Bridging the Gap: Information, Returns and Choices

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Abstract

How much of the gap in choices across social groups is driven by differences in returns or the ability to predict these returns? To formalize this question, we use a decomposition exercise and a structural model to quantify the role of information quality and differences in returns in driving this gap. Focusing on the college attendance decisions of White and Hispanic high school students in Texas, we use administrative data to understand what drives their differing choices. First, we show that the average monetary returns for college for Hispanics are almost zero compared to being positive for Whites. We then estimate the extent to which differences in returns and information quality are contributing to the gap in choices. Under reasonable assumptions, we find that differences in information quality narrow the choice gap in college attendance, where most of the gap is explained by differences in returns. We also show that to achieve parity in choice between the two groups, a policymaker would need to provide highly accurate additional information, potentially explaining between 19% and 35% of post-college earnings.

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1 Introduction

In social systems, where individuals' life trajectories are shaped by choices, understanding the determinants of these choices is crucial, particularly in the pursuit of equality. Standard economic models assume that individuals weigh the costs against the benefits of their decisions. However, it is rarely the case that individuals can perfectly predict the outcomes of their choices. In reality, they operate under significant uncertainty and have limited predictive capabilities about the consequences of their actions. This gap in information and prediction abilities affects the choices different people make, potentially widening or narrowing societal inequities. Therefore, it is essential to assess the extent to which these frictions contribute to differences in decision-making processes and choices.

In this paper, we focus on how differences in information and the ability to predict outcomes contribute to differences in choices. To do this, we adopt a structural approach. Our structural model follows a basic choice framework (Roy (1951)), where individuals participate when they perceive the potential returns to be exceed their individual threshold. We assume that individuals receive informative signals on their returns and use them to make a binary decision on whether to opt in or out. Within this model, we define the quality of individuals' information as the individuals' prediction quality, measured by the coefficient of determination (R-Squared), a commonly used measure in statistics to quantify the quality of statistical models.

In this simple model, choices are driven by two components. The first is the distribution of returns. The second is the quality of information on these return. This bifurcation of the choice problem motivates us to adopt a decomposition method akin to that of Kitagawa (1955), Blinder (1973), and Oaxaca (1973) to explore what drives the choice gap. Our method breaks the choice discrepancy into two channels: the information channel and the composition channel. The information channel quantifies how much of the gap is driven by the fact that the two groups have access to different information sources. It does so by equalizing the information quality across the two groups, holding the returns distribution fixed, and examining how the choice gap changes. The residual difference, as captured by the composition channel, examines what the choice gap would be if we equalized the net returns between the two groups while maintaining their distinct information qualities on those returns.

We apply a decomposition approach to examine the 9% gap in college attendance rates between Hispanic and White students in Texas. To do so, we use administrative data from Texas containing data on whether individuals attend a 4-year college, or not, and their post-high school earnings. We then assume that high school graduates are self-selecting into college based on their posterior beliefs about the monetary returns from college, opting in if their beliefs are higher than their threshold. In our analysis, we restrict attention to the case of Gaussian model, where signals and earnings are drawn jointly from a Gaussian distribution. The Gaussian distribution has the benefit of being fully characterized by the first and second moments, therefore allows us to characterize counterfactual choice fully using our measure of quality of information, R^2.

Although in our model we assume Gaussian structure, key components of the model are nonparametrically identified. in our model, beliefs dictate choice patterns, this allows us to use choice data to nonparametrically identify the distribution of beliefs and earnings for each group. Specifically, building on the marginal treatment effect literature (Heckman and Vytlacil (2005)) we show how in our model the beliefs distribution is identified. We assume that we have a continuous instrument that shifts the cost of attendance. In our empirical exercise, this instrument is the distance to a 4-year college. We assume that, conditional on a set of controls, distance to college is independent of both information and earnings and affects only the threshold (Card (1995), Carneiro et al. (2011), Nybom (2017), Kapor (2020), Walters (2018), Mountjoy (2022)). We then trace how small changes in the instrument change the conditional expectation of earnings. A small increase in the cost of attendance pushes out those individuals whose new cost is higher than their beliefs. Using the assumption of rational expectations, tracking these changes in the expected earnings tells us about the beliefs of these marginal individuals who are responding to the small cost change. Similarly, tracking how changes in the cost affect the propensity of attending college reveals the share of people with those beliefs. A similar argument also allows us to identify the distribution of earnings for individuals who go to college and for those who do not.

As is well known, we cannot identify the correlation between these two potential earnings values using only observational data, and hence we cannot identify the distribution of returns and point-identify our decomposition components. Using the parametric assumption on the Gaussian structure allows us to explore the full set of returns distributions, by exploring the full possible range of correlation values between potential earnings. We also discuss different approaches to better bound the value of information.

Using our decomposition approach as outlined above, and estimation of the Gaussian model, we find that in general, with no restrictions on the correlation values of earnings, the information differences can contribute to either enlarging or shrinking the gap. Restricting attention to the more realistic case where the correlation between earnings is similar between Whites and Hispanics, we find that information differences between the two groups contribute to reducing the gap. Equating information across the two groups, by providing Whites with the information quality of Hispanics, would increase the gap somewhere between 3 and 8 percentage points, under our imposed assumptions.

In the Appendix, we introduce an additional decomposition approach that builds on tools from the robust mechanism (Bergemann et al. (2022), Bergemann and Morris (2016), Bergemann and Morris (2013)) literature to allow us to bound the full set of counterfactuals, nonparametrically, allowing us to depart from the normal distribution assumption.

The decomposition exercise teaches us how much of the observed gap is driven by information differences, but does not tell us the extent to which information interventions would affect the gap. Therefore, we next examine by how much we need to improve Hispanics information quality in order to close the choice gap. We find for different correlation values Hispanics need much better quality information to achieve choice parity with Whites. This is due to the average low prior on returns these individuals have. We find that to achieve parity, the accuracy of information provided to Hispanics must be significantly increased; specifically, restricting attention to the case where correlation is positive, we find that the share of explained variance should be increased substantially by 24% to 59%, depending on the correlation value.

We then turn to ask how parity in choice can be achieved by considering a policymaker that wants to close the gap by providing Hispanics with additional new information. We postulate that this policymaker, acting as a statistician with access to information on earnings for individuals who attend college and those who do not, could provide an informative signal to each high school student about their potential income. We then ask how accurate must this additional information be? Our findings suggest that to effectively close the gap, this new information must be able to explain either 19% of the variance in college earnings or 35% of the variance in non-college earnings. These levels of precision are relatively high and challenging to achieve using standard datasets, as the NLSY97 or our administrative data, containing detailed information on schooling. This indicate that closing the gap through information provision requires the generation of better sources for predicting earnings.

1.1 Related Literature

This paper contributes to an extensive body of literature on human capital investment decisions, anchored by the foundational work of Ben-Porath (1967). Our study intersects with research focused on the impact of monetary returns on such choices, as explored in studies by Willis and Rosen (1979), Walters (2018), Abdulkadiroğlu et al. (2020), and Freeman (1971). These papers typically make assumptions about how individuals form beliefs about returns—often measured based on observable factors—and analyze how these beliefs factor into decision-making processes. Our approach differs by examining how variations in the information available to individuals influence their choices.

Another significant aspect of our research aligns with studies that investigate the nature of individuals' beliefs, such as those by[1] Manski (2004), Wiswall and Zafar (2015), Zafar (2011), Wiswall and Zafar (2021), and Diaz-Serrano and Nilsson (2022). These works delve into systemic differences and biases in beliefs among groups defined by socio-economic status. Our paper extends this inquiry, utilizing these findings to illuminate not just the distribution of beliefs but also the quality and extent of information available to these groups.

As discussed above, methodologically, our study builds upon the Marginal Treatment Literature, particularly the work of Heckman and Vytlacil (2005). This approach has previously been employed to examine the marginal treatment effects on returns to schooling, as demonstrated by Carneiro et al. (2011), Carneiro and Lee (2009) and Mountjoy (2022). Similar to some of these studies, we link the marginal treatment effect to beliefs. Eisenhauer et al. (2015) employed this structure to conduct a cost-benefit analysis of programs, focusing on agents' ex-post and ex-ante costs—closely paralleling our usage. Canay et al. (2020) and d'Haultfoeuille and Maurel (2013), in the context of college decisions and discrimination, demonstrate how the Roy model can identify ex-ante beliefs and preferences, aligning with our methodological approach.

Our work related to recent research by Bohren et al. (2022) on systemic discrimination. Their study, akin to ours, identifies two main sources of systemic differences between social groups. The first, termed 'technological systemic discrimination', aligns with our focus on differences in return distributions and captures disparities across groups in certain outcome variables. The second, 'informational discrimination', pertains to disparities arising from varied information available to decision-makers across groups. Our research differs in its concentration not on discrimination towards individuals but

[1] See overview of the literature on beliefs elicitation in Giustinelli (2022)

on the decisions individuals make about themselves and how these systemic forces shape it, with a specific focus on the quality of information rather than its structure. We further explore a distinct measure related to this in our Appendix.

While our primary focus is on educational decisions, our decomposition approach has broader applications. It can illuminate how information asymmetries contribute to decision-making disparities across various contexts. Recent studies, including those by Arnold et al. (2018), Arnold et al. (2022), and Canay et al. (2020), have explored the influence of judicial preferences and biases in decision-making. There is a growing interest in understanding how decision-making signals contribute to these disparities. Our decomposition methodology seeks to address these nuanced aspects of decision-making processes.

The remainder of the paper proceeds as follows. Section 2 describe our framework and decomposition approach. Section 3 describe the data and some descriptive statistics. Section 5 discuss the estimation results and section 6 concludes.

2 Framework

We consider a population of high school graduates, denoted by i. At the end of high school, each graduate must decide whether to attend college. The objective of individual i is to maximize earnings. Denote by Y^1 earning for an individual who attends college and by Y^0 their earnings if they do not. We assume that earnings are generated according to

$$\begin{aligned} Y^1 &= \alpha_1 + u_1, \\ Y^0 &= \alpha_0 + u_0 \end{aligned}$$

where α^d, with $d \in \{0, 1\}$, is the structural component of earnings and u_d is an unpredictable component of earnings, satisfying $E[u_d|\alpha_1, \alpha_0] = 0$. Before deciding whether to attend college, each individual i receives an informative signal about the structural component of earnings. Specifically, we denote by $\boldsymbol{S}_i \in \mathcal{S}$ the vector of realized signals that individual i observes. We further assume that $\boldsymbol{S} \perp u_d|\alpha_1, \alpha_0$. Our model separates earnings into two main components. The first is a structural component, α_1 and α_0, which agents can know and form beliefs about. The second component is u_d, which is unknowable at the time of the decision. These components of earnings include idiosyncratic shocks that can only be known ex-post. Henceforth, we will treat α_1 and α_0 as earnings.

In our model, signals link outcomes to beliefs; thus, we need to establish how individuals use signals to form beliefs. We adopt the standard approach in economics and model individuals as Bayesian agents with rational expectations (Muth (1961), Lucas (1972), Sargent and Wallace (1971)). Being Bayesian means that agents observe the signal, know the correct likelihood function, and update their priors to form new beliefs over the outcomes. Our second assumption, rational expectations, implies that an individual's prior is anchored to the observed distribution of outcomes. In Appendix H, we discuss how this assumption can be relaxed if a researcher believes that individuals have inaccurate priors or beliefs (Bohren et al. (2023)) and have access to data on beliefs, in addition to choice and outcome data, but our analysis from now on is restricted to Bayesian agents with rational expectations.

Finally, we assume that individuals incur some cost when attending college, that is a function of observables. Denote by X a set of observed variables, by $c(x)$ the cost of attendance and by $\mathcal{R} = \alpha_1 - \alpha_0$ the structural part of the returns, then individual i's decision rule is given by

$$D = \mathbb{1}\left[E[Y^1 - Y^0|\boldsymbol{S}] \geq c(x)\right] = \mathbb{1}\left[E[\mathcal{R}|\boldsymbol{S}] \geq c(x)\right].$$

Our decision rule suggests that individuals derive risk-neutral utility from earnings but allows high school graduates to possess any utility function that strictly increases with expected returns (Vytlacil (2006)). Modeling utility as an increasing function of returns includes also the standard linear indirect utility function, that has been used in models of school and education choices (Willis and Rosen (1979), Walters (2018), Abdulkadiroğlu et al. (2020)) In our framework, we standardize this utility function to be the identity function. Therefore, $c(x)$ serves as a composite of individual preferences, known monetary and non Monterrey costs, and other barriers to college attendance, such as credit constraints, social norms, and additional limitations.

2.1 Gaussian Scalar Model

The previous setup is quite general and allows for complex signal structures. However, we will narrow our focus to the simple scalar Gaussian model to enable further discussion on our measurement of the role of information frictions and our estimation approach. Let X represent a set of observables that characterize a high school student. We begin by assuming that returns, condition on X, follow a normal distribution

$$\mathcal{R}|X = x \sim N\left(\mu_{\mathcal{R},x}, \sigma^2_{\mathcal{R},x}\right)$$

We assume that agents receive a noisy measurement on their returns, therefore their signals are distributed as

$$s|\mathcal{R}, X = x \sim N\left(\mathcal{R}, \sigma^2_{\epsilon,x}\right)$$

As the signal is normally distributed around the truth, and returns are normally distributed, individuals with signal realization s, forms the following posterior mean belief

$$E[\mathcal{R}|s,x] = \left(\frac{\mu_{\mathcal{R},x}\sigma^2_{\epsilon,x} + \sigma^2_{\mathcal{R},x}s}{\sigma^2_{\epsilon,x} + \sigma^2_{\mathcal{R},x}}\right) \tag{1}$$

As we can see, if the variance of the noise in the signal, σ^2_ϵ is higher, agents put higher weight on their prior, and pay less attention to the information. We can now write explicitly the decision rule for individuals with cost $c(x)$ and signal realization s

$$D = \mathbb{1}\Big[E[\alpha_1 - \alpha_0|s,x] \geq c(x)\Big] = \mathbb{1}\left[\frac{\mu_{\mathcal{R},x}\sigma^2_{\epsilon,x} + \sigma^2_{\mathcal{R},x}s}{\sigma^2_{\epsilon,x} + \sigma^2_{\mathcal{R},x}} \geq c(x)\right]$$

From now on we subsume x, unless it adds something substantial. We can also calculate the share of individuals with cost c who choose to go to college. We first notice that as s is normally distributed and the linearity of the posterior means, the beliefs are also normally distributed with the following mean and variance

$$E[\mathcal{R}|s] \sim N\left(\mu_{\mathcal{R}}, \frac{\sigma^4_{\mathcal{R}}}{\sigma^2_{\mathcal{R}} + \sigma^2_\epsilon}\right)$$

and the share of individuals who would choose to attend college, given cost $c(x)$ is given by

$$P(D=1|c) = \Phi\left(\frac{\mu_{\mathcal{R}} - c}{\sqrt{\frac{\sigma^4_{\mathcal{R}}}{\sigma^2_{\mathcal{R}}+\sigma^2_\epsilon}}}\right) \tag{2}$$

where Φ is the standard Gaussian Cumulative Distribution Function (CDF).

2.2 The Effect of Information and Returns on Choice

In the framework discussed, individual choice is influenced by two factors: the net returns $\mathcal{R} - c(x)$, and the individuals' knowledge of these returns. Our analysis aims to discern how these elements affect decision-making across different social groups. We begin by detailing the measurement of individuals' knowledge of their returns. Our focus is on 'information quality,' which is quantified by the coefficient of determination, often denoted by R^2 (R-Squared). This measure is commonly used in statistical literature to assess the predictive accuracy of models. Specifically, we define the quality of information as

$$R^2_{\mathcal{R}} = \frac{\mathrm{Var}(E[\mathcal{R}|s])}{\mathrm{Var}(\mathcal{R}|x)} \tag{3}$$

This coefficient quantifies the portion of explained variance of returns, captured by the variance of the predictor $E[\mathcal{R}|s]$, relative to the total variance in returns. In statistical contexts, we commonly use R^2 to evaluate the quality of statistical models by assessing how much of the observed variance the model can account for. In our specific context, we employ R^2 to compare the informational value held by individuals from different groups. The R^2 value ranges between 0 and 1, with a value close to one indicating that the information can explain nearly all the variance in returns. Conversely, an R^2 value close to zero suggests that the information possessed by agents is of limited utility in explaining returns.

How does the quality of information and returns affect the decision on going to college? By rearranging (3) as follows:

$$\mathrm{Var}\left(E[\mathcal{R}|s]\right) = \sigma^2_{\mathcal{R}} R^2_{\mathcal{R}}$$

We observe that as information quality increases, beliefs dispersion increases as well. Intuitively, if individuals have access to better quality, more accurate, information, then they would respond to it more, and rely on it more when updating their beliefs, this would cause an increasing belief dispersion. Therefore, better information implies higher variance in beliefs.

Whether higher beliefs dispersion implies that more individuals would attend college is contingent upon the relationship between the cost of attendance and the mean returns in the population, $\mu_{\mathcal{R}}$. Figure 1 illustrates the interaction between the prior, information quality, and cost and how they affect choices. The black line represents the cost. The two red lines represent the survival functions for priors lower than the cost, while the two blue lines represent the survival functions for priors higher than the cost. Dashed lines indicate the posterior beliefs distribution for an agent with high-quality information, and solid lines represent the posterior mean distribution with low-quality information. The figure shows that if the cost is lower than $\mu_{\mathcal{R}}$, increasing the precision of the signal—or enhancing information quality—would actually reduce college attendance. Conversely, if the mean returns exceed the cost, a reduction in information quality could prove actually increase the share of individuals who opt in to college.

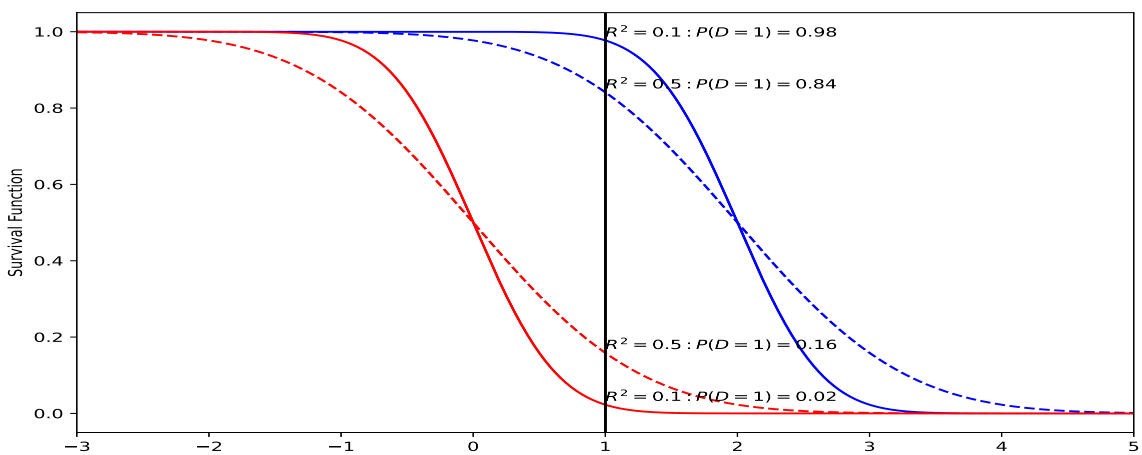

Figure 1: Cost, information and Beliefs interaction

Notes: This figure illustrates the interaction between the prior, information quality, and cost and how they affect choices. The black line represents the cost. The two red lines represent the survival functions for priors lower than the cost, while the two blue lines represent the survival functions for priors higher than the cost. Dashed lines indicate the posterior beliefs distribution for an agent with high-quality information, and solid lines represent the posterior mean distribution with low-quality information. The figure demonstrates that if the priors are higher than the cost, providing additional information reduces the share of participants from 98% to 0.84%. Conversely, if the prior is lower than the cost, improving the quality of information increases the share of individuals who opt in.

2.3 Two Measures of Information and Choice Gaps

Building on our model in the previous section we now discuss how to measure the role differences in information quality affect choices. We assume that individuals are categorized into two mutually exclusive groups, denoted by $g \in \{a, b\}$. Additionally, we posit that the proportion of college attenders in group b exceeds that in group a, formally $Pr(D = 1|b) \geq Pr(D = 1|a)$. In our empirical framework, these groups correspond to Whites and Hispanics, respectively. To understand the impact of information frictions on the choice gap, $Pr(D = 1|b) - Pr(D = 1|a)$, we introduce two metrics. The first metric explores how the dispersion in the information quality among the two groups affects their gap. The second metric assesses the level of accuracy required in the information accessible to the disadvantaged group (group a) to achieve choice parity. In the Appendix we consider two more measures of the role of information. First, in section F of the Appendix, we investigate an alternative measure for information's role. In this measure, we equate the signals received by individuals with the same returns in both groups. We also ensure that they update their beliefs correctly based on their own distribution of returns. We then examine how this affects the choice gap. Then in section E we consider additional decomposition method, that consider the quality of information with respect to the individuals priors and what they possibly know on the joint distribution.

2.3.1 Differences in information quality

Our first measure focuses on how much of the gap is driven by differences in the quality of information, by equating the information quality across groups. Equating information qualtiy can be thought of, in our Gaussian model, as equating the "normalized" signal dispersion, that is scaled by the variance of the underlying returns, i.e. equating the Signal-to-Noise Ratio ($SNR = \frac{\sigma^2_{\mathcal{R}}}{\sigma^2_\epsilon}$). To illustrate, note that we can write the R^2 in the Gaussian model as:

$$R^2 = \frac{\frac{\sigma^4_{\mathcal{R}}}{\sigma^2_{\mathcal{R}} + \sigma^2_\epsilon}}{\sigma^2_{\mathcal{R}}} = \frac{\sigma^2_{\mathcal{R}}}{\sigma^2_{\mathcal{R}} + \sigma^2_\epsilon} = \frac{1}{1 + \frac{1}{SNR}}$$

A nice property of equating the R^2 in our model is that the two groups update beliefs similarly, while observing a with respect to the same information. To see that, using equation 1, we have

$$E[\mathcal{R}|s] = \frac{\sigma_\epsilon^2 \mu_\mathcal{R} + \sigma_\mathcal{R}^2 s}{\sigma_\mathcal{R}^2 + \sigma_\epsilon^2} = \frac{\mu_\mathcal{R} + SNR \times s}{1 + SNR}$$

This suggests that once we adjust for the signal-to-noise ratio between the two groups, each member of the group gives the same importance to the signal s when updating their information. Consequently, the impact of the same piece of information s on returns influences the beliefs of both groups equally after we equate the R^2.

To quantify the role of information in exploring the gap, we suggest using a decomposition method à la Kitagawa (1955), Blinder (1973), and Oaxaca (1973). In it, we break down the differences in choices into two components, stemming from the varying predictability of returns between groups and the a residual term we call composition effects. Specifically, we investigate what proportion of individuals would choose to attend college if individuals with the same cost x had access to the same quality of information. Denote $R^2_{\mathcal{R},g,x}$ the information quality on returns of individuals from group g and observables x. Similarly, denote the cost function of group g as $c_g(x)$. Within the context of the Gaussian model we can decompose the observed gap in choices to

$$\begin{aligned} P(D=1|b) - P(D=1|a) = & \underbrace{\int_X \Phi\left(\frac{\mu_{\mathcal{R},b,x} - c_b(x)}{\sqrt{\sigma^2_{\mathcal{R},b,x} R^2_{\mathcal{R},b,x}}}\right) dF_b(x) - \int_X \Phi\left(\frac{\mu_{\mathcal{R},b,x} - c_b(x)}{\sqrt{\sigma^2_{\mathcal{R},b,x} R^2_{\mathcal{R},a,x}}}\right) dF_b(x)}_{\text{Information Channel}} \\ & + \underbrace{\int_X \Phi\left(\frac{\mu_{\mathcal{R},b,x} - c_b(x)}{\sqrt{\sigma^2_{\mathcal{R},b,x} R^2_{\mathcal{R},a,x}}}\right) dF_b(x) - \int_c \Phi\left(\frac{\mu_{\mathcal{R},a,x} - c_a(x)}{\sqrt{\sigma^2_{\mathcal{R},a,x} R^2_{\mathcal{R},a,x}}}\right) dF_a(x)}_{\text{Composition Channel}} \end{aligned}$$

where we use $F_g(x)$ to denote the CDF of x for group g. The information channel quantifies the extent to which the gap in choices arises from individuals having access to information of differing quality, despite equal cost, which affects their ability to predict the outcomes of their choices. Another perspective is to consider cases where members of group a and group b employ different models to predict the outcomes of their choices. The quality of these models originates either from the information they possess or from the underlying data-generating process of outcomes. We inquire how much of the gap stems from differences in the quality of these predictive models, where quality is measured using R^2. We postulate a counterfactual world in which we equalize the quality of these two models and examine how that would impact choices. In the Gaussian model, this has a straightforward interpretation as reducing the variance of the noise in the signal observed by individuals, σ_ϵ^2. In more general settings, with unrestricted data-generating processes providing more nuisance information, equalizing R^2 does not yield a closed-form solution. In many cases, different joint distributions of signals and outcomes may produce the same R^2 but induce complex choice patterns that contribute to gaps in choices influenced by information. In section F in the Appendix, we discuss another decomposition approach that equalizes the information structure across groups. This approach does not equalize the ability to predict across groups, but rather equalizes the signals that individuals with similar outcomes receive.

It is important to recognize that our analysis is a partial equilibrium exercise, where we use comparative statics to equalize the information quality between the two groups. Typically, information quality is determined endogenously within an equilibrium framework (Coate and Loury (1993), Lundberg and Startz (1983)), and is driven by choices individuals make that form the information environment and what agents can know. Furthermore, the information quality that individuals possess could be influenced by the effort they invest in acquiring it, a concept central to the standard rational inattention model (Caplin et al. (2022); Maćkowiak et al. (2023)). In this decomposition exercise, we do not delve into the underlying factors that drive these information discrepancies; rather, we accept them as given and investigate their extent of contribution to the observed disparity.

We now turn to discuss the second channel. The residual component, denoted as the composition effect, poses the inverse question: How much would the share of high school graduates from group a change if we held their information quality $R^2_{a,c}$ constant, but altered the composition of their returns and costs to match those of group b? This component informs us how much of the gap is driven by differences in the outcome distribution itself. Therefore, we interpret this component as quantifying the portion of the gap driven by the fundamentals themselves.

The two components of the distribution carry distinct policy implications. If the majority of the gap is driven by differences in predictive ability, policymakers aiming to close this gap should consider transferring the superior information or modeling techniques from group b to group a. This could involve educational interventions, information dissemination, or providing improved prediction tools for group a. Conversely, if the gap primarily stems from variations in the outcome distribution, policymakers concerned with narrowing the disparity should focus on policies that directly influence this distribution. This could include measures such as altering tax structures, providing targeted subsidies, or implementing regulatory changes that affect the underlying returns and costs for both groups. Identifying the primary driver of the gap not only enhances our understanding of its structural roots but also provides actionable insights for policymakers committed to fostering equal opportunities across different groups.

2.3.2 Quantifying how much additional information is needed for Equality

Our second measure takes a practical approach, focusing on how much we would need to alter the information accuracy for the disadvantaged group to achieve choice parity. If significant modifications are needed, this suggests that improving information quality may not be the most effective way to close the choice gap. Instead, policymakers aiming to reduce disparities may need to directly address differences in returns. For this measure, we find the quality of information that group a needs to have in order to achieve parity in choices with group b. Specifically we look for $R^2 \in [0, 1]$ that solves the following equation

$$P(D = 1|\mathrm{b}) = \int_X \Phi\left(\frac{\mu_{\mathcal{R},a,x} - c_a(x)}{\sigma^2_{\mathcal{R},a,x} R^2}\right) dF_a(x) \tag{4}$$

We then define the importance of information to the gap by

$$R^2 - \int_X R^2_{\mathcal{R},a,x} dF_a(x) \tag{5}$$

That captures the difference between the needed R^2 and the average current quality of information group a has. A large difference between these values suggests that policymakers would need to deliver considerably more accurate and informative signals to bridge the choice gap effectively. This metric also has ramifications in terms of policy cost. In most information cost models (Pomatto et al. (2023), Caplin et al. (2022), Maćkowiak et al. (2023)), the cost function respect the Blackwell order. In our Gaussian model, a high R^2, implies a more precise signals and these signals are ranked by Blackwell order, therefore, higher R^2 implies that the policymaker would incur higher costs to generate this supplemental information[2].

