Assessment 2

Context Free Grammars and Turing Machines

Norbert Logiewa
nl253

December 2017

Context Free Grammars

1. Consider the language

(a) Give a word that is in the language $\{a^i b^k c^m \mid i \geq 0, k \geq 0, m \geq 0, k = i + m\}$ and a word that is not in the language

Answer:
a word not in the language: aaaaaaaaa
a word in the language: aabbbbcc

(b) Give a context-free grammar for the language above.

Answer:
$S \rightarrow B \mid aSc$
$B \rightarrow \epsilon \mid bBc$

(c) Use the CYK algorithm to determine whether **abbaa** is a word of the language of the following grammar. Give the table. State in one sentence whether the word is a word of the language of the grammar and how you obtain this conclusion from the table.

$$S \Rightarrow AX \mid BY \mid SS \mid BA$$
$$X \Rightarrow AS$$
$$Y \Rightarrow BS$$
$$A \Rightarrow a$$
$$B \Rightarrow b$$

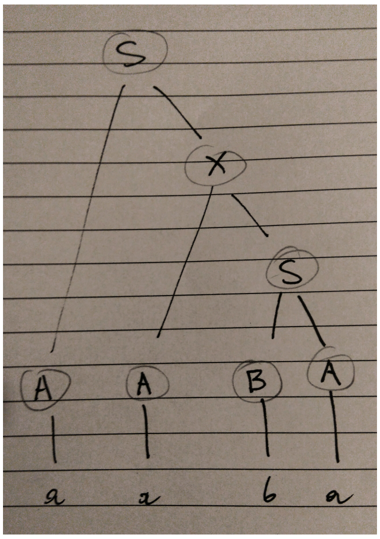
Answer:

5	–	—	—	—	—
4	–	–	—	—	—
3	–	Y	–	—	—
2	–	–	S	–	—
1	A	B	B	A	A
–	a	b	b	a	a

Explanation:
It's not, there is no way to parse it as S doesn't appear in the top row.

(d) Give a parse tree for the word **aaba** with respect to the grammar above (for part c))

Answer:



(e) What is $FIRST(SS)$ with respect to the grammar above (for part c))

Answer:
$FIRST(SS) = \{a,\, b\}$

2. Consider the following two context-free grammars

G_2

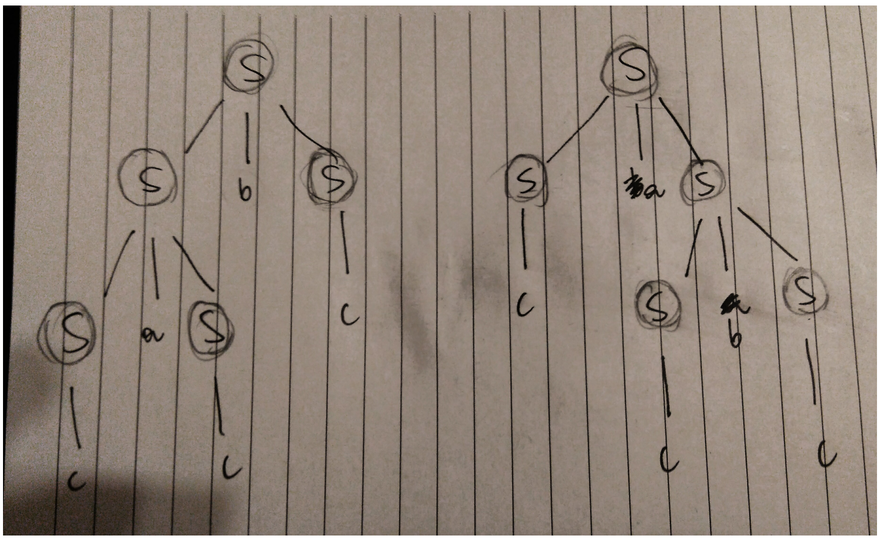
$$S \Rightarrow DAd$$
$$A \Rightarrow aS \mid \epsilon$$
$$B \Rightarrow bD \mid \epsilon$$
$$D \Rightarrow cB$$

G_1

$$S \Rightarrow SaS \mid SbS \mid c$$

(a) Draw two different parse trees for the word cacbc and the grammar G_1

Answer:



(b) Give the $LOOKAHEAD$ set for every rule of grammar G_2

Answer:

rule $R \Rightarrow t$	$NULLABLE(R)$	$FIRST(t)$	$FOLLOW(R)$	$LOOKAHEAD(R)$
S $\Rightarrow DAd$	$false$	$\{c\}$	$\{d\}$	$\{c\}$
A $\Rightarrow aS \mid \epsilon$	$true$	$\{a, \epsilon\}$	$\{d\}$	$\{a, d, \epsilon\}$
B $\Rightarrow bD \mid \epsilon$	$true$	$\{b, \epsilon\}$	$\{a\}$	$\{a, b, \epsilon\}$
D $\Rightarrow cB$	$false$	$\{c\}$	$\{a\}$	$\{c\}$

(c) Is the grammar G_2 $LL(1)$?

Answer:
No it's not, there are overlapping lookahead sets.

(d) Give the set of nullable non-terminals for the grammar G_2

Answer:
$\{A, B, D, S\}$

(e) Give the context-free grammar that you obtain from replacing all ϵ-rules in grammar G_2

Answer:

$\mathbf{G}_2$	**after replacing ϵ rules**
$S \Rightarrow DAd$	$S \Rightarrow DAd \mid Dd \mid Ad \mid d$
$A \Rightarrow aS \mid \epsilon$	$A \Rightarrow aS \mid a$
$B \Rightarrow bD \mid \epsilon$	$B \Rightarrow bD \mid b$
$D \Rightarrow cB$	$D \Rightarrow cB \mid c$

Turing Machines

Consider the following Turing machine with input alphabet {a, b} and tape alphabet $\{a, b, \_\}$

(a) Give computations for the words **ab** and **bb**. State for each word whether the machine accepts it, rejects it or loops. If the machine loops, then give the first five configurations of the computation.

Answer:

input word	computation	outcome
ab	$a/a/R$ ⊢ b/b/R ⊢ _/_/R ⊢ _/_/R	reject
bb	$b/b/R$ ⊢ a/a/R ⊢ b/b/R ⊢ b/b/R ⊢ b/b/R	loop

(b) Draw a Turing machine that decides the language of all words over the alphabet {a, b} that have an odd number of as and an odd number of bs.

Answer: