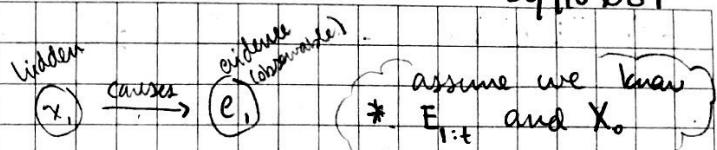


① Filtering

$$1.1) P(x_1 | e_1 = \text{"not flooded"}) = ?$$



* Recursive Definition : $P(x_t | e_{1:t}) = \alpha P(e_t | x_t) \sum_{x_{t-1}} (P(x_t | x_{t-1}) P(x_{t-1} | e_{1:t-1}))$

@ $e_1 = \text{"not flooded"} :$

$$P(x_1 | e_1) = \alpha P(x_1, e_1) \longrightarrow \text{"alpha trick"}$$

$$= \alpha \sum_{x_0} P(x_0, x_1, e_1)$$

$$= \alpha \sum_{x_0} P(x_0) P(x_1 | x_0) P(e_1 | x_1) \longrightarrow \text{Bayes Net! } P(x | \text{Parents}(x))$$

$$= \alpha P(e_1 | x_1) \sum_{x_0=\text{low,med,high}} P(x_0) P(x_1 | x_0)$$

$$@ x_0 = \text{low} : \rightarrow P(x_0 = \text{low}) = P(x_0 = \text{med}) = P(x_0 = \text{high}) = \frac{1}{3}$$

\therefore all same values

$\rightarrow @ x_1 = \text{low} :$

$$P(\text{low} | e_1) = \alpha (1) \left[(\frac{1}{3})(0.6) + (\frac{1}{3})(0.2) + (\frac{1}{3})(0) \right] = \underline{\underline{\alpha(0.2667)}}$$

$\rightarrow @ x_1 = \text{med} :$

$$P(\text{med} | e_1) = \alpha (0.95) \left[(\frac{1}{3})(0.35) + (\frac{1}{3})(0.6) + (\frac{1}{3})(0.5) \right] = \underline{\underline{\alpha(0.4592)}}$$

$\rightarrow @ x_1 = \text{high} :$

$$P(\text{high} | e_1) = \alpha (0.6) \left[(\frac{1}{3})(0.05) + (\frac{1}{3})(0.2) + (\frac{1}{3})(0.5) \right] = \underline{\underline{\alpha(0.15)}}$$

* find α * solve w/ α *

$$\sum_{x_1} P(x_1 | e_1) = 1 = \alpha \sum_{x_1} P(x_1, e_1)$$

$$\alpha < 0.2667, 0.4592, 0.15 > = 1$$

$$\underline{\underline{\alpha = 1.1417}} \longrightarrow P(x_1 = \text{low} | e_1) = 0.3045$$

$$P(x_1 = \text{med} | e_1) = 0.5243$$

$$P(x_1 = \text{high} | e_1) = 0.1713$$

1.2) $P(x_2 | e_1 = \text{"not flooded"}, e_2 = \text{"not flooded"}) = ?$

@ $e_1 = e_2 = \text{"not flooded"}$:

$$P(x_t | e_{1:t}) = P(x_t | e_1, e_{1:t-1}) \rightarrow t=2$$

$$P(x_2 | e_1, e_2) = \alpha P(e_2 | x_2) \sum_{x_2=\text{low,med,high}} P(x_2 | x_1) P(x_1 | e_1)$$

\star from 1.1

@ $x_2 = \text{low}$:

$$P(x_2=\text{low} | e_1, e_2) = \alpha(1) [(0.6)(0.3045) + (0.2)(0.5243) + (0)] = \underline{\alpha(0.2876)}$$

@ $x_2 = \text{med}$:

$$P(x_2=\text{med} | e_1, e_2) = \alpha(0.95) [(0.35)(0.3045) + (0.6)(0.5243) + (0.5)(0.1713)] = \underline{\alpha(0.4815)}$$

@ $x_2 = \text{high}$:

$$P(x_2=\text{high} | e_1, e_2) = \alpha(0.6) [(0.05)(0.3045) + (0.2)(0.5243) + (0.5)(0.1713)] = \underline{\alpha(0.1234)}$$