To better understand how a policymaker can influence individual choices and increase the R^2, we explore the quality of additional information needed to equate attendance rates across groups. Here, "additional information" refers to new signals that are orthogonal to an agent's existing information set; that is, we focus exclusively on previously unknown information that a policymaker could introduce. In practice, a policymaker is likely to disseminate information that correlates with what individuals already know, potentially overlapping with their private information. Therefore, in our thought exercise, we consider the case in which individuals first residualize the policymaker's signal and use only their existing information and the additional residualized information to inform their beliefs. We examine how the information quality of this additional information affects the choice gap. Specifically, we engage in three thought exercises for this purpose. First, we consider providing information that is solely informative about earnings if the individual chooses to go to college. Second, we examine the opposite scenario where the additional information is informative only about earnings if they opt not to go to college. Finally, we consider providing information that is relevant to both types of earnings. In our thought experiments, we assume that the policymaker, akin to an econometrician, can only provide information on the marginal distributions of α_1 and α_0, as she cannot know their joint distribution. For example, the policymaker could offer students a series of tests, then provide predictions on potential earnings depending on whether they attend college. To measure the precision of this additional new information, we quantify it by its ability to explain the marginals of α_1 and α_0, therefore we describe these additional signals in terms of R^2.

To formally introduce this new information, let s_n be the additional signal that a policymaker provides to Hispanics, after it has been partialled out from the agent's existing information. We assume that the signals are drawn from a Gaussian distribution and are correlated with the residual U_1 and U_0. The fact that the signal is partialled out implies that $\text{Cov}(s_n, \boldsymbol{S}) = 0$. Furthermore, as the signals and state are jointly normal, the agent's beliefs are additive. Specifically, we can write

[2]In fact, the Shannon entropy cost, commonly employed in rational inattention, is an increasing function of R^2 in the scalar Gaussian model introduced in Section 2. Specifically,

$$H(\mathcal{R}, S) = -\frac{1}{2}\ln(1 - R^2)$$

where H represents the mutual information function of returns and the signal.

the agent's beliefs, given their current signals and the additional information, as

$$E[U_1 - U_0|\mathcal{S}, s_n] = E[U_1 - U_0|\mathcal{S}] + \frac{\text{Cov}(U_1 - U_0, s_n)}{\text{Var}(s_n)} s_n$$

As the agents are Bayesian, their mean beliefs are determined by the law of iterated expectations. Since we assume that s_n is Gaussian, we only need to derive the variance of the new beliefs with the additional information. Denote $R^2_{1,n}$ and $R^2_{0,n}$ as the information quality of the new signals on U_1 and U_0, respectively. Then note that we can express the additional component to beliefs as

$$\text{Var}\left(\frac{\text{Cov}(s_n, U_1)}{\text{Var}(s_n)} s_n\right) = \frac{\text{Cov}^2(s_n, U_1)}{\text{Var}(s_n)} = \sigma_1^2 R^2_{1,n}$$
$$\text{Var}\left(\frac{\text{Cov}(s_n, U_0)}{\text{Var}(s_n)} s_n\right) = \frac{\text{Cov}^2(s_n, U_0)}{\text{Var}(s_n)} = \sigma_0^2 R^2_{0,n}$$

Without loss of generality, we fix $\text{Var}(s_n) = 1$ and set $\text{Cov}(s_n, U_1)^2$ to meet the required R^2 and the variance of new beliefs are distributed with the following variance

$$\begin{aligned} \text{Var}(E[\mathcal{R}|\boldsymbol{S}, s_n]) &= \sigma^2_{\text{E}} + \text{Cov}^2(U1, s_n) + \text{Cov}^2(U_0, s_n) - 2\text{Cov}(U1, s_n)\text{Cov}(U1, s_n) \\ &= \sigma^2_{\text{E}} + \sigma_1^2 R^2_{1,n} + \sigma_1^2 R^2_{0,n} - 2\sqrt{R^2_{1,n} R^2_{0,n}}\sigma_1\sigma_0 \end{aligned} \tag{6}$$

We can then calculate the counterfactual share of students who would attend college if they were provided with this new information. Notice that in order to calculate the counterfactual shares we do not need to know the correlation between α_1 and α_0, as we consider how the new information is informative on the marginals, but not on the difference.

2.3.3 Other Empirical Applications

While our focus is on educational decisions, the decomposition can also shed light on how information availability shapes behavior.

Example 2.1 (Discrimination)**.** Recent research has focused on the factors contributing to disparities in decision-making across groups, largely attributing these to individual preferences. Works by Arnold et al. (2018), Arnold et al. (2022), and Canay et al. (2020), explore the role of judicial preferences and biases. There is also increasing interest in understanding how decision-making signals contribute to these disparities. The decomposition approach aims to tackle these issues.

Example 2.2 (Changes over Time)**.** The decomposition method can reveal trends in decision-making over time by comparing two different time periods, labeled as groups A and B. This enables researchers to quantify behavioral changes attributable to better information access, either through belief identification or surveys.

Example 2.3 (Policy Decomposition)**.** Researchers can use the decomposition approach to analyze the behavioral consequences of policy shifts, particularly within the rational expectations framework prevalent in economics[3]. The method quantifies the impact of policies by isolating changes in returns and information access. For example, a tax code revision that simplifies the return process (Caldwell et al. (2023)) would alter both the amounts and predictability of returns. The decomposition approach allows researchers to quantify each factor's influence on tax return applications.

2.4 Model Identification and Empirical Specification

In this section we discuss how the Gaussian model can be partially identified using data on choices and outcomes. In Appendix C, we show how the choice model can be identified nonparametrically with a continuous instrument and results from the Marginal Treatment effect literature (Heckman and Vytlacil (2005)) and identification of discrete choice models (Matzkin (1992),Matzkin (1993)). In what follows we briefly go over the identification of the normal model and it's important components for our analysis.

2.4.1 Identifying the Gaussian model Parameters

We assume we observe a set of covariates X, a continuous instrument Z and outcomes Y. Although it's not imperative for identification argument, we parametrize the cost function as a linear function of covariates

$$c(x, z) = Zb_z + Xb_x$$

We assume that the distribution of $\alpha_1|X, D = 1$ and $\alpha_0|D = 1, X$ is observed. In the Appendix we discuss how it can be identified using Panel data and additional assumptions on the wages. For our

[3]For instance, Bhandari et al. (2021) analyzes welfare effects due to changes in consumption information.

discussion the α_1 and α_0 can be thought of as fixed effect, and are identified from panel data. We also assume that α_1 and α_0 are linear in covariates

$$\alpha_1 = X\beta_1 + U_1$$
$$\alpha_0 = X\beta_0 + U_0$$

Following our discussion on the Gaussian model, we assume that beliefs and residuals U_1 and U_0 are jointly normal, X operates only as a mean shifter and Z is independent from the potential outcomes, $Z, X \perp\!\!\!\perp U_1, U_0$.

$$\begin{pmatrix} U_1 \\ U_0 \\ E[\mathcal{R}|s,x] \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ X(\beta_1 - \beta_0) \end{pmatrix}, \begin{bmatrix} \sigma_1^2 & \rho_{1,0}\sigma_1\sigma_0 & \sigma_{1,\mathrm{E}} \\ \rho_{1,0}\sigma_1\sigma_0 & \sigma_0^2 & \sigma_{0,\mathrm{E}} \\ \sigma_{1,\mathrm{E}} & \sigma_{0,\mathrm{E}} & \sigma_{\mathrm{E}}^2 \end{bmatrix} \right)$$

The individuals decision rule is then given by

$$D = \mathbb{1}\left[E[\alpha_1 - \alpha_0 \mid s, x] \geq c(z,x)\right] = \mathbb{1}\left[E[U_1 - U_0 \mid s, x] \geq c(x,z) - X(\beta_1 - \beta_0)\right]$$

Using the fact that beliefs and U_1 and U_0 are jointly normal, we have that the choice probability is given by

$$P(D = 1|x,z) = \Phi\left(\frac{c(x,z) - X(\beta_1 - \beta_0)}{\sigma_{\mathrm{E}}}\right) \tag{7}$$

Notice that in general this is not enough to identify the cost function of parameters, as all parameters are identified up to scale. In addition, covariates can play a dual role, both affecting the outcome variable and controlling the cost. Therefore, we need to identify the scale parameter and the coefficients β_1 and β_0. To identify β_1 we use the standard Heckman Correction argument for Gaussian selection model (Heckman (1979)). Specifically, using the fact that U_1, U_0 and beliefs are jointly Gaussian, we have that

$$E[\alpha_1|D = 1, X] = E[\alpha_1 + \epsilon^1] = E[\alpha_1 + \epsilon^1] = X\beta_1 + E[U_1|D = 1, X]$$

where $E[U_1|D = 1, X] = \frac{\sigma_{1,\mathrm{E}}}{\sigma_{\mathrm{E}}} \frac{\phi\left(\frac{Xc_x - X(\beta_1 - \beta_0)}{\sigma_{\mathrm{E}}(x)}\right)}{1 - \Phi(\left(\frac{Xc_x - X(\beta_1 - \beta_0)}{\sigma_{\mathrm{E}}(x)}\right)}$. We can follow the same argument to identify β_0, and using the fact that $E[\alpha_0|D=0,X] = \frac{\sigma_{0,\mathrm{E}}}{\sigma_{\mathrm{E}}} \times -\frac{\phi\left(\frac{\gamma z - E[\theta]}{\sigma_\eta}\right)}{\Phi\left(\frac{\gamma z - E[\theta]}{\sigma_\eta}\right)}$. Denote the coefficient of the mills ratio as $\gamma_1 = \frac{\rho_{1,\mathrm{E}}\sigma_1}{\sigma_{\mathrm{E}}}$ and $\gamma_0 = \frac{\sigma_{0,\mathrm{E}}}{\sigma_E}$, and notice that we can identify σ_1 and σ_0 using the joint distribution of choice and earnings

$$f(D=1,\alpha_1,z,x) = \left(1 - \Phi\left(\frac{\frac{\mu(z,x)-\mu_\eta}{\sigma_\eta} - \frac{\gamma_c^1}{\sigma_1}\left(\frac{y_1-\mu_1}{\sigma_{y_1}}\right)}{\sqrt{(1-(\frac{\gamma_c^1}{\sigma_1})^2))}}\right)\right)\phi\left(\frac{\alpha_1-\mu_1}{\sigma_1}\right)\frac{1}{\sigma_1} \tag{8}$$

and similarly for σ_0. Finally, in order to get σ_{E}, we can use the fact that the covariance of beliefs and returns equal to the variance of returns, $\mathrm{Cov}(\mathcal{R},\mathrm{E}[\mathcal{R}|s,x]) = \mathrm{Var}(\mathrm{E}[\mathcal{R}|s,x])$. To see that notice that we can decompose returns as

$$\mathcal{R} = E[\mathcal{R}|s,x] + r$$

where $\mathrm{Cov}(E[\mathcal{R}|s,x],r) = 0$, and therefore $\mathrm{Cov}(E[\mathcal{R}|s,x],\mathcal{R}) = \mathrm{Var}(E[\mathcal{R}|s,x])$. Therfore, using the coefficient on the control function in the regression we have

$$\gamma_c^1 - \gamma_c^0 = \frac{\sigma_{1,\mathrm{E}} - \sigma_{0,\mathrm{E}}}{\sigma_{\mathrm{E}}} = \frac{\sigma_{\mathrm{E}}^2}{\sigma_{\mathrm{E}}} = \sigma_{\mathrm{E}}$$

Finally, the correlation between α_1 and α_0, is known to be un identified, as we can not observe both α_1 and α_0 simultaneously. Hence, our analysis explores the bounds on how this unobserved correlation affects the role of information. Then, for a given correlation parameter $\rho_{1,0}$, we can identify the R^2 needed for our decomposition by

$$\mathcal{R}^2 = \frac{\sigma_E^2}{\sigma_1^2 + \sigma_0^2 - \rho_{1,0}\sigma_1\sigma_0} \tag{9}$$

2.4.2 Testable implication of the model

In the model we assume that agents select on on outcome, therefore, this implies a testable restriction, which is that

$$\sigma_{\mathrm{E}} = \gamma_c^1 - \gamma_c^0 \geq 0$$

as σ_{E} standard error. In the section C in the section C.1.1 in the Appendix we show that this restriction also hold nonparametrically, and implies that the Marginal Treatment effect (Heckman and

Vytlacil (2005)) is decreasing. This is known implication of the Extended Roy Model (d'Haultfoeuille and Maurel (2013), Canay et al. (2020))

2.4.3 Restrictions on the correlation parameter

Our theoretical framework implies some constraints on the correlation between U_1 and U_0, that is informed by our model that implies some selection on returns. As implied by 3, the variance of beliefs about returns is bunded by the actual variance of returns. This is expressed by the inequality:

$$\sigma_{\mathrm{E}}^2 \leq \sigma_1^2 + \sigma_0^2 - 2\rho\sigma_1\sigma_0$$

This restriction is a generalization of the known fact in the standard Roy model (Roy (1951)) with complete outcome information, where the joint distribution of potential outcomes is point-identified (Heckman and Robb (1985)). If we assume agents have complete information, the inequality becomes an equality. If we maintain that agents select based on outcomes but have incomplete information, we can use the above inequality to bound the correlation between potential outcomes.

We can further restrict the bounds using the fact that we can identify the covariance between beliefs, $\mathrm{E}[\alpha_1 - \alpha_0|s, x]$ and U_1 and U_0. To do so we use the fact that the covariance matrix must remain positive semi-definite, we therefore restrict the set of possible $\rho_{1,0}$ to values that keep the following covariance matrix positive semi-definite.

$$\mathrm{Cov}(\boldsymbol{\alpha}, \mathbb{E}) = \begin{bmatrix} \sigma_1^2 & \rho_{1,0}\sigma_1\sigma_0 & \sigma_{1,\mathbb{E}} \\ \rho_{1,0}\sigma_1\sigma_0 & \sigma_0^2 & \sigma_{0,\mathbb{E}} \\ \sigma_{1,\mathbb{E}} & \sigma_{0,\mathbb{E}} & \sigma_{\mathbb{E}}^2 \end{bmatrix}$$

2.5 Estimation

We now turn to describe how we estimate the Gaussian choice model. As discussed in the , for non-parametric estimation, we need to employ an instrumental variable z. We therefore include it here in the discussion. We first start by estimating α_1 and α_0 by averaging wages over periods of time

$$\hat{\alpha}_{di} = \frac{1}{T-t}\sum_{a=t}^{T} Y_{i,a}^d$$

Then, given our $\hat{\alpha}_1$ and $\hat{\alpha}_0$, we estimate the model in three steps. In the first step, we estimate the propensity score using a probit model, the covariates X, and the instrument Z. In the second step, we use the Heckman control function approach (Heckman (1979)) to estimate β_1 and β_0. As discussed in the previous section, we obtain the standard deviation of beliefs from the coefficients on the control function. Next, we show how we can estimate the cost function. Using equation 7, we see that the probit regression coefficients, standardized by the standard deviation of beliefs, are impacted by both beliefs and costs. To adjust for this, we rescale the coefficients and add the conditional expectations, estimated using the control function approach:

$$\hat{c}(x,z) = \hat{\sigma}_\eta \times (z\hat{b}_z + X\hat{b}_x) + X(\hat{\beta}_1 - \hat{\beta}_0)$$

Finally, to get σ_1 and σ_0, we solve the maximum likelihood function as shown in equation 8.

To estimate our first measure of information contribution to the gap we simply calculate the $\hat{R}^2$, as discussed in 9 for both groups. We then estimate the information channel as

$$\underbrace{\hat{P}(D=1|b)}_{\text{Observed}} - \underbrace{\frac{1}{N}\sum_i \Phi\left(\frac{X(\hat{\beta}_1 - \hat{\beta}_0) - \hat{c}(x_i, z_i)}{\sqrt{\hat{\sigma}^2_{\mathcal{R}}\hat{R}^2_a}}\right)}_{\text{Counterfactual}}$$

Where the first part is just the observed share and the second part is the counterfactual share of individuals who choose to attend, if they had the same R^2 as group a. To estimate this part we simply average over $\Phi\left(\frac{X(\hat{\beta}_1 - \hat{\beta}_0) - \hat{c}(x_i, z_i)}{\sqrt{\hat{\sigma}^2_{\mathcal{R}}\hat{R}^2_a}}\right)$ for all the observation of group b. Estimation of the composition channel is done the same. To estimate the quantity in 4, we again fix a correlation parameter and solve for the R^2 that satisfies the sample approximation of the integral in 4:

$$\sum_i \Phi\left(\frac{X_i(\hat{\beta}_1 - \hat{\beta}_0) - \hat{c}(X_i, z_i)}{\sqrt{\hat{\sigma}^2_{\mathcal{R}} R^2}}\right) = \hat{P}(D=1|b)$$

We solve the the above equation numerically using bi-section algorithm. As our empirical specification of the model assumes that $U_1, U_0 \perp\!\!\!\perp X, Z$, we then report discuss the difference between $R^2 - \hat{R}^2$, as this measure.

3 Data

Our empirical application investigates the factors contributing to the college attendance gap between Hispanic and White students. We concentrate on Texas, where there are large and comparable Hispanic and White populations, but they differ substantially in their choices. Utilizing the methods described in Sections 2.3.1, we decompose the attendance choices and assess the influence of informational differences. We start by describing the data and then discuss the model results. The following section describes the data and variables we use throughout our analysis

3.1 Data Sources and Sample Construction

Our empirical study leverages a series of confidential administrative databases from the state of Texas, the second most populous in the U.S. with a sophisticated higher education system that engages a substantial portion of its populace, including over one million high school students (Agency (2023)). Additionally, Texas have a significant Hispanic demographic, comprising around 12 million individuals in 2022, or about 40% of the state's total population, matched by a 40% representation of White population.

The study combines data from several Texas agencies. The primary dataset is procured from the Texas Education Agency (TEA), offering demographic details of all Texan high school students. This dataset is enriched with school characteristics from the National Center for Education Statistics (NCES), which provides a broader picture of Texas high schools. We incorporate assessments from the Texas standardized testing program, which evaluates public primary and secondary school students' competencies in various grades and subjects. Further, we integrate data concerning college enrollment decisions from the Texas Higher Education Coordinating Board (THECB), supplemented by information from the Integrated Postsecondary Education Data System (IPEDS). Finally, the Texas Workforce Commission (TWC) supplies data on post-high school earnings, completing our comprehensive dataset.

In constructing our control variables, we follow the approach used by Mountjoy (2022), utilizing three types of covariates: student-level demographics, school characteristics, and neighborhood characteristics. For student-level demographics, we include categorical variables for gender, eligibility for free or reduced price lunch as a proxy for economic disadvantage, and an indicator for graduation under one of three programs: the Distinguished Achievement Program, Recommended High School

Program, or the Minimum High School Program, which reflect the various graduation tracks in Texas. In some of our analyses, we use test scores from Texas Assessment of Knowledge and Skills (TAKS) tests. We consider test scores from the exit exams in English-Language-Arts (ELA), which capture language skills, and Math test scores, these tests were held consistently across our three cohorts of interest. We then create a single measure of test scores by combining them in a one-factor model separately by cohort and normalize this factor to within-cohort percentiles. These high-stakes tests, which imply that they are likely to be indicative of student ability .Passing these exit-level test is a graduation prerequisite for Texas high school seniors in their junior and senior years.

For high school-level controls, we utilize NCES Common Core data, which incorporates the geographic locale code. This code categorizes urbanization into twelve detailed categories using Census geospatial data. Additionally, we include the distance to two-year colleges and an indicator denoting whether the school is classified as a Vocational Education School. Vocational schools are identified as those that provide formal training for semi-skilled, skilled, technical, or professional occupations to students of high school age who may opt to enhance their employment prospects, possibly instead of preparing for college admission. Controls also account for the local influence of the oil and gas industry, by measuring the long-term share of oil and gas employment at the high school level, employing NAICS industry codes from TWC workforce data. We normalize this measure of oil and gas employment by ranking it and control for its effects using a third-degree polynomial in our analysis of school characteristics.

Neighborhood characteristics include the 62 Texas commuting zones using the year-2000 mapping provided by the U.S. Department of Agriculture's Economic Research Service. We also construct an index of neighborhood quality, akin to the test score measure: We combine the tract-level Census measures of median household income and the percentage of households below the poverty line with the high school-level percentage eligible for free/reduced-price lunch into a one-factor model, then normalize this neighborhood factor to the within-cohort percentile. When controlling for neighborhood characteristics in the following discussion, we control for the third-degree polynomial of the neighborhood factor.

As outlined in section C in the Appendix, nonparametric identification necessitates an instrument. We employ the measure of proximity to the nearest 4-year colleges, calculating ellipsoidal distances between the coordinates of all Texas public high schools (sourced from NCES CCD) and those of all Texas postsecondary institutions (from IPEDS). We determine the minimum distances within 4-year sectors for each high school. To supplement some missing distances, we refer to Mountjoy (2022), which involved manual collection of location data by verifying each college's institutional profile. We adopt the same methodology for the variable of distance to 2-year colleges.

We limit our sample to cohorts from 2003 to 2005 to ensure a long time horizon. This approach, leveraging our earnings data, allows us to observe outcomes 16 (for the cohort of 2003 and 2004) and 15 years (for the cohort of 2005) into the future, thus better understanding the incentives faced by these students. Additionally, the Texas Higher Education Coordinating Board (THECB) has provided data on students attending four-year colleges, including both private and public institutions, starting from 2003. We further narrow our sample to high school students who are not enrolled in special education programs, are between the ages of 17 and 18 in the 12th grade, and have graduated from high school with at least the minimum requirements. As with any study focused on a specific state, there is a risk of out-migration; however, Texas has one of the lowest out-migration rates in the U.S. (Times (2014)). Following Mountjoy (2022), we also limit our test factor to individuals with grades below the 80s percentile. As Mountjoy (2022) discusses, high school students with a test score factor higher than the 80th percentile are more likely to enroll in out-of-state colleges. Figure 13 in the Appendix further illustrates that these individuals are more likely to have missing earnings data.

4 Summary Statistics and Empirical Patterns

Table 1 presents summary statistics for the analysis cohort. The table indicates substantial disparity in socio-economic backgrounds among the groups. A significant proportion of Hispanics originate from low-income families, necessitating reduced-price or free meals. They also live in census tracts with higher unemployment rates and a greater proportion of families below the poverty line. Over 58% of Hispanics attend Title I schools, markedly more than their White counterparts. Conversely, regarding the programs offered at these schools, there is no substantial difference in the distribution. Similarly, there is no significant difference in how schools inform students about the oil industry; the proportion of high school graduates working in the oil and gas industries over the long term is similar. Geographically, Hispanics are more likely to reside in urban areas, while Whites predominantly live in suburban and rural areas. Furthermore, in terms of proximity to colleges, Hispanics tend to live nearer to both four-year and two-year colleges compared to non-Hispanic Whites.

Next, we examine the discrepancy in college attendance. The initial row of Table 1 indicates that the choice gap in the decision to attend a four-year colleges in the first year after high school graduation between Hispanics and Whites is 9%. Table 9 in the Appendix assesses the extent to which observable factors contribute to this disparity. Adjusting for individual, school, and geographical characteristics, as discussed in section 3.1, reduces the gap to between 6% and 7%. Figures 2 and 3 show that there is high dispersion for both Whites and Hispanics likelihood of attending college. The figure plots propensity scores for Hispanics and Whites attending college, estimated using a Probit model with our control set and the distance to a four-year college. The figures show that there's a large overlap between college attendance and large variation, Additionally, the distribution of propensity scores for both college attendees and non-attendees among Whites is more right-skewed than that of Hispanics. This skewness shows that Whites have a higher likelihood of college attendance than their Hispanic peers.

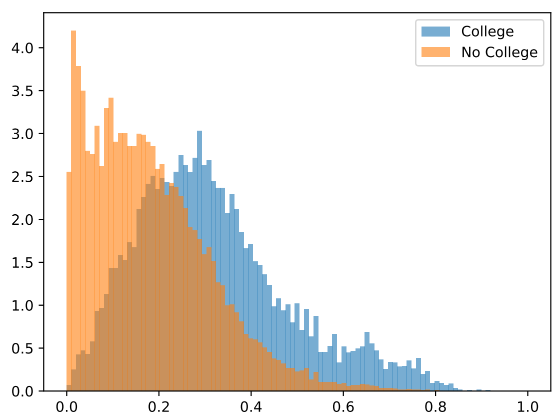

Figure 2: Raw Mean in Wages, without controls

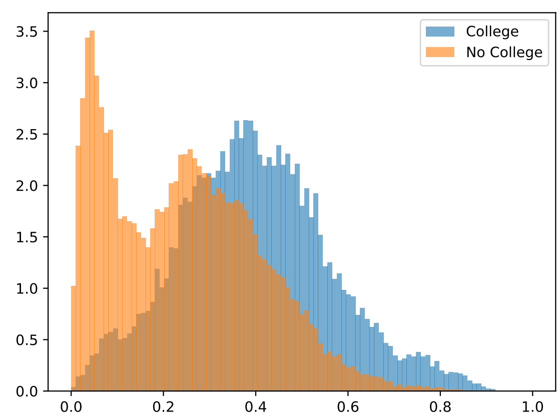

Figure 3: Raw Mean in Wages with controls

As discussed in the framework in Section 2, differences in choices can stem from variations in returns or disparities in information regarding these returns. Therefore, we delve into exploring these differences. Our exploration starts with an examination of discrepancies in test scores.

In our framework, test scores provide insights into the two key components driving decision-making. Firstly, grades serve as indicators of the distribution of human capital accumulated prior to the decision to attend or not. If we consider college education to be complementary to the accumulated human capital (Cunha and Heckman (2007), Heckman and Mosso (2014)), this suggest that differences in test scores are likely to imply also differences in college returns for Hispanics and Whites. However, grades offer more than just insights into returns; they also act as sources of information and signals available to agents. From this perspective, agents receive grades and employ them to form projections about the utility of these grades. Consequently, we also examine whether grades convey informative signals about returns and whether disparities exist between Whites and Hispanics.

Table 1 reveals a notable gap in academic readiness between Hispanics and Whites, as evidenced by exit exam grades. To what extent does this gap contribute to the overall disparity? The final row in Table 9 in the Appendix demonstrates that when we account for our measure of test scores, the gap narrows to 4.8%. Furthermore, Table 17 demonstrates that grades are significant in explaining choices. In a Probit model predicting these choices, the inclusion of grades increases the Area Under the Curve (AUC) from 0.74 to 0.77 for Whites and from 0.75 to 0.8 for Hispanics. This magnitude of increase is comparable to that observed when adding school and neighborhood characteristics to individual characteristics (rising from 0.68 to 0.75 for Hispanics and from 0.67 to 0.74 for Whites). These findings imply that high school graduates likely consider exit exam grades and their informational value in their college enrollment decisions.

Are grades informative on returns? We explore whether grades are likely to contain information about returns. To ascertain whether grades predict earnings and returns, Figure 14 in the Appendix illustrates the relationship between earnings and grades for both college attendees and non-attendees. The figure shows that for both Hispanics and Whites, higher grades correlate with increased earnings, irrespective of college attendance. Additionally, as grades increase, the earnings gap widens between those who attend college in their first year and those who do not. This is supported by the regression in Table 10 in the Appendix, which reveals that a one-unit increase in test scores raises the raw gap by approximately $16, controlling for our set of controls. Both figures and regression table suggest that the differential informativeness of grades on the gap is relatively minor, as the gap escalates nearly proportionately with grades.

The relationship between school informativeness is further examined in Table 12 in the Appendix. This table presents the out-of-sample R^2 from a model that employs Extreme Gradient Boosting to predict earnings based on students' course-taking patterns and the pass-fail indicator for Hispanics and Whites. The R^2 values are remarkably similar for both groups. This implies that the quality of information from school performance measures is comparable for Whites and Hispanics.

These results indicate that Hispanics and Whites encounter varying distribution of returns. However, the quality of information available to them through the school system does not significantly differ. This motivates the utilization of our model to gain a deeper understanding of how these differences contribute to the choice gap.

