CSCI 5512 · HW 3

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① Filtering

1.1) $P(x_1 \mid e_1 = \text{"not flooded"}) = ?$

hidden $x_1 \xrightarrow{\text{causes}} e_1$ evidence (observable)

* assume we know $E_{1:t}$ and X_0

* Recursive Definition of Filtering : $P(x_t \mid e_{1:t}) = \alpha P(e_t \mid x_t) \sum_{x_{t-1}} \left(P(x_t \mid x_{t-1}) \underline{\underline{P(x_{t-1} \mid e_{1:t-1})}} \right)$

previous step

@ e_1 = "not flooded":

$P(x_1 \mid e_1) = \alpha P(x_1, e_1)$ ⟶ "alpha trick"

$= \alpha \sum_{x_0} P(x_0, x_1, e_1)$

$= \alpha \sum_{x_0} P(x_0) P(x_1 \mid x_0) P(e_1 \mid x_1)$ ⟶ Bayes Net! $P(X \mid \text{Parents}(X))$

$= \alpha P(e_1 \mid x_1) \sum_{x_0 = \text{low, med, high}} P(x_0) P(x_1 \mid x_0)$

@ x_0 = low: ⟶ $P(x_0 = \text{low}) = P(x_0 = \text{med}) = P(x_0 = \text{high}) = 1/3$
∴ all same values

→ @ x_1 = low:

$P(\text{low} \mid e_1) = \alpha\,(1) \left[(1/3)(0.6) + (1/3)(0.2) + \cancel{(1/3)}(0) \right] = \underline{\underline{\alpha(0.2667)}}$

→ @ x_1 = med:

$P(\text{med} \mid e_1) = \alpha\,(0.95) \left[(1/3)(0.35) + (1/3)(0.6) + (1/3)(0.5) \right] = \underline{\underline{\alpha(0.4592)}}$

→ @ x_1 = high:

$P(\text{high} \mid e_1) = \alpha(0.6) \left[(1/3)(0.05) + (1/3)(0.2) + (1/3)(0.5) \right] = \underline{\underline{\alpha(0.15)}}$

* find α * solve w/ α *

$\sum_{x_1} P(x_1 \mid e_1) = 1 = \alpha \sum_{x_1} P(x_1, e_1)$

$\alpha \langle 0.2667, 0.4592, 0.15 \rangle = 1$

$\underline{\underline{\alpha = 1.1417}}$ ⟶

$P(x_1 = \text{low} \mid e_1) = 0.3045$
$P(x_1 = \text{med} \mid e_1) = 0.5243$
$P(x_1 = \text{high} \mid e_1) = 0.1713$

1.2) $P(x_2 \mid e_1 = \text{"not flooded"}, e_2 = \text{"not flooded"}) = ?$

@ $e_1 = e_2 =$ "not flooded":

$$P(x_t \mid e_{1:t}) = P(x_t \mid e_t, e_{1:t-1}) \rightarrow t = 2$$

$$P(x_2 \mid e_1, e_2) = \alpha\, P(e_2 \mid x_2) \sum_{x_1 = \text{low, med, high}} P(x_2 \mid x_1)\, \underbrace{P(x_1 \mid e_1)}_{\star \text{ from } 1.1}$$

@ $x_2 = \text{low}$:

$$P(x_2 = \text{low} \mid e_1, e_2) = \alpha(1)\left[(0.6)(0.3045) + (0.2)(0.5243) + (0)\right] = \underline{\alpha(0.2876)}$$

@ $x_2 = \text{med}$:

$$P(x_2 = \text{med} \mid e_1, e_2) = \alpha(0.95)\left[(0.35)(0.3045) + (0.6)(0.5243) + (0.5)(0.1713)\right] = \underline{\alpha(0.4815)}$$

@ $x_2 = \text{high}$:

$$P(x_2 = \text{high} \mid e_1, e_2) = \alpha(0.6)\left[(0.05)(0.3045) + (0.2)(0.5243) + (0.5)(0.1713)\right] = \underline{\alpha(0.1234)}$$

* find α values & solve *

$$\sum_{x_2} P(x_2 \mid e_1, e_2) = 1 = \alpha \sum_{x_2} P(x_2, e_1, e_2)$$

$$\alpha \langle 0.2876, 0.4815, 0.1234 \rangle = 1$$

$\underline{\alpha = 1.1205}$ $\longrightarrow$

$$\boxed{\begin{aligned} P(x_2 = \text{low} \mid e_1, e_2) &= 0.3223 \\ P(x_2 = \text{med} \mid e_1, e_2) &= 0.5395 \\ P(x_2 = \text{high} \mid e_1, e_2) &= 0.1383 \end{aligned}}$$

1.3) $P(x_3 \mid e_1 = \text{"not flooded"}, e_2 = \text{"not flooded"}, e_3 = \text{"flooded"}) = ?$

@ $e_1 = e_2 =$ "not flooded" , $e_3 =$ "flooded" :

$$P(x_3 \mid e_1, e_2, e_3) = \alpha P(e_3 \mid x_3) \sum_{x_2 = \text{low, med, high}} P(x_3 \mid x_2) \underbrace{P(x_2 \mid e_1, e_2)}_{\text{* from (1.2)}}$$

@ $x_3 = \text{low}$:

$$P(x_3 = \text{low} \mid e_{1:3}) = \alpha(0) = \boxed{0}$$

@ $x_3 = \text{med}$:

$$P(x_3 = \text{med} \mid e_{1:3}) = \alpha(0.05)\left[(0.35)(0.3223) + (0.6)(0.5395) + (0.5)(0.1383)\right]$$
$$= \alpha(0.0253)$$

@ $x_3 = \text{high}$:

$$P(x_3 = \text{high} \mid e_{1:3}) = \alpha(0.4)\left[(0.05)(0.3223) + (0.2)(0.5395) + (0.5)(0.1383)\right]$$
$$= \alpha(0.0773)$$

* find α and solve *

$$\sum_{x_3} P(x_3 \mid e_1, e_2, e_3) = 1 = \alpha \sum_{x_3} P(x_3, e_1, e_2, e_3)$$

$$\alpha \langle 0, 0.0253, 0.0773 \rangle = 1$$

$$\alpha = 9.7479 \longrightarrow \boxed{\begin{aligned} P(x_3 = \text{low} \mid e_{1:3}) &= 0 \\ P(x_3 = \text{med} \mid e_{1:3}) &= 0.2466 \\ P(x_3 = \text{high} \mid e_{1:3}) &= 0.7534 \end{aligned}}$$

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(2) Smoothing : $e_1 = e_2 =$ "not flooded", $e_3 =$ "flooded"

$P(x_k \mid e_{1:t})$, $k < t$ $\longrightarrow$ smooth the function / fill in the holes using what we've observed ($e_{1:t}$)

* Definition using Filtering : $P(x_k \mid e_{1:t}) = \alpha \underbrace{P(e_{k+1:t} \mid x_k)}_{\star\text{ backwards message}} \underbrace{P(x_k \mid e_{1:k})}_{\star\text{ filtering!}}$

* $P(x_1 \mid e_{1:3}) \rightarrow k = 1, t = 3$

$= \alpha P(e_2, e_3 \mid x_1) P(x_1 \mid e_1)$

* $P(x_2 \mid e_{1:3}) \rightarrow k = 2, t = 3$

$= \alpha P(e_3 \mid x_2) P(x_2 \mid e_1, e_2)$

* $P(x_3 \mid e_{1:3}) \rightarrow k = 3, t = 3$

$= \alpha \underbrace{P(e_{4:3} \mid x_3)}_{\times \text{ b/t } k \not< t} P(x_3 \mid e_1, e_2, e_3)$

} filtering, not smoothing :

$$\boxed{\begin{aligned} P(x_3 = \text{low} \mid e_{1:3}) &= 0 \\ P(x_3 = \text{med} \mid e_{1:3}) &= 0.2466 \\ P(x_3 = \text{high} \mid e_{1:3}) &= 0.7534 \end{aligned}} \quad f(3)$$

* Final Smoothing recursive definition : $P(x_k \mid e_{1:t}) = \alpha f(k) * \underline{h(k)}$

$\Rightarrow \underline{h(k)} = \sum_{x_{k+1}} P(e_{k+1} \mid x_{k+1}) \underline{h(k+1)} P(x_{k+1} \mid x_k)$ } backwards message!

f(1): $P(x_1 = \text{low} \mid e_1) = 0.3045$

$P(x_1 = \text{med} \mid e_1) = 0.5243$

$P(x_1 = \text{high} \mid e_1) = 0.1713$

f(2): $P(x_2 = \text{low} \mid e_{1:2}) = 0.3223$

$P(x_2 = \text{med} \mid e_{1:2}) = 0.5395$

$P(x_2 = \text{high} \mid e_{1:2}) = 0.1383$

(a) $h(2) = \sum_{x_3 = \text{low, med, high}} P(e_3 \mid x_3) h(3) P(x_3 \mid x_2)$ $\longrightarrow$ let $h(3) = \langle \overset{\text{low}}{1}, \underset{\text{med}}{1}, \overset{\text{high}}{1} \rangle$

$x_2 = \text{low}$: $= (0) + (0.05)(1)(0.35) + (0.4)(1)(0.05) = \underline{0.0375}$

$h(x_2 = \text{med}) = (0) + (0.05)(1)(0.6) + (0.4)(1)(0.2) = \underline{\underline{0.11}}$

$h(x_2 = \text{high}) = (0) + (0.05)(1)(0.5) + (0.4)(1)(0.5) = \underline{\underline{0.225}}$

} h(2)

$h(x_1 = \text{low}) = \sum_{x_2} P(e_2 \mid x_2) h(2) P(x_2 \mid x_1) = (1)(0.0375)(0.6) + (0.95)(0.11)(0.35) + (0.6)(0.225)(0.05) = \underline{\underline{0.0658}}$

$h(x_1 = \text{med}) = (1)(0.0375)(0.2) + (0.95)(0.11)(0.6) + (0.6)(0.225)(0.2)$
$= \underline{0.0972}$

$h(x_1 = \text{high}) = (1)(0.0375)(0) + (0.95)(0.11)(0.5) + (0.6)(0.225)(0.5)$
$= \underline{0.11975}$

* Point-wise product and solve : $P(x_k | e_{1:t}) = \alpha f(k) * h(k)$

$\alpha \langle f(x_1 = \text{low}) * h(x_1 = \text{low}),\ f(x_1 = \text{med}) * h(x_1 = \text{med}),\ f(x_1 = \text{high}) * h(x_1 = \text{high}) \rangle$

$= 1 = \alpha \langle (0.3045)(0.0658),\ (0.5243)(0.0972),\ (0.1713)(0.11975) \rangle$

$1 = \alpha \langle 0.0200,\ 0.05096,\ 0.02051 \rangle$

$\underline{\alpha = 10.9276} \longrightarrow$

$P(x_1 = \text{low} \mid e_{1:3})$	$= 0.2190$
$P(x_1 = \text{med} \mid e_{1:3})$	$= 0.5568$
$P(x_1 = \text{high} \mid e_{1:3})$	$= 0.2241$

@ x_2 : $1 = \alpha \langle (0.3223)(0.0375),\ (0.5395)(0.11),\ (0.1383)(0.225) \rangle$

$= \alpha \langle 0.01209,\ 0.05935,\ 0.03112 \rangle$

$\underline{\alpha = 9.7515} \longrightarrow$

$P(x_2 = \text{low} \mid e_{1:3})$	$= 0.1179$
$P(x_2 = \text{med} \mid e_{1:3})$	$= 0.5787$
$P(x_2 = \text{high} \mid e_{1:3})$	$= 0.3034$

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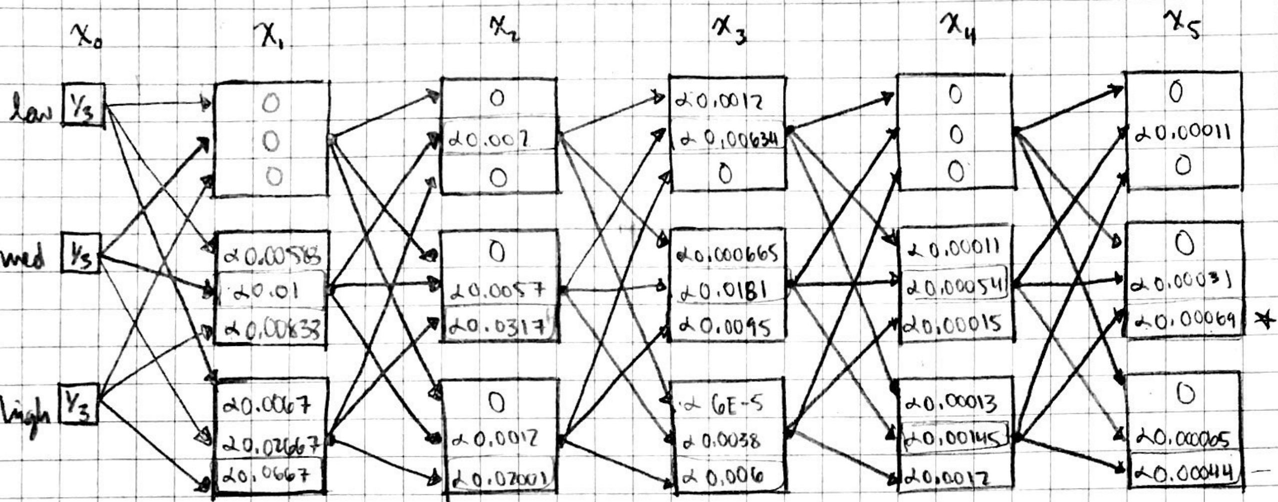
③ Most Likely Explanation