* find α values ; solve *

$$\sum_{x_2} P(x_2 | e_1, e_2) = 1 = \alpha \sum_{x_2} P(x_2, e_1, e_2)$$

$$\alpha < 0.2876, 0.4815, 0.1234 > = 1$$

$$\underline{\alpha = 1.1205} \longrightarrow$$

$$\boxed{\begin{aligned} P(x_2=\text{low} | e_1, e_2) &= 0.3223 \\ P(x_2=\text{med} | e_1, e_2) &= 0.5395 \\ P(x_2=\text{high} | e_1, e_2) &= 0.1383 \end{aligned}}$$

1.3) $P(x_3 | e_1 = \text{"not flooded"}, e_2 = \text{"not flooded"}, e_3 = \text{"flooded"}) = ?$

@ $e_1 = e_2 = \text{"not flooded"}, e_3 = \text{"flooded"}:$

$$P(x_3 | e_1, e_2, e_3) = \alpha P(e_3 | x_3) \sum_{x_2=\text{low, med, high}} P(x_2 | x_3) P(x_2 | e_1, e_2)$$

★ from 1.2

@ $x_3 = \text{low}:$

$$P(x_3 = \text{low} | e_{1,2,3}) = \alpha(0) = \boxed{0}$$

@ $x_3 = \text{med}:$

$$P(x_3 = \text{med} | e_{1,2,3}) = \alpha(0.05)[(0.35)(0.3223) + (0.6)(0.5395) + (0.5)(0.1383)] \\ = \underline{\underline{\alpha(0.0253)}}$$

@ $x_3 = \text{high}:$

$$P(x_3 = \text{high} | e_{1,2,3}) = \alpha(0.4)[(0.05)(0.3223) + (0.2)(0.5395) + (0.5)(0.1383)] \\ = \underline{\underline{\alpha(0.0773)}}$$

* find α and solve *

$$\sum_{x_3} P(x_3 | e_1, e_2, e_3) = 1 = \alpha \sum_{x_3} P(x_3, e_1, e_2, e_3)$$

$$\alpha < 0, 0.0253, 0.0773 > = 1$$

$$\underline{\underline{\alpha = 9.7479}} \rightarrow P(x_3 = \text{low} | e_{1,2,3}) = 0$$

$$P(x_3 = \text{med} | e_{1,2,3}) = 0.2466$$

$$P(x_3 = \text{high} | e_{1,2,3}) = 0.7534$$

(2) Smoothing: $e_1 = e_2 = \text{"not flooded"}$, $e_3 = \text{"flooded"}$

$P(x_k | e_{1:t})$, $k < t$ → smooth the function / fill in the holes using what we've observed ($e_{1:t}$)

* Definition using Filtering: $P(x_k | e_{1:t}) = \alpha \underbrace{P(e_{k+1:t} | x_k)}_{\star \text{backwards message}}, \underbrace{P(x_k | e_{1:k})}_{\star \text{filtering!}}$

* $P(x_1 | e_{1:3}) \rightarrow k=1, t=3$

$$= \alpha P(e_2, e_3 | x_1) P(x_1 | e_1)$$

* $P(x_2 | e_{1:3}) \rightarrow k=2, t=3$

$$= \alpha P(e_3 | x_2) P(x_2 | e_1, e_2)$$

* $P(x_3 | e_{1:3}) \rightarrow k=3, t=3$

$$= \alpha P(e_4 | x_3), P(x_3 | e_1, e_2, e_3)$$

$\times b/t \quad k \leq t$

} filtering, not smoothing :-

$$P(x_3 = \text{low} | e_{1:3}) = 0$$

$$P(x_3 = \text{med} | e_{1:3}) = 0.24666$$

$$P(x_3 = \text{high} | e_{1:3}) = 0.7534$$

* Final Smoothing: $P(x_2 | e_{1:t}) = \alpha f(k) * \underline{h(k)}$
Recursive definition

$$\Rightarrow \underline{h(k)} = \sum_{x_{k+1}} P(e_{k+1} | x_{k+1}) \underline{h(k+1)} P(x_{k+1} | x_k)$$

} backwards message!

$$f(1): P(x_1 = \text{low} | e_1) = 0.3045$$

$$f(2): P(x_2 = \text{low} | e_{1:2}) = 0.3223$$

$$P(x_1 = \text{med} | e_1) = 0.5243$$

$$P(x_2 = \text{med} | e_{1:2}) = 0.5395$$

$$P(x_1 = \text{high} | e_1) = 0.1713$$

$$P(x_2 = \text{high} | e_{1:2}) = 0.1383$$

② $h(2) = \sum_{x_3 = \text{low}, \text{med}, \text{high}} P(e_3 | x_3) h(3) P(x_3 | x_2) \rightarrow \text{let } h(3) = \begin{pmatrix} \text{low} \\ \text{med} \\ \text{high} \end{pmatrix}$

$$= (0) + (0.05)(1)(0.35) + (0.4)(1)(0.05) = \underline{0.0375}$$

$$h(x_2 = \text{med}) = (0) + (0.05)(1)(0.6) + (0.4)(1)(0.2) = \underline{0.11}$$

$$h(x_2 = \text{high}) = (0) + (0.05)(1)(0.5) + (0.4)(1)(0.5) = \underline{0.225}$$

$$\overline{h(x_1 = \text{low})} = \sum_{x_2} P(e_2 | x_2) h(2) P(x_2 | x_1) = (1)(0.0375)(0.6) + (0.95)(0.11)(0.35) + (0.6)(0.225)(0.05) = \underline{0.0658}$$

$$h(x_1 = \text{med}) = (1)(0.0375)(0.2) + (0.95)(0.11)(0.6) + (0.6)(0.225)(0.2)$$

$$= \underline{0.0972}$$

$$h(x_1 = \text{high}) = (1)(0.0375)(0) + (0.95)(0.11)(0.5) + (0.6)(0.225)(0.5)$$

$$= \underline{0.11975}$$

* Point-wise product and solve: $P(x_i | e_{1:i}) = \alpha f(k) * h(k)$

$$\alpha < f(x_1 = \text{low}) * h(x_1 = \text{low}), f(x_1 = \text{med}) * h(x_1 = \text{med}), f(x_1 = \text{high}) * h(x_1 = \text{high}) >$$

$$= 1 = \alpha < (0.3045)(0.0658), (0.5243)(0.0972), (0.1713)(0.11975) >$$

$$1 = \alpha < 0.0200, 0.05096, 0.02051 >$$

$$\alpha = \underline{0.9276} \rightarrow P(x_1 = \text{low} | e_{1:3}) = 0.2190$$

$$P(x_1 = \text{med} | e_{1:3}) = 0.5569$$

$$P(x_1 = \text{high} | e_{1:3}) = 0.2241$$

$$@ x_2: 1 = \alpha < (0.3223)(0.0375), (0.5395)(0.11), (0.1383)(0.225) >$$

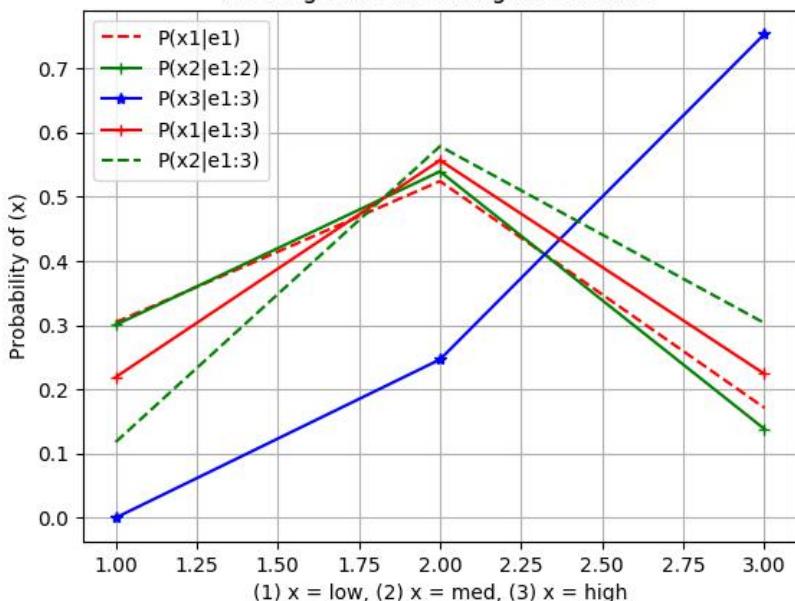
$$= \alpha < 0.01209, 0.05935, 0.03112 >$$

$$\alpha = \underline{0.7515} \rightarrow P(x_2 = \text{low} | e_{1:3}) = 0.1179$$

$$P(x_2 = \text{med} | e_{1:3}) = 0.5787$$

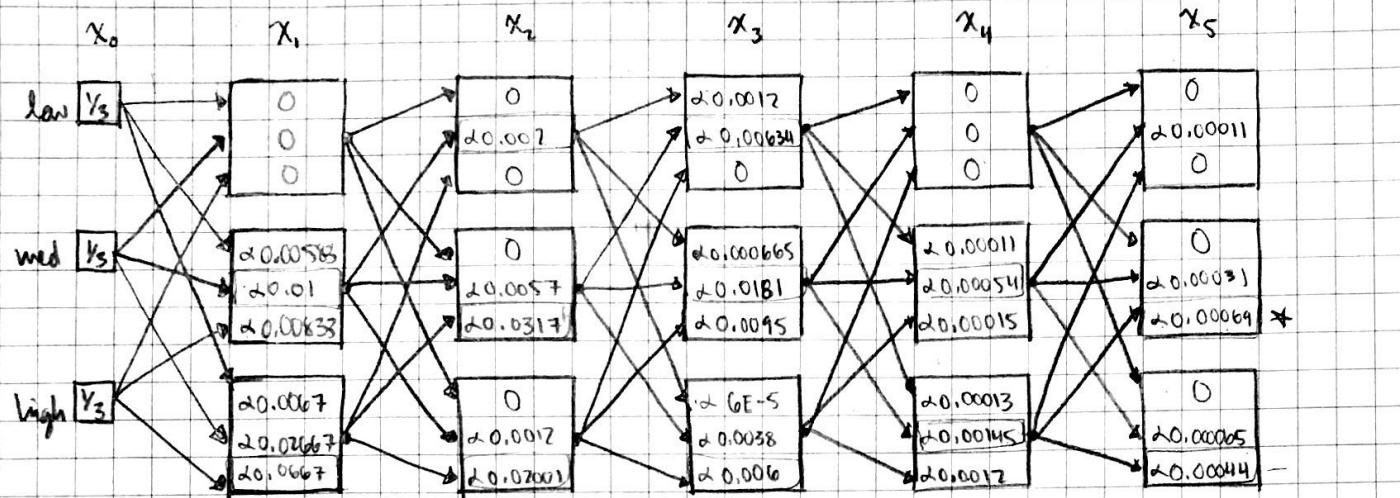
$$P(x_2 = \text{high} | e_{1:3}) = 0.3034$$

Filtering and Smoothing Probabilities



③ Most Likely Explanation

$e_1 = \text{"flooded"} = e_{\text{u}}$ $e_2 = e_3 = e_5 = \text{"not flooded"} \rightarrow P(x_{1:5} | e_{1:5}) = ?$



* Filtering, but with a max function instead of summation *

$$P(x_t | e_{1:t}) = \alpha P(e_t | x_t) \max_{x_{t-1}} (P(x_t | x_{t-1}) P(x_{t-1} | e_{1:t-1}))$$

$$\begin{aligned} @ x_1 = \text{law}: P(x_1 = \text{law} | e_1) &= \alpha P(e_1 | x_1) \max_{x_0} (P(x_1 | x_0) P(x_0)) \\ &= \alpha(0) = \underline{0} \end{aligned}$$

$$\begin{aligned} x_1, @ x_1 = \text{med}: P(x_1 = \text{med} | e_1) &= \alpha(0.05) \max_{x_0} \left\{ (0.35)(\text{law}), \underline{(0.1)(\text{med})}, (0.5)(\text{high}) \right\} \\ &= \underline{\alpha(0.01)} \end{aligned}$$

$$\begin{aligned} @ x_1 = \text{high}: P(x_1 = \text{high} | e_1) &= \alpha(0.4) \max_{x_0} \left\{ (0.05)(\text{law}), (0.2)(\text{med}), (0.5)(\text{high}) \right\} \\ &= \underline{\alpha(0.0667)} \end{aligned}$$

$$\begin{aligned} @ x_2 = \text{law}: P(x_2 = \text{law} | e_{1:2}) &= \alpha P(e_2 | x_2) \max_{x_1} \left\{ P(x_2 | x_1) P(x_1 | e_1) \right\} \\ &= \alpha(1) \max_{x_1} \left\{ (0.6)(0), \underline{(0.2)(0.01)}, (0) \right\} \\ &= \underline{\alpha(0.002)} \end{aligned}$$

$$\begin{aligned} x_2, @ x_2 = \text{med}: P(x_2 = \text{med} | e_{1:2}) &= \alpha(0.05) \max_{x_1} \left\{ (0.35)(0), (0.6)(0.01), (0.5)(0.0667) \right\} \\ &= \underline{\alpha(0.0317)} \end{aligned}$$