	All	Hispanic	Whites
College Attendance	0.23 (0.42)	0.18 (0.38)	0.27 (0.44)
Test Factor Percentile	43.18 (22.02)	36.37 (22.02)	48.11 (20.67)
Math Score	45.62 (23.88)	40.29 (24.12)	49.49 (22.94)
Reading Score	47.5 (25.41)	40.11 (25.41)	52.87 (24.03)
No Disadvantage	0.7 (0.46)	0.41 (0.49)	0.91 (0.29)
Elig. Free Meals	0.22 (0.41)	0.44 (0.5)	0.06 (0.24)
Elig. Reduced Price Meals	0.06 (0.23)	0.09 (0.29)	0.03 (0.16)
Other Disadvantage	0.03 (0.16)	0.06 (0.23)	0.0 (0.05)
Distiguish	0.06 (0.24)	0.07 (0.25)	0.05 (0.23)
Minimal	0.22 (0.41)	0.19 (0.39)	0.24 (0.43)
Required	0.72 (0.45)	0.74 (0.44)	0.7 (0.46)
CT Median Income	44027.0 (21371.0)	36265.0 (15939.0)	49663.0 (22986.0)
CT Families Below Poverty Line	14.5 (10.82)	20.08 (12.19)	10.44 (7.42)
CT Share of Employed	63.21 (9.97)	59.92 (10.01)	65.6 (9.23)
Title I schools	0.34 (0.47)	0.58 (0.49)	0.17 (0.38)
No Participation in Tech Program	0.24 (0.43)	0.22 (0.41)	0.26 (0.44)
Enroll in Career Tech Elective (6-12)	0.23 (0.42)	0.2 (0.4)	0.24 (0.43)
Participate in Tech Prep Prog (9-12)	0.32 (0.47)	0.33 (0.47)	0.32 (0.47)
Participate in Tech Prep Prog	0.21 (0.41)	0.25 (0.43)	0.18 (0.38)
Share in Oil Industry	52.73 (28.53)	49.21 (29.14)	55.29 (27.79)
City	0.37 (0.48)	0.52 (0.5)	0.25 (0.44)
Suburb	0.32 (0.47)	0.24 (0.43)	0.38 (0.49)
Town	0.11 (0.31)	0.11 (0.31)	0.1 (0.31)
Rural	0.2 (0.4)	0.13 (0.34)	0.26 (0.44)
Distance to 4-Year College	19.82 (18.8)	18.19 (20.5)	21.04 (17.25)
Distance to 2-Year College	9.65 (11.49)	8.23 (11.65)	10.73 (11.24)

Table 1: Summary Statistics

Note: The Columns include 12th-grade analysis cohorts from 2003-2005. NCES geographic categories are condensed into four types (city, suburb, town, rural). Distance from College is measured using the geodesic distance from the student high school to near by college. CT stands for the School Census Tract. Distinguish, minimal and required are the share of studnets with the Distinguished Achievement Program, Recommended High School Program, or the Minimum High School Program, respectivly. College Attendacne capture the share of high school students who attended college in the first year after high school graduation year

We now turn to focus on earnings differences between Hispanics and Whites. Table 5 in Appendix shows the average quarterly earnings for Whites and Hispanics at various intervals post-graduation. Generally, wages are on an upward trend over time, albeit at a decreasing rate. It is also evident that the earnings of Whites are higher and more variable than those of Hispanics. We can also see a Greater variance of Whites earnings, implies that they are harder to predict, and in our Gaussian model, implies that Whites would need a more precise signal for accurate prediction. Figure 4 explores the implied differences in earnings between college attenders and non college attenders, across Hispanics and Whites. The figure plots the coefficient for attending a four-year college for both Hispanics and Whites, controlling on for cohort fixed effect. The mean differences increase over time for both groups. Notably, this difference widens in the first five years post-graduation, then stabilizes at around \$500, approximately 6% of the average quarterly earnings for Hispanics 14-16 years after graduation. Figure 5 introduces our set individuals, school level and neighborhood level controls. We see that this reduces the level, but does not affect the gap, which stays relatively similar. These mean differences suggest that Hispanic high school graduates may have less incentive to attend

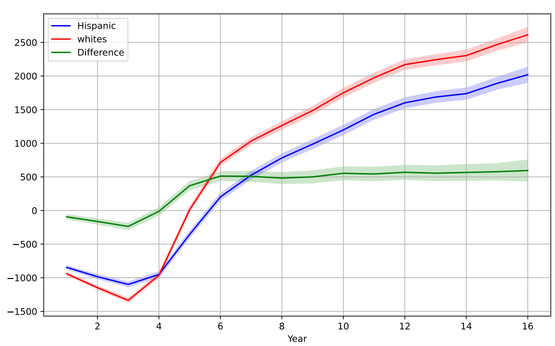

Figure 4: Raw Mean in Wages, without controls

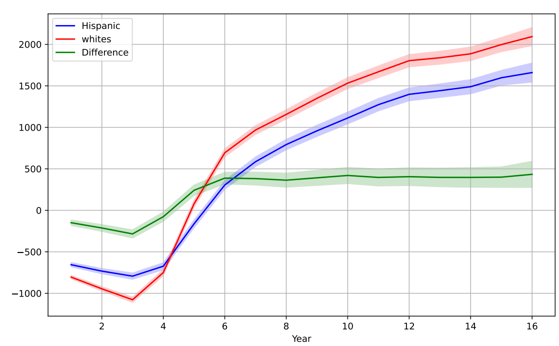

Figure 5: Raw Mean in Wages with controls

Notes: Figure 4 plots the coefficient for attending a four-year college for both Hispanics and Whites, controlling only for cohort fixed effect. The Coefficient for 16 years after college is using only two cohort, 2003-2004. Figure 5 plots the same coefficient, with all the added controls, as discussed in section 3

college compared to their White counterparts. However, this observed gap could be attributable to selection bias rather than reflecting the actual returns considered by the graduates. To investigate this, we utilize the distance to college from high school as an instrument in a Two-Stage Least Squares (TSLS) analysis. The use of this instrument follows the precedent set by Card (1995) and its subsequent application in works such as Carneiro et al. (2011), Carneiro and Lee (2009)

Kapor (2020), Abdulkadiroğlu et al. (2020) Mountjoy (2022). Table 6 in Appendix, examines the correlation between the instrument and test scores. Initially, without additional controls, test scores show a significant correlation with the instrument. After including individual characteristics, this correlation persists, which might indicate that spatial sorting is non-random and likely tied to other factors that influence both outcomes and information. Subsequent rows in the table introduce more controls for school and neighborhood characteristics, which largely account for the initial correlation, rendering the coefficient on distance nearly null. Revisiting table 6, we note that the inclusion of these controls does not markedly alter the instrument's effect.

Table 7 in the Appendix shows a strong first stage regression. The influence of distance to college on on the likelihood of attending a four-year college immediately after graduation. Controlling for our set of controls, we see that an increase of one mile in distance to college decreases the likelihood of college attendance by 0.2% for Hispanics and 0.1% for Whites. The magnitude of this effect remains relatively stable upon inclusion of different controls.

Finally, Table 2 presents the results from the TSLS regression that instruments the treatment effect using the distance to college instrument and includes all controls. It shows that, after adjusting for selection, the average effect for Hispanics is negligible, persisting up to 16 years post high school graduation. For White, on the other hand, there is a gradual effect that mirrors the earnings dynamics depicted in Figure 5, with significance emerging around 14-16 years. These findings suggest that the returns for Hispanics are generally much lower, potentially diminishing the incentive to pursue higher education. This underscores the need for a model that quantifies how these differences in returns influence decisions and contribute to the educational attainment gap.

5 Model Results

In this section, we estimate the model outlined in section 2.1 and discuss the implications of the estimated parameter for the role of information in determining the gap. Our analysis assumes that individuals are primarily concerned with their quarterly earnings 12-15 years post-graduation. As demonstrated in table 2, positive returns to college education starts approximately after 12 years. Consequently, we average the quarterly earnings within this 12-15 year period. This approach enables us to use data from our three cohorts and effectively capture the structural components, averaging over a long period.

	All	Whites	Hispanics
Avg. Wage 8-10	245.0	707.0	-1108.62
	(1194.0)	(1237.0)	(2028.0)
	245206	103198	142008
Avg. Wage 10-12	875.0	305.0	521.0
	(1436.0)	(1468.0)	(2295.0)
	239307	101284	138023
Avg. Wage 12-14	1552.0	255.0	2380.0
	(1531.0)	(1550.0)	(2370.0)
	233091	99428	133663
Avg. Wage 14-16	2605.0	377.0	5156.0
	(1632.0)	(1745.0)	(2424.0)
	149498	63271	86227

Table 2: Returns - Two Least Squares

Note: This table presents the results from a Two-Stage Least Squares (TSLS) regression of college attendance on earnings. Earnings are measured in periods of 8-10, 10-12, 12-14, and 14-16 years after the students' high school graduation. We instrument college attendance using the distance to the nearest college and control for individual, school, and neighborhood characteristics, as discussed in Section 3. For the 8-14 year period post-graduation, we include cohorts from 2003-2005. For the 14-16 year period, we include only the 2003-2004 cohorts due to data limitations.

We start our analysis by examining the relationship between the perceived cost of attending college and beliefs among Hispanic and White students. Figures 7 and 6 present histograms of the estimated costs for these groups, revealing that Hispanic students generally face lower attendance costs. As delineated in section 2, these costs encompass barriers to entry, such as credit constraints and discrimination, and are also influenced by preferences shaped by social norms, among other factors. Table 3 further shows this point, indicating that the average cost for Hispanic students corresponds to $1,199 of their quarterly earnings, compared to $2,879 for White students. Notably, while the average cost is higher for White students, the standard deviation for Hispanics, at 889, surpasses that of Whites, which is 693. This suggests a greater variability in the costs encountered by Hispanic students. These findings underscore the complexity of the cost dynamics in college attendance and its varied impact across different ethnic groups

In addition to cost analysis, Figures 6 and 7 also explore the distribution of conditional returns $\mathrm{E}[\alpha_1 - \alpha_0 | x]$, which represent the mean beliefs about returns for individuals with characteristic x. This measure captures the expected benefit of attending college, given certain attributes or circumstances.

The figure a noticeable disparity: White students exhibit significantly higher expected returns than Hispanic students.

Table 3 further complements this analysis, showing that the average beliefs on returns are lower than the actual average return for both groups. Specifically, the gap between the mean costs and mean beliefs about returns is narrower for White students (949) than for Hispanic students (2256). This implies that the share of White students would shift less in response to additional information. Following our discussion in section 2.1, this disparity suggests that, given the same quality of information, White students are more inclined on average to pursue college education compared to their Hispanic counterparts.

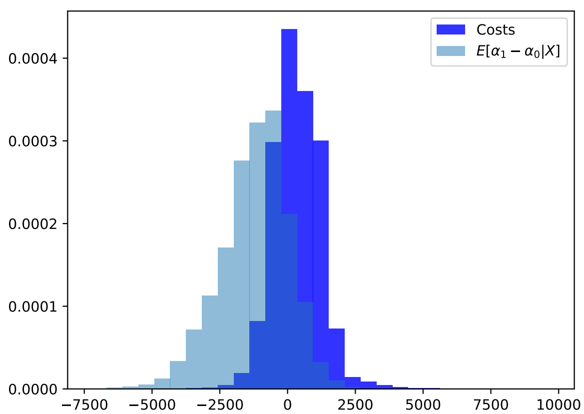

Figure 6: Hispanics Costs and Beliefs

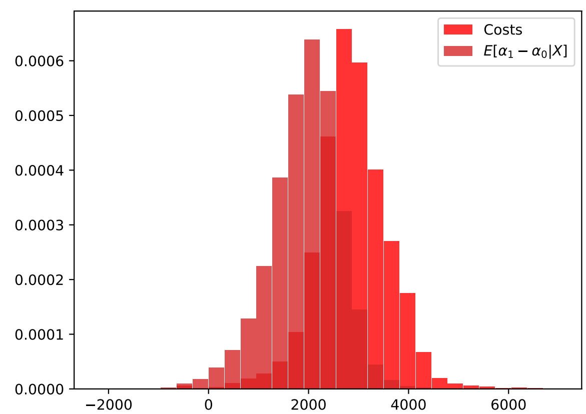

Figure 7: Whites Costs and Beliefs

Note: Figures 7 and 6 present histograms of the estimated costs and conditional priors $E[\alpha_1 - \alpha_0|X]$ for Hispanics and Whites, respectively. These estimates are derived according to the model discussed in Section 2. The parameters α_1 and α_0 represent the average quarterly earnings of high school students 12-15 years after graduation.

We now turn to look at the estimated distributions of beliefs on $U_1 - U_0$. Table 3 presents the estimated variance of α_1, α_0 residuals, and beliefs across different wage periods for the two groups. The variance in beliefs among Hispanics is notably higher than that among Whites, and the variances for the residuals of α_1 and α_0 the means are generally higher among Whites, although we can't rule out statistically that they are equal. These differences suggest that the quality of information, as measured by R^2, is the same or lower for Whites than for Hispanics. If the residual variance of returns for Whites is higher or the same as that for Hispanics, this implies that choice outcomes are less predictable for Whites. In both cases, the quality of information on returns hinges on the covariance structure of U_1 and U_0. Figure 16(a) in the Appendix shows plots the estimated cumulative distribution function (CDF) of the beliefs distribution, conditioned on the average covariates, and figure 16(b) shows the CDF where we fix all covariates and constant to zero. It is observed that for both Hispanics and Whites, the beliefs are systematically higher for the average White high school student. Concentrating on the CDF's shape when $X = 0$, we can again see that for White and Hispanics Students with the same observables, the beliefs of Whites are less dispersed.

	P(D=1)	σ_{E}	σ_1	σ_0	Avg. Cost	$E[\alpha_1]$	$E[\alpha_0]$	$E[\alpha_1 - \alpha_0]$	$Corr(E[\alpha_1\|x], E[\alpha_1\|x])$
Hispanics	0.21	2381	4490	6264	1199	6658.0	7715	-1057	0.76
		(657.0)	(125.0)	(818.0)	[889]	(1795.0)	(1843.0)	[2573.0]	(NA)
Whites	0.29	1414	5577	6316	2879	10871	8942.0	1930.0	0.95
		(873.0)	(155.0)	(491.0)	[693]	(2149.0)	(2211.0)	[3083.0]	(NA)

Table 3: Model Parameters

Note: The table displays model parameters estimated using average quarterly earnings 12-15 years after high school graduation. Standard errors for these parameters are presented in round parentheses ( ). Standard deviations of the costs and beliefs are indicated in square brackets [ ].

Next, we look at the implied information quality. As discussed in section 2, the correlation between U_1 and U_0 is not identified from the data. Therefore, to assess the quality of each group's information, we fix a correlation value, denoted as $\rho_{1,0}$, between U_1 and U_0, and calculate the implied R^2 for each value

$$R^2 = \frac{\hat{\sigma}^2_{\hat{E}}}{\hat{\sigma}_1^2 + \hat{\sigma}_0^2 - 2\rho_{1,0}\hat{\sigma}_1\hat{\sigma}_0}$$

Figure 8 shows the results of this comparison. It shows that as the correlation between U_1 and U_0 increases, the variance of returns declines, leading to higher implied information quality. For all correlation levels between U_1 and U_0, Hispanics display higher quality information on potential returns. Figure 9 preforms a similar exercise, fixing the standard errors returns, and compare the implied information quality. Despite higher information quality for Hispanics at a given correlation, the actual correlation between outcomes for Hispanics and Whites may differ. Although it is well-known that we cannot directly ascertain the correlation between the residuals of α_1 and α_0, we can make an educated guess by examining the impact of observed variables on these parameters. The last columns of Table 3 indicate that the correlation between $E[\alpha_1|X]$ and $E[\alpha_0|X]$ is notably high—ranging from 0.74 to 0.8 for Hispanics and from 0.91 to 0.97 for Whites, depending on the years used for wage measurement. This high correlation suggests that the observed variables affect both α_1 and α_0 similarly, as X increases. If unobserved variables also influence these outcomes similarly, the residuals of α_1 and α_0 would likely be highly correlated as well, implying high information quality. Notably, the correlations are much higher for Whites than for Hispanics, suggesting that while the correlation between residuals is likely positive for both groups, it is probably not identical. If we assume that the conditional expectation correlations mirror those for unobserved variables, the information quality for Whites could actually be much higher ($R^2 = 0.32$ for Hispanics with correlation 0.74 and $R^2 = 0.67$ for Whites with correlation 0.91).

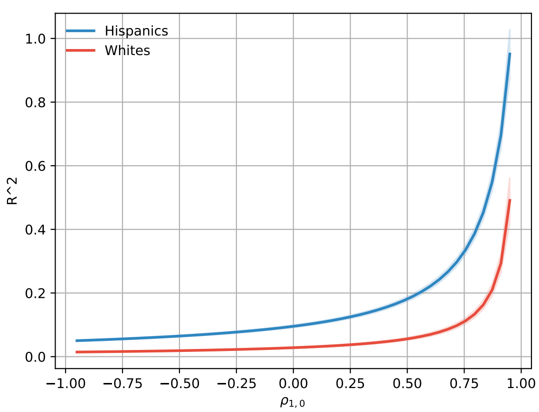

Figure 8: Information Quality

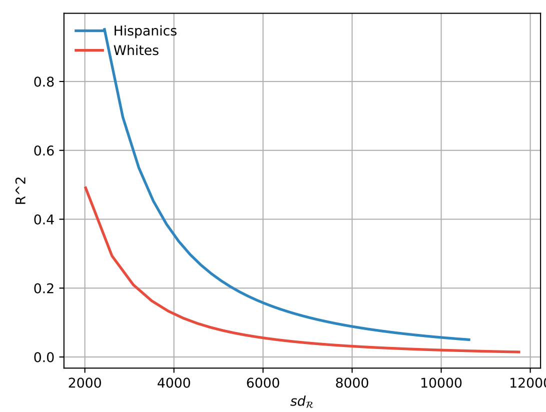

Figure 9: Information Quality

Notes: Figure 8 plots the variation in the quality of information regarding average returns to college attendance 12-15 years post high school graduation for Whites and Hispanics, as a function of the correlation between U_1 and U_0. Figure 9 plots how the quality of information fluctuates with changes in the standard deviation of $U_1 - U_0$.

5.1 The role of information

We now turn to show the results of the decomposition exercise, discussed in section 2.3.1. Our objective is to explain the 8% gap in college attendance decisions between Hispanics and Whites. The first row in table 4 shows the bounds on the role of quality of information in explaining the choice gap. The first row shows the bounds on the proportion of White high school graduates attending college if they were to access the same level of information quality as their Hispanic counterparts, without restrictions on the possible correlations. The limits on the information's contribution to the gap are notably wide, ranging from -215%, which suggests that equipping Whites with Hispanic-level information quality could raise White college attendance by 0.17 to 0.46 percentage points, to 161%, indicating that such information could entirely eliminate the gap by reducing White attendance by 13 percentage points or increase the gap by 15 percentage points. These bounds are achieved at the extreme values of $\rho_{1,0}$, where the difference between the correlation parameter is maximized. As previously discussed, these extreme values correspond to scenarios wherein one group has the highest possible information quality, while the other group has the lowest. Table 4 also shows that the composition channel is equally wide, capturing the residual effect, which can be interpreted as how much of the gap is affected by changing the returns and cost distribution of Hispanics to Whites. The second row in table 4 considers the role of information when under the assumption that correlations are positive. This is motivated by our earlier estimation, which demonstrated that conditional expectations based on observables are highly and positively correlated. This restriction doesn't significantly alter the bounds, placing the role of information somewhere between -195% and 130%. In section D in the Appendix, we consider another set of restrictions, by restricting the quality of information individuals have on the marginal distributions. These bounds, under realistic assumption on the quality of information on the marginals, generate wide bounds as well. Figure 15 in the Appendix explore the full set of counterfactual share of Whites endowed with Hispanic information quality, under varying values of $\rho_{1,0}$ for both Hispanics and Whites.

Assuming that the correlation of the two groups is at the opposite extreme or that these correlations are markedly different, is unlikely. It implies that the wage structure and the population distributions are fundamentally different between the two groups. In Figure 10, we focus on the case where the correlation parameters between the two groups are similar. The black line shows the case where the two correlation values are the same. As we demonstrate in section I in the Appendix,

	Information Channel	Composition Channel
Avg. Wage 12-15		
0.29-0.21=0.08		
1) No Assumptions		
LB, CF= 0.46,($\rho_w = -0.98, \rho_h = 0.9$)	-0.17 (-215.79%)	0.25 (316.0%)
UB,CF= 0.16,($\rho_w = 0.96, \rho_h = -0.96$)	0.13 (161.0%)	-0.05 (-60.76%)
2) Positive Correlation		
LB,($\rho_w = 0.01, \rho_h = 0.9$)	-0.16 (-195.67%)	0.23 (296.0%)
UB,($\rho_w = 0.96, \rho_h = 0.01$)	0.1 (130.0%)	-0.02 (-30.14%)

Table 4: Bounds on the role of information

Note: This table presents bounds on the information channel under various assumptions regarding the correlation between α_1 and α_0, utilizing average earnings data 12-15 years post high school graduation.

this can correspond to the case where all differences in wages between Hispanics and Whites are associated with different treatment of equivalent types, and no pre-labor market differences. We can see that in this case, information only contributes to decreasing the gap. For most values of $\rho_{1,0}^{Whites}$, we observe the gap increasing by approximately 7-8 percentage points, suggesting that providing Whites with the information quality available to Hispanics would double the size of the gap. The shaded area around the black line considers the case where we allow the correlation values to differ by a certain amount. Specifically, the red shaded area considers the case where the copula is allowed to differ by no more than 0.05 points. For all values considered, the gap suggests that reducing the information disparities between Hispanics and Whites contributes to narrowing the choice gap, thereby contributing equality in choice. The other shaded regions show how the bounds widen as we allow correlation values to differ by more. We also note that the bounds widen when the correlation is high, aligning with Figure 8, which demonstrates that an increase in correlation parameter for both Whites and Hispanics corresponds to a widening of gap in information quality.

In Appendix E, we consider a similar exercise, where instead of equating the R^2, we equate the average quality of information R^2 over all feasible ρ values, under the assumption that ρ is distributed uniformly across the set of feasible ρ values. Under this modified definition of information quality we find a similar result, of reduction gap increase of 7.3%. In this Appendix, we discuss how to impose further restrictions on the set of ρ values, motivated by the assumption that our agents are Bayesians.

Our analysis shows that under reasonable assumptions on the similarly between the correlation

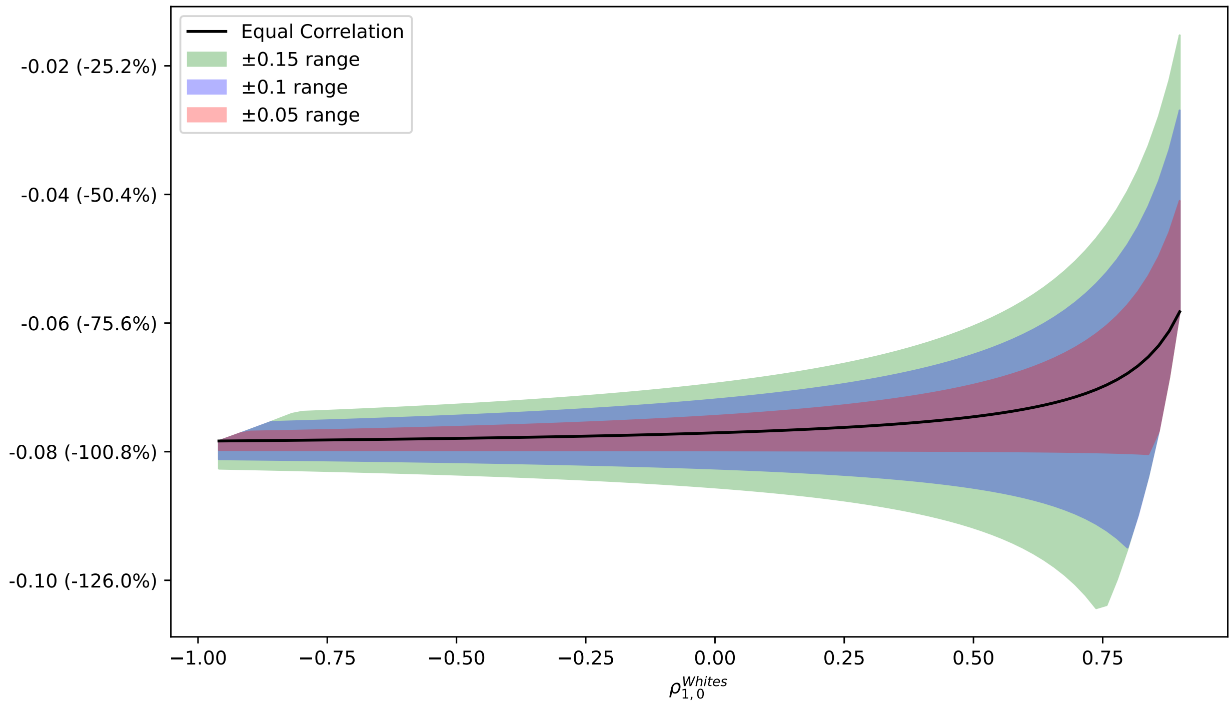

Figure 10: Bounds on the role of information

Note: The plot displays the information channel's bounds, with the X-Axis representing the White's potential earnings correlation and the Y-Axis indicating the information channel size. Parenthetical numbers denote the information channel's share in the 8% gap. The black line represents equal correlations for Hispanics and Whites. The red, blue, and green shaded areas depict bounds for maximum correlation differences of 0.05, 0.1, and 0.15, respectively, between Hispanics and Whites.

structures of Whites and Hispanics, information differences are reducing the choice gap and contributing to equality in choice. On the other hand, it seems that differences in the benefits and costs across groups are driving most of the gap. From a policy perspective, this implies that interventions aiming to either reduce costs or increase returns may have a better chance of reducing disparities in choices.

5.2 Second Measure of information gaps and Introducing Additional Signals

We now turn to our second measure of uncertainty, where we ask how much the information quality for Hispanic high school students should be changed to induce them to attend college at the same rate as White students. The blue line in Figure 11 plots the implied quality of information for each level of the correlation parameter, $\rho_{1,0}$, the red line plots the required quality of information to achieve equality in choice and the black line describes our second measure, described in equation 5, of information frictions, which is difference between the red and blue line. The figure shows that as as $\rho_{1,0}$ increases, the gap between the implied and the required quality of information for Hispanics to achieve parity with Whites share of college attenders grows as well. Intuitively, as the variance in returns increases with $\rho_{1,0}$, if there is less variance in the outcome variable, a larger share of it must be explained to induce sufficient variation in beliefs to prompt action. For positive values of rho it seems that the required quality of information needed to achieve parity is rather high, ranging form 12% in the case of the most negative, correlation between U_1 and U_0, 24% in the case of independence, and 98% at the top feasible ρ. This implies an increase of between 24% to 59% in accuracy for positive values of rhos.

The fact that we need to increase information quality by so much implies that information may not be most efficient way of achieving parity in choice. We now explore the process of adding this information. We consider a policymaker focused on achieving equality in college attendance between Hispanics and Whites. Typically, a policymaker aiming to maximize welfare from the agent's perspective would provide the most detailed information it can. However, the goals of policymakers often differ from those of decision-makers. For example, a policymaker might prioritize equality, value the positive externalities of education, or consider long-term effects that do not align with individuals' current objectives. As we discussed in section 2, we ask how precise should additional signal, containing new information, should be in order to achieve parity. As discussed in 2, the distribution of returns is generally unknown, we therefore, assess this precision by measuring the explained variance from U_1 and U_0.

Our initial focus is on adding information exclusively to either U_1 or U_0. To achieve parity, we consider additional information previously unknown to the agent about earnings if he opts for college, which necessitates that the quality of this signal be at $R^2 = 0.19$. This implies that the additional information must independently explain almost 20% of the variance in U_1. In a similar vein, for a signal on U_0 that aims to achieve parity in choices between Whites and Hispanics, it must be capable of explaining 38% of the total variance in U_0.

Figure 12 explores further the counterfactual college attendance changes rate for different quality levels of additional information on U_1 and U_0, as quantified by $R^2_{1,n}$ and $R^2_{0,n}$. This figure illustrates that focusing the information predominantly on one outcome tends to enhance participation more effectively than offering a signal informative about both U_1 and U_0. This is due to the fact that information on both U_1 and U_0 reduces the variance in beliefs, as shown in equation 6.

Can policymakers achieve the level of accuracy as discussed above? Table 11 in the appendix explores the Share of explained variance for Hispanics and Whites in the NLSY97. We use a linear regression to predict earnings for college goers and non-college goers at ages 34 or 35, utilizing detailed information on gender, cohort, urban location indicators, ability (as measured by the ASVAB tests), parents' education and earnings, and high school grades. We find that we can explain up to 10% (as measured by adjusted R^2) of the variance in earnings for both Hispanics and non-Hispanics, regardless of college attendance. Similarly, in our larger and richer administrative data set, we perform a similar exercise. We use Extreme Gradient Boosting (XGBoost) to predict average quarterly earnings between 12 and 14 years after high school, for Whites and Hispanics, aiming to effectively forecast future income based on individual covariates, exit exam scores, and a full set of indicators for the courses each student took and whether they passed them. This approach captured the information schools might use to inform a student. Again, we find that we can explain roughly 10% of our sample variance in earnings for both college and non-college goers. Including fixed effects for schools does not significantly change the quality of predictions. This information, potentially already known to the individuals, does not reveal entirely new insights. This suggests that achieving equality through informational interventions might be challenging.

In this exercise, we investigated the required precision of a signal to achieve parity in choice. In section G of the Appendix, we examine the necessary precision of signals on either U_1 or U_0 when the policymaker's goal shifts from equality of choice to equality in unconditional mean earnings. This exercise also considers the impact of these additional signals on selection patterns, thereby influencing wage disparities between Hispanics and Whites. Our analysis indicates that the effect of these interventions on income inequality is relatively minor. Consequently, merely providing a normally distributed signal correlated with U_1 and U_0 proves insufficient to bridge the average

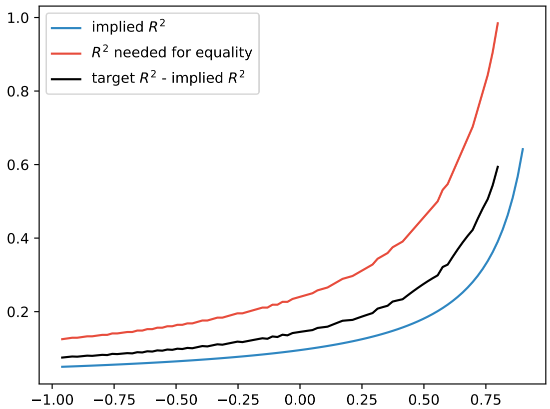

Figure 11: R^2 needed to achieve parity

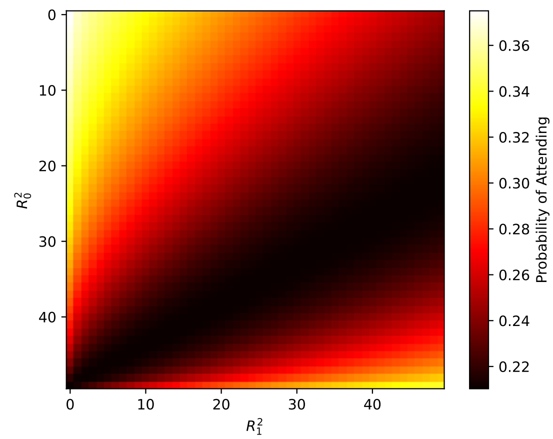

Figure 12: The effect of additional information on Earnings

Notes: The blue line in Figure 11 plots the implied quality of information for each level of the correlation parameter, $\rho_{1,0}$. The red line plots the required quality of information to achieve equality in choice. The black line describes our second measure of information frictions, which is difference between the red and blue line. Figure 12 shows the counterfactual share of Hispanics who would attend college after providing them with an additional signal of information quality on U_1 of R_1^2 and information quality on U_0 of R_0^2. For both figures, the quality of information is measured based on the ability to explain quarterly earnings 12-15 years after high school graduation.

earnings gap. A more flexible information structure than the simple normally distributed signal might be required, as considered for example in Vatter (2022).

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6 Conclusions

Individuals from diverse backgrounds have unique upbringings that significantly shape their later life and subsequent choices. Such experiences are pivotal in defining the constraints and opportunities they encounter, along with the outcomes and their information on these outcomes. This project explores how differences in returns and information affect different groups' college-going decisions. In this context differences in the outcomes and information can be driven by disparities that take place prior to time the decision is made (Neal and Johnson (1996)). For example, differences in information can arise if members of one group are coming from affluent backgrounds, have access to higher qualit y information on the monetary outcomes of college, unlike their less fortunate counterparts. These differences in where individuals grew up can similarly drive differences in the returns, if, for example, children from richer parents can make the cost of college lower or more obtainable. Information also accumulates through learning prior to the decision. For instance, dynamic models (Cunha and Heckman (2007),Cunha et al. (2021)) of investment illustrate three ways through which early life disparities can shape future opportunities. First, inadequate early investment can limit future choices. Second, dynamic complementarity suggests that boosting investments at one stage can enhance the value of subsequent investments. Lastly, early experiences influence the information individuals possess about the value of future investments. Therefore, discrepancies in pre-choice environments can manifest as changes in the potential returns or in the returns in the information available to individuals.

Information, in the context of decision-making, is not solely a byproduct of past experiences and accumulated knowledge, but is also a function of the future setup individuals would interact with, as affected by their choice. This is particularly evident when we consider the challenges associated with predicting outcomes like earnings, for different social groups. For instance, earnings for different groups can vary widely due to factors like industry sector trends, geographic economic conditions, and social biases. These disparities are not only affecting the returns distribution, but also affect the ability to predict future returns. This unpredictability has real implications for individuals making life-altering decisions. Therefore, our decomposition exercise explores how disparities across various realms accumulate and influence the decision processes of different groups.

This project introduces a new approach to analyze how these factors impact choice disparities among different groups, where we do not focus on individuals components, but observe how systemic differences are affecting choices. We do so by exploring how equating information across groups and introducing additional information affects choice gap. In our empirical exercise, we find that the information gaps between Hispanics and Whites is reducing the gap, therefore, disparities in later stages of lifetime, in our case, are not contributing to increasing inequality but contribute to reducing it. We also find that in order to achieve parity in choice through policy interventions in our setup, high-precision information is required, which is not typically available in standard data sources. This suggests that while information-based initiatives may have limited effectiveness, strategies directly targeting outcomes may be more effective in the long term to achieve parity in choices.

The approach proposed in this paper could be applied to other scenarios where information about outcomes plays a critical role in creating disparities, such as cases of discrimination, healthcare, and decisions related to investing in human capital and skill development. The central idea we present here is that to comprehend the drivers of behavioral differences and choices, as well as why these disparities persist, we must describe and quantify the information individuals possess, how they acquire it, and how they utilize it. Understanding the informational environment in which people operate is essential for comprehending the existence and persistence of differences across social groups.

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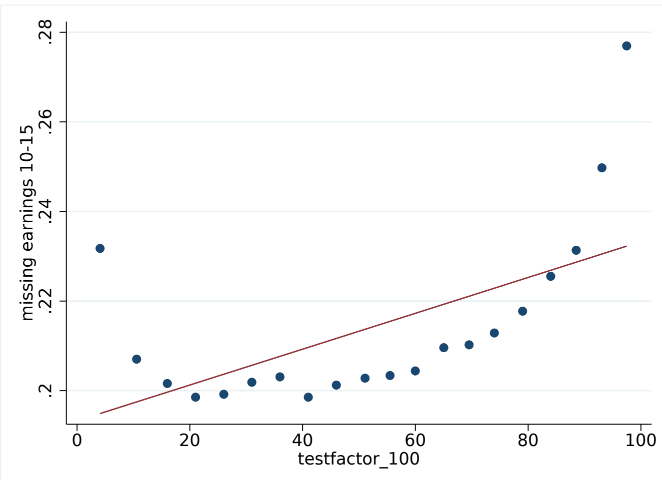
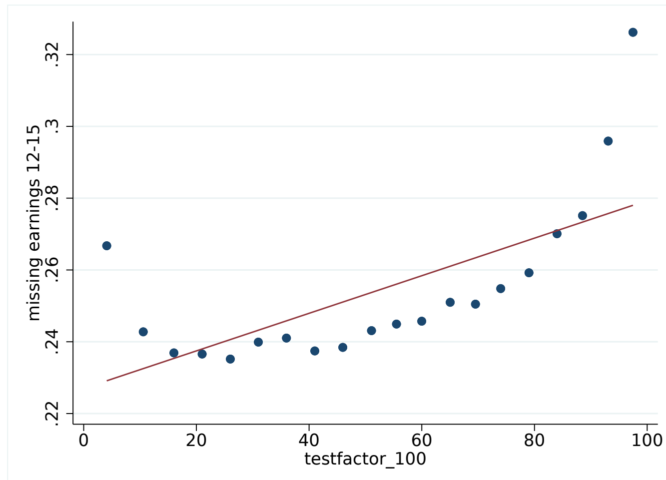
Appendix Table of Contents



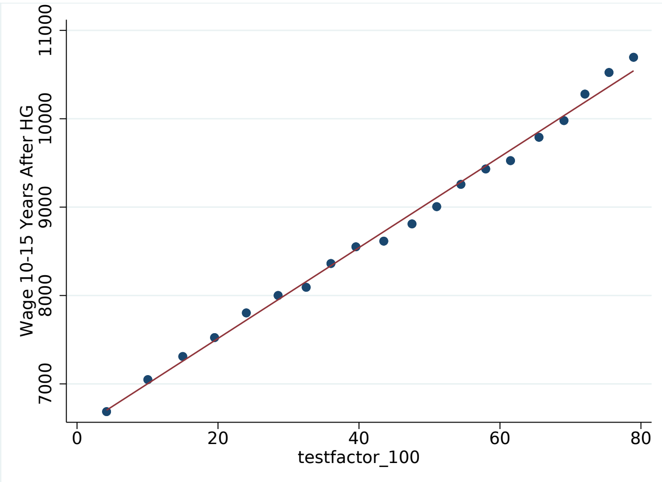
(a) Relation between test scores and missing Earnings for earnings 10-15 years after high school graduation



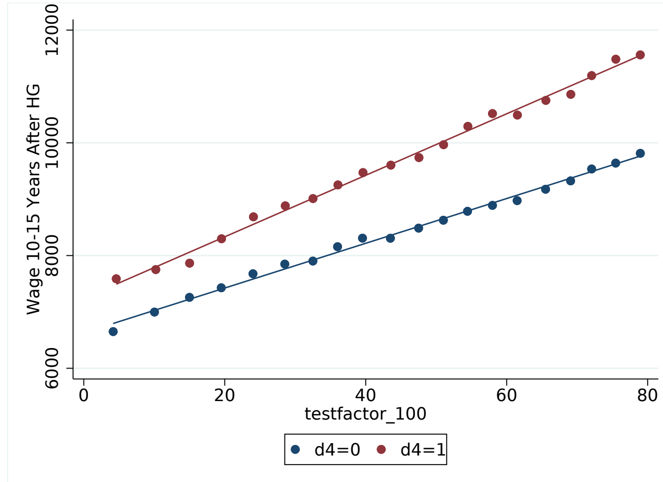
(b) Relation between test scores and missing Earnings for earnings 12-15 years after high school graduation

Figure 13: Relation Between test scores and Missing Earnings

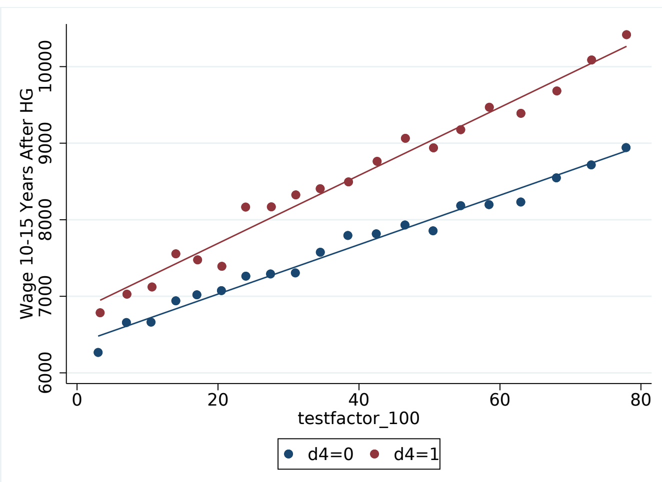
Notes: The above figures plot the share of missing earnings by test score factor, as described in Section 3. The first figure presents the missing earnings for the period of 10-15 years after high school graduation. Figure (b) illustrates the share of missing earnings for the period of 12-15 years after high school graduation. The red line indicates the expected trend line.



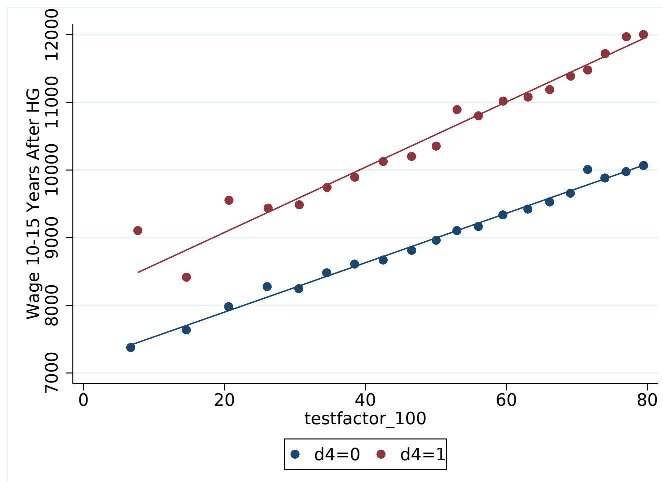
(a) Relation between test scores and earnings



(b) Relation between test scores and earnings, by college attendance



(c) Relation between test scores and earnings, by college attendance - Hispanics

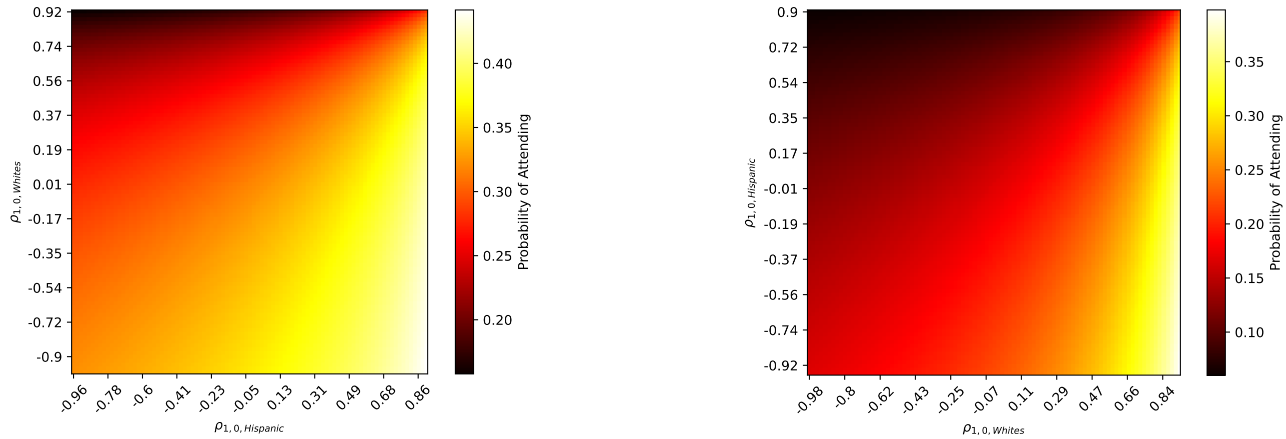


(d) Relation between test scores and earnings, by college attendance - Whites

Notes: This figure illustrates the relationship between test score percentile, as calculated in Section 3, and the expected average earnings 10-15 years after high school graduation. Figure (a) depicts the correlation between test scores and earnings for all individuals. Figure (b) presents this relationship, separated for individuals who attended college (red line) and those who did not (blue line). Figure (c) displays the same data but specifically for Hispanic individuals, while figure (d) focuses exclusively on White individuals.

Figure 14: Relation Between Test Scores and Earnings

A Additional Figures



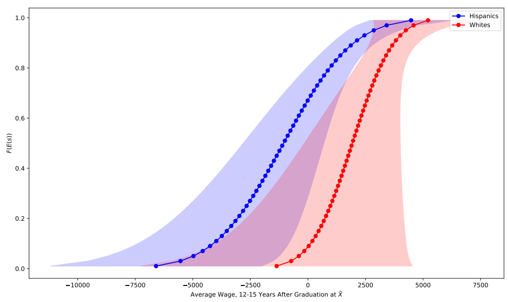
(a) Counterfactuals share of Whites with Hispanic Information

(b) Counterfactuals share of Hispanics with Whites Information

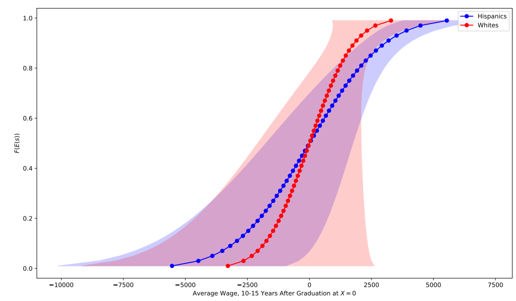
Figure 15: Counterfactuals share

Notes: The figures illustrate the counterfactual share of White and Hispanic college attendance for various potential earnings correlation values. Figure (a) depicts the share of White individuals under the scenario where they are provided with the same quality of information as Hispanics, with information quality measured across different correlation values. Figure (b) presents the counterfactual shares of Hispanic college attendance, assuming they received the information quality of Whites, as gauged by varying correlation values.

Counterfactuals Shares of Hispanic Attendance for Different Correlation Values



(a) Counterfactuals share of whites with Hispanic Information



(b) Beliefs CDF - Demean

Figure 16: Beliefs Cumulative Distribution for Whites and Hispanics

Notes: The figure displays the Cumulative Distribution Function of beliefs on α_1 and α_0 for both Hispanics and Whites. The shaded area represents the 95% Confidence Interval. Figure (a) illustrates these beliefs for the case where covariates are set to their mean. Figure (b) depicts the same graphs with all covariates, including the constant, set to zero.

B Additional Tables

	All	Hispanic	Whites	Difference (Whites - Hispanic)
Wage 8-10	7117.0 (4533.0)	6393.0 (3974.0)	7627.0 (4823.0)	1234.0 (6249.3)
Wage 10-12	8215.0 (5194.0)	7348.0 (4509.0)	8852.0 (5558.0)	1504.0 (7157.0)
Wage 12-14	9079.0 (5808.0)	8046.0 (4952.0)	9823.0 (6249.0)	1777.0 (7973.2)
Wage 14-16	9838.0 (6280.0)	8721.0 (5383.0)	10658.0 (6748.0)	1937.0 (8632.0)
Wage 12-15	9214.0 (5807.0)	8209.0 (4993.0)	9959.0 (6239.0)	1750.0 (7990.9)

Table 5: Wages Summary Statistics

Note: The table presents the mean earnings for Hispanics and Whites across various periods, spanning 8-16 years after high school graduation. For the period of 14-16 years post-graduation, data is exclusively from the 2003-2004 cohort. For all other time frames, data includes all cohorts from 2003-2004. Standard deviations are provided in parentheses.

	All	Hispanic	Whites
No Controls	-0.0156	-0.0232	-0.0436
	(0.0054)	(0.0053)	(0.0038)
Ind. Controls	-0.0277	-0.0151	-0.0392
	(0.0044)	(0.0066)	(0.0045)
+ School Char.	-0.0061	0.0074	-0.0177
	(0.004)	(0.0057)	(0.004)
+ Neighborhood Char.	-0.0014	0.0009	-0.0036
	(0.0018)	(0.0022)	(0.0021)

Table 6: Instrument Diagnostics

Note: The table displays coefficients on distance to a 4-year college, derived from a regression of test score factors, as defined in section 3, on distance to college. Each row introduces additional controls for individual student characteristics, school characteristics, and neighborhood characteristics. Standard errors, provided in parentheses, are clustered at the school-cohort level.

	All	Hispanic	Whites
No Controls	-0.0008	-0.0007	-0.0013
	(0.0001)	(0.0002)	(0.0001)
	317278	136581	180697
Ind. Controls	-0.0008	-0.0006	-0.0011
	(0.0001)	(0.0001)	(0.0001)
	317278	136581	180697
+ School Char.	-0.0014	-0.001	-0.0019
	(0.0002)	(0.0002)	(0.0002)
	317278	136581	180697
+ Neighborhood Char.	-0.0016	-0.0023	-0.0012
	(0.0002)	(0.0003)	(0.0002)
	317278	136581	180697

Table 7: First Stage

Note: The table presents the first-stage regression results, analyzing the effect of distance to a 4-year college on college attendance in the first year post-graduation. Each row adds additional controls for individual student characteristics, school characteristics, and neighborhood characteristics, as defined in section 3. Standard errors are given in parentheses and are clustered at the school-cohort level.

		All			Hispanic			Whites				
Wage Avg	8-10	10-12	12-14	14-16	8-10	10-12	12-14	14-16	8-10	10-12	12-14	14-16
No Controls	7.2078	3.1346	-0.6775	-3.4851	6.7659	4.1617	1.9655	1.4303	3.362	-3.329	-9.8946	-15.7508
	(1.5094)	(1.4327)	(1.4515)	(1.8191)	(1.5053)	(1.2071)	(1.1988)	(1.5494)	(1.2177)	(1.4513)	(1.6605)	(2.2097)
Obs.	245206	239307	233091	149498	103198	101284	99428	63271	142008	138023	133663	86227
Ind. Controls	4.9314	0.5937	-3.5101	-6.5359	5.8931	3.3429	1.2646	0.7636	4.1198	-2.1051	-8.4963	-14.2128
	(1.0258)	(0.946)	(1.0365)	(1.4211)	(1.4953)	(1.2306)	(1.1954)	(1.5131)	(1.1556)	(1.3397)	(1.5095)	(1.9959)
Obs.	245206	239307	233091	149498	103198	101284	99428	63271	142008	138023	133663	86227
+ School Char.	-1.7881	-2.9513	-4.1313	-5.7861	-1.7207	-1.0352	-0.8267	-0.6872	-1.758	-4.7777	-7.6157	-11.5689
	(1.2276)	(1.2815)	(1.3991)	(1.8907)	(1.384)	(1.4014)	(1.5289)	(2.0046)	(1.4649)	(1.7076)	(1.8884)	(2.5429)
Obs.	245206	239307	233091	149498	103198	101284	99428	63271	142008	138023	133663	86227
+ Neighborhood Char.	-0.504	-1.7866	-3.2171	-5.9322	-1.9984	-0.8557	-0.7281	-1.1675	1.6792	-0.7804	-3.5928	-8.7372
	(1.6783)	(1.9833)	(2.2052)	(2.8903)	(2.4266)	(2.8765)	(3.114)	(4.094)	(2.1061)	(2.4762)	(2.7454)	(3.6005)
Obs.	245206	239307	233091	149498	103198	101284	99428	63271	142008	138023	133663	86227

Table 8: Reduced Form

Note: The table presents the reduced-form results of regressing the distance to a 4-year college on earnings for the periods 8-10, 10-12, and 14-16 years after high school graduation. Each row incorporates additional controls for individual student characteristics, school characteristics, and neighborhood characteristics, as defined in Section 3. For all periods, the data includes the three cohorts from 2003-2005. Specifically for the 14-16 year period, only the 2003-2004 cohorts are used. Standard errors, provided in parentheses, are clustered at the school-cohort level.

	Hispanics Coefficient		
0	Baseline	-0.0891	(0.0092)
1	+ Individual Chars.	-0.0579	(0.0076)
2	+ School Char	-0.0587	(0.006)
3	+ Neighborhood Char.	-0.0776	(0.0038)
4	+ Test Score	-0.0428	(0.0035)

Table 9: College Attendance Gap

Note: The table displays the coefficient for Hispanics from a regression analysis, where the dependent variable is an indicator of first-time college attendance and the independent variable is the indicator of being Hispanic. Each row adds additional controls. The first row represents the raw difference with a cohort fixed effect. Subsequent rows include additional controls for individual student characteristics, school characteristics, and neighborhood characteristics, as defined in Section 3. Standard errors are shown in parentheses and are clustered at the school-cohort level.

Waiting for Data Release Approval

Table 10: Relation Between Earnings and Grades

	All	Hispanics	Whites
Ind Chr.	0.7	0.68	0.67
+ School Char.	0.72	0.71	0.71
+ Neighberhood Char.	0.75	0.75	0.74
+ Test Scores	0.78	0.8	0.77
N	321411	137551	183860

Figure 17: Area Under the Curve Analysis of Predicting College Attendance Decisions

Note: This table presents the Area Under The Curve (AUC) from a Probit model, predicting college attendance in the first year after high school among graduates. Each row progressively includes additional controls. The first row incorporates individual characteristics, the second includes school characteristics, the fourth integrates neighborhood characteristics, and the final row additionally accounts for the test score factor. For a detailed description of these controls, refer to section 3.

	Baseline			Ability			Ability + Parental Income			Ability + Parental Income + Parental Educ			Ability + Parental Income + Parental Educ+ Grades		
	R^2	$R^2 - Adj.$	N	R^2	$R^2 - Adj.$	N	R^2	$R^2 - Adj.$	N	R^2	$R^2 - Adj.$	N	R^2	$R^2 - Adj.$	N
All	0.139	0.137	3568.0	0.153	0.150	2965.0	0.163	0.159	2092.0	0.158	0.151	1490.0	0.135	0.117	910.0
Whites	0.129	0.126	2554.0	0.137	0.134	2192.0	0.150	0.145	1578.0	0.147	0.138	1185.0	0.123	0.102	749.0
Hispanics	0.131	0.125	1014.0	0.183	0.174	773.0	0.204	0.189	514.0	0.234	0.202	305.0	0.289	0.205	161.0
Whites- No College	0.085	0.081	1404.0	0.106	0.100	1171.0	0.122	0.112	848.0	0.132	0.115	587.0	0.148	0.097	284.0
Whites - College	0.046	0.041	1150.0	0.057	0.050	1021.0	0.080	0.068	730.0	0.093	0.076	598.0	0.105	0.073	465.0
Hispanics- No College	0.083	0.076	757.0	0.116	0.105	569.0	0.145	0.124	382.0	0.180	0.135	215.0	0.300	0.172	104.0
Hispanics - College	0.048	0.025	257.0	0.113	0.081	204.0	0.104	0.038	132.0	0.177	0.061	90.0	0.250	-0.050	57.0

Table 11: NLSY97 - R^2

Note: This table uses data from the National Longitudinal Survey of Youth 1997 (NLSY97), focusing on students who were 16 and 17 years old in 1997, to show the prediction quality of their income in 2015 using pre-decision variables. It presents the R^2 and adjusted R^2 values, from regression for All, Hispanics and whites and by college attendance. The "Baseline" column accounts for social group, gender, birth year, and college attendance. Subsequent columns incrementally introduce additional variables: the "Ability" columns include ASVAB test results; the third column incorporates household income data; the fourth column integrates information on parental education levels; and the final column incorporates high school grade information.

FE	educDecision	ethnic	In Sample R^2	Out of Sample R^2
FE	No College	All	0.19	0.11
		Hispanic	0.17	0.09
		Whites	0.18	0.10
	College	All	0.15	0.09
		Hispanic	0.14	0.09
		Whites	0.11	0.06
No-FE	No College	All	0.20	0.10
		Hispanic	0.18	0.09
		Whites	0.20	0.09
	College	All	0.15	0.10
		Hispanic	0.16	0.09
		Whites	0.13	0.08

Table 12: School Informativeness - R^2

Note: This table displays the in-sample and out-of-sample R^2 values for a model predicting average earnings 12-14 years post high school graduation. The No-FE rows ("No Fixed Effect") incorporates individual characteristics (as detailed in Section 3), test scores from exit exams in math and English comprehension, and indicators for each course taken during the three years of high school, including pass/fail status, taken from the Texas Education Agency data. The FE rows ("Fixed Effect") additionally includes a high school indicator, controlling for the impact of different high schools. Estimation is conducted using XGBoost, with parameter selection via Parallelizable Bayesian Optimization, as implemented in the R package "Parallelizable Bayesian Optimization."

C Non parametric identification of the choice model

We explore the non-parametric identification of choices. First, we identify the distribution of structural components, α_1 and α_0, by leveraging panel data, an instrumental variable, and specific wage structure assumptions. Next, we establish the identification of both the cost function and the beliefs distribution. While panel data aids in identifying α_1 and α_0. This step can be skipped if if one assumes that outcomes are observed without measurement error.

