi(\mathcal{R})} (\mathcal{R} - \tilde{E}) d\mathcal{R} = 0 \\
&\forall \tilde{E}, \mathrm{E}_{cf} \quad \int_{\mathcal{R}} \pi(\mathcal{R}, \tilde{E}, \mathrm{E}_{cf}) \frac{\pi_{IP,b}(\mathcal{R})}{\pi(\mathcal{R})} (\mathcal{R} - \tilde{E}) d\mathcal{R} = 0 \\
&\forall \mathcal{R}, \tilde{E} \quad \int_{\mathrm{E}_{cf}} \pi(\mathcal{R}, \tilde{E}, \mathrm{E}_{cf}) d\mathrm{E}_{cf} = Pr_a(\mathcal{R}, \mathrm{E})
\end{aligned}$$

and the objective is given by

$$max \text{ or } min \int_{\mathrm{E}_{cf} > c} \pi(\mathcal{R}, \tilde{E}, \mathrm{E}_{cf}) \frac{\pi_b(\mathcal{R})}{\pi_a(\mathcal{R})} d\mathcal{R} d\tilde{E} d\mathrm{E}_{cf}$$

[13] The following arguments are straightforward if researchers possess information on posterior beliefs, as elaborated in E.1.3.

Here, $\pi_{IP,g}$ is the wrong belief of group g. The first rule makes sure we use the info from group a, considering their wrong belief. The second rule is for group b to update their info with the same wrong belief. The third equation makes sure our data matches to the joint distribution, just like before. The objective just integrate over the new beliefs that are higher than the cost.

for the information quality decomposition in section 2.3.1, we maintain the Gaussian model assumption[14], which implies we assume that beliefs are Gaussian, and inaccurate prior is Gaussian as well. We then do as follows

1. Reweight the true distribution of $\mathcal{R}$ to match the wrong priors, to get the joint belief and returns distribution from the agents perspective.
2. Use the methods from E.1.2 to identify the out what the joint distribution would be if agents had the right priors.
3. Calculate the implied R^2 to assess information quality.
4. Assume there's a standard signal that shows the truth and matches this R^2.
5. Figure out the wrong beliefs and their spread with this R^2.

This way, we compare the quality of information across groups, considering how it affects choices, while keeping the assumption of incorrect priors.

[14]Without the Gaussian assumption, similar approach to the one above can be taken, considering the set of possible signals that induce R^2 at a given level