$$@ x_2 = \text{high}: P(x_2 = \text{high} | e_{1:2}) = \alpha(0.6) \max_{x_1} \left\{ (0.05)(0), (0.2)(0.01), (0.5)(0.0667) \right\}$$

$$\begin{aligned}
 @ x_3 = \text{law} : P(x_3 = \text{law} | e_{1:3}) &= \alpha P(e_3 | x_3) \max_{x_2} \left\{ P(x_3 | x_2) P(x_2 | e_{1:2}) \right\} \\
 &= \alpha (1) \max \left\{ \underbrace{(0.6)(0.002)}_{\approx 0.0012}, \underbrace{(0.2)(0.0317)}_{\approx 0.00634}, 0 \right\} \\
 &= \underline{\alpha (0.00634)}
 \end{aligned}$$

x_3

$$\textcircled{a} \quad x_3 = \text{med} : P(x_3 = \text{med} | e_{1,2,3}) = \alpha (0.95) \max \left\{ \frac{(0.35)(0.002)}{2(0.000665)}, \frac{(0.6)(0.0317)}{2(0.018)}, \frac{(0.5)(0.02)}{2(0.0095)} \right\} = \underline{\alpha (0.0181)}$$

$$\text{④ } x_3 = \text{high}: P(x_3 = \text{high} | e_{1,3}) = \alpha (0.6) \max \left\{ \frac{(0.05)(0.002)}{6 \times 10^{-5}}, \frac{(0.7)(0.0317)}{0.0038}, \frac{(0.5)(0.02)}{0.006} \right\}$$

1

$$\text{② } x_4 = \text{law}: P(x_4 = \text{law} | e_{1:n}) = \alpha P(e_4 | x_4) \max_{x_3} \left\{ P(x_4 | x_3) P(x_3 | e_{1:n-3}) \right\}$$

$$= \underline{\alpha(0)}$$

74

$$\text{② } x_4 = \text{med} : P(x_4 = \text{med} | e_{1..4}) = \alpha(0.05) \max \left\{ \begin{array}{l} (0.35)(0.00634), (0.6)(0.018), (0.5)(0.006) \\ \times 0.00011 \end{array} \right. \left. \begin{array}{l} \geq 0.00054 \\ \geq 0.00015 \end{array} \right\}$$

$$\text{② } x_4 = \text{high}: P(x_4 = \text{high} | e_{\text{...}}) = \alpha(0.1) \max \left\{ \begin{array}{l} (0.05)(0.00634), (0.2)(0.0181), (0.5)(0.006) \\ \alpha 0.00013 \quad \alpha 0.00145 \quad \alpha 0.0012 \end{array} \right\}$$

1

$$\begin{aligned}
 @ x_5 = \text{law} : P(x_5 = \text{law} | e_{1..5}) &= \alpha P(e_5 | x_5) \max_{x_u} \left\{ P(x_5 | x_u) P(x_u | e_{1..u}) \right\} \\
 &= \alpha (1) \max \left\{ (0.6)(0), \underbrace{(0.2)(0.00054)}_{\approx 0.00011}, (0) \right\} \\
 &= \alpha (0.00011)
 \end{aligned}$$

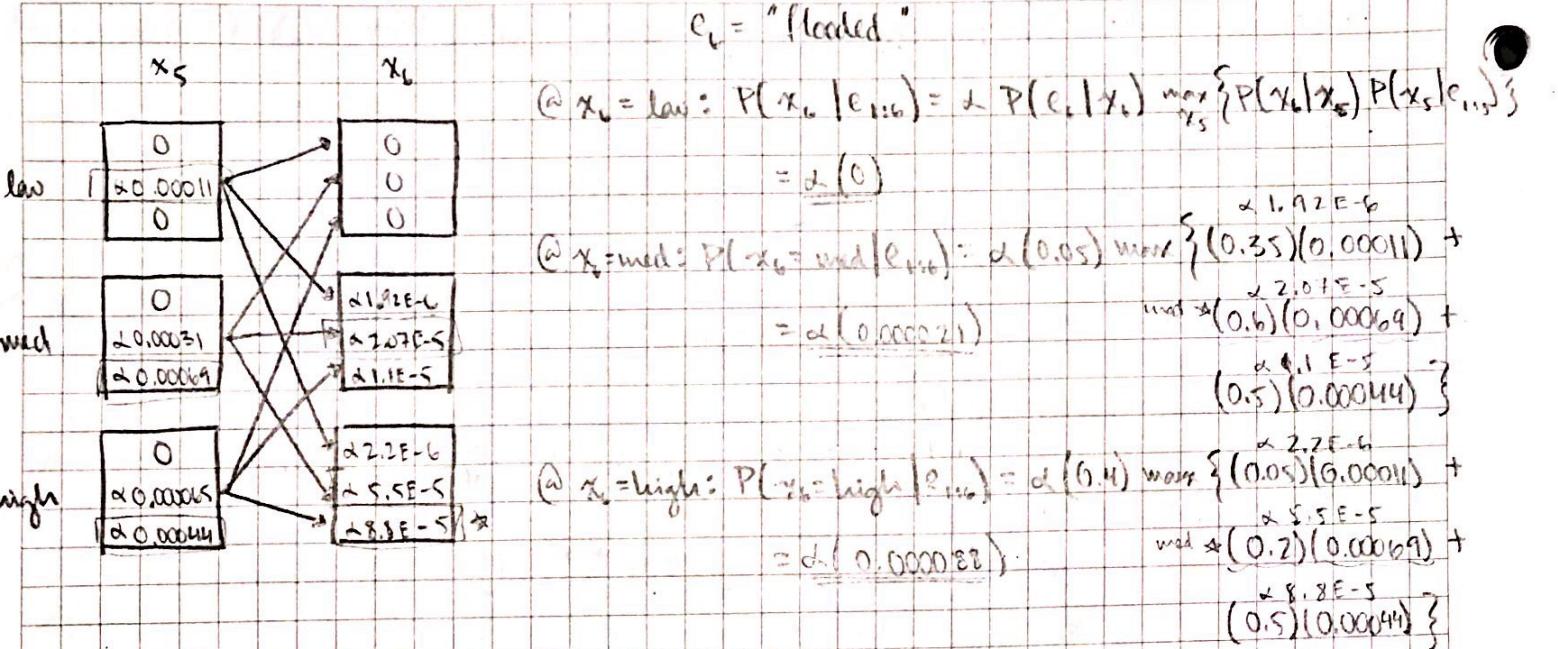
N5

$$\text{④ } x_5 = \text{med} : P(x_5 = \text{med} | e_{1, \dots, 5}) = \alpha(0.95) \max \left\{ \begin{array}{l} (0.35)(0), \\ 0 \\ (0.6)(0.00054), \\ \alpha 0.00031 \\ (0.5)(0.00145) \end{array} \right\} \xrightarrow{\text{high}} \alpha 0.00069$$

$$\text{④ } x_5 = \text{high} : P(x_5 = \text{high} | e_{1,5}) = \alpha(0.6) \max \begin{cases} (0.05)(0), (0.2)(0.00054), (\cancel{0.5})(\cancel{0.00045}) \\ 0 \quad \cancel{+ 0.00065} \quad \cancel{+ 0.00044} \end{cases}$$

* Trace path backwards, from largest value *

$x_5 = \text{med}$, $x_4 = \text{high}$, $x_3 = \text{med}$, $x_2 = \text{med}$, $x_1 = \text{high}$, $x_0 = \text{high}$



* Trace path backwards, from largest value *

$x_6 = \text{high}, x_5 = \text{high}, x_4 = \text{high}, x_3 = \text{med}, x_2 = \text{med}, x_1 = \text{high}, x_0 = \text{high}$

④ Particle Filtering — see particle-filter.py

python3 particle-filter.py 1,000,000

State for x_{10} : low = 60,891 \rightarrow $P(x_{10} = \text{low} | e_{1..10}) = 0.060891$
 med = 621,035 \rightarrow $P(x_{10} = \text{med} | e_{1..10}) = 0.621035$
 high = 318,074 \rightarrow $P(x_{10} = \text{high} | e_{1..10}) = 0.318074$

with 1,000,000 particles

⑤ After 10 throws, variance $\sigma^2 \leq 10 \rightarrow t=10$

$$P(x_t) = N(0, 1) \quad P(e_t | x_t) = N(x_t, 0.75)$$

$$N(x_t, \sigma_x^2)(z_t) = P(z_t, x_t) = \alpha e^{-\frac{1}{2} \left[\frac{(z_t - x_t)^2}{\sigma_x^2} \right]} = N(\mu_t, \sigma_t^2)$$

$$F_t = P(z_t | x_t) \int_{-\infty}^{\infty} P(x_t | x_{t-1}) P(x_{t-1} | z_{1..t}) dx_{t-1}$$

$$= \alpha e^{-\frac{1}{2} a (z_t + \frac{b}{2a})^2} \quad \left. \begin{array}{l} a = \frac{1 + \sigma_x^2 + \sigma_z^2}{\sigma_x^2 (1 + \sigma_x^2)} \\ b = -2 \frac{z_t}{\sigma_x^2} \end{array} \right\} \quad c = \frac{z_t^2}{\sigma_x^2}$$

