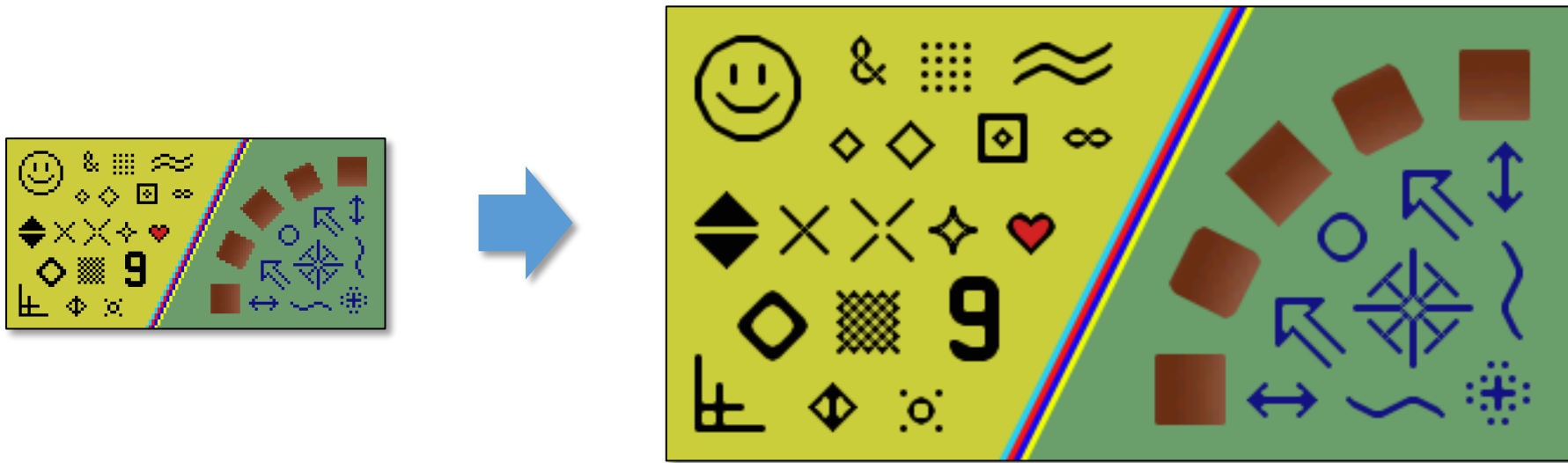


# Image Pyramids

## Image Resampling & Interpolation



# Overview

- Image downsampling.
- Aliasing.
- Gaussian image pyramid.
- Laplacian image pyramid.

# Slide Credits

Most of these slides are adapted from:

- Prof. Ioannis Gkioulekas, Carnegie Mellon University, (16-385 Computer Vision, Spring 2019).
- Prof. Noah Snavely, Cornell University(CS5670 Introduction to Computer Vision)

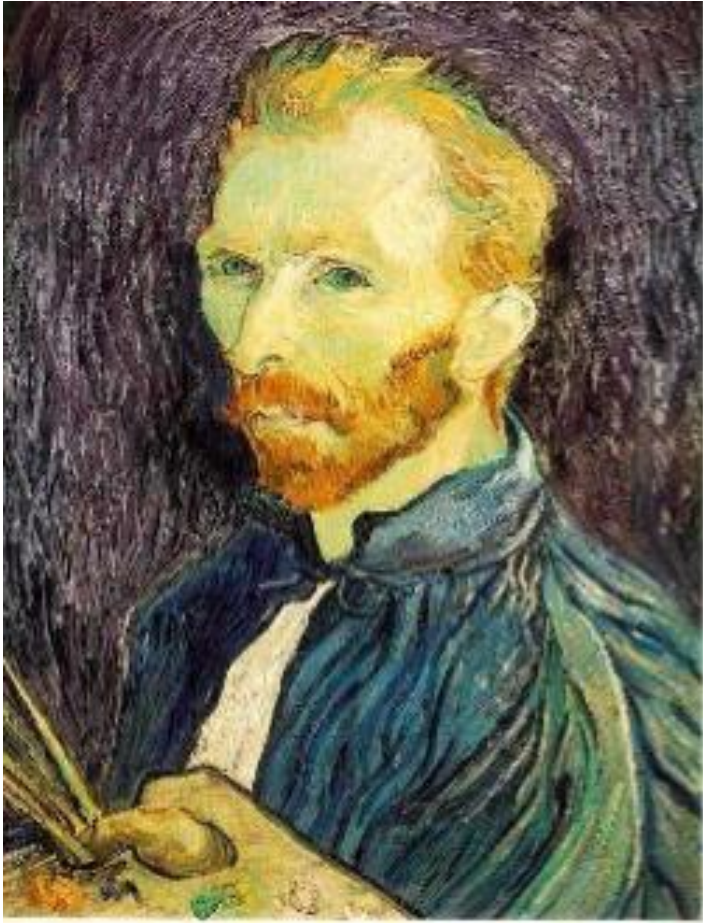
Image downsampling

A close-up portrait of Vincent van Gogh, showing his face and upper torso. He has a thick, reddish-brown beard and mustache, and his eyes are a pale, yellowish-green. His hair is a mix of yellow and brown. He is wearing a dark blue jacket over a white shirt. The background is a dark, textured brown. The painting is done in Van Gogh's characteristic style, with visible, expressive brushstrokes.

**This image is too big to fit on the screen.  
How would you reduce it to half its size?**



# Naïve image downsampling



1/2

Throw away every other row and column to create a 1/2 size image  
- called *image sub-sampling*

delete even rows  
delete even columns



1/4

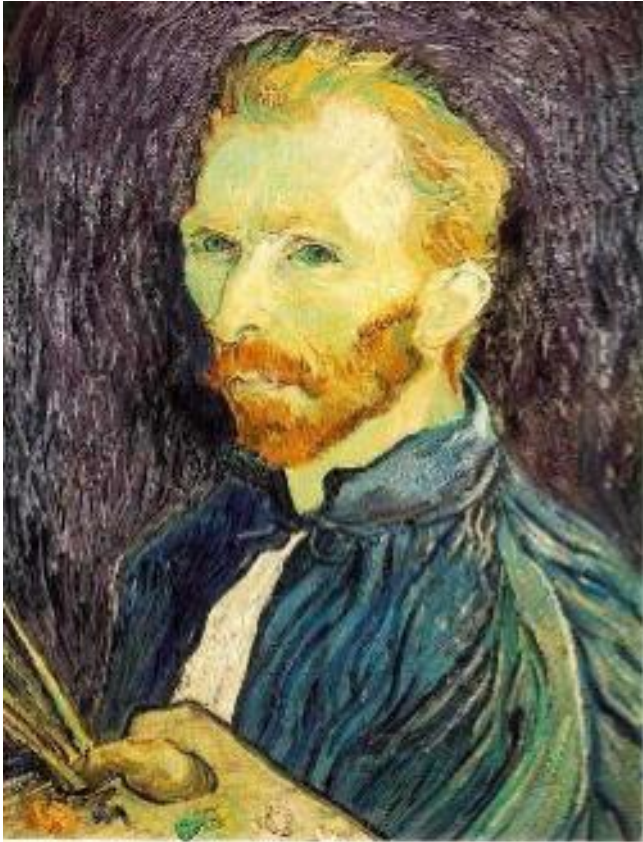
delete even rows  
delete even columns



1/8

What is the problem with this approach?

# Naïve image downsampling



1/2



1/4 (2x zoom)

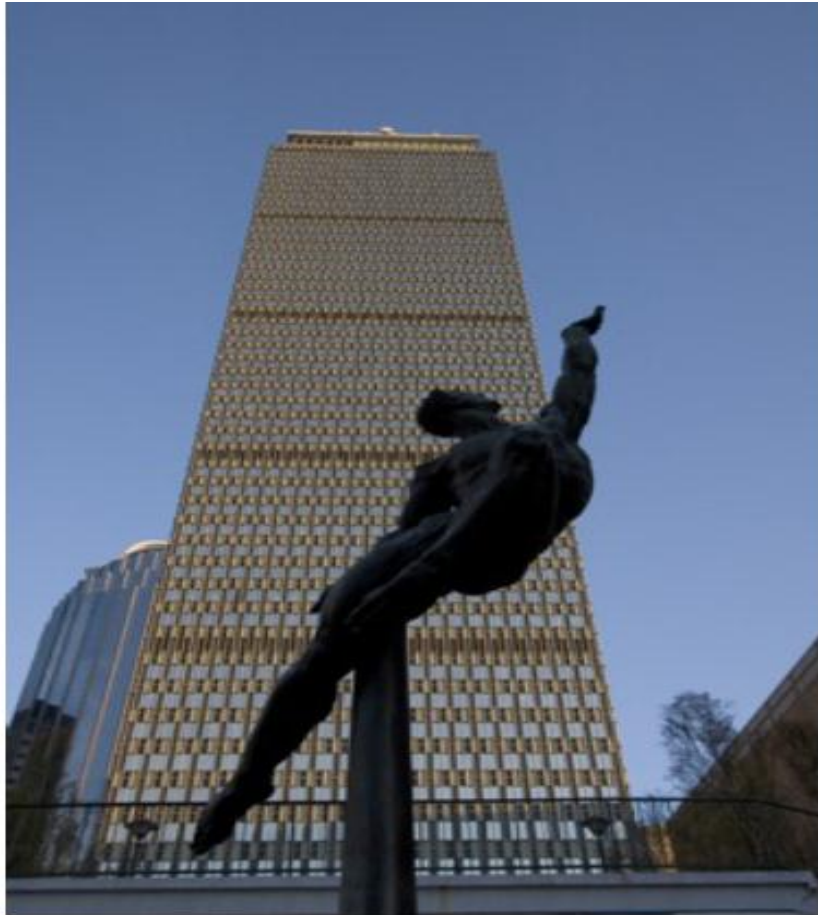


1/8 (4x zoom)

What is the 1/8 image so pixelated (and do you know what this effect is called)?