$e_1 =$ "flooded" $= e_4$ $\quad$ $e_2 = e_3 = e_5 =$ "not flooded" $\longrightarrow P(x_{1:5} \mid e_{1:5}) = ?$



* Filtering, but with a max function instead of summation *

$$P(x_t \mid e_{1:t}) = \alpha\, P(e_t \mid x_t) \max_{x_{t-1}} \left(P(x_t \mid x_{t-1})\, P(x_{t-1} \mid e_{1:t-1}) \right)$$

x_1

@ $x_1 = \text{low}$: $P(x_1 = \text{low} \mid e_1) = \alpha\, P(e_1 \mid x_1) \max_{x_0} \left(P(x_1 \mid x_0)\, P(x_0) \right)$

$= \alpha\,(0) = \underline{0}$

@ $x_1 = \text{med}$: $P(x_1 = \text{med} \mid e_1) = \alpha\,(0.05) \max\left\{ \underset{\alpha 0.00583}{(0.35)(1/3)},\ \underset{\alpha 0.01}{\overset{\star\text{med}}{(0.6)(1/3)}},\ \underset{\alpha 0.00833}{(0.5)(1/3)} \right\}$

$= \underline{\underline{\alpha\,(0.01)}}$

@ $x_1 = \text{high}$: $P(x_1 = \text{high} \mid e_1) = \alpha\,(0.4) \max\left\{ \underset{\alpha 0.00667}{(0.05)(1/3)},\ \underset{\alpha 0.02667}{(0.2)(1/3)},\ \underset{\alpha 0.0667}{\overset{\star\text{high}}{(0.5)(1/3)}} \right\}$

$= \underline{\underline{\alpha\,(0.0667)}}$

x_2

@ $x_2 = \text{low}$: $P(x_2 = \text{low} \mid e_{1:2}) = \alpha\, P(e_2 \mid x_2) \max_{x_1} \left\{ P(x_2 \mid x_1)\, P(x_1 \mid e_1) \right\}$

$= \alpha\,(1) \max\left\{ \underset{0}{(0.6)(0)},\ \underset{\alpha 0.002}{\overset{\star\text{med}}{(0.2)(0.01)}},\ \underset{0}{(0)} \right\}$

$= \underline{\underline{\alpha\,(0.002)}}$

@ $x_2 = \text{med}$: $P(x_2 = \text{med} \mid e_{1:2}) = \alpha\,(0.95) \max\left\{ \underset{0}{(0.35)(0)},\ \underset{\alpha 0.0057}{(0.6)(0.01)},\ \underset{\alpha 0.0317}{\overset{\star\text{high}}{(0.5)(0.0667)}} \right\}$

$= \underline{\underline{\alpha\,(0.0317)}}$

@ $x_2 = \text{high}$: $P(x_2 = \text{high} \mid e_{1:2}) = \alpha\,(0.6) \max\left\{ \underset{0}{(0.05)(0)},\ \underset{\alpha 0.0012}{(0.2)(0.01)},\ \underset{\alpha 0.02001}{\overset{\star\text{high}}{(0.5)(0.0667)}} \right\}$

X_3

@ $x_3 = low$: $P(x_3 = low \mid e_{1:3}) = \alpha P(e_3 \mid x_3) \max_{x_2} \{ P(x_3 \mid x_2) P(x_2 \mid e_{1:2}) \}$