σ_x^2 = variance of hidden state (actual position?)

σ_z^2 = variance of observed state (car guess / estimate?)

$$P(x_t) = N(0, 1) \therefore \mu_{x_t} = 0 \quad ; \quad \sigma_{x_t}^2 = 1$$

$P(z_t | x_t) = N(x_t, 0.75) \therefore \sigma_z^2 = 0.75 = \text{variance in observed state}$

* Let $\sigma_x^2 \leq 10$:

* Let $a = 0.35$ (from slides) \rightarrow we thought we observed them to be @ 0.75

$$F_t = a' e^{-\frac{1}{2}a(x_t - \frac{z_t}{\sigma_z})^2}$$

$$= N\left(\frac{z_t}{\sigma_z}, \frac{1}{a}\right) \rightarrow a = \frac{1 + (10) + (0.75)}{0.75(1+10)} = 1.424$$

iterate until $t=10$?

$$\sigma_{t+1}^2 = \frac{(\sigma_t^2 + \sigma_z^2) \sigma_z^2}{\sigma_t^2 + \sigma_z^2 + \sigma_z^2} \quad @ \sigma_0^2 = 1, \sigma_z^2 = 0.75, \sigma_x^2 \leq 10?$$

$$\sigma_{t=1}^2 = \frac{(1 + 10) 0.75}{1 + 10 + 0.75} = \frac{1}{a} = 0.7021$$

$$a = \sigma_z^2 @ \sigma_0^2 = 1$$

$$\sigma_{t=2}^2 = \frac{(0.7021 + 10) 0.75}{0.7021 + 10 + 0.75} = 0.7009$$

$$\sigma_{t=3}^2 = \frac{(0.7009 + 10) 0.75}{0.7009 + 10 + 0.75} = 0.7009$$

$$\sigma_{t=4}^2 = " = 0.7009$$

$$\sigma_{t=5}^2 = " = 0.7009$$

$$\sigma_{t=6}^2 = " = 0.7009$$

$$\sigma_{t=7}^2 = " = 0.7009$$

$$\sigma_{t=8}^2 = " = 0.7009$$

$$\sigma_{t=9}^2 = " = 0.7009$$

* can't calculate yet without observed z_t values

0.700877

70% accuracy

⑥ $P(x_0) = \frac{1}{2}$ = uniformly distributed b/t $[-1, 1] \rightarrow$ (weight = $\frac{1}{1+0} = \frac{1}{2}$)

$P(x_{t+1} | x_t) = N(x_t, 1) \rightarrow$ transition probability

$P(e_t | x_t) = N(x_t, 0.5) \rightarrow$ evidence probability

$$\sigma_e^2 = \sigma_x^2$$

($P(x_1 | e_1=0) = ?$)

$$\begin{aligned} F_1 &= P(x_1 | e_1) = P(e_1, x_1) \int_{-\infty}^{\infty} P(x_1 | x_0) P(x_0) dx_0 \\ &= N(x_1, 0.5) \int_{-\infty}^{\infty} N(x_1, 1) P(x_0) dx_0 \\ &= N(x_1, 0.5) \cdot \underbrace{\int_{-\infty}^{\infty} N(x_1, 1) dx_0}_{(1)} \cdot \underbrace{\int_{-\infty}^{\infty} P(x_0) dx_0}_{(1)} \\ &\quad , \quad \text{b/c Normal} \quad \text{uniformly distributed } [-1, 1] \\ &\quad \text{with weight } \frac{1}{2} \\ &= N(x_1, 0.5) \cdot 1 \cdot 1 \end{aligned}$$