In our analysis, we work under the assumption that the researcher has access to a random, independently and identically distributed sample of observations, each denoted by $(Y_{a,i}, D_i, X_i, Z_i)$.

All analyses are conditional on the covariates vector X, so we omit the X notation for simplicity.

C.1 Identification of $P(\alpha_1, \mathrm{E}[\alpha_1 - \alpha_0|s])$ and $P(\alpha_0, \mathrm{E}[\alpha_1 - \alpha_0|s])$ and the cost function

We impose the following assumptions on the wage data generating Process. Wages are set according to

$$Y_{i,a} = D_i(\alpha_1 + \epsilon^1_{i,a}) + (1 - D_i)(\alpha_0 + \epsilon^0_{i,a}))$$

where $Y_{i,a}$ is individual i's income at age a, D_i is a dummy variable indicating whether the HG i attended four years college or not. One can think of α_d as individual fixed effect, if that individual goes to college or not. We further impose the following assumptions on the wage process

Assumption 1. *(1) for all a we have $E[\epsilon^D_{i,a}|\alpha_D] = 0$ (2) $\alpha_1, \alpha_0 \perp\!\!\!\perp \epsilon^D_{i,a}$ and (3) there exist at least two periods a^D, a'^D for each $D \in \{0, 1\}$ such that $\epsilon^D_{i,a} \perp\!\!\!\perp \epsilon^D_{i,a'}|X$*

Denote by $P(Z) = E[D = 1|Z]$ the propensity score conditional on Z. We then employ the following assumption

Assumption 2. *The characteristic functions of the conditional distribution $\alpha_1|D = 1, P(Z) = p$, $\alpha_0|D = 1, P(Z) = p$, $\epsilon^D_{i,a}|D = 1, P(Z) = p$ and $\epsilon^D_{i,a'}|D = 1, P(Z) = p$ are non vanishing*

The first part of Assumption 1 is standard and implies that any constant is absorbed into α^D, ensuring that deviations from the structural component are independent of the fixed effects. The second restriction mandates the existence of at least two periods in which the shocks are mutually independent, given the covariates X. While this condition is restrictive, it accommodates complex correlation structures, such as finite moving averages or other forms of multi-period correlations. The Assumption 2 stipulates that the characteristic functions of the conditional distributions for $\alpha_1|D = 1, P(Z) = p$, $\alpha_0|D = 1, P(Z) = p$, $\epsilon^D_{i,a}|D = 1, P(Z) = p$, and $\epsilon^D_{i,a'}|D = 1, P(Z) = p$ are non-vanishing[4]. This is a standard assumption that is used for non parametric identification of factor models and assures us that we can use the characteristic functions to back-out the distribution of α_d.

[4] The non vanishing assumption can be further relaxed, as shown in Evdokimov and White (2012)

Next, we impose restrictions on the agent information set. In the spirit of rational expectations, we assume that there are two parts to wages; a structural component, on which individuals have information on, and an unpredictable shock component that is not known to the high school gradutes.

Assumption 3 (Information Restriction)**.** *The signals individuals obtain do not contain any information on the non structural part of the wage, $\epsilon^1_{i,a}, \epsilon^0_0$.*

$$s_i \perp\!\!\!\perp \epsilon^1_{i,a}, \epsilon^0_{i,a} | \alpha_1, \alpha_0$$

This implies that individuals can only receive information on the structural component of the wage, but may not have information on time varying shocks. Finally we impose the following assumptions on the instrument Z

Assumption 4 (Instrument Restrictions)**.** *We assume that the instrumet satisfies the following conditions*

1. $\epsilon^1_{i,a}, \epsilon^0_{i,a}, \alpha_1, \alpha_0 \perp\!\!\!\perp Z$
2. $S \perp\!\!\!\perp Z | \alpha_1, \alpha_0$
3. Z *is continuously distributed on* $\mathcal{Z} \subseteq \mathbb{R}$
4. $E[\alpha_1 - \alpha_0 | s]$ *continiously distributed*
5. $c(z)$ *is differentiable and covers the entire support of* $E[\alpha_1 - \alpha_0 | s, x]$

The assumptions are akin to standard Instrumental Variable (IV) assumptions (Heckman and Vytlacil (2005)), but they incorporate additional structure through the modeling of the choice equation. The first assumption establishes the instrument's independence from the outcome variables. The second dictates that information is independent of the instrument, conditioned on the structural components. Notably, these first two assumptions collectively imply that $E[\alpha_1(t_i) - \alpha_0(t_i) | s, x] \perp\!\!\!\perp Z$, aligning with standard IV assumptions where the selection variable is uncorrelated with the instrument. The final part of Assumption 4 is a technical requirement ensuring that we can recover the cost function by monitoring the derivative, as demonstrated in the proof.

Denote $E[\alpha_1 - \alpha_0 | s] = \text{E}$. We show the following proposition.

Proposition 1. Under assumptions (1)-(4), $P(\alpha_1, \mathrm{E})$,$P(\alpha_0, \mathrm{E})$ and the cost function $c(z)$ are identified

Proof. Let a, a' be two periods such that $\epsilon^D_{i,a} \perp\!\!\!\perp \epsilon^D_{i,a}$. We start by showing how to identify $\mathrm{P}(\alpha_d|\mathrm{E})$ First, using assumption 1, 3, and 4 we have that $\epsilon^D_{i,a} \perp\!\!\!\perp \alpha_1|p(Z) = p, D = 1$ as

$$\begin{aligned}
\mathrm{P}(\epsilon^D_{i,a}, \alpha_1|p(Z) = p, D = 1) &= \mathrm{P}(\epsilon^D_{i,a}, \alpha_1|p(Z) = p, \mathrm{E} \geq c(z)) \\
&= \mathrm{P}(\epsilon^D_{i,a}|\alpha_1, p(Z) = p, \mathrm{E} \geq c(z))\mathrm{P}(\alpha_1|p(Z) = p, \mathrm{E} \geq c(z)) \\
&= \mathrm{P}(\epsilon^D_{i,a}|p(Z) = p, \mathrm{E} \geq c(z))\mathrm{P}(\alpha_1|p(Z) = p, \mathrm{E} \geq c(z)) \\
&= \mathrm{P}(\epsilon^D_{i,a}|p(Z) = p, D = 1)\mathrm{P}(\alpha_1|p(Z) = p, D = 1)
\end{aligned}$$

where the first equality stems from the choice model, the second stems from Bayes rule, and the third equality is due to the contraction rule and the decomposition rule of conditional Independence. We have an equivalent result for α_0 and $\epsilon_{i,a'}$. Last, notice that as $\epsilon_{i,a} \perp\!\!\!\perp \epsilon_{i,a'}$, $\epsilon_{i,a}, \epsilon_{i,a'} \perp\!\!\!\perp m_{\mathcal{R}}(s)$ and $\epsilon_{i,a}, \epsilon_{i,a'} \perp\!\!\!\perp Z$ we have that $\epsilon_{i,a} \perp\!\!\!\perp \epsilon_{i,a'}|p(Z) = p, D = 1$

Therefore $\epsilon^D_{i,a}$ and $\epsilon^D_{i,a'}$ and α_D are mutually independent, conditional on D and P, and we can now utilize Kotlarski's Lemma (1967) to identify the conditional distribution of α_1 and α_0. We first show how to identify the conditional distribution of α_1. Let $\Psi(y_a, y_{a'})$ be the conditional characteristic function of $(Y_{i,a}, Y_{i,a'})$ given $(P(z) = p, D = d)$. Let $\Psi_{\alpha_1}(t), \Psi_{\epsilon_a}(t)$ and $\Psi_{\epsilon'_a}(t)$ be the conditional characteristic functions of $\alpha_1, \epsilon_{i,a}, \epsilon_{i,a'}$, given $(P(z) = p, D = d)$, then we can show that (Rao (1992), page 21 and Gilraine et al. (2020))

$$log\Psi_{\alpha_1}(t) = iE[\alpha_1|D = 1, P(Z) = p]t + \int_0^t \frac{\partial}{\partial y_a}\left(log\frac{\Psi(y_a, y_{a'})}{\Psi(y_a, 0)\Psi(0, y_{a'})}\right)_{y_a=0} dy_{a'}$$

Noticing that

$$\frac{\partial}{\partial y_a}\left(log\frac{\Psi(y_a, y_{a'})}{\Psi(y_a, 0)\Psi(0, y_{a'})}\right)_{y_a=0} = \frac{\frac{\partial\Psi(0, y_{a'})}{\partial y_a}}{\Psi(0, y_{a'})} - iE[Y_{i,a}|D = 1, P(Z) = p]t$$

and that by assumptions 1 and 3 we have $iE[Y_{i,a}|D = 1, P(Z) = p]t = iE[\alpha_1|D = 1, P(Z) = p]$ we then get

$$\log \Psi_{\alpha_1}(t) = \int_0^t \frac{\frac{\partial \Psi(0, y_{a'})}{\partial y_a}}{\Psi(0, y_{a'})} dy_{a'}$$

as the characteristic function fully defines the distribution and $\Psi(y_a, y_{a'})$ is observed in the data, we have identified $\mathrm{P}(\alpha_1|D = 1, P(z) = p)$. Similar argument shows that we can identify $\mathrm{P}(\alpha_0|D = 0, P(z) = p)$.

Next, denote by $F_{\alpha_1}(\cdot|D = 1, P(Z) = p)$ the conditional CDF of α_1. Denote by $V = F_{\mathrm{E}}(\mathrm{E})$ the quantile of the beliefs in the beliefs distribution. Then following the arguments in Carneiro and Lee (2009) we have that for all k on the support of α_1 we have that

$$\begin{aligned} F_{\alpha_1}(k|P(z), D = 1) = E[\mathbb{1}\{\alpha_1 \leq k\}|P(Z) = p, D = 1] &= E[\mathbb{1}\{\alpha_1 \leq k\}|P(Z) = p, V > p(Z)] \\ &= \frac{1}{p}\int_{1-p}^{1} E[\mathbb{1}\{\alpha_1 \leq k\}|V = v]dv \end{aligned}$$

rewriting the equation gives us

$$pE[\mathbb{1}\{\alpha_1 \leq k\}|P(Z) = p, D = 1] = \int_{1-p}^{1} E[\mathbb{1}\{\alpha_1 \leq k\}|V = v]f(v)dv$$

Using assumption 4 we can take derivative from both sides, with respect to p, and get

$$E[\mathbb{1}\{\alpha_1 \leq k\}|V = 1 - p] = E[\mathbb{1}\{\alpha_1 \leq k\}|P(Z) = p, D = 1] + p\frac{E[\mathbb{1}\{\alpha_1 \leq k\}|P(Z) = p, D = 1]}{\partial p}$$

Therefore we have that $\mathrm{P}(\alpha_1|V)$ is identified. Following similar steps we have that $\mathrm{P}(\alpha_0|V)$ is also identified

$$E[\mathbb{1}\{\alpha_0 \leq k\}|V = 1 - p] = E[\mathbb{1}\{\alpha_0 \leq k\}|P(Z) = p, D = 0] - (1 - p)\frac{E[\mathbb{1}\{\alpha_0 \leq k\}|P(Z) = p, D = 0]}{\partial p}$$

Next, observe that we can construct the probabilities $P(\alpha_1|\mathrm{E})$ and $P(\alpha_0|\mathrm{E})$ using the law of iterated expectations we have

$$e = E[\alpha_1 - \alpha_0|\mathrm{E} = e] = E[\alpha_1 - \alpha_0|F_{\mathrm{E}}(\mathrm{E}) = V] = \int \alpha_1 P(\alpha_1|V)d\alpha_1 - \int \alpha_0 P(\alpha_0|V)d\alpha_0. \quad (10)$$

Therefore we can identify the inverse $F_{\mathrm{E}}^{1}(V)$ and consequently the cumulative distribution function (CDF) of beliefs, $F_{\mathrm{E}}(e)$. As the CDF is strictly increasing by assumption 4 we can also identify $P(\alpha_1|\mathrm{E})$ and $P(\alpha_0|\mathrm{E})$ as needed. Therefore we've identified the joint $P(\alpha_1, \mathrm{E})$ and $P(\alpha_0, \mathrm{E})$. Finally, to identify the cost function, observe that

$$P(z) = \Pr(\mathrm{E} > c(z)) = 1 - F(c(z)) \implies F_{\mathrm{E}}^{-1}(1 - P(z)) = F_{\mathrm{E}}^{-1}(F(c(z))) \implies F_{\mathrm{E}}^{-1}(1 - P(z)) = c(z).$$

Finally, in order to identify the cost function, we notice that

$$P(z) = \mathrm{P}(\mathrm{E} > c(z)) \quad = 1 - F(c(z)) \implies F_{\mathrm{E}}^{-1}(1 - P(z)) = F_{\mathrm{E}}^{-1}(F(c(z)))F_{\mathrm{E}}^{-1}(1 - P(z)) = c(z)$$

which concludes the proof. □

C.1.1 Testable Implication

As discussed in Canay et al. (2020), the model implies that the marginal treatment effect should be decreasing. To see that notice that we use 10 to identify the CDF of V, therefore, if we get that this is not increasing function of v, this implies that our model is mispecificed.

D Bounds on the Information Channel with Information Restrictions

In this section, we consider how restrictions on what individuals may possibly know can impact the set of possible correlation values. We focus on assumptions about individuals' ability to predict marginal distributions. Our identification strategy establishes agents' beliefs about $E[\alpha_1 - \alpha_0|s, x]$, but we can not identify the separate beliefs on of U_1 or U_0. By incorporating external knowledge, we can make educated guesses about the quality of agents' information on their marginals - U_1 and U_0. This could be then used to narrow down the plausible range of correlations.

Let R_1^2 and R_0^2 denote the quality of information individuals have on U_1 and U_0 respectively. Using the covariance matrix between $E[\alpha_1|s, x]$, $E[\alpha_0|s, x]$, α_1, and α_0, we can derive bounds on $\rho_{1,0}$.

To see this, first, notice that we can write the identified beliefs variance, σ_E^2, as:

$$\begin{aligned}
\sigma_{\mathrm{E}}^2 &= \mathrm{Var}(E[\alpha_1 - \alpha_0|s, x]) \\
&= \mathrm{Var}(E[\alpha_1|s, x]) + \mathrm{Var}(E[\alpha_0|s, x]) - 2\mathrm{Cov}(E[\alpha_1|s, x], E[\alpha_0|s, x]) \\
&= \sigma_1^2 R^2 + \sigma_0^2 R^2 - 2\mathrm{Cov}(E[\alpha_1|s, x], E[\alpha_0|s, x])
\end{aligned}$$

Then, using our guess on R_1^2 and R_0^2, we can solve for $\mathrm{Cov}(E[\alpha_1|s, x], E[\alpha_0|s, x])$. Similar to our identification discussion in Section 2.4, we know that the covariance between α_d and the beliefs on it $\mathrm{E}[\alpha_d|s, x]$ satisfies

$$\mathrm{Cov}(\alpha_1, \mathrm{E}[Y_1|s, x]) = \mathrm{Var}(E[\alpha_1|s, x]) = \sigma_1^2 R^2$$

and similarly for $\mathrm{Cov}(y_0, E[y_0|s])$. Finally, we can also derive $\mathrm{Cov}_{0,\mathrm{E}_1} = \mathrm{Cov}(\alpha_0, E[\alpha_1|s, x])$ and $\mathrm{Cov}_{0,\mathrm{E}_0} = \mathrm{Cov}(\alpha_0, E[\alpha_1|s, x])$ using our identified $\mathrm{Cov}(E[\alpha_1 - \alpha_0|s, x], \alpha_d)$. Specifically, we have that

$$\begin{aligned}
\gamma_1 \times \sigma_{\mathrm{E}} &= \mathrm{Cov}(E[\alpha_1 - \alpha_0|s, x], \alpha_1) \\
&= \mathrm{Cov}(E[\alpha_1|s, x], \alpha_1) - \mathrm{Cov}(E[\alpha_0|s, x], \alpha_1) \\
&= \sigma_1^2 R_1^2 + \mathrm{Cov}_{1,\mathrm{E}_0}
\end{aligned}$$

where γ_1 is the coefficient on the control function as discussed in section 5.1. We can then solve for $\mathrm{Cov}_{1,\mathrm{E}_0}$. Using a similar argument, we can solve for $\mathrm{Cov}_{0,\mathrm{E}_1}$. Further refinement of the free parameters is possible by observing two key aspects. Firstly, agents utilize identical signals in predicting both U_1 and U_0. Secondly, within the Gaussian model framework, the posterior mean is a linear function of the signals. This leads us to the following relationship:

$$\mathrm{Cov}(\alpha_1, \mathrm{E}[\alpha_0|s, x]) = \mathrm{Cov}(\mathrm{E}[U_1|s, x] + \nu, \mathrm{E}[U_0|s, x]) = \mathrm{Cov}(\mathrm{E}[U_1|s, x], \mathrm{E}[U_0|s, x])$$

In this equation, ν denotes the uncorrelated residual from projecting U_1 on s, and is thus uncorrelated with $E[U_0|s]$, which is a linear function of s. Using this we can express R_0^2 in terms of R_1^2

$$R_0^2 = \frac{\mathrm{Cov}_{\mathrm{E}_1,\mathrm{E}_0} - \gamma_0 \sigma_\eta}{\sigma_0^2}$$

With the above calculations, for a guess of R_1^2, we can derive the following covariance matrix between beliefs and the U_1 and U_0

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_0 & \sigma_1^2 R_1^2 & \mathrm{Cov}_{\mathrm{E}_1,\mathrm{E}_0} \\ \rho\sigma_1\sigma_0 & \sigma_0^2 & \mathrm{Cov}_{\mathrm{E}_0,\mathrm{E}_1} & \sigma_0^2 R_0^2 \\ \sigma_1^2 R_1^2 & \mathrm{Cov}_{\mathrm{E}_0,\mathrm{E}_1} & \sigma_1^2 R_1^2 & \mathrm{Cov}_{\mathrm{E1,E0}} \\ \mathrm{Cov}_{\mathrm{E}_1,\mathrm{E}_0} & \sigma_0^2 R_0^2 & \mathrm{Cov}_{\mathrm{E1,E0}} & \sigma_0^2 R_0^2 \end{pmatrix}$$

where all parameters are known beside ρ. Feasible values for ρ are values between -1 and 1 that assure that the covariance matrix is positive semi-definite, that allows us to restrict the possible quality of information on the returns.

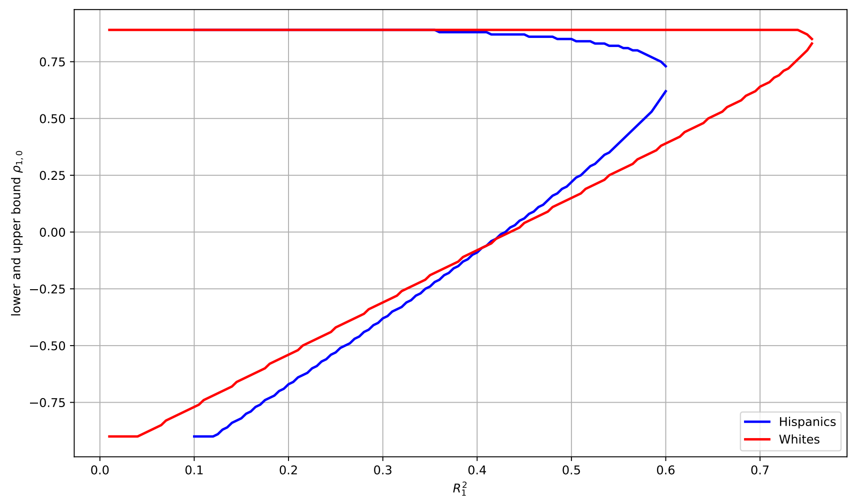


Figure 18: Lower and Upper bound for $\rho_{1,0}$

Notes: The figure plots the upper and lower bound of possible $\rho_{1,0}$ *for both Hispanics and whites for residuals from the residual of average income between 12-15 years after high school graduations for both Hispanics and Whites.*

Figure 18 plots the lower and upper bounds for $\rho_{1,0}$ for a fixed R_1^2 for both Hispanics and whites. The figure shows that increasing the R_1^2, shrinks the set of possible $\rho_{1,0}$. To understand the intuition, consider the extreme case in which agents have perfect knowledge of U_1. In this situation, $\rho_{1,0}$ cannot equal one because that would imply agents have perfect knowledge of $U_1 - U_0$. This is only possible if $\sigma_{\mathrm{E}}^2 = \sigma_1^2 + \sigma_0^2 - 2\sigma_1\sigma_0$, which is not the case in our estimates. The restrictions we get from the covariance matrix ensure that there exists some signal structure s that can beliefs with quality R_1^2 and R_0^2 on the marignals and R^2 on the returns. Notice that R_1^2 does not go from 0 to 1, as some values of R^2 are not feasible for any correlation value. The figure shows that Whites may potentially can have much more information on the U_1 than Hispanics.

	Gap	Information Channel	Composition Channel
Avg. Wage 12-15	0.29-0.21=0.08		
1) $R_1^2 = 0.1$			
LB,($\rho_w = -0.76, \rho_h = 0.88$)		-0.17 (-209.52%)	0.25 (310.0%)
UB,($\rho_w = 0.88, \rho_h = -0.9$)		0.09 (109.0%)	-0.01 (-9.03%)
2) $R_1^2 = 0.1$ and $\rho_{1,0} > 0$			
LB,($\rho_w = 0.01, \rho_h = 0.88$)		-0.15 (-191.42%)	0.23 (291.0%)
UB,($\rho_w = 0.88, \rho_h = 0.01$)		0.05 (65.93%)	0.03 (34.07%)

Table 13: Bounds on correlation, implied by beliefs restrictions

What are the possible values of R_1^2, and consequently R_0^2? We conduct two exercises to better understand the proportion of variance explained. First, in the appendix, we examine how well our covariates and school choice predict future earnings. Using Extreme Gradient Boosting, we forecast future income based on our set of covariates, exit exam grades, and indicators for the courses each student took and whether they passed them. This model explains approximately between 0.1 and 0.14 of the earnings variance for both college and non-college students among Hispanics and Whites. We conduct a similar exercise using NLSY97 data to predict earnings at age 34/35. Using a standard regression model, we explain roughly 20% of the earning variance for Hispanics and 10% for Whites. Row 1 in table 13 uses uses these estimates to inform our bounds. We fix the $R_1^2 = 0.1$ and examine how this affect the set of possible $\rho_{1,0}$.. We observe that information restrictions do not have in our case strong effect on the bounds and the bounds remain wide, ranging from increasing the gap by 17 percentage points to reducing it by 9 percentage points. In Row 2, we impose also that the correlation is positive. This does not affect the bounds size by much.

E Another Measure of Information Quality - A Bayesian Approach

In our analysis in Section 2.3.1, we examine the quality of information with regard to the true underlying variance of $U_1 - U_0$. This approach yields wide bounds due to the necessity of considering all possible correlation values between U_1 and U_0. In this section, we employ a different approach and consider another definition for the quality of information. We do this by considering the expected explained variance under various correlation values, taking into account the individual prior beliefs regarding the correlation parameter. Specifically, we assume that agents cannot determine the correlation between U_1 and U_0, treating it as part of the uncertainty they encounter. We posit that agents hold a non-informative prior over the feasible correlation values, distributed as $\rho_{1,0} \sim U(\rho_{min}, \rho_{max})$. We quantify information quality as the proportion of explained variance relative to the total variance of the outcome, which encompasses the variance resulting from uncertainty over $\rho_{1,0}$. Let

$$R^2(\rho) = \frac{\mathrm{Var}(E[\mathcal{R}|s])}{\sigma_1^2 + \sigma_0^2 - 2\rho\sigma_1\sigma_0}$$

and $f(\rho)$ denotes the prior distribution over the set of possible correlation parameters, we then define the the Bayesian R-Squared as the mean R^2, given the prior

$$BR^2 = \int_\rho R^2(\rho) f(\rho) d\rho \tag{11}$$

This approach is in line with the discussion by Gelman et al. (2019), which considered a similar measure for assessing model performance under uncertainty of the parameters. Within our framework, we limit the model uncertainty of agents exclusively to the correlation parameter, maintaining the the assumption that agents are confident in their predictions and know the likelihood functions $P(S|\mathcal{R})$.

Notice that we can write the identified variance of beliefs using (11) as

$$\mathrm{Var}(E[\mathcal{R}|s]) = \frac{BR^2}{\int_\rho \sigma_{\mathcal{R}}^{-1}(\rho) f(\rho) d\rho}$$

where we let $\sigma_{\mathcal{R}}^{-1}(\rho)$ denote the inverse of the implied variance of returns for a given correlation value

ρ. Denote by BR_g^2 the Bayesian R^2 of group g. We can now define our alternative information quality decomposition, similar to (2.3.1), as

$$P(D=1|b)-P(D=1|a)=\int_X \Phi\left(\frac{\mu_{\mathcal{R},b,x}-c_b(x)}{\sqrt{\frac{BR_b^2}{\int_\rho \sigma^{-1}_{\mathcal{R},b,x}(\rho)f(\rho)d\rho}}}\right)dF_b(x)-\int_X \Phi\left(\frac{\mu_{\mathcal{R},b,x}-c_b(x)}{\sqrt{\frac{BR_a^2}{\int_\rho \sigma^{-1}_{\mathcal{R},b,x}(\rho)f(\rho)d\rho}}}\right)dF_b(x)$$
$$+\int_X \Phi\left(\frac{\mu_{\mathcal{R},b,x}-c_b(x)}{\sqrt{\frac{BR_a^2}{\int_\rho \sigma^{-1}_{\mathcal{R},b,x}(\rho)f(\rho)d\rho}}}\right)dF_b(x)-\int_X \Phi\left(\frac{\mu_{\mathcal{R},a,x}-c_a(x)}{\sqrt{\frac{BR_a^2}{\int_\rho \sigma^{-1}_{\mathcal{R},a,x}(\rho)f(\rho)d\rho}}}\right)dF_a(x)$$

Therefore, our decomposition asks how much of the gap is driven by differences in information quality, taking into account fundamental model uncertainty on the correlation, from the agents perspective.

We next discuss different approaches to determine ρ_{min} and ρ_{max}. The first is to consider the set of feasible values, as discussed in section 2.4.3. This is the set we as Econometricans can consider. We can also consider the set of values that the individuals considers. Individuals observe know their information strucutre, and can use it to restrict the feasible set of ρs even further, and therefore, from their perspective the average value of information may be different. Unfortunately, we do not observe the information structure and their signals, but we only know the Var$(E[U_1-U_0|s,x])$. In section D we discuss how, given a guess on the quality of information on the marginals R_1^2, we can derive the covariance matrix of $U_1, U_0, E[U_1|s], E[U_0|s]$. The next claim shows, that knowing this is sufficient to identify the set of possible correlation values that the agent who know their information strucutre find feasible. Let $\boldsymbol{S}$ denote the vector of signals agents observe. Let the matrix

$$A=\begin{bmatrix}\sigma_{S_1,S_1} & \cdots & \sigma_{S_1,S_n} & \sigma_{S_1,U_1} & \sigma_{S_1,U_0}\\ \vdots & \ddots & \vdots & \vdots & \vdots\\ \sigma_{S_n,S_1} & \cdots & \sigma_{S_n,S_n} & \sigma_{S_n,U_1} & \sigma_{S_n,U_0}\\ \sigma_{U_1,S_1} & \cdots & \sigma_{U_1,S_n} & \sigma_{U_1,U_1} & \sigma_{U_1,U_0}\\ \sigma_{U_0,S_1} & \cdots & \sigma_{U_0,S_n} & \sigma_{U_0,U_1} & \sigma_{U_0,U_0}\end{bmatrix}$$

be the covariance matrix between the vector of signals S, U_1 and U_0. And let

$$B = \begin{bmatrix} \sigma^2_{\mathrm{E}_1} & \sigma_{\mathrm{E}_1,\mathrm{E}_0} & \sigma_{\mathrm{E}_1,U_1} & \sigma_{\mathrm{E}_1,U_0} \\ \sigma_{\mathrm{E}_0,\mathrm{E}_1} & \sigma^1_{\mathrm{E}_0} & \sigma_{\mathrm{E}_0,U_1} & \sigma_{\mathrm{E}_0,U_0} \\ \sigma_{U_1,\mathrm{E}_1} & \sigma_{U_1,\mathrm{E}_0} & \sigma^2_1 & \rho\sigma_1\sigma_0 \\ \sigma_{U_0,\mathrm{E}_1} & \sigma_{U_0,\mathrm{E}_0} & \rho\sigma_1\sigma_0 & \sigma^2_0 \end{bmatrix}$$

be the covariance matrix between $\mathrm{E}[U_1|s]$, $\mathrm{E}[U_0|s]$, U_1 and U_0.