$= \alpha (1) \max \{ \underbrace{(0.6)(0.002)}_{\alpha\, 0.0012}, \underbrace{(0.2)(0.0317)}_{\alpha\, 0.00634}, \underbrace{(0)}_{0} \}$ ★ med

$= \alpha (0.00634)$

@ $x_3 = med$: $P(x_3 = med \mid e_{1:3}) = \alpha (0.95) \max \{ \underbrace{(0.35)(0.002)}_{\alpha\, 0.000665}, \underbrace{(0.6)(0.0317)}_{\alpha\, 0.0181}, \underbrace{(0.5)(0.02}_{\alpha\, 0.0095} \}$ ★ med

$= \alpha (0.0181)$

@ $x_3 = high$: $P(x_3 = high \mid e_{1:3}) = \alpha (0.6) \max \{ \underbrace{(0.05)(0.002)}_{\alpha\, 6E-5}, \underbrace{(0.2)(0.0317)}_{\alpha\, 0.0038}, \underbrace{(0.5)(0.02)}_{0.006} \}$ ★ high

$= \alpha (0.006)$

X_4

@ $x_4 = low$: $P(x_4 = low \mid e_{1:4}) = \alpha P(e_4 \mid x_4) \max_{x_3} \{ P(x_4 \mid x_3) P(x_3 \mid e_{1:3}) \}$

$= \alpha (0)$

@ $x_4 = med$: $P(x_4 = med \mid e_{1:4}) = \alpha (0.05) \max \{ \underbrace{(0.35)(0.00634)}_{\alpha\, 0.00011}, \underbrace{(0.6)(0.0181)}_{\alpha\, 0.00054}, \underbrace{(0.5)(0.006)}_{\alpha\, 0.00015} \}$ ★ med

$= \alpha (0.00054)$

@ $x_4 = high$: $P(x_4 = high \mid e_{1:4}) = \alpha (0.4) \max \{ \underbrace{(0.05)(0.00634)}_{\alpha\, 0.00013}, \underbrace{(0.2)(0.0181)}_{\alpha\, 0.00145}, \underbrace{(0.5)(0.006)}_{\alpha\, 0.0012} \}$ ★ med

$= \alpha (0.00145)$

X_5

@ $x_5 = low$: $P(x_5 = low \mid e_{1:5}) = \alpha P(e_5 \mid x_5) \max_{x_4} \{ P(x_5 \mid x_4) P(x_4 \mid e_{1:4}) \}$

$= \alpha (1) \max \{ (0.6)(0), \underbrace{(0.2)(0.00054)}_{\alpha\, 0.00011}, (0) \}$ ★ med

$= \alpha (0.00011)$

@ $x_5 = med$: $P(x_5 = med \mid e_{1:5}) = \alpha (0.95) \max \{ \underbrace{(0.35)(0)}_{0}, \underbrace{(0.6)(0.00054)}_{\alpha\, 0.00031}, \underbrace{(0.5)(0.00145)}_{\alpha\, 0.00069} \}$ ★ high

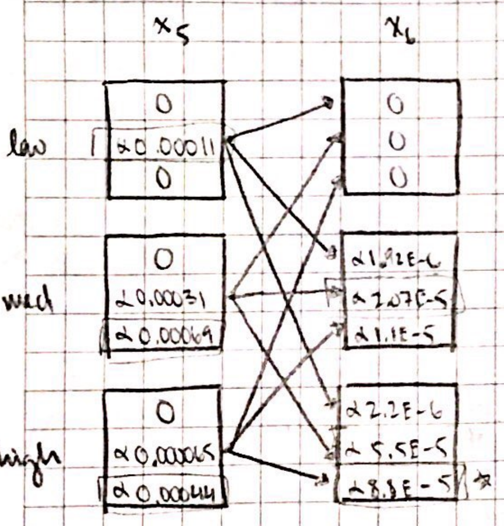
$= \alpha (0.00069)$

@ $x_5 = high$: $P(x_5 = high \mid e_{1:5}) = \alpha (0.6) \max \{ \underbrace{(0.05)(0)}_{0}, \underbrace{(0.2)(0.00054)}_{\alpha\, 0.000065}, \underbrace{(0.5)(0.00145)}_{\alpha\, 0.00044} \}$ ★ high