* $e_1=0 \therefore$ observed value to be 0 @ t=1

$$\sigma_{x_0}^2 = 2, \sigma_z^2 = 0.5, \sigma_x^2 = 1?$$

$$F_1 = N\left(-\frac{b}{2a}, \frac{1}{a}\right) \rightarrow a = \frac{1 + \sigma_x^2 + \sigma_z^2}{\sigma_z^2(1 + \sigma_x^2)} = \frac{1 + 1 + 0.5}{0.5(1 + 1)} = 2.5$$

$$b = -\frac{2e_1}{\sigma_z^2} = 0$$

$$\boxed{F_1 = N(0, 0.4)} \rightarrow \mu_1 = 0, \sigma_{t=1}^2 = 0.4$$

@ $e_2=0$:

$$\mu_2 = \frac{(\sigma_z^2 + \sigma_x^2)e_2 + \sigma_x^2\mu_1}{\sigma_t^2 + \sigma_x^2 + \sigma_z^2} = 0$$

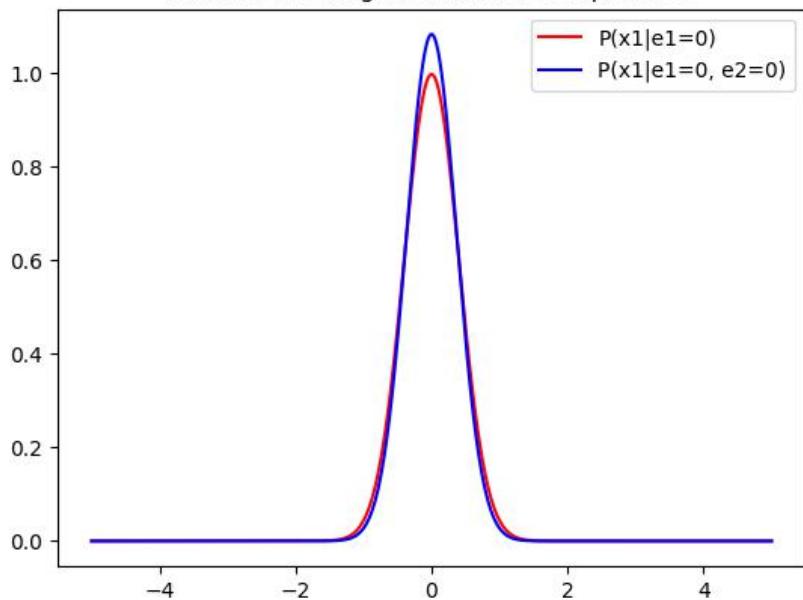
$$\sigma_z^2 = \frac{(\sigma_z^2 + \sigma_x^2)\sigma_z^2}{\sigma_z^2 + \sigma_x^2 + \sigma_z^2}$$

$$= \frac{(0.4 + 1)(0.5)}{0.4 + 1 + 0.5}$$

$$\boxed{F_2 = N(0, 0.3684)}$$

as $t \rightarrow \infty$, and $e_t = \emptyset$, the distribution will continue to narrow around \emptyset , with $\sigma_t^2 \rightarrow 0$

Kalman Filtering with Evidence Equal to 0



```
31 """
32 plot_normal:
33     https://emredjan.github.io/blog/2017/07/19/plotting-distributions/
34 """
35 def plot_normal(x_range, mu=0, sigma=1, cdf=False, **kwargs):
36     """
37         Plots the normal distribution function for a given x range
38         If mu and sigma are not provided, standard normal is plotted
39         If cdf=True cumulative distribution is plotted
40         Passes any keyword arguments to matplotlib plot function
41     """
42     x = x_range
43     if cdf:
44         y = ss.norm.cdf(x, mu, sigma)
45     else:
46         y = ss.norm.pdf(x, mu, sigma)
47     plt.plot(x, y, **kwargs)
48
49 def plot6():
50     # F1 = N(0, 0.4)
51     x = np.linspace(-5, 5, 5000)
52     #plot_normal(x, 0, 0.4, color='red', lw=2, ls='--')
53     y1 = ss.norm.pdf(x, 0, 0.4)
54
55     # F2 = N(0,0.3684)
56     #plot_normal(x, 0, 0.3684, color='blue', lw=2, ls='--')
57     y2 = ss.norm.pdf(x, 0, 0.3684)
58
59     plt.plot(x, y1, 'r--', x, y2, 'b--')
60     plt.legend(['P(x1|e1=0)', 'P(x1|e1=0, e2=0)'])
61     plt.title('Kalman Filtering with Evidence Equal to 0')
62
63     plt.show()
```