The following proposition shows that the set of ρ that keep A Positive Semi-Definite (PSD) is the same is the set of ρ that keeps matrix B PSD.

Proposition 2. A correlation value ρ is feasible according to matrix A if and only if it is feasible according to matrix B

Proof. Without loss of generality, we restrict attention to the case where signals are independent. This is without loss as we can always residualized signals, and as conditional distribution in the Gaussian case is linear, we do not change the information content of the signals. We start by showing that if matrix B is PSD for some ρ, then matrix A is also PSD for that ρ. We consider the contrapositive case and show that if matrix A is not PSD for ρ then B is not PSD for that ρ. Fix correlation value ρ, and assume that it makes matrix A non PSD. Then, there exists a vector t such that $t'At < 0$. Denote t_{s_i} the value in vector t that corresponds to signal s_i. and by t_1 and t_0 the value in vector t that correspond to U_1 and U_0. Using the fact that signals are uncorrelated, we can write

$$t'At = \sum_i t^2_{s_i} + t_1 \left(\sum \sigma_{s_i,1} t_{s_i}\right) + t_0 \left(\sum \sigma_{s_i,0} t_{s_i}\right) + \sigma^2_1 t^2_1 + \sigma^2_0 t^2_0 + 2\rho\sigma_0\sigma_1 t_1 t_0 < 0 \tag{12}$$

We now show that there must exists a vector k, such that $k'Bk < 0$. Denote k_{E_d}, k_1 and k_0, similar to before, then

$$\begin{aligned} k'Bk = {} & 2k_1(\sigma^2_{\mathrm{E}_1} k_{\mathrm{E}_1} + \sigma_{\mathrm{E}_1,\mathrm{E}_0} k_{\mathrm{E}_0}) \\ & + 2k_0(\sigma^2_{\mathrm{E}_0} k_{\mathrm{E}_0} + \sigma_{\mathrm{E}_1,\mathrm{E}_0} k_{\mathrm{E}_1}) \\ & + (2\sigma_{1,0} k_{\mathrm{E}_1} k_{\mathrm{E}_0} + \sigma_{\mathrm{E}_1} k^2_{\mathrm{E}_1} + \sigma_{\mathrm{E}_0} k^2_{\mathrm{E}_0}) \\ & + \sigma^2_1 k^2_1 + \sigma^2_0 k^2_0 + 2\rho\sigma_1\sigma_0 k_1 k_0 \end{aligned}$$

As we restricted attention to the case where signals are uncorrelated, and the conditional distribution of Gaussian model is linear function of signals, we can rewrite these expressions as

$$\begin{aligned}
k'Bk &= 2k_1(k_{\mathrm{E}_1}\sum_{s_i}\sigma_{s_i,1}^2 + k_{\mathrm{E}_0}\sum_{s_i}\sigma_{s_i,1}\sigma_{s_i,0}) \\
&+ 2k_0(k_{\mathrm{E}_0}\sum_{s_i}\sigma_{s_i,0}^2 + k_{\mathrm{E}_1}\sum_{s_i}\sigma_{s_i,1}\sigma_{s_i,0}) \\
&+ (2k_{\mathrm{E}_1}k_{\mathrm{E}_0}\sum_{s_i}\sigma_{s_i,1}\sigma_{s_i,0}) + k_{\mathrm{E}_1}^2\sum_{s_i}\sigma_{s_i,1}^2 + k_{\mathrm{E}_0}^2\sum_{s_i}\sigma_{s_i,0}^2) \\
&+ \sigma_1^2k_1^2 + \sigma_0^2k_0^2 + 2\rho\sigma_1\sigma_0k_1k_0 \\
&= 2k_1(\sum_{s_i}\sigma_{s_i,1}(\sigma_{s_i,1}k_{\mathrm{E}_1} + \sigma_{s_i,0}k_{\mathrm{E}_0})) \\
&+ 2k_0(\sum_{s_i}\sigma_{s_i,0}(\sigma_{s_i,0}k_{\mathrm{E}_0} + \sigma_{s_i,1}k_{\mathrm{E}_1})) \\
&+ \sum_{s_i}(\sigma_{s_i,1}k_{\mathrm{E}_1} + \sigma_{s_i,0}k_{\mathrm{E}_0})^2 \\
&+ \sigma_1^2k_1^2 + \sigma_0^2k_0^2 + 2\rho\sigma_1\sigma_0k_1k_0
\end{aligned}$$

We now show how to find values of the vector k that makes this expression negative. We set $k_1 = t_1$ and $k_0 = t_0$. We use the additional two values of k to equate the remaining values such that $k'Bk = t'At < 0$. To do so, we notice we have two equation for two parameters

$$\sum_{s_i}(\sigma_{s_i,1}k_{\mathrm{E}_1} + \sigma_{s_i,0}k_{\mathrm{E}_0}))(k_1\sigma_{s_i,1} + k_0\sigma_{s_i,0}) = \sum_{s_i}t_{s_i}(k_1\sigma_{s_i,1} + k_0\sigma_{s_i,0}) \tag{13}$$

and

$$\sum_{s_i}(\sigma_{s_i,1}k_{\mathrm{E}_1} + \sigma_{s_i,0}k_{\mathrm{E}_0})^2 = \sum_{i}t_{s_i}^2 \tag{14}$$

Using the first equation, we can then solve for k_{E_1} in terms of known values and k_{E_0}

$$k_{\mathrm{E}_1} = \frac{\sum_{s_i}t_{s_i}(k_1\sigma_{s_i,1} + k_0\sigma_{s_i,0}) - k_{\mathrm{E}_0}(k_1\sigma_{s_i,1} + k_0\sigma_{s_i,0})}{\sum_{s_i}\sigma_{s_i,1}}$$

plug this back into equation 14, we see that we have continuous function of k_{E_0}. This function goes from from 0 to infinity, the right hand side is a finite and positive expression, then by the Intermediate value theorem there exists a solution, which implies that there exists a vector for which $k'Bk < 0$ and B is not PSD. To show the reverse, we can show following the same steps that if B is not PSD then A is not PSD as well, which concludes the proof. □

E.1 Results of the Bayesian Information Quality Decomposition

Table 14 shows the result of the effect of equating BR^2 as we just discussed. The first row shows the effect with restricting the set of possible ρ according to the discussion we had in section 2.4.3. We can see that equating the information quality as such, would result in increasing the gap by 7 percentage points. This happens as the information quality of Hispanics is better than the information quality of Hispanics. Therefore, the differences in information are pushing the choice gap down. The next row shows the quality of information, restricting the set of possible values of ρ to positive ones.

	Gap	Information Channel	Composition Channel
1) Feasible ρ			
($\rho_{min} = -0.96, \rho_{max} = 0.9$)		-0.073 (-91.41%)	0.152 (191.0%)
2) Feasible ρ, $\rho \geq 0$			
($\rho_{min} = 0, \rho_{max} = 0.9$)		-0.052 (-65.59%)	0.131 (166.0%)

Table 14: Information Decomposition - Average quality

F Another Measure for Information Differences - Equating Information Structure Across Groups

F.1 Decomposition Approach with Information Structure

In the main text we considered two ways to measure the role of information frictions on choice. We now consider an additional one that aims to equate the information structure across groups. Information structure is a tuple $\mathcal{S} = (P(s|\mathcal{R}), S)$ containing a set of conditional density function, that describes the probability of observing signal s, for an individual with return $\mathcal{R}$, and the support of these signals S. Information structures are widely used in economics and captures the mapping between the the state variables and beliefs (Bergemann and Morris (2016),Bergemann and Morris (2019)). In the following exercise we want to understand how the fact that different groups have access to different information structures, affect the choice gap. We therefore consider equating the information structure across the two groups. We then preform similar decomposition exercise as we did in section 2.3.1. In this decomposition exercise we decompose the choice gap to differences in choice that are attributed to differences arising from differences in the information structure and differences in the returns distribution

$$
\begin{aligned}
&\underbrace{P(D=1|\text{Group } b) - P(D=1|\text{Group } a)}_{\text{Total Effect}} = \\
&\underbrace{P(D=1|\text{Group } b) - P(D=1|\text{Group } b \text{ with information structure of Group } a)}_{\text{Information Channel}} \\
+&\underbrace{P(D=1|\text{Group } b \text{ with information structure of Group } a) - P(D=1|\text{Group } a)}_{\text{Composition Channel}} = \\
&\underbrace{\int_{\mathcal{R}\times c} \mathcal{P}(E_{b,b}(s) \geq c|\mathcal{R}, c, b)\pi_b(\mathcal{R}, c)d\mathcal{R}dc - \int_{\mathcal{R}\times c} \mathcal{P}(E_{a,b}(s) \geq c|\mathcal{R}, c, a)\pi_b(\mathcal{R}, c)d\mathcal{R}dc}_{\text{Information Channel}} \\
&+\underbrace{\int_{\mathcal{R}\times c} \mathcal{P}(E_{a,b}(s) \geq c|\mathcal{R}, c, a)\pi_b(\mathcal{R}, c) - \int_{\mathcal{R}\times c} \mathcal{P}(E_{a,a}(s) \geq c|\mathcal{R}, a)\pi_a(\mathcal{R}, c)d\mathcal{R}dc}_{\text{Composition Channel}}
\end{aligned}
$$

where

$$
E_{a,a}(s) = \int_{\mathcal{R}} \mathcal{R}\frac{P(s|\mathcal{R}, a) \times \pi_a(\mathcal{R})}{\int_{\tilde{\mathcal{R}}} P(s|\tilde{\mathcal{R}}, a) \times \pi_a(\tilde{\mathcal{R}})d\tilde{\mathcal{R}}}d\mathcal{R}
$$

is simply the beliefs of group b, when they have access to information of group b and prior of group b,[5] and

$$
E_{a,b}(s) = \int_{\tilde{\mathcal{R}}} \tilde{\mathcal{R}}\frac{\overbrace{P(s|\tilde{\mathcal{R}}, a)}^{\text{Information}} \times \overbrace{\pi_b(\tilde{\mathcal{R}})}^{\text{earnings}}}{\int P(s|\tilde{\mathcal{R}}, a)\pi_b(\tilde{\mathcal{R}})d\tilde{\mathcal{R}}}d\tilde{\mathcal{R}}
$$

is a counterfactual beliefs for group of members b, is they have the information structure of group a, but returns distribution of group b.

[5]Remember that we assume rational expectations, hence the prior is the true distribution of returns

The information channel measures the extent to which the gap in choices would change if both groups had access to the same information structure as group a. Disparities in information structure can arise from various environmental factors affecting the decision-maker. For instance, if members of group b typically have more academically inclined parents than those in group a, they are likely to receive more accurate information about the benefits of college for an individual, thus providing clearer signals about potential earnings post-college. Additionally, if the social networks of group b members are closely connected to a specific industry that requires certain information, this could create differences in individuals' abilities to predict returns. Therefore, the information channel quantifies the extent of the gap in choices that is attributable to individuals in the two groups receiving different signals, despite having equal potential returns.

It's important to note two things. First, the information structure captures not only 'measurement' type signals but also what individuals understand about the data-generating process. Second, in our decomposition exercise, we impose that individuals update their beliefs correctly. They use the new signals and their correct priors to adjust their understanding. In other words, we examine how they would update their beliefs knowing that the distribution of signals they receive comes from a new source.

The following example demonstrates two points. First, how information structure incorporates the underlying data generating process that govern the returns and is not only a function a "measurement" type signals. Second, the example shows that what's important is not equating the access to signals, but equating the meaning that these signals have, captured by the information structure.

Example F.1 (Occupation and Earnings)**.** The informational content of the signals individuals might be more dependent on the structure of the economy itself. For instance, consider the case where the earnings of non-college-goers are zero for both members of groups a and b, and there are two occupations in the economy: lawyers and accountants. Both lawyers and accountants are paid either a high or low wage, $H > 0 > L$, with equal probability. Prior to deciding to go to college, individuals receive an informative signal on their potential returns if they end up being lawyers. Denote these signals as $\tilde{H}_{\text{law}}$ and $\tilde{L}_{\text{law}}$. The distributions of occupations, earnings, and the signal for each group are given below.

Group b

		H	**L**
Lawyer	$\tilde{H}_{law}$	$\frac{6}{20} \times \frac{5}{6}$	$\frac{6}{20} \times \frac{1}{6}$
	$\tilde{L}_{law}$	$\frac{6}{20} \times \frac{1}{6}$	$\frac{6}{20} \times \frac{5}{6}$
Accountant	$\tilde{H}_{law}$	$\frac{4}{20} \times \frac{1}{2}$	$\frac{4}{20} \times \frac{1}{2}$
	$\tilde{L}_{law}$	$\frac{4}{20} \times \frac{1}{2}$	$\frac{4}{20} \times \frac{1}{2}$

Group a

		H	**L**
Lawyer	$\tilde{H}_{law}$	$\frac{4}{20} \times \frac{5}{6}$	$\frac{4}{20} \times \frac{1}{6}$
	$\tilde{L}_{law}$	$\frac{4}{20} \times \frac{1}{6}$	$\frac{4}{20} \times \frac{5}{6}$
Accountant	$\tilde{H}_{law}$	$\frac{6}{20} \times \frac{1}{2}$	$\frac{6}{20} \times \frac{1}{2}$
	$\tilde{L}_{law}$	$\frac{6}{20} \times \frac{1}{2}$	$\frac{6}{20} \times \frac{1}{2}$

Table 15: Demonstration of Information Structure

In this economy, the share of high earners and low earners is $\frac{1}{2}$ for both groups. The share of individuals in both groups with signals $\tilde{H}_{\text{law}}$ and $\tilde{L}_{\text{law}}$ is also $\frac{1}{2}$. Moreover, for both groups, individuals who end up as lawyers and received a high signal have a $\frac{5}{6}$ probability of having high earnings. The only difference between the two groups is the share of individuals who end up being lawyers, versus those ending up being accountants. This difference implies that the signals each individual from each group receives have different information content, generating differences in the distribution of beliefs. For members of group b, the information structure is given by

$$P(\tilde{H}|H) = P(\tilde{H}|\text{Lawyer}, H)P(\text{Lawyer}|H) + P(\tilde{H}|\text{Acc}, H)P(\text{Acc}|H) = \frac{7}{10} \tag{15}$$

$$P(\tilde{H}|L) = P(\tilde{H}|\text{Lawyer}, L)P(\text{Lawyer}|L) + P(\tilde{H}|\text{Acc}, L)P(\text{Acc}|L) = \frac{3}{10} \tag{16}$$

Similarly, for members from group a we have

$$P(\tilde{H}|H) = P(\tilde{H}|\text{Lawyer}, H)P(\text{Lawyer}|H) + P(\tilde{H}|\text{Acc}, H)P(\text{Acc}|H) = \frac{19}{30} \tag{17}$$

$$P(\tilde{H}|L) = P(\tilde{H}|\text{Lawyer}, L)P(\text{Lawyer}|L) + P(\tilde{H}|\text{Acc}, L)P(\text{Acc}|L) = \frac{11}{30} \tag{18}$$

which implies that even when the marginal distribution of the signal and returns is the same, the implied beliefs given the same signal are different

$$m_{R_{\langle b,b\rangle}}(\tilde{H}) = H \times \frac{7}{10} + L \times \frac{3}{10} \tag{19}$$

$$m_{R_{\langle a,a\rangle}}(\tilde{H}) = H \times \frac{19}{30} + L \times \frac{11}{30} \tag{20}$$

Therefore, although the marginal distribution of signals and returns is the same in the economy, the information structure is different, and the same signal would be interpreted differently in both cases. What does it mean to switch the information structure between group a and group b in this environment? In the thought experiment we perform here, we ask what would be the observed behavior if we provided a signal with the same informational content on the returns as the other group. In this sense, our decomposition approach is "reduced form" in spirit, as we do not describe what drives the differences in information. Instead, we explore the ways in which systemic differences in information on earnings are provided to individuals and how they affect the observed gaps in behavior. These differences can arise from various channels, some due to the way the economy is structured, others might be due to differences in individuals, such as the ability to process information or the financial ability to acquire information.

The following example shows that the same component can play a role as both a piece of information and part of the data generating process of the outcomes.

Example F.2 (Knowledge of some structural components)**.** In some cases, individuals may know specific parts of the data-generating process of earnings. For example, assume that the earnings are determined by a function with a known component to the decision-maker, x, and some unknown component ν_d:

$$\alpha^1 = m_1(x, \nu_1) \tag{21}$$

$$\alpha^0 = m_0(x, \nu_0) \tag{22}$$

Here, x could represent known ability, latent cost of effort, or parental connections in the labor market. In this case, the information structure is simply the probability of observing x, given the earnings $P(x|\alpha^1, \alpha^0)$. This assumption is common in economic models where we believe that some variables affecting the outcomes are known to the decision-makers while making choices, and they use them to form beliefs about the outcomes. Therefore, in our thought experiment of switching the information structure between groups, we separate the two roles of x. Specifically, we fix the distribution of x in the population, thereby keeping the distribution of earnings fixed. But we ask what would happen if the agent did not know x, but instead had access to a similar information environment as group a, and how that would change choice patterns.

We now proceed to investigate the second component of decomposition - the composition channel. We can express this channel as:

$$\underbrace{P_{\langle a|b\rangle} - P_{\langle a|a\rangle}}_{\text{Composition Channel}} = \int_{\alpha^1,\alpha^0} P(m_{R_{\langle a,b\rangle}}(s) \geq 0|\alpha^1,\alpha^0,a)\frac{\pi(\alpha^1,\alpha^0|b)}{\pi(\alpha^1,\alpha^0|a)}\pi(\alpha^1,\alpha^0|a) \tag{23}$$

$$- \int_{\alpha^1,\alpha^0} P(m_{R_{\langle a,a\rangle}}(s) \geq 0|\alpha^1,\alpha^0,a)\pi(\alpha^1,\alpha^0|a)d\alpha^1 d\alpha^0 \tag{24}$$

In the composition channel, we maintain the information structure of group a, yet re-weight the population of group a to align with the distribution of group b. This thought experiment explores how the share of college attenders from group a would change if we modified the composition of the group, so that their distribution of earnings would align with that of group b. In this counterfactual, we are not breaking the connection between information and earnings, as we did in example 2.3, but merely shifting the proportion of individuals at certain earnings levels, ensuring that they take the change into account while forming their beliefs. As we alter the distribution of earnings, while keeping the information structure fixed, we also modify the marginal distribution of signals within the population. This means that if, for instance, we increased the proportion of potential students with high α_1, α_0, we are also enlarging the population's share of those receiving signals tied to higher earnings. Consequently, maintaining the information structure fixed means that we are transforming the distribution of signals in the population, but keep it's meaning.

Example F.3 (Knowledge of some structural components-Continued)**.** In this example, the composition channel involves adjusting the share of members in group a with specific earnings levels, to align with those from group b. It's important to note that we are not necessarily equalizing the share of variable x between the two groups. If x represents, for example, ability, and the function $m(.,\nu)$ varies between groups, our hypothetical scenario doesn't balance the share of high and low ability across both groups. If m differs, matching the share of high and low ability could result in significantly different distributions. Since the HGs are not concerned with ability itself but as an indicator of their returns, aligning individual parts across groups doesn't provide insight into how the distribution of outcomes influences choice.

Similar to our discussion in the main text, it's crucial to understand that our analysis offers a partial equilibrium perspective on changing information structure. The information structure in many cases changes endogenously.For example, individuals may exert effort to generate better information in response to the distribution of returns. It also may be that differences in information could arise due to selection and equilibrium effects. For example, if information influences labor market selection patterns, and employers respond to these patterns, our counterfactuals won't address this. Our analysis assumes that the existing information structure is a given and demonstrates further details in the appendix.

F.1.1 Gaussian Scalar Interpretation

In the scalar Gaussian case we can write the decomposition explictly as

$$\begin{aligned} P(D=1|b)-P(D=1|a) = & \underbrace{\int_X \Phi\left(\frac{\mu_{\mathcal{R},b,x}-c_b(x)}{\sqrt{\frac{\sigma^4_{\mathcal{R},b,x}}{\sigma^2_{\mathcal{R},b,x}+\sigma^2_{\epsilon,b,x}}}}\right) dF_b(x) - \int_X \Phi\left(\frac{\mu_{\mathcal{R},b,x}-c_b(x)}{\sqrt{\frac{\sigma^4_{\mathcal{R},b,x}}{\sigma^2_{\mathcal{R},b,x}+\sigma^2_{\epsilon,a,x}}}}\right) dF_b(x)}_{\text{Information Channel}} \\ & + \underbrace{\int_X \Phi\left(\frac{\mu_{\mathcal{R},b,x}-c_b(x)}{\sqrt{\frac{\sigma^4_{\mathcal{R},b,x}}{\sigma^2_{\mathcal{R},b,x}+\sigma^2_{\epsilon,a,x}}}}\right) dF_b(x) - \int_c \Phi\left(\frac{\mu_{\mathcal{R},a,x}-c_a(x)}{\sqrt{\frac{\sigma^4_{\mathcal{R},a,x}}{\sigma^2_{\mathcal{R},a,x}+\sigma^2_{\epsilon,a,x}}}}\right) dF_a(x)}_{\text{Composition Channel}} \end{aligned}$$

Therefore, in the scalar Gaussian case, equating information structure across two groups essentially means equalizing the level of uncertainty surrounding true returns.

Remark. Notice that in our discussion here we fixed the information structure, as signals conditional on returns. We did this, as returns are what agents care about, and for the decision process they are indifferent between two pairs of earnings with the same difference. Therefore from the perspective of the agents, the payoff relevant value for the decision is the difference. Another approach can be to define the information structure on earnings. This would imply a different interpretation of information.

In the following parts we discuss how this decomposition measure can identified under different assumption on the data or the type of fundamentals and information.

F.2 Non-parametric point Identification of the Decomposition Components

Fix two groups $g \in \{a, b\}$. In the subsequent sections, we demonstrate how to identify the quantity

$$P_{\langle a,b \rangle} = \int_{\mathcal{R} \times c} \mathcal{P}(E_{a,b}(s) \geq c | \mathcal{R}, c, a) \pi_b(\mathcal{R}, c) \tag{25}$$

required for decomposition. As outlined in Section C, the primary challenge lies in constructing the distribution of posterior means that incorporates both the counterfactual distribution of signals and returns. This must be achieved despite having access only to the conditional expectations distribution, rather than the complete information structure available to agents. We first establish conditions for point identification, then extend our analysis to more general cases for identifying this quantity. Throughout the analysis we assume that $\pi(c)$ is identified, and implicitly condition on the cost.

F.2.1 Point identification under increasing beliefs function

We start by showing that if we are willing to assume that the information is scalar, and that beliefs are increasing function of that signal, then the quantity in 25 is identified.

Proposition 3. Let $E[\mathcal{R}|s]$ be a strictly increasing function of s, then equation 3 is identified.

Proof. The claim follows trivially from the fact that a strictly monotonic transformation is merely a renaming of the signal but does not alter its information content. Therefore, individuals update beliefs in the same manner, using either the information structure's likelihood functions $P(s|\mathcal{R})$ with support $\mathcal{S}$ or $P(E[\mathcal{R}|s]|\mathcal{R})$ with support given by the posterior means, for any prior. To illustrate this in our continuous density of signals case, we have

$$E_{a,b}(s) = \int \mathcal{R} \frac{P_a(s|\mathcal{R})\pi_b(\mathcal{R})}{\int P_a(s|\mathcal{R})\pi_b(\mathcal{R})d\mathcal{R}} d\mathcal{R} = \int \mathcal{R} \frac{\left|\frac{1}{\frac{\partial E_a}{\partial s}}\right| P_a(E_a|\mathcal{R})\pi_b(\mathcal{R})}{\left|\frac{1}{\frac{\partial E_a}{\partial s}}\right| \int P_a(E_a|\mathcal{R})\pi_b(\mathcal{R})d\mathcal{R}} d\mathcal{R} = E[\mathcal{R}|E_a(s); b]$$

where E_a denotes the beliefs of group a, with their information structure and prior, and $E[\mathcal{R}|E_a(s); b]$ is the belief induced by observing the signal $E_a(s)$ and prior π_b. As demonstrated in section C, for a given correlation between α_1 and α_0, we can identify the joint distribution of $E[\mathcal{R}|s]$ and $\mathcal{R}$ for groups a and b. Therefore, as each signal corresponds to a unique belief, we can calculate the implied counterfactual beliefs distribution directly from the identified distribution of beliefs. Consequently, $P(E_{a,b}(s)|\mathcal{R})$ is identified, and equation 25 is trivially identified. □

Under what conditions can we expect the conditional expectation to be a strictly increasing function of returns? A sufficient condition for this is that the joint distribution of $\mathcal{R}$ and s satisfies the Monotone Likelihood Ratio Property (MLRP). The following corollary formalizes this claim.

Corollary 1. Let $P(\mathcal{R}, s)$ satisfy the strict Monotone Likelihood Ratio Property,

$$\forall s > s', x > x' \quad P(\mathcal{R}|s)P(\mathcal{R}'|s') > P(\mathcal{R}'|s)P(\mathcal{R}|s') \tag{26}$$

then the quantity in equation 3 is identified.

Proof. The corollary follows from the preceding proposition and the fact that MLRP implies First-Order Stochastic Dominance,

$$F_s(\mathcal{R}) \leq F_{s'}(\mathcal{R}) \tag{27}$$

which implies that the conditional expectation is strictly increasing,

$$E[\mathcal{R}|s] = \int_{\mathcal{R}} (1 - F_s(\mathcal{R}))\, d\mathcal{R} > \int_{\mathcal{R}} (1 - F_{s'}(\mathcal{R}))\, d\mathcal{R} = E[\mathcal{R}|s'].$$

Here, F_s denotes the CDF of $\mathcal{R}$, conditional on s. □

F.2.2 Identification under the general Gaussian Model

We first introduce a general Gaussian model with a finite number of signals. Throughout the discussion, we fix the cost c and make the identification argument conditional on c. We assume that α_1 and α_0 are jointly normally distributed. Further, we assume that individuals observe a scalar signal S, and the structural components of earnings α_1, α_0 are drawn from a joint normal distribution.

$$\begin{pmatrix} \boldsymbol{S} \\ \alpha_1 \\ \alpha_0 \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \boldsymbol{\mu}_s \\ \mu_1 \\ \mu_0 \end{pmatrix}, \begin{bmatrix} \Sigma_{\boldsymbol{S}}, \Sigma_{\boldsymbol{S},1}, \Sigma_{\boldsymbol{S},0} \\ \Sigma_{\boldsymbol{S},1}, \sigma_1, \sigma_{1,0} \\ \Sigma_{\boldsymbol{S},0}, \sigma_{1,0}, \sigma_0 \end{bmatrix}\right)$$

Using the properties of the normal distribution, we can write the joint distribution of the signals and the returns, where $\Delta = \alpha_1 - \alpha_0$, as

$$\begin{pmatrix} \boldsymbol{S} \\ \alpha_1 - \alpha_0 \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \boldsymbol{\mu}_s \\ \mu_1 - \mu_0 \end{pmatrix}, \begin{bmatrix} \Sigma_{\boldsymbol{S}} & \Sigma_{\boldsymbol{S},\Delta} \\ \Sigma_{\boldsymbol{S},\Delta}^T & \sigma_1^2 + \sigma_0^2 - 2\sigma_{1,0} \end{bmatrix}\right)$$

Where $\Sigma_{\boldsymbol{S},\Delta} = \Sigma_{\boldsymbol{S},1} - \Sigma_{\boldsymbol{S},0}$. Given a signal realization $\boldsymbol{S}$, the information structure, $\Pr(\boldsymbol{S}|\theta)$, is then given by

$$\Pr(\boldsymbol{S}|\mathcal{R}) = \mathcal{N}\left(\mu_{\boldsymbol{S}} + \Sigma_{S,\mathcal{R}}\sigma_{\mathcal{R}}^{-2}(\mathcal{R} - \mu_{\mathcal{R}}), \Sigma_{\boldsymbol{S}} - \Sigma_{\boldsymbol{S},\mathcal{R}}\sigma_{\mathcal{R}}^{-2}\Sigma_{\boldsymbol{S},\mathcal{R}}^T\right)$$

An individual with signal realization $\boldsymbol{S}$ forms the following posterior mean:

$$E[\Delta|\boldsymbol{S}] = \mu_{\mathcal{R}} + \Sigma_{S,R}^T\Sigma_{\boldsymbol{S}}^{-1}(\boldsymbol{S} - \mu_S)$$

This implies that individuals i with cost c and signal realization $\boldsymbol{S}$ would choose to go to college if

$$D = \mathbb{1}\left[E[\alpha_1 - \alpha_0|\boldsymbol{S}] \geq c\right] = \mathbb{1}\left[\mu_{\mathcal{R}} + \Sigma_{S,R}^T\Sigma_{\boldsymbol{S}}^{-1}(\boldsymbol{S} - \mu_S) \geq c\right]$$

We can calculate the share of students who attend college with cost c. First, we note that the beliefs distribution is given by

$$E[\mathcal{R}|\boldsymbol{S}] \sim \mathcal{N}\left(\mu_{\mathcal{R}}, \Sigma_{S,\mathcal{R}}^T\Sigma_S^{-1}\Sigma_{S,\mathcal{R}}\right)$$

Therefore, the share of individuals who would go to college is given by

$$P(D=1|c) = \Phi\left(\frac{\mu_\Delta - c}{\Sigma_{S,\Delta}^T \Sigma_S^{-1} \Sigma_{S,\Delta}}\right)$$

Now, again, we assume that individuals are divided into two groups $g \in \{a, b\}$. Fixing a copula parameter between α_1 and α_0 for each group, and using results from section C, we know we can identify the joint distribution of returns and beliefs for groups a and b, $P_a(\mathcal{R}, E(s))$ and $P_b(\mathcal{R}, E(s))$. We now show that this is sufficient to identify the quantity in 3 and solve for the decomposition.

Given the information structure of group a, we can derive the counterfactual joint distribution of signals and returns as follows[6]

$$\begin{pmatrix} \boldsymbol{S}_a \\ \mathcal{R}_b \end{pmatrix} \sim N\left(\begin{pmatrix} k_a + m_a \mu_{\boldsymbol{S}_a} \\ \mu_{\mathcal{R}_b} \end{pmatrix}, \begin{bmatrix} m_a \sigma_b^2 {m_a}^T + \Sigma_{\boldsymbol{S}_a} - m_a \Sigma_{\boldsymbol{S}_a,\mathcal{R}_b}^T & m_a \sigma_{\mathcal{R}_b}^2 \\ {m_a}^T \sigma_{\mathcal{R}_b}^2 & \sigma_{\mathcal{R}_b}^2 \end{bmatrix}\right)$$

where $k_a = \mu_{S_a} - \Sigma_{\boldsymbol{S}_a,\mathcal{R}_b} \sigma_{\mathcal{R}_b}^{-2} \mu_{\mathcal{R}_b}$ and $m_a = \Sigma_{\boldsymbol{S}_a,\mathcal{R}_b} \sigma_{\mathcal{R}_b}^{-2}$ and subscript $g \in \{a, b\}$ indicates that the parameters are from the distribution of group g.

We can now derive the counterfactual posterior mean belief, given a signal realization $\boldsymbol{S}$.

$$E_{a,b} = \mu_b + m_a^T \sigma_{\mathcal{R}_b}^2 \left(\Sigma_{\boldsymbol{S}_a,\mathcal{R}_a} \sigma_{\mathcal{R}_a}^{-2} \sigma_{\mathcal{R}_b}^2 \sigma_{\mathcal{R}_a}^{-2} \Sigma_{\boldsymbol{S}_a,\mathcal{R}_a}^T + \Sigma_{\boldsymbol{S}_a} - \Sigma_{\boldsymbol{S}_a,\mathcal{R}_a} \sigma_{\mathcal{R}_a}^{-2} \Sigma_{\boldsymbol{S}_a,\mathcal{R}_a}^T\right)^{-1} \left(\boldsymbol{S}_a - k_a - m_a \mu_{\boldsymbol{S}_a}\right)$$

and the counterfactual belief distribution is given by

$$E_{a,b} \sim N\left(\mu_b, \sigma_{\mathcal{R}_b}^4 m_a^T \left(\left(m_a \sigma_b^2 m_a^T + \Sigma_{\boldsymbol{S}_a} - m_a \Sigma_{\boldsymbol{S}_a,\mathcal{R}_a}^T\right)^{-1}\right)^T m_a\right)$$

Denote by OV_a the identified variance of beliefs for group a

$$OV_a = \Sigma_{S_a,\mathcal{R}_a}^T \Sigma_{S_a}^{-1} \Sigma_{S_a,\mathcal{R}_a}$$

The following proposition assert that we can identify the variance of the counterfactaul beliefs distribution

[6] We slightly abuse notation here setting $\mathcal{R}_b$ to denote that it's distributed as in group b

Proposition 4. Let $\mathcal{R}$ and signal vector $\boldsymbol{S}$ be jointly Gaussian-distributed, conditional on the cost c, for members of both group a and b. Then we we can point identify the counterfactual quantity

$$\int_{\mathcal{R}\times c} \mathcal{P}(E_{a,b}(s) \geq c|\mathcal{R}, c, a)\pi_b(\mathcal{R}|c)p(c)d\mathcal{R}dc$$

Proof. The proof follows from the following derivation:

$$\begin{aligned}
\mathrm{Var}(E_{a,b}) &= \sigma^4_{\mathcal{R}_b} m_a^T \left(\left(\Sigma_{\boldsymbol{S}_a,\mathcal{R}_a} \sigma^{-2}_{\mathcal{R}_a} \sigma^2_{\mathcal{R}_b} \sigma^{-2}_{\mathcal{R}_a} \Sigma^T_{\boldsymbol{S}_a,\mathcal{R}_a} + \Sigma_{\boldsymbol{S}_a} - \Sigma_{\boldsymbol{S}_a,\mathcal{R}_a} \sigma^{-2}_{\mathcal{R}_a} \Sigma^T_{\boldsymbol{S}_a,\mathcal{R}_a} \right)^{-1} \right)^T m_a \\
&= \frac{\sigma^4_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} \Sigma^T_{\boldsymbol{S}_a,\mathcal{R}_a} \left(\Sigma_{\boldsymbol{S}_a,\mathcal{R}_a} \Sigma^T_{\boldsymbol{S}_a,\mathcal{R}_a} \left(\frac{\sigma^2_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} - \frac{1}{\sigma^2_{\mathcal{R}_a}} \right) + \Sigma_{\boldsymbol{S}_a} \right)^{-1} \Bigg)^T \Sigma_{\boldsymbol{S}_a,\mathcal{R}_a} \\
&= \frac{\sigma^4_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} \Sigma^T_{\boldsymbol{S}_a,\mathcal{R}_a} \left(\Sigma^{-1}_{S_a} - \frac{\left(\frac{\sigma^2_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} - \frac{1}{\sigma^2_{\mathcal{R}_a}}\right) \Sigma^{-1}_{S_a} \Sigma_{S_a,R_a} \Sigma^T_{S_a,R_a} \Sigma^{-1}_{S_a}}{1 + \left(\frac{\sigma^2_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} - \frac{1}{\sigma^2_{\mathcal{R}_a}}\right) \Sigma^T_{S_a,R_a} \Sigma^{-1}_{S_a} \Sigma_{S_a,R_a}} \right)^T \Sigma_{\boldsymbol{S}_a,\mathcal{R}_a} \\
&= \frac{\sigma^4_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} \left(OV_a - \frac{\left(\frac{\sigma^2_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} - \frac{1}{\sigma^2_{\mathcal{R}_a}}\right) OV_a^2}{1 + \left(\frac{\sigma^2_{\mathcal{R}_b}}{\sigma^4_{\mathcal{R}_a}} - \frac{1}{\sigma^2_{\mathcal{R}_a}}\right) OV_a} \right) \\
&= \frac{\sigma^4_{\mathcal{R}_b}}{\sigma^2_{\mathcal{R}_b} + \frac{\sigma^2_{\mathcal{R}_a}(\sigma^2_{\mathcal{R}_a} - OV_a)}{OV_a}}
\end{aligned}$$

where in the third row we used the Sherman-Morrison formula and the definition of OV_b. □

Remark. Notice that in the normal case, where both the returns distribution and signals are normally distributed, there is no loss of generality in assuming that high school graduates receive only a scalar noise of the form

$$s = \mathcal{R} + \epsilon$$

where $\epsilon \sim \mathcal{N}(0, \sigma^2_\epsilon)$. Following the same steps as before, we can show that the observed variance of beliefs is given by

$$OV = \frac{\sigma^4_{\mathcal{R}}}{\sigma^2_{\mathcal{R}} + \sigma^2_\epsilon}$$

which implies that the information structure $P(S|\mathcal{R}) = \mathcal{N}(\mathcal{R}, \sigma_\epsilon^2)$ is identified by

$$\sigma_\epsilon^2 = \frac{\sigma_\mathcal{R}^2(\sigma_\mathcal{R}^2 - OV)}{OV}$$

Given the information structure, the counterfactual distribution is simply given by

$$\frac{\sigma_a^4}{\sigma_a^2 - \frac{\sigma_{\mathcal{R},b}^2(\sigma_{\mathcal{R},b}^2 + OV_b)}{OV_b}}$$

which aligns with the counterfactual quantity when agents have a richer signal structure.

F.2.3 Identification of the Information Structure Decomposition with Data on the Full Belief Distribution

In some cases, researchers may hope to elicit information on the probabilities that an agent put on each outcome realization (Manski (2004), Wiswall and Zafar (2015), Zafar (2011), Wiswall and Zafar (2021), Diaz-Serrano and Nilsson (2022)). We now turn to show that this information is sufficient for point identification of our choice gap decomposition, with respect to the information structure.

We assume that individuals from group b have earnings distribution π_b and access to the information structure $(\mathrm{P}(S|b, \mathcal{R}), \mathcal{S})$, and for group a have returns distribution π_a and access to the information structure $(\mathrm{P}(S|a, \mathcal{R}), \mathcal{S})$. Denote by $q_{s,g} \in \Delta(\mathcal{R})$ the posterior beliefs induced by a signal realization $s \in S$ and prior π_g. We let $q_{s,g}(\mathcal{R})$ be the assigned density that this posterior puts on state $\mathcal{R}$. Furthermore, we assume that we observe for each group the joint distribution, $\phi(\mathcal{R}, q_s)$, of returns $\mathcal{R}$ and the posterior beliefs q_s.

We start by noting that within the framework, knowing beliefs allows us to identify a richer notion of costs. Specifically, denote by $B_i = \int_\mathcal{R} \mathcal{R} q_i(\mathbb{R}) d\mathcal{R}$ the measured posterior mean for individuals with beliefs q_i and notice that

$$P(D = 1|x, B) = E[\mathbb{1}[B_i \geq c(x, \nu)]] \tag{28}$$

where ν is additional cost heterogeneity, not included in our identifying discussion in section 2.4. Under some regularity conditions and the assumption $B \perp\!\!\!\perp \nu|X$, we can identify the distribution of $c(x, \nu)$ for each x and B, using variation in B. The identification here relies on B as a "special regressor" needed for identification, as discussed in (Lewbel (2012)). From now on we assume we know the joint distribution of $P(q_i, c_i|x)$, and omit the cost c.

To identify the outcomes distribution, we can use two approaches. The first is simply be able to observe the realization distribution if possible. The other is to use the measured beliefs and simply integrate over beliefs, i.e.

$$\pi_g(\mathcal{R}) = \int_i q_i(\mathcal{R})di \tag{29}$$

Under the assumption that rational expectations are held, this should provide the initial prior[7]

We start by showing the following lemma that shows that for a fixed information structure, there's a mapping from the posterior, given prior π_g' to a posterior under a different prior. Let π_g and $\pi_{g'}$ be two priors with the same support, then for each s, information structure $P(s|\alpha)$ prior π_g and implied posterior q_s, the counterfactual posterior with prior $\pi_{g'}$ is given by $q_{s,g'} = \frac{\frac{q_s(\mathcal{R})}{\pi(\mathcal{R})}\pi_{g'}(\mathcal{R})}{\int_{\mathcal{R}} \frac{q_s(\mathcal{R})}{\pi(\mathcal{R})}\pi_{g'}(\mathcal{R})d\mathcal{R}}$

Proof.

$$\begin{aligned} q_{s,g'}(\mathcal{R}) &= \frac{P(s|\mathcal{R})\pi_{g'}(\mathcal{R})}{\int_{\mathcal{R}} p(s|\mathcal{R})\pi_{g'}(\mathcal{R})d\mathcal{R}} \\ &= \frac{\frac{P(s)q_s(\mathcal{R})}{\pi(\mathcal{R})}\pi_{g'}(\mathcal{R})}{\int_{\mathcal{R}} \frac{P(s)q_s(\mathcal{R})}{\pi(\mathcal{R})}\pi_{g'}(\mathcal{R})d\mathcal{R}} \\ &= \frac{\frac{q_s(\mathcal{R})}{\pi(\mathcal{R})}\pi_{g'}(\mathcal{R})}{\int_{\mathcal{R}} \frac{q_s(\mathcal{R})}{\pi(\mathcal{R})}\pi_{g'}(\mathcal{R})d\mathcal{R}} \end{aligned}$$

□

Lemma F.2.3 demonstrates that the counterfactual posterior can be calculated from the known posterior π_g and the counterfactual distribution $\pi_{g'}$, without requiring explicit knowledge of the information structure. Given the counterfactual posteriors, one can also derive the counterfactual means and thus identify all components of the decomposition. We proceed to establish that all parts of the decomposition are identified.

Recall that for our decomposition we needed to identify the distribution of counterfactual posterior mean, if the returns were drawn according to group b, information according to group a and updated

[7]If one also has access to outcomes data, maintain the common prior assumption, and assume that agents are Bayesian with inaccurate beliefs (Bohren et al. (2023)). We discuss this further in section H

correctly in this new counterfactual world.

$$P_{\langle a,b\rangle} = \int_{\mathcal{R}} \mathcal{P}(E_{a,b}(s) \geq 0|\mathcal{R}, a), \pi_b(\mathcal{R})d\mathcal{R}$$

Proposition 5. Assume we know $\phi_a(q_{s,a}, \boldsymbol{\mathcal{R}})$ and $\phi_b(q_{s,a}, \boldsymbol{\mathcal{R}})$ then the conditional distribution $\mathcal{P}(E_{a,b}(s)|\mathcal{R}, a)$ is identified and so is $P_{\langle a,b\rangle}$ in 25

Proof. The proof follows from Lemma F.2.3. Notice that according to Lemma F.2.3, every two signals that generate the same posterior for group a, also generate the same posteriors in the counterfactual case where $\mathcal{R}$ is distributed according to π_b; therefore, it's enough to know the posterior without requiring the information structure. Further, using Lemma 3, we can identify the distribution of the counterfactual posteriors by calculating the implied distribution of the composition $\frac{\left(\frac{q_s(\mathcal{R})}{\pi(\mathcal{R})}\right)\pi_{g'}(\mathcal{R})}{\int_{\mathcal{R}}\left(\frac{q_s(\mathcal{R})}{\pi(\mathcal{R})}\right)\pi_{g'}(\mathcal{R})d\mathcal{R}}$. Finally, to obtain $P(E_{a,b}|\mathcal{R})$, we only need to map each posterior to its implied mean. As $P(E_{a,b}|\mathcal{R})$ is identified, $P_{\langle a,b\rangle}$ is trivially identified, along with the decomposition components values. $\square$

One implication of Proposition 5 is that in the case where we have binary outcomes $Y \in \{1, 0\}$, and we know the joint distribution of $\phi(E[Y|s], Y)$, the decomposition is point identified using simply the conditional mean beliefs.

Corollary 2. If outcomes are binary $Y \in \{1, 0\}$ and we observe the joint $\phi(E[Y|s], Y)$, then $P_{\langle a,b\rangle}$ in 25 is point identified

Proof. Simply follows from proposition 5 and the fact that in the bianry case $E[Y|s]$ is the posterior distribution. $\square$

The case of binary outcomes is prevalent in many applications within the discrimination literature. For instance, in bail decisions, judges are often modeled as agents attempting to predict the likelihood of reoffense (e.g., Arnold et al. (2018)), Researchers may wish to quantify the extent to which disparities in decisions made for Black and White defendants are driven by the information available to judges or by the underlying distribution of reoffending rates. The above corollary demonstrates that we can decompose this gap and precisely identify the role each component plays. Similar arguments can be extended to other contexts, such as hiring decisions (Bertrand and Mullainathan (2004),Kline et al. (2022)) or treatment allocation in medical settings (Chan et al. (2022)).

F.3 Nonparametric Partial Identification

Researchers often have data on outcomes and posterior mean beliefs, accessible via the identification strategy outlined in Section C or through surveys querying individual beliefs. However, access to this data alone in general does not suffice for the point identification of the counterfactual beliefs distribution. Building on insights from the empirical information robustness literature(Bergemann and Morris (2019, 2013, 2016); Bergemann et al. (2022) Syrgkanis et al. (2017) Gualdani and Sinha (2019) Magnolfi and Roncoroni (2023)) we demonstrate a methodology to identify the counterfactual distribution of beliefs. Our proof in this section relies on Bergemann et al. (2022).

Our objective is to describe the identified set of the first and second parameter of interest. Following the last section, we assume that everything is conditioned on x, z, and subsume x and z for brevity, and assume to know the joint distribution $\tilde{H}(\mathcal{R}, \mathrm{E})$. Before we start we redefine and define some of the notation we would be using in the discussion. We assume that individuals have access to information structure $\mathcal{S}$, with support s and density function $f(s|\mathcal{R})$ and the corresponding CDF $F(s|\mathcal{R})$. We denote by $\mu \in \Delta(\text{supp}(\mathcal{R}))$ the prior distribution. The posterior mean beliefs, given information structure $\mathcal{S}$ and prior μ is given by $E[\mathcal{R}|s; \mathcal{S}, \mu]$. Throughtout most of the discussion we would fix $\mathcal{S}$, and indicate it only when it matters. We further define the conditional distribution of beliefs, that are generated for a given prior and information structure, conditioned on $\mathcal{R}$ as

$$P^{\mu}_{\mathcal{S}}(\mathrm{E}|\mathcal{R}) = \int_{s:E[\mathcal{R}|s;\mathcal{S},\mu]=\mathrm{E}} dF(s|\mathcal{R})$$

Before moving to the main identification argument we show the following two trivial claims.

Claim 1. Let $\mathrm{E}(s, \mu)$ be

$$\mathrm{E}(s, \mu) = \arg\min_{\mathrm{E}} \int_{\mathcal{R}} (\mathcal{R} - \mathrm{E})^2 \frac{\mu(\mathcal{R}) f(s|\mathcal{R})}{\int_{\mathcal{R}} \mu(\mathcal{R}) f(s|\mathcal{R}) d\mathcal{R}} d\mathcal{R} \tag{30}$$

then $\mathrm{E}(s, \mu) = E[\mathcal{R}|s; \mu]$

Proof. The results are simply implied by the first order conditions. □

Claim 2. Fix two prior distributions, $\mu, \mu' \in \Delta(\mathcal{R})$, where μ is absolute continuous with respect to

μ' and let $\mathcal{E}(s,\mu)$ and $\mathcal{E}(s,\mu')$ be

$$\mathcal{E}(s) = \arg\min_{\mathrm{E}} \int_{\mathcal{R}} (\mathcal{R}-\mathrm{E})^2 \frac{\mu(\mathcal{R}) f(s|\mathcal{R})}{\int_{\mathcal{R}} \mu(\mathcal{R}) f(s|\mathcal{R}) d\mathcal{R}} d\mathcal{R} \tag{31}$$

and

$$\mathcal{E}'(s) = \arg\min_{\mathrm{E}} \int_{\mathcal{R}} \left[(\mathcal{R}-\mathrm{E})^2 \frac{\mu(\mathcal{R})}{\mu'(\mathcal{R})}\right] \frac{\mu'(\mathcal{R}) f(s|\mathcal{R})}{\int_{\mathcal{R}} \mu'(\mathcal{R}) f(s|\mathcal{R}) d\mathcal{R}} d\mathcal{R} \tag{32}$$

Let $\Gamma(\mathcal{R},\mathcal{E})$ be the joint distribution of $\mathcal{R}$ and $\mathcal{E}(s)$, where $\Gamma(\mathcal{R},\mathcal{E};\mu) = \mu(\mathcal{R}) \int_{\{s:\mathcal{E}(s)=\mathcal{E}\}} dF(s|\mathcal{R})$, and let $\tilde{\Gamma}(\mathcal{R},\mathcal{E})$ be the joint distribution of $\mathcal{R}$ and $\mathcal{E}'(s)$, where $\tilde{\Gamma}(\mathcal{R},\mathcal{E}) = \mu'(\mathcal{R}) \int_{\{s:\mathcal{E}'(s)=\mathcal{E}\}} dF(s|\mathcal{R})$, then

$$\Gamma(\mathcal{R},\mathrm{E}) = \frac{\mu(\mathcal{R})}{\mu'(\mathcal{R})} \tilde{\Gamma}(\mathcal{R},\mathrm{E}) \tag{33}$$

Furthermore,

$$P^{\mu}_{\mathcal{S}}(\mathrm{E}|\mathcal{R}) = \frac{\tilde{\Gamma}(\mathcal{R},\mathrm{E})}{\mu'(\mathcal{R})} \tag{34}$$

Proof. Notice that for each signal realization $s \in S$ we have

$$\begin{aligned}
\mathcal{E}'(s) &= \arg\min_{\mathrm{E}} \int_{\mathcal{R}} (\mathcal{R}-\mathrm{E})^2 \frac{\mu(\mathcal{R})}{\mu'(\mathcal{R})} \frac{\mu'(\mathcal{R}) d(s|\mathcal{R})}{\int_{\mathcal{R}} \mu'(\mathcal{R}) d(s|\mathcal{R}) d\mathcal{R}} d\mathcal{R} && (35)\\
&= \arg\min_{\mathrm{E}} \int_{\mathcal{R}} (\mathcal{R}-\mathrm{E})^2 \mu(\mathcal{R}) f(s|\mathcal{R}) d\mathcal{R} && (36)\\
&= \mathcal{E}(s) && (37)
\end{aligned}$$

Therefore, $\int_{\{s:\mathcal{E}'(s)=\mathrm{E}\}} dF(s|\mathcal{R}) = \int_{\{s:\mathcal{E}(s)=\mathrm{E}\}} dF(s|\mathcal{R})$, for all E, which implies

$$\begin{aligned}
\phi(\Gamma(\mathcal{R},\mathcal{E})) &= \mu(\mathcal{R}) \int_{\{s:\mathcal{E}(s)=\mathcal{E}\}} dF(s|\mathcal{R}) && (38)\\
&= \mu(\mathcal{R}) \int_{\{s:\mathcal{E}'(s)=\mathcal{E}\}} dF(s|\mathcal{R}) && (39)\\
&= \frac{\mu(\mathcal{R})}{\mu'(\mathcal{R})} \tilde{\Gamma}(\mathcal{R},\mathcal{E}) && (40)
\end{aligned}$$

Finally, notice that $\mathcal{E}'(s) = \mathcal{E}(s) = E[\mathcal{R}|s;\mathcal{S},\mu]$, therefore we have that

$$P^{\mu}_{\mathcal{S}}(\mathrm{E}|\mathcal{R}) = \int_{s:E[\mathcal{R}|s;\mathcal{S},\mu]=\mathrm{E}} dF(s|\mathcal{R}) = \int_{s:\mathcal{E}(s)=\mathrm{E}} dF(s|\mathcal{R}) = \int_{s:\mathcal{E}'(s)=\mathrm{E}} dF(s|\mathcal{R}) = \frac{\tilde{\Gamma}(\mathcal{R},\mathrm{E})}{\mu'(\mathcal{R})} \tag{41}$$

□

We can now proceed to the identification argument. We want to describe the identified set of the information channel. The first component, which is observed share, is clearly identified, we therefore need only to show that the counterfactual share is identified. Fix an observed joint distribution of beliefs and states, induced by an unknown information structure $\mathcal{S}$ and μ', $\phi(\mathcal{R}, E) = \mu'(\mathcal{R})P^{\mu'}_{\mathcal{S}}(E|\mathcal{R})$. We want to characterize the set of possible joint distributions of beliefs and states for the counterfactual case where we change the state distribution to μ, but leave the information structure $\mathcal{S}$ unchanged. Throught the discussion we assume that both priors have common support and that $\forall \mathcal{R}\mu(\mathcal{R}) \gg 0 \iff \mu'(\mathcal{R}) \gg 0$, such that our counterfactual would be well defined.

Denote by $\mathcal{C}(\phi(\mathcal{R},\mathrm{E}),\mu')$ the set of joint distributions, $\tilde{\phi}(\mathcal{R},\mathrm{E})$, that can be induced by the information structure $\mathcal{S}$, which induces $\phi(\mathcal{R},\mathrm{E})$, and the returns distribution μ. i.e.

$$\mathcal{C}(\phi(\mathcal{R},\mathrm{E}),\mu) = \left\{ \tilde{\phi}(\mathcal{R},\mathrm{E}) \in \Delta(\mathrm{supp}(\mathcal{R}), cl(\mathrm{supp}(\mathcal{R}))) \middle| \right.$$
$$\left. \exists \mathcal{S} \text{ s.t } \mu'(\mathcal{R})P^{\mu'}_{\mathcal{S}}(\mathrm{E}|\mathcal{R}) = \phi(\mathcal{R},\mathrm{E}), \mu(\mathcal{R})P^{\mu}_{\mathcal{S}}(\mathrm{E}|\mathcal{R}) = \tilde{\phi}(\mathcal{R},\mathrm{E}) \right\}$$

where $cl(\mathrm{supp}(\mathcal{R}))$ is the support of beliefs. Our objective is to find a tractable characterization of this set. Let $\pi(\mathcal{R},\mathrm{E}_{\mu'},\mathrm{E}_{\mu}) \in \Delta(\mathcal{R}, cl(\Theta), cl(\Theta))$ be a joint distribution that satisfies

$$\int_{\mathrm{E}_{\mu}} \pi(\mathcal{R},\mathrm{E},\mathrm{E}_{\mu}) d\mathrm{E}_{\mu} = \phi(\mathcal{R},\mathrm{E}) \tag{42}$$

$$\forall \mathrm{E}_{\mu'},\mathrm{E}_{\mu} \quad \mathrm{E}_{\mu'} = \arg\min_{\mathrm{E}} \int_{\mathcal{R}} (\mathcal{R}-\mathrm{E})^2 \pi(\mathcal{R},\mathrm{E}_{\mu'},\mathrm{E}_{\mu}) d\mathcal{R} \tag{43}$$

$$\forall \mathrm{E}_{\mu'},\mathrm{E}_{\mu} \quad \mathrm{E}_{\mu} = \arg\min_{\mathrm{E}} \int_{\mathcal{R}} (\mathcal{R}-\mathrm{E})^2 \frac{\mu(\mathcal{R})}{\mu'(\mathcal{R})} \pi(\mathcal{R},\mathrm{E}_{\mu'},\mathrm{E}_{\mu}) d\mathcal{R} \tag{44}$$

and denote the set of implied joint distribution of $\mathcal{R}$ and E_μ as

$$\mathcal{M}(\phi(\mathcal{R},\mathrm{E}),\mu) = \Bigg\{\tilde{\phi}(\mathcal{R},\mathrm{E}) \in \Delta(\text{supp}(\mathcal{R}), cl(\text{supp}(\mathcal{R})))\Bigg| \\ \tilde{\phi}(\mathcal{R},\mathrm{E}) = \int_{\mathrm{E}_{\mu'}} \frac{\mu(\mathcal{R})}{\mu'(\mathcal{R})}\pi(\mathcal{R},\mathrm{E}_{\mu'},\mathrm{E}_\mu)d\mathrm{E}_{\mu'}, \pi \text{ satisfies } (42), (43), (44)\Bigg\}$$

Claim 3. For any observed distribution $\phi(\mathcal{R},\mathrm{E}) \in \Delta(\text{supp}(\mathcal{R}), cl(\text{supp}(\mathcal{R})))$ and $\mu \in \Delta(\text{supp}(\mathcal{R}))$ that is absolute continuous with respect to μ', we have

$$\mathcal{C}(\phi(\mathcal{R},\mathrm{E}),\mu) = \mathcal{M}(\phi(\mathcal{R},\mathrm{E}),\mu) \tag{45}$$

Proof. We start by showing that $\mathcal{M}(\phi(\mathcal{R},\mathrm{E}),\mu) \subseteq \mathcal{C}(\phi(\mathcal{R},\mathrm{E}),\mu)$. Let $\tilde{\phi}(\mathcal{R},\mathrm{E}) \in \mathcal{M}(\phi(\mathcal{R},\mathrm{E}),\mu)$ and let $\pi(\mathcal{R},\mathrm{E}_{\mu'},\mathrm{E}_\mu)$ be the corresponding joint distribution that satisfies (42),(43),(44). Then define the the information structure $\mathcal{S}_{\mathrm{E}_{\mu'},\mathrm{E}_\mu}$ as

$$P(\mathrm{E}_{\mu'},\mathrm{E}_\mu|\mathcal{R}) = \frac{\pi(\mathcal{R},\mathrm{E}_{\mu'},\mathrm{E}_\mu)}{\int \pi(\mathcal{R},\mathrm{E}_{\mu'},\mathrm{E}_\mu)d(\mathrm{E}_{\mu'},\mathrm{E}_{\mu'})} = \frac{\pi(\mathcal{R},\mathrm{E}_{\mu'},\mathrm{E}_\mu)}{\mu'(\mathcal{R})} \tag{46}$$

where the denominator follows from condition (42). Notice that as π satisfies condition (43), claim 1 and claim 2 implies

$$P^{\mu'}_{\mathcal{S}_{\mathrm{E}_{\mu'},\mathrm{E}_\mu}}(\mathrm{E}|\mathcal{R}) = \int_{\mathrm{E}_\mu} P(E,\mathrm{E}_\mu|\mathcal{R})d\mathrm{E}_\mu \tag{47}$$

hence, using constraint (42), we have

$$\mu'(\mathcal{R})P^{\mu'}_{\mathcal{S}_{\mathrm{E}_{\mu'},\mathrm{E}_\mu}}(\mathrm{E}|\mathcal{R}) = \mu'(\mathcal{R})\int_{\mathrm{E}_\mu} P(\mathrm{E},\mathrm{E}_\mu|\mathcal{R})d\mathrm{E}_\mu = \phi(\mathcal{R},\mathrm{E}) \tag{48}$$

Next, notice by constraint (44) and claim 2 we know that $\frac{\int_{\mathrm{E}_{\mu'}} \pi(\mathcal{R},\mathrm{E}_{\mu'},\mathrm{E}_\mu)d\mathrm{E}_{\mu'}}{\mu'(\mathcal{R})} = P^{\mu}_{\mathcal{S}_{\mathrm{E}_{\mu'},\mathrm{E}_\mu}}(\mathrm{E}|\mathcal{R})$, then

$$\tilde{\phi}(\mathcal{R},\mathrm{E}) = \int_{\mathrm{E}_{\mu'}} \frac{\mu(\mathcal{R})}{\mu'(\mathcal{R})}\pi(\mathcal{R},\mathrm{E}_{\mu'},\mathrm{E}_\mu)d\mathrm{E}_{\mu'} = \mu(\mathcal{R})P^{\mu}_{\mathcal{S}_{\mathrm{E}_{\mu'},\mathrm{E}_\mu}}(\mathrm{E}|\mathcal{R}) \tag{49}$$

therefore, we showed that there exist an information structure as needed, which implies $\tilde{\phi}(\mathcal{R},\mathrm{E}) \in$

$\mathcal{C}(\phi(\mathcal{R}, \mathrm{E}), \mu)$

To see the reverse inclusion, $\mathcal{C}(\phi(\mathcal{R}, \mathrm{E}), \mu) \subseteq \mathcal{M}(\phi(\mathcal{R}, \mathrm{E}), \mu)$. Fix $\tilde{\phi}(\mathcal{R}, \mathrm{E}) \in \mathcal{C}(\phi(\mathcal{R}, \mathrm{E}), \mu)$ and let $\mathcal{S}$ be the information structure that satisfies

$$\mu'(\mathcal{R}) P_{\mathcal{S}}^{\mu'}(\mathrm{E}|\mathcal{R}) = \phi(\mathcal{R}, \mathrm{E}) \tag{50}$$

$$\mu(\mathcal{R}) P_{\mathcal{S}}^{\mu}(\mathrm{E}|\mathcal{R}) = \tilde{\phi}(\mathcal{R}, \mathrm{E}) \tag{51}$$

Define the functions $\mathrm{E}_{\mu} : S \to cl(Supp(\mathcal{R})), \mathrm{E}'_{\mu} : S \to cl(Supp(\mathcal{R}))$ as

$$\mathrm{E}_{\mu'}(s) = \arg\min_{\mathrm{E}} \int_{\mathcal{R}} (\mathcal{R} - \mathrm{E})^2 \mu'(\mathcal{R}) f(s|\mathcal{R}) d\mathcal{R} \tag{52}$$

$$\mathrm{E}_{\mu}(s) = \arg\min_{\mathrm{E}} \int_{\mathcal{R}} (\mathcal{R} - \mathrm{E})^2 \frac{\mu(\mathcal{R})}{\mu'(\mathcal{R})} \mu'(\mathcal{R}) f(s|\mathcal{R}) d\mathcal{R} \tag{53}$$

and define the joint probability $\pi(\mathcal{R}, \mathrm{E}_{\mu'}, \mathrm{E}_{\mu})$ as

$$\pi(\mathcal{R}, \mathrm{E}_{\mu'}, \mathrm{E}_{\mu}) = \mu'(\mathcal{R}) \int_{s:\mathrm{E}_{\mu'}(s) = \mathrm{E}_{\mu'}, \mathrm{E}_{\mu}(s) = \mathrm{E}_{\mu}} dF(s|\mathcal{R}) \tag{54}$$

Next, using claim 2 we know that $\mathrm{E}'_{\mu}(s) = E[\mathcal{R}|s; \mathcal{S}, \mu']$ and therefore

$$\int_{\mathrm{E}_{\mu}} \pi(\mathcal{R}, \mathrm{E}, \mathrm{E}_{\mu}) d\mathrm{E}_{\mu} = \mu'(\mathcal{R}) \int_{\mathrm{E}_{\mu}} \pi(\mathrm{E}, \mathrm{E}_{\mu}|\mathcal{R}) d\mathrm{E}_{\mu} = \mu'(\mathcal{R}) \pi(\mathrm{E}|\mathcal{R}) = \mu'(\mathcal{R}) P_{\mathcal{S}}^{\mu'}(\mathrm{E}|\mathcal{R}) = \phi(\mathcal{R}, \mathrm{E}) \tag{55}$$

To see that π satisfies condition (43), we can use the law of iterated expectations

$$\begin{aligned}
\forall \mathrm{E}_{\mu'}, \mathrm{E}_{\mu} \quad & \arg\min_{\mathrm{E}} \int_{\mathcal{R}} (\mathcal{R} - \mathrm{E})^2 \pi(\mathcal{R}, \mathrm{E}_{\mu'}, \mathrm{E}_{\mu}) d\mathcal{R} && (56)\\
& = \arg\min_{\mathrm{E}} \int_{\mathcal{R}} (\mathcal{R} - \mathrm{E})^2 \int_{s} \pi(\mathcal{R}, \mathrm{E}_{\mu'}, \mathrm{E}_{\mu}, s) ds d\mathcal{R} && (57)\\
& = \arg\min_{\mathrm{E}} \int_{s:\mathrm{E}_{\mu'}(s) = \mathrm{E}_{\mu'}, \mathrm{E}_{\mu}(s) = \mathrm{E}_{\mu}} \int_{\mathcal{R}} (\mathcal{R} - \mathrm{E})^2 \pi(\mathcal{R}, \mathrm{E}_{\mu'}, \mathrm{E}_{\mu}, s) && (58)\\
& = \mathrm{E}_{\mu'} && (59)
\end{aligned}$$

where we used the fact that E'_{μ} minimizes the expression by construction. A similar argument shows that (44) also holds. Finally, by claim 2, condition (44), and the way π is constructed, we have that

$$\int_{\mathrm{E}_{\mu'}} \frac{\mu(\mathcal{R})}{\mu'(\mathcal{R})} \pi(\mathcal{R}, \mathrm{E}_{\mu'}, \mathrm{E}_{\mu}) d\mathrm{E}_{\mu'} = \mu(\mathcal{R}) P^{\mu}_{\mathcal{S}}(\mathrm{E}|\mathcal{R}) = \tilde{\phi}(\mathcal{R}, \mathrm{E}) \tag{60}$$

which implies that $\tilde{\phi}(\mathcal{R}, \mathrm{E}) \in \mathcal{M}(\phi(\mathcal{R}, \mathrm{E}), \mu)$ □

To conclude the identification argument, we introduce the following assumption:

Assumption 5. *μ_a is absolutely continuous with respect to μ_b.*

We fix cost c, and denote the set of the possible probabilities

$$P_{\langle a,b\rangle}(c) = \mathcal{P}(E_{a,b} \geq c|\mathcal{R}, c, a)\pi_b(\mathcal{R}|c) \tag{61}$$

as

$$\mathcal{I}(\phi_a(\mathcal{R}, \mathrm{E})) = \left\{ p \in [0,1] \middle| p = \int_{\mathcal{R}} \int_{\mathrm{E} \geq c} \tilde{\phi}(\mathcal{R}, \mathrm{E}) d\mathrm{E} d\mathcal{R} \right. \\ \left. \text{s.t } \tilde{\phi}(\mathcal{R}, \mathrm{E}) \in \mathcal{C}(\phi_a(\mathcal{R}, \mathrm{E}), \mu_b(\mathcal{R})) \right\}$$

The following claim shows an easy characterization of this set

Claim 4. The identified set is given by

$$\mathcal{I}(\phi_a(\mathcal{R}, \mathrm{E})) = \left\{ p \in [0,1] \middle| p = \int_{\mathcal{R}} \int_{\mathrm{E} \geq c} \tilde{\phi}(\mathcal{R}, \mathrm{E}) d\mathrm{E} d\mathcal{R} \right. \\ \left. \text{s.t } = \tilde{\phi}(\mathcal{R}, \mathrm{E}) \in \mathcal{M}(\phi(\mathcal{R}, \mathrm{E}_{\eta_a}), \mu_b(\mathcal{R})) \right\}$$

Proof. Follows from claim 3 and assumption 5. □

Proposition 6. The quantity in 25 is partially identified given the distribution of $\phi(\mathcal{R}, c, \mathrm{E})$

Proof. follows trivially from claim 4. □

Notice that we can further simplify the characterization of the identified set by using the fact that constraint (43) and (44) are satisfied if and only if the first order conditions hold. Therefore, we can rewrite the constraints (43) and (44) as

$$\forall \mathrm{E}_{\mu'}, \mathrm{E}_{\mu} \quad \mathrm{E}_{\mu'} = \int_{\mathcal{R}} (\mathcal{R} - \mathrm{E}) \pi(\mathcal{R}, \mathrm{E}_{\mu'}, \mathrm{E}_{\mu}) d\mathcal{R} \tag{43a}$$

$$\forall \mathrm{E}_{\mu'}, \mathrm{E}_{\mu} \quad \mathrm{E}_{\mu} = \int_{\mathcal{R}} (\mathcal{R} - \mathrm{E}) \frac{\mu(\mathcal{R})}{\mu'(\mathcal{R})} \pi(\mathcal{R}, \mathrm{E}_{\mu'}, \mathrm{E}_{\mu}) d\mathcal{R} \tag{44a}$$

Now, as the constraints (42), (43a) and (44a) are linear, the identified set is convex, and we can define it as an interval bounded between $[\underline{p}, \overline{p}]$, such that

$$\underline{p}, \overline{p} = \min_{\pi}, \max_{\pi} \int_{z} h_a(z) \int_{\mathcal{R}} \int_{\mathrm{E}_a} \int_{\mathrm{E}_b \geq c(z)} \pi(\mathcal{R}, \mathrm{E}_a, \mathrm{E}_b) \frac{\mu_b(\mathcal{R})}{\mu_a(\mathcal{R})} d(\mathcal{R}, \mathrm{E}_a, \mathrm{E}_b, z) \tag{62}$$

s.t

$$\forall \mathrm{E}, \mathcal{R} \int_{\mathrm{E}_b} \pi(\mathcal{R}, \mathrm{E}, \mathrm{E}_b) = \phi(\mathcal{R}, \mathrm{E}) \tag{63}$$

$$\forall \mathrm{E}_a, \mathrm{E}_b \quad \mathrm{E}_a = \int_{\Theta} (\mathcal{R} - \mathrm{E}_a) \pi(\mathcal{R}, \mathrm{E}_a, \mathrm{E}_b) d\mathcal{R} \tag{64}$$

$$\forall \mathrm{E}_a, \mathrm{E}_b \quad \mathrm{E}_b = \int_{\Theta} (\mathcal{R} - \mathrm{E}_b) \frac{\mu_b(\mathcal{R})}{\mu_a(\mathcal{R})} \pi(\mathcal{R}, \mathrm{E}_a, \mathrm{E}_b) d\mathcal{R} \tag{65}$$

G The Effect of Additional Information on Earnings Inequality

In this section, we explore how adding additional information on potential returns would affect the observed wage gaps. Specifically, we examine how the differences

$$\mathrm{E}[Y|Whites] - \mathrm{E}[Y|Hispanics]$$

alter as we introduce more precise information on either U_1 or U_0. As discussed in the text, we focus on additional information that has been residualized from other information individuals may possess. Since the posterior mean of a normal distribution is linear, the covariance matrix between $(s_n, E[Y_1 - Y_0|s, x], U_1, U_0)$ is as follows:

$$\begin{pmatrix} \sigma^2_{s_n} & 0 & \sigma_{s_n,U_1} & \sigma_{s_n,U_0} \\ 0 & \sigma^2_{\mathrm{E}} & \sigma_{\mathrm{E},U_1} & \sigma_{\mathrm{E},U_0} \\ \sigma_{s_n,U_1} & \sigma_{\mathrm{E},U_1} & \sigma^2_1 & \rho\sigma_1\sigma_0 \\ \sigma_{s_n,U_0} & \sigma_{\mathrm{E},U_0} & \rho\sigma_1\sigma_0 & \sigma^2_0 \end{pmatrix}$$

As seen in section 5.2, the most substantial changes in selection patterns occur when we introduce a signal on either U_1 or U_0. Consequently, we confine our focus to these signals.

Assuming information is provided solely on U_1, it follows that $\mathrm{Cov}(s_n, U_0) = 0$. The derivation for the case with information only on U_0 is analogous. Let $\tilde{E}$ and $\tilde{P}(d = 1|x, z)$ represent the counterfactual conditional mean earnings, given the new signal, and the counterfactual propensity score, respectively. Denote by $\sigma_{\tilde{E}} = \sigma^2_{\mathrm{E}} + \sigma^2_{s_n,1}$, the counterfactual variance in beliefs. Then, the new mean earnings are given by

$$\tilde{E}[Y|g] = \int_X [\tilde{E}[Y|x, z, d = 1, g]\tilde{P}(d = 1|x, z, g) + \tilde{E}[Y|x, z, d = 0, g]\tilde{P}(d = 0|x, z, g)]f(x, z)d(x, z)$$

where f is the density of X, Z. We can further simplify $E[Y_1|x, z, D = 1, g]$ and $E[Y_1|x, z, D = 0, g]$

$$E[Y|x, z, D = 1, g] = x\beta^g_1 + \frac{\sigma_{\tilde{E},1}}{\sigma_{\tilde{E}}} \frac{\phi\left(\frac{\mu_x - c(x,z)}{\sigma_{\tilde{\mathrm{E}}}}\right)}{1 - \Phi\left(\frac{\mu_x - c(x,z)}{\sigma_{\tilde{\mathrm{E}}}}\right)}$$

$$E[Y|x, z, D = 0, g] = x\beta^g_0 - \gamma_0 \frac{\phi\left(\frac{\mu_x - c(x,z)}{\sigma_{\tilde{\mathrm{E}}}}\right)}{\Phi\left(\frac{\mu_x - c(x,z)}{\sigma_{\tilde{\mathrm{E}}}}\right)}$$

where we can use γ_0, as our additional signal is not correlated with U_0.

Figure 19 plots the implied mean earnings of Hispanics, given a signal's effect on average earnings for different information quality R^2_1. As seen in the table, the earnings gap between Hispanics and whites stands at \$1750. As the signal quality increases, Hispanic income also increases. At the extreme possible case, we provide only information on U_1 and $R^2_1 = 0.97$, the earnings are \$9572, which are still lower than the measured average earnings of whites. The main takeaway from this figure is that providing even highly accurate information does not substantially mitigate inequality. This implies that policymakers wishing to address inequality by relying on selection[8] need to provide a more complex signal. This would deviate from the normative model and push into college those individuals who would benefit most, and push out those with negative returns. Generally, this would require leaving the Gaussian model which we are using here, and constructing a more complicated signal.

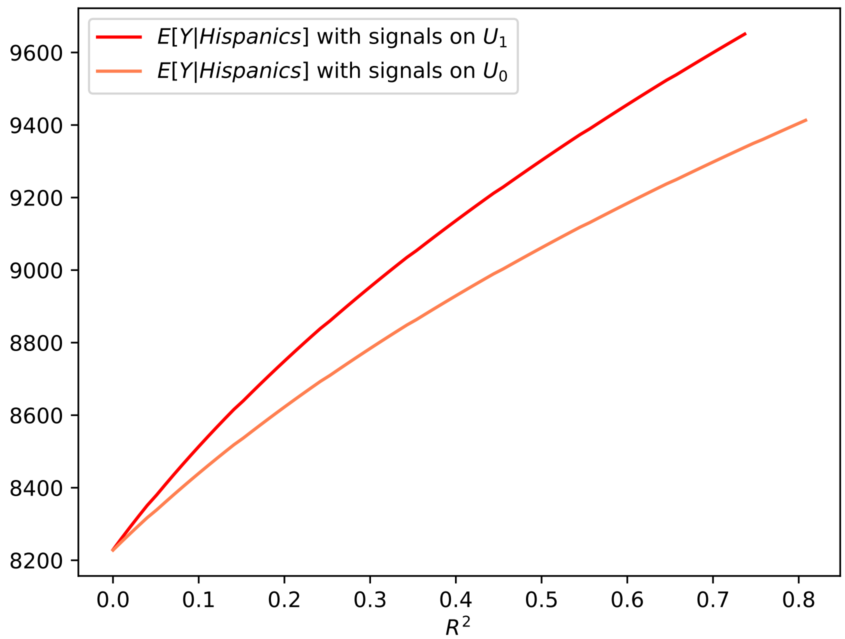


Figure 19: Cost, information and Beliefs interaction

Note: This figure plots the effect on the observed mean earnings of Hispanics when an additional informative signal with information quality R_d^2 is introduced, informing on either U_1 or U_0. The red line depicts the change in mean earnings 12 to 15 years after high school graduation, following the introduction of additional information, with quality R^2, on earnings for those who choose to attend college the first year after high school graduation. The orange line conducts the same exercise showing the changes in mean earnings, due to introduction of additional information with information quality R^2 on earnings for individuals who do not attend college.

[8] Under the assumption that large earnings are invariant to significant changes in selection patterns

H Discussion on Rational Expectations

Our analysis in the main text rests on the assumption that individuals adhere to Bayesian principles and have rational expectations. We incorporate this into our model and identification strategy by assuming that (1) individuals interpret signals accurately using the correct likelihood function, and (2) their prior distribution on returns is accurate. Maintaining the Bayesian perspective, our model of belief formation could be violated in two ways[9]. Agents might employ incorrect likelihoods or hold erroneous priors, or exhibit both inaccuracies. We recognize that generally, any deviation from rational expectations can be viewed as model mis-specification. Specifically, our model posits that agents' beliefs about $\mathcal{R}$, or an increasing function thereof, are formed correctly. However, if agents are Bayesian but derive an incorrect posterior, this too is indicative of a mis-specified utility function[10]. To illustrate, denote $\tilde{\mathrm{E}}(s)$ as the subjective belief and $\tilde{q}(\mathcal{R}|s)$ as the subjective posterior given signal s, and let $q(\mathcal{R}|s)$ represent the accurate posterior conditional on signal s. We then have[11]

$$\tilde{E}(s) = \int_{\mathcal{R}} \mathcal{R}\tilde{q}(\mathcal{R}|s)d\mathcal{R} = \int_{\mathcal{R}} \mathcal{R}\frac{\tilde{q}(\mathcal{R}|s)}{q(\mathcal{R}|s)}q(\mathcal{R}|s)d\mathcal{R} = \mathrm{E}\left[\mathcal{R}\frac{\tilde{q}(\mathcal{R}|s)}{q(\mathcal{R}|s)}|s\right]$$

Thus, a violation of the rational expectations assumption essentially represents a re-weighting of returns, analogous to a misclassified utility function. It's important to note that as long as this reweighting implies an increasing relation with the true posterior mean, i.e. we can write $E[\mathcal{R}|s]$ as some increasing function of $\tilde{E}(s)$, then this is not an issue for identification, as we discussed in section 2.

Disentangling mis-specified utility, beliefs, and priors presents a significant challenge, obscuring the influence of information, as captured by signals, on decision-making. We thus focus on a specific violation of rational expectations where agents hold an incorrect prior but interpret signals accurately. This concept, referred to as inaccurate beliefs[12]. As discussed in C, we use the accurate

[9] We presuppose a population with a shared prior and access to the same information structure. The notion of a common prior is restrictive, which we acknowledge but do not explore alternatives to this here.

[10] This concept aligns with discussions in Bohren et al. (2023) regarding identification issues, and with the concept of omitted payoff bias in Kleinberg et al. (2018).

[11] We assume that individuals know correct support, such that their posterior might update incorrectly, but it has the same support as the correct posterior

[12] Inaccurate beliefs, as discussed in Bohren et al. (2023), particularly in the context of discrimination, reflect how agents may use biased priors that lead to erroneous belief updating.

beliefs assumption identify the distribution of beliefs. With a more comprehensive data set encompassing both outcomes and surveyed beliefs, researchers can point identify systematic biases in belief formation and presence of common priors.[13] For clarity, we postulate that researchers have access to the joint distribution $P(\tilde{E}, \mathcal{R})$, where $\tilde{E}$ denote a subjective belief not necessarily derived from rational expectations[14]. Additionally, we assume that researchers can estimate or access a common the agents mis-specified prior. As discussed in section F.2.3, one might deduce the subjective common prior either by eliciting and averaging the full belief distributions or through a structural method that assumes the subjective prior is some function of the underlying observed distribution of outcomes.

Two components are essential for the decomposition exercise. The first concerns how a mis-specified prior might change if we alter the underlying distribution of returns. One of the benefits of rational expectations arises from its linkage of true outcomes to beliefs. Should this link be disrupted, the researcher must posit how these elements interact. We proceed under the assumption that mis-specified priors remain constant with changes in the distribution of $\mathcal{R}$. Other assumptions by researchers could be made, and the framework of analysis would remain unchanged. The second aspect to consider is the value of information in the presence of biased beliefs. Here, we assume that the value of information is the R^2 that could would be implied if an agent had the same information structure and used the correct prior.

We start by looking at the way we measure information issues, as we talked about in section F. Here, we can use the ideas from that section but modify the IC constraints. This rule makes sure the information setup meets the inaccurate beliefs agents have :

$$\begin{aligned}
&\forall \tilde{E}, \mathrm{E}_{cf} \quad \int_{\mathcal{R}} \pi(\mathcal{R}, \tilde{E}, \mathrm{E}_{cf}) \frac{\pi_{IP,a}(\mathcal{R})}{\pi(\mathcal{R})} (\mathcal{R} - \tilde{E}) d\mathcal{R} = 0 \\
&\forall \tilde{E}, \mathrm{E}_{cf} \quad \int_{\mathcal{R}} \pi(\mathcal{R}, \tilde{E}, \mathrm{E}_{cf}) \frac{\pi_{IP,b}(\mathcal{R})}{\pi(\mathcal{R})} (\mathcal{R} - \tilde{E}) d\mathcal{R} = 0 \\
&\forall \mathcal{R}, \tilde{E} \quad \int_{\mathrm{E}_{cf}} \pi(\mathcal{R}, \tilde{E}, \mathrm{E}_{cf}) d\mathrm{E}_{cf} = Pr_a(\mathcal{R}, \mathrm{E})
\end{aligned}$$

[13] Vatter (2022) examines an alternative method for identifying posterior distributions, using known changes in available signals. Specifically, it utilizes changes over time in quality metrics and demonstrates that adjustments to the threshold can be used to identify the prior distribution, before observing the signal, under linear preferences assumption.

[14] The following arguments are straightforward if researchers possess information on posterior beliefs, as elaborated in F.2.3.

and the objective is given by

$$max \text{ or } min \int_{\mathrm{E}_{cf}>c} \pi(\mathcal{R}, \tilde{E}, \mathrm{E}_{cf}) \frac{\pi_b(\mathcal{R})}{\pi_a(\mathcal{R})} d\mathcal{R} d\tilde{E} d\mathrm{E}_{cf}$$

Here, $\pi_{IP,g}$ is the wrong belief of group g. The first rule makes sure we use the info from group a, considering their wrong belief. The second rule is for group b to update their info with the same wrong belief. The third equation makes sure our data matches to the joint distribution, just like before. The objective just integrate over the new beliefs that are higher than the cost.

for the information quality decomposition in section 2.3.1, we maintain the Gaussian model assumption[15], which implies we assume that beliefs are Gaussian, and inaccurate prior is Gaussian as well. We then extract the true R^2 that is implied by the information, then to equate the R^2 we assume that in the coutnrefactual world agents have access to a scalar signal with the implied R^2. Specifically, we follow these steps:

1. Re-weight the marginal distribution of $\mathcal{R}$ to match the incorrect prior, and get the joint distribution of belief and returns from the agents perspective.

2. Use the methods from F.2.2 to pin down what would be the variance of the beliefs distribution if individuals had correct prior.

3. Calculate the implied R^2 to assess information quality using the variance of beliefs from 2, with the true variance of the returns, $\sigma^2_{\mathcal{R}}$.

4. Calculate the variance of beliefs of beliefs if agents had the correct prior in the counterfactual world - $R^2\sigma^2_{CF}$. Notice that this is enough to know the entire joint distribution of beliefs and returns.

5. Calculate the implied distribution of beliefs using the incorrect prior and results from F.2.2.

6. Re-weight the marginals of the incorrect prior to match those the counterfactual distribution.

This approach equate the information quality, but maintain the wrong priors.

[15] Without the Gaussian assumption, similar approach to the one above can be taken, considering the set of possible signals that induce R^2 at a given level

I Simple Model of Wage Differences with Equal Correlation value

Individuals from two distinct groups—white and Hispanic—are characterized by a tuple (t_1, t_0), representing their attributes or 'types' in situations with and without a college education. These types might include factors such as ability, personality traits, and other characteristics crucial for the labor market. The model posits that for both groups, these types follow a bivariate normal distribution:

$$(t_1, t_0)|\text{white} \sim (t_1, t_0)|\text{Hispanics} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix} \middle| \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right)$$

Here, ρ represents the correlation coefficient, suggesting an equal correlation in the abilities and traits distribution across both groups. Despite this similarity in the characteristics distribution, members of different groups are treated differently in the labor market. Specifically, Wages are determined by the following linear function for each group g:

$$U_1^g = \gamma_1^g + \beta_1^g t_1, \quad U_0^g = \gamma_0^g + \beta_0^g t_0$$

In this equation, U_1^g and U_0^g represent the wages for college-educated and non-college-educated individuals, respectively, within group g. The coefficients γ_1^g, γ_0^g, β_1^g and β_0^g determine how individuals types are translate into wage levels. In this model indicates that wage disparities arise not from pre-labor market differences between the groups but from how similar types are treated differently in the labor market.

Remark. In this model, linear functions are used to maintain the normality used in empirical exercise. However, this approach can be generalized. One could consider a more flexible specification, where types are jointly distributed uniformly, and the copula is the same across the two groups. In that case the functions can be nonrestrictive in the way types are mapped to income.