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A Probabilistic Framework for Robust and Accurate Matching of Point Clouds

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Abstract. We present a probabilistic framework for matching of point clouds. Variants of the ICP algorithm typically pair points to points or points to lines. Instead, we pair data points to probability functions that are thought of having generated the data points. Then an energy function is derived from a maximum likelihood formulation. Each such distribution is a mixture of a bivariate Normal Distribution to capture the local structure of points and an explicit outlier term to achieve robustness. We apply our approach to the SLAM problem in robotics using a 2D laser range scanner.

1 Introduction

Matching point clouds is an important problem in computer vision and robotics. For example 3D range scans taken from different positions have to be integrated to build 3D models. In this case a six-vector of parameters has to be estimated. The case of 2D point clouds with three parameters to be estimated (a 2D-translation and a rotation) is significant for the simultaneous localization and mapping (SLAM) problem in robotics when using a 2D laser range scanner as sensor. Such a device measures range values of a 2D slice of the environment and is a common input sensor for mobile platforms. Scenarios in large buildings, where long cycles have to be closed, require both accurate scan matching results and good estimates of uncertainty to distribute accumulated errors accordingly. At the same time the scan matcher should be robust, because real environments tend to be non-static: People are moving, doors are opening and closing and so on. Such events give raise to outliers and the better the scan matcher tolerates this the better it can be used for mapping and localization.

This paper focuses on 2D point clouds with an Euclidian 2D transformation to be estimated. Nevertheless, we claim that our view of the problem is also relevant for other cases, for example for the full 3D case. The emphasis is on the probabilistic framework that is used to derive our scan matching algorithm. In previous work we have shown experimentally the capability of our algorithm to build accurate maps in realtime using range scan matching [2]. Here we give both a theoretical justification and improvements.

2 Previous Work

The standard approach to the rigid registration problem is the *Iterated Closest Point (ICP)* algorithm or variants thereof. The name ICP was introduced in the seminal paper by Besl and McKay [1], but similar ideas were developed independently at the same time by Zhang [21] and Chen and Medioni [5]. We will now repeat this algorithm briefly and thereby introduce the notation utilized in the rest of the paper. Although the role of the two point clouds to be matched is exchangeable we speak of data points which are to be registered with a model or model points. Let $\mathbf{x}_i$ be a data point and assume that an initial estimate of the parameters is given. Then map each $\mathbf{x}_i$ according to these parameters into the coordinate frame of the model. Without explicitly noting, the $\mathbf{x_i}$ are always thought of as being mapped by the current parameter estimation in the rest of the paper, that is $\mathbf{x}_i$ is in effect a function of the transformation parameters. The ICP algorithm now iterates two subsequent steps:

1. Correspondence: For each $\mathbf{x}_i$, find the closest point $\mathbf{m_i}$ of the model.
2. Estimate new parameters, such that the sum of squared distances between each $\mathbf{x}_i$ and $\mathbf{m_i}$ pair is minimized. Update the $\mathbf{x_i}$ according to the new parameters.

There is a closed form solution for the second step. Several researchers noted that the convergence properties of this point-to-point approach is poor and that point-to-plane correspondences (as used by Chen [5]) perform much better (e.g. [17]). In the field of mobile robotics Lu and Milios estimated normals from the scan [13] and this way incorporated kind of a point-to-line metric. Robust methods typically consist of leaving out points that have too large residuals.

Fitzgibbon [9] proposed to replace the closed-form solution by a non-linear energy function and to use the Levenberg-Marquard (LM) algorithm to minimize it. This allowed him to incorporate a robust kernel, as it is used in M-Estimators [16, 19]. The immediate problem here is the calculation of first derivatives that are needed by the LM algorithm. His solution is to calculate *distances* to closest points on a regular grid (instead of the closest points themselves). This way, the spatial derivatives can be calculated numerically at each grid point and interpolated in between. Interestingly, a similar approach using even an octree (but without using robust kernels) was already proposed 1992 [4]. Our solution is similar in that we use a regular spatial grid. But the information at each grid point is much more sophisticated in that it contains also a model of the local environment going beyond simple interpolation. For optimization we use Newton's algorithm instead of LM. This algorithm has better convergence properties at the cost of requiring "real" second derivatives (compared to the Gauss-Newton approximation of the Hessian used in LM). One benefit of our method is that these can be calculated analytically.

3 A Probabilistic Approach

This section presents a probabilistic interpretation of the ICP algorithm, constituting the base for the design of a robust energy function that captures the

local structure of points. As mentioned earlier, the correspondence step pairs each point $\mathbf{x_i}$ to a model point $\mathbf{m_i}$. Assuming that the location of points which correspond to $\mathbf{m}_i$ were generated by a normal distributed random process with parameters $\mathbf{m_i}$ and σ_i, the likelihood of having measured $\mathbf{x_i}$ is:

$$p(\mathbf{x_i}) \sim \exp -\frac{d_i^2}{2\sigma_i^2}, \tag{1}$$

with $d_i = |\mathbf{x_i} - \mathbf{m_i}|$. So the Normal Distribution $N(\mathbf{m_i}, \sigma_i)$ can be interpreted as a *generative process* for $\mathbf{x_i}$: It is assumed that the location of $\mathbf{x}_i$ has been generated by drawing from this distribution.

Now the problem of estimating the transformation parameters can be formulated as a Maximum Likelihood problem: The parameters are optimal if the resulting transformed data points $\mathbf{x}_i$ maximize the following likelihood function:

$$\Psi = \prod_i \exp -\frac{d_i^2}{2\sigma_i^2}. \tag{2}$$

Equivalently the negative log-likelihood of Ψ can be minimized:

$$-\log \Psi = \sum_i \frac{d_i^2}{2\sigma_i^2}. \tag{3}$$

If the variances are equal for each i this is the energy function that is minimized by ICP. In a similar manner a point-to-line measure can be formulated. Then d_i denotes the distance to the line. Solely the domain has to be limited to a finite range. Otherwise the integral over the probability function is unlimited and cannot be normalized.

Fig. 1(a) and (b) show the underlying probability density functions (*pdf*s). Both can be generalized by a bivariate Normal Distribution:

$$p(x) \sim \exp -\frac{1}{2}(\mathbf{x_i} - \mathbf{m})^t \mathbf{C}^{-1} (\mathbf{x_i} - \mathbf{m}), \tag{4}$$

where $\mathbf{C}$ is a symmetric 2×2 matrix. If $\mathbf{C}^{-1}$ has two equal eigenvalues, it becomes the Point-to-Point measure. If one eigenvalue is near zero, the pdf becomes the Point-To-Line measure. Figure 1(d) illustrates such a distribution and how it can be understood as a product of two univariate Normal Distributions.

Our view is now as follows: In the correspondence step each data point is assigned a generative process in the form of a pdf that is considered to have generated its coordinates. This probability function is not necessarily restricted to be a Normal Distribution, any pdf that captures the structure of points locally well is a candidate pdf.

Now we deal with how to choose a good probability density function: The pdf should be able to approximate common structures (like lines) well and should at the same time be robust against outliers. This paper proposes to use a mixture of a bivariate Normal Distribution and an uniform distribution. That mixture reads:

$$p(\mathbf{x}) = \xi_1 \exp \frac{1}{2}(\mathbf{x} - \mathbf{m})^t \mathbf{C}^{-1} (\mathbf{x} - \mathbf{m}) + \xi_2 \, p_{\text{outlier}}, \tag{5}$$

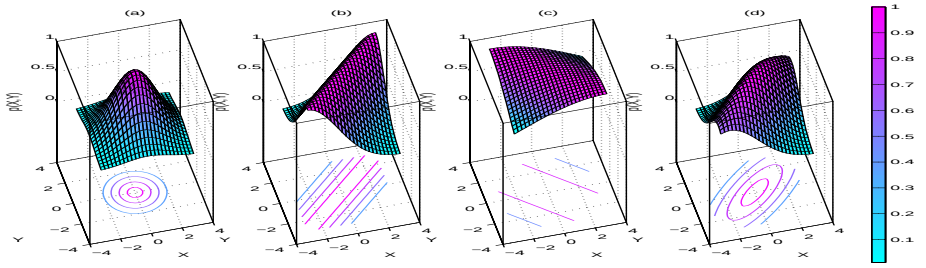

Fig. 1. Some probability density functions which are interpreted as generative processes for the points of the point cloud. **(a)** Point-To-Point **(b)** Point-To-Line **(c)** Point-To-Line, orthogonal to the line of (b) with larger variance **(d)** Product of (b) and (c): A bivariate Normal Distribution that is determined by a point and a covariance matrix.

where p_{outlier} is the expected ratio of outliers.

The constants ξ_1 and ξ_2 can be determined by requiring that the probability mass of p must equal one in a finite region (for example one by one meter). As shown above, the use of a bivariate Normal Distribution allows the modelling of points, lines and anything in-between with inclusion of expected variances. The influence of data points is therefore weighted in a sound way. On the other hand, the log-likelihood of this mixture probability fulfills the requirements for robust functions used in M-estimators: It grows subquadratically and the influence of outliers vanishes at some point. Figure 2 illustrate these claims. Together, this mixture leads to an accurate and robust energy function.

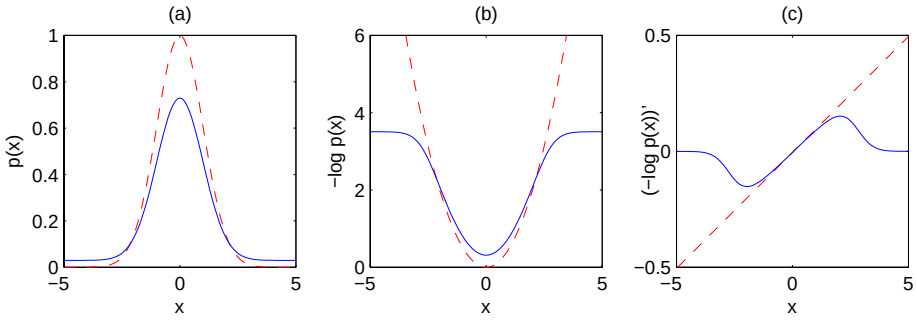

Fig. 2. Comparison between Gaussian generative process (red dashed) and a mixture process with outlier model (blue solid). Likelihood **(a)**, negative log-likelihood **(b)** and influence function **(c)** which is the derivative of (b). For parameter estimation the negative log-likelihood of the generative process is used to build the function that is to be minimized. The influence function characterizes the bias that a particular measurement has on the solution [3]. For the Gaussian the influence of outliers grows without bounds.

4 Registration

To perform the registration, at first generative processes are determined from the model's points in a preprocessing step. For this purpose the 2D plane is subdivided into a regular grid. Now each grid point gets a pdf assigned (the mixture, see eq. 5) that locally represents the structure of the model point cloud around the grid point. For each grid point the parameters of a bivariate Normal Distribution are determined by taking into account all the points of the model cloud that are in a certain range around it. These parameters are simply determined by the mean and the covariance matrix of all these points. The expected outlier ratio is set to a constant (we use 0.3). If a density function around a grid point should not be approximated well, it will become rather uniform (large variances in both principal directions). If it can be approximated well it will provide strong constraints by small variance in at least one principal direction. At the same time it will still be robust through the outlier term. Figure 3 shows some examples.

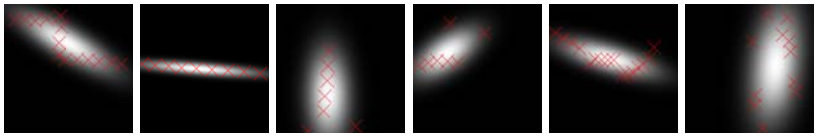

Fig. 3. Bivariate Normal Distributions calculated around several example grid points in real laser range scans. The cells shown here have a dimension of one by one meter. One such distribution is determined by the covariance matrix and the mean of the contained points.

Now the preprocessing is finished and the iterative registration can begin. Establishing the correspondence between a point of the data point cloud and a pdf is now a simple lookup in the regular grid (which is possible in constant time in contrast to finding a closest point). As a point of the data point cloud does typically not fall onto a grid point, the pdfs of the four closest grid points are assigned to it. This is handled by adding up all the four respective log-likelihood terms. These are weighted bi-linearly, which assures a continuous log-likelihood function. Summing up all the data point's log-likelihood terms results in an energy function which is to be optimized with respect to the transformation's parameters.

5 A Computational Advantageous Approximation for Optimization

The summands of the energy function to be optimized consist of terms that have the form $\log(c_1 \exp -\frac{1}{2}(\mathbf{x}-\mathbf{m})^t\mathbf{C}(\mathbf{x}-\mathbf{m}) + c_2)$. These have no simple first and second derivatives. This section presents an approximation that allows a cheap analytical computation of gradient and Hessian (needed by Newton's algorithm). A look at fig. 2(b) suggest, that a robust log-likelihood function

of the form $p(x) = -\log(c_1 e^{-\frac{x^2}{\sigma^2}} + c_2)$ could be approximated by a Gaussian: $\widetilde{p}(x) = d_1 e^{-\frac{d_2 x^2}{\sigma^2}} + d_3$. Parameters d_i are fitted by requiring that $\widetilde{p}(x)$ should behave like $p(x)$ for $x = 0$ and $x \to \infty$, and additionally $p(\sigma) = \widetilde{p}(\sigma)$ (in the bivariate case the function's values are required to be equal at the one sigma contour). The derivatives of this approximation do now have an extremely simple form and can be calculated cheaply. Main computational effort is the evaluation of only one exponential function per data point to calculate both gradient and Hessian.

6 Some Experimental Results

First we present some examples. Fig. 4 shows the log-likelihood functions generated by several typical laser range scans with bright values meaning high probabilities, approximated by the method of the last section. Here, black values do not stand for zero probability but for the logarithm of the expected outlier probability! The typical scan match time (including calculation of the log-likelihood functions' parameters) is under 10 ms if the initial guess by odometry is taken into account. The initial error is then only around a few centimeters and some degrees in rotation. Newton's algorithm converges in the majority of cases in two to five iterations. The distance between grid points is 50 cm and an environment of one by one meter is used to calculate means and covariance matrices.

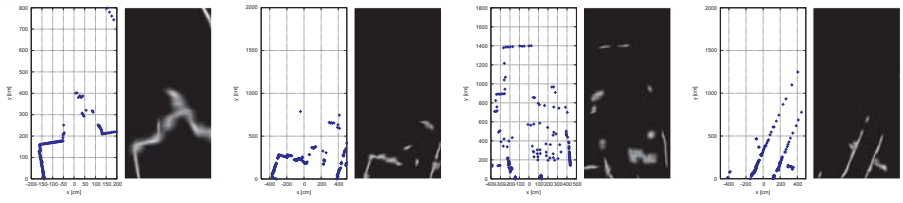

Fig. 4. Some example log-likelihood functions around grid points.

We use the scan matcher as the basis for building maps using a mobile robot equipped with a laser scanner as data acquisition platform. Our approach here belongs to a family of techniques where the environment is represented by a graph of spatial relations obtained by scan matching [14, 11, 10]. A spatial relation consists of the parameters estimated by our technique and a measure of uncertainty. This measure is provided by the Hessian of the energy function at the optimum and is used to distribute errors accordingly through the graph. Details and more experimental results can be found in [2]. Figure 5 gives an impression of the accuracy of our method. The data set there consist of 600 scans taken in a large (around 50 by 60 meters) environment with two long loops to be closed.

7 Conclusion

This paper presented a probabilistic framework for the registration of point clouds. The main contributions were:

1. The concept of explicitly pairing points to probability distributions.
2. Using a mixture of a bivariate Normal Distribution and an outlier term to model local structure of points.

We further proposed a computational advantageous approximation that allows simple calculation of gradient and Hessian. In our approach, each "magic" number has a clear defined meaning thanks to the probabilistic framework.

We applied our technique to the SLAM problem in robotics with excellent experimental results on various data sets. Some of the data sets consisted of several tens of thousand scans and we take the succesful processing of these sets as an experimental proof for the claimed robustness.

Future work will focus on techniques for robust estimation of the probability functions in the regular grid and on higher order models to derive probability functions from. Another challenge is the design of mixtures with components that adapt over time like it has been done in computer vision applications for tracking [12], perhaps also integrating the simultaneous tracking of moving objects.

Acknowledments

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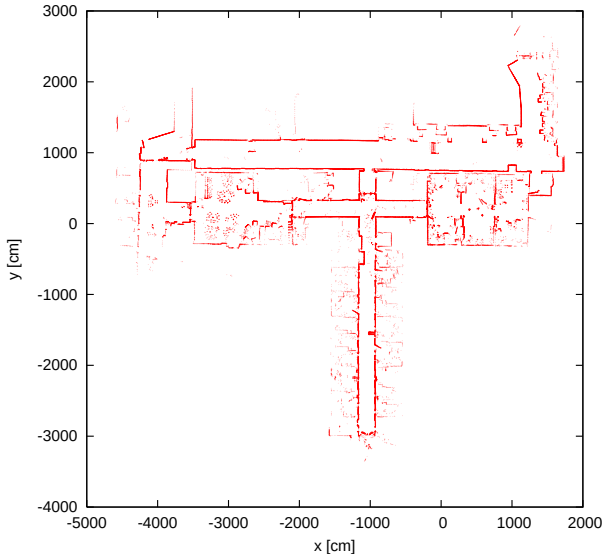

Fig. 5. A map built using our registration technique with data acquired from a 2D laser range scanner, visually demonstrating the accuracy of our results. Range scan data courtesy of T. Duckett, University of Örebro.

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