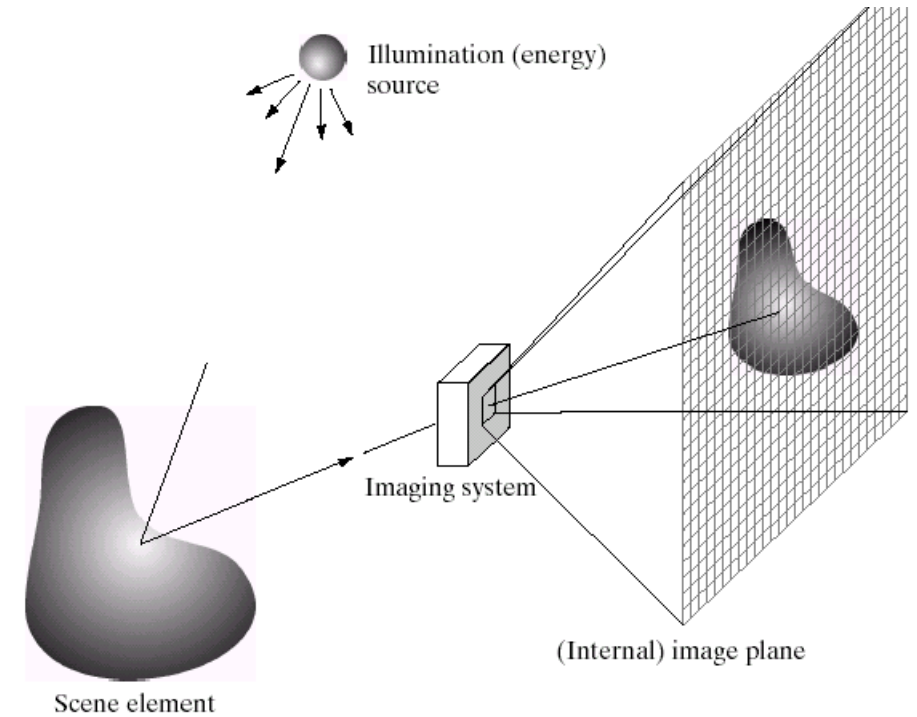
# Image downsampling – another example



What this effect is called? **Aliasing**



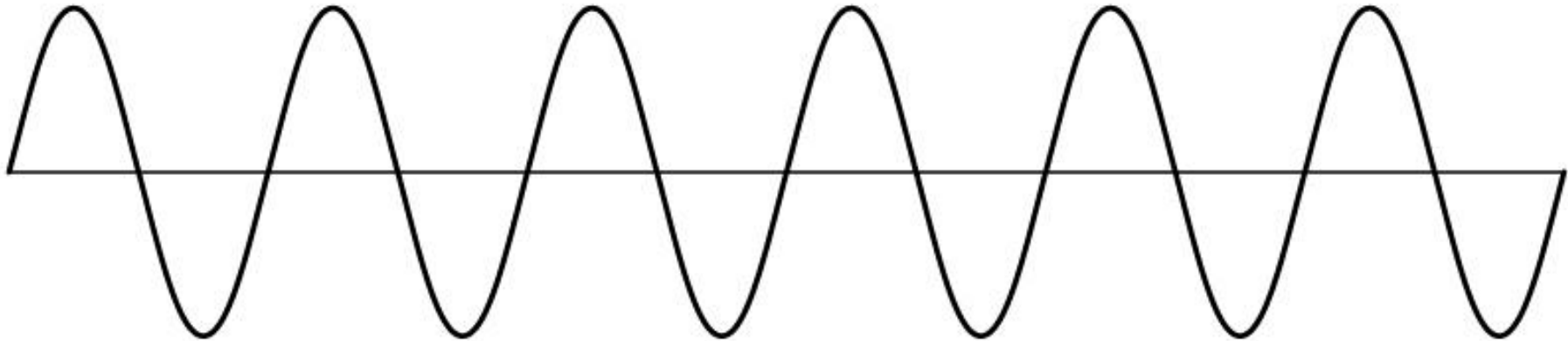
# Reminder



Images are a *discrete*, or *sampled*, representation of a *continuous* world

# Sampling

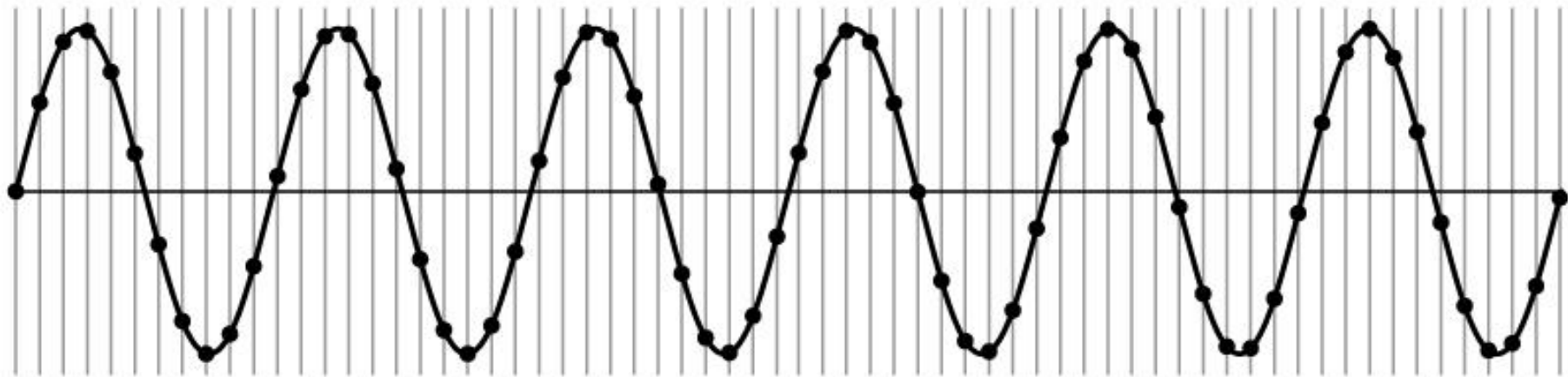
Very simple example: a sine wave



How would you discretize this signal?

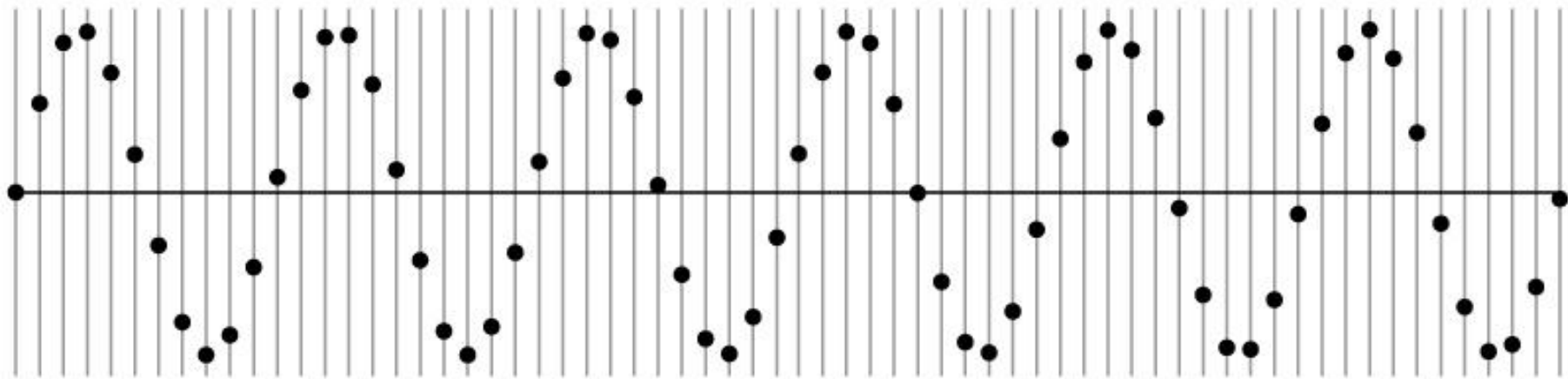
# Sampling

Very simple example: a sine wave



# Sampling

Very simple example: a sine wave



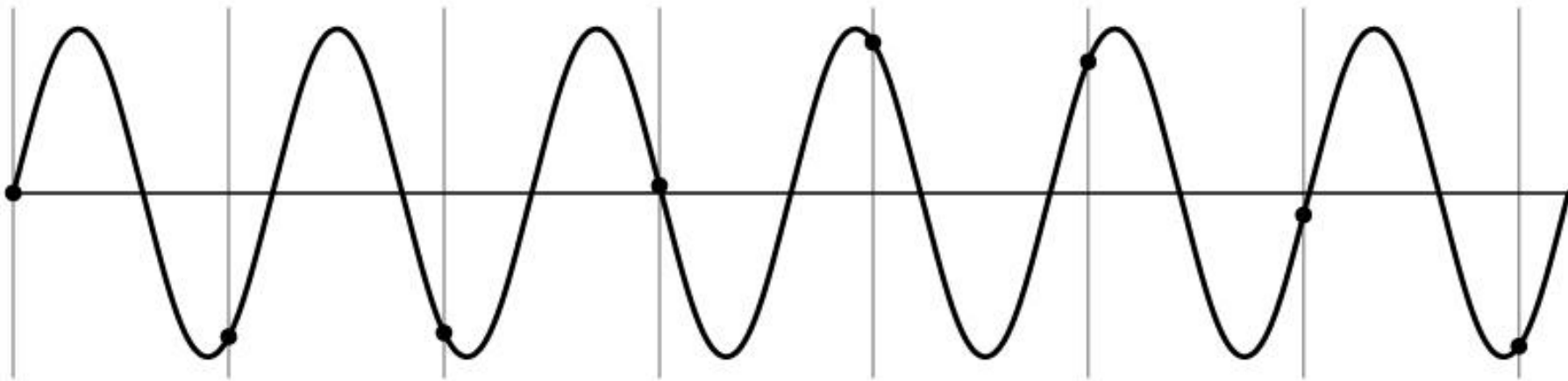
How many samples should I take?

Can I take as *many* samples as I want?



# Undersampling

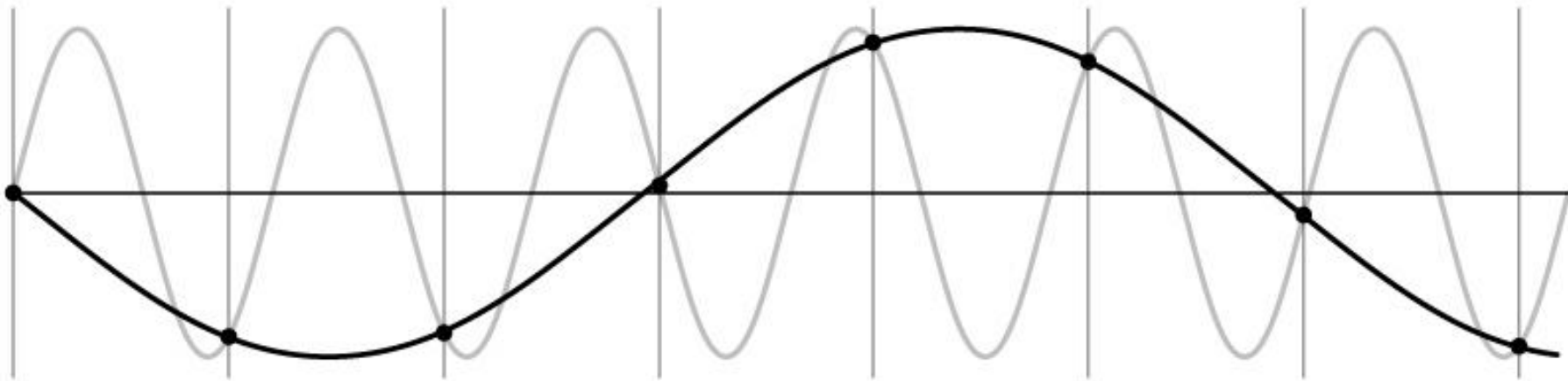
Very simple example: a sine wave



Unsurprising effect: information is lost.

# Undersampling

Very simple example: a sine wave

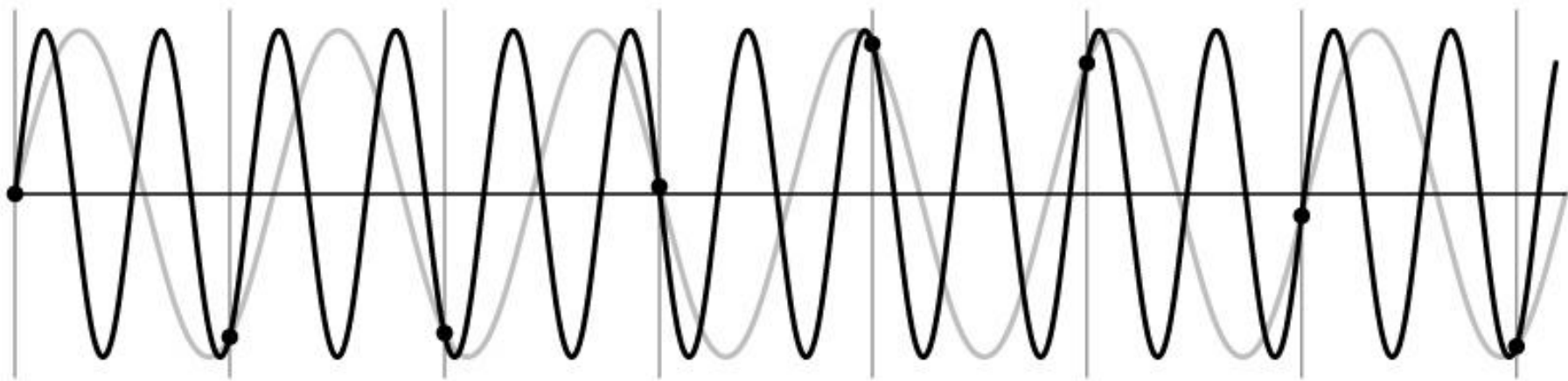


Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of *lower* frequency.