$= \alpha (0.00044)$

* Trace path backwards, from largest value *

$\boxed{x_5 = med,\ x_4 = high,\ x_3 = med,\ x_2 = med,\ x_1 = high,\ x_0 = high}$



e_6 = "flooded"

@ x_6 = low: $P(x_6 \mid e_{1:6}) = \alpha P(e_6 \mid x_6) \max_{x_5} \{ P(x_6 \mid x_5) P(x_5 \mid e_{1:5}) \}$

$= \alpha (0)$

@ x_6 = med: $P(x_6 = \text{med} \mid e_{1:6}) = \alpha (0.05) \max \{ \overset{\alpha\, 1.92E\text{-}6}{(0.35)(0.00011)} + \overset{\alpha\, 2.07E\text{-}5}{(0.6)(0.00069)} + \overset{\alpha\, 1.1E\text{-}5}{(0.5)(0.00044)} \}$ med

$= \alpha (0.0000021)$

@ x_6 = high: $P(x_6 = \text{high} \mid e_{1:6}) = \alpha (0.4) \max \{ \overset{\alpha\, 2.2E\text{-}6}{(0.05)(0.00011)} + \overset{\alpha\, 5.5E\text{-}5}{(0.2)(0.00069)} + \overset{\alpha\, 8.8E\text{-}5}{(0.5)(0.00044)} \}$ med

$= \alpha (0.000088)$

* Trace path backwards, from largest value

$\boxed{x_6 = \text{high},\ x_5 = \text{high},\ x_4 = \text{high},\ x_3 = \text{med},\ x_2 = \text{med},\ x_1 = \text{high},\ x_0 = \text{high}}$

(4) Particle Filtering — see particle-filter.py

$ python3 particle-filter.py 1,000,000

State for x_{10}: low = 60,891 ⟶ $P(x_{10} = \text{low} \mid e_{1:10}) = 0.060891$
med = 621,035 ⟶ $P(x_{10} = \text{med} \mid e_{1:10}) = 0.621035$
high = 318,074 ⟶ $P(x_{10} = \text{high} \mid e_{1:10}) = 0.318,074$

with 1,000,000 particles

(5) After 10 throws, variance $\sigma^2 \leq 10 \longrightarrow t = 10$

$P(x_0) = N(0,1) \quad P(e_t \mid x_t) = N(x_t, 0.75)$

$N(x_t, \sigma_z^2)(z_t) = P(z_t, x_t) = \alpha e^{-\frac{1}{2}\left[\frac{(z_t - x_t)^2}{\sigma_z^2}\right]} = N(\mu_t, \sigma_t^2)$

$F_t = P(z_t \mid x_t) \int_{-\infty}^{\infty} P(x_t \mid x_{t-1}) P(x_{t-1} \mid z_{1:t})\, dx_{t-1}$

$= \alpha e^{-\frac{1}{2} a \left(x_t + \frac{b}{2a}\right)^2}$ $\Bigg\}$ $a = \dfrac{1 + \sigma_x^2 + \sigma_z^2}{\sigma_z^2 (1 + \sigma_x^2)}$ $\quad b = \dfrac{-2 z_t}{\sigma_z^2}$ $\quad c = \dfrac{z_t^2}{\sigma_z^2}$

σ_x^2 = variance of hidden state (actual position?)

σ_z^2 = variance of observed state (car guess / estimate?)

$P(x_0) = N(0,1) \therefore \mu_{t=0} = 0 \; ; \; \sigma_{t=0}^2 = 1$

$P(z_t | x_t) = N(x_t | 0.75) \therefore \sigma_z^2 = 0.75$ = variance in observed state

* let $\sigma_x^2 = 10$:

* let $x_1 = 0.75$ (from slides) $\longrightarrow$ we thought we observed them to be @ 0.75

$F_1 = \alpha_1' e^{-\frac{1}{2}a(x_1 - \frac{b}{2a})^2}$

$= N\left(\frac{-b}{2a}, \frac{1}{a}\right) \longrightarrow a = \frac{1 + (10) + (0.75)}{0.75(1+10)} = 1.424$

ⓐ $\sigma_{t=0}^2 = 1$, $\sigma_z^2 = 0.75$, $\sigma_x^2 = 10$?