# Undersampling

Very simple example: a sine wave

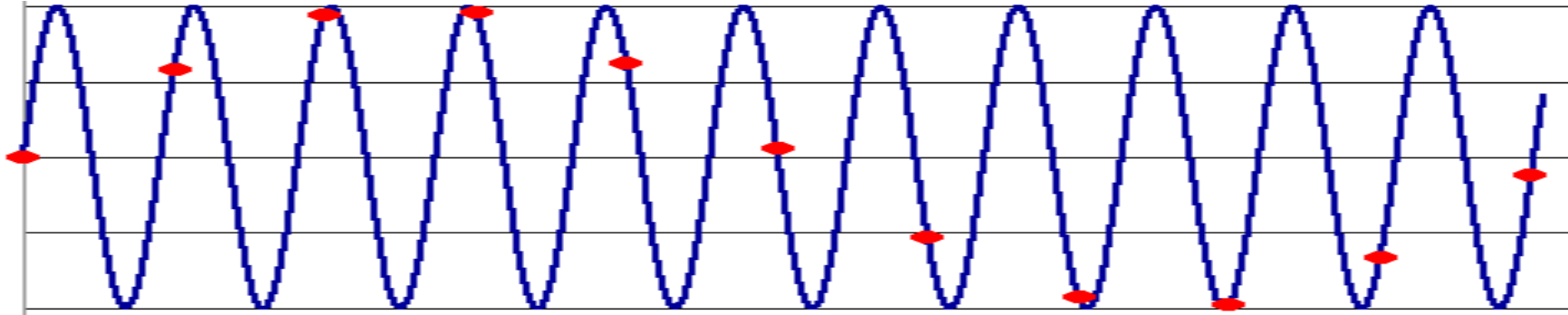


Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of *lower* frequency.

Note: we could always confuse the signal with one of *higher* frequency.

# Aliasing



- Occurs when your sampling rate is not high enough to capture the amount of detail in your image
- Can give you the wrong signal/image—an *alias*
- To do sampling right, need to understand the structure of your signal/image
- To avoid aliasing:
  - sampling rate  $\geq 2 * \text{max frequency in the image}$ 
    - said another way:  $\geq$  two samples per cycle
  - This minimum sampling rate is called the **Nyquist rate**



# Aliasing in photographs

This is also known as “moire”

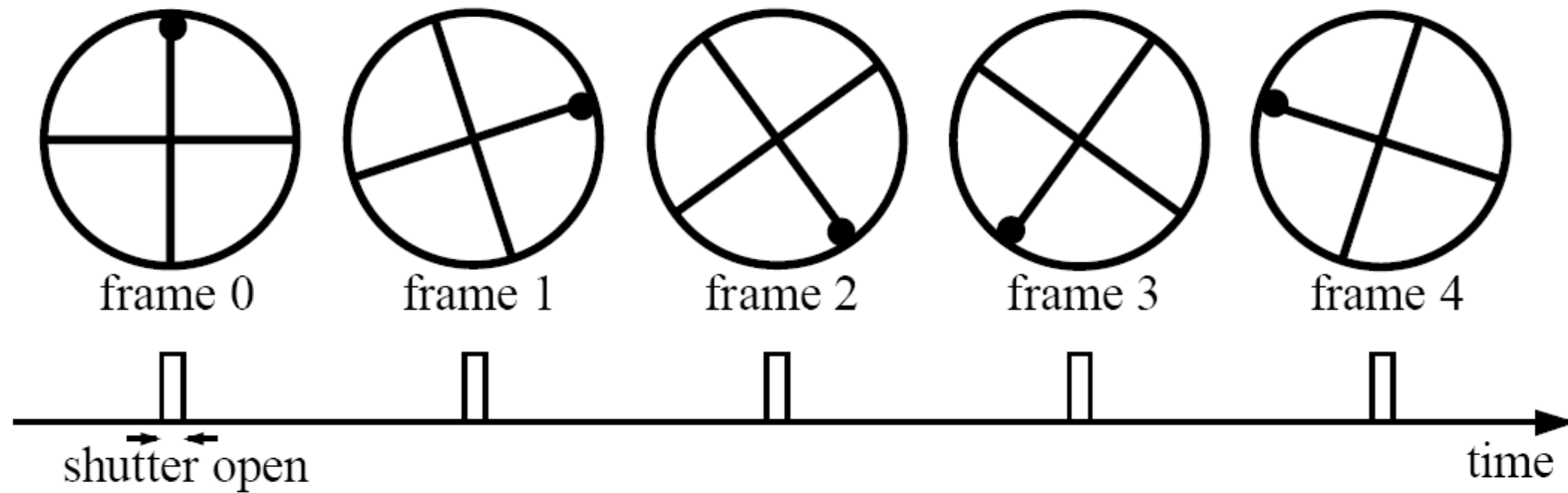


# Temporal aliasing

Imagine a spoked wheel moving to the right (rotating clockwise).

Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time =  $1/30$  sec. for video,  $1/24$  sec. for film):

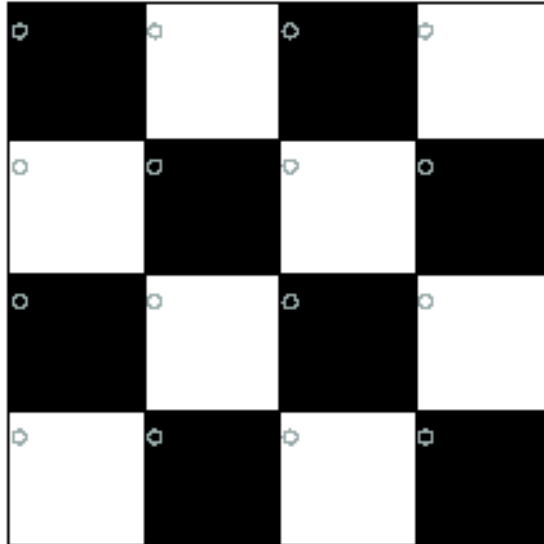
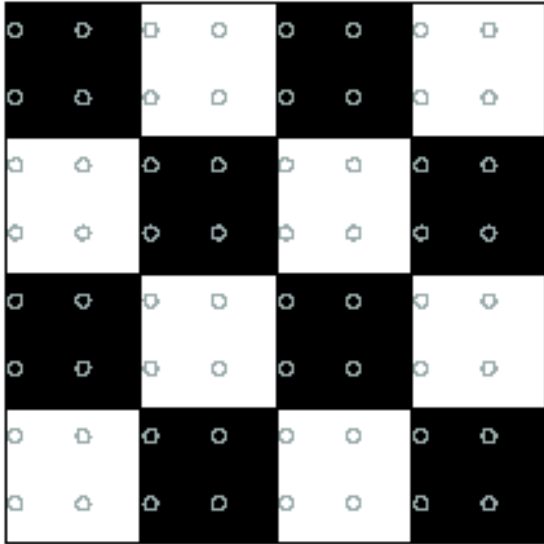


Without dot, wheel appears to be rotating slowly backwards!  
(counterclockwise)

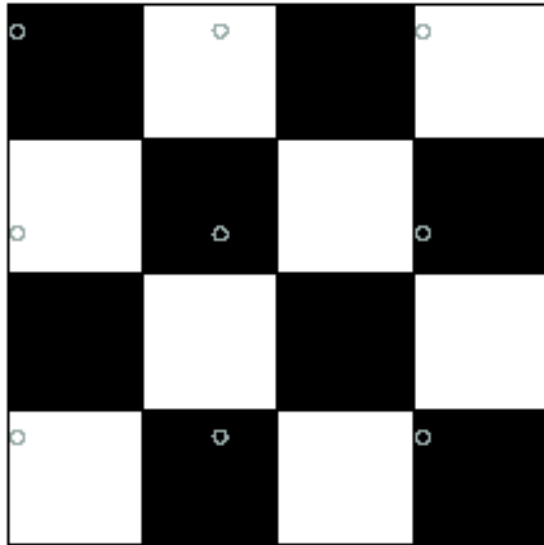
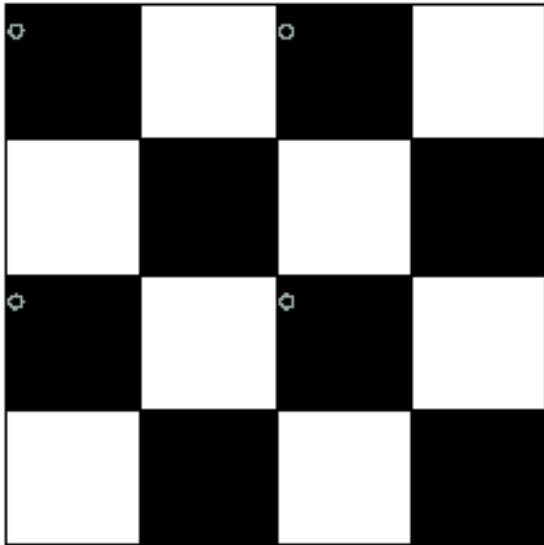
# Wagon wheel effect



# Nyquist limit – 2D example



Good sampling



Bad sampling



# Anti-aliasing

- When downsampling by a factor of two
  - Original image has frequencies that are too high

How would you deal with aliasing?

Approach 1: Oversample the signal

- This is how camera manufacturers started focusing so heavily on number of megapixels.

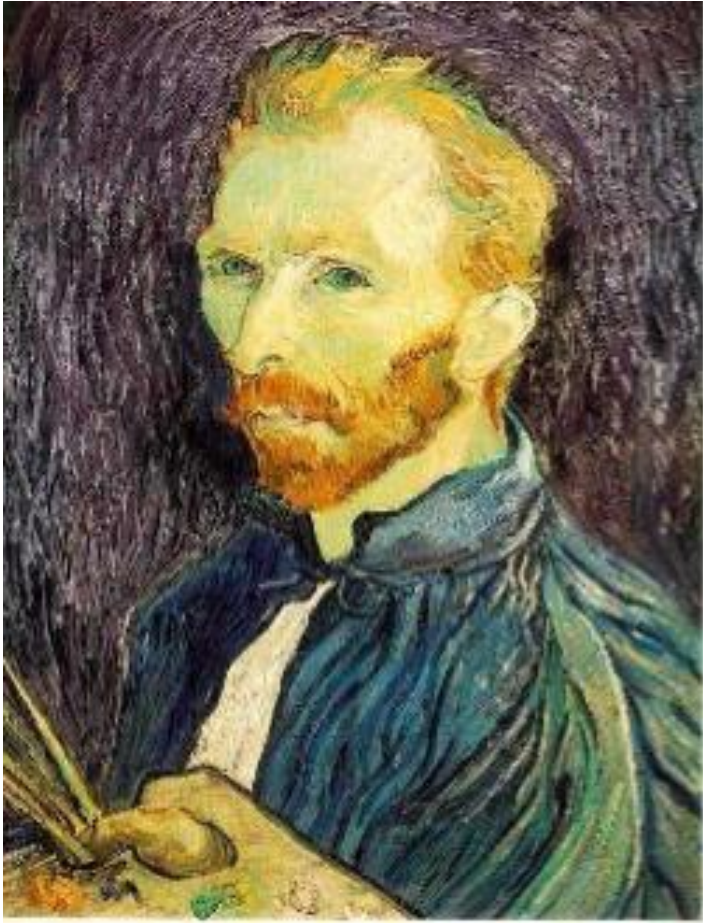
Approach 2: filter the image, *then* subsample

- Remove some of the detail effects that cause aliasing.
- Lose information, but better than aliasing artifacts.

How would you smooth a signal?

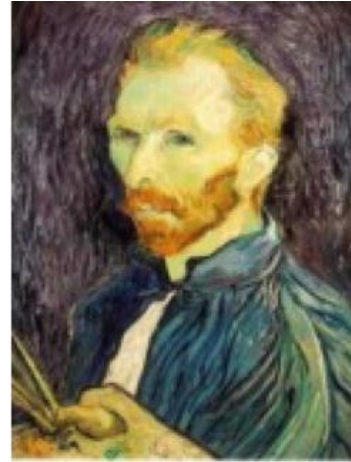
# Better image downsampling

Apply a smoothing filter first, then throw away half the rows and columns



1/2

Gaussian filter  
delete even  
rows  
delete even  
columns



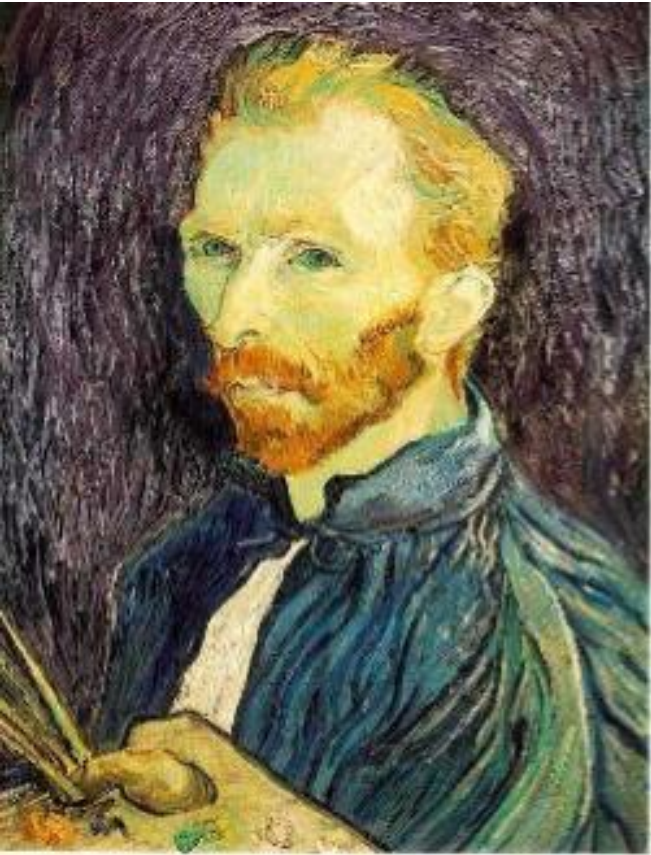
1/4

Gaussian filter  
delete even  
rows  
delete even  
columns



1/8

# Better image downsampling



1/2



1/4 (2x zoom)



1/8 (4x zoom)



# Compare with Naïve image downsampling



1/2



1/4 (2x zoom)



1/8 (4x zoom)



# Anti-aliasing

Question 1: How much smoothing do I need to do to avoid aliasing?

Question 2: How many samples do I need to take to avoid aliasing?

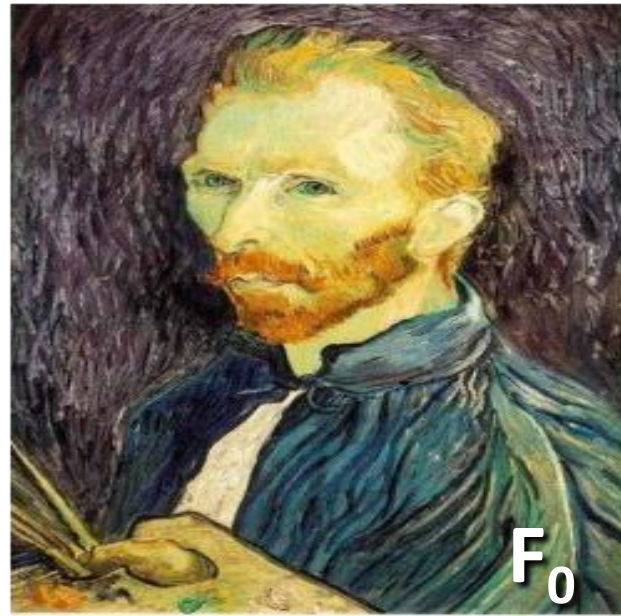
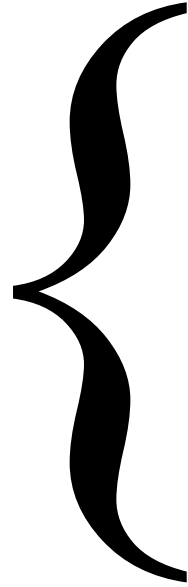
Answer to both: Enough to reach the Nyquist limit.

We'll see what this means soon.

Gaussian pre-filtering

Gaussian image pyramid

# Gaussian image pyramid



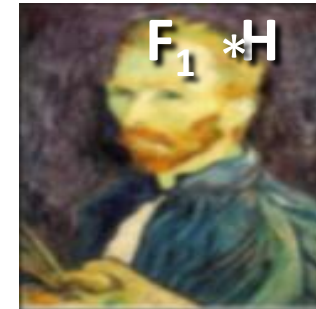
The name of this sequence of subsampled images

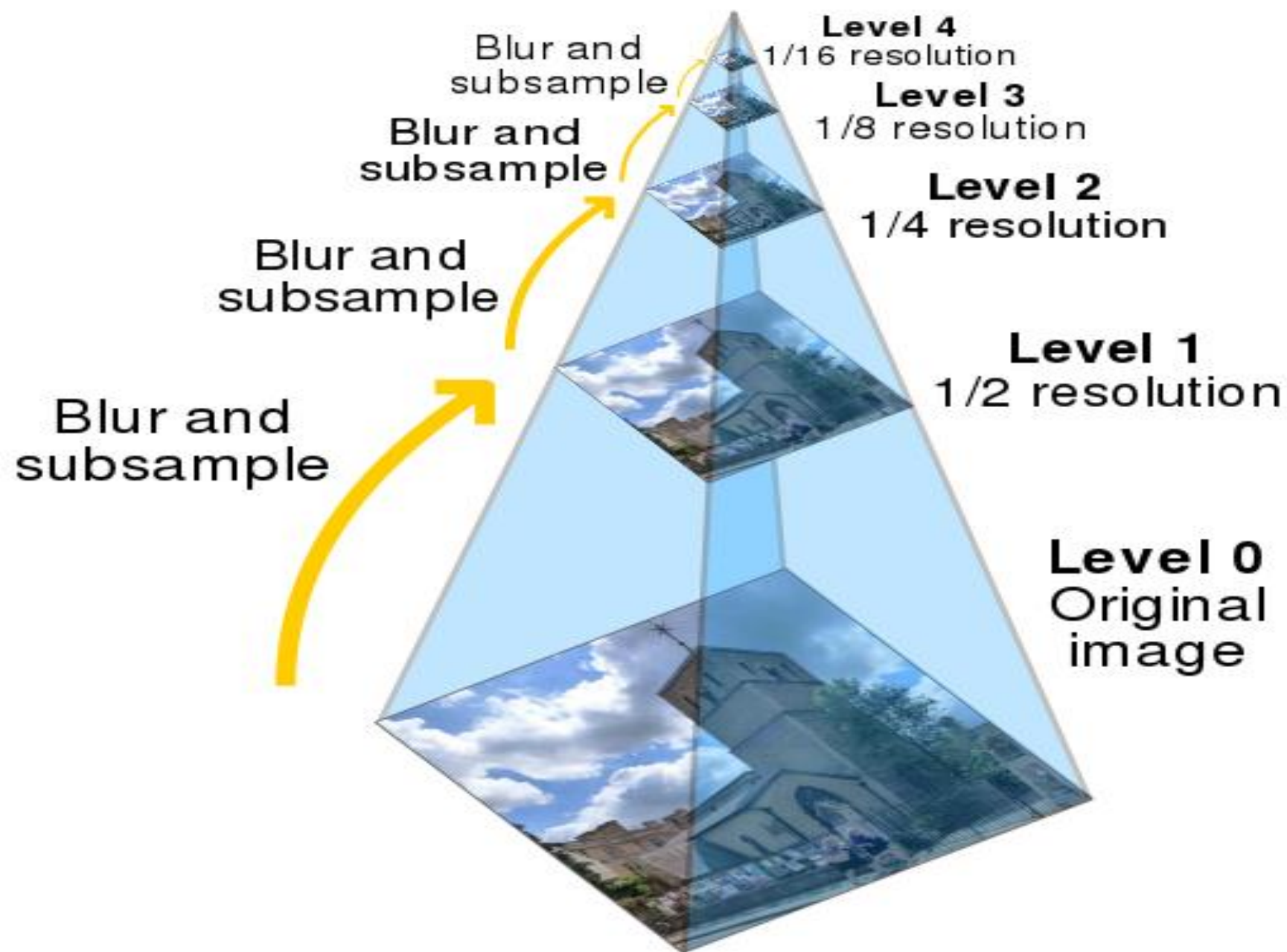


blur      subsample

blur      subsample      ...

- Solution: filter the image, *then* subsample





# Constructing a Gaussian pyramid

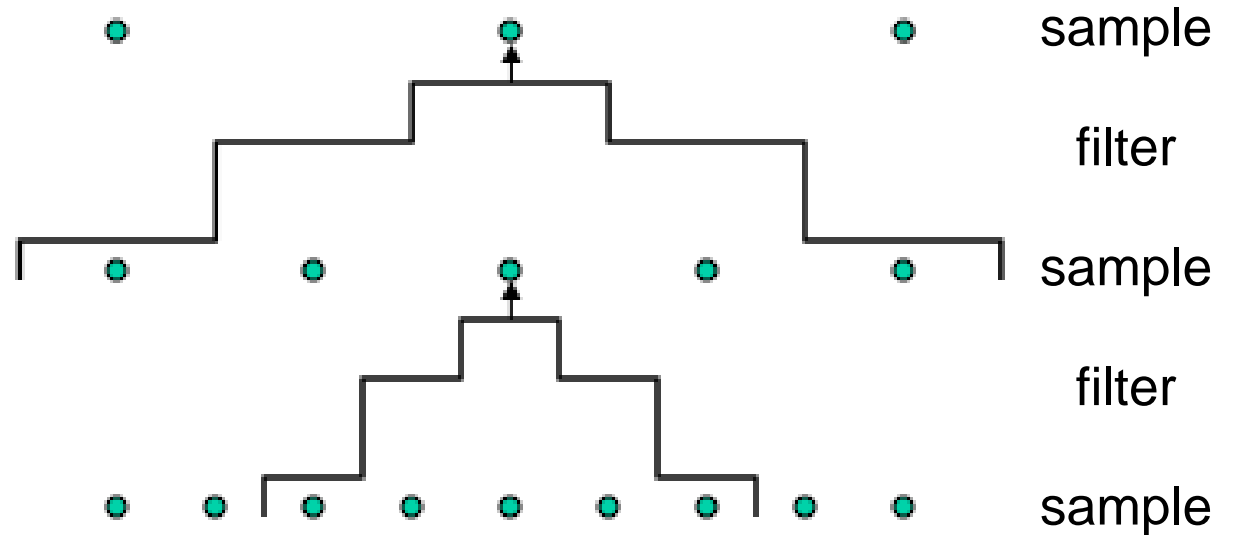
## Algorithm

repeat:

    filter

    subsample

until min resolution  
reached

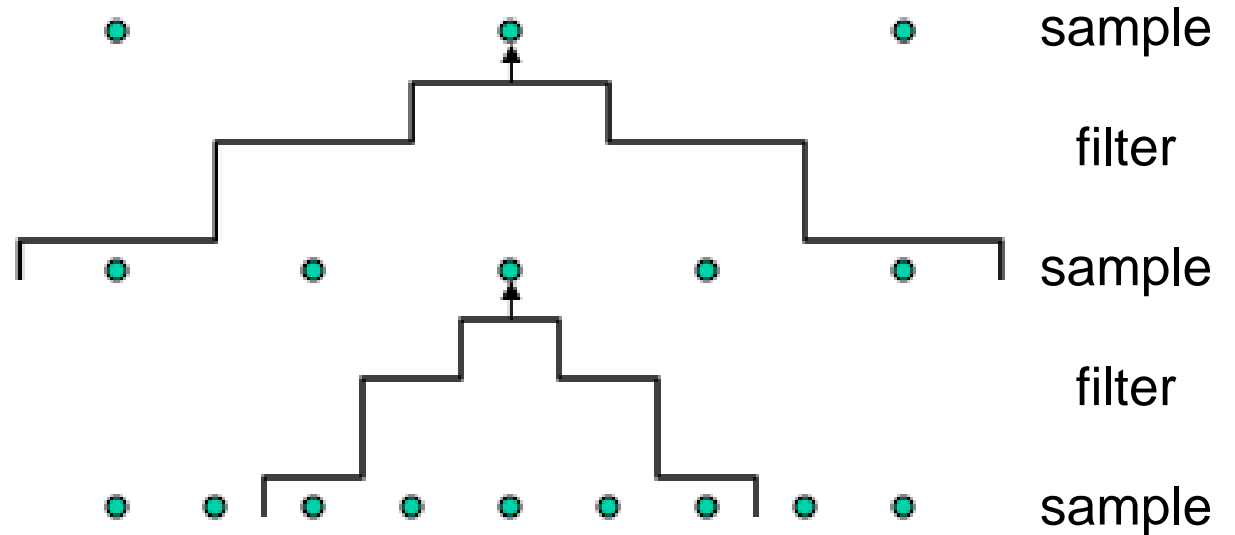


Question: How much bigger than the original image is the whole pyramid?

# Constructing a Gaussian pyramid

## Algorithm

```
repeat:  
    filter  
    subsample  
until min resolution  
reached
```

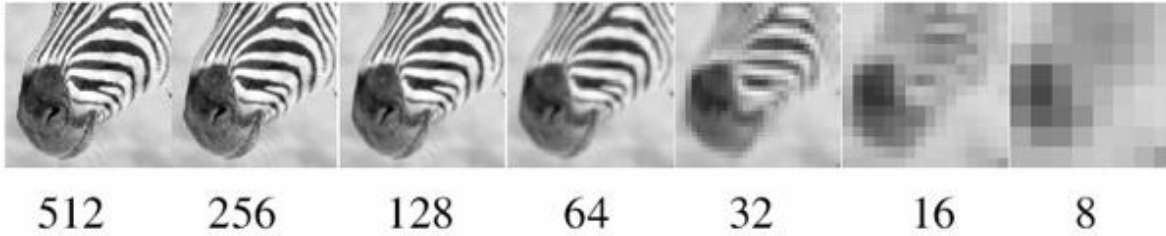


Question: How much bigger than the original image is the whole pyramid?

Answer: Just  $\frac{4}{3}$  times the size of the original image!  
(How did I come up with this number?)



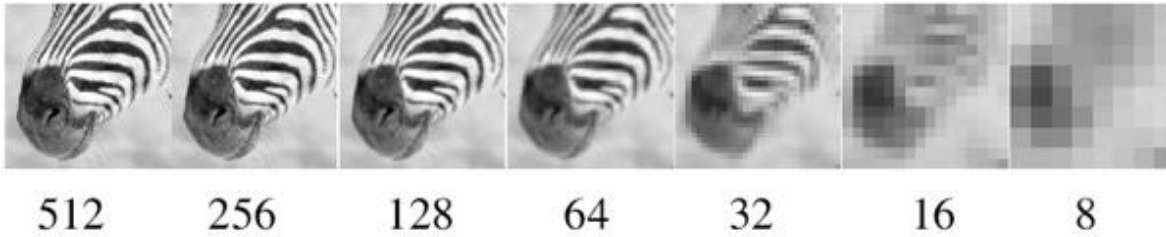
# Some properties of the Gaussian pyramid



What happens to the details of the image?



# Some properties of the Gaussian pyramid



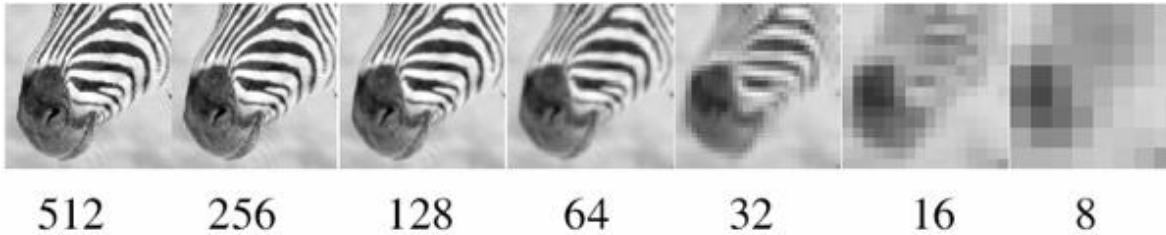
What happens to the details of the image?

- They get smoothed out as we move to higher levels.

What is preserved at the higher levels?



# Some properties of the Gaussian pyramid



What happens to the details of the image?

- They get smoothed out as we move to higher levels.

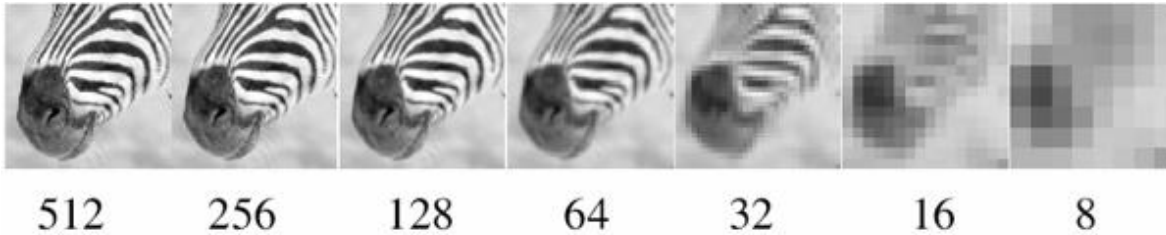
What is preserved at the higher levels?

- Mostly large uniform regions in the original image.

How would you reconstruct the original image from the image at the upper level?



# Some properties of the Gaussian pyramid



What happens to the details of the image?

- They get smoothed out as we move to higher levels.

What is preserved at the higher levels?

- Mostly large uniform regions in the original image.

How would you reconstruct the original image from the image at the upper level?

- That's not possible.



# Blurring is lossy



level 0

-



level 1 (before downsampling)

=



residual

What does the residual look like?



# Blurring is lossy



level 0

-



level 1 (before downsampling)

=



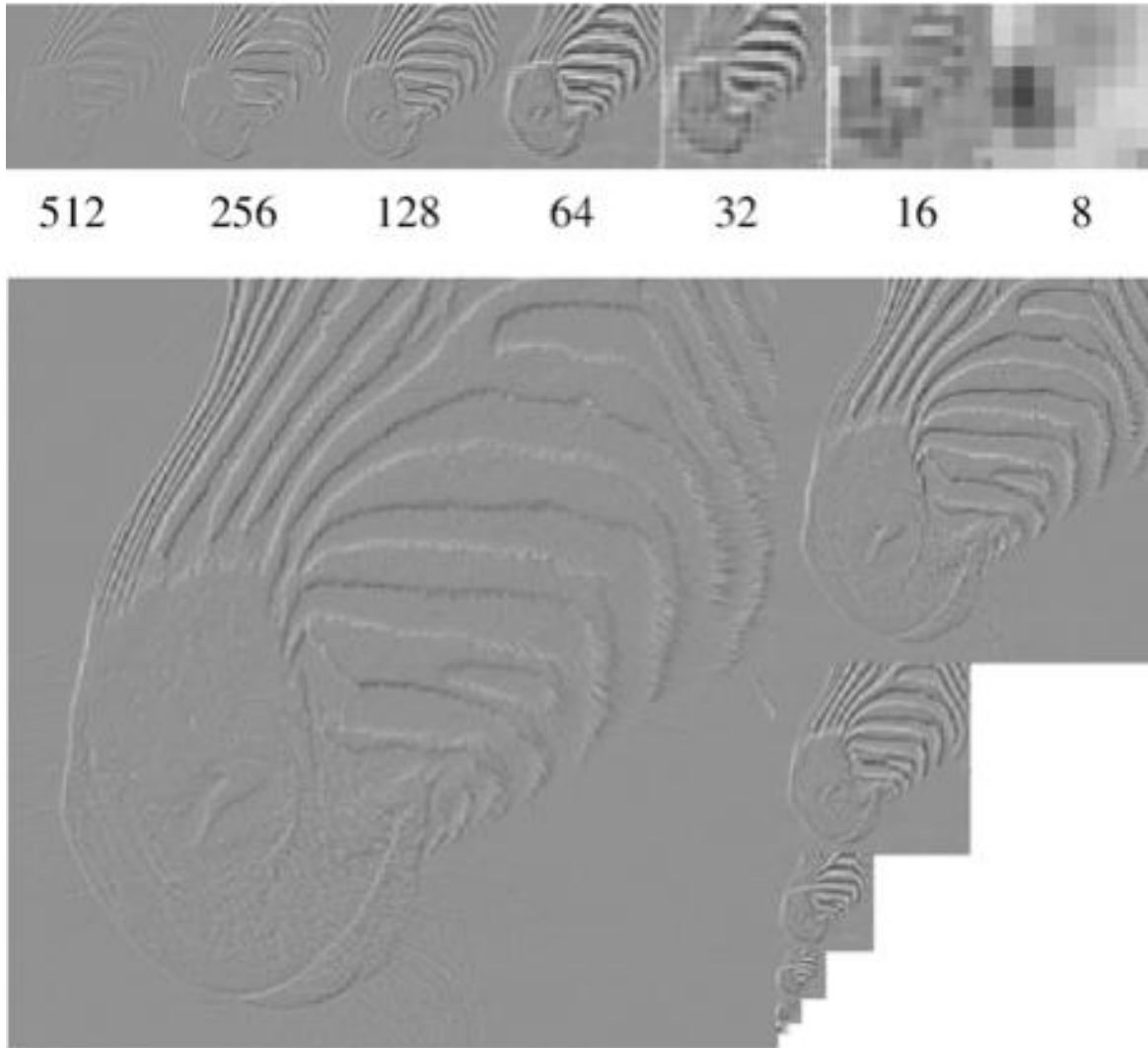
residual

Can we make a pyramid that is lossless?



# Laplacian image pyramid

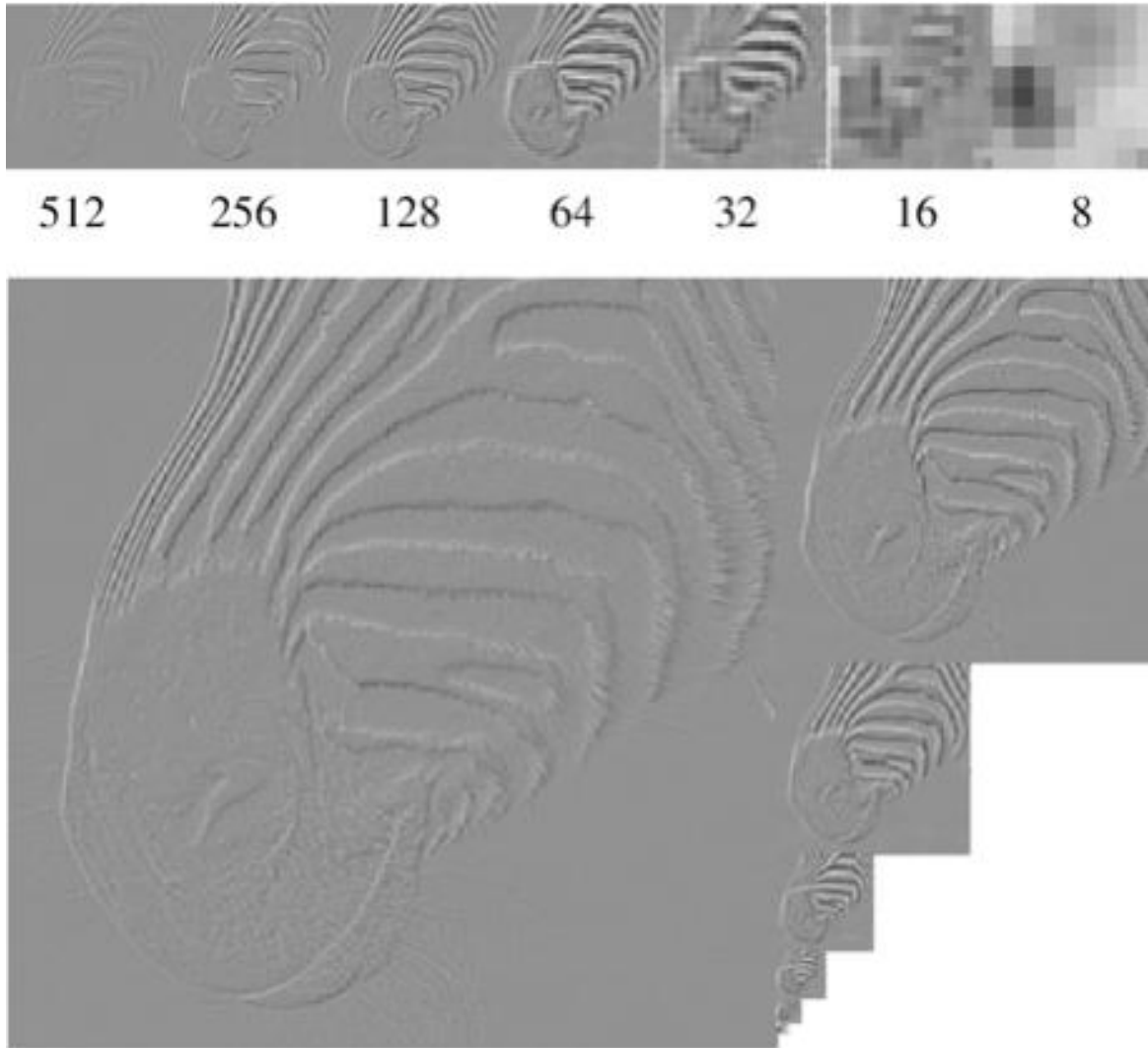
# Laplacian image pyramid



At each level, retain the residuals instead of the blurred images themselves.

Can we reconstruct the original image using the pyramid?

# Laplacian image pyramid



At each level, retain the residuals instead of the blurred images themselves.

Can we reconstruct the original image using the pyramid?

- Yes we can!

What do we need to store to be able to reconstruct the original image?

# Let's start by looking at just one level



level 0

=



level 1 (upsampled)

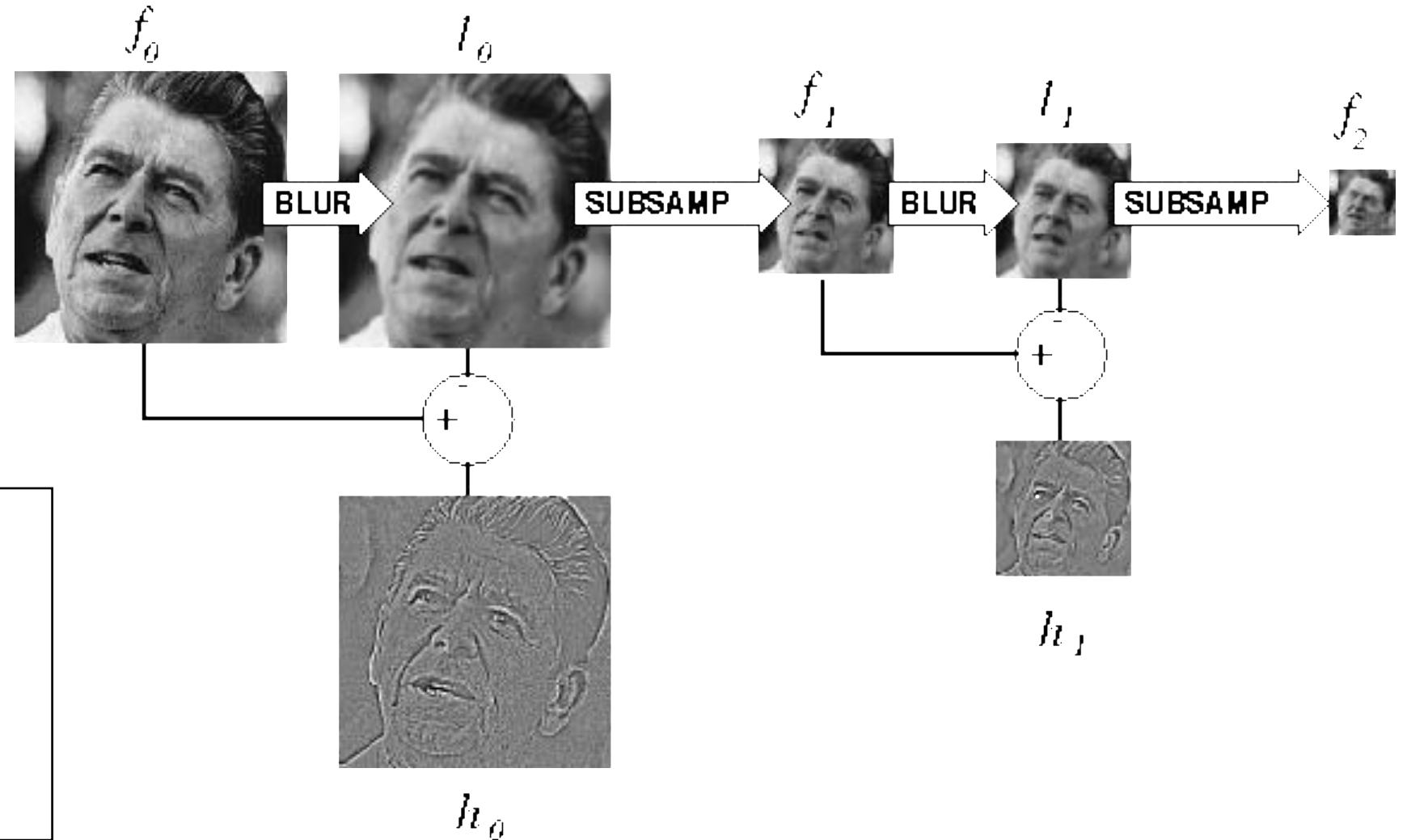
+



residual

Does this mean we need to store both residuals and the blurred copies of the original?

# Constructing a Laplacian pyramid

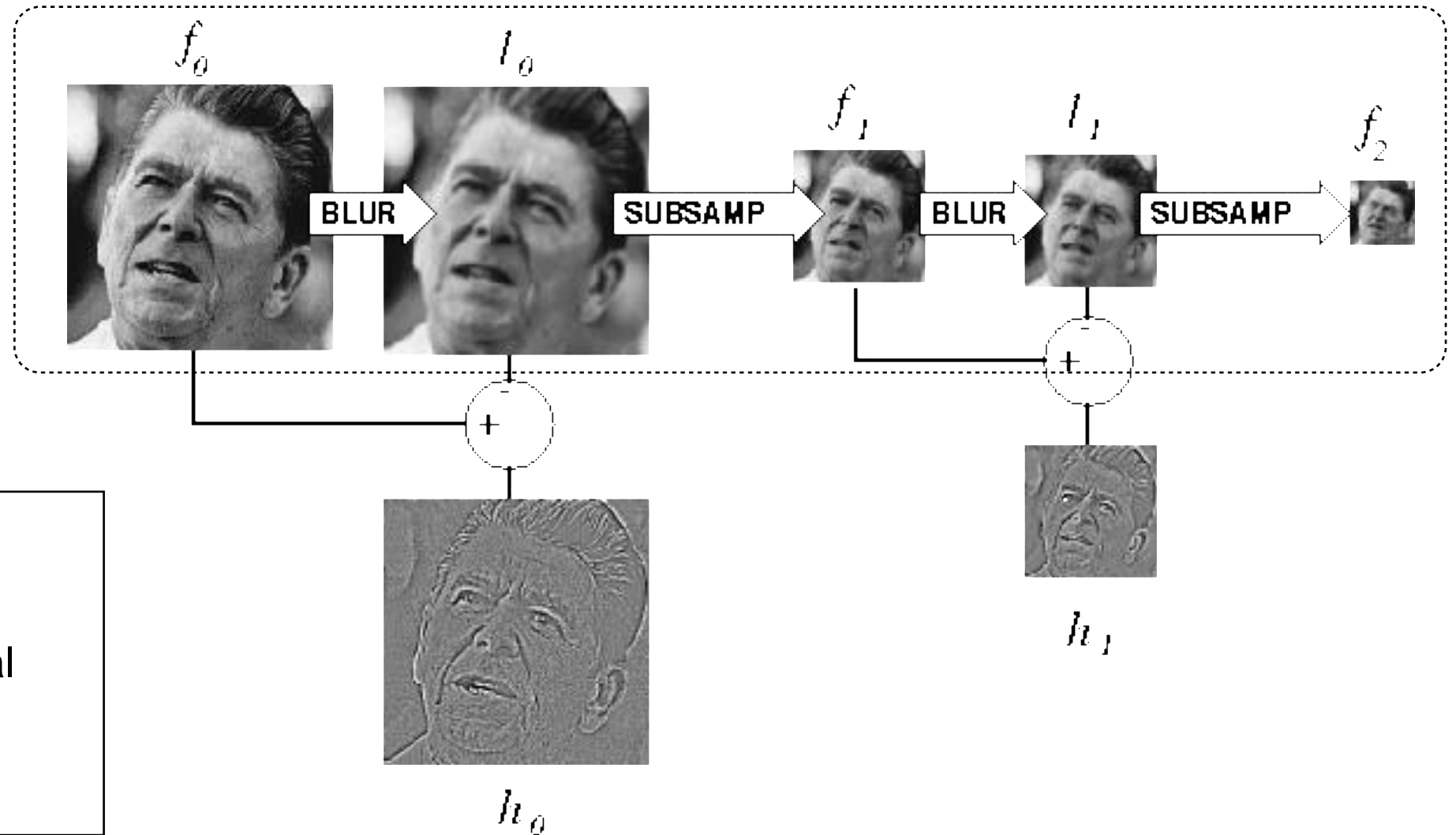


## Algorithm

repeat:  
    filter  
    compute residual  
    subsample  
until min resolution  
reached

# Constructing a Laplacian pyramid

What is this part?



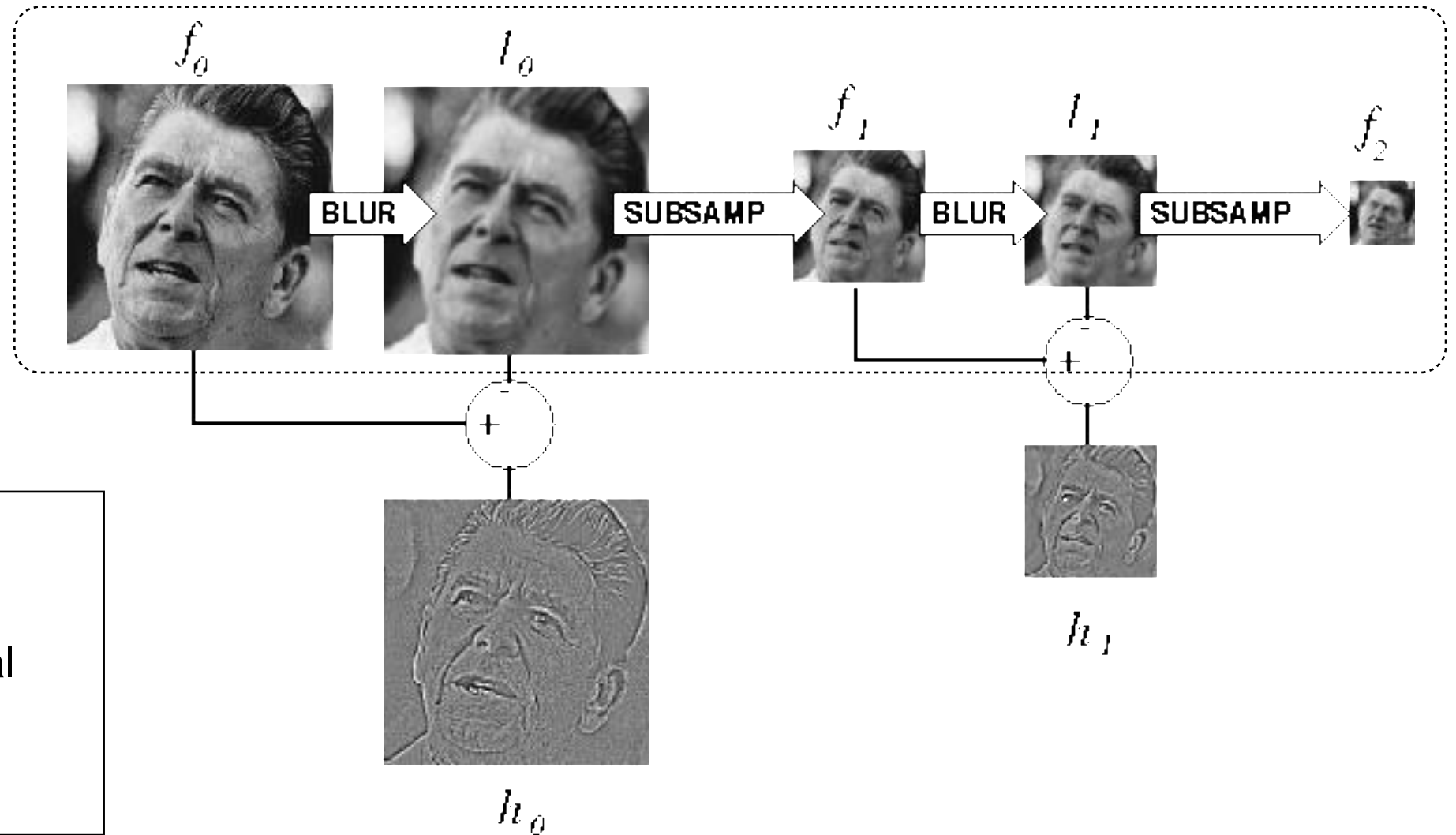
## Algorithm

repeat:  
    filter  
    compute residual  
    subsample  
until min resolution  
reached



# Constructing a Laplacian pyramid

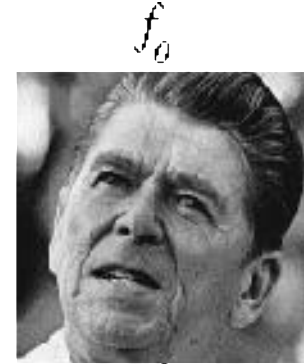
It's a Gaussian pyramid.



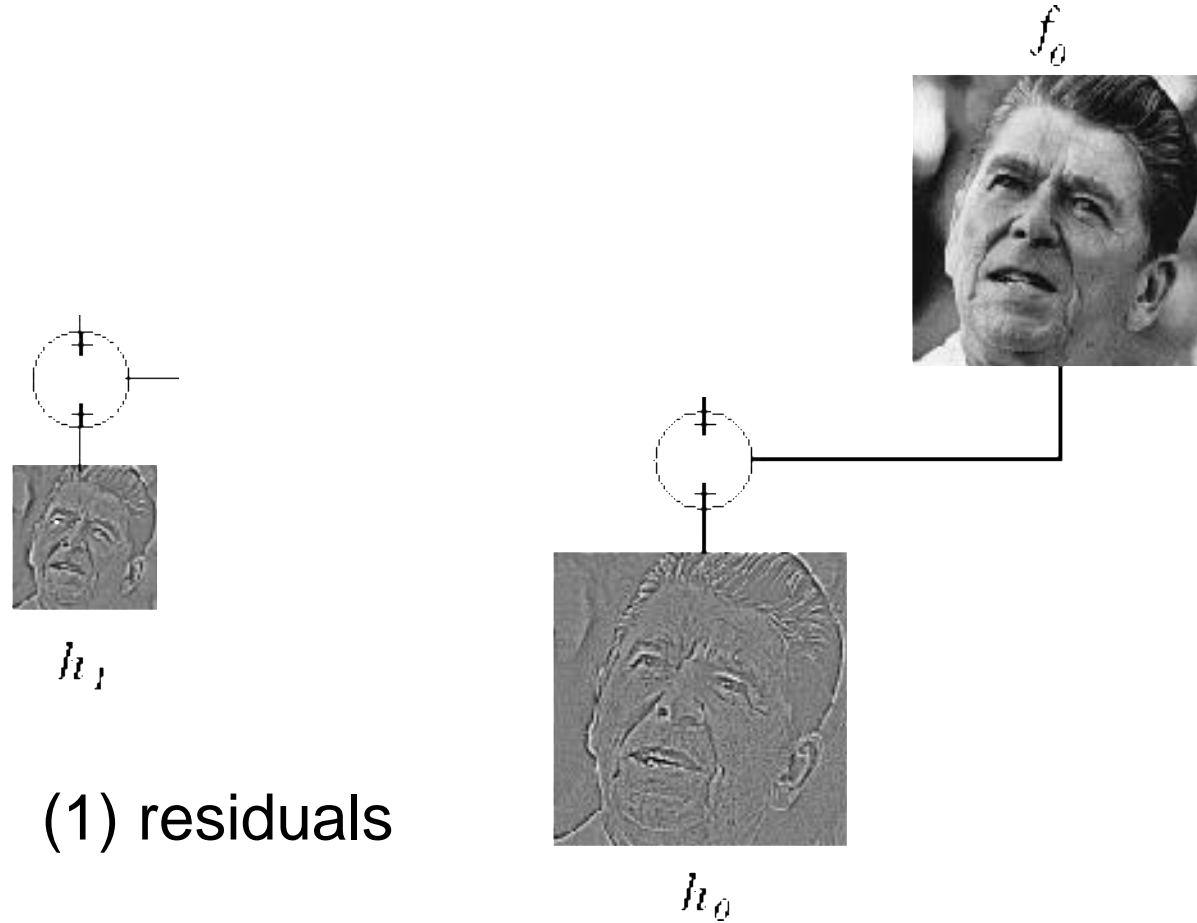
## Algorithm

repeat:  
    filter  
    compute residual  
    subsample  
until min resolution  
reached

# What do we need to construct the original image?

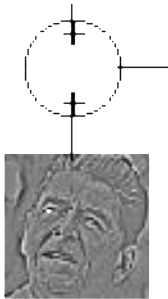


# What do we need to construct the original image?



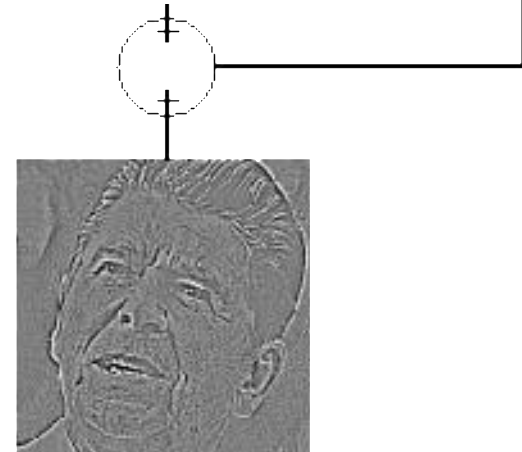
# What do we need to construct the original image?

(2) smallest image  $f_2$

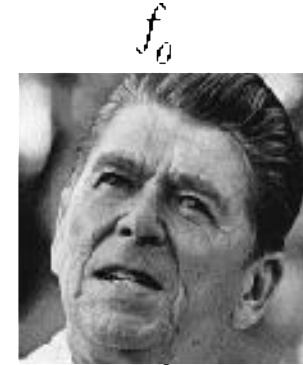


$h_1$

(1) residuals

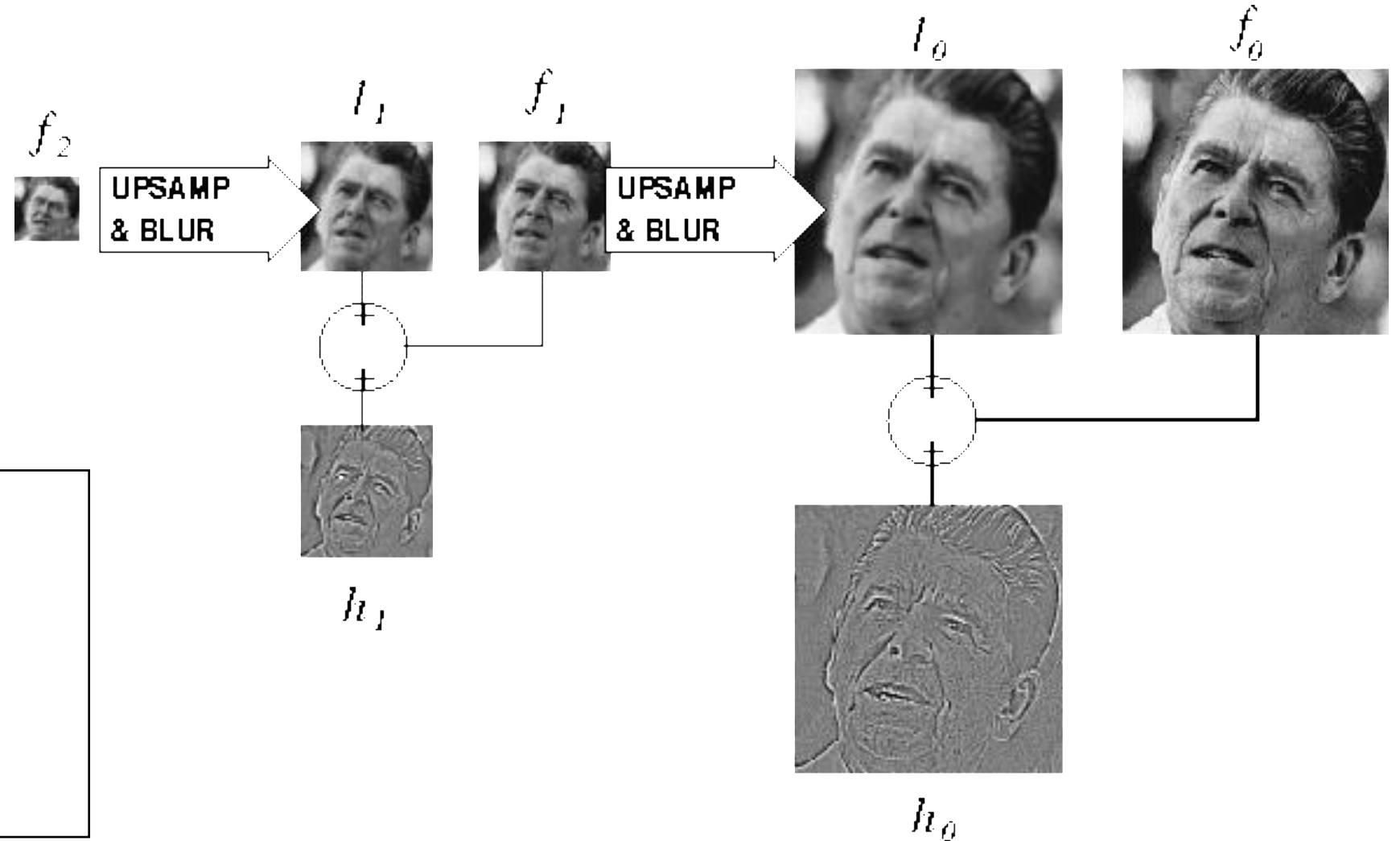


$h_0$



$f_0$

# Reconstructing the original image



## Algorithm

repeat:  
    upsample  
    sum with residual  
until orig resolution  
reached

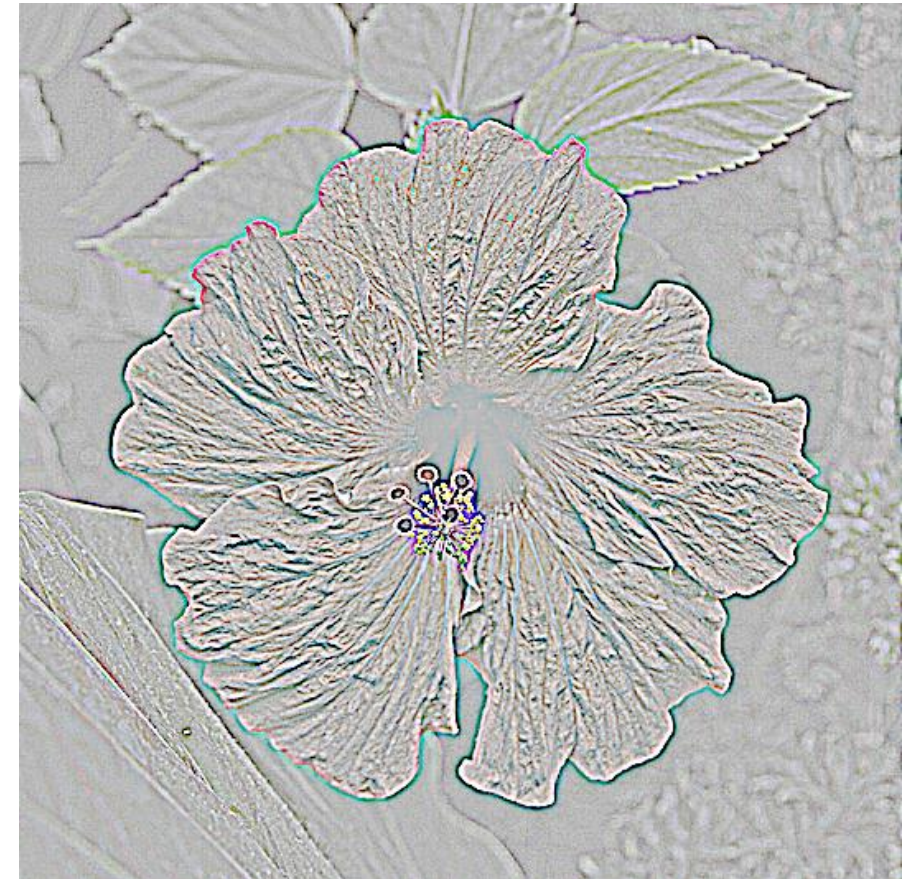
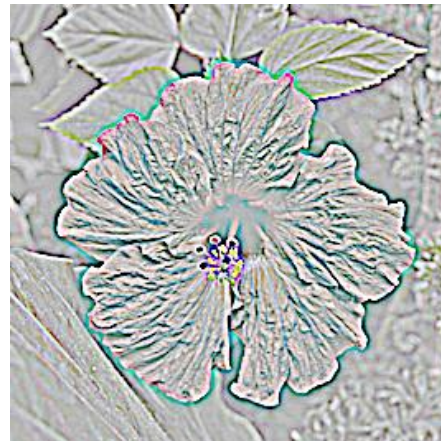


# Gaussian vs Laplacian Pyramid



Shown in opposite order for space.

Which one takes more space to store?





# Laplacian pyramid - Still used extensively





# Laplacian pyramid - Still used extensively



input image



foreground details enhanced, background details reduced user-provided mask

# Other types of pyramids

Steerable pyramid: At each level keep multiple versions, one for each direction.



Wavelets: Huge area in image processing





# What are image pyramids used for?

image compression



multi-scale  
texture mapping

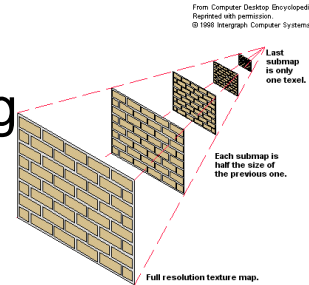
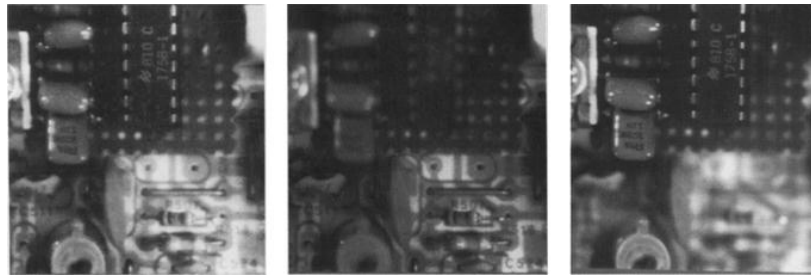


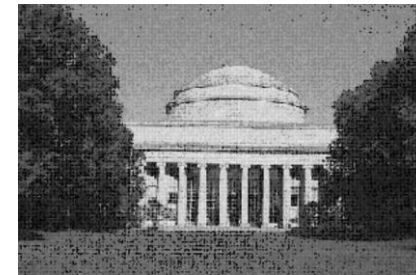
image blending



focal stack compositing



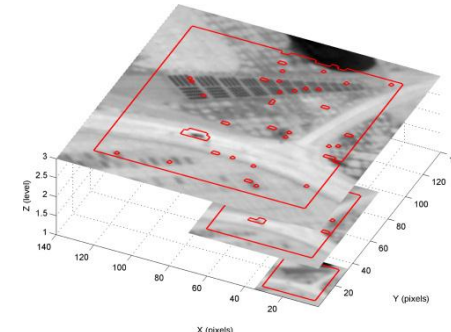
denoising



multi-scale detection



multi-scale registration





# Multiresolution Processing

## motivation in the context of object recognition

- Images seen as connected regions of similar texture and intensity levels that combine to form objects
- If both small and large objects or low and high contrast objects are present simultaneously- advantageous to study them at several resolutions
- If objects are
  - small in size or low in contrast
    - examine them at high resolution
  - large in size or high in contrast
    - a coarse view is required



- Image with information at different **scales** or levels of **detail** (e.g., people vs buildings)
- Analyzing information at the same scale will not be effective.

# Multiresolution Processing

## motivation in the context of object recognition

- Use windows of different size (i.e., vary scale)



- Alternatively, use the same window size but analyze information at different resolutions:



High resolution



Low resolution

Small size objects should be examined at a high resolution

Large size objects should be examined at a low resolution

# Upsampling

- This image is too small for this screen:
- How can we make it 10 times as big?
- Simplest approach:  
repeat each row and column 10 times
- Nearest neighbor interpolation
- Bilinear interpolation
- Bicubic interpolation



# Image interpolation

Original image:  x 10



Nearest-neighbor interpolation



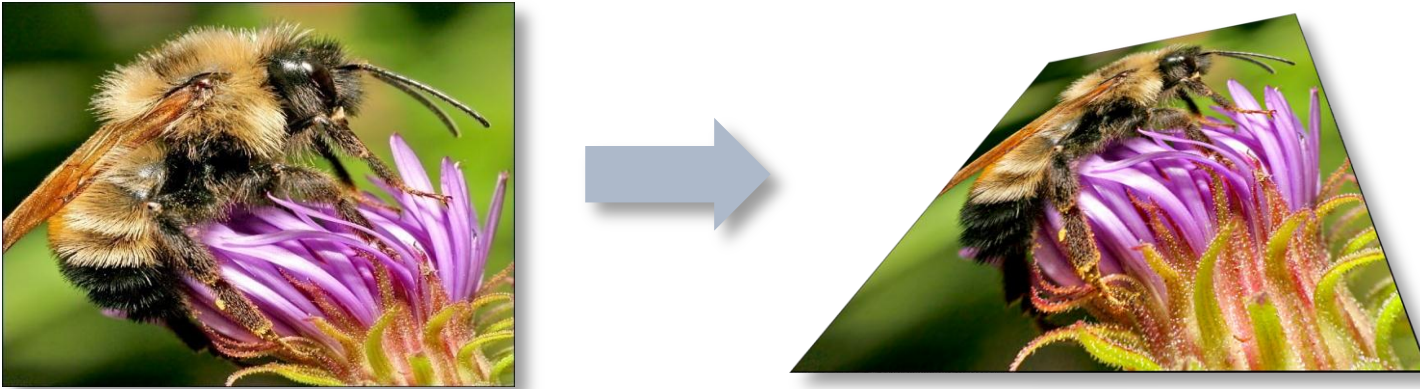
Bilinear interpolation



Bicubic interpolation

# Image interpolation

Also used for *resampling*



Depixelating Pixel Art



Nearest-neighbor interpolation



SIGGRAPH 2011

Raster-to-vector graphics





# Modern methods



(a) Bicubic



(b) SRCNN



(c) A+



(d) RAISR



(e) Bicubic



(f) SRCNN



(g) A+



(h) RAISR

From Romano, et al: RAISR: Rapid and Accurate Image Super Resolution, <https://arxiv.org/abs/1606.01299>

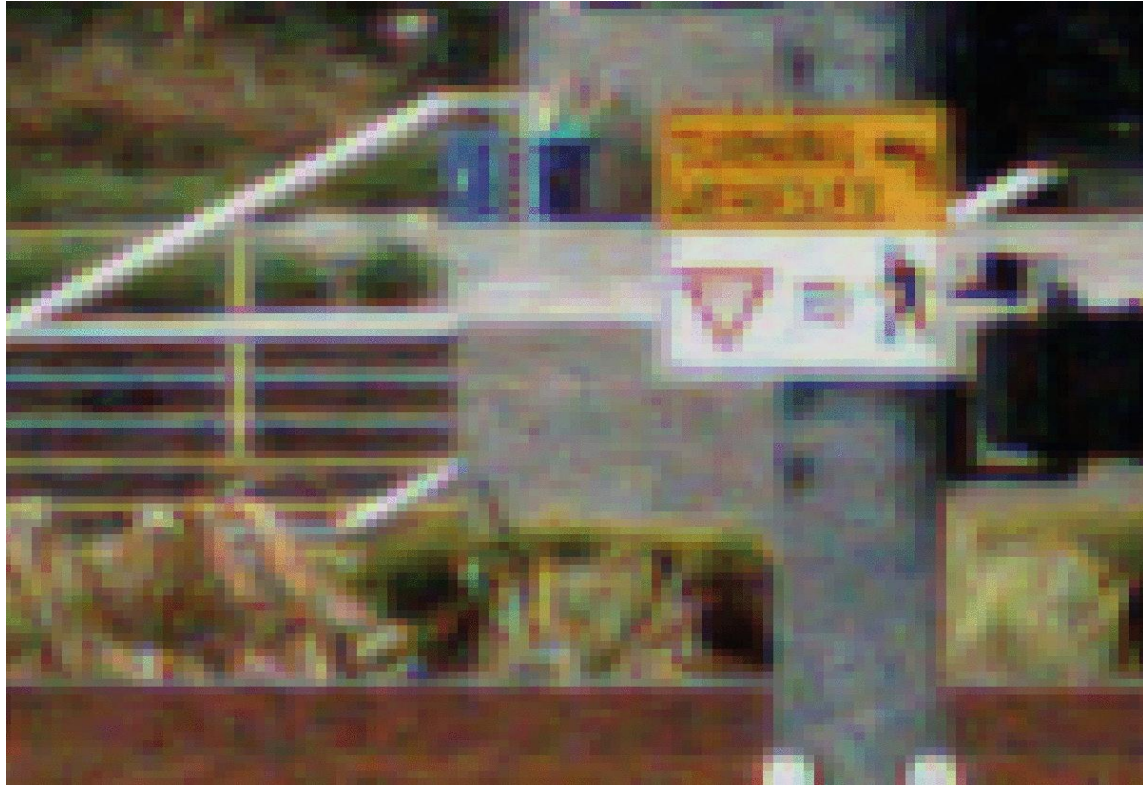
SRCNN: Image Super-Resolution Using Deep Convolutional Networks



# Super-resolution with multiple images

- Can do better upsampling if you have multiple images of the scene taken with small (subpixel) shifts
- Some cellphone cameras (like the Google Pixel line) capture a **burst** of photos
- Can we use that burst for upsampling?

# Google Pixel 3 Super Res Zoom



Effect of hand tremor as seen in a cropped burst of photos, after global alignment



Example photo with and without super res zoom (smart burst align and merge)