$$\sigma_{t+1}^2 = \frac{(\sigma_t^2 + \sigma_x^2)\,\sigma_z^2}{\sigma_t^2 + \sigma_x^2 + \sigma_z^2} \quad \Bigg\} \text{ iterate until } t = 10?$$

$$\sigma_{t=1}^2 = \frac{(1 + 10)\,0.75}{1 + 10 + 0.75} = \frac{1}{a} = \underline{\underline{0.7021}}$$

$a = \sigma_1^2$ @ $\sigma_0^2 = 1$

$$\sigma_{t=2}^2 = \frac{(0.7021 + 10)\,0.75}{0.7021 + 10 + 0.75} = \underline{\underline{0.7009}}$$

* can't calculate μ_t without observed z_t values

$$\sigma_{t=3}^2 = \frac{(0.7009 + 10)\,0.75}{0.7009 + 10 + 0.75} = \underline{\underline{0.7009}}$$

$\sigma_4^2 =$ " " $= \underline{\underline{0.7009}}$

$\sigma_5^2 =$ " " $= \underline{\underline{0.7009}}$

$\sigma_6^2 =$ " " $= \underline{\underline{0.7009}}$

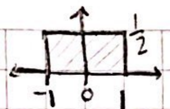
$\sigma_7^2 =$ " " $= \underline{\underline{0.7009}}$

$\sigma_8^2 =$ " " $= \underline{\underline{0.7009}}$

$\sigma_9^2 =$ " " $= \underline{\underline{0.7009}}$

$\sigma_{10}^2 =$ " " $= \underline{\underline{0.7009}} \longrightarrow \boxed{0.700877}$

70% accuracy

⑥ $P(x_0) =$  $=$ uniformly distributed b/t $[-1, 1] \longrightarrow$ height $= \frac{1}{1+|{-1}|} = \frac{1}{2}$

$P(x_{t+1} \mid x_t) = N(x_t, 1) \longrightarrow$ transition probability

$P(e_t \mid x_t) = N(x_t, 0.5) \longrightarrow$ evidence probability

$\sigma_z^2 = \sigma_e^2$

$P(x_1 \mid e_1 = 0) = ?$

$$F_1 = P(x_1 \mid e_1) = P(e_1 \mid x_1) \int_{-\infty}^{\infty} P(x_1 \mid x_0)\, P(x_0)\, dx_0$$

$$= N(x_1, 0.5) \int_{-\infty}^{\infty} N(x_0, 1)\, P(x_0)\, dx_0$$

$$= N(x_1, 0.5) \cdot \underbrace{\int_{-\infty}^{\infty} N(x_0, 1)\, dx_0}_{(1)\ \text{b/c Normal}} \cdot \underbrace{\int_{-\infty}^{\infty} P(x_0)\, dx_0}_{(1)\ \text{uniformly distributed } [-1,1] \text{ with height } 1/2}$$

$$= N(x_1, 0.5) \cdot 1 \cdot 1$$

* $e_1 = 0$ ∴ observed value to be 0 @ $t=1$

$\sigma_{x_0}^2 = 2$, $\sigma_z^2 = 0.5$, $\sigma_x^2 = 1$?

$$F_1 = N\left(\frac{-b}{2a}, \frac{1}{a}\right) \longrightarrow a = \frac{1+\sigma_x^2+\sigma_z^2}{\sigma_z^2(1+\sigma_x^2)} = \frac{1+1+0.5}{0.5(1+1)} = 2.5$$

$$b = \frac{-2e_1}{\sigma_z^2} = 0$$

$\boxed{F_1 = N(0, 0.4)} \longrightarrow \mu_1 = 0$, $\sigma_{t=1}^2 = 0.4$

@ $e_2 = 0$:

$$\mu_2 = \frac{(\sigma_1^2 + \sigma_x^2)\, e_2 + \sigma_x^2 \mu_t}{\sigma_t^2 + \sigma_x^2 + \sigma_z^2} = \underline{0}$$

$$\sigma_2^2 = \frac{(\sigma_1^2 + \sigma_x^2)\, \sigma_z^2}{\sigma_1^2 + \sigma_x^2 + \sigma_z^2} = \frac{(0.4+1)(0.5)}{0.4+1+0.5} = \underline{0.3684}$$

$\boxed{F_2 = N(0, 0.3684)}$

as $t \to \infty$, and $e_t = \emptyset$, the distribution will continue to narrow around $\emptyset$, with $\sigma_t^2 \to 0$